If Pythagoras does not present himself to our minds as a sharply outlined figure, standing in the bright light of history, this is not merely the result of accidents in the course of historical transmission. From the very beginning, his influence was mainly felt in an atmosphere of miracle, secrecy, and revelation. In that twilight period between old and new, when Greeks, in a historically unique achievement, were discovering the rational interpretation of the world and quantitative natural science, Pythagoras represents not the origin of the new, but the survival or revival of ancient, pre-scientific lore, based on superhuman authority and expressed in ritual obligation. The lore of number is multifarious and changeable. That which was later regarded as the philosophy of Pythagoras had its roots in the school of Plato. Outlines of an earlier reformulation of Pythagorean doctrine in the manner of the *Pythagorica* of the fifth century can be detected in the fragments of Philolaus. As the old and the new interpenetrated and influenced each other, the picture of Pythagoras became distorted until, with the victory of rational science, he came to seem its true founder.

To investigate these interrelationships is still a somewhat risky undertaking; but an attempt has been made to take more account than has previously been done of the variety in the kinds of evidence available and, above all, to clarify the ramifications and the divagations of the tradition.

I am indebted for much advice and encouragement to my teachers Otto Seler, Reinhold Merkelbach, and Helmut Berve, as well as to Dr. Ludwig Koenen and to Dr. Burkhardt Cardona, who also helped me with the proofs. To all of these I offer hearty thanks.

Erlangen
April 1962

Walter Burkert
In revising this book for translation, it has been impossible to add references to all the literature on the subject which has appeared since 1962. I have tried to concentrate on the ancient evidence, to cut down polemics, and to incorporate whatever I have learned in these years, notably from some reviews of the German edition, and from continuous discussions with B. L. van der Waerden. In the question of the “discovery of the irrational,” I have taken a stand which is less critical of the tradition; and more thorough acquaintance with ancient religion has pushed the concept of “shamanism” further into the background. But though a good number of passages have been revised, and though there are some small rearrangements in the order of treatment, still the book has remained, in all the main lines, the same.

New and comprehensive accounts of Pythagoreanism have been given by Kurt von Fritz, H. Dürrie, and B. L. van der Waerden in Pauly-Wissowa (XXIV 171–390; Supp. X 843–864), and by W. K. C. Guthrie in his History of Greek Philosophy, vol. I; and the same year (1966) saw the appearance of two books with the title Pythagoras and Early Pythagoreanism, by C. J. de Vogel and J. A. Philip. Discussion is certain to continue. I cannot claim to have provided a definitive history of Pythagoreanism, or a complete account of Pythagoras, the man and the genius. Still I trust that this book gives a full and perspicuous presentation of the evidence and thus will be useful even to those who are not inclined to draw the same conclusions from it.

My special thanks are due to Edwin L. Minar, Jr., who not only completed the laborious task of translation in a spirit of most pleasant collaboration, but to whom is due the original initiative which brought about the English edition. The responsibility for the content, and for all that may be wrong in it, remains mine.

Schwerzenbach-Zürich
August 1970

Walter Burkert
The “Pythagorean question” has sometimes been compared with the Homeric question. Not that the details of the problem would especially suggest this; what does remind one of that most famous of philological controversies is the difficulty of the argument and the lack of agreement on methodology, as well as the multiplicity and contradictory character of the solutions advanced. Another similarity, and not the least striking, lies in the tremendous importance of the questions about the life, activity, and influence of Pythagoras of Samos. Over the origins of Greek philosophy and science, as over the beginning of Greek literature, lies the shadow of a great traditional name. The attempts of scholarship to grasp the underlying historical reality keep getting entangled in contradictions; where some think they discern the figure of a world-historical genius, others find little more than empty nothingness.

Pythagoras’ influence was a lasting one. The ancient tradition of the history of philosophy made him the ancestor of the “Italian School” and therefore, after Thales, the second, and more important, originator of philosophia—in fact, the inventor of the word. The doctrine transmitted under his name, that numbers are the principles of what exists, that the “One” is its primal ground, became part of the amalgam of Neoplatonism. In the trend set by Iamblichus, Pythagoras was the high priest, par excellence, of the divine wisdom. He then became, in the trivializing school tradition of the Middle Ages, the master of the quadrivium, and in particular the inventor of arithmetic. The early modern period discovered Pythagoras as the creator of natural science, which was just then being reborn; what Copernicus and Galileo taught was regarded by their contemporaries as a revival of Pythagorean science.1

Subsequently, as research based on historical perspective increasingly

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replaced the uncritical appropriation of ancient culture, the traditional picture of Pythagoras, imposing though also vague in outline, inevitably gave way more and more before criticism. In the scholarly controversy that followed scarcely a single fact remained undisputed, save that in Plato’s day and then later, in the first century B.C., there were Pythagoreoi. The “wisdom of Pythagoras,” however, has also had passionate defenders, who opposed to criticism a countercritique: one-sided, self-sufficient methodology, they protested, had substituted hypothesis for tradition.

When we set out to survey the most important attitudes and trends in modern Pythagorean scholarship, the point of departure must be the work of Eduard Zeller. In it the material is not only collected, with a completeness scarcely to be surpassed, but sifted with uncommon methodological rigor. The criterion for the value of a tradition is its age, and Zeller arrives at the verdict, often cited since in agreement and disagreement: “The tradition about Pythagoreanism and its founder thus has more and more to tell, the further it lies, chronologically, from the events . . . ” (I 364). This “expansion of the tradition” (ibid.) arose basically from “dogmatic preconceptions, partisan interests, dubious legends, and spurious writings” (365). The most important source, nearly the only one which is left, is in the reports of Aristotle, in his surviving treatises. A second primary source is found in the fragments of Philolaus, in which August Boeckh, in his day, claimed to have found a firm foothold amidst the bog of Pythagorean pseudoepigrapha. But Aristotle speaks of “Pythagoreans,” not of “Pythagoras”; so the figure of the master fades off in a mist of nonhistorical legend. Pythagoras is still recognized as the “founder of a religious society” and teacher of transmigration (411), but all philosophical significance is denied his ethical and religious doctrines (557 ff). Alongside this stands, without connection, the number philosophy of the Pythagoreans. This can be reconstructed from Aristotle and is confirmed by Philolaus, though aside from him the Pythagoreans remain anonymous and scarcely datable. Sharply separated from all this ancient material is the neo-Pythagorean school, which (Zeller thinks) arose, compounded of Platonic and Aristotelian elements, not before the first century B.C. (III 2.92 ff).

Zeller’s work had a decisive influence, especially in Germany; it dominates Diels’s arrangement of testimonia in the Fragmenta der Vorsokratiker. In the chapter on Pythagoras (DK 14), the biographical reports are assembled, along with the reports as to whether or not Pythagoras wrote anything.4 In the separate chapter on “The Pythagorean School” (DK 58), the most important section is the collection of “ancient Periapatetic” material (38 B), following Iamblichus’s catalogue of Pythagoreans (58 A), and itself followed by the acusmata (58 C), the Pythagorikai apophthesa of Aristoxenus (58 D), and the allusions to “Pythagorists” in the Middle Comedy (58 E). Between these two chapters are found, in chronological order, the testimonies on individual Pythagoreans, notably Hippasus (DK 18), Philolaus (DK 44), and Archytas (DK 47). Hermann Diels follows Zeller also in the question of the genuineness of the Philolaus fragments.

Zeller’s solution, however, left a number of problems unsolved, and later research entered in with supplement, modification, and criticism. Above all, a gap had opened between Pythagoras the religious founder and the number philosophy of anonymous Pythagoreans; to connect these disparate elements and to show their original unity was bound to be an extremely enticing challenge. For at this time the tide of system building in philosophy, which had borne the work of Zeller, was ebbing, and this very change made possible a deeper understanding of cultural history. As the boundaries of philosophy became fluid, the connections with pre-scientific, religious-mythical thought became clearer. So the task appeared to be to comprehend how religion and philosophy could be united in Pythagoras: mysticism and science (or at least the germ from which science sprang). Then came the high tide of the evolutionary idea, and it began to seem possible, with its help, to explain the contradictions of the tradition and to give everything its place in an extended and detailed history of Pythagoreanism.

The direction was set by August Döring (1892); his thesis became most influential because he was followed by John Burnet in the later editions of his Early Greek Philosophy.6 The unity of science and religion is found in the ideal of catharsis; scientific activity is the highest form

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4 Two doxographical reports are arbitrarily added (14.20, 21; cf. below, ch. IV 1, mm. 19-41; I 3, n. 151-153).  
5 From the fifth edition on, the excerpt from the Hypomnemata has been added, as a result of the discussion of Max Wellmann (58 B); cf. below, ch. I 3, n. 1).  
6 Burnet refers to Döring (EGP 98 n. 1); as a result, the extent of what Burnet traced back to Pythagoras himself increased considerably after the first edition (London, 1892, 105 ff).
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of purification and conduces toward the salvation of the soul. Further, we have the principle that "The more primitive any Pythagorean doctrine appears, the more likely it is to be that of Pythagoras himself" (Burnet, EGP 99); in this way it is inferred that Pythagoras had not only a number theory but an astronomical system. A more advanced stage is discernible in the "Pythagoreans" of Aristotle, but his reports too are broken down into different categories and are supplemented from Plato as well as later sources.

The picture of Pythagoreanism built up in this manner is far more colorful than Zeller’s speculative account, but it depended so largely on inferences and hypotheses that opposition was inevitable. On the basis of the same leading idea, of development from religion to rational science, completely different and mutually irreconcilable reconstructions were offered. The most significant contribution, after Burnet, was F. M. Cornford’s article “Mysticism and Science in the Pythagorean Tradition”; but his conclusions were importantly modified by his own pupil J. E. Raven. Here should be mentioned also the independent accounts of Léon Robin, Abel Rey, and Pierre-Maxime Schuhl. The fact that each scholar had to erect a whole new superstructure shows the weakness of the foundation.

Nevertheless, the attempts to improve on Zeller are based on additional source material, which, though not newly discovered, was for the first time revealed in its true purport. For this was a further weakness in Zeller’s position; he had been too hasty in rejecting the later tradition. More exact study showed, here as elsewhere, the fallacy in the principle recentiores, ergo detiores. Erwin Rohde initiated the careful analysis of the sources of the lives of Pythagoras by Porphyry and Iamblichus, and this led back, for substantial portions, to authors of the fourth century B.C.—Aristoxenus, Dicaearchus, Heraclides Ponticus, then Timaeus. With growing optimism others followed along the path Rohde had pioneered. It was not only in the realm of political history that many new insights could thus be attained; the problem of the philosophy and science of Pythagoras also began to appear in a new light. The reconstruction of Aristotle’s Protrepticus from passages in Iamblichus seemed to yield statements by Aristotle himself about the philosopher Pythagoras. But most important of all, thirty-five years after the first appearance of Zeller’s Philosophie der Griechen, Diels had shown in his Doxographi graeci (1879) that Theophratus was the ultimate source of the wide-branching doxographical tradition. After that it was difficult to attribute the reports of the Placita to the neo-Pythagoreans, as Zeller had done (I 407ff). Diels classified them with the “ancient Peripatetic tradition” (DK 5B15), and they play their important role in all the above-mentioned reconstructions of a developing Pythagoreanism.

There was also an attempt to discover indirect sources; one sought to find Pythagorean material reflected in other pre-Socratic thinkers, whether by way of influence or of polemic. The most important step in this direction was the thesis of Paul Tannery, that Parmenides took the cosmology of the doxa section of his poem from the Pythagoreans and that Zeno’s polemic was directed against their number theory. This brought some very ancient evidence into the field, which could help to classify, to supplement, and even to test the reports of Aristotle. This hypothesis was developed in many ways, and only very seldom placed seriously in question. Similar endeavors were of course directed toward Plato, whose connection with Archytas is firmly fixed in the biographical tradition. To be sure, it is especially difficult in this case to achieve any certainty. The native capacity of Plato’s mind to remodel and reshape is too great, and in each specific instance it is a controversial question to what extent particular statements of the dialogues may be interpreted as historical facts. The theory of Burnet and Taylor, which takes every portrayal in the dialogues as historical fact, not only transforms the entire Timaeus to a Pythagorean document of the fifth century b.c., but presents us with a Socrates who is an advocate of the theory of ideas and an adept of Pythagorean wisdom. Though this radical solution has attracted no following, the Pythagorean origin of the theory of ideas, and especially of the

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2 Döring, AGP 1892, 505; Burnet, EGP 97ff; below, ch. II 6.
3 CQ 1922–1923; cf. PLin Pace 1–27 and CAH IV (1926) 544–552.
5 See below, ch. II 1. Special gratitude is due Armand Delatte, who completely studied through the later tradition, including even its most abstruse areas. All the parallel material is collected and set out very perspicuously by Delatte in his edition of Diogenes Laertius’ life of Pythagoras, and by Ludwig Deubner, building upon Delatte’s work, in his edition of Iamblichus’ life of Pythagoras.
6 Lam. Protr. 51, s1FF; Arist. Protr. fr. 11 Walter. On this, cf. Burkert, Hermes 1960, 106ff; below, ch. I 3, n. 77. T. Düring, Aristoteles’ Protrepticus (Göttingen, 1961) p. 189, holds to the Aristotelian origin of this passage in Iamblichus (B18–20 Düring); this forces him to smooth over the difficult transition at Lam. Protr. p. 51, s1F by conjecture and to ignore the characteristic τῆς ἐρωτήσεως; cf. Lam. Protr. 411, as a result, the hints toward source analysis that the text of Iamblichus provides are arbitrarily removed.
7 CQ below, ch. III 3 and VI 3, n. 40.
of the tradition against the attacks of critics. Even the translations and explications of the Pythagorean source material collected in the \textit{Fragmente der Vorsokratiker}, by Antonio Maddalena (1954) and Maria Timpanaro Cardini (1958–1964), have the goal of confirming the age and philosophical significance of Pythagoreanism; and chauvinistic enthusiasm for Pythagoras runs riot in the bulky works of Vincenzo Capparelli.\footnote{This thesis was advanced by Ernst Hofferth, too, in \textit{Essays on the History of Medicine Presented to Karl Sudhoff} (Zürich, 1924, pp. 61–72; see his very favorable discussion of Frank’s book, \textit{JAW} 107 [1923] 166ff), and worked out, though in a somewhat superficial manner, in the dissertation of Jenny Bollinger (Zürich, 1925).}

Over against all these attempts to achieve a more positive view than Zeller are energetic movements of scholarly criticism which have even called into question testimony accepted by that scholar. The genuineness of the Philolaus fragments was attacked by Carl Schaarschmidt in 1864, and in 1868 by Ingram Bywater. While Zeller’s authority held up for a while in Germany, Burnet followed Bywater and therewith ensured the predominance of the negative verdict on the Philolaus fragments which still holds in the English-speaking world.\footnote{La sapienza di Pitagora (Padua, 1941–1944; 2 vol. with 1,535 pages; \textit{Il contributo pitagorico alla scienza} (Padua, 1955); and \textit{Il tentore di vita pitagorico ed il problema della onomastica} (Padua, 1958).}

Rejection of these Philolaus fragments is an essential element in the thesis of Erich Frank, whose book \textit{Plato and die sogenannten Pythagoreer} (1923) towers over everything else that has appeared since Zeller on the history of Pythagoreanism, in the qualities of critical vigor, penetration, and firmness of judgment. To be sure, its merits are counterbalanced by one-sidedness and obvious perversities. Frank’s methodological contribution was that he consistently held to the history of the natural sciences—mathematics, music, and astronomy—as basis for the reconstruction of Pythagoreanism. The first result was to date the development much later; all Pythagorean science, he thought, had come into existence in the circle of Archytas, about 400 B.C., influenced by the fully developed atomism of Democritus. The philosophy of the “so-called Pythagoreans,” however, the number theory, was dependent on the late Plato, and was basically a creation of Speusippus, who had also himself forged the book attributed to Philolaus.\footnote{Cf. Zeller I 396–397 (vs. Schaarschmidt); below, ch. III 1, n. 14.}

The figure of Pythagoras fades into the mist,
and all the much-discussed Pythagoreanism of the fifth century becomes a mirage.

Frank’s book, teeming with arbitrary theories and ex parte judgments, has been severely criticized; and curiously enough, Frank himself expressed quite different views in later writings. Nevertheless, the book still has importance, above all because of the extreme way in which the problem is put: “Plato and the Pythagoreans”—their mutual relationship is in fact the central problem of any historical investigation of Pythagoreanism, and Frank was right in perceiving that the influence did not go entirely in one direction. There is Plutonic material which at a later date was wrongly labeled Pythagorean, and the generation of Plato’s immediate disciples—Speusippos, Xenocrates, and Heraclides—played the decisive role in this development. Just as correct, and important, is the idea of regarding “Pythagorean” mathematics and science in the context of non-Pythagorean science, which stems from the Ionian and Eleatic philosophy of nature, and inquiring to what extent the Pythagoreans may have been on the receiving side.

In fact, notwithstanding the reconstructions of “Pythagorean mathematics,” the date and importance of Pythagorean influence on Greek mathematics had been called into question as long ago as 1907 by Gustav Junge, then by Heinrich Vogt, and above all by the profound and influential work of Eva Sachs, *Die fünf platonischen Körper* (1917). The reports relating to Pythagoras himself are eliminated, what is firmly attested is dated later, and the non-Pythagorean science of the Greeks is revealed. Finally, William Arthur Heidel tried to push further on in this direction. If we add that the reports of the “religious leader” Pythagoras were subjected by Walther Rathmann to so devastating

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23 As late as 1931 a direct attack was published: G. de Santillana and W. Pitts, “Philolaus in Limbo, or: What Happened to the Pythagoreans?” *Isis* 42 (1951) 112–120.

24 Though one read in 1923 that “all those discoveries attributed by later writers to Pythagoras himself were in fact achievements of certain Southen Italian mathematicians of the time of Plato” (vii), he wrote later (Knowledge, *Will and Belief: Wissen, Wollen, Glauben: Collected Essays*, ed. L. Edelestein [Zurich, 1955] p. 82) “it can hardly be doubted that Pythagoras was the originator of this entire scientific development. He was a rational thinker rather than an inspired mystic.” Though he had in 1923 held it to be impossible that Plato had taken over Pythagorean material (Philolaus, in *Philebus* 56a—x) if one reads the *Philebus* without prejudice, one has the impression, throughout, of being in the presence of the mature result of a long philosophical career” (p. 304)—still, he declares quite plainly in 1940 that Plato was dependent, in this passage, on Pythagorean ontology (APh 61 1940 49 *Knowledge, Will and Belief* 100). On the other hand, his radical hypercriticism comes to the fore again in a review of von Fritz’s *Pol., APh* 64 (1943) 220–225.

25 APh 1940; cf. below, ch. VI.

26 See now Theile, *Texts* (245 pages!); as against 94 lines of text from Archytas, which Diels regarded as genuine, there are 46 pages of ps-Archytas material.

27 E.g., Pythagoras was taken prisoner in Egypt by Cambyses (525 B.C.), stayed 12 years in Babylon, and then returned to Samos in time to leave the island, in 532 B.C., because of the tyranny of Polycrates (cf. below, ch. II, n. 16). He met Phalaris about 570, and was the teacher of Empedocles, who was born about 490. The musical experiments which are attributed to Pythagoras are physically impossible (below, ch. V 1).
should be thought of as corporeal or incorporeal, whether Pythagoreans believed that the earth moves or not. Each side of each question is advocated by ancient authorities, sometimes with explicit polemic. Even the most extreme credulity comes to a limit here; it is simply impossible to accept "the Pythagoras of tradition," because there is no single tradition.

Finally—less striking but scarcely less disquieting—in many cases late tradition gives the name of Pythagoras, where older tradition, dealing with the same topic, does not do so. This circumstance alone is more suspicious when a number of apparently ancient testimonies crumble at the touch of analysis—the name of Pythagoras is interpolated in Aristotle’s Metaphysics, added by Iamblichus in a passage from Aristotle in his Protrepticus, supplied by Proclus, after Iamblichus, in a fragment of Eudemos, inserted by Porphyry in a passage stemming from Theophrastus. When observations of this kind multiply, Zeller’s suspicion about the “expansion of the tradition” seems justified.

On the other hand, criticism is always subject to counter criticism. Though many sources may be late and not very reliable, more must lie behind them all than a simple zero. “Pythagoreanism without Pythagoras,” without chronological position or a place in the history of thought, is not only unsatisfying to the scholar, but impossible in itself. A minimalism that eliminates every aspect of tradition which seems in any respect questionable cannot help giving a false picture.

These very difficulties in evaluating the Pythagorean tradition reveal certain characteristic differences between Pythagoras and the other pre-Socratics, which inevitably set the direction for a new study of the problem. Just as a city which was continuously inhabited over a period of time, by changing populations, presents to the archaeological investigator far more complicated problems than a site destroyed by a single catastrophe and then abandoned, the special difficulty in the study of Pythagoreanism comes from the fact that it was never so dead as, for example, the system of Anaxagoras or even that of Parmenides. When their systems had been superseded and lost all but their philolog-
cal and historical interest, there still seemed to be in the spell of Pythagoras’ name an invitation to further adaptation, reinterpretation, and extension. And at the source of this continuously changing stream lay not a book, an authoritative text which might be reconstructed and interpreted, nor authenticated acts of a historical person which might be put down as historical facts. There is less, and there is more: a “name,” which somehow responds to the persistent human longing for something which will serve to combine the hypnotic spell of the religious with the certainty of exact knowledge—an ideal which appeals, in ever changing forms, to each successive generation.

Scholarship cannot succumb to this spell. Its first task must be, since the original phenomenon cannot be grasped directly, to interpret the interpretations, to single out and identify the different strata of the tradition and to look for the causes that brought transformation to the picture of Pythagoras. Zeller, in his day, had seen the task and solved it in his own way, blaming neo-Pythagoreanism for all the “late” distortions. Since the results of source analysis are hardly compatible with this thesis, inquiry must take a new start. Perhaps the chances of success are better than before; our knowledge of the development of fourth-century philosophy has grown, thanks to studies of Academic and Peripatetic tradition, notably by Werner Jaeger and Fritz Wehrli, and to the elucidation of the Platonic system of first principles carried on, after Julius Stenzel, by Hans Joachim Krämer and Konrad Gaiser. The flood of works on the pre-Socratics has provided us with means of distinguishing fifth-century thinking, in the wake of Parmenides, from later philosophy. And in relation to this, we should be able to identify an even earlier stratum.

At the same time, the intention in which follows is to give its full value to each of the various aspects of the Pythagoras tradition. One

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28 Cf. below, ch. II 2, n. 46-47; I 3, n. 52; III 1, n. 65ff.
29 Cf. below, ch. I 2, n. 76; II 6; IV 1, n. 19, 32, 38; VI 1, n. 34-37.
30 Cf. above, n. 6.
31 Cf. above, ch. I 2, n. 11.
32 Cf. below, ch. VI 1.
33 Ps. Abs. 2.28, p. 158.4 14 Nauk; cf. J. Bernays, ‘Theophrastos’ Schrift über die Frommigkeit (Heilbr. 1866) 119f.
34 The Pythagorean movement still lingers on today. The book of Jean Mallinger, Pythagore et les mystères (Paris-Brussel, 1944) is dedicated to the “revered head of the Pythagoreans of Belgium,” and his conviction is that “Pythagoras answers, in an amazing way, all the questions and all the needs which today’s anxiety brings forth” (p. 7). The same author has written a Note sur les secrets ésotériques des Pythagoriciens (Paris, 1946).
35 In July and August, 1955, there was held in Brussels, Athens, and Samos a “Pythagorean World Congress” (cf. Schottlander 333). Since then, Tignani, the site of ancient Samos, has been renamed Pithagorion.
36 The task of disentangling the various branches of the tradition has been emphasized especially by Olof Gigon; see Entretiens sur l’antiquité classique I (Vandoeuvres-Geneva, 1952) p. 141.
37 Jaeger showed how the Old Academy and Peripatos projected their own ideal, at each period, upon Pythagoras (Arist. 96f; SHBH 1928, 395f, 415ff).
38 Cf. his commentary on the relevant fragments of Aristotle, Dicalearchus, Heraclides, and Clearchus.
can take science seriously as science and at the same time comprehend the meaning and function of that which was present before science, and which continues to influence it. One can acknowledge the structural unity of philosophy and still recognize that the history of thought is not exhausted in the reconstruction of doctrinal formulae. It is inherent in the forms of life, it cannot be separated from the unique individual, and it cannot be repeated.

Most studies of Pythagoreanism have dealt with only one restricted aspect; even Zeller confined himself to the development of philosophical concepts, left mathematics aside, and bracketed out religious and ethical questions; and later works have been even more specialized, whether in the philosophical area, in that of mathematical, astronomical, and musical problems, or that of religion. The very thing that might seem rash and hasty, in view of the fundamental differences of interpretation, is what the nature of the situation demands: as many-sided a treatment of the problem as is possible. For many of the contradictory conclusions have come from investigating and tracing the course of single paths of development, with no thought of the way in which these may converge with other, equally important lines. Any attempt to date Pythagorean philosophy and science back as far as possible, even to Pythagoras himself, must take account of its connection with the religious-cultic and primitive sides attested for the movement; and any investigation of Pythagoras the “shaman” must endeavor to make sense of the later development of Pythagorean science. The division of labor which seems so reasonable brings along with it the danger of a vicious circle. It can happen that the historian of science builds his reconstruction on a philologically inadequate foundation; the philologist takes over the seemingly exact result of the historian of science; the philosopher, on the basis of this criterion, rejects contradictory evidence—and so on. The many-sided treatment which can prevent misunderstandings of this kind must be the objective of the classical scholar; collection, interpretation, and critical analysis of the ancient evidence is the necessary common foundation. Mathematical and philosophical thought, as well as parallels from comparative religion, can never yield more than possibilities; as to the historical facts, the sources are decisive.

At the beginning of the following studies stands, not the analysis of the oldest evidence, but discussion of the tradition of Pythagorean philosophy; the task turns out to be not to separate older and more reliable from later and more dubious material, but to judge between traditions which are contradictory but of equal antiquity. In this matter a remarkable stroke of luck has provided us with an important new piece of source material, a fragment of Speusippus on Pythagorean philosophy first published in 1953. This makes certain, what a careful analysis of the sources would in any case make likely, that a Platonizing interpretation of Pythagoreanism, which had a decisive influence on the later tradition, goes back to Plato’s immediate disciples and differs sharply from the reports of Aristotle. The latter’s evidence thus becomes more important than ever; for he alone warns us to separate Pythagorean and pre-Platonic from Platonic material. Only from the point of view gained by the revelation of this contrast is it possible to evaluate Plato’s own testimony, and in particular the allusions in the Philebus. In this way we find incontrovertible evidence—as against Frank and Howald—of a pre-Platonic and pre-Socratic philosophy of the Pythagoreans.

Study of the oldest, pre-Platonic tradition can thus be supplemented by those pieces of evidence which stand outside the Platonic influence, and were not affected by the reinterpretations mentioned above. Once more the reports of Aristotle become especially important, the fragments of his lost monograph on the Pythagoreans. The Pythagoras story, which used to be, for the most part, written off as the unfortunate product of the obfuscation of historical facts, may be understood as the expression, precisely, of a definite historical reality. Pioneers of this line of interpretation were Karl Meuli and E.R. Dodds. To the legend belong the acusmata, in their essence doubtless extremely old. The result that emerges is a rather distinctive picture of a shamanistic “Wise Man” and a Life, or Way of Life (bios), dominated by ritual—a Pythagoreanism foreign to all exact science.

The study of the Philolaus fragments harks back to the results of the first chapter. A point of view is achieved from the distinction of Platonic and Aristotelian traditions about Pythagoras, from which one can see clearly the genuineness of at least part of these fragments. The attempt is made, then, to make these understandable in the framework of fifth-century thought, as the attempt of a Pythagorean to come to terms with the natural science (physiologia) of the Ionians and the Eleatics.

The history of the natural sciences can confirm this result, for on one hand the Philolaus testimonia take their position in the history of fifth-century thought, while on the other the origin of the exact
sciences is seen to lie outside the Pythagorean realm. The renown of Pythagoras as the inventor of mathematics and mathematical natural science is explicable as a distortion of perspective; a pre-rational interpretation of the cosmos, along with arithmological speculation, is seen in hindsight as rational science.

My aim has been to take full account of the ancient evidence, and the modern literature has been consulted as fully as was practicable. As far as possible the originator of each particular thesis or argument has been named, but it was out of the question to try to provide an encyclopedic doxography of all pertinent views on each problem. I hope nothing decisive has been overlooked.

I. Platonic and Pythagorean Number Theory

Πλάτων πυθαγορικὸς—from the time of Aristotle, this finding has often been repeated, but there is little clarity as to the extent and the manner in which Plato borrowed Pythagorean doctrine, or as to what Pythagoreanism was like before Plato. This applies especially to those attempts to derive the Ideas from numbers, to equate them with numbers, or even to replace Ideas with numbers, which Aristotle and others attribute to Plato and his pupils, in particular to Speusippus and Xenocrates. These theories looked somewhat like a capitulation to Pythagoreanism on the part of the aging Plato, until Julius Stenzel set the task "to understand the concepts of Plato’s late philosophy in their own context and to incorporate into Plato’s philosophical development, inseparably, those ‘Pythagorean mystical’ elements that have up to now been traced back to external, foreign influence." Since then, much scholarly work has been devoted to the Platonic "doctrine of principles," and to its interpretation in Platonic terms, starting from Platonic presuppositions. Still the question of pre-Platonic sources, of pre-Platonic Pythagorean doctrine, persists. The crucial point is to grasp firmly what the difference is between this and the later, Platonic philosophy. An investigation must start from a sketch of the Platonic theory of principles, since we know more about its context.

The foundation for the study is the evidence of Aristotle; he alone sets up Platonic and Pythagorean doctrines side by side, specifying

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1 Esp. Met. 987a29 ff.; cf. e.g. Aët. 2.6.6; Cic. Rep. 1.16, Fin. 5.87, Apul. Flor. 15, p. 60, Apul. Plat. 1.3, ch. 1.4, n. 47.
2 ZG 108.
3 Handy bibliography in the German reprint of Stenzel ZG and in Gaiser, who also gives a convenient collection of the sources. The most comprehensive work remains L. Robin, La théorie platonicienne des idées et des nombres d’après Aristote (Paris 1908, repr. 1963). New evidence was adduced by Merlan (1934) and Wilpert (1940, 1941). The paper of H. Compercz (1930) deserves special mention, because it has been generally overlooked. A new, energetic attempt at reconstruction and interpretation has been made in the books of Krämer (1959) and Gaiser (1963); among more recent contributions, see Thielek T Homon; Dönt; K. H. Illing, Phileus 13 (1968) 1-31.
their points of agreement and—what is almost more important—their differences.  

This might appear an extremely unfavorable point of departure. Precisely as the result of the amount of recent research in this field, it remains one of the most controversial in Platonic studies, without any prospect of early agreement. There is controversy as to how the connection of ideas and numbers is to be understood, how the whole matter is related to the well-known philosophical theories expressed in Plato’s dialogues, and what is to be attributed to Plato himself and what to his various pupils. And the most learned studies in the field, the books of Harold Cherniss,\(^6\) have shaken the very foundation of the discussion with the thesis that most of the supposed evidence is wrong inference, or even falsification, by Aristotle, and that the true Plato is to be found only in the dialogues and has nothing to do with these speculations. At the other extreme, Krämer and Gaiser hold that the backbone of Plato’s philosophy is a “doctrine of principles” (Prinzipienlehre) which can be found even in allusions in the dialogues. The lively, sometimes acid, discussion which has been going on for years will not easily be brought to a solution.\(^7\)  

Fortunately for the reconstruction of Pythagorean doctrine, not much depends on the solution of these problems. What the relation is between Plato’s dialogues and the “doctrine of principles,” whether it represents an early or only a late stage in Plato’s philosophy, whether a particular doctrine is to be traced to the historical Plato, to Speusippus, Xenocrates, or even to Aristotle’s interpretation, becomes irrelevant when the issue is to establish the difference between Platonism in general and earlier Pythagoreanism. In what follows, the adjective “Platonic” may be understood as applying to “Platonists” and “Platonism,” regardless of the question of their relation to the philosophy of Plato himself.  

For, though Cherniss has rightly emphasized the independence of Plato’s pupils and the differences between their doctrines,\(^7\) still it is not to be denied that there is a common tendency in the area of “first philosophy”—to use Aristotle’s term—in their essays at basic ontological doctrine. The aim is to trace ideal Being to its basic principles (ἀρχαί) and to understand this Being in terms of these principles, just as the Platonist learned to understand the world in which we live from the point of view of the ideas. Heinrich Gomperz proposed the convenient term “system of derivation,” though this characterizes the movement of thought in a one-sided manner. It is rather the ascent from the empirical to the Higher, the vision of the Idea through the world of experience that surrounds us, and the realization of the ἀγαθόν, the One, in the complexity of the ideas that Platonists strive for, than conceptual “deduction” or “derivation” (Arist. EN 1095a32).  

The problem of unity and multiplicity of ideas, of their interpretation and their distinction by the method of diaeresis was, as the dialogues show, of increasing concern to Plato.\(^\ast\) The borderline between dialectical exercise and metaphorical seriousness seems to be deliberately blurred. There occurs a curious reticence when the conversation touches upon the most essential questions, even in early dialogues.\(^\ast\) In the Timaeus, Plato gives more definite indication that it is the question of the πρῶτων ἀρχῶν (the first principle of all things) that is being bracketed out (48c): Plato has reduced the multiplicity of the world to four elements, the elements to regular solids, and these to triangular surfaces; “but the principles that are still prior to these god knows, and he among men who is dear to him” (53d). The Republic introduced the Good as the Highest, the Sun in the realm of ideas, “beyond being” (509b). The opposite to the good, in all the later dialogues, is described as an indefinite oscillation in two directions, toward “great and small,” “more and less,” “stronger and weaker,” and it is the Good that constitutes measure and definiteness in this continuum.\(^\ast\)  

It is from this point of view that we can approach the reports about Plato’s oral teachings,\(^\dagger\) in particular the lecture On the Good. Our

\(^4\) Astonishingly little attention has been paid to Aristotle’s distinction of Platonism from Pythagoreanism, though Zeller (I 463ff) collected the evidence. Thus down to Raven (KR, nos. 320, 405, 406) and Guthrie (I 256–262; cf. below, ch. I 2–3), Platonic doctrines are included among Pythagorean documents. Frank denied himself a “more exact source-analysis of the Aristotelian and post-Aristotelian reports,” because of the inadequacy of “the space available” (n. 388); thus he missed what would have refuted his thesis (cf. below, ch. I 2) that all “Pythagorean” philosophy is post-Platonic.\(^\dagger\)

\(^6\) Cherniss, Plato, vol. I—the second volume is not expected to appear—and Riddle, esp. pp. 20ff.

\(^7\) Riddle 60ff.

\(^\ast\) Parm. 139a4ff (and passim), Philo. 15a, Soph. 251a4ff. Cf. Krämer 429ff.

\(^\text{9}\ootnote{Prot. 357b. Cf. Krämer 389ff.}\

\(^\ast\) This has been worked out in a convincing way by Krämer.

\(^\dagger\) In one passage Aristotle cites explicitly the “unwritten doctrines” of Plato (Phys. 209b15; cf. Cherniss, Plato 113ff, 160ff; Krämer 416ff), and he clearly refers to them in other passages (Ross, PTh 143ff); once (92a22) he speaks of Plato’s “repeated” pronouncements.
I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

I. The Platonian Theory of Ideal Numbers

12 Fr. 27–31, on which see esp. Wilpert, Hermes 1940. Even Cherniss concedes that Alexander of Aphrodisias used the book directly, though obviously no one did so after him (Plato 1115n. 77; Riddle 27ff).


15 Phys. 247.30ff; see esp. Wilpert, Hermes 1941, 227ff, and Zw Fr 183ff; De Vogel, Mnemosyne 1949, 205ff; Cherniss, Plato 285ff; Krämer 282ff.

16 Harm. 2, p. 30 M.: the logos had to do “with mathematics, numbers, geometry, astronomy, and, finally, that the good is single” (translating Macrae’s conjecture κατά παράδειγμα and taking τό πέρας as adverbial. Cf. Cherniss, Riddle 87 n. 2, Ross, PTI 148 n. 1, 244, Krämer 423; differently interpreted in De Vogel, GP 1 274 n. 1).

17 Cherniss tries to undermine this foundation with the observation that in the reports about the lecture On the Good the point corresponds to the One (Alex. Met. 55.20ff, SimpI. Phys. 454.19ff; also Sext. Exp. Math. 10.259ff), while according to Aristotle Plato eliminated the concept of the point (99a20ff; Cherniss, Plato 167ff, Riddle 28f).

18 De Vogel’s attempted compromise, Mnemosyne 1949, 306ff, is not completely convincing. In the passages of Aristotle, however, which refer to Plato, there is no mention of the point as a “monad having position”; the first number is 2 (cf. 1081a2ff), and the first geometrical entity is the line, whereas Speusippus adopts the point corresponding to One. Thus there is some inexactness in the later reports (cf. Krämer 418 n. 76: the commentators are using mathematical terminology). This tends to confirm the reliability of Aristotle.

19 While Cherniss seeks to isolate the lecture On the Good as much as possible (Riddle 12), Ross speaks of a “course of lectures” (PTI 48), and Krämer states: “The discourses on the Good provide . . . the standard expression for the teaching activity of Plato in general” (409); he could refer to στάδαν Ep. 7. 342a and Arist. Met. 922a22, but his argument from the “imperfect of repetition” (407) in the passage from Athenaeum (Iamn. 2 p. 30 M.) is not conclusive; he would understand “every time when the lecture turned out to be on mathematics . . . this would seem, I think, paradoxical to the audience.” But δὲ δὲ φανερών may as well be optative in indirect discourse (W. Thiller, AGP 50 [1968] 29 n. 1). Aristotle used to tell the anecdote as a warning how not to proceed in lecturing, how to avoid disappointment by correct advertising. Evidently not the audience alone had been disappointed. Thus this experience has—against Krämer—nothing to do with the “test” described at Ep. 7. 340b, the deliberate discouragement of merely curious auditors by demonstrating the difficulty of philosophy.

20 The Platonian Theory of Ideal Numbers is a cornerstone of Platonism, especially in Germany, since Wilamowitz’ Platon (1918), but Cherniss pronounced a negative verdict (Riddle 13), and proofs of inauthenticity were undertaken by G. Müller, Archiv Philos. 3 (1949) 231–276 (contra, H. Patzer, ibid. 5 [1954] 19–36, and B. Stenzel, AP 74 [1953] 381–397)—and above all by L. Edelstein, Plato’s Seventh Letter (Leiden, 1966) (contra, K. von Fritz, Platon in Sizilien, [Berlin, 1968]). The aid of the computer has been invoked: M. Levison, A. Q. Morton, A. D. Wimppear, Mind 77 (1968) 309–325; but to attribute not only the Seventh Letter but also the production to the Timaeus and the Critias to Speusippus can hardly be the final solution.


his discussions of “first philosophy,” he has a tendency to put a disproportionate emphasis on number theory: “If the Ideas are not numbers, it is impossible for them to exist at all; for from what kind of principles will the Ideas come?” (1081a12ff). The most important hints at these unwritten doctrines in the corpus Platonicum are to be found in famous passages of the seventh Letter; no wonder therefore that the discussion about the “esoteric” Plato is intimately bound up with the question of the genuineness of that letter. Again, however, the answer is of limited importance for the investigation undertaken here: it is generally agreed that the letter is either by Plato, or by one of his immediate pupils, and, even in the latter case, the doctrines contained, especially the “philosophic digression” have about the same degree of authenticity as the reports on the lecture On the Good. Dionysius, the letter says, has learned from Archytas (338c) and Dion something of the doctrines which Plato himself had not yet wanted to communicate to him, parakousmata (383d), chance bits of information about “the greatest matters” (341b), “something of that which is highest and first in the philosophy of nature” (344d; cf. 341d). It concerns “truth about virtue and vice” (349a-b), and at the same time “truth and falsity in the whole of being” (349b), one singular object of learning (mathema), which is not easy to communicate (341c). The “greatest teaching” is the Good (Rep. 505a), thus the close relation between the comments in the seventh Letter and the later memoranda of his pupils On the Good is obvious. As the audience was startled that “the Good” should concern mathematics and astronomy, the letter binds up “philosophy of nature” with “virtue and vice.” Plato states however, “There is not any writing of mine on these matters, nor will there ever be, for this is a thing which cannot be put into words like other doctrines” (341c)—a sentence as famous as it is controversial in its interpretation. While Cherniss believes that anyone who takes these words as genuine and
relates them to the *On the Good* must in consistency give up trying ever
to understand Plato. Krämer emphasizes that these “ultimate matters”
are not “inexpressible” in an absolute sense, but only for the great
mass of mankind.\(^{22}\) Yet Plato himself shows how serious he is about
this “inexpressibility” by tracing its cause, in an excursus, to the
relation between Being and the means of knowing. There are four means
or steps of “knowing” an object: by name, by definition, by image,
or by knowledge to which mind and right opinion are added (*ἐπιστήμη
cαὶ νοῦς λόγις τε δόθη, 342c*). But the fifth, Being itself, stands apart as
that which is the object of knowledge and truly exists” (*342b*). Mind
(*nous*) comes closest to this (*342d*) but even mind does not grasp it
completely and unambiguously. Each of the four kinds of knowing
comprehends a qualitative aspect (*poion tī*) as much as the Being which
the soul is seeking (*342c*):\(^{23}\) “There are two things, the Being and the
qualitative aspect, and it is not the ‘what kind’ but the ‘what’ that the
soul seeks to know. But each of these four proffers to the soul the thing
that is not being sought, and thus fills every man’s mind with puzzle-
ment and unclarity” (*343b-c*). Because of this inadequacy—even on
the part of mind and knowledge—it is easy to contrast and refute
where the “fifth” is concerned: “When we are under the necessity of
separating out and revealing the fifth element, anyone who likes to do
so has the means of confuting us” (*343d*). The ring of disappointment
at the “contempt” (*341e, 345b*) to which the most sublime is exposed
may echo the contempt (*ἐποκαταφρόνησις* Aristox.; above, n. 16) shown
by hearers of the lecture On the Good. Therefore the only course
possible, the letter says, is to take those who are already on the right
path and are “related to” truth (*344a*), and lead them on, in patient
practice, to the point where Being reveals itself to them in a sudden
illumination: “Knowledge of each thing, and Mind, blaze forth in
his view as he strains to the limit of human capacity” (*344c*).\(^{24}\)

If this is taken seriously as Plato’s view, the dialogues are devaluated
as lacking seriousness, being only preliminary guideposts to what ought
to happen in a philosophical life. But also, the oral teaching can never
be a final “system,” made up out of non-contradictory rules and
terms. For not even knowledge firmly grasps its object. Plato is not
satisfied with elegant “principia mathematica,” he asks for the Good—
not in the sense of an “ought” imposed by some authority, but as the
goal of all striving, the prop and the meaning of existence. This can
only be perceived in an individual experience similar to religious reve-
lation. But the only way to it is the common quest of indefatigable
dialogue, which necessarily leads to the most stringent rules of dialectic,
to logic and mathematics. There will be no final result, which can be
taken down on paper or papyrus, so that any written account must be
misleading. When Plato’s students wrote about “the Good,” they
deviated from this attitude;\(^{25}\) but it was doubtless a necessary task, once
the discussion had grown into a variety of conflicting interpretations,
to put down what Plato himself had taught. Independent philosophiz-
ing passed over very quickly, in the Academy, into history of philosophy
—interpretation of the authoritative thought of the dead Master.\(^{26}\)

For the historian, it is exact doxography that matters; so what follows
is an outline of *doxai* on principles attributed to Plato. The highest
principle of Platonic ontology is the One; alongside it\(^{27}\) stands the
Indefinite Dyad, a principle that is also described as great-and-small,
many-and-few, exceeding-and-exceeded, and unequal.\(^{28}\) It is responsi-
ble for every kind of multiplicity, contrast and change in the realm

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\(^{22}\) Cherniss, *Riddle 13*; Krämer 23ff, 401, 457ff; Gaiser 4f. According to Pl. *Leg*. 968a, the most important mathematica are not secret, (*ἀπόρρητα*), but cannot be told in advance (*ἀπόρρητα*).

\(^{23}\) The distinction *τί-ποίος* appears as early as *Gorg*. 444c, *Men*. 71b. To modern logic, the *τί* question seems to be almost void of sense (R. Robinson, *Plato’s Earlier Dialectic* [Oxford, 1953] 49–53) which shows that it is not modern logic for which “the soul is longing.” According to *Tim*. 49d (cf. *Arist*. *Met*. 1033b21), the object of the senses, as contrasted to the idea, is only *a poioi τί*, never a *τί*, but this statement is not identical (against Wannamowitz, *Plato II* 294; Krämer 304, 459 n. 155) with the doctrine of the *seventh Letter*, where even *ἐπιστήμη* cannot grasp the *τί*.

\(^{24}\) *αυτόνα ονείρωσι* MSS, cm. E. Sachs (Wannamowitz, *Plato II* 295 n. 2): *αυτερίως*.

\(^{25}\) Krämer (412) thinks that the seventh Letter would allow *ἐπιστήμη* in the sense of the *Phaedrus* (276d, 278a), but 344e expressly rejects *ἐπιστήμη*. Ross, *PTI* 158, on the other hand, thinks that Plato forbids prose writings, and that the semioptic dialogues are left untouched; but 344e seems to allude to the main works of Plato.

\(^{26}\) Aristotle, of course, is the exception; he combined criticism with his notes. Fr. 27: “one ought to remember that we are but men [i.e. subject to failure] not only in the pursuit of happiness, but also in carrying out a demonstration.”

\(^{27}\) There is no word about any derivation of the “Indefinite Dyad” from the One; the later, so-called Pythagorean tradition presents monistic as well as decidedly dualistic interpretations, cf. below, ch. 1 3.

\(^{28}\) Arist. fr. 28 = *Alex*. *Met*. 56.8ff, *Simipl. Phys*. 453.33ff; *γιὰ τοῦ ἀνῖνον δαίμον*, *Met*. 1082b17; *μή δέ*, *Phys*. 192a27. There was some doubt whether the term *δόμασα* *δαίμον* could be traced to Plato himself (Ross, *PTI* 264; *Met*. 169), until the new fragment of Speu-
sippus (*Plato Latinus* III 38; below, ch. 1 3) was published. Since Speusippus himself called the second principle *δαίμονα*, it is from older tradition that he took the term “inter-
mensurabilis dualismus,” i.e. from Plato. Aristotle expressly refers to the “unwritten doc-
tines” for the concept of *μεταλπτικόν* (*μέγαν και μικρόν*), *Phys*. 200b16. *Epin.* 1900–901b, a passage intentionally obscure, alludes to the role of the Dyad (on this passage, A. R. Lacy, *Phronesis 1* [1955–1956] 81–104). Ross (*PTI*) tries to render the concept by “bare plurality” (which, strictly speaking, is rather *Speusippus*’ *πάθιον*); Becker (ZwL 18) by “logical extension,” though Plato wants us to think of this “extension” as a duality, a deviation in two directions from the center, the measure. The One, Krämer has shown that, this pattern of thought is discernible in ethical discussions of the late dialogues (140ff, 444ff).
of Being, as against the unity, identity, constancy brought about by the One. The One is identical with the Good; the Indefinite Dyad is the ground of all evil. It is also called Not Being (Arist. Phys. 192a7). In Aristotle’s terminology, the two principles are also related as form and matter.

Both principles “beget” numbers from themselves. First 2 comes to be, as the “indefinite dyad” is limited by the One and transformed to the definite number 2 which consists of two equal units. According to Aristotle, the other numbers arise in the natural succession 3, 4, 5, etc., but Aristotle emphasizes the difficulty—nay, impossibility—of deriving in this way other numbers than those of the type 2. The numbers that arise in this way are independent entities, which cannot be combined in arithmetical calculations; their units are ὁδὸς ἱμάλητος, as Aristotle puts it.

The relation of these numbers to the mathematical numbers which are used in calculation is hard to establish. Speusippus and Xenocrates put forth different solutions for this question: the former eliminated ideas and only recognized mathematical numbers as the ground of reality; Xenocrates equated ideal and mathematical numbers. The main concern of these philosophers, however, is not to lay the foundations of mathematics, but to explain the world by means of its principles. The ideal numbers are not only the ideas of particular numbers—“twoness,” “threeness,” etc., but somehow govern the structure of reality: they are ideas themselves. It is not clear how this connection

of ideas and numbers is to be understood, in detail. While Aristotle says simply that the ideas are numbers, Theophrastus speaks of an attaching (anapetem) of the ideas to certain numbers, and thus allows us to imagine a looser relationship. The attempt of Stenzel and Becker to understand the ideas as “numbers” on the basis of the method of division (diaréze) prominent in Plato’s late dialogues is not convincing.

The most familiar example, the first step in the derivation of the world from numbers, often cited by Aristotle, is the sequence of geometrical dimensions: the line corresponds to the number 2, the plane surface to 3 (the triangle being the simplest conceivable plane), the solid (tetrahedron) to 4. It is recognized that Speusippus and Xenocrates worked out this derivation. Speusippus has the point at the beginning as corresponding to the One. Aristotle also quite clearly attributes to Plato the derivation of line, plane, and solid “after the numbers”; here the “unlimited dyad,” the “great-and-small,” appears as “short and long,” “broad and narrow,” “deep and shallow.” One text derived from the lecture On the Good tells us that the numbers 2, 3, 4 correspond to this derivation, but the testimony is unreliable insofar as, like Speusippus, it has the point corresponding to the One. That Aristotle attributed to Plato himself the

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29 This is why the lecture was entitled On the Good; cf. Aristoxenus (above, n. 10), Arist. Met. 98b14, who, according to his own philosophy, says Plato “assigned” the Good to the One, as if this were a second step, whereas for Plato there was basic identity.
30 γεννάω is used as a technical term, notwithstanding its mythological connotations. The ontological priority is established according to the principle ἀναπετέμενον τὸν πέλαν ἀναπαραστηθάνει: of two concepts, the one which can be thought without presupposing the other is “prior”; thus number antedates geometrical magnitudes, the line antedates the plane, etc. (101a14; Alex. Met. 55.22).
31 101a23, 101b23, 101b24, fr. 28 = Alex. Met. 56.8-35.
32 101b24, 101b21, 101b20, 100a33, Ross, PTI 191.
33 101a10. The Indefinite Dyad “produces duality” 101a15; on the generation of the number Four, 101a21, 102a13. At 98b33 Aristotle says numbers can be “easily” generated out of Indefinite Dyad, ἐξοτικῶς πρῶτον, a much debated expression (Wilpert, ZfA 207; Ross, PTI 188; Becker, ZfA 8), since the meaning “prime number” does not fit 101a10. The One is said to be the cause of the Odd (101a40), “falling into” the even number (101a44), constituting the middle of even numbers (101b28). Thus it would seem that the series of integers is produced by doubling and by adding one (cf. Alex. Met. 57.24), and still Aristotle states there can be no addition in the sphere of ideal numbers (101b24, 35, 100a33; Ross, PTI 192ff, cf. Wilpert, ZfA 214ff).
34 100a23, 100b35.
35 The evidence is collected by Ross, PTI 151.
36 Ross, PTI 216 n. 1, has collected the passages of Aristotle; Theophr. Met. 661, who is followed by Ross 218: Plato “assigned numbers to ideas,” without identification; other attempts at interpretation: De Vogel, Mnemosyne 1949, 311; Wilpert, ZfA 170.
37 After Stenzel’s vague suggestions, Becker tried to give a precise solution in QSt 1 (1931) 454-501, defended ZfA; criticism in Cherniss, Riddle 54ff; Ross PTI 191ff. The main argument against Becker is that the “ideal number” would change with every new definition of an idea, and the same number would be attributed to quite different ideas. It is true that Alex. Met. 57.6 says that διαρεῖας γίνεσθαι ἀδύνατον: but the number 2 is not generated by diaréze (above, n. 33), nor is the sequence line-plane-solid a diaréze.
38 Fr. 39: Th. ar. 84.10 πρώτη μὴ γὰρ ἄρχη εἰς μέγεθος στηρίγμα, δεύτερα γραμμή, τρίτη γραμμάτων, τέταρτον στερεόν.
39 Fr. 39: cf. fr. 34 = 102a24.
40 101b32: ἔτερον δὲ ἐν τῇ στιγμῇ (γεγονός τά μεγεθύν) ἢ τῇ στιγμῇ αὑτοῦ δοκεῖ ἵνα οὐδὲ ἐν ἀλλ' ὄνομα τῷ ἐν, cf. Top. 1081b7, 2b6; Cherniss, Plato 131 n. 82.
41 This is clear from the passages where questions are raised about the relation of the ideal magnitudes so produced to the “intermediate” realm of mathematical magnitudes (902b13ff, 108b23ff, 108d27ff). Since neither Speusippus nor Xenocrates accepted this intermediate realm, the reference must be to Plato (Ross, PTI 206ff).
42 902b10ff, 108a27ff, 108b24ff, 108b21ff, Ἐπικολ., p. 75.20ff Ross.
43 sect. Emp. Math. 10.27ff; Alex. Met. 55.18ff and Simpil. Phys. 454.19ff do not mention this relation to the numbers.
44 See above, n. 17. At 101a47, 101b17, 108a27ff, Aristotle distinguishes between those who begin with the “great-and-small” and “others” (Speusippus: above, n. 38) who begin with the point.
derivation of the line from 2, the plane from 3, and the solid from 4, seems probable on the basis of two passages, though both are very controversial. The well-known reduction of the physical world in the *Timeus*, adequately evaluated by modern natural scientists, seems to find here its natural sequel. If the elements are traced back to polyhedra, and polyhedra, the simplest of which is the tetrahedron, to triangular planes, and the question of their “origins,” the “more ultimate principles” (ἄρχαι ἀρχαι) is explicitly excluded (53d), the reduction in the present passage leads further, to the line and finally to the ultimate principles, the One and the Indefinite Dyad.

Aristotle further explicates the doctrine of ideal numbers in connection with psychological theory: Plato is said to have formed the soul out of “elements,” in the *Timeus*, in pursuance of the thought that like is known by like;47

47 (a) At 190b20ff, after his criticism of Speusippus, Aristotle turns his attention to those “who posit ideas” (οἱ τὰς ἱδέως ἀφημένους): ποιοῦν γὰρ τὰ μεγέθη ἐκ τῆς ἱδέας καὶ ἀρμόδιον ἐκ μὲν τῆς διάδοσις τὰ μήκη, ἐκ πρῶτοι δὲ ἐκ τῶν ἐπίσημων, ἐκ δὲ τῆς τετραγώνης τὰ τετράγωνα, ... διὰ τοῦτο γαὶ πάντων ἱδεῶν ἡμεῖς καὶ τῷ συμβαλλόντος τούτῳ συνήθει, οὕτω γαρ, ὅστε τὰ μαθηματικά (cf. 113) οὐδὲ τά συνήθει αὐθάλονται, ἀλλά μὴν οὐδὲ ὑπάρχει καὶ αὐτῶν οὐδὲν θεωρήματι, ἐκ μὲν τῆς βούλησις κινεῖ τὰ μαθηματικά καὶ ποιεῖ τίς ἱδία τῶς δόξας ... οὕτω μὲν οὐδὲν βασιλεύει τοιαύτῃ τῆς ἱδεῶς τὰ μαθηματικά διαμετράνωσον οἱ δὲ πρῶτοι ... (and here follows criticism of the connection of number and mathematical number in Plato). This passage has most often been interpreted as applying to Xenocrates (fr. 38 Heinez; Ross, *M., 2.448;* Chenciss, *Plato, 595a, Gnomon 1959, 45ff; *De Vogel, Miemosyne 1949, 303). On the other hand, Ross (*Plato 208f*) and Saffrey (25ff) hold that Xenocrates is referred to individually only from ἐκ μὲν τε αὐτῶν, and that in the earlier part both Xenocrates and Plato were meant. Ross’s argument that in 524f the “mathematical” is the “intermediate realm” of Plato and not of Xenocrates, will not hold up against Chenciss (Gnomon 1959, 47), who has recognized here an allusion to the expanded discussion of number by Saffrey. Aristotle argues that the question whether line, surface, and solid are ideas or a separate class must be asked with reference to Plato, because Xenocrates equated ideas and mathematics. Aristotle poses the same question at 992b13ff (cf. above, n. 41), with an unambiguous reference to Plato. The clause οὐδέν᾿ ὑπάρχην γαρ καὶ αὐτῶν οὐδὲν θεωρήματος cannot be directed against Xenocrates, who was precisely the one who wanted to force ideas and mathematical together (προσαγωγικά), thus. After all the interpretation of Ross and Saffrey is preferable.

(b) In 193β2ff, the question is discussed: What belongs to the Form (εἶδος)? Do flesh and bones belong to the Form of Man? Does the line, or the continuous, belong to the Form of triangle or circle? Some deny this, καὶ ἄνωτερον πάντα εἰς τούτου ἀρμόδιον, καὶ γραμμή τέους λόγου τῶν δύο ἐστιν φασιν καὶ τῶν ἱδεῶς λέγοντων ἃ ἂν αὐτογραμμάτων τοῦ διδάσκων, ὅ ἂν τοῦ εἶδος τῆς γραμμῆς, ἐκάκα μὲν εἶναι εἶναι τέους εἶδος καὶ ὁ τὸ εἶδος (ὁ λόγος διδάσκων), ἐπὶ γραμμήν δὲ ἀνακόπη. . . . Once more he deals first with Speusippus, who did not accept ideas, and then distinguishes two directions in the theory of Ideas, and the probability is that they are those of Plato and Xenocrates. Chenciss (*Plato, 597f, Gnomon 1959, 44*) takes αὐτογραμμάτων, because it lacks an article, as a predicate noun with both διδάσκων and τοῦ εἶδος τῆς γραμμῆς, appealing to “the elementary rules of Greek grammar.” Thus do not apply, however, in technical philosophical language (cf., just previously, 113: γραμμής τοῦ λόγου τῶν δύο ἔστιν). The “name” is first, he thinks, Xenocrates designated the dyad, while Plato designated the “idea of line” as “line in itself”; thus the doctrine of ideal numbers and the ordinary theory of ideas stand side by side. In this case, however, the explanation beginning μὲν γὰρ must be understood as applying to the first instead of the second solution (Gnomon 1959, 44 n. 3), and the phrase beginning οἱ δὲ is not only a tautology but destroys the continuity. In 1043b3ff, where the same problem is treated, there is a consistent connection of dyad and line. Therefore the translation must be, “Of those who accept the ideas, some call the dyad the line itself, others the form of line” (this is consistent with Ross, *Plato 203* and van der Wielen 144ff). For, to paraphrase the justification, with the dyad, for example, the form is nothing else than the dyad itself, whereas with the line—the ideal—must distinguish between its form—the dyad—and a second constituent (the short-and-long). The equation of line and dyad belongs to Xenocrates, and we may attribute the other version of the theory of ideal numbers to Plato. To be sure, the argumentation of the μὲν γὰρ sentence seems to be directed against Xenocrates, and would thus have to go back to a second Platonist, who may have defended Plato’s proposal. There were too many partners to the discussion, too many varieties of the doctrine, for our reconstruction to attain absolute certainty.


47 Tim. 35af; cf. ch. V, 1, below.


49 So Chenciss, *Plato 565f*, Riddle 14f; Theiler, *Arist. 94*. Kukkhari sought to prove that Pythagorean doctrine was in the background here, relying on the late evidence of Aetius and Theo Smyrnaeus (below, ch. 13, n. 109) and on the alleged “caractere artificel et natt” of the doctrine (Arch. 43)—though it does not seem to have been too primitive for Xenocrates. First he dismisses of the theory of Ideas (Tfr. 36f, 47f), though it is unmistakably presocratic (οἱ μὲν γὰρ ἀγαθὸν τὰ ἔστιν ... ἀλήθειαν, then later (Arch. 33f, 39f) seems willing to consider whether this “point neutralique” may after all show Platonist influence. But according to Aristotle’s explicit testimony the theory of ideas is Platonist and not Pythagorean. Cf. also Saffrey 88f.


51 In 1992a10ff, Aristotle presents the derivations of line, plane, and solid in the first person. (On such expressions in the early books of the *Metaphysics* see Jaeger, *Arist. 171, 188*, Eng. ed.) In addition, Alexander refers for this (Met. 117, 24) to the *Pepi ἀρχαῖοι*.
(ετι δὲ καὶ ἀλλως) adds still another version of the doctrine, without citation of the source. The identification of the One with mind (νοος) is attested for Xenocrates, as well as the series knowledge-opinion-sensation, and he knows the derivation-series line-plane-solid (above, n. 39). But then Xenocrates’ definition of soul is introduced as something new, a conclusion of “some”; therefore what precedes can not refer to Xenocrates, and it is a tempting conclusion that what follows the citation of the Timaeus and its interpretation in the work On Philosophy is a reference to “unwritten doctrines”.

The close connection of all this with Plato is shown by a remarkable passage in the Laws (804a): The origin of each thing takes place ... when a first principle, taking on increment [the line], passes into its second transformation [the plane] and from this to its neighbor [the solid], and having made three transformations makes perception possible to those who perceive it.

Plato is dealing with the relation of the soul to the physical world, the priority of soul over matter. Thus there are present in Plato both the series line-plane-solid and the equation of the last stage with perception, though to be sure there is nothing about the application of numbers.

Finally, we must take account of Theophrastus’ statement that “most” of those who posit the One and the Indefinite Dyad as first principles only carry the derivation to a certain point:

for having generated numbers and planes and solids, they practically omit the rest, except for a brief mention, just enough to make clear that some are from the Indefinite Dyad ... and some from numbers and the One, like soul ...

Speusippus treats the matter similarly, he says, but Xenocrates differently (Met. 625bf). Again we have an ontology, separated from Speusippus and Xenocrates, in which the series number-line-plane-solid is present alongside the connection of numbers and soul. Thus for the pioneering exposition of the theory of ideal numbers, with its implications for the physical world and the soul, we find ourselves led back again and again to Plato, and apparently to the lecture On the Good.

The commentators on Aristotle, from Alexander of Aphrodisias on, are unanimous that the doctrines developed in On the Good were Pythagorean, and Aristotle also says that in his theory of the first principles Plato “mostly” followed the Pythagoreans, though he did have “something of his own” to add. The seventh Letter, in the passage on these doctrines (338c), makes reference to Archytas: from his circle, it is suggested, Dionysius might have had knowledge of doctrine which Plato, himself, had not imparted to him. But, even if this relationship may be regarded as certain, still the question becomes all the more urgent what it may have been that suggested this course to Plato. That he did not take over someone else’s system unchanged may be assumed from the start, and not only because of its psychological improbability. When Plato went to Magna Graecia for the first time, at the age of nearly forty, his intellectual attitudes must have been fairly well established. The main background for Plato’s ontology is clearly Eleatic dialectics: it is not by chance that Parmenides and the “Eleatic master” play a leading part in the late dialogues; the One and the Good had been equated already by Euclides of Megara, a doctrine

15 Fragments 15 and 5. Cf. Heinze 2ff, Chernis, Plato 570f. The series νοος-ἐπιστήμη-δόξα-ἀισθήσεων is familiar to Aristotle himself (De an. 428af, Met. 1074b33).

59 Frs. 60-65 Heinze.

80 Set forth by De Vogel, Mnemosyne 49, 304. Chernis, Plato 571f, emphasizes the ὁμοιός, which he refers to γνωριστικόν, as a connection with what precedes, and he does not cite the “some” in the middle. But the word γνωριστικόν introduces a new topic; and, besides, ὁμοιός is to be referred to the main clause (Theiler, Aris. 94). Theiler (ibid.) acknowledges that “Aristotle would have had to express himself differently, if he had already cited Xenocrates”; but then he separates off the last clause as an addendum, and still, because of topob20ff, attributes what precedes to Xenocrates. Yet he recognizes that there is a difference from the definition of soul, which was “perhaps only formulated later.” So we have both the early and the late Xenocrates—too complicated a solution.

52 Cf. Phys. 209b1ff.

56 The connection was already noted in England’s commentary. Cf. Cornford, PLatt 108; Kucharski Tht. 71ff. Also Pl. Leg. 819c7: ὁμοιός ... ὁμοιός, δόξα, ἀισθήσεως.

53 The series ἐπιστήμη-δόξα-ἀισθήσεως is also found at Parm. 153f. Cl. 124a, 164b, Tim. 28a, 37bc, 52a.
Plato is echoing in the *Protagoras*. Above all, Aristotle clearly distinguishes between Platonism and Pythagoreanism; starting from his statements, it is possible to judge the originality of Plato, as well as to find out what Pythagorean philosophy was like before Plato.

It is true that Aristotle’s reports on the history of philosophy harbor the numerous and serious sources of error which Charniss, in particular, indefatigably exposes. But in default of other sources, there is nothing else to do than—with due caution—to follow Aristotle’s hints. Perhaps we may hope that those distortions, and the faulty perspective, which is present to an equal degree in the reports on Platonists and on Pythagoreans, will prove negligible in assessing the differences between the two.

2. The Philosophy of the Pythagoreans According to Aristotle

Aristotle does not present his reports on the Pythagoreans as an impartial historian of philosophy, but always in the context of his own exposition, which has purposes of its own. The thing that lends the confrontation with the Pythagoreans its special immediacy for Aristotle is their connection with the Academic teaching on first principles, which he criticizes vigorously while at the same time making it his own point of departure. And, in the organization of his inquiries about first principles (αρχαι) in the first two books of the *Metaphysics*, the Pythagoreans are not only treated separately, among the “pre-Socratics” (983b23ff.), but are even more often treated in comparison with Plato. Their doctrines are set forth in detail in the discussion of the difficulties in the Academic number theory (in books M and N). The latter is the real target of Aristotle’s polemic; Pythagorean material may be cited for its own sake, in order to achieve completeness, or it may be used to win a point from the Platonists. There are further detailed reports in the *Physics*, in the discussion of the concept of *άρμονιον* (202b30ff.). Isolated reports on items of scientific theory are also found in other works; the famous cosmic system, with its moving earth and harmony of the spheres, is discussed in the book *On the Heavens*.

In addition, Aristotle devoted two special books to the exposition and criticism of Pythagorean doctrines, and also wrote on Pythagoras. He himself alludes once (Met. 986a12) to his “more exact” discussions. Plutarch, Alexander of Aphrodisias, Aelian, and especially Iamblichus have preserved important material from these books, which supplements the reports of the didactic treatises.

It has been emphasized repeatedly that Aristotle, in his extant works, consistently speaks of *Πυθαγόρεια*, not of *Pythagoras*, and this seems...
I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

also to have been the case in the lost works, as far as philosophical
doctrines are concerned. Often he adds the word καλοικότερον, and the
extent to which this expression, “the so-called Pythagoreans,” shows
some kind of reservation has been vigorously debated. There lies in
the Greek expression about the same kind of nuance that is expressed
nowadays by quotation marks. But the studious avoidance of
Pythagoras’ name is intentional.

The doctrine of the Pythagoreans usually appears as an undifferen-
tiated unity; or in any case there is no sure foothold for the modern
ttempts to discern various stages in its development. To be sure,
“others of this same group” (ἔτεροι τῶν αὐτῶν τούτων) are named as
originators of the “table of opposites,”16 differences on specific points
are mentioned twice,17 and several times there is a restrictive “some”
(τινές).18 Yet there is never a distinction between “early” and “late”
Pythagoreans.

For the comparison of Plato and the Pythagoreans we may begin
with the first book of the Metaphysics.19 Plato’s doctrine is at first con-
ected closely with the Pythagoreans: “in most respects following
them, possessing also some features of its own which set it apart

7 Citation of the title as Προ τῆς Πυθαγόρας φιλοσοφίας, Αγρ. 1.18.6, proves nothing
in spite of Guthrie I 200): the later tradition often has “Pythagoras” instead of “Pytha-
goreans”; cf. D.L. 3.8, with Arist. Met. 987a32ff., Olympiod. In Meteor. 50.9ff., on
342b30, and n. 5 above.
8 Frank uses the adjective οὐσιατικόν in the title of his book, as does Bollinger (and see
410b27, with Philoponus’ comment ἡ ἀρχή τοῦ ἐν τῷ Ὀρθωντὶ ἐν τῷ ἐν καὶ Contra,
Cherniss (Pres. 384ff.) calls attention to ὡ τοὺς καλοίκων γενώσσις (Pol. 129b40), and speaks of
“designations in the currently recognized sense” (Gnomon 1959, 37ff.); cf. ZM 354. Von
Fritz (AnImath 1945, 249.38) suggests that there may be a certain reserve intended, be-
cause it was unusual for a philosophical school to be named in this way after its founder;
but cf. Ἀνάλογος, Pl. Crat. 400b, Dialect. 6.8 (DK II 444.13), Ἡμιλείας, Pl. Thit. 179c,
D.L. 9.6. Zeller’s observation is important, that there are names of political factions in
ξένοι (II 446. n. 1: Minar 21). E.g., Διονύσιος, Pl. Ep. 7.334c; Διογένης, Hell. Oxy. 10;
Since the formations in -ειον are regular only for s-stems (βίον ἴδιον, but न्यूदा-
εψरास), the form Πυθαγόρας must depend on earlier examples, but probably in the
realm of the εὐκρίνοι.

Cornford himself admits that it is difficult to divide Aristotle’s reports between the
two “radically opposed” schools he reconstructs.

986a22. Cf. the end of this chapter.

10 Met. 345a14ff. (Milky Way), De an. 424a16ff. (notes in the air and souls). The theory
of comets in Metr. 342b30ff. and the system of the 10 heavenly bodies is mutually
exclusive (below ch. IV 1).

11 τοῖς is even used in relation to the theory of numbers in CaL 90017 (glossed over by
Zeller I 450ff., 451 n. 2), and in relation to particular questions CaL 299b21, Metr. 345b30,
Sens. 444a16. In Aristotle, τοῖς can also refer to a single author, as Pol. 129b14 (Hdt. 3.20),
Gen. an. 750a5 (Hdt. 2.95).

12 On the problems of the analysis of 987a22ff., in connection with 987b11ff., see
Cherniss, Plato 177 n. 100, and 191.

2. The Pythagoreans according to Aristotle

from the philosophy of the Italians” (987a30f). To be sure—and we
must be mindful of this qualification—this relates exclusively to the
question of what principles (ἀρχαί) the early thinkers proposed, and
how they are related to the Aristotelian classification. Plato and the
Pythagoreans both accepted numbers as the principles (987b24)—
number not as the number of other assumed objects, but as an inde-
pendent entity, ὁδός. In this sense Pythagoreans and Plato are regularly
mentioned in conjunction, and are set over against all other thinkers
caller Aristotle,20 and in this conjunction Aristotle regularly
attacks them. At the same time, though, Aristotle emphasizes the
difference that, while Plato separates the numbers, as ideas, from the
sensible world and even sets between them the mathematical realm as
a realm of its own, for the Pythagoreans things “are” numbers, they
“consist of” numbers.21 Aristotle is puzzled at this: the Pythagoreans
introduced principles, he says, that would have been quite suited to
lead beyond the merely perceptible into the higher realms of Being,
but then they never speak of anything but what is perceptible, and
“squerder” (παναλοίκουσει) their principles on this world of ours,
as though there did not exist anything but what is perceptible, what
the sky encloses (980b29ff.). What differentiates Plato from the Pyth-
agoreans is “separation” (χωρισμός), the “introduction of the Forms”
(987b31), which Aristotle traces back to dialectic developed by
Socrates.22

If the numbers are identical with things, they are space- and
timebound. Aristotle actually speaks of their origin, in a cosmogenic
sense,23 and says that they are extended, that their units possess mag-
nitude.24 It is naturally against this thesis that the principal logical
and physical objects are directed,25 and Zeller wanted to ex-
clude the report about “extended units” completely, as Aristotelian

13 τοῖς μέσωσε γε ὡν οὐσίαν εἶναι, καὶ μὴ ἄριστο τι ὁ λόγος ἂν ἐργεῖται εἰ... 987b22 (cf.996a6ff.,
993b19ff., 1053b1ff., Phys. 1024a (also on the ἐπόμενον).

14 987b28 (cf. τῶν ἀριθμῶν τὰ ὀντα, 1083b17); 1090a22; ἀριθμός τὸν ἰδίον αὐτὸς, 980a21 (cf. 986b3); τὰ εἰκόνα ἐξ ἀριθμῶν, 1083b21ff. (cf. CaL 3001a1ff., 986b6; 990021;
1082b, 16.18. ἐξ ἀριθμῶν ὡν ἄντα, 1090a23, 32. But we also find ἐξ ἔκτισιν (ἐλ θεμάτων)
ἀς ἀριθμῶν, 1083b18; τοὺς ἀριθμοὺς ἐν τοῖς ἀσθενῶς, 1080b. The lack of
χωρισμός, 1080b16, 1083b10, 1090a23, Phys. 303a20ff; on this point the Pythagore-
ans are to be praised (above, n. 4)—Aristotle himself argues against χωρισμός!
174 212.
16 below, n. 35.
17 τὰς μέσωσεν ἐκεῖν μέγας, 1080b19; cf. b33, 1083b15, 990012.
18 Logical objections: mathematical number has no magnitude, 1080b1ff., CaL 3001b1ff.,
1080b41ff.; physical: the force of gravity (990014f, 1090a33ff., CaL 3001a19) and the pheno-
momenon of movement (99008b) both remain unexplained.
masculine, the "unlimited" as feminine, and correspondingly the odd number is also masculine, and the even feminine. Aristotle gives a complicated explanation of the correspondence of odd limit, even and unlimited, at the basis of which lies the representation of numbers by arrangements of pebbles. Since Burnet the significance of this


28 This is why the number 5 (= 2 + 3) is "marriage" (γάμους). Aristotle alludes to this at Met. 1071b23 and fr. 203—Alex. Met. 39.8ff. Also Plut. *De E 388a-c*, *Quaest. Rom.* 288c. Cf. below, n. 31.


odd

even

Met. 1.148f; *Ross, Phys.* 542ff; *Raven, PyE* 130f, 188ff; *KR* 243ff; *Becker, Grdl.* 34ff; *Guthrie I*, 242ff. Aristotle's expression is not clear: καὶ χωρίς is an old *dus in* interpretation; but the inclusion of the opposition square-rectangle in the table of opposites favors the usual interpretation; with the odd numbers results, with the even rectangles. For one, more complicated explanations, see *Taylor, CR* 40 [1926] 140ff. and J. Timpanaro Cardini, *Phys.* 3 (1961) 105ff.— *Simpl. Phys.* 455.20ff.— (DK 58B38); cf. *Philop. Phys.* 189.11, 391.25. *Themist. Phys.* 80.9f attributes a different interpretation to certain *εἴδη*: the even number can be halved, but halving continues to infinity κατὰ τὴν δύομονα. Precisely because of the apparent irrationality of this, some have sought to find here an ancient doctrine, like W. A. Heidel, *AGP* 1901, 395ff (followed by *Burnet, IV* 288f; *Ross, Met.* 1.149), and with variations *Raven in PyE* 193 and *KR* 244f. What the exegetes mean, however, can be deduced from Porphry ap. *Simpl. Phys.* 453.15ff: the number 2 is the principle of division which proceeds to infinity, and only to this extent it is true that there is a connection between even numbers and the principle of infinity. These observations derive from the thought of the Indefinite Dyad, and are therefore not early Pythagorean.— *Raven, PyEl* 130f, and *Kucharski ("Les principes des Pythagoriciens et la dyade de Platon,"
*Archives de philos.* 22 [1959] 175–191, 385–391) have stated, not incorrectly, that in the gnomon procedure One and Dyad are given equal rank as principles of the odd and even numbers respectively; but further this is, that the Dyad in this function could have been called Indefinite, and that therefore all the later tradition was right in designating the concept of the Indefinite Dyad as Pythagorean, cannot rest on the formulations cited from *Nicomachus*, *Theo*, and *Iamblichus*. All the detailed accounts ascribe to Pythagoras the unequivocally Platonic concept of the Indefinite Dyad, καθ' ἐπιρρήματι καὶ πληρω (see below, ch. 1), but Aristotle speaks in this passage of *ἀπειρομένου, not of ἀδιάμετρον, and nowhere else makes any mention of an important role played by "two" in Pythagoreanism. When *Philo* says (165), "the illustration is Aristotle's," he is forgetting that it is reported in indirect discourse.

28 Zeller I 486. On "number atomism" and Ephemer, see below, nn. 66, 74.

29 So also Raven in *KR* 247 n. 1. This makes it impossible that the word διάμετρος was coined by the Pythagoreans, as H. Gomperz tried to show (*Hermes* 67 [1932] 155–167). (On Philolaus B22, see above, ch. III 2, n. 45.) We may speak of the "innameness of number," but it is impossible to equate Pythagorean doctrine with the conception of Aristotle, whose theory of abstraction was possible even after the development of the theory of ideas.

30 Syrian. Met. 80.20ff., cf. 83.12ff.


32 τοῦ δὲ διαμέτρου τοῦ τοῦ ἔρημος καὶ τοῦ περίτου, 986b18. At 1004b31 the two odd-even and limit-unlimited are separated, but this may be merely a slip. The point is enumeration of oppositions of any kind. (Chrennius, *Pres.* 47, 186–188, assumes that for the second pair Platonists are included, but there is no evidence for this.) Limit-unlimited comes first at 990b28 and 986b23 (the table of opposites); only limit-unlimited at 1091a17 (cf. *EN* 1106b18, fr. 47): only odd-even at *Phys.* 203a10ff, the explicit equation of even and unlimited at *Phys.* 203a10ff. On 987a15 cf. below, n. 38.

33 The basic importance of this opposition of limit and unlimited was set forth by Heidel (*AGP* 1901), in opposition to Zeller. He leaves the question open whether the opposition of odd and even was originally equal or subordinate to it (1901); but if one considers that it must have been from a Platonic perspective that Aristotle brought the
allusion to the graphic basis of Pythagorean number speculation has been recognized; it is not the cipher or numeral (like “5”) which serves as pictorial representation of a number, but the shape of an area—the sort of thing we are familiar with from dice or dominoes. Léon Brunschvicg introduced the apt allusion to the constellations, where, too, in each case a limited number of points of light defines an object: Orion, the Bear, the Lion, etc. One need not believe, however, that any one pebble arrangement was the sole point of departure for the basic identification of limit and odd. Even numbers can of course be halved, until one comes to odd numbers, and then the halving process is “at its limit.” More general considerations also enter in: like the positive evaluation of limit, the habit of prizes odd numbers has for long been deeply rooted in popular feeling. Plutarch, what is more, in a passage where at least for part of the way he is following Aristotle, gives a perspicacious explanation, also from the viewpoint of the pebble figure, of the masculine character of the odd number and the feminine of the even. The even number, he says, has at its midst an empty space, capable of reception, whereas the odd number has a middle member with procreative power. This direct symbolism must be regarded as old, and not only because of its attestation; it has a connection, at least subliminally, with the general Greek association of masculinity with the word ἔρως. From the two primal principles rises the One; from it, number. The similarity of this to the Platonic system, to the derivation of

number from the One and the Indefinite Dyad, is obvious. Aristotle, too, sets Unlimited alongside Indefinite Dyad, but at the same time emphasizes what is “peculiar” (ἴδιον) in Plato’s formulation: “the fact that in place of the Unlimited, treated as singular, he posits a dyad, and derives the Unlimited from the great-and-small—this is peculiar to him.” (987b25). The term Indefinite Dyad is thus by no means Pythagorean in origin; but the difference goes still deeper. The Unlimited, unitary and undifferentiated, was according to Pythagorean conceptions “outside the heaven” and penetrated the world by being “breathed in” by the heaven to separate natural things (φύσεις) from one another, being “enclosed and partitioned off” (ἐναπολαμβανόμενον) in the limited. Obviously the concept of empty space, as well as that of air, is present here. In any case the Unlimited of the Pythagoreans is principle and constituent of the cosmos at the same time, whereas the Unlimited Dyad of the Platonists is conceived not cosmetrically but ontologically, even logically. It is not in any place, but is a transcendental principle, in which everything in the realm of experience takes part, and even, in a different way, in the realm of Ideas; it belongs to the theory of ideas, just as the cosmic Unlimited of the Pythagoreans fits consistently into a world without a conception of incorporeal being.

Insofar as ἀπειρὸν and ἀδρόμος δύνα are comparable, the One falls naturally into the company of πέρας, and Aristotle occasionally speaks

34 Similarly, Phys. 203a15.
35 Phys. 203a6: "ei te t' evo tov oúfran to ἀπειρον... to ἀπειρον eivo to t' evo tov oúfran t' av tov περιττον... to περιττον evo tov oúfran t' av tov περιττον... περιττον... περιττον..." ἀπειρον... to περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... αὐτον... περιττον... α guardar oves. —Cherniss (Pros. 17 n. 68, 38) conjectures that Aristotle first thought of the equality of even and unlimited under the influence of the Platonic concept of the Unlimited Dyad. But not only the connections shown here, but also the pebble procedure, which Aristotle surely did not invent, would count against this. Moreover masculine and feminine are also found in Xenocrates (fr. 15), in a mythological form of expression. Cf. also Philolaus fr. 204.

28 EGP 101ff. Zeller (I 48ff) had denied the spatial character of numbers in Pythagoreanism. Cf. above, n. 20.
30 On this, see below, ch. VI 4.
31 Plut. De E 888–C. The first part, the explanation of the ἤρως ἐρμής, corresponds to Arist. fr. 199 and 203 (Alex. Met. 4018ff), Quaest. Rom. 288c; Stob. 1 proem 10; cf. Aristox. fr. 23. The One also has a mid-position in odd numbers according to the Platonic doctrine (Arist. Met. 1083b28).

even odd • • • marriage • • • • • • •

32 I.e., in the sexual sense of ἔρως.—Cherniss (Pros. 17 n. 68, 38) conjectures that Aristotle first thought of the equality of even and unlimited under the influence of the Platonic concept of the Unlimited Dyad. But not only the connections shown here, but also the pebble procedure, which Aristotle surely did not invent, would count against this. Moreover masculine and feminine are also found in Xenocrates (fr. 15), in a mythological form of expression. Cf. also Philolaus fr. 204.

33 B 47: τὸ 0 ἄρθρον στεξίεν τὸ 0 ἄρθρον καὶ τὸ περιττόν, τούτοις δὲ τὸ μὲν περιττόμενον τὸ δὲ ἄρθρον, τὸ δὲ ἐν 0 ἄρθρον περιττόμενον ἐν τούτω (καὶ γὰρ ἄρθρον ἐναντίον καὶ περιττόν), τὸ δὲ ἄρθρον ἐν τῷ ἐναντίῳ.
as though the two were to be identified. Yet precisely because this is so obvious—even absolutely necessary—from the point of view of Platonism, one must believe the definite testimony of Aristotle that in the Pythagorean view the One had its origin from Limit and Unlimited together: τὸ δὲ ἐν ἐξ ἀμφιτέρων ἐκεῖν τοῦτου (986a19). For the Platonist this is no less a vexation than the lack of incorporeality; μᾶλλον δὲ ἀμφιτέρω κατὰ τὸ εὖ ἀνίκερος are the words with which Asclepius impatiently interrupts his paraphrase of this passage of the Metaphysics (38.25). The One has a share in each of the opposite forces; it is “even” and “odd” at the same time, ἀρτιοπερίττων. It is perfectly in keeping that it should be, as late sources say, bisexual, ἄρσενκόρπως. This One comes into being and develops further—it is nothing else than the world before its further evolution. The “first principles,” Limit and Unlimited are, then, what was there before the world came into being. To the Pythagoreans, number philosophy is cosmogony: κοσμοποιοῦσα (1091a18). The further development of number is also cosmogonic:

They say clearly that when the One had been constructed—whether of planes or surface or seed or something they cannot express, then immediately the nearest part of the Unlimited began “to be drawn and limited by the Limit.”

The One becomes a Two as the Unlimited penetrates it. Here is one

38 Aristotle characterizes the One (εἷς) and the Unlimited (ἀμφίτερον) as ὀδηγοῦ (above, n. 14), and to this extent sets them side by side (987a27); he calls the One “element and principle” at 1080b31, and only names the Unlimited as “principle” at 982a26 (differently, 986b5f). At 985a26, if one strikes out κατὰ τὸ ἔν, with MS E, the identification of ἔν and πέρας is perfect, for then the pair τὸ πεπερασμένον καὶ τὸ ἄπεραν at 951 and αὐτὸ τὸ ἀπερασμένον καὶ αὐτὸ τὸ ἔν at 957 correspond; but, since Alex. Met. 47.11 read κατὰ τὸ ἔν (whereas the equivalence of One and πέρας would seem natural to any later, Platonic-minded reader), we must keep the full text. Aristotle is enumerating unsystematically: both the primary pair of opposites, and the One, and number in general, are ὀδηγοῦ for Pythagoreans; in his repetition, Aristotle leaves out the πέρας that goes with the ἔν, following, conjecturally or not, Platonic ways of thinking (cf. Cherniss, Princ. 45 n. 175, 224ff). In Aristotle’s own philosophy, πέρας is ὀδηγοῦ and πεπερασμένον is δῆλον (Calcl. 29bc13); and he can, under the influence of Platonic terminology, even ascribe to Empedocles the doctrine that τὸ ἔν is a στοιχεῖον (Gen. corr. 312a22).—One cannot derive from the table of opposites the equation one = limit = good (as KR 241). This would imply that a rectangle has crooked sides, since “rectangular” is in the same column as “crooked.”

39 Fr. 199 and 203 Rose (the latter, Alex. Met. 40.188f); Plut. De E 388a–e; “for it is both even and odd,” Arist. Met. 986a20. Raven (Kr 317f) concludes from the table of opposites that “one” was originally regarded as odd; against this, cf. n. 38.

40 Nicom. Th. ar. 4.1, 4.17f; Macrob. Somn. Sc. 1.6.7.

41 ἀπεράος γὰρ λέγετον ὡς τὸ ἔν ἀναβαθμένος, ἐκ ἐξ ἐπεραστεῖς ἐκ τῶν ἀπερασμένων εἰς τὸν ἀπερασμένον, καὶ τῶν ἀπερασμένων ἐκ τῶν ἀπερασμένων ἐκ τῶν ἀπερασμένων ἐκ τῶν ἀπερασμένων (1091a18ff). The ὀδηγοῦ cannot be deleted; it shows a citation is to follow, and probably separates paraphrase from quotation. Cf. below ch. III 1, n. 92; Met. 1080b21fr. Phys. 21b22ff, with fr. 201, and above, n. 35; also 986b14: γεν φάνα καὶ τὸ γεν λόγον ἀπερασμένου.

of the most widespread cosmogonic themes, “the separation of Heaven and Earth”:

άως ὀδηγεῖ τα γειά τ’ ἐν μορφή μία.
ἐπεὶ δὲ ἐξελειπθησαν ἀλλήλων δίκη...

The process was modified by the Pythagoreans, with their ideas of the earth in motion and the central fire. But in these very ideas is apparent a complete equivalence of the things separated: the “Hearth” of the universe and its fiery envelope. “Zeus’s castle” and Olympus. This separation has happened in the past: when Aristotle quotes exactly, verbs in past tenses suddenly appear.

The growth of the cosmos is described, in the manner of the pre-Socratics, as the growth of a living being, and embryological concepts form part of the background. The One begins to breathe, and, as the breath flows in, it assumes a more complicated structure. Similarly, the Hippocratic book De natura pueri teaches that the embryo develops as the seed, in the womb, takes in air, and is divided and articulated by it: “Each of these [bodily structures] is articulated by the breath; for as they are aerated by it they separate according to their natural affinities.” In Aristotle’s account the origin of the One from seed is at least mentioned as a possibility. Thus it becomes obvious that the ancient idea of macrocosm and microcosm is at work in Pythagorean teaching. It is not a matter of an ontology featuring the
I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

derivation of number-in-itself; their number theory is cosmogony.50

Here Pythagorean doctrine is quite in line with pre-Socratic traditions. The cosmogony of Leucippus is closely related: he too has two kinds of entity at the “beginning”: the many atoms and the empty space between that makes possible their separation. The first step toward organization in the cosmic whirl is that a “membrane” or “caul” (ὑμήν—here too a term used in embryology) detaches itself from the nucleus, and the cosmos grows as matter flows in from the exterior.53 But beyond the pre-Socratic horizon a mythical one also becomes visible: the separation of Heaven and Earth, and above all the double nature of the primally existent. For what can only be explained artificially, in arithmetical terms, is familiar to the mythologist—the primal being that is bisexual.52

Perhaps a quite specific mythical cosmogony forms the background of the Pythagorean number theory. There are striking similarities of detail in the Orphic cosmogony which in the romance of Pseudo-Clement is given by Apion as an example of pagan theology. The problems of transmission are exceedingly complicated,58 but the basis is unquestionably a hexameter poem ascribed to Orpheus. Allegorical interpretation of Orphic poems, from a philosophical point of view, goes back at least to the fourth century B.C., as the papyrus from Dherweni has proven;54 so it is quite possible that in the tradition of philosophical exegesis ancient material has been preserved. In specific

50 Cherniss (Pres. 39f, 44f, 224f, 387f) tries to show that the Pythagoreans had no doctrine of the origin of number, and that Aristotle only produces this impression by his projection of Platonic ways of thinking. To establish this he must, at Met. 1091a18ff, make a radical separation of the cosmic One (“the universe itself”) and the “numerical unit”: “Aristotle is confusing . . . the cosmogony with the number-theory” (p. 39). But Aristotle says unequivocally that the Pythagoreans knew only one kind of number, the cosmic (9902a1), that is, that they thought of number theory as cosmogony, of cosmogony as the development of arithmetic.

51 Leucippus ΑІ = D.L. 9.32: τὸν θ’ ὧν ἡμένα ἀκέστωσα . . . αὐτὸν τὸ πόλιν τῆς περιτήρεις ὧν ἡμένα αδενεοδικας τῇ τῆς ἐπικορέας (ἐπικορέας MSS) τῶν ἐξεθαν σωμάτων. (On the text and interpretation see J. Kerschshtein, Hermes 87 [1959] 441-448, esp. p. 446.) The agreement is still greater when the question is not of an abstract πρᾶς but of a plurality of περιτήρεις (below, ch. II 3).


Eusebius knew the Apion dialogues (Hist. eccl. 3.38.5).

54 Arch. delt. 19 (1964) 17ff. Interpretation of Orphic material in Cleanthes and Chrysippus, SVP II 906, 917-178.

55 Apion’s source is not to be identified with the cosmogony of “Hieronymus and Hellanicus” (Orph. frag. 54, 57ff), where Kern placed the Apion testimonia. The “Rhapsodic Cosmogony” is closer, if πνεύμα is a synonym for αἰθή (Orph. frag. 70). There, however, Chaos and Aether are “begotten” by Chronos (Orph. frag. 60), whereas, according to Apion, Chaos had always been there (Hom. 6.3.1, 6.4.1, Rec. 10.30.3), and Chronos plays no active role. But Chronos is also missing in the Orphic cosmogony known to Aristotle (Met. 1071b27) and Eudemus (fr. 150), and in the parody of Aristophanes (Av. 395ff). The bisexual Phanes seems to be reflected in the myth of Plato’s Symposium (189d; cf. K. Ziegler, Ntb 1913, 520ff; RE s.v. Orphische Dichtung 1361f). The breathing in of a πνεύμα seems already to have a part in the Egyptian story of the world egg (S. Morenz, in Aus Antike und Orient: Festschr. W. Schubart, 1950, pp. 71ff; cf. also Quellen des alten Orients: Die Schöpfungsmynen, Zürich 1964, p. 90) and in Mochus, l'Hist. 784A; Gen. 1.2.

56 Hom. 6.4.1ff (Orph. frag. 58): ὄπειρον τὸν βοθύνοντα ἀἷμαν . . . σωμάτων ἀθρογοῦν . . . ἀπότομοι θάλασσαι . . . ἀποκινοῦσιν νόμος καὶ μείζον τὰς ὀφθαλμούς . . . ὀποῖα ἡ πολιτεία καὶ τὰ περιχεῖσι πνεύμα ἐπιστρέφεται . . . ἐκ τούτων περιλαβότητος πνεύματος αναφοράς προέκυψεν ἐκ Φαίησις μεγάλιτον τὸ τούτο ἀγαπητό . . . καὶ τῇ περιφερείᾳ τῶν ὁμολογών καὶ τών τάκτων τῆς πτερών . . . 6.5.4 (Orph. frag. 35): ἦν ὁ θαρακτήρ . . . 6.6.2 (Orph. frag. 56): τοῦ καὶ τῆς δυναμικῆς λαμβάνει . . . τῇ παρακήπασθα ἀρχή, αὐτὸς δὲ ὁ σωπερ ἐπὶ ἀκραίωσον σοφίματος προκατεβαίνει. Rec. 10.32.1: ["Chaos] primo omnium tempore multo concreto genuit quandam quasi bullam, quam . . . aliquanto tempore circumcincta per superficiem maternae, ex qua quasi ex vulva processerat, rigore frigoris objurata et glaciibus augmentis semper increasens . . . 10.30.4: "perisse ac proutisse ex sic duplici quandam speciem, quam illi melioriuis . . . et hoc esse principium omnium.

57 Arist. Met. 968A9ff; cf. below, ch. IV 3.
complaining, in his day, that very different things were being equated here. No wonder that historians of philosophy, with so many different points of departure, have arrived at quite disparate results. Some interpret the Pythagorean number theory as a radical materialization of number and find it a kind of atomism, while others see in it a philosophy of mathematical form, an idealism closely akin, if not identical to, the Platonic theory of ideas.

The “number atomism” interpretation goes back to Cornford. In his account of Pythagorean doctrine, Aristotle speaks of a plurality of extended monads, and he often alludes to the definition of the point as a “monad having position.” If we interpret this as a comprehensive key idea, to be taken along with the pebble figures, the “star pictures” (constellations), and the procedure of Eurytus, who would determine the “number” of man or horse by making an outline picture with pebbles—the result is the thesis that the Pythagoreans understood the materialized point as a kind of atom. They thought of all bodies as consisting of such point-atoms, and therefore things “are” numbers in the most literal sense; that is, they are the number of atom-point-units which they at any given moment contain. Does not Aristotle himself say that the Atomists “in a way” claim that things “are numbers or composed of numbers” (Cael. 303a)?

Still, though Aristotle’s refutation sometimes presupposes an atomistic view, Cornford’s theory cannot claim to give the final answer. Every atomic theory operates with invisibly small “units,” so that every visible body consists of an incalculably large number of them; and in fact Cornford speaks throughout of “indefinite plurality.” But is an incalculably large number still a “number”? Would

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83 900a22ff.: one should distinguish, he says, between the number “out of which the world is composed,” the “magnitudes being composed” (60), on one hand, and numbers as causes of properties like opinion or opportunity. (Cf. 986a21f., 987a2ff., 103b17, and 109a28ff.; and 109a11ff., where the opponents in mind are mainly the Platonists.)

84 Cornford, CQ: Philarm 50ff. Cherniss, especially, is in agreement (Pres. 305, 387); he thinks this interpretation gives him a basis from which to criticize Aristotle’s reports. Raven expresses doubts. Pyth. 69ff. KR. 247 (though on pp. 248 and 249 objects are said to be “aggregations of unit-point-atoms”)

85 108a9, 108b15f.; cf. above, nn. 15, 18. 108b26ff. is aimed at the Platonists.

86 Below, ch. I 1, nn. 86-87.

87 Arist. Met. 109a20ff.; Theophr. Met. 6a14ff. after Archytas. Further reports by P. Alex. Met. 827.19 (DK 45) are of dubious authenticity.

88 At Met. 1083b8ff., Aristotle asserts, as refutation of Pythagorean views, that there are no “indivisible magnitudes.” But, on his own principles, Cherniss ought not to have assumed from this (Pres. 395) that the doctrine attacked had precisely the content which the refutation presupposes.

89 CQ 1922, 137, Philarm 60.
it not be a better conclusion—on the sories principle—that a unit amounts just about to nothing? If this doctrine is supposed to be early Pythagorean, we must reject the majority of the testimonies of Aristotle. According to all the sources, the fundamental thing for the Pythagoreans was the numbers from 1 to 10, not myriads; the important thing is form, not statistics, and this includes the method of Eurytus, who looked for the significant points, the specific criteria of the shape.72 Nor can Pythagorean atomism be deduced from the arguments of Zeno;73 and how could anyone put more trust in conjectures about putative opponents of Zeno, whom he does not himself name, than in what Aristotle ascribes explicitly to the Pythagoreans? If Echphantus was indeed “the first” to attempt an atomistic interpretation of the number theory, this is an attempt to modernize the theory, rather than a revelation of its original significance.74

Another line of thought mentioned by Aristotle is almost always taken as Pythagorean: the “limit” (πέρας) is the essential nature (οὐσία) of a thing, so that a plane surface is more oüsiα than the physical object, a line more than a plane, a point—and thus a limit—still more than a line. Since this argument is contrasted by Aristotle, in one passage, to that of Plato, Speusippus, and Xenocrates (though Xenocrates is not named), it has been thought that by a kind of process of elimination we may decide for the Pythagoreans.75 In addition, a doxographical report names μαθηματικοι as adherents to the limit doctrine, and people have assumed without hesitation that “mathematicians” must mean Pythagoreans.76

72 So Raven, PyEl 104f, KR 313f.
73 Below, ch. III 3.
74 DK 51,2; G. Vlastos, Gnomen 25 (1953) 32f; below, ch. IV 3, n. 17.
75 The passages are 1002a4ff, 1028b16ff, 1060b12ff, 1096b1ff, and perhaps Cael. 298b33. Diels included this material (DK 38B23-24), as did Kirk and Raven (320, as “pre-Parmenidean,” and 405, in the chapter on Philolaus and Eurytus; the phrase “by a process of elimination,” p. 316). See also Ross, Met. II 162, Kucharski, Tèr. 27f, Guthrie I 259. Zeller saw the right answer (I 484 n. 1: “Plato”): to be sure, the problem of the “points” arises again; cf. above, ch. I 1, n. 20, as did Cherniss (Pres. quatt. Plato 132ff) and Saffrey 27ff.
76 Ps-Galen Phil. hist. 18 (Dox. 611.2, 23: 613.1), Sext. Emp. PH 3.32, Math. 9.364, 367f. For the assumption that references to μαθηματικοι are to Pythagoreans, cf. Frank, n. 174, and Logos 9 (1920-1921) 246 n. 1; Rouger 23, 64, etc. Examination of the data reveals that μαθηματικοι are advocates of astronomical theories in Act. 2.15.5, 2.16.2, 2.16.7, 2.30.7, 2.31.2, 2.29.6; astrological, 5.18.6. The statement in Act. 2.31.2 on the distance of the moon from the earth is decisive: it is the view of Aristarchus of Samos (Dox. 61). At Act. 4.14.3 Pythagoreans and μαθηματικοι are named as sponsors of the theory of mirror images, and with this belongs the theory of vision in 4.13.9 (the same comparison with the stretching out of the hand): The astronomer Hipparchus is named here: ἐνω δὲ καὶ Πυθαγόρις τῇ δόξῃ ταύτῃ συνεπεργάσαντο ἐνδε δὲ βεβαιωθή τῶν

To be sure, the underlying consideration, that there can be a surface without a body, but no body without surface, line, and point77—this “conceptual experiment” is fundamental for the system of derivation found in Aristotle’s On the Good.78 But for the Pythagoreans, who know of nothing else than what is sensually perceptible, there can scarcely be a “limit” without a body (ἀνευ σώματος, 1002a6). In fact the limit doctrine appears once where the point of view is that of χωραμένος,79 and this shows it cannot be Pythagorean. There is a passage in which the thought seems to be that the limit is immanent: To a mention of the limit doctrine, Aristotle adds the question, whether things of this kind exist inside or (as Plato, Speusippus, and Xenocrates have it) outside the realm of the sensible.80 Yet some Platonists maintained the immanence of mathematical.81 For the Pythagoreans even the primary One is three-dimensional, or corporeal. The reduction of the physical world in the schema of body-surface-line-(point) belongs to the Platonists, not to the Pythagoreans, who knew only the one world of the sensible.

Now it is true that a much-cited passage of Aristotle does seem to confirm an idealistic interpretation of the Pythagorean number doctrine, the famous sentence in which the difference between Plato and the Pythagoreans seems to shrink down to nothing but an alteration of terminology: τὸν δὲ μεῖκνει τούνομα μεταβάλει Πλάτων ἢ τὸν γὰρ Πυθαγόρις μερισάει τὸ ὑπάτα φαίνει τῶν ἀριθμῶν, Πλάτων δὲ μεῖκνει, τούνομα μεταβάλλων (987b10ff). If it was only that “he changed the word,” then, it is suggested here, the relation of number and things in Pythagorean doctrine was of just the same kind as that of

μαθηματικοι . . . That is, even in antiquity, we have a conjecture based on the postulate that mathematics is Pythagorean. Ps.-Plutarch (Aet. 4,14,3) left out the μαθηματικοι and speaks only of Pythagoreans; this is how the tradition burgeons. What we must hold to is that the μαθηματικοι of the doxographical tradition are professional mathematicians and astronomers of the Hellenistic age. (This is also true for Por. ap. Stob. 1.49,61, in quod of Delatte, Litt. 126. Cf. also Cic. Div. 2.91 Sext. Emp. Math. 10.174 Plut. Prim. fit. 953a.)

77 1002a6: καὶ τὸ μὲν (καὶ ἐπίφανες, γραμμή, στιγμή) ἄνευ σώματος ὀδήγεσθαι δοκεῖ ἐνοικία, τὸ δὲ σῶμα ἄνευ τόνων ἀδύνατον.
79 1096b1ff, διὰ τὸ οὐν χαράλατο ἤστατα τῆν b11.
80 1028b18: ἐπὶ παρά τὰς αἰσθήτας οἱ μὲν υἱοὶ οὐκ εἶναι οὐδὲν τούτους, οἱ δὲ πλείον καὶ μᾶλλον ὄντα ἀδίκα, ὅπερ Πλάτων . . . Ἐπικουρίας . . .
81 987b10ff (immanence of mathematical, separation of forms); cf. 1076a3ff. Other Platonists, not precisely identifiable, are included along with Plato, Speusippus, and Xenocrates at 1085a13, 1087b13ff, 1088a2ff.
I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

idea and thing in Plato; and all attempts to prove a Pythagorean origin for the theory of ideas make use of this argument.\textsuperscript{83}

The difference between this and the reports elsewhere that for the Pythagoreans things “are” numbers, seems insuperable. The concept of development might seem to offer a solution of the dilemma, but its sponsors offer opposite formulae. If the identification of thing and number seems to Burnet primitive and therefore old,\textsuperscript{83} to Cornford the idea of “imitation” seems “mythical,” and therefore original.\textsuperscript{84}

Others have decided to eliminate one of the two versions as a faulty tradition, but, here again, sometimes one and sometimes the other is rejected.\textsuperscript{85}

Amazingly enough, Aristotle himself does not seem to have noticed the discrepancy; the phrase ἀρίθμος ἐστι... αὕτε τὰ πράγματα stands just a little further on (b28). It is quite unlikely that Aristotle was following different sources; could the contradiction be only apparent?\textsuperscript{86}

Evidently Aristotle knew the Pythagoreans had used the word μιμητος. He surely must have intended to belittle Plato’s originality in the doctrines which Aristotle himself disliked. Thus the thought that this Pythagorean μιμητος was the same as μεθεζ, and therefore implied a theory of ideas, will be seen to be interpretation. Aristotle does give some clues as to how imitation can be understood without a theory of ideas. The word “immanence” is no help (above, n. 21), but he does say that the Pythagoreans supposed they saw ὀμοιωματα between things and numbers,\textsuperscript{87} and it is only natural to express such similarity or correspondence by the word μιμητος. Similarly, in the Hippocratic writings the relation of microcosm and macrocosm becomes a matter of “imitation,” but—and here is the surprising fact—this imitation may be turned either way. One may just as well say that the human body “imitates” the cosmos as that the parts of the cosmos “imitate” human organs.\textsuperscript{88} In the same way, either the arts imitate nature or nature imitates the arts.\textsuperscript{89}

Imitation is a two-sided correspondence, which makes it possible to interpret separate things following the same pattern, but without implying differences of rank or a relationship of ontological priority. Indeed, it scarcely makes a difference whether one says that stone “imitates” bones, or that earth “is” flesh.\textsuperscript{90} When we place the Pythagorean theory in this pre-Socratic context, Aristotle’s statement about “imitation” falls into place with the rest. Nothing more is meant than the correspondence of cosmos and number, in the sense that one explains and illuminates the other.\textsuperscript{91}

In post-Platonic thought one can scarcely speak of imitation without assuming that it implies a gradation of kinds of Being, especially since Plato often characterizes the relation of sensible object and idea as μιμητος.\textsuperscript{92} To this extent the surprising report of Aristotle is comprehensible. For the historical placement of the Pythagorean number doctrine, the result is more a confirmation than a correction, to say nothing of an earthquake contradiction.\textsuperscript{93}

Again and again it becomes clear that the Pythagorean doctrine cannot be expressed in Aristotle’s terminology. Their numbers are “mathematical” and yet, in view of their spatial, concrete nature, they are not.\textsuperscript{94} They “seem” to be conceived as matter (ἐνεργα) and yet they are something like form (ἐνεργος).\textsuperscript{95} They are, in themselves, being


\textsuperscript{84} EGP 99ff, 307ff; now add Philip 71f.

\textsuperscript{85} CQ 1922, 143f; 1923, 5.

\textsuperscript{86} Since the notion of imitation seemed more fruitful philosophically, it is accepted by Gilbert, AJP 1939, 35f; by Jaeger, Paideia 1 (Eng. tr.), for whom, in the other reports, “he is no doubt making the mistake of translating into material terms their theoretical identification of numerness and existence”; by Ross, PTh 217 (only “a primitive savage” could simply identify thing and number); and by Maddalena, 265 n. 24. Taking the opposite position, Cherniss, Pres. 386ff; says the imitation-statement came from a source who wished to contest Plato’s originality, perhaps Aristoxenus (392: cf. Aristox. fr. 21). Similarly, Frank thinks of Speusippus (256).

\textsuperscript{87} See the attempts at compromise by Zeller (I 451f) and Reck (367ff), who ingeniously suggest that things “are” numbers when one considers their basic nature, but “imitate” numbers when one considers their properties. Also see Raven Pyl. Catt., Guthrie I 230f.

\textsuperscript{88} 985b17: ἀριθμοὶ ἵναι λέγωνται, περὶ τῶν ἀριθμῶν (Στιχ., 1092b11). These references are added by Zeller (I 451f) and also missed by Mondolfo (ZM 134f).

\textsuperscript{89} Tim. 38a, 39c, 48e-f, etc.

\textsuperscript{90} Cherniss (Pres. 386ff) finds a third version of the relation of thing and number in the expression, “the elements of numbers are the elements of all things” (986at; cf. 987b25, and for the formulation 987b18; contra, Guthrie I 229f).

\textsuperscript{91} The Pythagorean number is “mathematical” (108b16) but “not unitary” (μοναδικός, 108b19, b2a); still one can say, “an arithmetical number is unitary” (108b16); so that the corporeal number of the Pythagoreans is not mathematical (108b12).

\textsuperscript{92} 986a6: ἐκάστῳ β’ ὁ γὰρ ἔλθεν ἔσθη τὰ στοιχεῖα τάττων; 986a16: καὶ ὃς ἔλθεν τούτων οὐδέν καὶ ὃς πάθη τε καὶ ἔξωθε; cf. also above, n. 65.
(οὐσία), and yet are not quite so. They cannot be expressed in the Aristotelian framework of the four principles, or in the categories of form and matter; great as the temptation has been, both for Aristotle and for modern scholars, to understand the opposition of Limit and Unlimited as identical to that of form and matter, the explicit statements that the One partakes of both Limit and Unlimited, and that number is a kind of material (δύνα), stand in the way. Missing are the impact of the theory of ideas and the dialectic, the classification of Being into stages of differing reality (οὐσία), the reduction of the sensible world to immaterial principles. Neither the system of Aristotle nor the conceptual framework developed by the Academy forms any part of the background of these Pythagorean doctrines; rather, they obstruct our access to them and impede our understanding of them. When one puts these observations alongside the traces of cosmogenetic myths which dominate the apparently abstract pattern of the genesis of the “numbers,” there can remain no doubt. What Aristotle presents as the philosophy of the Pythagoreans is truly pre-Socratic, unaffected by the achievements of Socratic-Platonic dialectic, and not to be measured by their standards. Οὐ γὰρ πρῶτοι διαλεκτικὴς οὐ μετέχουν (Arist. Met. 987b32).

Thus the thesis of Frank and Howald, that the whole number philosophy of the “so-called Pythagoreans” was developed only within the precincts of the Old Academy, is refuted from within the theory itself. Indeed, Aristotle says, repeatedly and unambiguously, that the Pythagoreans are to be dated earlier than Plato. Elsewhere, to be sure, Aristotle’s chronological indications are imprecise enough to make one think that he was not himself quite clear about the order of events. The surprising anonymity of the doctrines is of a piece with this chronological indefiniteness. The only personal name that certainly belongs with this philosophy is “Pythagoras”; but Aristotle purposely avoids it. The “Pythagoreans” belong among the “later” pre-Socratics.

As to the sources to which Aristotle owes his knowledge, there are two different clues. The story about Eurytus is expressly referred by Theophrastus to an account given by Archytas, and here we obviously have oral tradition transmitted through Academic connections in Magna Graecia; Speusippus and Xenocrates had been in Sicily with Plato. In the second place, Aristotle occasionally plays off the Pythagorean doctrines against the Academy in such a way as to make the conclusion unavoidable that he is using written sources without Academic coloring. Therefore he must have had at least one original Pythagorean document.

The scanty indications about Archytas given by Aristotle and his pupil Eudemus reveal an advanced stage of Pythagorean doctrine, not far from Platonism. Archytas proposes definitions whose purpose is differentiation between matter and form, while Pythagorean definitions had been “superficial,” and their number theory made no distinction between form and matter. Archytas designated the “irregular,” or the “indefinite,” as cause of motion, whereas in the Pythagorean number theory the cause of motion remained unclear (990a8). If we add that the procedure of Eurytus, too, seems to be an attempt at a more comprehensive systematization of the number theory, and that Eurytus is to be dated earlier than Archytas, the terminus ante quem recedes still further back into the fifth century. But Aristoxenus and later authors mention one name in conjunction with that of Eurytus—namely, Philolaus (below, ch. III).

When one attempts to understand the Pythagorean number theory in the context of fifth-century pre-Socratic thought, it is impossible not to become aware, along with the mythical-cosmogonical

96 ἀκέφαλος, μέρος, ἄνω, ἄνωθεν ἀπὸ σινθα, above, n. 14; nevertheless, σαφῶς μὲν οὐ δυσχέρατο παρ᾽ ἑλείνου, 986b5; cf. Phys. 204a33ff.
97 Frank 258: “Thus there is nothing in the whole of Aristotle’s account of the Pythagorean philosophy which could not be derived from Speusippus or similar Platonists.” Cf. his next pages; similarly Howald, Fr. Sudhoff 70ff; with amplification, Bollinger 40ff.
98 987a20ff, b32, 105b12, 107b21.
99 Their relation to the Atomists is expressed very vaguely at 985b23, in the words τὸ δὲ τούτων καὶ περὶ τούτων (cf. Alex. Met. 17.6ff). At 107b21 the Pythagoreans are dated earlier than Democritus.
100 At 100a21ff Platonists and Pythagoreans are mentioned as “those who came later and second wise” (οἱ δὲ δεύτεροι καὶ μελλόντες . . . Σικερατικοί). This counts against Philip’s thesis that the whole doctrine goes back to Pythagoras himself (141).
background and their proximity to Leucippus, of a certain kinship with the Eleatics. In terms of Aristotle’s criticism, the Eleatics and the Pythagoreans find themselves near neighbours. Both schools, and they alone among the pre-Socratics, pressed forward to the brink of immaterial being, but in neither case were they able to make any real use of this advance. For the Eleatics, too, though “transcending sense perception” (Gen. corr. 325a13), “still conceived of ‘existing things’ as being only sensible things” (Met. 1010a1ff; cf. Cael. 298b21), just as the Pythagoreans squander their principles on the sensible world (98b29ff), while at the same time mistreating it in the process: “They seem to be speaking of another heaven and other bodies than the perceptible” (1090a34), and the theses of the Eleatics about our world actually seem to “border on madness” (Gen. corr. 325a19).

A striking fact about the examples that Aristotle gives for the Pythagorean equation of numbers and things is that it is never a question of the relation of individual thing and individual number—aside from the isolated fooleries of Eurytus—but of the correspondence of a plurality of things to the system of numbers, and in particular the correspondence of alterations in things to alterations in the number series. As Aristotle puts it, πάθη of things correspond to πάθη of numbers. To the alternation of even and odd in the number series corresponds an alternation of Unlimited and Limit in the world. The πάθη και μέρη of the heaven are reflected in numbers (986a5), as is the structure of the musical scale (τῶν ἀρμονῶν τὰ πάθη, 985b32). It is not, after all, transcendent Being that the Pythagoreans are thinking about, but that-which-is, thought of as something coming to be, and many-shaped: ὄντα καὶ γεγονόμενα (985b28, 990a20).

Plurality and becoming: these were the problems that Parmenides had set for philosophy. He called both of them unthinkable and unsayable, because they were incompatible with his basic principle, ēstia. The logical postulate of Truth, according to which any meaningful thought or speech presupposes a Truth, seemed to require the existence of an absolute Being, and “Truth” could only be expressed as “Being,” óv. To the philosophers after Parmenides this seemed irrefutable; but still, few followed him to the logical conclusion, of denying becoming and plurality. But how was Parmenides to overcome? Faced by this problem, the Pythagoreans were able to enlist the help of the technique of calculation. It must have seemed difficult even to a Zeno to deny that the thing “is valid” or “is” which we use as the paradigm of elementary correctness—namely, that 1 and 1 is 2, and that 2 and 2 is 4. And yet in this procedure something apparently new is gained out of the presuppositions, multiplicity and alteration emerge. Out of a few fundamental figures, the numbers from 1 to 10, there develops an inextricably complicated system. The “generative” character of mathematics broke through the inflexibility of the Eleatic system: plurality and becoming are not unthinkable and unsayable after all, but they can be thought and expressed in the form of numerical operations. These thus become, of course, the only legitimate form of expression about that—which-is; and the cosmogonic myth becomes the dry calculation of a process of division or addition. By contrast with Parmenides or Melissus, however, this is a relaxation of the stiffness of the denial of reality, and a new attempt to get back to everyday reality.

One could even see the foundation of mathematical science as latent in the principle that only mathematical expression is valid expression—only that its content, in empirical knowledge, is scanty; mathematics is still in its earliest stages, ringed about by all sorts of connotations and arbitrary conventions. It was only the development of mathematics from elementary arithmetic to deductive geometry, at a later time, that made a mathematical science possible—a development that leads, by way of Archytas, to Eudoxus (cf. below, chs. IV, VI).

“The Pythagoreans were the first to take up mathematics ... and, having been brought up in it, came to believe that its principles are the principles of existing things.” This is Aristotle’s psychological explanation of the origin of the Pythagorean doctrine. Modern scholars, too, have inclined toward psychological explanations, except that the driving force is thought to be not habituation, but the overwhelming impression of a discovery made: that of the numerical proportions in musical intervals. On the other hand, a passage of Iamblichus, obviously derived from Aristotle’s book on the

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108 Cornford and Raven tried by different means to show a relation with the Eleatics. Cf. also below, ch. III 3. The interpretation which follows is indebted to C. H. Kahn, “The Greek Verb ‘To Be’ and the concept of Being,” Foundations of Language 2 (1966) 245-265.
109 Cf. 985b29, 32, 986a5, 990a19, 1088a17, 1090a21.
110 985b29ff; cf. ch. VI 1.
111 Guthrie I 239.
I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

Pythagoreans,\(^{112}\) speaks of the factual justification that the Pythagoreans had to offer: “Whoever wishes to comprehend the true nature of actual things, should turn his attention to these things, the numbers and proportions, because it is by them that everything is made clear.” Number is ósía, that about things which can, with a claim to truth, be expressed (Arist. Met. 987b19); nothing is known without number.\(^{113}\)

A perfectly certain interpretation of a philosophy is impossible when it is known to us only indirectly and mostly in the context of polemic. Its place in the history of thought, however, can be satisfactorily assessed when we are clear about the traditional background (the cosmogony) and the philosophical problems current at the time (Elaticism).

\(^{112}\) Lam. Comm. math. sc. 25 p. 78.8–21. (On the deletion of the intrusive reference to geometry, see below, ch. VI 3, nn. 2–5, with context.) Compare the passages from Aristotle in the right-hand column.

\(^{113}\) This passage cannot have been formulated by Iamblichus himself (following Arist. Met. 985b24ff), for there is not the slightest hint of the immateriality of the numbers, which was so important to any Platonic (cf. Comm. math. sc. p. 74.9ff). Since in general the later tradition does not usually follow Aristotle, an intermediary source (a re-working of 985b24ff, but free of Platonicizing touches) would also be unlikely. Rather, since both the preceding material in Iamblichus (below, ch. II 5) and that which follows (Arist. frs. 52–53, guaranteed by Procl. In Eucl. 28.13ff and Cic. Tus. 3.60) come from Aristotle, the answer must be that here too we have an independent fragment of Aristotle.\(^{114}\) Alex. Met. 40.12– Arist. fr. 203; cf. below, ch. III 2, and above, n. 91.

Finally we come to the “table of ten opposites,” which Aristotle sets apart from the rest of the Pythagorean number theory which he treats: “Other members of this same school say there are ten principles, which they arrange in two columns of cognates” (986a22, Ross tr.):

(1) limit (πέρας) : unlimited (ἄπειρον)
(2) odd (περιττός) : even (άρτιον)
(3) one (όσι) : plurality (πλῆθος)
(4) right (δεξίον) : left (άριστερόν)
(5) male (ἄρρεν) : female (θηλή)
(6) resting (ήρεμον) : moving (κινούμενον)
(7) straight (εὐθά) : crooked (καμάτιλον)
(8) light (φῶς) : darkness (σκότος)
(9) good (ἀγαθόν) : bad (κακόν)
(10) square (τετράγωνον) : oblong (τετράμερης)

Though the good has the second-to-last position, the arrangement is clearly made from a normative point of view. In the Nicomachean Ethics Aristotle speaks explicitly of a “column of goods”\(^{115}\) there, too, he attributes the system to the Pythagoreans, but adds, “and Speusippus, too, seems to have followed them” (1096b6). The inclusion of movement in the same column as the unlimited or indefinite (ἄπειρον, ἀόρατον) is alluded to by Aristotle in the general remark that “the principles in the second column, because they are negative (στερητικαὶ) are indefinite (ἀόρατοι).”\(^{115}\) This is in a context which, according to the definite statement of Eudemus, applies especially to Plato, or better, to the interpretation and systematization of Plato in the Academy. At the end of the Metaphysics, Aristotle alludes again to the “column of the beautiful (καλόν),” sets forth his opinion of the right point of view, and in his final sentence shows again what the real object of his criticism is: the “separation” of the Academics.\(^{116}\)

Thus the “table of opposites” is quite closely connected with Academic doctrines; we have here a continuous transition between Pythagorean and Platonic. There is one small suggestion of Archytas (n. 115). Hermodorus’ report of the On the Good reveals a similar train of thought: an initial threefold division of Being is traced back to a

\(^{114}\) 1096b5; 1106b28f. Here the Platonic Unlimited Dyad is brought into close connection with Pythagoreanism (Krämer 147).

\(^{115}\) Phys. 201b25 Met. 1066a14. Cf. above, n. 106; on Cael. 284b6f, above, n. 49.

\(^{116}\) 1093b1ff; b27: μὴ χωριστέα εἶναι τὰ μαθηματικά. Further, cf. 1072a31, Gen. corr. 310a15, 1054a22ff, with a reference to Ἡπείρου ἱερών.
twofold division, in which one side has “equal, abiding, harmonized,” (ἰσον, μένον, ἥμομοιον) and the other “unequal, moving, unharmonized” (ἀνίσον, κυνομένον, ἀνάμορφον). The connection with Speusippus is particularly close. Therefore it is not surprising if later Platonists, and also pseudo-Pythagorean works, keep introducing similar “tables of opposites.”

On the other hand, Aristotle considers it possible that Alcmaeon already knew the “table of opposites.” This does not mean, however, that Aristotle dates it to the beginning of the fifth century, but that he cannot, or will not, make a positive statement on the chronology; the table may be older or later than Alcmaeon. It may be that he knew it only from the oral tradition, passed on, later, by Speusippus. To think in terms of polarities, of antithetical pairs of concepts, is an old human habit. The fact that there are ten pairs, however, and the inclusion of square and rectangle, do not give the impression of being ancient; and Alcmaeon is close to Ionian philosophers like Heraclitus. There is even more uncertainty latent in the possibilities of interpretation than in the chronology. Is such a rigid schema the result of true philosophical reflection, or of a primitive way of thinking? Is it the expression of a strictly regimented way of life? We can see what the Platonists made of it, but it is not a helpful foundation for a reconstruction of Pythagorean philosophy.

118 The idea that “good” does not belong at the beginning, but only appears as a later development, comes from Speusippus (107b3ff), applied to the Pythagoreans and Speusippus in common; cf. 1075a3ff, 1091a2ff, 434: τῶν ὁμοίων, ὁμοίως, ἱσομοιωτάτων, ἡμομοιοῦν. Herm. Phys. 241 n. 111. In addition, the pair one-plurality, which occupies third place in the “table of opposites” is fundamentally important to Speusippus.
119 The following (a and b) are probably based on Aristotle: (a) Plut. De Is. et Os. 48.170e (the order, compared with Aristotle’s, 9 3 1 6 7 2 10 4 8; the “good” is at the beginning; instead of “plurality” he has “dyad”). (b) Por. VP 38 (from Diogenes Antoninus): monad-dyad, light-darkness, right-left, equal-unequal, abiding-moving, straight-circular. (c) Eudorus ap. Simpl. Phys. 187.22ff: ordered-disorderly, definite-indefinite, known-unknown, male-female, odd-even, right-left, light-darkness. (d) “Pythagoras” in Varro Ling. s.11: finitum-infinitum, bonum-malum, vivum-mortem, diem-noctem, status-status. (e) Ps.-Archytas p. 19, 5–13 Thesleff: ordered-disorderly, limited-unlimited, speakable-inexpressible, rational-irrational, binding-bondspoil, etc. (f) Eurylus, Stob. 1.6.19: speakable-inexpressible, ordered-disorderly, rational-irrational. (g) Philo Qu. in Exod. 2.33: odd and even—odd and even, equality-inequality, similarity-dissimilarity, same-different, unification-dissolution, better-worse. Cf. also Tim. Locr. 1.
120 So Raven, PyEl 10ff, KR 263.1, De Vogel, Rev. philos. 1935. 345, Guthrie I 223, 224ff, on the other hand, the table is dated in the second half of the fifth century by Zeller 460 and Philip 17.
121 G. R. R. Lloyd, Polarity and Analogy (Cambridge, 1966), esp. 31–51. There is a tempting similarity to Iranian dualism, noted, as it seems, by Aristoxenus (fr. 13 Hippol. 1.212).

3. The Later Tradition and Its Sources

The most important of the later sources for Pythagorean philosophy are the Pythagorean Memoirs (Πυθαγορικά υπομνήματα) excerpted by Alexander Polyhistor, the Life of Pythagoras (Περί Πυθαγόρου βίος) excerpted by Photius, the reports of Aetiou, and (most extensive of all) those of Sextus Empiricus.

It will become increasingly clear as we proceed that the so-called Memoirs (or Hypomnemata) can surely not be an original Pythagorean writing of the fourth century b.c., as Wellman and Delatte tried to prove. After the careful study of Festugière, we may date them with some confidence to the end of the third century b.c.1

The life of Pythagoras which Photius read has now been attributed by Theiler, on good grounds, to Eudorus.2

Sextus Empiricus gives four rather comprehensive accounts of Pythagorean doctrines, of which two pairs are more closely connected with each other than with the rest. The discussion “on number” (PH 3.115–167) is repeated in much greater detail at Math. 10.248–309.3 There is a detailed exposition of the doctrine of ideal numbers, followed by a refutation of the concept of participation and “separation”, and because of a remarkable agreement with Hermogenes’ account of Plato’s doctrine of principles, the text has rightly been employed in the reconstruction of these Platonic theories; it must derive from one account of Plato’s lecture On the Good.4 It must be remembered,

2 In Panarion (1965) pp. 209ff, Immisch (SBHedd 1919, 7) had sought to show that the author was Agatharchides. Against this, see Wilamowitz, Plato IV 84,1, Ueberweg-Prachter 155*, and K. Reinhardt, RE XXII 763–768 (on the influence of Platonism). Ps.-Justin Coh. ad gent. 10 cites the book (cf. 43833, 43939). Von Fritz (SBM1 1966, 6) considers the word σεβαστικός (43819) to be of imperial date, but as early as Epicurus we find σέβασμα and σεβασμός (fr. 141. Sent. Vat. 32).
3 Raven, PyEl 10ff, rashly attributes all four reports to the same source. In the following argument we need not cite more than Math. 10; PH does not add anything. At PH 3.152 the irrelevant πάντα δύομεν should be emended (comparing Math. 10.522ff) to πάντα δύομεν (Mutschmann should have taken this conjecture of Pappenheim into the text).
4 Heinze, 37ff; Merlan, Philologus 1934, 37ff, Wilpert, Hermes 1941, Zowit 121ff; De Vogel, Muenszyn 1949, 209ff, Krämer 282ff, above, ch. 11.
however, that Sextus ascribes the whole, unambiguously, to the Pythagoreans, and even to Pythagoras himself. It is scarcely possible to determine the immediate source Sextus is following;4 it is later than Epicurus, who is cited in it, and even later than Asclepiades of Bithynia.7

A differently organized exposition, under the heading “On the Criterion,” deals with the relationship between mathematical proportion and nature (Math. 7); and the same subject is treated much more briefly, in a different order, and with some supplementary material, in the book “Against the Arithmeticians” (Math. 4).8 In the former is found the famous citation of “Posidonius in his exposition of Plato’s Timaeus” (Math. 7.93). The bitterly debated question, whether this means that Posidonius wrote a commentary on the Timaeus, we may here leave unsolved;9 Reinhardt and Schmnel agree that there is much more from Posidonius than a single sentence.10 Thus we have, in this passage, an exposition of Pythagoreanism as it was seen by Posidonius. It is an important question, nevertheless, whether Reinhardt was right in singling out the explanation of the Pythagorean Tetractys (94–100) as “a piece of book-learning that Sextus could have gotten from anywhere.”11

This is in fact the most popular exposition of Pythagorean doctrine,

which has very close parallels in Philo,12 Theo,13 and Anatolius.14 In Sextus the passage merely serves the purpose of clarification, not of development of the thought; but since Posidonius comes first to mind if we think of a possible common source of Sextus and Philo, it is natural to ask whether Posidonius himself may not have been the one to take over and work in a piece of “book learning.”

The elimination of the “interpolation” does make difficulties. Reinhardt himself had trouble deciding where it begins, and in any case this must be in the middle of a sentence.15 At its end lies an incontestably Posidonian train of thought, set off by the phrase καὶ ἄλλως (101),16 but section 99 is introduced in just the same way, so that the transitional flourish merely demonstrates the unity of the passage’s structure. The controversial section is referred to twice in the sequel. The concluding sentence (109)17 repeats a verse cited there,18 though one cannot simply regard it as separable from the Stoicizing commentary. Further, in the Posidonian passage (101ff), there are mentioned as incorporeal point, line, and plane, “which we also discussed

12 Op. 47, cf. Sext. Emp. Math. 7.94; Philo 48, cf. Sext. Emp. Math. 7.95–98; Philo 49, cf. Sext. Emp. Math. 7.99–100. The agreement in order and formulation is striking, even though in some parts Sextus is fuller, and in some parts Philo. Philo’s mention (50) of the game of putting a nut on top of three others to make a pyramid (κατατέμνειν τὸν κύκλον, cf. Anat. 22.31ff), provides an explanation for the word ἐπισώφρονος in Sext. Emp. Math. 7.100. (Consequently, at Math. 4.5, instead of the inappropriate ἐπισώφρονος, whose correct use is illustrated by PH. 1.154, we must read ἐπισωφρονος.)—A short summary of the same train of thought is given by Philo V. Mos. 2.115.

13 Sext. Emp. Math. 7.95–98, cf. Theo Sm. 93.21ff. In particular the “first tetractys” of Sextus 7.95 is only comprehensible through Theo 94.10ff. For “the nature of the whole,” Sext. Emp. Math. 7.93, cf. Theo Sm. 94.4, for the oath by the tetractys (Sext. Emp. Math. 7.94), cf. Theo Sm. 94.6ff; in addition the explanation τῶν μὲν παραβουλῶν λέγουσιν Πυθαγόρα, in Sextus, and in Theo ἀναβαίνει Πυθαγόρα λέγουσι. Theo’s source here is obviously Thrasyllus (61.18 refers to the Thrasyllian quotation at 87.4 cf. Schmkel, Miatt. Stoa 200 n. 3). Theo’s passage is also relevant in the Timaeus discussion: “the second tetractys” is the sequence of numbers in the Timaeus.

14 Pages 31f (abbreviated in Th. ar. 29.10ff); esp. p. 32 — Th. ar. 34.14 corresponds almost word for word with Sext. Emp. Math. 4.6–9. Also cf. Th. ar. 29.10: πρῶτη τῆς τούτων τοῦ στροκευός φύσις, with Philo Op. 49: πρώτη γάρ αὕτη (ἢ τετράδες) τῶν τούτων στροκευών φύσεως.

15 Reinhardt, Posidonius 416,3, places the end of the citation of Posidonius in the middle of section 93, but in his paraphrase (p. 418) he finds it necessary to add the concluding sentence of 93 about number and reason. In RE XXII 725, he adds the beginning of 94 to the Posidonius citation.

16 Above, p. 10.

17 καὶ γὰρ τὸ λόγον πᾶσα τέχνη ἀκατάστατα ἐκ καταθέσεως (SVF I.73), τὸ δὲ σύντομον ἀρμένως τὸν ἀρμόδιον δὲ τὰ παρὰ ἑπάνων.

18 In the same way, the end of section 98, and 100, allude to the tetractys oath in 94. Reinhardt (Posidonius 416 n. 1) interprets this as characteristic of “the interpolated passage.”
a little earlier” (104). Now since in a later passage, which is certainly Posidonian, there are mentioned precisely the “incorporeal ideas . . . consisting in the borders of bodies,”19 the present passage, too, will have to be attributed to Posidonius. The continuity of the whole passage is irrefutable: the criterion of truth for the Pythagoreans is the “reason that is achieved from mathematics” (93), since like is known from like; the next logical step is to show to what extent nature is mathematically constructed, and thus akin to mathematical reason.20 The solution is in the thesis that “everything is like number,” which is then explained in various instances. If Posidonius, à propos of the Timaeus, wished to speak about the recognition of like through like, he could not ignore the fact that in the Timaeus the soul, which recognizes, is created as a number pattern, as the physical world is made up of mathematically determined triangles. Obviously, Posidonius is in part dependent on Aristotle, who, following a similar line of thought, brought the Timaeus and the theory of ideal numbers into connection, and also treated of this matter in the dialogue On Philosophy.21

Thus it remains probable that the whole section in Sextus, that is to say the most prevalent type of exposition of the Pythagorean theory of numbers, was transmitted by way of Posidonius.22 The additional material in the parallel passage23 is probably to be explained by the fact that Posidonius was abridged twice, in Sextus’ work, in different ways. It is important to see that not only in the attention he pays to the Timaeus, but in details of his interpretation, Posidonius goes back to the Old Academy.24

 Sextus Empiricus occasionally uses the name Πυθαγόρας in addition to Πυθαγόρειος; but the doxographer Aëtius even uses Πυθαγόρας more often than Πυθαγόρειος. The two coalesce into the expression οἱ ἀπὸ Πυθαγόρας, in the later tradition,25 but some trace of the difference has remained: one branch never has Πυθαγόρειος where the other has Πυθαγόρας.

Now, we immediately notice a fact of great importance: the majority of the reports about Πυθαγόρειος can be confidently referred to Aristotle as source,26 but this is not so of even one of the reports about Πυθαγόρας. Pythagoras is frequently named in the same breath as Plato, Pythagoreans never.27 Thus from the external form of the tradition itself, it is clear that alongside the Aristotelian tradition about the Pythagorean philosophy there was another, which dared to name Pythagoras himself and connects him closely with Platonism. And to be sure, in discussing Sextus’ reports we constantly kept finding it necessary to refer to the Academy.

“The first principle of all things is the Monad; from the Monad comes the Indefinite Dyad to serve as matter for the Monad which is cause, and from the Monad and the Indefinite Dyad comes the numbers.” So runs the beginning of the Memoirs excerpted by Alexander (D.L. 8.5). Aëtius’ phrasing is similar: “Among their principles are the Monad and the Indefinite Dyad. For him, one of the principles is directed toward the active or formal cause . . . and the other toward

19 Math. 7.119 repeats almost word for word what “Posidonius in his exposition of Plato’s Timaeus” had said in 93. Point, line, and plane are incorporeal in 90–100; and in 104 they as well as time can be given as examples of incorporeality.
20 Reinhardt is wrong in missing, in the “interpolation,” the relation to the problem of knowledge (Posidonius 416).
21 De an. 402b4ff has the same quotation from Empedocles as Sextus 7.92 (above, ch. 11, nn. 47–53); cf. Chalcid. 50.
22 Cf. F. E. Robbins, “Posidonius and the Sources of Pythagorean Arithmology,” CP 15 (1920) 309–322; “The Tradition of Greek Arithmology.” CP 16 (1921) 97–123; V. DeFalco, “Sui trattati aritmetici di Nicomaco ed Anatolo.” Riv. indo-grecita-italica 6 (1922) fasc. 3–4, pp. 11–60. Robbins assumes a pre-Posidonian source; DeFalco points out the fundamental importance of Posidonius’ exposition. These studies may be consulted for the individual problems of filiation in the very complicated tradition; cf. also W. Theiler, Gnomon 28 (1956) 284ff, and P. Boyancé, Râg 76 (1963) 82ff.
23 Above, n. 8.
24 Excess of the Timaeus by Posidonius is also attested by Plat. De an. proor. 1023b (a passage that would also fit into a discussion of the criterion of truth; cf. Merlan, Philologus 1934, 41ff); and Theo names Posidonius, at 103.18, in the course of an account of number speculation taking off from the Timaeus. Cf. Merlan, Philologus 1934, 197ff, PIneoqf 34ff; below, n. 75.
25 Aët. 3.2.1: Πυθαγόρειος, Stob. τῶν ἀπὸ Πυθαγόρας, Plut.; 1.15.7: οἱ ἀπὸ Πυθαγόρας, Plut., but Stobaeus Πυθαγόρειος continuing from 1.15.2; see Diels’s note; 1.9.2: οἱ ἀπὸ Πυθαγόρας, Stob., Plut.; but Theodorot has, in more detail, Θαλῆς καὶ Πυθαγόρας... (cf. Diels, Dex. 40); 4.14.3 belongs with 1.13.10: Πυθαγόρας, as does 1.16.1 to 1.24.3, but 2.9.1 to 1.18.6: Πυθαγόρειος. 1.14.2 remains problematic. (Plutarch has the correct text, as Diels says in his note. There is a relationship to Cleanthes: Aët. 1.14.5, 2.14.2, cf. 2.2.1; Por. ap. Euseb. Praep. evang. 3.7.4)
26 Mostly particular questions in natural philosophy: Aët. 1.15.2, cf. Arist. Sph. 455a40. The material appended in 1.15.7 is simply attribution of the doctrine of Empedocles (1.15.3), who was considered a Pythagorean, to Pythagoreans generally.—At 1.18.6 Aristotle is explicitly cited (Phys. 312b33, fr. 201); 2.9.1 belongs in the same context. At 2.29.4 Aristotle is cited along with Philip of Opus (not in Rose; Ross p. 142). Cf. 3.1.2 with Mete. 345a14ff, b10ff, and 3.2.1 with Mete. 342b30ff; the doctrine of Hippocrates of Chios (342b36) and his pupils is attributed to the “Pythagoreans,” as well as the anonymous doctrine of 345b1ff, which belongs also to Hippocrates. There remain 2.13.15 (“Heracleides [fr. 113a Wehril] and the Pythagoreans”), 2.30.1 (“some of the Pythagoreans, including Philolaus”) [DK 44A20], and 2.24.5, which probably goes with 2.13.15. Of 10 lemmata, 6 are surely taken from Aristotle; others are from Heracleides, Empedocles, and Philolaus.
27 Pythagorox and Plato: 1.23.1, 4.4.1, 4.7.1, 4.7.5, 4.9.10, 5.20.4, Pythagorases, Plato, and Aristotle: 2.10.1, 4.20.1, 5.4.2. Pythagorx and Plato in contrast: 1.10.2-3, 4.2.3-5, 2.4.1-2.
the passive or material.”28 Further on the Monad is also called mind, god, and good, and the Dyad is called divinity (δαίμων) and evil.29 The long passage in Sextus builds on the same foundation: “Pythagoras said that the first principle of existing things is the Monad . . . and being added to itself . . . it produces the so-called Indefinite Dyad . . .” (Math. 10.261; cf. 276) “. . . and the rest of the numbers were produced from these . . . The Monad had the position of the active cause, and the Dyad that of the passive matter” (277). In the Photius passage the continuity is broken, but the same general outline is discernible: alongside a “Monad, i.e. the one that is comprehended by intellect,” which is distinguished from the number 1 (438b14), and which is “first principle of all things” (439a19), stands the Indefinite Dyad, which is perceived “according to excess and defect”30 and “from the Monad and the Dyad” come the numbers (439a2).

The doctrine attested in these four passages is as homogeneous in content and terminology31 as it is flagrantly contradictory to everything that Aristotle ascribes to the Pythagoreans. There is not a word about Limit and Unlimited, about odd and even, about the One as a bisexual entity, or of cosmogonic development. On the other hand, every aspect corresponds to the system of derivation which Aristotle ascribes to Plato and his pupils, in distinction from the Pythagoreans. The pair of Monad and Indefinite Dyad is Platonic,32 the concept of the Indefinite Dyad is purely Platonic, the idea of excess and defect (great-and-small)33 is explicitly characterized by Aristotle as peculiar to Plato,34 the matching of form and matter is Platonic-Aristotelian, like that of active and passive,35 and Aristotle labored in vain to apply these categories of his to the “so-called Pythagoreans.”36

The one question remaining in abeyance is that of “separation” (χωρόμαι)—hardly surprising, since we are dealing with Hellenistic and largely Stoicizing tradition. Once Aëtius expressly denies separation in the Pythagoreans,37 but in another paragraph he speaks of “participation” (μετοχή),38 and in Sextus too the concept of participation plays a central role in both exposition and criticism.39 In any case, all agree that the causes are, according to Pythagorean teaching, incorporeal40—a formulation that Poseidonius, too, could accept.41 There is thus unanimity that the sensible world is derived from higher principles.

28 1.3.8. This report is linked by the expression πάλιν δέ to another one: “The first principles, he said, are the numbers and the proportions contained in them, which he also calls harmonies, and the compouds of these two are the elements, which are called geometrical figures (γεωμετρεῖα)” (cf. Cic. Acad. 2.118). Similarly in 1.10.3, “Pythagoras posited the so-called forms . . . in the numbers and in their harmonies and in the so-called geometrical figures, as inseparable from bodies.” This is confronted with Plato’s “separation” (1.10.3), so here the numbers of Pythagoras are being interpreted as immanent. The “geometrical” construction of the elements out of numerical proportions presupposes Tim. 53b. Now the whole section begins with the φαντασία legend of Heracleides (fr. 87–88 Wehrli; Burkert, Hermes 1960, 159ff), which also had something to do about “numbers” in the world (at least if Tim. 53b is even partly based on Heracleides [Burkert, ibid. 161ff]). Thus one may conjecture that this passage of Aëtius is based ultimately on material from Heracleides—who was influenced by Platonic ways of thinking.

29 Aët. 1.7.18 with ps.-Galen. Phil. hist. 35 (Dox. 618.12).

30 438b14: μονάς μὲν γὰρ . . . ἄνωθεν ἕν τοῖς γονίστις άθανάτα, ἐν δὲ τὸ εὐ τοῖς ἀρθροῖς ἄθανάτων διάδομα δὲ καὶ δύο τὸ ἐν τοῖς ἀρθροῖς. Immnisch omitted this second sentence as extraneous and commentary, obviously by oversight. The discrepancy between ἀρθροῖς and ἀρθροῖνς cannot be removed by a simple correction. There ought to be a three-part gradation: (1) the principles Monad and Dyad, (2) the numbers 1, 2, 3, etc. (for this differentiation between Monad and Dyad on one hand and the one and the two “in the numbers [ἐν τοῖς ἀρθροῖς],” Sext. Emp. Math. 10.276, Lydus Mens. 2.6, Philo Qu. in Gen. 4.110), and (3) countable objects, ἀρθροῖνς. (Detailed differentiation of numbers and countables in Sext. Emp. PhI 3.157ff, Math. 10.284ff, 4.11ff; Thea Sm. 19.157ff.)

31 To be sure, the Indefinite Dyad is derived from the Monad at Sext. Emp. Math. 10.261, and less clearly at Hypomn. 25, though there is no suggestion of this in Photius and Aëtius, any more than in the reports about the ὁ θεὸς. Cf. above, n. 6.

32 In the reports on Plato the word is ἐν, though Xenocrates fr. 15 has ἔοικε.


34 Met. 98b26ff.

35 For the concept of ἐνδόθ Πυthagoras is grouped with the Stoics at Aet. 1.9.2: “Thales, Pythagoras, Anaxagoras, Heraclitus, and the whole flock of Stoics (ῥημάδος, Theodoret) say that matter can be turned and altered and transferred, and that all of it is always flowing and intermingling.” Cf. Hypomn. 25, on the elements; Aët. 1.4.3: “matter is passive”; cf. 1.16.1; Sext. Emp. Math. 10.277, 9.366, etc. This too suggests the Timeus; cf. Arist. fr. 207 (below, n. 164), Xenocrates fr. 28 (below, n. 124). At Aet. 1.23.1 the definition of movement as “a difference or otherness in matter” is ascribed to Pythagoras and Plato; Eudemus fr. 60 attributes this definition to Plato, and distinguishes him in this from Archytas (above, ch. 1, 2, n. 106).

36 Above, ch. 1.2, n. 95.

37 1.10.2, cf. above, n. 28. In the anonymous Prolegomena in Platonem 5.36–38 (p. 201 Hermann) it is stated that Plato put his ideas ἐν τοῖς παραδειγματικοῖς, Pythagoras and Aristotele ἐν τοῖς ποιητικοῖς.

38 1.11.13.

39 Sext. Emp. Math. 10.261: “(the monad) by participation in which each thing is called one (ἐν ἕλεγχῳ).” From a similar source, Theo. 21.2 has “(the monad) . . . for by participation in it each thing is called one.” (In the sequel this is contaminated with different matter. The whole may come from Modateus, to whom Stob. 1 prom. 8 attributes the beginning, and Theo. 21.2 the passage 18.3ff.) Its model is to be found in Encl. 7.1.1: "monad: that according to which each existing thing is called one." The "Pythagorean sources" introduce the idea of participation, and then Sextus’ criticism (Math. 10.284ff) concentrates on this point.

40 “The first causes are incorporeal,” “Aet. 1.11.5; more fully Sext. Emp. Math. 10.250ff, 10.261, 104; “the monad . . . that is among mentally apprehended things,” Anon. Phot. 438b14: “all bodies come from the monad,” 439a3ff. In Hypomn. 25, the "perceivable bodies" only come into being after point, line, plane, and (geometrical) body. Philop. (De an. 82.19) significantly names Pythagoreans and Xenocrates (fr. 29) together as proponents of "incorporeal principles" (ἀπαράπτωτα ψυχή).

41 Sext. Emp. Math. 7.119; cf. above, n. 19.
and there is presupposed an ontological gradation like that of Plato’s system of derivation. This too contradicts the statements of Aristotle, according to which the “so-called Pythagoreans” did not go beyond the sensible.\(^4\) What the Hellenistic tradition presents as the philosophy of Pythagoras is, according to Aristotle’s positive pronouncement, Platonism and not Pythagoreanism.

This tradition about Pythagoras, inconsistent with Aristotle, apparently came to dominate the field completely.\(^4\) A few indications will show this well enough.\(^4\) The Platonist Eudorus of Alexandria (first century B.C.) says that, “according to the highest logos,” the first principle of the Pythagoreans was the One, and “according to the second logos,” an “opposite nature” along with the One. The opposition is developed with a “table of opposites,” and summarized in the formula “One (or Monad) and Indefinite Dyad.”\(^4\)

Moderatus transposes the Pythagorean number doctrine into logical-conceptual language,\(^6\) but starts from the presupposition that One and Dyad are the principles. He elicits from the Parmenides a doctrine of the stages of the One which Plato is supposed to have taken from the Pythagoreans.\(^7\) According to Hippolytus, Pythagoras posited the “unbegotten (ἀδέλφος) Monad” as first principle, from which the Dyad and the other numbers had their origin.\(^8\) On the other hand Numenius ascribes a radical dualism to Pythagoras: the

Indefinite Dyad (duitas indeterminata) was coeval with the divinity, the Monad, and the relation of the two is that of form and matter (species and sylva).\(^9\) It is clear that even the more detailed expositions of the neo-Pythagoreans rely on the same tradition; the unmistakable Indefinite Dyad shows that the whole construct is, if Aristotle was right, Ἐνθα Πλάτωνος.

In fact, people did realize the inconsistency with the reports of Aristotle. This appears not only from the polemics of the neo-Platonists,\(^5\) but from a curious document, attributed to Theano, who was usually known as the wife of Pythagoras.\(^5\)

I have learned that many of the Greeks suppose Pythagoras said that everything came to be from number. This statement, however, involves a difficulty—how something that does not even exist is even thought to be real. But he did not say that things came to be from number, but according to number. For in number is the primary ordering, by virtue of whose presence, in the realm of things that can be counted, too, something takes its place as first, something as second, and the rest fall in order.\(^5\)

Here we have the neat separation of ἀριθμός and ὁ ἄριθμος, the concept of μετονομή ("presence" above), in almost verbal agreement with formulations in Sextus and Theo,\(^8\) and in addition direct polemic against a tradition according to which things come to be "from numbers." Many of the Greeks "have learned this false representation of the Pythagorean doctrines; here there is unmistakable polemic against Aristotle.\(^4\) The method is subtly indirect: an "original" document is witness against his interpretation. And who is qualified to offer authentic exegesis, if not Pythagoras' wife and student herself? To be sure, Syrianus was in a still more favorable situation; he read the Περὶ διώκου of Pythagoras himself, which named Ἐρατόσθενος (Aristotle) and Δαύς

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\(^{4}\) Above, ch. I 2, nn. 15.

\(^{5}\) Theo, ch. 17, 20ff, that the "later ones" had introduced Monad and Dyad as principles, but that the "disciples of Pythagoras" (οἱ ἀπὸ Πυθαγόρου) had posited "all the terms set out in series, by virtue of which even and odd are conceived," i.e. the system of number concepts. The sharp distinction between τὸ δὲ αὐθορυσία τρία καὶ δύο, between ἀριθμόστατον and ἀριθμός (above, n. 30), shows that this is not in agreement with Aristotle. Rather, these are Moderatus' own views (cf. Por. VP 49–51). The historical form of his statement is obviously based on the accidental collocation in the doxography (Aët. 1.3.8; above, n. 28).—Limit and Unlimited became important again especially after Porphyry's commentary of the Philebus; cf. Iam. Comm. math. sc. pp. 12ff, and passim, Procl. In Eucl. 5.14ff and passim; Nicomachus (Ar. 2.18.4) brings in Philolaus (below, ch. III 1).

\(^{6}\) Collected by Zeller 146ff, cf. De Vogel, Pythagoras, 207ff. See esp. Plat. V. Hom. 145 (on the source question, Diels, Doss. 88ff); Nicom. Th. ar. 11.16, 12.3ff (the Indefinite Dyad as ἀλέθεια καὶ πληναυσμός καὶ ἀθάνατος, Por. VP 38, Iam. In Nic. 61.21. Cf. also Festugière, Rév. IV 18ff.


\(^{8}\) Por. VP 48–51; cf. above, n. 43.

\(^{9}\) Por. ap. Simpl. Phys. 230.34ff.

\(^{10}\) Cf. below, ch. II 2, n. 28.


\(^{12}\) Above, ch. I 2, nn. 22–23.

\(^{13}\) Stob. 1.10.13 = p. 195.12 Thesleff. It is a remarkable idea, that number "does not even exist," in blatant contradiction of all neo-Pythagoreanism and Neoplatonism. The unheistating equation of being and corporeal being shows that the passage is Hellenistic in date.

\(^{14}\) Above, n. 39.

\(^{15}\) Probably against the lost writings on the Pythagoreans rather than the didactic works, which can scarcely have had much influence outside the school. The word "beget" is a barbed reference to Aristotle's accounts (Met. 1091a13ff).
as first principles and made number, the “ruler of shapes and forms,” the basis of the “origin of all things.” Aristotle, accordingly, must have been mistaken... This is how the gap between the different traditions created the demand for apocrypha.

It is possible that the Pythagoras items of the Vetusta placita were “reconstructed” in the circle of Posidonius or Antiochus, with the help of apocryphal writings. But these are only intermediate members in a chain of tradition that goes back further, to weighty authorities. Theophratus remarks in his Metaphysics, on the problem of the good:

Plato and the Pythagoreans make the distance (ἀπόστασις) a great one, but they think everything wants to imitate it. And yet they set up a kind of antithesis, of the Indefinite Dyad and the One. In the former resides the Unlimited, the disorderly, and, so to speak, all kinds of formlessness, yet the nature of the whole would not be possible without it.57

Theophrastus’ starting point is that Plato and the Pythagoreans, both, posited a “good” at a “great distance” from nature as a whole. Everything strives toward it, “and yet” they cannot get along without an opposite principle. The opposition is explained as that between the One and the Indefinite Dyad, in which all is unbounded and unordered, by its nature, is pitted up. Both the term Indefinite Dyad, the whole idea of a principle “in itself,” and the opposition of “formlessness” and the form-giving One are according to Aristotle Platonic and not Pythagorean; still Theophrastus sees the two as a unity.58 So the non-Aristotelian tradition, which is orthodoxy for later genera-

55 Syrian. Met. 9.37ff, 123.1ff. On the reconstruction of this Sacred Discourse, see Delatte, Litt. 191ff, p. 164.21ff Thieslff.
56 Diels (Dox. 181), following Zeller (I 467ff), derives Atticus’ testimonies, so far as they are not from Aristotle, from the pseudepigrapha. (On Posidonius, cf. below, n. 133).
57 Met. 1127ff. The Ross-Forets translation takes ἀπόστασις as “the distance between the real and the things of nature,” but the point of the whole thing is the discussion of ἀδιάβολο and καλόν. Speusippus has it that ἀδιάβολο is σπάνιον and the καλόν is greatly preponderant (1121ff); Plato and the Pythagoreans make the distance between the two great, though the second principle is indispensable in the world, ἰπποτέρως ἧ καὶ ἀπόρφηρες, 1126. Theophrastus sets his own conviction against both of these, that τά μὲν οὖν ὄντα καλόν ἑτερον ὀντα (1125ff).—Eudemus, too, after first distinguishing the two, comprehends both Plato and the Pythagoreans under the concept of the indefinite (above, ch. I 2, n. 100).
58 Theler, Parva p. 204, reads κατὰ τοὺς καθαρὸς ἀντίθετος τῶν ποιῶν, and opines that this formulation does not include the Pythagoreans. However, it also does not exclude them, and in content the κατὰ sentence belongs very closely with the preceding. The ἀδιάβολο is the model of everything real, “and yet” (read ἀδιάβολ not instead of ἡ ἄκολο) it is not sufficient by itself; the opposite principle is equally necessary. (Cf. below, n. 66.)

...tions, is already present in Theophrastus, even though he speaks of Pythagoreans and not of Pythagoras. His contradiction of Aristotle may not be explained away as concision of language or hasty composition.59

Since Theophrastus was surely only a transmitter of this conception of Pythagorean doctrine, he must be dependent on predecessors who went still further than Aristotle60 in connecting Plato with Pythagoreanism—to the point of identification. The only candidates would be Plato’s immediate pupils, the members of the Old Academy. As a conjecture, this suggestion was made many years ago;61 the proof came to light with a fragment of Speusippus which was first published in 1953. It had made its way into Proclus’ commentary on the Parmenides, whose final portion is only preserved in the Latin translation of William of Moerbeke.

For the good neo-Platonic thesis that the One is still higher than Being, Proclus refers to what Speusippus had reported as the opinion of “the ancients” (“tamquam placentia antiquis,” i.e. ὃς ἀρέσκοντα τοῖς παλαιοῖς):

le unum enim melius ente putantes et a quo le ens, et ab ea qua se secundum principium habitudine (i.e. καὶ τῆς κατ’ ἄρχην Ξεισου) ipsum liberaventer. existimantes autem quod, si quis le unum ipsum securum et solum meditatum sine alio secundum se ipsum ponat, nullum alterum elementum ipsi apponens, nihil utique fict aliorum, interminabilem dualitatem entium principium inducerunt (i.e. τῆς ἀρχῆs τῆς τῶν ἄρχης εἰσόηγου).62

It is clear that as far as Proclus could tell, Speusippus was speaking simply of “the ancients (οἱ παλαιοὶ).”63 But there is only one possible interpretation of this. Speusippus is not presenting his own system, in

59 This is Zeller’s proposal. Speaking of Theophrastus, he says (I 472 n. 4), “... in the whole report Pythagorean and Platonic material is combined in such a way that it appears impossible to determine, from it alone... what belonged to each.” Aristotle does say, however, that the Indefinite Dyad is not at all to be attributed to the Pythagoreans.
60 Met. 9872aoff (above, ch. 1 2).
61 Cf. Frank 260.1, though he overlooks the difference from Aristotle.
62 Plato Latinus III: Parmenides: Procli commenationis in Parmenides interpr. G. de Moerbeke, ed. R. Klambi and C. Lobowski (London, 1953) 38ff. Klambi reconstructs the Greek text, p. 86. He suspects that the source is Speusippus’ On Pythagorean Numbers (I 4 Lang), with Nicomachus as intermediary; but this cannot be proven. Merlan has shown (PHNepol 96-140) that Lamblichus, in Comm. math. sc. 15ff, was able to use Speusippus in detail, perhaps through some intermediary.
63 Proclus repeats, at the end of the quotation: "testatur et iste hanc esse antiquorum opinionem."
which the Indefinite Dyad was replaced by the concept of "plurality (πλήθος)," along with other modifications. 64 But he cannot be using the expression "the ancients" of Plato either, even disregarding the plural form; he only outlived Plato eight years. Therefore Speusippus was referring to the Pythagoreans, and possibly thinking of Pythagoras himself. Plato himself speaks of Pythagorean views as an "ancient" revelation. 65

Speusippus then, quite contrary to the statements of Aristotle, attributes to the "ancient" Pythagoreans the opposition of "One" and "Twoness," the concept of the Indefinite Dyad, and in general a principle ἀπλός καθ' ἀπλό καὶ χωρίς οικόμενον ("ipse scorsum et solum meditatum"). The line of thought is so close to that of Theophrastus, who was a generation younger, 66 that we can no longer harbour any doubt as to the source from which the latter acquired his non-Aristotelian conception of Pythagorean doctrine. Plato's nephew and successor claimed that the basic thought of the Platonic doctrine of ideal numbers was Pythagorean.

Speusippus was not alone in this trend. Xenocrates, too, wrote Πρωταγόρεια, 67 and we have one statement about Pythagoras from his pen: "Pythagoras, as Xenocrates says, discovered that the musical intervals did not originate without number, either." 68 Xenocrates, then, attributed to Pythagoras himself a specific scientific discovery about numbers in music, but even more than this: the word καὶ ("either," above) indicates that he traced to him some other number theory as well. 69 Now Xenocrates interpreted the origin of the world soul in the Timaeus as derivation of number from One and Indefinite Dyad, 70 and the connection of number and music, which according to Xenocrates Pythagoras also discovered, is definitely presupposed in this section of the Timaeus. 71 Furthermore, Xenocrates developed from the Timaeus his definition of soul as "number moving itself," 72 and precisely this definition of soul is ascribed by the doxographers to Pythagoras, 73 as well as the doctrine of the One and the Indefinite Dyad. The later tradition about Pythagoras is largely based on the exegesis of the Timaeus by Xenocrates, who understood the ideas contained in Plato's dialogue as the teaching of Pythagoras.

Speusippus, too, interpreted the Timaeus and developed a definition of soul out of it, 74 which was taken over by Posidonius, 75 namely that the soul is ἑδα τούτι πάντω διαστατοῦ, and Posidonius adds καὶ ἀρμόνια συνεστώσα ἄρμονίας περιέχουσα. Posidonius interprets the Timaeus as evidence for Pythagorean doctrine, 76 and in this too he is following Speusippus, for the new fragment is proof that the latter saw Platonism and Pythagoreanism as a unity.

A third writer deserves mention in this context, Heraclides Ponticus. He attributed to Pythagoras the invention and definition of the word φιλοσοφία, and this ascription made its way, via the doxographers, into all the ancient handbooks. Yet it is disproved by the semantic history of the word. It had meant close acquaintance and familiarity with φιλοσοφία; and Plato was first to define it as in insatiable striving, and set it in opposition to the possession of wisdom. This was after the Sophists and their claims had roused popular animosity. 77 Heraclides probably combined with this a number theory that took its direction from the Timaeus; at least he ascribed to Pythagoras the sentence, "The knowledge of the perfection of the numbers of the soul is happiness." 78

Interpretation of the Timaeus and the orally transmitted theory

64 Speusippus did, however, come out for placing the One above Being (fr. 346 — Arist. Met., 1082a14ff., and the testimony discovered by Merlan, Phileg. 96ff., in lamblichus). A point of departure for this line of thought can be seen in Pl. Rep. 509b, and better in Plato's Parmenides; cf. Dodds, CQ 1928, 129ff.
65 1082b16; below, ch. 1.4
66 The elevation of the One in Speusippus corresponds to the "great distance" of Theophrastus. Both emphasize the necessity of the second principle, without which "nihil utique fieri ahorum," οὐχ οἷς τε ἄνευ ταύτης τῆς τῶν ὅλων φύσεως, as in Arist. Met. 1088b3ff.
67 101.4.13
68 Πρωταγόρεια, ὁς φησί ξενοκράτης, ἐνταξεὶ καὶ τὰ ἐν μουσικῇ διαστάσει τῆς χωρίς ἀρμονίας ἐναντία, fr. 9 Henne — Proc. in Phil. 30.8ff. It is unlikely, in view of Henne's note, that more than this sentence goes back to Xenocrates; cf. below, ch. V.3.
69 Cf. Sext. Emp. Math. 10.266, who gives it as Pythagorean teaching that ὁ ἁπλὸς γραμμή ὑπὸ χωρίς ἀρμονίας νεκρήσεια...
70 Fr. 6B II. Plut. De an. procr. 1012b-c; following this, Tim. Lect. 95c.
71 Cf. below, ch. V.1.
73 Aetius 4.2-3, from which is derived Theodoret 5.17. Also Nemesius 102 M., Meletius An. Ox. III 146.30; cf. Cic. Tus. 1.120 Aetius adds that Xenocrates is following Pythagoras (4.2.4, from Stob.).
74 Fr. 40 Lang — lam. ap. Stob. 1.49.32; cf. the discussion of Merlan, Philologus 1934, 197ff.; Cherniss, Plato 598ff.; Merlan, Phileg. 36ff. Cherniss sets out the relationship to the Timaeus.
76 Reinhart has to concede this, since the citation of Posidonius at Sext. Emp. Math. 7.93 is made in the section on the Pythagoreans.
78 Heraclides fr. 44 Wehrli — Clem. Al. Strom. 2.130.3; above, n. 28.
of ideal numbers combine, for Plato’s pupils (including Aristotle), into a system which for the Platonists is regarded as the doctrine of Pythagoras, which they themselves follow, with more or less radical modifications; Aristotle, on the other hand, characterizes the same system as Platonic and contrasts it with other, obviously older, less advanced pronouncements which he calls the doctrine of the “Pythagoreans.” The later tradition, beginning with Theophrastus, follows the Platonists, and to a great extent forgets the reports of Aristotle. Not completely, to be sure: that which Aristotle ascribes to the “Pythagoreans” in general, appears in the doxographical reports under the name of Philolaus.

This relationship of the different branches of tradition can also be shown in the further development of the doctrines. The derivation of the world from number takes place, according to “Pythagoras,” by the development of the point (“a monad having position”) out of unity, and then of the perceptible world from the point in the hierarchical succession line, plane, solid. As a rule the number 2 is assigned to the line, 3 to the plane, and 4 to the solid. Sextus emphasizes the difference from another derivation, according to which the line comes to be through a continuous movement (βόης) of the point, the plane through movement of the line, and the solid through movement of the plane. Here there is no need for any other “first principle” than the One. Yet the two ideas get mixed together.

We have shown that this derivation of the physical world is firmly anchored in the Platonic number theory; it remains to show that none of all this is ascribed to Pythagoreans by Aristotle, and that it even contains contradictions of what is—according to Aristotle—Pythagorean.

In his philosophical glossary Aristotle takes over as valid the definition of the point as “a monad having position” and mentions it often, from the point of view of “separation,” as a formula used by the Platonists. He never connects it with Pythagoreans; and he cannot do so consistently with his allegation that the “units” of the Pythagoreans possess “magnitude,” for “point” and “magnitude” are mutually exclusive terms. For this very reason the derivation of the physical world through the series surface, line, point cannot belong to these Pythagoreans; the world of perceptible bodies is for them the only reality, beyond which they do not go. Thus Aristotle never connects the line—plane—solid formula with Pythagoreanism, but always with Platonism. Indeed there is one passage in which he clearly points out the difference. In his De caelo he explicitly disagrees with all those who “make up all bodies from planes” (290a1ff), and an express reference (300a1) shows that he has the Timaeus especially in mind. He asserts that to be consistent one must carry such an analysis further, to the point, not only as far as the ἀπὸ πρῶτου διαστασιν, this is directed against Plato and Xenocrates. The refutation revolves about the problem of weight, which remains unexplained in these geometrical constructions; and he appends the casual remark, “and the same thing happens to those who construct the world out of numbers: for there are some who see nature as composed of numbers, like some of the Pythagoreans”—here too the problem of weight is untouched. The Pythagoreans who “compose the world out of numbers” are thus contrasted with “those who compose bodies of

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79 Cf. connection of the Timaeus with the δύραδα δύσματα at Phys. 205b11ff, and the sequence Timaeo—ideal numbers, De an. 404b18ff, which is found again in Posidonius (above, n. 21).
80 Procl. In Iul. 95.21ff, Schol. Arist. Met. 40144 (as Pythagorean), and the passages cited in n. 82.
81 Hypomnem. 25, Anon. Phot. 439a10ff. (Skipped by Act. I.3.8.)
84 Thus in Posidonius, already, the “nut game” and the “flowing” obviously belong together. Cf. Sext. Emp. Math. 7.291f, Philo Op. 49f, Deal. 25, Plut. De F 390c-d.
85 Above, ch. I 1, nn. 41-45.
87 Phys. 212a22ff: “if, as they say, the point and the monad have a separate existence of their own,” Met. 1069a12, 1072b30, 1084b23 ("if number is separable"). Top. 108b7ff applies to Speusippus (cf. below, n. 105); against attribution to the Pythagoreans, Heidel, AJF 1940, 27, n. 36, Wilpert Zfr 1918 218 n. 46.
88 Above, ch. I 2, n. 18.
89 Above, ch. I 2, n. 15.
90 The evidence for Speusippus and Xenocrates, above, ch. I 1, nn. 38-39. That the πρότερον doctrine of Met. 1028b10ff is not applied to the Pythagoreans was shown above, ch. I 2, nn. 77-81. The passage Met. 1036b24ff is almost always thought to refer to the Pythagoreans: καὶ ἀνάγωγον πάντα ἐκ τῶν ἀριθμῶν, καὶ γραμμής τῶν λόγων τῶν τῶν δύο ἐννοιας. (Ps.-Alex. Met. 512.23ff—"two" as ἐν τοῖς ἀριθμοῖς, line 37. —Ross, Met. II 202 sees the source as Aristotle’s lost book. KR accepts it [as pre-Parmenidean!], no. 317 [cf. 316]. Also called Pythagorean by Zeller I 511 n. 1; von Fritz, ABG 1935, 83; Chremis, Psev. 225; Guthrie I 257.) One reason is that this is set up against of ἀρίθμων ἔννοιας; but it is Speusippus who does not believe in ideas, but only in mathematical numbers. In the context, the point is the separation of form and matter (ὅλωσις, 334, 84b7; ἐφαρμοσμένα, b2), so that the Pythagoreans cannot be meant.
91 Above, ch. I 1, n. 17.
The related thought, that the point in movement makes the line, the line in movement the plane, and the plane in movement the solid, appears in Aristotle’s polemic against Xenocrates. He seeks to reduce the latter’s definition of soul to absurdity by pointing out that according to these geometrical doctrines—introduced with the words ἐπὶ τὸν ἑκάτερον (409a4)—even the “self-moving number” would have to consist of lines. In order to demonstrate a contradiction, Aristotle seizes upon a statement of the opposite party—the Platonists.

Archytas had evidently gone deeply into stereometric problems; Aristotle’s statement that “the Pythagoreans” called a surface χρωμα (Plato is said to have introduced the term ἐπὶ τὸν ἑκάτερον) may belong in this context. According to Diogenes Laertius, Archytas introduced the idea of movement into geometry, and in fact Archytas’ solution of the “Delian problem,” reported by Eudemus, makes good use of such concepts: a right-angled triangle, revolved about one leg, produces a cone, the end of a straight line, when turned about, describes an arc, etc. Archytas, according to Eudemus’ evidence, also concerned himself with the explanation of movement in general. Thus the conjecture seems justified that this comprehensive treatment of geometrical magnitudes by the use of the idea of movement comes precisely from

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88 Bywater (31) already perceived this difference, and used it against the authenticity of Philolaus fr. 12.
89 Cf. above, n. 83.
90 De an. 409a1ff.; cf. Cherevis, Pres. 389, Plato 396 n. 322, Theiler, Arist. 101, Guthrie 260ff. Whether Aristotle is playing off the conception of Speusippus against Xenocrates (Cherevis), or whether Xenocrates could have spoken in this way himself (Theiler refers to fr. 39, though it is “not quite uncontaminated”)—this is a question we need not answer.
92 D.L. 3.24. The concept of ἐπὶ τὸν ἑκάτερον is presupposed by Plato, Meno 76b and Pol. 290e; in a different sense Th. 173c – Pi. fr. 292, as well as Democrit. fr. 11d Aetius ἐπὶ τὸν ἑκάτερον (“terrestrial,” as distinguished from 11c “aerial”; 11b is not a direct quotation). The idea of χρωμα is alluded to at Pl. Meno 75b-c. C. Mugler, “Sur l’histoire de quelques définitions de la géométrie grecque, II: La surface,” AC 27 (1958) 76-91, presents an ingenious but ill-founded reconstruction; he puts the χρωμα idea earlier than Parmenides.
93 8.83 (referring to the problem of doubling the cube): πρῶτος τῷ μηχανικῷ ταῖς μαθηματικῶς προσαρμισμέναις ἁρμαία μεθοδίαι καὶ πρῶτος κόσμος ὑπόκλισιν διαγραμμάτων γεωμετρικῆς προσάρμοσις.
94 Eudemus fr. 141 Welthu. DK 47A14.
95 Above, ch. I 2, n. 100.

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100 Above, ch. I 2, n. 100.
101 Frank 370ff, accepted by Wilpert ZbF 174.6. Frank goes on, in an imaginative vein, to construct a “dynamic system of Archytas” (124ff), but far exceeds the limits of what can be known of the matter. Cf. Cherevis, Pres. 388ff.
102 Cf. above, ch. I 2, nn. 104-106.
104 “Pure” mathematics, divorced from ontological problems, is hardly to be assumed in Archytas. Arnothius 2.9 (Dox. 172) names Pythagoras and Archytas: “causam in numeris posita.” But Plato’s philosophy, through dialectics and theory of ideas, was bound to change whatever he took over. The connection of the numbers 2, 3, and 4 with line, plane, and solid, which seems more primitive than the motion idea of Archytas, might be an attempt to derive these concepts by purely logical means, without introducing motion.
105 For Speusippus the point is οὐκ ὀν μετὰ (Arist. Met. 1085a33): ὅτι σημεῖον ἐν γραμμῇ καὶ μονᾶς ἐν ἀριθμῷ ἐκτέταρτον γιὰ δῆλον (Top. 108b26; cf. above, ch. I 1, n. 40). Quite similarly, Sext. Emp. Math. 10.278 has τὸ σημεῖον κατὰ τὸν τῆς μονᾶς λόγον τετάρατόν. 
106 Fr. 4. Lang = Th. ar. 82.10ff (= DK 44A13, KR 319); the sentence cited is at Th. ar. 84.10ff. KR, p. 355, have “little doubt” that Speusippus is reproducing genuine and early Pythagorean doctrine, and Guthrie agrees (I 269). Saffrey would prefer to brand the fragment a neo-Pythagorean forgery, but there are no arguments for this—not even the absence of the title from the list of Speusippus’ writings in D.L. 4.4, for this is expressly labeled as incomplete.
107 575.
108 Cf. above, n. 62.
cognition, which in Platonism is set up as parallel to the hierarchy of geometrical solids, and also with the numbers from 1 to 4 (1 νοῦς, 2 ἐπιστήμη, 3 δόξα, 4 αἰσθήματα), likewise makes its appearance as the teaching of Pythagoras, the Platonic system, in all its aspects, is being passed off as Pythagorean. Here too, along with partial agreement, we find a contradiction to the statement of Aristotle that the Pythagoreans equated δόξα with the number 2. The relationship between Xenocrates and the series of types of cognition, and a certain kinship between them and the Timaeus, have already been mentioned.

But we must go over the same ground once more. “Pythagoras” taught, according to Aëtius, that earth came to be from the cube, fire from the pyramid (or tetrahedron), air from the octahedron, water from the icosahedron, and finally, from the dodecahedron, the “sphere of the All.” The agreement with the Timaeus, extending to actual verbal coincidence, is obvious. Less striking, but no less clear, is the contradiction of what Aristotle says about the Pythagoreans: “They have not said anything at all about fire or earth or the other material things of this sort, because, I suppose, they did not have anything distinctive to say about perceptible things.” Here we may add an item from the scholia to Euclid, possibly from Eudemus, whose significance was seen by Eva Sachs. The Pythagoreans, it says, treated of only three regular solids—pyramid, cube, and dodecahedron, and Theaetetus was the first to add the octahedron and icosahedron. But again Speusippus, contra Aristotle, treated the derivation of the elements from the regular solids as Pythagorean; he wrote in his book On Pythagorean Numbers, “... and about the five shapes, which are given to the cosmic elements” (Th. ar. 82.17 DeFalco).

The keystone of the argument is the declaration of Aëtius, “Pythagoras says that the cosmos is a created thing (γεγονός) in the realm of thought but not in that of time (και ἐπίνοοι ... στὸ κατὰ χρόνον).” “Pythagoras” is here taking a position in the controversy which developed over interpretation of the Timaeus, whether the creation of the world by the Demiurge is to be understood literally. Speusippus and Xenocrates asserted that it was merely a mode of exposition διδασκαλία χάρων, an essay in mental construction (ἐπιστροφή). It is this that gives point to Aristotle’s polemic, “There ought to be no difference of opinion as to whether or not the Pythagoreans posit generation (γένεσις) or not; for they say clearly ...” Plainly, Speusippus and Xenocrates gave up their interpretation of the Timaeus as Pythagorean doctrine, and Aristotle is refuting them with the actual words of a Pythagorean document. Aristotle already presupposes and rejects the conception of Pythagoreanism that makes its appearance in the later doxographical tradition. It is that of Speusippus, Xenocrates, and Heraclides, which amalgamates interpretation of the Timaeus with the Platonic “system of derivation.”

There remain to be discussed a number of reports in the doxography, most of them concerning less vital questions, that cannot be placed in direct connection with the evidence of Aristotle; the latter did not have occasion, of course, to mention every detail of Pythagorean teaching. For these additional testimonies too, however, it can be shown that they go back to Plato’s disciples, or, insofar as they preserve older material, were transmitted by them.
The kernel of Pythagorean wisdom is the "tetractys," or "four-group," made up of the numbers, 1, 2, 3, 4, which add up to 10. They are represented in a pebble figure, in the form of the "perfect triangle;"

and the available sources, from Posidonius on, show how these four numbers contain not only the basic intervals—fourth, fifth, octave, and double octave—but also, according to the Platonic pattern, point, line, plane, and solid.126 The harmonic ratios, the "perfection" of 10, and the role of the pebble figures are all part of what Aristotle attributes to the Pythagoreans; but the derivation of spatial shapes is in implicit contradiction to what he says.121 Speusippus' book On Pythagorean Numbers, on the other hand, presented this series; and half of his presentation was devoted to the number 10, with emphasis on the asseveration which recurs again and again in the literature on the tetractys, that all people, Greeks and barbarians alike, count to 10 and then return to 1.122 But there is a relation to Xenocrates as well. It is frequently mentioned that the Pythagoreans, in their obsession with the tetractys called it the "fourth of ever flowing nature (σωγάν δενάν φύσεως)."123 But Xenocrates called matter δέναν.124 and, though this may also hold an allusion to an expression of Plato's, it is tempting to believe that the verse on the tetractys was known to him.125 For another verse, too, is commonly cited in the literature along with this one: ὁρμή ἐκ τε πάντων ἐπόμενος126 and Themistius cites this very line in a passage where he refers to Xenocrates' book On Nature.127 Though a good many bits of Themistius' own wording may have worked their way in, alongside the paraphrase of Aristotle, still he was well informed about Xenocrates.128

Thus from an element of Pythagorean tradition which at first seems to contain an ancient, pre-Platonic kernel, a clear trail leads us back to Speusippus, and a somewhat less clear one to Xenocrates. It is the latter to whom the connection seems closer in the matter of the daimones. It is quite probable that Pythagoras himself spoke of δαίμονες, but the process of transition from religious experience and teaching to systematic philosophy is obscure. In any case, the expressions in the later tradition, about daimones as souls or as some kind of intermediate being, were the work of Platonists. Aëtius (1.8.2) names Thales, Pythagoras, Plato, and the Stoics as advocates of doctrines about daimones. Plutarch is more cautious: in two parallel passages he names, first, "Plato, Pythagoras, Xenocrates, and Chrysippus" (De Is. et Os. 25.360d), and then again only "Plato, Xenocrates, and Chrysippus" (De def. or. 17.419a). That Pythagoras is not named first, in despite of chronology, or even may be left out, suggests the conjecture that what is meant is really "Pythagoras as cited by Xenocrates."129 If, as Aristotle says (fr. 193), the Pythagoreans considered it the most natural thing in the world for a person to "see" a daimon, and if they also saw the myriad of souls dancing as motes in a sunbeam,130 this is a long way from the Platonic concept of immateriality. There is also the graded series god-daimon-hero-man, which was attributed to the

120 Sext. Emp. Math. 7,94–100, with the parallels (above, nn. 8, 12–14). Aët. 1,3,8, Theo Sm. 87,57, 93,71ff, etc. The most detailed treatment is that of Delatte, Litt. 499–568. Sometimes it is also called τέταρτος ὁρμή (cf. Thule, Cion 28 [1965] 284f.). Lucian (V. auct. 4) represents Pythagoras as saying, "What you suppose is four is really ten, and a perfect triangle, and our Oath." Cf. also Burnet, EGP 102f, below, ch. II 4.

121 Above, ch. II 4.

122 Th. ar. 83,7: όρμη τε και κατά φύσιν εἰς τοῦτον κατανόμον παντόσον ἀρθρωμένης ἐκλήρην τε κει πάντες ἀνθρώποι, (Arist.) Pr. 15,3,910b23ff: διὰ τα χάρια τὰ τούτα τε μυστήρια, καὶ βασιλέα καὶ Ελλήνης, εἰς τὰ δέκα καταρθήματα... ἐν πάλιν ἀναδιούντως; Sext. Emp. Math. 4,3: ἐπὶ τοῦτον φύσιν παλιν ἀναλύομεν εἰς τὴν μονάδα (at 7,94 the corresponding sentence has dropped out); Philo Op. 47: ἐπὶ τὰ δέκα καταρθηματα εἰς τὴν μονάδα καὶ ἀναδιούντως. Quite similar is Anat. p. 39 = Th. ar. 86,3; cf. Theo Sm. 93,19, 99,17. Anon. Phot. 4302, Hippol. Ref. 6,23,3. There is a remarkably close resemblance between the text from the Pythagoreans and the following of Aët. 1,3,8: μέχρι γάρ τῶν δέκα πάντων ἐκλήρην, πάντες βασιλέα ἀρθρωμένους, ἐφ’ ἂν ἑλέντος πάλιν ἀναδιούντως εἰς τὴν μονάδα. The common source is obviously Speusippus.

123 Below, ch. II 4.

124 Fr. 28 HI. Aët 1,3,2.

125 For this combination, see Zeller II 1,1014, n. 3; Chernes, Plato 484f, Cion 1999, 41, against n. 11, Heinze 14. Delatte is dubious, Litt. 2541. Pl. Leg. 906f. - cf. also the homographic reports on Pythagoras' teaching about ὄνειρα: above, n. 35.

126 Sext. Emp. Math. 7,94, 4,2, Theo Sm. 99,16, Plut. De an. proo. 1020f, Iam. VP 162, etc. Delatte, Litt. 14f, would like to show that the line is from an ancient hexameter Ἴπως δέος, but the paremiae (with a catastrophic dependence on II. 22,71 and Tyrtaeus 7,27) may be independent.


128 Themistius knew this book of Xenocrates, as is shown by fr. 61 Heinze (Cherniss, Plato 399 n. 325, vs. Heinze p. 65). Saffrey (38ff) would like to posit a "neo-Pythagorean" version of the physics of Xenocrates, for to him "Pythagorean" means "non-Xenocratic." But the connections shown here leave no room for doubt; Xenocrates himself "Pythagorizes.”


I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

Pythagoreans from the time of Aristoxenus.131 There is no direct way of checking this; nevertheless, this hierarchical series is common in Plato,132 while Aristotle ascribes to the Pythagoreans a three-term series, with Pythagoras himself as middle member between god and man (fr. 192).

In addition to such beliefs about souls and daimones, the doxographical tradition assigns to Pythagoras or the Pythagoreans the threefold division of the soul into νοῦς, θυμός, ἔπιθυμα,133 and the authority of Posidonius is cited for this.134 But it is apparent by now that this is no isolated phenomenon; in other respects as well Posidonius appears as a link in the Pythagoras tradition.135 Now, this tripartition of the soul is closely related to the theory of the “three ways of life,” which Heraclides Ponticus cites as coming from Pythagoras.136 It is possible that the soul doctrine insinuated its way into the tradition by the same route. Anyway, here too Platonic and Pythagorean material is seen as a unity, and Aristotle’s reports lead in a different direction.137

The Hypomnemata present a somewhat different division of the psyche (30) “into νοῦς and φρένες and θυμός. νοῦς and θυμός are also present in the other animals, but φρένες only in human beings.” Von Fritz has tried to show, on the basis of a thorough semantic investigation, that in this respect the author is reproducing a completely un-Platonic and therefore pre-Platonic doctrine.138 From Empedocles on, φρένες and νοῦς were generally equated, and in all post-Platonic philosophy νοῦς is a specific criterion of humanity; therefore a theory which assigns νοῦς to animals as well must go back to the first half of the fifth century B.C. Also, φρένες, he says, was not used from the fourth century on.

Still, φρένες does occasionally appear in later writings,139 and it remains arguable that at least a certain kind of νοος belongs to animals.140 More important, however: the evidence of the Hypomnemata is not isolated. Aëtius tells us (5.20.4), “Pythagoras and Plato say that the souls of the so-called irrational animals have reason too, but they do not act in accordance with reason because of the faulty composition [mixture] of their bodies, and because they do not have the faculty of speech (τὸ φραστικὸν). We can see this in the case of apes and dogs; for they reason (νοοῦσι) but do not speak (φράζουσι).” What Aëtius says about thinking (νοοῦσι) and speaking (φράζουσι) now makes clear the distinction between νοος and φρένες in the Hypomnemata. There are other ways, too, in which the two sources resemble one another in their treatment of “Pythagorean” psychology.141 In addition, Aëtius speaks of Pythagoras and Plato in a single breath, in this specific context, so that we cannot single out particular un-Platonic or pre-Platonic elements. Here, too, we have an interpretation of Plato that, like Aristotle, restricts immortality to the highest psychic activities, and therefore divides the intellectual segment of the psyche in the same way as the Stoic differentiation between λόγος ἐνδιάβετος and λόγος προφαρμοκός.142 This is not a trace of pre-Platonic Pythagoreanism.

There remain two ancient-sounding reports of Aëtius on doctrines of Pythagoras. The first says that necessity hedges the world about: ἀνάγκην περικείεται τῷ κόσμῳ,143 and the second is a definition of time...

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131 D.L. 8.23, Iamb. VP 100, 144, 175, Por. VP 38; cf. Aristox. fr. 34, Iamb. VP 37, Zaleucus ap. Stob. 4.2.19, Carm. aur. 1-4.
132 Pl. Crat. 397c-e, Rep. 392a, 427b, Leg. 717b, 738d, 801e, 818c, 910a, Epin. 984d.—Inscriptions from Dodona (Collitz II 1582, 1566, 1585b), apparently from Epitrep, Corecyra, and Aetolia, point toward a ritual background.
133 Posidonius in Galen Plac. Hipp. et Plut. 4.7, 5.6 (V 425, 478 K.), with the cautious formulation αὐτῶν μὲν τῶν Πυθαγόρας συγγράμματος οὖν οἶδα, ἐν μὲν δυσανησθάνιον, τεκμαρμάρισμος δὲ ἐπὶ τοῖς μαθηταῖς αὐτῶν γεγραφότοι. Aet. 4.4.1, cf. 7.5.5 (“Pythagoras and Plato”). It is common in the pseudo-Pythagorean literature: Archytas fr. 64 Nolle = Iam. ap. Stob. 1.49.34; Assara (or Aresia) ap. Stob. 1.49.27 = Thesleff, Texts pp. 48f; Metopus ap. Stob. 3.1.113 = Thesleff, Texts pp. 116; Theages ap. Stob. 3.1.117 = Thesleff, Texts p. 190. Cf. also Pollux 2.226.
134 For this reason the tradition is often defended: Burnet, IGP 296 n. 2, Taylor, Tim. 263ff, 406ff, Cornford, CQ 1922, 147. Joly hesitates, pp. 78f, with refs. Philolaur fr. 13 has nothing to do with this (below ch. III 2, nn. 149-157).
135 Above, nn. 8-22, 74-75.
137 Not only the mutes in the sunlight, but also the allusion to metempsychosis at De an. 407b122 (below, ch. II 9). On ψυχή and δαιμόνια, see below, ch. III 2, nn. 165-168.
138 "Νοος, νομον, The Derivatives in Pre-Socratic Philosophy," Phr. 41 (1946) 33f; similarly Wellhünn, Hermes 1919, 235ff; Delatte, V 222.

139 Cf. Festugière REG 1945, 43f; e.g., Chrysippus, SVF III p. 27.8; Anon. Lond. 4.14f, Philo. Po. 5.19, Rhe. 1.240f. When the "diaphragm" is misplaced in the head (Hypomn. 30), we scarcely get the impression of having the ancient or original version ("correctly") Heb. 6: moon — φρή — mid-point — seat of intelligence.
140 According to Aet. 4.5.12, Democritus followed Parmenides and Empedocles in equating νοος and φωτις (cf. Arist. De an. 404a27f). Strato (fr. 107f Wehrli) equates νοος and αἴσθησις; in such circumstances one could scarcely deny the lower animals any share in νοος.
141 Aet. 4.5.10: "Πυθαγόρας τὸ μὲν ψυχικὸν περὶ τὴν καρδίαν, τὸ δὲ λογικὸν καὶ νοερὸν περὶ τὴν κεφαλὴν. Cf. Hypomn. 30: νοος and φρένες in the brain, θυμος in the heart.—Aet. 4.7.5: "Πυθαγόρας Πλάτων τὸ μὲν λογικόν ἀβάρτον, τὸ δὲ λογικόν φάραγον. Cf. Hypomn. 30: τὸ μὲν φράσμαν ἀβάρτον τὸ δὲ λοιπά λατήτια.
142 Aetius cannot be drawing from the Hypomnemata, since he would not have found the references to Plato there. The Hypomnemata are probably older than the Vetiurata platica (cf. the Stoic Συσκεύαση, Aet. 5.20.4).
143 Aet. 1.25.2: Zeller (I 542 n. 2) is inclined to equate Necessity with the fire that surrounds the world.
as “the sphere of the encompassing (σφαιρα του περιέχοντος).” This seems to be known to Aristotle, who refers to it in connection with the definition of time abstracted from the *Timaeus* and dismisses its reasoning, “that everything is in time and likewise in the sphere of the all” as “somewhat simple-minded.” Indeed the association of the celestial sphere and all-embracing time was widespread in the fifth century and evidently reflects a pre-philosophic way of thinking. It makes its way into Pythagorean doctrine as attested by Aristotle: the cosmos breathes in time, along with the Unlimited; it is still discernible in Aristotle’s allusion to the divine *aion* encompassing the world, though transformed by Plato’s distinction between *aion* and *chronos* (*Tim.* 37d). Later tradition points to Orpheus for this imaginative concept: Chronos hatching the world egg. And Ananke, too, occurs in Orphic mythology, expressly associated with Chronos surrounding the world. It is possible that Orphic cosmogonies were exercising some influence as early as the fifth century and that they were in closest connection with Pythagoreanism. The way by which

144 *Aet.* 1.21.1, *Arist.* *Phys.* 218a33 (τὸν χρόνον) οί μὲν γὰρ τὴν τοῦ διὸ κινηον εἶναι φασι, οί δὲ τὴν σφαιραν αὐτόν. For the attribution of the first definition to Plato (*Tim.* 37c-d, 39b-c), cf. *23b21*, *Theophr.* *Phys.* op. fr. 15 (*Dox.* 492), *Eudemus* fr. 82 W., *Aet.* 1.22.1, *Plot.* 3.7.8, *Simplicius* (*Phys.* 700.16ff., 785.14) thinks the definition of “Pythagoras” is to be found in the Categories of the *pr*-Archytas (24,15 Thesleff) (χρόνος) καθ’ ὑδάτι διάστημα τὸ τῶν πατων φύσις. *Speusippus* (fr. 53 L.) and *Xenocrates* (fr. 40 H.) have different definitions of time.—According to Plutarch (*Quaest.* *Plat.* 1007b) Pythagoras defined time as *διὸν ψυχή*. (Cf. the pr.-Archytas definition of time with *Speusippus’* definition of soul, above, n. 74.)

145 *Hermeippus* the comic poet, fr. 4 (= *Stob.* 1.8.36; ca. 430–420 B.C.):

> εἶκεν δ’ ἄτοι αὐτοῖς τῷ διὸν, ὅ ποιρή
> ἄτο οὖσα περιέρχεται κύκλῳ τα ἄστεν οὖν αὐτὸι . . .
> 4 ὁμοίωτα δ’ ἐναντίων, ὅν ὁ δε περιέρχης ταλεύτης
> οὐθένων οὐκ ἄρθρον ἔχει . . .

Cf. *Hebl.* 16: “annus autem, in quo omnia circumvent, habet in se ipsa omnia.”—The heaven is χρόνον ποιεῖται in *Critias* B18 (cf. B15, 33f). In Indian mythology too, Kala or Time is described as being the starry heaven (*Olerud* 135).

146 *Arist.* fr. 201 (cf. *Carl.* 279a11). This is not identical with the Pythagoras testimony *Aet.* 1.21.1: according to these Pythagoreans, the sphere of the universe came to be, but nobody spoke of coming-to-be of time before Plato (*Arist.* *Phys.* 25b17); and the role of the celestial sphere for measuring time was to change radically through the introduction of a moving earth (ch. IV 3).

147 *Carl.* 279a18–83.

148 *Kern,* *Orph.* frg. 54, 57, 70.

149 *Kern,* *Orph.* frg. 54, 126, 162; *Nicomachus* *Th.* ar. 81.19: τὴν Ἀράσογεν οἱ θεόλογοι τῆς πατών οἰκονομήν ἑωτητῆς ἄτος στροφής. The Ananke of Parmenides (B 8.30, 10.6) is assigned to the mid-point of the universe (*A37*). On Orphism and Pythagoreanism, above, ch. I 2, nn. 53–56; below, ch. II 1.

these two isolated statements came to be handed down in the doxography as lore of “Pythagoras” is not clear; but there is one obvious guess: the Plato interpretation of the Old Academy played a decisive role. For as the definition of time (chronos) is treated by Aristotle in the context of exegesis of the *Timaeus*, so ananke belongs to the *Republic*, where the universe is whirling on the knees of Ananke (*616c*).

Finally, there is the basically important doxographical statement that Pythagoras coined the term “cosmos,” ἐκ τῆς ἐν αὐτῷ ἀτέχους (Aëtius). This makes him the creator of one of the most influential concepts of Greek natural philosophy. The history of the word in pre-Socratic philosophy, as far as we can trace it, speaks, however, decisively against the thesis of such a programmatic invention of the concept at the very beginning of Greek philosophy; we rather see the word gradually developing, from the prosaic “arrangement” of things in parts of the universe to the “ordering” of the whole world. There is, nevertheless, again an obvious guess how the alleged invention came to be attributed to Pythagoras: it fits very well into the anecdote of Heraclides Ponticus which had Pythagoras explaining the word *philosophia*—an invention of Pythagoras which also made its way into all the doxographical handbooks.

Nevertheless, it is *communis opinio* that the Pythagoreans at least, if not Pythagoras himself, played a decisive role in the development of the Greek idea of “cosmos.” One always cites Plato, who introduces, against Callicles, the doctrine of “wise men” that friendship and equality hold sway in the world, and that this is why it is called

150 *Clearthus* (fr. 3–4 Wehrli) wrote a commentary on the mathematical passages of the *Republic*.

151 *Aet.* 2.1.1 = *DK* 14.21 (τὴν τῶν διων περιέχοντος), D.L. 8.48 (τὸν ὤρανον); the additional notice about Parmenides relates only to the spherical shape of the earth, as *Diels* saw, *Dox.* 492; Anon. *Phont.* 440a27ff. (τὸν ὤρανον); *Achilles* Is. 5 (τὸ πάν); *Schol.* A 2.1 (ὃ τῶν διον ταῖς); *Iam.* VP 162. On the cosmology attributed to “Pythagoras” in the doxography, below, ch. IV 1.


153 So *Kirk, Heraklitis 313.1*, Kerscheneinerer 229; cf. above, n. 77, and below, ch. IV 1.

"cosmos." Here Plato adduces the evidence of geometry, in a manner that cannot be well explained from the dialogue itself, and there is much to be said for the conjecture that Plato is alluding to the close affinity between geometry and the study of first principles which he had learned from Archytas. It is also possible, however, that he was already foreshadowing ideas of his own, later formulated in the Timaeus; and the close relationship of this passage of the Gorgias to other thinkers of the fifth century, especially Empedocles and Euripides, ought not to be overlooked because of concentration on the Pythagorean allusions. In spite of the distinctly Pythagorean flavor of the

discussion, the mention of geometry certainly does not point back to the oldest stratum of Pythagorean mathematics, in which arithmetic was central. Thus in no instance does the doxography on Pythagoras the philosopher bring us back to solid ground; it all seems to derive from the Platonic exegesis of the Old Academy.

Zeller's solution of the problem of the Pythagorean tradition must, then, be modified. It is not true that we can make a chronological distinction between the oldest and therefore most reliable accounts, those of Aristotle, and a later accretion which cannot be checked and is therefore suspect; nor is it true that neo-Pythagoreanism, no matter how early one makes it begin, is to be held accountable for the changing course of the tradition. In the earliest evidence available two conceptions of Pythagorean philosophy are in sharp contrast with each other. The non-Aristotelian tradition is even the older of the two, insofar as it can be traced to Speusippus. This tradition from the Old Academy equates the Pythagorean philosophy with the doctrines of the Timaeus and with the Platonic number philosophy. The highest principles, immaterial, are the One and the Indefinite Dyad; from them the pure, incorporeal numbers; the numbers produce the pure geometrical shapes, line, plane, and solid, as well as the perceptual functions of the "Living Creature in itself"; from the mathematical regularity of the regular polyhedra come the elements and therewith the multifariousness of the empirical world; and this whole process takes place, in this order, only in thought, which traces back changeless Being to its ultimate principles and understands it by means of them.

In each of these points the exposition of Aristotle offers contradiction. The Pythagoreans do not think of separating the numbers from the physical world, but stay in the realm of the perceptible; the numbers are spatially extended shapes, and things "are" numbers; order, correspondence, harmony within the empirical world are comprehensible in terms of number, which is the key to understanding and shows forth its power in diverse ways. The world, which is single, has come to be, as harmony, out of Limit and the Unlimited, and its articulation expresses the perfection of number. There are hints of mythical ways of thought, and obviously there is little logical or systematic consistency.

That in fact the interpretation of the Old Academy does form part of the background for Aristotle is shown by his clearly polemical form
I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

of expression: there ought to be no difference of opinion as to whether or not they attributed generation to the cosmic numbers. Aristotle at times, to be sure, shows that he has roots in the other tradition. In the *De caelo* he joins in the Pythagorean number juggling, in his *On Philosophy* he follows right along in the Pythagorean tracks in explaining the perfection of musical harmony, taking his departure from ἀσέφος and παραϊνον φύσις; in the first book of the *Metaphysics* he treats Plato and Pythagoreans as quite closely connected, whereas in books *M* and *N* he scarcely mentions anything but their disagreements.

In consideration of these matters, the question of a possible Platonic "early period" in Aristotle's thought is bound to rise. The *De caelo* and the first book of the *Metaphysics* are thought to be early works, it is possible that in his understanding of the Pythagorean philosophy, as well as in other matters, Aristotle only freed himself gradually from the picture of it drawn in the Old Academy. But, given the complicated and controversial character of the problem of Aristotle's philosophical development, this cannot be anything but a conjecture. It may help us, however, to understand the surprising fragment 207, without taking refuge in atheosis: “In his writings on Archytas, Aristotle says that Pythagoras too called matter 'other,' because it is in flux (ῥεωτή) and always becoming 'other' than it was.” This obviously has connections with the "manner in flux (ἐν ῥεωτη)" which according to the doxographers was part of the doctrine of Pythagoras and Plato.

If the doxography goes back, in general, to the Old Academy, it is possible that Aristotle too, either in an early work, or in an exoteric discussion, perhaps in a looser, dialogue form, adopted the Pythagoras of his colleagues. Maybe he was attempting a philosophical interpretation of a traditionary pronouncement of Pythagoras, in which the word "'other' (ἄλλο) occurred.

But if the tradition rejected by Zeller proves to be that of the Old Academy, it is by no means thereby rehabilitated. It would be rash to assume that if we merely accepted tradition the problem would be solved. For what we have is not a unitary tradition, but irreconcilable contradiction, and one cannot endorse one side without rejecting the other. Scholarship must decide between the Platonic and the Aristotelian line, for only one of them can be historically correct.

It is natural to try to settle the matter by use of the idea of development. The accounts of Aristotle have a decidedly antique air; they seem "pre-Socratic," while the doxography stemming from the Platonists presupposes a sophisticated use of dialectic. But it is out of the question to suppose that Aristotle is referring to Pythagoreans of the fifth century, the Platonists to contemporaries. Aristotle says explicitly that not Pythagoreans but Plato introduced the notion of the Indefinite Dyad, and Speusippus speaks of the "ancients," Xenocrates and Heraclides of Pythagoras himself. For the same reason we must reject the hypothesis, occasionally considered as a last resort, that Pythagoreans of the fourth century could have taken over the theory of ideas and the dialectic developed in the Academy. Plato’s disciples are not talking about younger contemporaries, but about Pythagoras. Either Aristotle arbitrarily reinterpreted Pythagoreanism as primitive

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150 Met. 1091a213ff.
151 Cael. 268a120.

153 We cannot here go into the question of the attempt of Cherniss to reconstruct a common original for the *Metaphysics* *A* and *M* (Plato 180ff).


155 Fr. 207 = Damasc. Prius. 2.172.16ff. Because of its incoherence with Aristotle's other statements, the authenticity of this fragment was denied by Zeller I 470 n. 3, Cherniss, Pres. 17 n. 68, De Vogel, *Pythagoras* 214ff. Roslopp, whose reconstructions of Pythagoreanism follow Platonic paths, defended its genuineness (Verher 43.1), citing *Met.* 108b26, whose contrast of "other" and "one" is attributed by ps.-Alexander 798.23 to Pythagoreans (cf. Ross, Met. II 471). But, since the question is that of the characterizations of matter (ἄτη), Aristotle cannot be speaking of the Pythagoreans (cf. above, ch. I 2, n. 95). What we have is exegesis of Plato (cf. esp. *Parm.* 157b).

156 Above, n. 35.

157 Zeller explained MM 1182a11 (cf. 1194a29) as a cruder version of 1078b21 with Ποθογράμα instead of Ποθογράμμα. Dirnmeier defends the authenticity of MM (Aristoteles Werke 8, Berlin 1958), and it might well transmit early Academic tradition.

158 Zeller saw clearly the contradiction of the traditions (I 465ff.), but did not come to this conclusion, for several reasons: He did not originally perceive the dependency of the doxography on Theophrastus, and this led him to explain away the direct testimony of Theophrastus (above, n. 39). Also, he relied solely on the dialogues for his interpretation of Plato, rejected the reports of Aristotle, and therefore did not see the inner connection of the "system of derivation." Finally, it is only the new fragment of Speusippus that proves the traditions are in contradiction even in the matter of the ἄτη, and not only in relation to line–plane–solid and the doctrine of the elements.

159 Above, ch. I 2.

160 E.g., Ross, Met. II 471 (cf. above, n. 164), Kurcharski (above, ch. I 1, n. 49).
speculation, or the tradition of the Old Academy is, from a historian's point of view, fiction.\footnote{The contradiction of the two traditions has mostly been overlooked, except by Zeller; it was only the interest devoted to the Platonic number philosophy that brought it clearly into the foreground. To be sure, people have often perceived the connection between the later tradition and Platonism. (On Sext. Emp. above, n. 45 but Paven. CQ 1951, 147ff, see in Sext. Emp. Math. 10.24ff.) at least partially Pythagorean material, perhaps going back to Archytas. On Aëtius, cf. Frank 260 n. 1.) Less clearly seen was the contradiction with Aristotle. Delatte (Vie 234) asserts that the Hypomnemata show some influence of Pythagorean doctrine, in spite of the Indefinite Dyad in section 25; on the other hand Wellmann (Hermes 1919, 227) thought he could prove by Aët. 1.3.8 that the Indefinite Dyad was genuine Pythagoreanism. The two testimonies stand or fall together. (Righlty, on Hypom. 25. Wierus, Mm. 1943, 97; Festugière REG 1945, 10ff, with refs.) Often, too, the difference is simply ignored, as by Confort, PPAram. (A) 171 Similarly, the Platonists always present Parmenides as simply Platonic, while Aristotle emphasizes the Eleatics' "primitive" inability to get beyond sense perception (nousynaios) (DK 28A24-25).

\footnote{A phrase coined by A. Diès, Auteur de Platon (Paris, 1927) 40ff.}

\footnote{Throughout antiquity, and beyond, the interpretation of Pythagorean doctrines offered by Plato and his disciples held the field; but, though the stream that was later accepted as Pythagoreanism came from Plato, one ought not to designate Plato and his pupils as Pythagoreans. They are thinkers sui generis, and later "Pythagoreanism" is very largely a dimitiatu Plato. This book comprehends as Pythagoreanism that which was not "transposed" by Plato, that which existed before Plato and contemporary with him as the intellectual position and the way of life of persons who called themselves Pythagoreans. (On the origins of "neo-Pythagoreanism" see below, ch. 1.4.)}

\footnote{We shall see in chapters IV and VI that the character of the tradition is no different in the fields of mathematics and astronomy.}

what is Pythagorean, aside from pre-Platonic testimonies and Aristotle, is in statements that agree with them and are not contaminated with Platonic material.\footnote{This result is decisive in the case of the PhiloIous fragments (ch. Ill, below). To be sure, bits of genuine Pythagoreanism make their appearance in the Platonists' tradition, as with the tetractys (above, n. 120). But in each case authenticity must be proven.\footnote{Cf. Diès, Ed. of PhiloIous xxii; Wilpert, Philob. 576.}}

One might still hesitate to accept a conclusion that leads to disagreement with the overwhelming majority of the sources. But it is supported by Plato himself; the significance of this evidence becomes clear only when it is seen as foreshadowing the conflict of interpretation which arose among Plato's successors.

4. **Pythagoreanism in Plato and the Origin in Platonism of the Pythagorean Tradition**

Modern scholarship on Plato has taught us to see, in the totality of his work and in the individual dialogues, the inner consistency and the necessary unity. The tendency to look for concrete historical relationships, for the origin of particular inspirations, or the target of specific allusions, has receded into the background.\footnote{Also Empedocles and the Sophists. It is true that the "stranger from Elea" was in later times called a Pythagorean (Clem. Al. Strom. 1.48.2).} Not the least important cause of this is a certain resignation, for it is quite impossible to determine and delimit, from the study of Plato alone, all of his "sources." In Plato, every thesis or argument derives its importance from its truth value, not from its origin in one source or another. Foreign material is no longer foreign, but an integral part of the Platonic structure. This is why the question of the nature of historical Pythagoreanism is perhaps hardest of all to answer, from Plato alone.

Plato's dialogues do not suggest strongly that Pythagoreanism was the determining influence upon him. Aside from Socrates, three thinkers in particular stand out: Heraclitus, Anaxagoras, and Parmenides.\footnote{The importance of Parmenides seems to increase in the later dialogues: alongside the one Timaeus stand Parmenides, Sophist, and Politicus. All the same, the few references to Pythagoreanism are of special relevance in their Platonic context.} Pythagoras is named in a single passage in the Republic, in the final reckoning with Homer. Has Homer, Socrates asks, earned the laurels of a lawgiver like Lycurgus, Charondas, or Solon, or even even...
speculation, or the tradition of the Old Academy is, from a historian’s point of view, fiction.\footnote{The contradiction of the two traditions has mostly been overlooked, except by Zeller; it was only the interest devoted to the Platonic number philosophy that brought it clearly into the foreground. To be sure, people have often perceived the connection between the later tradition and Platonism. (On Sext. Emp., above, n. 4; but Raven, CQ 1951, 147ff, saw in Sext. Emp. Math. 10.248ff at least partially Platonic material, perhaps going back to Archytas. On Aëtius, cf. Frank 260 n. 1.) Less clearly was seen the contradiction with Aristotle. Delatte (\textit{Vie 234}) asserts that the \textit{Hypomnemata} show an influence of Platonic doctrine, in spite of the Indefinite Dyad in section 25; on the other hand Wellmann (\textit{Hermes} 1919, 227) thought he could prove by Aët. 1.3.8 that the Indefinite Dyad was genuine Pythagoreanism. The two testimonies stand or fall together. (Rightly, on \textit{Hypom.} 25: Wiersma, \textit{Menomysa} 1942, 97; Festugière \textit{REG} 1945, 10ff, with refs.) Often, too, the difference is simply ignored, as by Cornford, \textit{PlParm} 4ff. \footnote{A phrase coined by A. Diès, \textit{Autour de Platon} (Paris, 1927) 400ff.} Similarity, the Platonists always present Parmenides as simply Platonic, while Aristotle emphasizes the Eleatics’ “primitive” inability to get beyond sense perception (\textit{ανόητος}) (\textit{DK} 28A24–25).} 

The decision can scarcely be in doubt: Speusippus, Xenocrates, and Heraclides equate the doctrine of their master Plato, and therewith also their own philosophical positions, with the wisdom of Pythagoras, whereas Aristotle sees both Academic and Pythagorean philosophy from a certain distance, and even from the stance proper to an active polemic.\footnote{Throughout antiquity, and beyond, the interpretation of Pythagorean doctrines offered by Plato and his disciples held the field; but, though the stream that was later accepted as Pythagoreanism came from Plato, one ought not to designate Plato and his pupils as Pythagoreans. They are thinkers sui generis, and later “Pythagoreanism” is very largely a \textit{dimidiatos} Plato. This book comprehends as Pythagoreanism that which was not “transposed” by Plato, that which existed before Plato and contemporary with him as the intellectual position and the way of life of persons who called themselves Pythagoreans. (On the origins of “neo-Pythagoreanism” see below, ch. 1.4.)} Plato’s pupils ignore the intellectual developments of a century and a half, a period of tremendous intellectual turmoil; their picture of Pythagoras is, in the history of thought, an impossibility.

The consequences of this decision must be faced, without further attempts at compromise. The “derivation system” is an achievement of Plato and the Academy, a genuine \textit{transposition platonicien}\footnote{We shall see in chapters IV and VI that the character of the tradition is no different in the fields of mathematics and astronomy.} of an older, Pythagorean number philosophy, making use of some improvements introduced in the circle of Archytas. The tradition that attributes the “derivation system” to Pythagoreans, or even to Pythagoras himself, is to be taken for what it is: evidence for the Old Academy, for Speusippus, Xenocrates, and Heraclides, but not for historical Pythagoreanism.\footnote{This result is decisive in the case of the Philolaus fragments (ch. III, below). To be sure, bits of genuine Pythagoreanism make their appearance in the Platonists’ tradition, as with the tetractys (above, n. 120). But in each case authenticity must be proven.} This involves, as mentioned above, nearly the whole of the post-Aristotelian tradition on Pythagoreanism. At least in the philosophical realm,\footnote{1 Cf. Diès, \textit{Ed. of Philoile} xxii; Wilpert, \textit{Philob.} 576.} the only usable evidence for what is Pythagorean, aside from pre-Platonic testimonies and Aristotle, is in statements that agree with them and are not contaminated with Platonic material.\footnote{2 Also Empedocles and the Sophists. It is true that the “stranger from Elea” was in later times called a Pythagorean (\textit{Clem. Al. Strom.} 1.48.2).}

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Pythagoras is named in a single passage in the \textit{Republic}, in the final reckoning with Homer. Has Homer, Socrates asks, earned the laurels of a lawgiver like Lycurgus, Charondas, or Solon, or even
those of a practical adviser like Thales or Anacharsis? Or (600a-b), if not in public life, is Homer said in private life to have been, during his lifetime, influential in the education of any persons who cherished him and his association with them and passed on to their successors some kind of “Homericaue way of life,” as did Pythagoras? He was greatly loved, in this way, and his followers even to the present day speak of a “Pythagorean way of life,” and seem in some way to stand out from the rest of mankind.

What gives this passage its importance is its connection with Plato’s own lot. For him, too, the most desirable career would have been great influence in the polis; after this was denied him, he decided to be, at least “in private life... influential in the education” of individuals: he founded the Academy.

In contrast to this “private circle” of Plato, the character Timaeus in Plato’s dialogue is shown as one who combines political activity and experience with profound philosophical learning, so that he exemplifies the synthesis of philosopher and statesman which was Plato’s highest ideal. To be sure, he is accompanied by Critias and Hermocrates, and it is possible for the hypercritical to point out that he is not characterized *expressis verbis* as a Pythagorean. But for a man from Italy who has been deeply engaged in studying “the nature of the universe” (*περὶ φύσεως τοῦ παντός*, 27a), it is hard to think of any other identification than with the Ἰταλικοῖ, καλοῖμενοι Πυθαγόρειοι. It can also be inferred from his praise for the lot of the συνήκος of a great man in the *Laws* (711c) that Plato was inclined to see his ideal of the philosopher-king as realized in the circle of the Pythagoreans; for this ἄπαν λεγόμενον is obviously the Attic version of the Pythagorean ῥιμακῖος (cf. below, ch. II 4). There is no reason to regard Timaeus as a historical person. Frank speculated that he served as a mask for Archytas, but the fact that his home city is given as Locri Epizephyrii calls Philistion to mind. It has already been mentioned that even Speusippus and Xenocrates thought of the doctrine of the *Timaeus* as Pythagorean wisdom. All the same, for historical purposes the doctrine of the *Timaeus* can serve as a source for Pythagorean doctrine no more than—or rather, just as little as—the *Parmenides* can for the historical *Parmenides*; and the theory of the elements can be shown to be non-Pythagorean. The fact, though, that Plato uses an Italiote as the spokesman for his own cosmological scheme, is an indication that in Magna Graecia Plato had at least found an impulsion or an inspiration toward a view of the cosmos that seemed to him significantly different from the system of Anaxagoras.

As with “Pythagoras,” Plato names “Pythagoreans” in just one passage. They call music and astronomy “sister sciences,” he says, and try to find the numbers that represent musical intervals. What Aristotle says about the “harmony of the spheres” is here given additional depth; rather than criticizing absurdities, Plato sets forth the leading idea. We are also taken beyond Aristotle by the indication that the theory of the numerical nature of the intervals was peculiar to the Pythagoreans and distinguishes them from other, non-Pythagorean musical theorists.

Having come so far, we can confidently point out other Platonic allusions to Pythagoreans, like a passage in the *Cratylus* whose importance has been emphasized by Boyancé. More important, however, is a section of the *Philebus*.

Socrates-Plato attacks the problem of ἡδονή by relating it to the more general problem of the “one and many,” unity, plurality, and their mutual interpenetration, not only in the objects of experience but in the realm of ideas itself (14d et seq.). An ancient tradition, he says, shows us the way toward solution of the problem:

> There is a gift of the gods—so at least it seems evident to me—which they let fall from their abode; and it was through Prometheus, or

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8 The relationship between the Academy and the Pythagorean society has often been emphasized. Boyancé brought our the significance of their common cult of the Muses (Muses 249ff). Cf. also Morrison, CQ 1958, 211ff.

9 200c–d.


Frank 128f, with notes 375, 375a, 376. There is nothing among the testimonia collected in the *Timaeus* chapter of DK (49) which might not have been derived from Plato. The argument sometimes heard, that the characters of Plato’s dialogues, and especially the title characters, are regularly historical, is in all probability not valid for the *Philebus*, and therefore not generally. (Cf. also Wilamowitz, Platon II 84, Cornford, Tim. 2ε.)

11 Above, ch. I 3, nn. 115–116. At Tim. 48b (cf. 53c), the derivation of the elements is introduced as something completely new, which has never previously been discussed.

12 Crat. 405c; Boyancé, Sagen 97 n. 4, Muses 101, REG 1941, 147ff. (Though the reconstruction, from the *Cratylus*, of a Pythagorean “doctrine d’Euthyphron” goes too far. Here again the question rises whether the “Pythagoreanism” so disclosed, which is supposed to underlie nearly all the etymologies of the *Cratylus*, is not rather Platonism. Plato is playing, in a “Pythagorizing” vein, with various preexisting forms.)
One like him, that it reached mankind, together with a fire exceeding bright. The men of old, who were better than ourselves and dwelt nearer the gods, passed on this gift in the form of a saying: all things (so it ran) that are ever said to be consist of a one and a many, and have in their nature a conjunction of Limit and Unlimitedness ... (16c; tr. Hackforth).

Our task is, he says, not to proceed immediately from the One to the Many but to comprehend, stage by stage, the numerical structure that lies between the One and the Unlimited—and herein lies the difference between dialectics and eristic—just as the grammarian knows the number and nature of the sounds which, in the unitary realm of language, determine the multiplicity of linguistic expression, or as the musician becomes master of the infinite realm of tones by his knowledge of the limited number of the intervals. After the special problem of the Philebus is then once more formulated, the ontology based on the opposition of πέρας and ἀπερά is used profitably in its solution.13

In antiquity Syrius, Proclus, and Damascius saw a connection between the Limit-Unlimited pair of this passage and Pythagoreanism, and specifically with the fragments attributed to Philolaus.14 In general, modern scholars have assumed an almost self-evident Pythagorean origin for this material,15 although Frank16 defended the view that this was not a borrowing at all, but a purely Platonic argument, provided, in Plato’s way, with a poetic-mythical garb. Porphry, in fact, explained the Philebus by reference to Aristotle’s On the Good.17

Now, it can be shown from the reports of Aristotle himself that the doctrines of the Pythagoreans on Limit and Unlimited cannot have been developed out of Platonism, as Frank thought.18 But aside from that, we can find a much stronger argument in the Philebus itself. In the exposition of the example from music, “the men of old,” οἱ πρῶτοι, are mentioned again. They recognized the number and kind of the intervals which “we have learnt, conformably to the teaching of the men of old days who discerned them, to call ‘scales’ (άρμονίς)” (17d). They discovered in the bodily movements of the performers a similar organization, “that must, we are told, be numerically determined and be called ‘figures’ and ‘measures,’” and advanced the view that “this is always the right way to deal with the one-and-many problem.” Thus these musical theorists reach out beyond the boundaries of their specialty,19 adopting a postulate that takes in everything, and the latter is identical with that of the “men of old,” previously formulated (16c-d). Now, in the Republic,20 Plato distinguishes two tendencies in musical theory, and decides that only one of them, that of the Pythagoreans, is worth serious attention, because they measure musical intervals by numerical proportion. In the Philebus, Plato is at first less precise: τὰ διαστήματα ὕπόσω ἐστὶ τὸν ἄρμον τῆς φωνῆς ... καὶ ὑπότα (17c-d). This looks like mere classification, such as one also finds in non-Pythagorean musicology; but the following expression, καὶ τῶν ὀρῶν τῶν διαστήματων is a technical term in the theory of proportion.21 Rhythms and meters are measured “by numbers” (17d); and coming back to this theme later (23d-e), Plato states unmistakably that musical harmony depends on numerical proportions. Thus it is natural to suppose that from the beginning Plato was thinking of the same musical theorists;22 from what Plato himself says it emerges that the ontology of the Philebus has its roots in Pythagoreanism.

So we come into possession of a piece of pre-Aristotelian evidence for a Pythagorean philosophy of some scope, musically oriented. A prime necessity, however, is to distinguish precisely between what Plato inherited and what it became in the alembic of his own mind. But a complete answer to this problem cannot be derived from Plato’s words alone. The best we can achieve is a lower limit, so to speak: what is derivative is that which does not follow from the context and the purpose of the dialogue. Insofar as what we learn in this way

14 Syrius, Met. 9.37ff, Procl. In Tim. 1.84, 176.29, II 168.29, Theol. Phil. 1.1, 5, 3, 7, Damasc. Princ. I 101.3 R.
15 Zeller I 457 n. 1, 480.1; Ueberweg-Practcher 306; Taylor, Phileb. 52ff; Diels, Philol. xxii (with hesitation); Raven, Pyth. 110ff; Hackforth, Phileb. 20ff.
16 Ibid. 304. Similarly, Bollinger 74. Mondolfo’s citation of individual occurrences of the words πέρας and ἀπερά in pre-Socratic philosophy (Zeller Mondolfo 378f) does not constitute an adequate refutation of this. It would be conceivable that Plato himself followed hints from a number of different predecessors.
18 Above, ch. 1 2.
19 There is no indication of any such thing in the treatment of “grammar” (18b-d).
20 16c-d et seq.; above, n. 11.
21 In the system of Aristocles, too, one finds δος; they are the notes, with their names (Aristox. Harm. 49.20 M., Pl. Rep. 443d). But these are only “comprehensible” in the οὐσία; the same interval may be exemplified by quite different notes, while in Plato they are “comprehended” (ἐνείμαθεν λίπης, 17c) before the οὐσία. In the ratio theory the δος are determined by the interval peculiar to each case (e.g. 9 : 8 is a whole tone).
22 Richter (89ff) believes that at Phils. 17c Plato is rehabilitating the Pythagorean theory of music, and only at 23c goes into the ratio theory, from a new point of view.
I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

coincides with the testimony of Aristotle, the reconstruction may be regarded as confirmed.

The fundamental problem, that all being is at the same time one and many, had been formulated long ago (14c). It is also discussed in other dialogues and brought into connection with the problems raised by the Eleatics. Empirical things (γεγονόμενα) had already been designated, in passing, as unlimited (ἄπειρον, 13b), in contrast to the "unities" represented by the ideas. What is new in the pronouncements credited to "the ancients" (16c) is, first of all, the antithesis of πέρας and ἄπειρα. This feature alone is also found at the beginning of the later passage resuming the argument: τῶν θεῶν ἐλέγομεν πως τὸ μὲν ἄπειρον δεῖξα τῶν ὀντών, τὸ δὲ πέρας (23c). In the conclusion drawn from this (16d), the opposition of one and many again comes to the fore: the method put forward as ancient tradition is that of Platonic diaeresis, which Plato was already calling for in the Phaedrus. Still, the special emphasis on "number" as intermediary between the "One" and the "Infinite" is new. In the example from music theory this "number" suddenly appears with a different function: the measures and tones are not only "dialectically" divided, but "measured" in numerical proportions. When the argument is resumed, the "mixture" of Limit and Unlimited consists in the fact that "number" is introduced into things: numerical relations like equal, double, etc. (25d). It is not only musical harmony that depends on this process, but health of body and the proper rhythm of the seasons—thoughts already brought forward by Eryximachus in the Symposium. Still different is the function of "number" in another context of thought: everything dependent on τέχνη was discovered by its means (16c). Here it is a question of counting, measuring, weighing: one is reminded of an allusion in the Politicus to the thesis of πολλοί τῶν κομφών ... ὁς ἀρά μετρητική περὶ πάντων έστι τὰ γεγονόμενα. So right from the beginning there

is interpenetration of various factors. What is Pythagorean, because it is, in the Platonic milieu, novel and a bit awkward, is the pair πέρας-άπειρα as well as a certain emphasis on number and proportion. More we cannot say, from the analysis of Plato’s text; but if we add the testimony of Aristotle, his words and Plato’s complement one another. The conception of the world as a harmony of Limit and Unlimited, permeated by number, that great creator of order, was for Plato a point of departure and a guidepost on his quest for intellectual mastery of the riddle of the plurality and unity of being.

When the argument is resumed (23c), "a god" is again named as revealer of the pair of opposites ἄπειρον-πέρας. Plato, independently (τιθέμεθα), posits a "mixed" class and also introduces, as a novelty, a fourth class (the αἰτία). Here, then, we are on Platonic ground. This is where the Unlimited first receives its more precise specification, which could not have been foreseen. Whereas it was previously matched with plurality (πλεῖστος, 16d, 17c), it is now explained with the help of pairs of opposites like warmer and colder, strongly and mildly, more and less, rather and less rather (24a). There is emphasis on both the importance and the difficulty of this way of thinking: "if things are said again and yet again, there is some prospect of the two parties to a discussion being brought to a tolerable agreement" (24c). In every "more and less rather" lies "the nature of the Unlimited" (24c). The place of a simple conception of a spatial or numerical "Unlimited" is taken by the continuous "indefiniteness" on both sides of the limiting measure. The place of the Unlimited is taken by the Indefinite Dyad, as we may express it in the light of the reports about On the Good.

The analysis of Being in the Philebus is a foreshadowing, an early beginning which makes use of the thought of others, moving toward the ontology developed in On the Good. As Hermodorus expresses it, τῷ ἐδόν ὁ Πλάτων κατὰ τὸ ἄπειρον καὶ ἀδύνατον ὑποτιθέμενον ἀπ’ ἕκεινον

23 Ἐσπ. Παρισ. passim, Soph. 251a
24 186a (medicine), 187a (music), 188a (seasons). There it is Eros that is named, not number; the speech is not simply Pythagorean, but Platonic artistry in exposition. But Pythagorean themes do seem to be used.
25 Pol. 284c. Plato dismisses these people lightly and ironically, because they are not used to looking at things καθ’ ἐνότητα. He takes over their leading ideas, but only in a significantly refined form. What Plato develops out of Pythagorean thought in the Philebus is what he misses in the Politicus. All the same, it may be that Pythagoreans are in his mind here too (as scholars have mostly supposed, since Campbell; cf. Raven, Pyth. 186). In one case the fruitful development of the basic idea takes his attention, in another the more cautious, popularizing application of it is rejected (e.g., in medicine; see below, ch. III 3).

Kucharski, "Le Philèbe et les Éléments harmoniques d’Aristoxène," Rev. philos. 84 (1959) 41–72, and H. Koller, "Die diätetische Methode," Glotta 39 (1960) 6–24, believe that Plato is developing the method of diaeresis according to the model of music theory, but a contrary argument would be that there is nothing in the Phaedrus, Sophist, or Politicus to suggest any such connection (though in Phdr. 270d he speaks of "numbering" the forms), whereas the feature which is peculiar to the Philebus, πέρας-άπειρα is attested by Aristotle as Pythagorean.

Hermodorus’ evidence is at n. 29 below. Merlan has drawn attention, rightly, to the role of the problem of τὸ μάλλον καὶ ἴσον δέδειξα in Aristotle’s doctrine of the categories (Philebus 1034, 35ff). Even modern writers are unanimous that there is a relationship to the On the Good; cf. Taylor, Philb. 50ff; Ross, Met. I 171; Wilpert, Philib. passim.

What Plato presupposes as "source" is no more, and nothing else, than what Aristotle treats as the doctrine of the Pythagoreans. There is nothing of the One and the Indefinite Dyad, but Limit and Unlimited; and if we are right in combining with the Philebus the passage in the Politicus, no theory of ideas. At the same time, we see Plato's philosophy, emerging from these Pythagorean stimuli, moving in the direction of the ontology of the lecture On the Good. Our decision, regarding the tradition about Pythagoreanism, between that of the Old Academy and that of Aristotle, is thus irrevocably confirmed: it was not Speusippus, Xenocrates, and Heraclides, but Aristotle, who gave authentic information on the Pythagoreanism that was there before Plato. And what is more, we can understand, from Plato, how the former tradition arose, with its distortion of the historical picture.

For Plato's affirmation of the divine origin of the doctrine of Limit and Unlimited is more than a glinting sequin on the fabric of the exposition. It signifies that its truth is beyond doubt; and Plato feels that this imposes on him the obligation to grasp the truth of this idea and its all-encompassing significance. Such a divine revelation is not something finished and complete, but a task to fulfill—like that of "coming to the aid of the god" (βοηθείω τοῦ θεοῦ) in the Apology. It is precisely because of the authority of such a doctrine that interpretation must set in immediately; and in this process Plato's own thoughts and those of others become almost inextricably intertwined.

Here is a type of interpretation untroubled by minutiae of historical accuracy and only interested in the sense intended. Actually, the Greek question τι λέγει; does not mean, "What words are used?" but "What does this mean?" This method is characteristic of Plato, but surely not of him alone. When, in the dialogue named after him, Protagoras speaks of Homer and Hesiod, Orpheus and Musaeus as Sophists, Socrates overtrumps him with the claim that the most σοφοτατ, students of φιλοσοφία, are to be found in Cretan and Sparta. A similar passage in the Laws is more serious. At first the Spartan and Cretan constitution is represented as aimed toward warfare as its only goal (625d). This object is rejected as comprehending only one portion of υπηρ. The result of this, however, is not rejection of the Cretan and Spartan νόμοι—they are of divine origin, handed down by Zeus and Apollo. Therefore, ὄντως τῷ αληθείᾳ οἶμαι καὶ τῷ δικείουν ὑπὲρ γε θεῖας (πολιτείας) διαλεγομένους λέγειν (630d-c), the interpretation must be revised; and to the bewilderment of the Cretan and the Spartan, the object of their constitutions appears all of a sudden as much more comprehensive, and by no means restricted to the military aim.

This is the spirit in which, on the basis of the Philebus, we must judge the relationship with Pythagoreanism; here too the tradition is set before the philosopher as a task which has to be thought through. And as the ancient claim, that Sparta's constitution came from the god of Delphi, must be taken seriously, the "divine" origin of Pythagorean teachings, too, is more than a façon de parler. Aristoxenus tells us that Pythagoras got his doctrines from the Delphic oracle, but Aristotle says that Pythagoras himself was believed to be the "Hyperborean Apollo." Taken together with the Philebus, this can only mean that the Pythagoreans, even as early as those whom Plato knew, understood their own philosophical activities as developments of the basically identical doctrines of their master, the divine Pythagoras, "but that it all comes from that great man (ἔνας δὲ πάντα ἐκκόλοθος τοῦ ἀνδρός)."

We must suppose, then, that this way of thinking was already established in Plato's day. It explains the surprising uncertainty of Aristotle as to the chronological relationships of the Pythagoreans. He refused to accept the assertion that all this was Pythagoras' teaching, but he had no basis for more precise chronological determination (and perhaps no interest in it).

But where Aristotle looks upon it with critical aloofness, Plato's disciples join him in taking their place within the Pythagorean tradition. In this respect too, the Philebus confirms the conclusion we were able to reach from the contradiction of Aristotelian and Academic accounts of Pythagoreanism. Plato's school sees in its own philosophical treatment of the problem of ultimate principles a continuation of Pythagoreanism, so that ancient material is reinterpreted accordingly. This Platonic interpretation of Pythagorean philosophy became

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30 Ap. 21b.
Cf. the sharply ironical, but by no means entirely inaccurate, formulation of Chrysipus, referring to Aristotle (Riddle 30): "Aristotle is one of those who cannot be refuted by an author's words because he is sure that the author was unable to say what he really thought."
31 Prot. 316d, 342b.
dominant in the ancient tradition. Aristotle was the only one to con-
tradict it, and shows us thereby what had been there before Plato;
and in fact what Plato presupposes is what Aristotle criticizes.

It is not necessary at this point to pursue in detail the further allusions
to Pythagoreans in Plato, whether certain or conjectural. What has
been said is enough to serve at least as a sketch of the general trends.
The true problem of the Pythagorean tradition lies in Platonism, for
Platonizing interpretation took the place of historical reality. One
can only guess at the reasons why Plato and his pupils saw themselves
as continuators of Pythagoreanism. Personal contacts were certainly
important; they were already present in the Socratic circle. Simmias
and Cebo of Thebes, “hearsers” of Phileola, appear even in Xenophon
as pupils of Socrates, and we may believe that, as the Phaedo has it,
the Pythagorean Echecrates of Phlius had some connection with
Socrates. Plato’s friendship with Archytas is attested by the seventh
Letter, and the later biographies of Plato agree that a principal motive
of his first journey to Magna Graecia was to establish contact with
the Pythagoreans there. One must proceed cautiously in trying to deter-
mine what extent of doctrinal agreement there may have been between
Plato and the Pythagoreans. There is a considerable danger of project-
ing onto the Pythagoreans, unjustifiably, the unique style of thought and
presentation that were Plato’s. We must remember the possibility of
individual influences that could be at work in the impressiveness and
magnetism of such a personality as Archytas. It is worth remembering
that in the seventh Letter immortality figures as a belief not subject to
skepticism in 335b.5ab; cf. Rep. 498d). In addition to external influences

38 On Phileola, below, ch. III 2; on the ὁμορρομαίον doctrine, below, ch. III 2; on the
harmony of the spheres, below, ch. IV 4; on the “nuptial number,” below, ch. VI 4; on
the origin of the soul, ch. V; on Gorg. 493a, below, ch. III 2; on Gorg. 507e, above,
40 Echecrates as a Pythagorean: Aristox. fr. 18-19 (cf. DK 53); ps.-Plato, Ep. 9, where
Echecrates appears as a friend of Archytas, is doubtless a rhetorical exercise with no
historical authority. The Echecrates from whom Timaeus claims to have received
information about Locrians (FCHist 560F2-Polyb. 12.10.7) is in, spite of the chronolo-
gical problems, likely to be the same as the Pythagorean from Phlius (cf. Oldfather,
RL Supp. 3.41f; Jacoby I N4, K. 172, Nn. 316, 195; Echecrates of Locri as a teacher
of Plato: Cic. Fin. 5.87, Val. Max. 8.2.3). For Echecrates, below, ch. II 4.
41 335b, 335b: ἔτσι ἐγκατέστηκε οὐκ ἠκούσας δὲ τῶν ἐν Σκιάλες περὶ πλείων ποιήσεων, τῶν
τῆς ἀρχής συγγενέστερον ἰτα. According to this Archytas had “pupils.” Cf. al. 355a.
42 DL 3.6, Cic. Fin. 5.87 (from Antiochus), Rep. 1.16, Apul. Plat. 1.3, etc. (with
inconsistencies in details). Cf. also Wilamowitz, Plat. II 32f. In Anon. Phot. 43b18
Plato is called Ἀριστοτέλους τοῦ συμμαθήτου μαθήματος; this is polemic against the other view,
that Archytas had been a pupil of Plato (Immsch. 46). The meaningless συμμαθήτης
(Apel. Plat. 1.4) and Ἀριστοτέλους τοῦ συμμαθήτου (I 1.1 116a) is a misunderstanding of the
same source.

43 On the problem of the sources of the Vénus placita, see ch. I 2, n. 126, ch. I 3, nn. 56,
114, 147.
44 On the playing off of “more genuine” Pythagoreans against Aristotle, see below,
ch. III 1. With reference to the reports of Aristotle, Philonius clearly formulates his
principle for interpretation of Pythagoreanism (De an. 70.2): ἐν μνήμην τοῦ φαινομένου
ἐναγάλλησα, καταγάλλησα ἄπειρον δὲ καὶ σωφρόν, ἔλαχθος μεθισμένως: ἐν δέ ἐκείνῳ σωφρόν, δει
τὴν ἀλήθειαν τῶν φαινομένων συμπέτειν.
able to develop a life of its own. One cause for this lies in the fact that tradition about Plato’s oral teaching lost in importance, in comparison with the steady influence of the Platonic dialogues. The doctrines formulated in the latter were well known, while the theory of ideal numbers sank into desuetude.

The most important factor, however, was the effect of the direction of Academic development on the tradition about Pythagoras. From Aristotle’s time it had been customary to see Plato as a synthesis of Socrates and Pythagoras. In the generation of Plato’s pupils the “Pythagoreanism,” metaphysical speculation, carried the day. But, when a reaction set in and the Academy became predominantly “Socratic,” Pythagoras necessarily lost his place. This is just what happened, from the time of Arcesilaus and the “Middle Academy”; the school of Plato went over to skepticism, and there had to be a parting of the ways with Pythagoreanism.

There were necessarily two aspects to this breach. Insofar as the Academics following the Socratic-skeptical trend still felt themselves to be Plato’s successors, they had to push diligently aside everything “dogmatic,” and especially the mathematical-scientific and metaphysical teaching of the school, as not genuinely Platonic. They found another origin for it: Pythagoras. On the other hand, what this process discarded retained, even for the rationalistic Hellenistic world, a certain fascination. Those who were attracted by it could no longer attribute it to Plato, against the authority of the Middle Academy, but found it necessary to reach back for the authority of “Pythagoras.” And when he took the limelight, Plato and his pupils were stigmatized as plagiarists.

To Cicero, “Academic” and “skeptic” mean the same thing, and in fact Plato was often invoked by the skeptics. This tradition may have affected the report of Sextus Empiricus on the theory of ideal numbers; use was made of an exact transcript of the lecture On the Good, but the whole is presented as refutation of the Pythagoreans. Plato’s name first occurs only quite incidentally (10.258), but in the skeletal refutation he is cited at length (10.302ff.). Criticism of the theory of ideal numbers takes the form of a struggle, in alliance with Plato, against Pythagoras.

The contrary position is expressed in Numenius’ work On the Difference (διαστάσεις) between the Academics and Plato. For Numenius, Plato and “the great Pythagoras” are about the same thing, and whatever Socrates had to offer also came from Pythagoras (5.7). Plato’s direct disciples had followed their master, and “it was primarily because of them that Pythagoras came to be highly honored” (5.2); Arcesilaus and his pupils had wandered from this path, and with scorn and anger Numenius condemns this apostasy.

But long before Numenius this attitude had produced remarkable results. From the third century B.C. on, the apocryphal Pythagorean writings appear, each trying to outdo the other, presenting Platonic-Peiric-Pythagorean doctrines as original pronouncements of Pythagoras and his pupils. The interpretation of Pythagoreanism that Speusippus, Xenocrates, and Heraclides had given has hypostasized in this revival, and the purported originals make the intermediaries superfluous. Thus the criticism of Pythagoreans themselves is turned against the Platonists:

... that Plato and Aristotle, Speusippus and Aristoxenus and Xenocrates, as the Pythagoreans say, appropriated what was fruitful, with slight modification, but collected some superficial or inconsequent things, whatever is brought forward by those later malicious slanderers in an effort to refute and mock the school, and put these down as the special doctrines of the sect ...

Thus even these Pythagoreans admit, implicitly, that what was essential, or “fruitful,” in their doctrines agrees with Plato and Aristotle, and that they could make no good use of what historical tradition had to offer as the ἰδα οὐ πολύς of Pythagoreanism. Later Pythagoreanism is stamped

48 Cic. Acad. 2.74, 12.6, Sext. Emp. Philo 1.221, Prop. in Pl. 10. 3-8 (p. 205 Hermann). Cf. Burkert, "Cicero als Platoniker und Skeptiker," Gymnasion 72 (1965) 175-200. The first ps.-Xenophanic letter (Hecher p. 988) is also a polemic of the "Socrates" against Plato’s "Pythagoreanism." The reproach is made against the Platonists: Ἀφεῖναι δὲ ἦσαν τὸν Πυθαγόρα συγκαταράκτου τοῖς ἐπανεμεθραίησαν ἀλλότριοις.
49 Above, ch. 1; 3, nn. 4ff.
I. PLATONIC AND PYTHAGOREAN NUMBER THEORY

so deeply with Platonism that it has no longer any conception of its real origin.

One might therefore define later Pythagoreanism as Platonism with the Socratic and dialectic element amputated. In fact, Plato remained the principal source for all later Pythagoreans—Plato’s myths, and in particular the Timaeus. The apocrypha, presenting the supposed originals, could make no headway against this overwhelming influence. Apollonius of Tyana taught “doctrines about the physical world similar to the opinions of Plato’s Timaeus,” and though Proclus considered the “Timaeus Locrus” document genuine and put it at the head of his commentary on the Timaeus, it was not this sorry scribble but Plato’s Timaeus itself that he classified, along with the Chaldaean Oracles, as far excelling all other literature. Scholars have shown in different ways that Neoplatonism is quite closely dependent on the Old Academy, and “Pythagoreanism” too belongs in this category. It is also basically Platonism, existing at a time when Plato (as interpreted in Pythagorean fashion) had lost his position in the Academic school. Later, neo-Pythagoreanism converges, in the philosophical realm, with Neoplatonism.

And yet Pythagoreanism is not sufficiently characterized by that which is lacking, as compared with Plato. The inherited material that was lumped together under Pythagoras’ name, undigested or mutilated though it may have been, was in this process raised to a new dignity and endowed with unexampled authority. The whole body of apocryphal literature lies within the realm of religion. The ontology of the Old Academy was oriented toward the divine; the Pythagorean pseudopigrapha meet a subconscious religious need of the Hellenistic period. And when, from the first century B.C. on, people once more come forward to declare themselves Pythagoreans, their most noticeable characteristic is that they are seeking (or even, as for example in the case of Apollonius of Tyana, claiming to possess) a superhuman, divine wisdom. And it may be that in this very point—not in details of doctrine but in the claim to possess divine knowledge—we are most likely to find an element of its real origin, in the influence of Pythagoras of Samos.

II. Pythagoras in the Earliest Tradition

I. SOURCE PROBLEMS

Platonizing interpretation brought basic change to the Pythagoras tradition, so that a historical reconstruction of what it was like before Plato’s time must be based on the pre-Platonic evidence. This can be supplemented by reports that were not drawn into the process of Platonic transmogrification; but, to judge these with any approach to confidence, we must first survey the nature of the tradition as a whole. It is no longer admissible simply to reject what is late attested, as Zeller did; any such item might have its origin in an ancient and authentic source. After Rohde, the most important advances in the analysis of the sources were made by Delatte and Lévy. Significant results have been achieved; but the essential thing is to distinguish between what is certain and what is merely conjecture.

Most of our material on the life and activities of Pythagoras is collected in the eighth book of Diogenes Laertius, in Porphyry’s Life of Pythagoras, and especially in Iamblichus. We may add the tenth book of Diodorus, of which only fragments are preserved, and the very short sketch in Justin. Photius’ excerpt from an anonymous life of Pythagoras has very little that pertains to history or biography. Iamblichus is most detailed; his plan was to present the Πυθαγόρειον φιλοσοφία in a work designed to run to ten books, because ten is the perfect number. The Pythagorean Life (for his subject was the appropriate way of living, rather than the biography of Pythagoras as such) was followed by the Protrepticus, the book On General Mathematical Knowledge (De communi mathematica scientia), and, as a treatment of the special sciences, a commentary on Nicomachus’ Introduction to

An example of analysis carried too far is Bertram’s dissertation. In his concluding Table (57 ff), the whole text of Iamblichus’ Vita Pythagorica is traced back to sources by sentence, through two or three intermediaries for each, to sources of the fourth century B.C. But his evidence is mainly vague associations and scarcely ever attains the status of proof. For an example of the way in which hasty source analysis can lead to error, see my discussion of the “Letter of Lysis,” Philogous 1961, 17–24.

For convenience, the commonly accepted title Life of Pythagoras has been retained here.
Arithmetic. Later in the series came the book on numerical theology (Theologumena arithmeticae). Further parts are not preserved, and perhaps not all of them were written. It was Iamblichus who set the direction for the later Neoplatonists, toward a definite equation of Platonism and Pythagoreanism.

Our first question must be as to the direct sources of Iamblichus. Here analysis is easiest in the case of the Theologumena arithmeticae; one source, Anatolius On the First Ten Numbers, has been recovered, and the second, Nicomachus 'Αριθμητικών θεολογομικών βιβλία β', we have in an excerpt by Photius (Bibl. 187). The manuscripts of the Theologumena arithmeticae often name Anatolius and Nicomachus, and the book proves to be, essentially, a cento made up from the two older ones. What is not Anatolius is mostly quotation of Nicomachus, including his own quotations. The contribution of the author, or rather compiler—and in spite of doubts this was probably Iamblichus himself—is merely arrangement and introduction.

Certainty was reached by Erwin Rohde on another point, the relation between Porphyry and Iamblichus. The latter did not use Porphyry directly; instead of the general Philosophic History he used special works on Pythagoras. Thus we have access to important sections of Nicomachus' biography of Pythagoras, and in their original wording.

It is to be ascribed to the influence of Iamblichus that the chapter on Pythagoras in Porphyry's Philosophic History became separated from the rest and is therefore the only surviving section (cf. Harder xv).

V. de Falco, "Sui Theologumena arithmeticae," Riv. indo-greco-latino 6 (1922) fasc. 1/2, 49-61; H. Oppermann, Giomou 5 (1929) 548-558. If Iamblichus had inserted the source citations himself they would have been evenly distributed; but, as it is, they are found almost exclusively in sections based on Nicomachus.

That Iamblichus does use Porphyry is maintained, after Zeller I 65 n. 1, by E. Norden (Augustos theos [Berlin, 1913] 344 n. 2) and more recently by J. A. Philip (TAPA 90 [1959] 183-194). Rohde's proof to the contrary (Q 123ff) was based on the fact that the Nicomachus sections of Porphyry occur, without exception, as verbal quotations in Iamblichus, while apart from them there are only occasional points of contact. In addition, Iam. VP 233, compared with Por. VP 59, proves that Iamblichus is copying the wording of Nicomachus, not that of Porphyry. Iam. VP 253 has the Doric forms proper to the Lysis letter (Hercher, Epistologr. gr. p. 662), which are not preserved by Porphyry (58). Iam. VP 170 has Metapontum, correctly, where Por. VP 4 names Croton (Nauck liv, 36; below, ch. II 2, n. 18). Even Iamblichus could not have spoken of "general agreement" (VP 248) if he had had before him the contradictory versions of Por. VP 56-57.

Por. 20-31 Iam. 30, 31, (241), 34, 40, 61, 62, 36, 63, 134-135, (142), 136, 64-67; Por. 59-61: Iam. 233-237. The material promised in the sentence with which the extant text of Porphyry breaks off is in Iam. VP 189-194. In these cases Porphyry names Nicomachus; on the other hand, Iamblichus names Nicomachus at VP 251. Further, Iam. VP 254 p. 135, 10-17 Deubner: Por. VP 53 p. 47, 18-48. I Nauck; Iam. 135, 18-136, 11 Por. 577 pp. 49, 16 50, 11. Thus we can be certain that these passages are all from Nicomachus (contra Jager, 5ff), who supports the source is the "handbook," as well as Delatte, Pol. 219, and Minar 68 n. 40, who take Nealternis as the direct source; correctly Rohde, Q 115f. It also becomes obvious that Porphyry copies more mechanically than Iamblichus. Where Porphyry gives two connected excerpts from Nicomachus, Iamblichus may distribute the same material in different chapters, according to an artificial outline of his own. Without the parallels in Porphyry it would be a hopeless undertaking to attempt to identify in the mosaic those tiles that make up the contribution of Nicomachus.

Further, there is in Porphyry a connected passage from Moderatus on number theory, and twice he cites Antonius Diogenes' romance on The Wonders beyond Thule, though it is not quite clear where the quotations end. His basis for the remainder must be a handbook containing many citations of its sources, and for this part the points of contact with Diogenes Laertius are especially numerous.

Corssen, Philologus 1912, 332ff, Lévy, Sources 92 n. 1; the fact that Iamblichus puts passive for active at p. 135, 17, and thus has the persecuted being killed, is his own misunderstanding, (Lévy, Sources 97). Since, further, Iam. VP 248 pp. 133, 12-14=Por. VP 147, pp. 46, 34-47, 3 show verbal agreement, and in addition Iam. VP p. 133, 8 introduces the understanding of the Pherecydes episodes which is later (p. 135, 11) attributed to Nicomachus, the excerpt from Nicomachus must begin as early as Iam. VP 248, and Por. VP 54, and the Aristoxenos citation (fr. 18) must come from Nicomachus (as fr. 31 = Iam. VP 411. Por. VP 59). This had already been deduced by Rohde, from Iam. VP 251: Νεαράκως Μικρά μὲν άλλα συνολογετόν τούτους (Q 115f; Lévy, Sources 116, attributes the Aristoxenos citation to the third source of Iamblichus, the "handbook"; Dicks, DR 14, 16, n. names Apollonius as the source of the Aristoxenos citation, doubtless a mere slip, passed on by Ignatius Cardini, p. 55). Nicomachus cited Aristoxenos, but preferred to follow another version. Most authorities suppose this is Nealternis, who is cited by the same at Por. VP 147, 38, 8 (from Fritz, Pol. 5, after Corssen), but Lévy thinks it was Satyrus (Sources 51f; cf. D.L. 8, 40), and that Nealternis is the basis rather for D.L. 8, 39, = Hesychius. It is not likely that the citation of Dicaearchus, Por. VP 561 (fr. 34 W.) was in the text of Nicomachus (Corssen, Philologus 1912, 341, according to von Fritz, Pol. 7, already in Nealternis), since Iamblichus does not seem to know it (above, n. 5). Nor does Dicaearchus fr. 33 = Por. VP 17 appear in Iamblichus; Porphyry has inserted into the Nicomachus text a passage from the "handbook" source (the same citation at D.L. 8, 40 = fr. 35b W.).

Iamblichus' skill in combining disparate quotations can best be seen from his use of Plato: cf. Merlan, PLNEOPH. 148f. Small variations between Porphyry's and Iamblichus' wording are frequent. Generally, Iamblichus is fuller; and it may be that in the variability of his exposition he introduces expansion as often as Porphyry abbreviates.


Por. VP toff and 32ff, Lydus Mem. 4, 42 p. 99 W. shows that Porphyry's source in VP 44 is still Diogenes Antoninus. Rohde first (Q 126) assigned 10-17 and 32-45 to Antoninus, then later (Rommer 272 on), on the ground that in his novel Antoninus could not have made explicit citations of sources, only 10-14, 32-36, and 44. (In 15 Dionysophanes is cited, in 41 Aristotle). On the other hand Jager (36 ff, 43-47) and K. Rehling (Antoniustheologie, Diss. Tübingen, 1929, 31) hark back to Rohde's original thesis, excluding only the direct citations. To be sure, the mention of Abura connects Por. VP 17 and 34. Erythrid adds Por. VP 40-47, which is quite improbable, because Iam. VP 228 has copied the same text, but Iamblichus did not use Antonius Diogenes— and Por. VP 54-55, tightly compacting die àposkemenvonporos in 54 and 55, though this is squeezed into a context from Nicomachus (above, n. 6).
II. PYTHAGORAS IN THE EARLIEST TRADITION

Another direct source, Apollonius of Tyana,\(^{10}\) is once explicitly cited in Iamblichus (254–264), and once shown to be a source by parallel passages in Porphyry.\(^{11}\) Rohde showed convincingly that two other continuous sections, the speeches of Pythagoras in Croton\(^{12}\) and his meeting with Phalaris,\(^{13}\) have the same origin.

Rohde went on to propose a mechanical two-source theory, according to which Iamblichus drew material exclusively from Nicomachus and Apollonius; he tried to assign each chapter to one of the two, though allowing that Iamblichus may have worked out some individual passages on his own.\(^{14}\) This two-source theory was vigorously attacked by Méautis, and definitely refuted by Lévy.\(^{15}\) At least

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10 The identity of this Apollonius with the wonder-worker of Tyana was contested, after Wytenbach, by Méautis (91), but has probability on its side. Apollonius was a conscious and enthusiastic Pythagorean.

11 Iamb. VP 3–8, 11; cf. Por. VP 2. Rohde (Q 128f) and Lévy (Sources 105f) give the whole section 3–25 to Apollonius; but it is more probable that Iamblichus’ compulsory method begins at the very beginning, though perhaps in an especially careful vein. (Cf. the connection of Iamb. VP 19 with Nicom. Th. ar. 53.1ff.)

12 Iamb. VP 37–57 is bracketed with the Apollonius passage 254ff by 4 separate concordances: 49 and 262; the temple of the Muses 50 and 264; the Pytharum (or Pythium) 50 and 261; Thes. 53 and 255.

13 Iamb. VP 215–222; cf., for Apollonius before Domitian, Philostr. VA 8.1ff (Rohde, Q 164ff; Lévy, Sources 109, Lég. 50ff). Boyancé (REA 1934) tried to trace the section to the Abaris of Heracleides Ponticus. In refutation, Miss De Vogel points out the Stoic and late Platonic terminology (Mmnoymne 18 [1653] 388–396, Pythagoras, 304–306). Festugière (REG 1950.19) in 1950–51 reduces comparisons with hermetic-poetic material. The rapid summary at 239 shows that Iamblichus was not composer but compiler.

According to Rohde Iamblichus’ own hand can be seen in 103–105, 157–162, 167–186, 198–199, 214, 223–228, 240–241, 244–247. Where doubts occur, Rohde always gives the one version to Nicomachus, and the other to Apollonius; but we should bear in mind the possibility of free self-creation by Iamblichus.---Iamb. VP 155ff = In Nic. 5.27ff is attributed by Rohde to Nicomachus’ biography of Pythagoras (Q 156f); but are we to suppose that Iamblichus would really cite Nicomachus to comment on Nicomachus, and in addition that he would have been able to find such a detailed parallel to what Nicomachus says at Ar. 1.1? The passage is tolerably well integrated into Iamb. VP 155ff, while it seems a disconnected, interpolated block at In Nic. 5.27ff; Iamblichus is repeating his own paraphrase.

14 Méautis 87ff, Lévy, Sources 111ff. E.g., Nicomachus (Iamb. VP 30–Por. VP 20) names 2,000 authors of Pythagoras, Apollonius (Iamb. VP 254, 260) gives 300, but Iamb. VP 29, like D.L. 8.15, 600. (Iamb. VP 29 is marked as an interpolation by Deubner, but, given the compulsory habits of Iamblichus, incongruities are always to be expected.) Iamb. VP 30 is assigned to a “handbook” by Rohde (Q 131f). Further: Iamblichus presents the miracles of Pythagoras in two versions, one according to Nicomachus and one according to Aristotle. If it can be shown that the versions taken from Nicomachus are almost always altered in a distinctive manner, so as to blunt the paradoxes, it is quite improbable that Nicomachus himself recorded both versions (as Rohde says, Q 152f); and why should Porphyry always have chosen the non-Aristotelian formulation? (Cf. below, ch. II 3.)---The sentence about the 7 books of Pythagoras (Iamb. VP 199) cannot come from Nicomachus, who thought Pythagoras left no writings (VP 57; cf. Iamb. VP 146, where doubtles Nicomachus is to be understood as one of the ἀληθέοι κάθεις Pythagoreans who attributed the Προτείς λόγος not to Pythagoras but to Telegonus; cf. Euseb. Hist. eccl. 6.19.9). This sentence, however, shows verbal coincidence with D.L. 8.15 (below, ch. III 1).

15 A third source is to be recognized, of the “handbook” type, whose presence can be detected in coincidences between Diogenes Laertius and the “handbook” segments of Porphyry. Actually, there is no need to restrict Iamblichus to three books; it was more convenient, if he wanted to write on music (ch. 26), to find something in Nicomachus’ book on music than to seek out an appropriate passage in his life of Pythagoras. And Iamblichus could have read personally, as Stobaeus did, the Πυθαγορακη υποθαγωσεων of Aristoxenus.\(^{37}\)

Thus our analysis leads, in the first instance, mainly to sources of the first or second century: Moderatus lived under Nero.\(^{18}\) Apollonius under Domitian, and Nicomachus is dated by the fact that Apuleius translated his Introduction to Arithmetic into Latin.\(^{19}\) Yet these neo-Pythagoreans are themselves only intermediaries. Nicomachus cited his authorities meticulously; Apollonius did not. So Rohde judges Nicomachus very favorably: he is an intelligent compiler, and no forger. Toward Apollonius he is very negative: “one is best advised not to believe anything he says.”\(^{20}\) But this judgment can only be reckoned as true a parte potiori. Apollonius too used good sources, and it is precisely his material that has provided most opportunities for those who, from time to time, have tried to discover really ancient lore.\(^{31}\) Nicomachus, on the other hand, who calls himself a Pythagorean, is so intimately concerned in his narrative that, at least in selection, arrangement, and interpretation, his personal contribution must not be underestimated.

For the earlier stages of the tradition Diogenes Laertius is particularly important, though his work is hard to analyze. He has woven together material from various handbooks, and his “card-file” method makes it almost impossible to discern connections of any larger elements. Still, there is an example of parallel tradition in the Pythagoras
article of the Suda and a Platonic scholium (Rep. 600b) which is almost identical with it; both are referred to Hesychius of Miletus. The same exposition is discernible in Diogenes Laertius, though broken up by numerous insertions. So one of the main sources can be distinguished, even if its name is unknown.  

The work of Neanthes of Cyzicus must have been an important intermediary source. He brought together several versions of the origin of Pythagoras, including that of Aristoxenus; like Hesychius he names Pherecydes and Hermodamas as Pythagoras' teachers, and enumerates the brothers of Pythagoras in the same terms as the source common to Diogenes Laertius and Hesychius. Thus in Neanthes we find the earliest example of the handbook provided with source citations and variants. Perhaps it was the one that set the trend; in any case it furnished an exposition that was widely used—by the source of Diogenes Laertius, by Nicomachus-lamblichus, by Clement and Porphyry. Its striking that Neanthes is often mentioned along with Hippobotus. Obviously one had cited the other; probably Hippobotus, who wrote On Sects (D.L. 1.19), made use of the work of Neanthes, whose special concern was with the "mythical." Thus the sequence Neanthes-Hippobotus may be one link in the handbook tradition.

Callimachus' student Hermippus wrote several influential books on Pythagoras; Josephus calls him the "most distinguished" of the biographers of Pythagoras. The fragments we have contain the most eccentric material in the whole Pythagorean tradition; Rohde considered the book "a malicious satire on Pythagoras." But, although it can be shown occasionally that Hermippus distorts older source material in a rationalizing and ironical spirit, still it cannot all be arbitrary invention. A student of Callimachus will, with one degree or another of seriousness, be engaged in collecting precisely that which is antique and odd, so that Hermippus too may have some accurately preserved material.

Finally, the most important sources to which analysis can lead us, because they are the oldest, are Herodotes Ponticus, Aristoxenus, and Dicairachus, along with Aristotle. In their case, significant chapters can be discerned, documented by direct quotation; attempts to enlarge the evidence by building up more complicated reconstructions hardly get beyond the stage of hypothesis.

The greatest unknown is the historian Timaicus. He is cited by name

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28 Cf. Delatte, Vitr. 9-63, where the older literature is discussed. "Handbooks" are responsible for the coincidences between Diogenes Laertius and Clement, Hippolytus, Porphyry, and lamblichus. (On this, cf. Jäger passim.)

29 ForHist 84-84-33. The book in question is the fifth book of his Method (F29). This Neanthes is probably to be dated about 200 B.C., and distinguished from a historian of the same name of about 100 years earlier (Jacoby, ForHist IIC, 144; for the early date, von Fritz, Hist. 6.)

30 F29=Por. VP. 1-2; the emendation of Κλέανθης to Νεάνθης is guaranteed by Clem. Al. Str. 6.2; Aristox. fr. 114-4; Neanthes himself believed that Pythagoras came from Syria—the most unusual view; he must, however, have mentioned the canonical view, that he came from Samos, and in that case he must have had three versions side by side, as do Por. VP 1 and Clement, loc. cit.

31 Eléoucrates in Neanthes (Por. VP 2) and Diogenes Antiochus (Por. VP 10) and Eléoucrates in D.L. 8.2 and Hesychius can only be secondary variants.

32 Lévy, Sources (cf. above, n. 6) also attributes D.L. 8.30—Hesychius to Neanthes, comparing Neanthes F30 (Por. VP 55). Here Neanthes also is harking back to Aristoxenus (fr. 18. Ian. VP 249).

33 Nicon. (Por. VP 61 Ian. VP 189 Neanties F31) Th. ar. 52.8ff (Neanthes F33 Aristox. fr. 12). Hippobotus is named along with Neanthes for differing versions in Clem. Al. Str. 6.62.2; that of Hippobotus (Pythagoras a Samian, attributed by D.L. 8.1 to Hermippus) may have been in Neanthes (above, n. 24). At D.L. 8.72, also, Neanthes (F28) follows Hippobotus.

34 Ap. 1.16. This is a somewhat tendentious passage, for Josephus found in Hermippus indications of the dependence of Pythagoras on the Jews. The fragments are collected in ForHist III 41ff.
II. PYTHAGORAS IN THE EARLIEST TRADITION

for a series of very specific reports about Pythagoras,33 so that, a fortiori we must assume that he gave a general exposition of Pythagoreanism. He apparently showed a decided sympathy with Pythagoreanism, rooted in western Greek local patriotism.34 Suspected of holding Timaeus material are Justin,35 Diogoras,36 and especially Apollonius.37 In principle, the task is to define a kind of Vulgate tradition, not derived from either Aristocles or Dicaearchus. Delatte and Rostagni produced, independently, bold reconstructions, not without contradicting each other, particularly in the decisive matter of chronology. These constructs have, rightly, been subjected to sharp criticism. It cannot be proved that everything in Iamblichus and the other late accounts goes back to an authority of the fourth century B.C., nor that Timaeus would be the only eligible candidate for this position. There are an uncomfortable number of intermediate sources, passing the material on in a continuous process of variation, combination, and compilation; and other historians, like Theopompus28 and Duris,39 also made mention of Pythagoras. Also, it would be strange if Antiochus of Syracuse, Philistus, and Ephorus had nothing to say about Pythagoreanism in connection with the history of Magna Graecia; but not the slightest trace of such is discernible.

We cannot divide the whole of the later tradition nearly among the fourth-century sources.40 On the whole, the “later” tradition seems to be not so much the result of unscrupulous falsification as of simple-minded, naive collection and transmission of whatever could be found, contradictions and all. Still, even if the source analysis takes us back as far as we could hope, to the very beginning of biographical study in the early Peripatos, Zeller’s skepticism cannot be considered otherwise... Apolloniuss sections have this origin, viz. Iam, VP 259ff (Delatte, Litt. 25 n. 3; disagreeing, Buryan; cf. above n. 13), and especially Iam, VP 259ff (without hesitation Bythumann 27/28; Delatte, “La chronologie pythagoricienne de Timée,” Mèdes Belges 1920, 5–13: “Apollonius se revèle tributaire de Timée pour son histoire du Pythagorisme presque toute entière” [13]; cf. Litt. 86, Vie 169, Rostagni, “Pythagora et Pythagorici in timio,” Sélém. II 1, 3–50). On the other hand, Von Fritz, Pol. 53ff, shows that in Iam, VP 259ff Apollonius’ method is that of “what might almost be called large-scale historical black painting” (61), that he uses much old material but cannot be relied on for details. Cf. Jacoby, FGHist IIIb, Komm. 550ff, Noten p. 325 n. 19tc. The ambivalence of the arguments is shown, for example, by the repeated connection of Pythagoras with Anaximander (Iam. VP 33–Por. VP 21, from Nicomachus; Iam. 122, 130—but Anaximander was not founded till 407 B.C. Shall we say that the local patriotism of the historian from Tauroctonos is evidence here, or that Timaeus, of all people, is excluded by such a monstrous anachronism?

33 FGHist 369F3, 14, 16, 17, 131, 132.
34 One might conjecture that Timaeus was first to trace the origin of the name Magna Graecia (Μεγάλη Ελλάς) to the influence of Pythagoras (Nicom. in Iam. VP 30–Por. VP 20, Iam. 166, Cic. Tus. 5.10, Leul. 13).
35 20.4. For the attribution to Timaeus, cf. A. Enmann, Untersuchungen über die Quellen des Pompeius Trogus für die griechische und sÄ technique Geschichte (Dortpat, 1880). Justin 18 is equivalent to Timaeus F31, and Justin 11 corresponds to Iam. VP 56, which directly follows a fragment of Timaeus (F17). The different reports about the luxurious living of the people of Croton (Tim. F44–45. Justin 1–2) can be reconciled (von Fritz, Pol. 46f.) Still, this does not prove that Timaeus is Justin’s only source. Is the impression of the unity of Justin’s exposition perhaps only the result of its severe compression, and therefore to be credited to the epitomator rather than to the unitary source? Pompeius Trogus did use a number of different sources (O. See, Die Praefatio des Pompeius Trogus [Erlangen 1955] 107–12). Diodorus is following a highly rhetorical, moralizing source, which treats the Seven Sages in the same manner (9.4–15; Schwartz, RE V 679E). It was later than Callimachus (10.64). Among its sources was Aristocles (Schwartz 679; von Fritz, Pol. 22ff), but there is no distinctive trace of Timaeus. The relation of the Damon–Phintias story in Aristocles (fr. 31) to the version in Diodorus is (10.43) is controversial; in Aristocles, Dionysius wishes to test the steadfastness of a Pythagorean in danger of death, and therefore condemns the innocent Phintias, whereas in Diodorus the latter had really planned an attempt on the tyrant. Wehrli (Aristocles p. 37) finds that Diodorus version, since it is simpler, is the earlier; but Cobet (Collatione critica [Leiden, 1878] 433) and Nauck (LVII 40) stressed that the famous denouement, Dionysius’ wish to be admitted as a third member of their friendship, only makes sense if there was no deadly enmity but only a test engineered by the tyrant. Thus Aristocles’ account is after all the original, and Diodorus is not using pre-Aristoclean material.

87 In the speeches of Pythagoras, Iam. VP 47–56, 54 corresponds to Timaeus F17; there is a relationship between 42 and Timaeus F146; 56 agrees with Justin 20.4.11, and 37, 40, and 47 are related to D.L. 8.22f. Local tradition of south Italy is evident in 40, 44, 46, 50, and 52. Furthermore, Timaeus F13 corresponds to Iam. VP 71–72 (von Fritz, Pol. 39). It is inferred from the connection with Timaeus that Iam. VP 71–72 comes from Apollonius, Rohde, Q 137; Delatte, Litt. 85f. (The latter adds sections 74–78 and therefore misinterprets the Lysis letter; cf. Burkert, Philologus 1961). Timaeus F13 is to be compared with Iam. VP 35. On the basis of these correspondences, the conclusions are drawn that (1) the speeches of Pythagoras in Iam. VP 37–57 are taken entire from Timaeus (“selon toute vraisemblance textuellement Timaeus”), Huyane, RevHil 20 (1954) 182), and that (2) all...
refuted. It is precisely with the authors in the circle of Plato and Aristotle that the real problems begin.

Most obvious is the contradiction between Aristoxenus and Dicaearchus, regarding the catastrophe that overwhelmed the Pythagorean society. One of the two reports must be basically wrong: either Pythagoras withdrew to Metapontum before the outbreak of the unrest and died there (as Aristoxenus says) or he and his followers were hounded from city to city (as Dicaearchus has it). Like his doctrines, the life of Pythagoras also becomes a mirror image of real controversies in the schools. On the one hand there is the controversy over the primacy of the theoretical or the practical life (βίος θεωρητικός, βίος πρακτικός). In this respect Heraclides thinks of Pythagoras as the apostle of pure “theory”; to Aristoxenus and Dicaearchus he an active politician. Then, in the matter of his doctrines on the soul, Heraclides, following Plato, advocates the immortality of the soul and seeks proofs for its independence of the body. This explains his interest in the long, and the metempsychoses of Pythagoras, and the “apparently dead woman” of Empedocles. Like Xenocrates, he expands Plato’s myths into a system that comprehends both science and religion, and joins together astronomy and immortality. We need not be bound by Timaeus’ characterization of him as a παραδεισούχος, but in his treatment of the anecdote about φιλοσοφία we can show that, contrary to history, he projected Plato’s conception onto Pythagoras.

Dicaearchus said that “soul” is a mere word (frr. 7ff), and there is an unmistakable irony in his account of Pythagoras’ doctrine of metempsychosis. Pythagoras was once, he says, a beautiful courtesan. Dicaearchus has Pythagoras, at the end, stumbling from one catastrophe to another; and the Locrians, who were famous for their εννομία, denied him admittance to their city.

Aristoxenus dealt most fully with Pythagoras and his pupils. He was a native of Tarentum, and cited in evidence the acquaintance of his father with Archytas, as well as his own acquaintance with the “last” of the Pythagoreans. Clearly, he put himself forward as an expert in Pythagorean matters, just as he was, at the same time, an authority in musical theory. He considered false, however, the numerical theory of the tone intervals, which is specifically attested by Plato and Aristotle as Pythagorean, and did not believe in the immortality of the soul, which he called a δρομονία, with all the consequences drawn from that idea in Plato’s Phaedo. Thus Aristoxenus has to steer a complicated course.

In his Harmonics, Aristoxenus disposes in a single sentence of the music theory advocated by the Pythagoreans down to Archytas, without even naming the Pythagoreans; what does not suit him is not mentioned. Aristoxenus characterizes the Pythagoreans with whom he was acquainted, as the “last,” suggesting that with them the school ended—but there was a Pythagorean named Lycon who wrote against Aristotle, and there are also other traces that lead into the later fourth century. Apparently Aristoxenus does not recognize these men as Pythagoreans. Finally, the assertion that Pythagoras was fonder of beans than anything else can only be veiled polemic against the taboo on beans attested by Aristotle and Heraclides.

The Ποιηθηκαί ἀποφασεῖς show the lineaments of a rational, clearly articulated ethic, oriented toward practical needs. Its political precepts are surprisingly similar to those of Plato’s Republic, though the predecessors of Plato named by Aristotle in this connection are Phileas.

41 Cf. the introduction, above. Not only Capparelli (I 351 and passim) but also Morrison (CQ 1956, 135) maintain that Zeller’s skepticism can no longer be upheld in view of the results of source analysis.
42 Below, ch. II 2.
43 Cf. the discussion of Wehrli in his commentaries on the fragments of the Peripatetics.
44 Jaeger, SBBln 1928, 396 n. 1, 415ff, after Rohde, KISdr II 110.
45 Below, ch. IV 4.
46 PfrHist 506ff. Heraclides fr. 84.
47 Burkert, Hermes 1960, 159ff.
48 Fr. 36. The irony was noticed by Rohde, Pythi II App. 10 (Eng. tr.), and Wehrli, P. 53.
49 Fr. 14 fr. Here there is polemic against Aristoxenus (fr. 43), who made Zaleucus a pupil of Pythagoras.
50 Fr. 10; cf. frr. 47ff; Plut. De gen. 592f. on fr. 2, von Fritz, Gesammelt 32 (1960) 495.
51 Fr. 18–19; cf. fr. 1; below, ch. II 6.
52 Below, ch. V 1.
53 Fr. 118ff.
54 Aristoxenus is the fullest of the ancient sources for Pythagoreanism, and therefore the question of his credibility is especially important and much discussed. The very rationality which characterizes Aristoxenus’ Pythagoreans, far from mysticism and magic, seems a favorable sign to some scholars, suspicious to others. On the positive side, following Kirsch, who, for example, in Delattre (Litt. 159, Pol. 213), Restani (SermIn II 112ff), Howard (JAW 197 [1923] 163), with qualifications von Fritz (Pol. 278f), Weilhauer tentatively (602ff). On the negative side: Lévy (Sources 44ff), Frank (260, APJ 1943, 221 [where he wrongly depends on the formulation of Euseb. Praep. evang. 15.2 = Aristox. In 64, followed in this by van der Waarden, SA 107]).
55 Page 32 M., emphasized by Frank, APJ 1943, 221f. It should be kept in mind, in this connection, that, according to Aristoxenous in Eusebius, Aristoxenus makes malicious reference to Aristotle without calling him by name (fr. 64; cf. Wehrli, p. 68).
56 On this, below, ch. II 6.
57 Below, ch. II 4 n. 124. The report on Pythagoras as an eater of meat in fr. 25 is biased in the same direction; cf. below, ch. II 4 n. 111.
and Hippodamus, and not the Pythagoreans. When Aristotle says that none of the “ancients” had dealt in detail with the nature of τοῦχνη, whereas Aristothenes presents as “Pythagorean sayings” precisely what the Eudemon Ethics expounds as τοῦχνη, then it is obvious that fourth-century ethics, and not ancient Pythagorean tradition, is being set forth. Finally, Zaleucus and Charondas, contrary to any chronological scheme, are made into pupils of Pythagoras, and we learn that Pythagoras was the first to introduce weights and measures into Greece.

Not that Aristothenes invented it all; he had sources of information, but like the Platonists he interpreted Pythagoreanism in accordance with his own preconceptions. And this is the basic fact that finally emerges from analysis of the sources: though the late tradition may be traced back to the writers of the fourth century B.C., this does not lead us onto firm ground but into the precarious territory of that day’s controversies. Instead of reliable facts one finds the shifting claims to a Pythagorean tradition apparently already in a state of flux, for the benefit of the peculiar concerns of the pupils of Plato and Aristotle. One may attempt to discover historical truth from amidst the polemics— not, naturally, isolating the individual testimonies, but considering each along with the others that contradict it. But the foundation of historical research must be the pre-Platonic evidence because this belongs to a time before Pythagoreanism sank into the quicksand of the school controversies. We may reasonably expect information about the early school from those later authors who are critical of Pythagoreanism rather than from those who identify themselves with it and are therefore forced into radical reinterpretations of it. From this point of view the testimony of Aristotle again emerges as especially important, for he clearly, in his lost book on the Pythagoreans, was concerned to collect material, rather than to interpret it in such a way as to fit into his own system.

It is customary to complain about the sparseness of early testimonies. Still, whereas we have no explicit reference to Anaximander or Parmenides by any fifth-century author, there is a quite imposing array of references to Pythagoras. Interestingly, most of these had already been assembled in ancient times; we have them as quotations in the lives of Pythagoras. The history of Pythagoreanism was already, at that time, the laborious reconstruction of something lost and gone.

II. PYTHAGORAS IN THE Earliest Tradition

2. Historical Background

It is only in post-Aristotelian sources that biographical and historical details regarding Pythagoras and the Pythagoreans are to be found. The most important accounts are those of Aristothenes and Dicaearchus, but they differ diametrically on one vital point—the role of Pythagoras in the revolution in Croton. Considering these authorities and also the later tradition, of which some indeterminable portion comes from Timaeus, one is tempted to say that there is not a single detail in the life of Pythagoras that stands uncontradicted. It is possible, from a more or

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58 Cf. A. Rivaud, “Platon et la ‘politique pythagoricienne,’” Mél. Glotz (Paris, 1932) 779–792. “Aristothenes a reconstruit, avec le secours des textes de la République, une politique pythagoricienne qu’aucun auteur pythagoricien n’avait jamais formulée expressément” (784). M. Pohlenz (NGG, ph.-h. KL, 1924, 19–32) inferred the source of the py-Demosthenic first oration Against Aristoteletos a Pythagorean writing Περί νόμου. But aside from the problems of detail involved in such a reconstruction, it would not lead us to pre-Platonic terrain. The trial of Aristoteletos took place about 124 B.C., so that there might be influences from Socrates, Academics, or Peripatetics, not even excluding Aristothenes.

59 Aristox. fr. 41; EE 1246b37ff, 1248b2, 124a15; the testimony of Aristotle about the “ancients,” Phys. 195b3ff. But compare 196b5; H. Täger, De Aristothenes libro Pythagorico (Diiss. Göttingen, 1925). This unpublished dissertation has unfortunately remained almost unknown; even Wehrli does not use it. Yet Wehrli too says (59): “The hallmark of the Αριστοθελος is to lay claim to Academic-Peripatetic material on behalf of the Pythagoreans.”

60 Fr. 43; followed by Posidonius (Sen. Ep. 90.6).

61 Cf. 44; cf. below, ch. VI 1.

62 The Πεδικαρικοι ανδομαται seem to be a modern substitute for the ανευματα. Wehrli (61ff) refers, on fr. 37, to Arist. Met. 985b30. Concise formulations of popular “wisdom”—καρπος, 7, οικιστα χριτ ιμποται (987422) are analyzed and more fully developed by Aristothenes.
less critical selection of the data, to construct a plausible account; but it is bound to rest on shaky foundations, for no documentary evidence has appeared.2

A mainstay in discussion of the chronology3 is the report of Aristoxenus, according to whom Pythagoras left Samos for Italy in 532/531 B.C. because of the oppressive tyranny of Polycrates.4 On the other hand, Eratosthenes identified the philosopher Pythagoras with the Pythagorians of Samos who was an Olympic victor in 588 B.C.5 The

1 Cf. Zeller I 38ff. The most important of the older studies is Kriese. See further Delatte, Pol.; von Fritz, Pol. and RE; Minar; Ciceri II 86ff; Dumbabin 358ff, 366ff, 368ff.
2 A striking piece of original evidence for the extraordinary fame that surrounded Pythagoras as early as the 5th century is perhaps to be seen in the "Pythagoras" coins of Abdera. See Seltman, Greek Coins (London, 1931, 1955) 124ff, pl. xlviii 11 (cf. Numismatic Chronicle 6.9 [1949] 21). J. M. F. May, The Coinage of Abdera (London, 1966) pp. 157, 167, with pl. 13 no. 218 (P183); also, addendum p. 176 with text-cut and description of no. 218 (P183). The date is given as 430/420 B.C. On the obverse the coins of Abdera show the picture of a Griffin, after that of the mother-city Teos. The reverse has, after about 450 B.C., the name of the mintmaster in charge (who may have been the priest of Apollo, Abdera's patron deity), and in the quadratium incuse, which changes from one mintmaster to another and, at least in some cases, bears an obvious relationship to his name—as when Python uses a tripod, Eunagon a prize amphora, Nicostratus a soldier, or Molopagos a girl dancing (nos. 345-348, 293-295, 313-315 May). The coins of the mintmaster ΠΥΘΑΓΟΡΗΣ display an idealized, bearded head, in two different types. The design must be related to the person of the mintmaster, and may be related to his name. Seltman's bold conclusion, that the person represented is the famous Pythagoras himself, seems likely. G. M. A. Richter's suggestion that the mintmaster portrayed himself (Greek Portraits IV [Brussels, 1963] 17-19) has been disproved by the appearance of a second, different type. The representation must be that of an ideal person, not a living individual; see W. Schwabacher in Stockholm Studies in Class. Archaeology 5 (1968) 59-63. In any case, we know Pythagoras was known in Abdera at this epoch (Democritus, D.L. 9.38; Herodotus cites the Greeks living on the Black Sea and Hellespont for the relation between Pythagoras and Zalmoxis; cf. below, ch. II 3). To be sure, the interpretation cannot be proven, and there are other coins of Abdera with heads that cannot be identified. —Cf. Zeller I 381 n. 1; Rohde, Q 118ff; Jacoby, Apollodor 213ff; Lévy, Sources 14ff; von Fritz, Pol. 68ff and RE 179-185.
3 Aristox. fr. 16 W., followed by Apollod. FGrHist 244 F338-339; cf. IIib Noten p. 326 n. 198. In spite of this, Aristoxenus made Zaleucus and Charondas pupils of Pythagoras (fr. 43).
4 FGrHist 244 F1. D.L. 8.47; ibid. 48, an epigram on this Pythagoras by Thucydides of Cynoce (time of Callimachus), and (49) an anonymous epigram which gives his father's name as Crates. Eratosthenes' list of Olympic victors must have been reliable for the period in question; i.e. the Olympic victor of 588 is a historical figure (Rohde I 118). Lévy (Sources 151) sees the whole as part of the légende d'enfance that grew up about Pythagoras: even as a boy he won an Olympic victory.

8 Pliny the Elder (HN 2.37) dates an astronomical discovery of Pythagoras to 612/609; and at 36.71 he places him in the reign of Psmennetephesiris (=Psmennethes II, 669-650 B.C., or Psmennethes II, 594-589; naturally the two could easily be confused).—Eusebius (Ann. Chron. p. 1430 Karsae) places Pythagoras under the successor of Sanherib (i.e. after 751 B.C.; cf. Jacoby, FGrHist IIb 296, on 273 F9: not Alexander Polyhistor, but an addition by Eusebius). The Numa story (Jacoby, Apollod. 223ff; cf. Burkert, Philelogus 1661) and the encounter with Phalaris (Iam. VP 215ff) were invented without consideration of chronological matters.
9 Alcidamas ap. D.L. 8.56, Timeus FGrHist 566 F14—D.L. 8.54. There is no reason to doubt the text, as Lévy does at Sources 54 n. 3; neither the dating of Pythagoras nor that of Empedocles by Timeus is known. (For contradictory reconstructions, see Delatte, Musée Belge 1920; Rostagni, Scrinium II pp. 3-30; cf. von Fritz, Pol. 47ff; RE 180-184; Jacoby FGrHist IIbb Komm. 552.) At iam. VP 44 Pythagoras is represented as speaking of 7 Corinthian Olympic victors; and this could only apply to the years between 508 and 496 (cf. A. Moretti, Olympiakoi [Rome, 1957]). Can this come from Timeus (cf. ch. II, n. 37)?

8 Zeller fehlt [1640 n. 1; cf. Riedemann 38] that in fr. 7 Xenophanes speaks of Pythagoras as though he were already dead; and Heraclitus (fr. 40) speaks of Pythagoras in the same breath as Hesiod—though scarcely on chronological grounds. Cf. below, ch. II 6.
9 496: cf. below, ch. II 3.
10 Iam. VP 83: cf. below, ch. II 4.
12 Hdt. 4.95; Hellanicus FGrHist 473; Isoc. Bus. 28; Hermippus ap. D.L. 8.1; etc. He was, however, a "Tyrrhenian" according to Theopompus FGrHist 113 F52, Aristox. fr. 143 c. W., and Aristotle (MSS Aristarchus) fr. 190, from one of the "Tyrrhenian islands of the northern Aegean." (The statement in Plut. Quaest. conv. 727b—c that he came from Etruria, is P lurarchus' improvisation.) He was a Syrian from Tyre according to Hecanthes FGrHist 84 F29 (cf. Apollonius ap. Iam. VP 57, 7, 13). Lycus (?) ap. Por. VP 5 για μέγαν γιαντί αὐτοῦ μέγα Σάμων ὁ Βασίλειος, οὶ δὲ Μεσσηνικοὶ (cf. below, ch. II, n. 11). Philius appears in the genealogies, varying in detail, of D.L. 8.1 and Paus. 1.1.12. (For Pythagoreans from Philius, cf. Aristox. fr. 19 and Echecrates in Pl. Phd.; for Pythagoras and Philius, Heracleides fr. 87 W. A Philiou σωματερός is distinguished as a hero in D.L. 8.46; cf. below, ch. II 4.) Zeller's conciliatory suggestion (after K. O. Muller) that Pythagoras came from a "Tyrrhenian-Pelagian family that had migrated from Philius to Samos" (I 380.2) will scarcely find support any more.
13 Heraclitus fr. 129, Hdt. 4.93. In Iamblichus the name is consistently misspelled as "Mnemonarchus." It is given as Marinus in the genealogy of D.L. 8.1 (cf. von Fritz, RE XXIV 72; Maurocras as son of Pythagoras, Plut. Amen. Paul. 1); Demosthenes (or de Marato?) natus, Justin, 20.4.3.
in which Pythagoras figured. As early as Herodotus he is connected with Egypt. In the tales of his journeys to visit the Phoenicians, the Chaldaeans, and the Magi there is a good deal of imaginative conjecture, though he unquestionably had some sort of contact with the Orient. The principal site of his activity was southern Italy, and the traditions center about the cities of Croton and Metapontum. It is well attested that his death took place in the latter city.

Not surprisingly, Pythagoras' teaching is permeated with the kind of religious character of Magna Graecia. Typical of this is the prominence of the chthonian deities—Demeter, Persephone, Dionysus—and of eschatological beliefs, especially the type that produced the numerous representations of the journey of the defiled dead into the Beyond. This religious situation is probably older than Pythagoras.

18 Antiphon (Por. VP 9 = lam. VP 26f) speaks of Πυθαγόραν καλοῦμεν άτι καὶ νόμος ἡμῶν, ἐν τοῖς ἱεροῖς τῶν κοινῶν βουλεύονται, and an ἄρτιν βρέχοντα. Local patriotism may play a part in the report of Duris (FGrHist 760F2) about an epic and dedicatory offering of Pythagoras' son Arinomus in the Heraeum at Samos. A "Saman poet" may be Apollonius (Por. VP 2 = lam. VP 9). On Samian coins, see n. 2 above.

19 2.81 (below, ch. II 3); Isoc. Bus. 28; Hecataeus of Abdera FGrHist 654B35 = Diod. 16.60.4, 96.2, 98.2. The letter of introduction from Polykrates to Aias (Antiphan ap. Por. VP 7 = D.L. 8.3) is an invention based on Aigeus' letter to Nectanebo in behalf of Eudoxus (D.L. 8.87). See further Zeller I 387.1.

20 Aristoxenus alleged that he paid a visit to "Zaratas" (fr. 13 = Hippl. Ref. 1.1.2.12; on the extent of the Aristoxenian material see Wehrli sof and W. Spoerri, REA 57 [1955] 267-290. Alexander Polyhistor followed him in this (FGrHist 273F49; cf. Jacoby III 294ff). It was a chronologically impossible idea to imagine that Pythagoras got from Egypt to Babylon as a prisoner of Cambyses (IG XIV 1297, II 20 = FGrHist 252B7; Apul. Flor. 15, p. 56; Th. ar. 53.1ff; lam. VP 19; the epistle, see above Democritus). Further references in Zeller I 384ff, as also for his supposed contacts with the Thracians (Zalmoxis, below, ch. II 3, n. 202), with Arabs, Jews, Indians, and the Druids of Gaul. The god Men and the taboo on white roosters are certainly from Asia Minor (ch. II 4, n. 47), and the wearing of trousers is Persian or Scythian (Ael. VH 12.12).

21 Alcidamas in Arist. Rhet. 1398b10f: Ιталοί τοις Πυθαγόρας (τετελεσμένους). The legends are mainly localized in Croton (of which Caulonia was a colony) and Metapontum (ch. II 3). Pythagoras is Metapontine to Lykos (? = ap. Por. VP 5; Brothius (DK 17.1), Thcno (lam. VP 267, pp. 146.22 and 132), and Hippasus (DK 18.1-2) are sometimes from Croton, sometimes from Metapontum.

22 Arist. fr. 191, Aristox. fr. 18, Dicarchus fr. 34-35 (cf., for further material, Zeller I 417ff; Delatte, Pol. 203ff, Vat. 241ff, Lévy, Lévy, 657ff). In Metapontum, Cicero saw "Pythagoricae ipsum illum lucem, ubi vita ediscerat, sedemque" (Fin. 5.4.3; this may of course be a rediscovey for the benefit of tourists). According to Porphyry (VP 3 = FGrHist 666F115), Timaeus spoke of the house of Pythagoras in Croton, and the same report is given, but with Metapontum as the city, by Favorinus (D.L. 8.15), Justin (20.4.18), and Lamblichus (VP 170). Val. Max. 8.15.1 is corrupt. It is more likely that Porphyry made an error (or his source; cf. Rohde, Q 133.1; Delatte, Vat. 183; Lévy, Sources 54.1, Lévy 65) than that Timaeus purposely substituted Croton for Metapontum (as Jacoby, FGrHist Hh Komm. 551; Jager 20f).

23 Cf. Giannelli, Wielkimiester 450f. Wilamowitz (Plato II 84) had already called attention to the religion of the western Greeks and its connection with Pythagoreanism.

The prominence of Aphrodite beside Persephone in the votive tablets from Locri surely cannot be derived from Pythagoreanism, but there is nothing distinctly Pythagorean in the famous gold tablets, either. Pythagoras entered a religious world of a peculiar character, in which Mediterranean, Italian, and pre-Doric Achaean elements were amalgamated. It is remarkable that only Croton and Metapontum, among the south Italian cities, are noted for the worship of Apollo. There seem to have been quite ancient Apollo cults in Metapontum, as well as in Macalla, which belonged to Croton. The facts that Croton used the tripod of Apollo on its oldest coins (about 550 B.C.), that Caulonia,
a colony of Croton, showed on its coins Apollo καθαρτής with a stag, 26 that in about 470 Metapontum issued Apollo coins 27—these facts doubtless have implications about the soil in which Pythagorians' doctrine took root. His unique success had a quite individual kind of background, which combined the piety of chthonic mysteries with worship of Apollo the “purifier.”

The traditions about the persons surrounding Pythagoras scarcely take us out of the realm of legend. The name of Theano is famous; she is usually called Pythagoras' wife, but sometimes her daughter, or only his pupil; 28 but Theano is also the name of the wife of the mythical king Metapontus. 29 Bro(n)inus, named in the book of Alcmaeon, is sometimes called father and sometimes husband of Theano, 30 and both are placed sometimes in Croton and sometimes in Metapontum. 31 Pythagoras' son and successor is most often Telages, 32 but he also has other sons, daughters, and successors. 33 Of the other “Ancient Pythagoreans” collected in the Fragmente der Vorsokratiker, Cerclops is probably a figment of ancient philology, 34 Petron perhaps an invention, 35 Paron a misinterpretation of Aristotle, 36 and Parmiscus is only attested in the cult legend of Delos. 37

26 Dicaearchus, fr. 33 (Por. VP 18), names magistrates, youths, boys, and women; the speeches are given in a different order (youths, magistrates, children, women) in lambichus (VP 37-57, from Apollonius, after Timaeus: above, ch. II 1, nn. 12, 37). In an explanation by the Socratic philosopher Antisthenes of the word ποδόσφαιρος in Od. 11 (scholion on the line from the Homeric Questions of Porphyry), is found the sentence άστικα καὶ Ποθητώρας λέγεται πρὸς παιδῶν ἁρώμειας ποταμάκος λόγως διαθέτει πρὸς ἀπότομος λόγως παιδικοῦ, καὶ πρὸς γυναικῆς γυναίκος ἀρμοδίους, καὶ πρὸς ἄρχοντος ἀρχικοῦς, καὶ πρὸς ἐφίδιως ἐφίδιως. There is no cogent reason to strike this sentence of the Antisthenes material, as being an addition by Porphyry (with H. Schrader; L. Rademacher, Artium scriptores, SBDVem 227 [1951] 121f.): cf. also F. D. Caizzi, Antisthenis fragmena (Milan, 1966) p. 107. Interest of some Socratic in Pythagoras may be reflected in Plut. De curis. 516c. But the content of Pythagoras' teaching was unknown, according to Dicaearchus ap. Por. VP 19. The speeches given in extenso by lambichus (VP 37-57) are later a production (pace Rostagni, StFphil 1 12ff. De Vogel, Pythagoras 70-147; cf. Gymnasirom 74 [1967] 450).

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28 Dicaearchus, fr. 33 (Por. VP 18), names magistrates, youths, boys, and women; the speeches are given in a different order (youths, magistrates, children, women) in lambichus (VP 37-57, from Apollonius, after Timaeus: above, ch. II 1, nn. 12, 37). In an explanation by the Socratic philosopher Antisthenes of the word ποδόσφαιρος in Od. 11 (scholion on the line from the Homeric Questions of Porphyry), is found the sentence άστικα καὶ Ποθητώρας λέγεται πρὸς παιδῶν ἁρώμειας ποταμάκος λόγως διαθέτει πρὸς ἀπότομος λόγως παιδικοῦ, καὶ πρὸς γυναικῆς γυναίκος ἀρμοδίους, καὶ πρὸς ἄρχοντος ἀρχικοῦς, καὶ πρὸς ἐφίδιως ἐφίδιως. There is no cogent reason to strike this sentence of the Antisthenes material, as being an addition by Porphyry (with H. Schrader; L. Rademacher, Artium scriptores, SBDVem 227 [1951] 121f.): cf. also F. D. Caizzi, Antisthenis fragmena (Milan, 1966) p. 107. Interest of some Socratic in Pythagoras may be reflected in Plut. De curis. 516c. But the content of Pythagoras' teaching was unknown, according to Dicaearchus ap. Por. VP 19. The speeches given in extenso by lambichus (VP 37-57) are later a production (pace Rostagni, StFphil 1 12ff. De Vogel, Pythagoras 70-147; cf. Gymnasirom 74 [1967] 450).

The tradition of the political activity of Pythagoras is consistently related to Croton, rather than to Metapontum. When it is reported that Pythagoras made a series of speeches before the boys, the young men, the women, and the magistrates, 28 this may well reflect an archaic, club-like organization of society. 29 But the reconstruction of the political events is difficult.

Two events were basically important in determining the history of Croton during this period, its defeat by Locri at the Sagras River 40 and the victory over Sybaris which is dated 510 B.C. From that time on, according to the evidence of coins, 41 Croton exercised some kind of hegemony in southern Italy. It clearly ended, suddenly, about 450 B.C., shortly before the revival of Sybaris-Thurii. The Pythagorean tradition contains reports, with various small modifications, of a great catastrophe: the house of Milo, which was the meeting place of the Pythagoreans in Croton, was burnt down by their opponents, and only a few of those present escaped. 42 Since one of the survivors, Lysis,
II. PYTHAGORAS IN THE EARLIEST TRADITION

later became the teacher of Epaminondas in Thebes, this termination of Pythagorean dominance cannot have taken place before 450 B.C., and therefore cannot during the lifetime of Pythagoras. It is tempting to interpret these events along with the numismatic evidence, to the effect that from about 510 to 450 B.C., under the leadership of a Pythagorean oligarchy, Croton experienced a period of prosperity and power, which then was broken up by internal strife. After this catastrophe, Tarentum became the center of Pythagoreanism.

It is a question, however, what Pythagoras himself had to do with these events. After the defeat at the Sagras, according to Justin’s account, he succeeded in reforming the Crotoniates’ character to such a degree, by “daily praises of virtue,” that the victory over Sybaris became possible. Then, according to Timaeus, directly after this victory, they fell into luxurious living (*τροφή*). According to a detailed account in Diodorus, Pythagoras, functioning as the embodied conscience of Croton, had a part in bringing about the war against Sybaris. A number of factors conspire, however, to make this account seem very suspicious: the quite different account of Herodotus, the inherent improbabilities, and, most of all, the structure of Diodorus’ exposition, which is very like the plot of a tragedy. The destruction of Sybaris was the worst atrocity wrought by Greeks against a Greek city in that era; the attempt to make the unheard—of comprehensible


44 Justin 20.4.7ff, Timaeus FGrHis 566F44-45. According to Apollonius (lam. VP 255) the revolt against the Pythagoreans resulted from conflict over conquered Sybaris territory. Dünhabin (360) points out that no Olympic victors from Croton are listed for the years 548-532. This can be combined with the report of Justin and so used for dating the battle at the Sagras (above, n. 40). But is it certain that Croton’s recovery was due to Pythagoras?

45 Diodorus 12.9.2ff. The source is neither Euphorus (Diels, hesitantly, DK 14.14) nor Timaeus (Lévy, Sources 54f), but some later author (E. Schwartz, Re V 685f). Herodotus’ version (3.44f) is different. An illogical feature of Diodorus’ account is that the tyrant Telys of Sybaris (Herodotus calls him king) first banishes his opponents and then, after they have sought refuge at Croton, demands their return. What follows is modeled, down to details, on tragedies like Aeschylus’ Suppliantes and Euripides’ Heracleidae and Suppliantes. The exiles have taken refuge at the altars in the marketplace of Croton, emissaries of Telys demand their surrender and threaten war, the popular assembly is in doubt what to do until Pythagoras addresses them and reminds them of the obligation to protect suppliants, and the war and the famous victory follow. According to Andron (ch. II 3 below), Pythagoras foretold to a friend the conquest of Sybaris, which makes him a seer, without active participation in the events, as in lam. VP 113, 177 (Apollonius?) was bound to give rise to legends, and the contrast of “Sybaritic” luxury with Pythagorean sobriety was a strong stimulus to the creation of moralistic and edifying fiction.

According to Aristotle and Aristoxenus, Pythagoras withdrew to Metapontum before the outbreak of trouble and died there; but according to Dicaearchus, Pythagoras himself was in Croton at the time of the revolt, and in several accounts he is even said to have perished in the fire. Since the escape of Lysis cannot have occurred before 450 or so, the reports of Dicaearchus and the later writers cannot be accepted. Probably two anti-Pythagorean movements have been combined, one in Pythagoras’ lifetime and another about 450. It is clear, however, that there was no established tradition, no author seems to use documented evidence; everything depends on oral tradition, in which “Pythagoras” quickly comes to mean “the Pythagoreans,” in much the same way as the Ionians speak of Ion and the Dorians of Heracles.

When Plato contrasts Pythagoras with the lawgivers as one who became, ἡ δεύτερα, a guide to the good life, he is not thinking of him as a political figure. In some respects, Pythagoras is connected with

40 Arist. fr. 191, Aristoc fr. 18; similarly Apollonius (lam. VP 255), Philod. Rhet. II 180 Sudhaus, Justin 20.4. Others accounted for Pythagoras’ absence from Croton by saying he had gone to Delos to care for, and bury, his old teacher Pherecydes (Satyrus ap. D.L. 8.40, Nicom. ap. Por. VP 53, lam. VP 255). This was contradicted as early as Dicaearchus (fr. 34 = Por. VP 56; to judge by the form of the citation, in indirect speech, the sentence in question belongs with the Dicaearchus quotation, and not, as Lévy thought, Sources 51, n. 4, to Porphyry).

47 Fr. 34-35. Others expanded the story: his disciples made a bridge over the fire with their own bodies, so that the master might escape (Por. VP 57; Tzetzes Chil. 11.8ff). Hermippus’ version is unique: Pythagoras died in the war between Acragas and Syracuse, because he refused to escape by crossing a field of beans. (D.L. 8.40, Schol. Pl. Rep. 600b, Suda s.v. Pythagoras; at D.L. 8.93 this is combined with his flight from the fire. It is hardly possible to determine whether the similar story of Myllias and Timysha at the beanfield, recounted by Neanthes FGrHis 84F31 = lam. VP 189ff, is primary, as Lévy thought, Lög. 68, or derivative.)


49 Minar 53ff; with modifications Morrison, CQ 1956, 149.

50 Dicaearchus (fr. 34) refers expressly to oral tradition. The so-called ὁσιομαχία of the Crotoniates cited by Apollonius (lam. VP 262) are suspect. Reference to such a source may point to Timaeus (cf. Morrison, CQ 1956, 149); but the whole thing may be a fabrication, for the “memoirs” are supposed to prove that the arbitration judgment by the representatives of Tarentum, Metapontum, and Caulonia, which was unfavorable to the Crotoniates, was purchased by bribery; cf. Delatte, Vol. 218; von Fritz (Vol. 65f) thinks the “memoirs” may have been contained in a local chronicle which might have been embellished by legends. Cf. Lévy, Sources 115-9.

51 Rep. 600a. This is emphasized by Delatte, Vol. 17f; Frank, AJPh 1943, 222, n. 7; R. Joly, Mémo. Ac. R. de Belgique 51.3 (1958) 28.
Metapontum more closely than with Croton,\(^{52}\) while the political activity and the burning-episode belong to Croton.\(^{53}\) Some modern scholars have gone so far in a skeptical direction as to assert that Pythagorean political activity in Croton is an invention—Aristoxenus and Dicaearchus, they suppose, projected upon the Pythagoreans their own ideal of the βός πρακτικός, and thus invented Pythagorean politics together with an appropriate historical background.\(^{54}\)

There is, however, an often forgotten testimony of Theopompos, in the midst of a fragment of Posidonius about the tyrant Athenion of Athens.\(^{55}\) Athenion had been a member of the Peripatetic school, but at the first opportunity he cast aside the mask of philosophy and became a tyrant, thus illustrating "the Pythagorean doctrine regarding treachery, and the meaning of that philosophic system which the noble Pythagoras introduced, as recorded by Theopompos in the eighth book of his History of Philip, and by Hermippus the disciple of Callimachus."\(^{56}\) Theopompos must have said that the secret, but genuine, goal of the philosophy introduced by "the excellent Pythagoras" (ὁ καλὸς Πυθαγόρας) was tyranny.

There are other traces of an anti-Pythagorean tradition, representing Pythagoras and his pupils to be tyrants. Appian writes, in a context very similar to that of Posidonius, "also in Italy, some of the Pythagoreans, and in other parts of the Grecian world of those known as the Seven Wise Men, who undertook to manage public affairs, governed more cruelly, and made themselves greater tyrants than ordinary despots."\(^{57}\) Diogenes Laertius names a Πυθαγόρας Κροτωνάτης, τιραννικός ἀνθρώπος, who is said to have been a contemporary of the philosopher—a desperate attempt to get rid of the tradition of Pythagoras as tyrant;\(^{58}\) and elsewhere the revolt against the Pythagoreans is represented as a blow for freedom from tyranny.\(^{59}\)

The existence of this tradition as early as the fourth century helps us understand why it was so important to Aristoxenus to show Pythagoras as an opponent of Polycrates and an émigré in search of freedom, and to emphasize the readiness of the Greeks of southern Italy to accept Pythagorean dominance or leadership.\(^{60}\)

Theopompos' interpretation is as tendentious as that of Aristoxenus.\(^{61}\) But, if the Pythagorean dominance in Croton can be seen in two such different lights in the fourth century, as a model of free government under aristocratic guidance or as a detestable tyranny, the underlying reality cannot be an invention of Aristoxenus and Dicaearchus, nor one by Theopompos, but only a tradition that could be interpreted in more than one way. We must believe Dicaearchus' testimony that the memory of Pythagorean rule and of revolt against it was still alive in Magna Graecia in his day. Plato, too, alludes to the Pythagoreans' connection of politics and philosophy (above, ch. 1.4).

There is no inconsistency between this and the religious and ritual side of Pythagoreanism. In fact, cult society and political club are in origin virtually identical. Every organized group expresses itself in terms of a common worship, and every cult society is active politically as a σύντομο.\(^{62}\) Pythagoreanism fits into this picture and can be seen to have firm rootage in the social and political conditions of the time.

\(^{52}\) D.L. 8.46f. cf. Delatte, Vite 251. For similar concern about Pythagoras' "image," and the assertion that he authorized athletes to eat meat, cf. below, ch. II 4, n. 121.

\(^{53}\) D.L. 8.39f. Terr. Apol. 46.13; Arnob. 1.4v; cf. Delatte, Vite 251, Pol. 10. Apollonius, too, hints at this theme (fam. VP 257f).

\(^{54}\) Free choice as the basis of Pythagorean rule: fam. VP 249 = Aristox. fr. 18. Von Fritz (Pol. 188f), discussing Aristoxenus' basis, suggests that the whole passage from Nicomachus (Pol. VP 21f = fam. VP 33f) in which Pythagoras is represented as converting various tyrants and freeing many cities was drawn from Aristoxenus; and in this he is followed by Wehrli (ad Aristox. fr. 17). But considering the manner of citation, this seems uncertain: the sentence for which Aristoxenus is specifically cited, about Lucanians, Persians, Messapians, and Romans as pupils of Pythagoras, is missing in the parallel passage, fam. VP 33f but stands by itself at D.L. 8.14 and fam. VP 241. It is, then, an isolated citation, of interest to later generations because of the mention of the Romans; note also that it breaks the continuity at Pol. VP 21f. It is hard to attribute to Aristoxenus the anachronism of including Tauroeniun (above, ch. II 1, n. 37; Frank makes much of this, AJP 1943, 220).

\(^{55}\) His thrust at Pythagoras may be intended as a covert blow at the Academy, against which heathen胴 Παλαιάνας διαμαχής, Ath. 11.508c-d, and against which the reproach was so often directed, that Plato's pupils became tyrants (fam. Athen. loc. cit., from Herod.).

\(^{56}\) Thus in Athens the decease of the Hermis and the profanation of the mysteries constituted a political act, as the "Bacchanalia" were regarded in Rome as a comitatio (fam. 39.8.14.f). Plato's Academy was a cult organization (on this, Boyancé, Muses 254ff), and Dion's partisans were united, among other things, by initiation into the mysteries (Pl. Ep. 7.330e, Plut. Dion 56).
II. PYTHAGORAS IN THE EARLIEST TRADITION

In some reports the name of Pythagoras has an almost mythic function: a belief, a political act, or a mishap experienced by Pythagoreans is described by an assertion that Pythagoras taught, did, or suffered this or that. But we must resist the temptation to interpret Pythagoras as a mythical figure—the “speaker from Pytho” as an incarnation of the Hyperborean Apollo, his fiery death as deification, and the like. Even what is legendary has its relation to specific historical events. There is no doubt of the historical reality of the Pythagorean society and its political activity in Croton; but the Master himself can be discerned, primarily, not by the clear light of history but in the misty twilight between religious veneration and the distorting light of hostile polemic. Pythagoras and the Pythagorean legend cannot be separated.

3. METEMPSYCHOSIS AND “SHAMANISM”

That Pythagoras taught the doctrine of metempsychosis is generally regarded, and rightly, as one of the most certain facts in the history of early Pythagoreanism. For it is alluded to in the most ancient piece of evidence about Pythagoras, the well-known lines of Xenophanes,

καὶ ποτὲ μὴ στυφελιζόμενον σκίλακος παρώντα
φαινόμενον ἐποτήριον καὶ τὸδε φόβοι ἐποιεὶ,
πατὸν μηδὲ ῥάπτει, ἤπει η ἄλοιχ᾿ ἄνεος ἀστὶν
ψυχὴν, τὴν ἔγνων φυλετειαν ἄϊον.

63 This interpretation appears as early as Aristippus the younger (D.L. 8.21 = Diogenes Laertius A162): στὴ τὴν ἀλῆθεν ἁρώνῳ ὡς ἑκων τοῦ Πυθοῦ. Apollonius has it somewhat differently, in the context of the birth legends (fam. VP 7: δὴ τὸ τοῦ Πυθοῦ ποιμαντήθη). Pythagoras “sounds like a nom de guerre,” says Nilsson (A History of Greek Religion [Oxford, 1925, 1950] 202). But there are many personal names of similar form (Athenagoras, Diagoras, Hermagoras), and the name Pythagoras itself is not rare (cf. the RE articles).

64 C.F. Heracles on Octa, whom Perigeneus Proteus imitated in a.d. 167, as Timanthes of Cleonae had done (Paus. 6.8.4). Lévy, Lég. 711, refers to Croesus, who was transported from the pyre, according to the legend, to the land of the Hyperboreans, R. Eisler (Orpheus the Fisher [London, 1921] 118) rashly equates Pythagoras with Apollo, and G. Heracleous in turn with the sun-god, who makes his way to the west and is there burnt in his house...

1 D.K. 21187. D.L. 8.36 (A. P. 7.120; Suda svv. Xenophanes, στυφελιζόμενον). Whether Pythagoras was the person referred to was rejected by O. Kern, AGI 1 (1888) 499, by Rathmann, 371, and by Maddalena, 335ff. Maddalena adds that, even if Xenophanes did mean Pythagoras, the doctrine in question was not metempsychosis but only the equality of the human and the animal soul; but this overlooks the explicit formulation φιλοι ἄνεος ἄνεος ζωῆν. The passage is regarded as the best piece of evidence about Pythagoras by Zeller (I 557), Willkomm (CHLI II 190), Krantz (Hermes 1914, 226f; DK 1 490.38f), Mondello (in ZFM 314ff), Thomas (718), Long (17), Duddh (ib. 143 n. 55), and others.

Diogenes Laertius says these lines refer to Pythagoras. In the fragment itself there is no name mentioned, but Diogenes explicitly cites the beginning of the poem, so that at least his source, where Xenophanes is cited for evidence on Pythagoras, was based on the complete text. The first words (“And once . . .”) show that other, similar anecdotes had preceded this one; thus the subject was a well-known person, and not some anonymous “Orphic.” It is worth noting that Xenophanes also attacked Epimenides; not only the Homeric religion was subject to his criticism, but non-Homeric religious manifestations such as doctrines about the soul and about ritual purification.

An allusion of Aristotle is equally explicit. He complains of his predecessors that in their theories about the soul they paid far too little attention to the necessary presuppositions about the body: “They try to say what kind of thing the soul is, but do not go on to specify about the body which is to receive the soul, as though it were possible, as in the tales of the Pythagoreans, for just any soul to clothe itself in just any body.” In his critique of various philosophers, he introduces this ironical comparison with the Pythagoreans’ “myths” as though they were something well known. What he cares about here is not distinctions among individuals, but among species. The soul “clothes itself” (a common expression in the doctrine of metempsychosis) in any kind of body, human or animal; and this failure to distinguish between human and animal gives rise to the same scandalized tone that we hear in Xenophanes. Theophrastus, on the other hand, in arguing against sacrificing animals, tries to bring animals and human beings closer together. The former, too, have souls, quite like those of human

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Apollonius of Tyana imitated his Master, recognizing in a lion a later incarnation of King Amasis (Physiocr. V 5.42; see Lévy, Sources 2 n. 7.134).

1 D.L. 8.18 (DK 21A1), 1.11 (DK 21B20).

4 Arist. De an. 403b20 (DK 81119) and 414a22. Doubts that the passage refers to metempsychosis are expressed by: athmann 18f, and Maddalena 338ff. The latter claims that metempsychosis implies a judgment of the dead, so that it is the entrance of “just any” soul into “just any” body; but what Aristotle is talking about is the character of souls in general, and not of those of individuals. Maddalena suggests that he is thinking of the dust particles in the air (κοινόωρα), alluded to at De an. 404b18, though this doctrine is not so far compatible with other metempsychosis. Cherniss (Pers. 325 n. 130) leaves the matter undecided. See also Nilsson, Op. II 665, and Theiler Arist. 98, and the Mylius anecdote, Arist. fr. 131.

5 τιθῆς is used by Aristotle in a sense very similar to the modern one. Cf. 107432.

6 Krantz, DK 1 594.7; οἰδαίος ἡμέρας Hdt. 2.123; similarly D.L. 8.77, Por. Arist. 1.1, Max. 114.112, Diod. 5.286. Demonet can also speak of atoms as ὠσειόδες eis τοὺς τιποὺς (Arist); cf. Hippocr. Vic. 1.25; but the ancient commentators have no hesitation about the reference of the present passage to metempsychosis. Philop. De an. 140.3f cites Empedocles fr. 117 in explanation of it; cf. Olympiod. In Phd. A 1.1. p. 56.17ff Norvin.
II. PYTHAGORAS IN THE EARLIEST TRADITION

beings, or perhaps even identical with them, “as Pythagoras taught.”

Porphyry gives a description, taken from Dicaearchus, of the arrival of Pythagoras in Croton: Pythagoras had great success, and was invited to give lectures before the civic leaders, the young men, the boys, and the women.

As a result of these events, a great reputation grew up about him, and he won many disciples from the city itself; not only men but women too . . . [including Theano], as well as many from the non-Greek territory nearby, kings and nobles. Now the content of his teaching to his associates no one can describe really, for the secrecy [μυστηρίο] they maintained was quite exceptional. But the doctrines that became best known to the public were, first, that the soul is immortal, then that it migrates into other species of animals, in addition that at certain intervals what has once happened happens again, so that nothing is really new, and finally that we ought to regard all living things as akin. Pythagoras is said to have been the first to introduce these opinions into Greece.

Whether all this is from Dicaearchus is controversial and cannot be definitely decided by philological means.² It is in Porphyry’s manner, however, to quote long passages; and his second citation from Dicaearchus is an extensive section, as can luckily be proven from the parallels.³ The skeptical tone of the expression is noteworthy; no one, he says, knows for sure. This cannot come from Porphyry, who cites, from different sources, details about the mathematical knowledge and procedures of Pythagoras;⁴ nor from a source like the “handbook” used by Diogenes Laertius, which knows of writings by Pythagoras.⁵ In fact, such an expression is scarcely conceivable after the image of Pythagoras that originated with Speusippus, Xenocrates, and Heracleides had become canonical. This points, then, to the antiquity of the source, and specifically to Aristotle’s pupil Dicaearchus, who was so sceptical about the Platonic-Pythagorean doctrine of the soul.⁶ One of the doctrines designated here as “best known to the public,” that of the cyclical recurrence of all things, is attested as Pythagorean by Dicaearchus’ colleague, Eudemus.⁷

Ion of Chios had already named Pythagoras as the one who knew all about the soul, in an epigram which, according to Diogenes Laertius, refers to Pherecydes of Syros.⁸

Likewise, the tradition of the Greeks that lived on the Black Sea and the Hellespont, as reported by Herodotus,⁹ connects the Περί της Μητέρας with Pythagoras. In neither of these cases is anything said about metempsychosis, but Pythagoras is obviously the best-known name in the realm of immortality and the afterlife.

It is no less important to recognize, however, that the later tradition frequently attempts to ignore or interpret away the doctrine. There is not a word about metempsychosis in Aetius; and, what is more, he projects the Platonic doctrine of the soul onto Pythagoras. The

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⁴ Por. A. 3.56: οἱ φαντάσματα κατὰ Πυθαγόρας καὶ τὴν φύσιν τῆς αἰώνιας κληρονόμον (καὶ ζώον); from the concluding sentence of the argument attributed at 3.35 to Theophrastus. The sentence is hardly an addition by Porphyry, who expressly denies the extension of metempsychosis to animals (De regressu am. fr. 11 Bidez).

⁵ Por. VP 19; section 18 is given as fr. 33 by Wehrli. The transition from indirect to direct discourse, which may seem suspicious, has already taken place in 18. Section 19 was attributed to Dicaearchus by Rohde, Q 126, Jäger 40, Burnet, EGP 92, Restagno, Sämtliche 42 n.1, Levy, Sources 50, Mondofo in ZM 36.5. The attribution was contested by Rammann 36, on the ground that the report is incompatible with Dicaearchus’ participation in the “religion” (though we know at least, from fr. 36, that Dicaearchus did treat of Pythagoras’ metempsychosis doctrine). Madalena (76 n. 21) suggests that the uncertainty expressed does not concern the report itself, but rather a fragment, as Dicaearchus himself. Wehrli, following Rammann, omits section 19.

⁶ Por. VP 36: the conclusion 3.14. 8.40: the conclusion 3.14 W.

⁷ Por. VP 36, citing the Memoirs. In the next sentence Eudoxus is cited (fr. 36 Ginsinger, 125 Laverre). Ginsinger is wrong in assigning the whole passage, including the citation of the Memoirs, to Eudoxus.


⁹ 1. ch. II 1, n. 48. A similar agnostic expression in Por. VP 5 is probably also old: “If you happen not to know the country or the city whose citizen this man was, do not be concerned; for some say he was from Samos, some from Phlius, some from Metapontum.” The name of the author of this is, however, uncertain: MSS Λεόντει (Λεόντεις? ιδ.). See below, ch. II 5, n. 66.


¹¹ DK 501B4 fr. 30 Blumenthal = fr. 5 Diehl (D.L. 1.120). Rammann disputes the attribution to Pherecydes (44f), but this cannot be checked. He says the last verse is an interpolation, because the μόνος of line 1 has no corresponding μόνος, per line 3; but for a fragment of this kind of argument is inconclusive. Madalena (348f) emphasizes that metempsychosis is not explicitly mentioned. In line 3 the manuscript reading is εἰπέρ Πυθαγόρας εὐτύμως αὐτός, per line 2; the obvious correction is that of Sandbach, Proc. of the Philol. Soc. 185 n.5 (1958-1959) 36. The wording of line 4 is awkward. For the subject matter Kranz (Hermes 1934, 227) compares Heraclitus fr. 129 (followed by Mondofo in ZM 217f, and Sandbach, loc. cit.). A similar expression about Pythagoras is found at Ov. Trist. 3.3.62.

¹² 4.95 967, below, n. 283.
The doctrine of metempsychosis is set forth, with some detail, in works of Pindar,\textsuperscript{22} Empedocles,\textsuperscript{23} Herodotus (2.123), and Plato.\textsuperscript{24} The question how much of this can be accepted as testimony on Pythagoreanism depends on one's judgment of the complicated and much disputed phenomenon of "Orphism." Scholars' conceptions of Pythagoreanism and of Orphism are inevitably as interdependent as the parts of a balance. A "minimalist" attitude to the Orphic tradition rapidly raises the importance of Pythagoreanism,\textsuperscript{25} while hypercriticism toward Pythagoreanism peoples Greece with Orphoeteleia.\textsuperscript{26} There is no such thing as a communis opinio on Orphism, especially since the sensational discovery of the papyrus of Derveni\textsuperscript{27} has shaken many established views.\textsuperscript{28}

A few details, however, are likely to survive examination. There were 'Orphikai, purported poems of Orpheus, perhaps circulating in differing versions, and including at least a theogony and cosmogony, of the sort for which the papyrus of Derveni provides a philosophical commentary.\textsuperscript{29} There were 'Orphoeteleiai,\textsuperscript{30} who with reference to these writings gave private initiations to mysteries, in which the punishments in store for the unintiated in the next world were vividly depicted. For the initiates there was the bios 'Orphikos, an ascetic life featuring specific abstinences, and especially vegetarianism.\textsuperscript{31}

\footnotetext[22]{11} 11; see above, ch. I. In the Memoirs, as in Aetius, only the highest part of the soul is immortal. The neo-Pythagorean Alexander of Abonutichus rejected metempsychosis in somewhat similar terms: the "soul" develops and then perishes, but the important thing is the spirit (\phi\si\nu\tau\rho\alpha) which emanates from the mind of "Zeus." (Lucian Alex. 40; in the background is Pl. Tim. 10b; cf. Plut. De fac. 944c.)

\footnotetext[23]{13} 13 Sext. Emp. Math. 9.127f; on this pneuma theory which was attributed to Pythagoras see Cic. Nat. d. 1.27.

\footnotetext[24]{14} Fr. 98 H. = Por. Abst. 4.22. Xenocrates also speaks of the danger of "assimilation" to the souls of irrational creatures through the eating of meat (fr. 100 H. = Clem. Al. Strom. 7.32-39; here Polemo is named as also Xenocrates). For Theophrastus, see above, \textsection 6.

\footnotetext[25]{15} Sext. Emp. Math. 9.127, 129 cites these same lines of Empedocles, but does not mention metempsychosis.

\footnotetext[26]{16} On the theory of daimones, see Heinze 78ff; for the rejection of metempsychosis, 147.

\footnotetext[27]{17} Fr. 22 H. = Ian. VP 7 (cf. Por. VP 2). Eudoxus and Epimenides are also cited for this view; cf. the statement of Speusippus about the birth of Plato, fr. 27 L. - D.L. 3.4. Cf. below, n. 143.

\footnotetext[28]{18} Clearchus, like Heraclides, championed the immortality of the soul and its independence of the body (Welch) p. 47). In fr. 38 (Ath. 4.147c, 13c 441b14) he represents the Pythagorean Euphrosyne as a terrifying person as speaking about the imprisonment of the soul in the body and the prohibition of suicide (cf. Pl. Phd. 61c).

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Metempsychosis is not attested directly for Orphism in any ancient source—only the preexistence of the soul. It is undergone punishment in its confinement to the body, which is both a prison and a protection. Souls are borne by the wind into the body; it is an almost unavoidable supplement to suppose that other living creatures, at their death, have "breathed out" these same souls. According to Plato, priests of the mysteries teach reincarnation: "priests and priestesses who have paid careful attention to being able to explain their ministry" (Men 81a).

Herodotus ascribed to the Egyptians a fully developed theory of metempsychosis; and, since the Egyptians never had such a doctrine, it is clear that Greek ideas are being projected onto the foreigners. His manner of introducing the doctrine suggests south Italy. Herodotus makes the ominous addition: "There have been some Greeks, both earlier and later, who have subscribed to this doctrine, giving it out as their own. Though I know their names, I am not putting them down." All possible combinations have been tried in the attempt to decide what Herodotus is concealing, but with no certain result.28

28 Wilamowitz, GliH II 194: "eine orphische Seelenlehre soll erst einer nachweisen." That Orphism taught metempsychosis is contested by Krüger, Orph. 37f, Thomas 35f, Long 89f. On the other side of the argument are Rohde, Psyche II 121ff (Eng. ed. 341ff), Nilsson, Op. II 663ff, GrR I 611ff, Dodds, Irr. 149 with n. 94.

29 Plato Crat. 400 b-c; also Phd. 62b, Xenocrates fr. 20. Neither the Attic φρονούν ποτέ περιστάλερα can have occurred in a hexameter poem.


31 Leg. 898d-c; cf. Wilamowitz, Platon I 249. Perhaps we may add the verse of Rhadamantyes, Arist. EN 1132b27.

32 Cf. H. Bonnet, in Realelexicon der ägyptischen Religionsgeschichte (Berlin, 1952) 796f. This is regarded as a closed question; Kees, for example, does not even mention it.

33 2.123: "The Egyptians say that Demeter and Dionysus are the rulers of the underworld. And also, they were the first to proclaim the following doctrine..." Herodotus adds the doctrine of metempsychosis, because the preceding context reminded him of Greek ideas. Demeter and Dionysus in close connection point to southern Italy; it is there from that the triad of Ceres, Liber, and Libera was introduced to Rome in 493 BC.

34 Available for choice are the Orphics, Pythagorians, and Empedocles. (Plerocides is hardly eligible; cf. Long 13f.) The possible combinations are (1) Orphics and Pythagorians (Zeller, SBl 3889, 991; Nilsson, I 701; Morrison, CQ 1956, 137; and others), (2) the Orphics and Empedocles (Rathmann 48ff, though he does not exclude the first possibility), (3) Pythagorians and Empedocles (Long 22, Timpanaro Cardini 21ff). Maddalena (346f) concludes that Pythagoras is not intended, from the fact that Herodotus does name him at 4.95. It has been thought that it was Empedocles whom Herodotus did not wish to name, for the reason that he was still alive (Stein in his note on the passage; Burnet, EGP 88 n. 5). Herodotus is reticent in the realm of cultic ἀπόφθεγμα (e.g. 2.61, 2.47, 2.170f), but also as to what he considers inferior (4.41, 1.51; thus Long, 23 n. 60, thinks Herodotus does not wish to mention a plagiarist; Morrison, CQ 1956, 137, suspect he is thinking of Eukles). If one adopts the longer version at 2.81, Orphism is "it." Pythagorean for Herodotus, so that Pythagorians and Empedocles must be intended; and the echo of a theory of the elements (a journey through creatures of land, water, and air) does remind one of Empedocles fr. 115, even though there are details that do not match exactly (no mention of fire; 1,000 instead of 30,000 years; 100 years Kern, Orph. frg. 115).

35 4.81. Wilamowitz believes in interpolation (GliH II 189 n. 1 = 187 n. 1), as do Rathmann 54ff and PhW 54 (1934) 117ff, Linforth 38ff, Long 24f, Timpanaro Cardini 44. The longer text is defended by Krüger, Orph. 135, Boyancé, Musée 94.1, Dodds, Irr. 169 n. 80. "Hericthus (Orph. frg. 16, 272 n. 4) accepts the contested textus receptus without question."

36 C. Schmidt 2.672 and Jacoby, RE Supp. II 515ff. According to the evidence of the papyri, the twofold tradition does not go back to ancient times (A. H. Paap, De Herodotus papyri in papyris et monumentis Egyptis servatis [Leiden, 1948] 95ff), and in Byzantine times there was no longer any reason for interpolation. There is no way to tell whether, in his brief reference to the passage (Apol. 50), Apuleius was using the shorter text (as Linforth 43) or was himself abridging.

37 That is, the longer version but with the verb ἀναμιλουμαι. After Zeller SBl 3889, 991 and Burnet, EGP 88 n. 4, had separated the words and Πυθαγορευομαι by a comma, I corrected and modified to καὶ ἐξάγομαι (Coll. Budé, 1936, Maddalena, 236f, added omits as interpolated everything after Orphicoi).

38 More reasonable and intelligible," says Linforth (43); but this is merely the result of its brevity.

39 Linforth would like to deny this (43); but see 2.80: ἀμφιθεομαι... Λυκηπτος... ἀναμιλασθενομαι. Ἑλληνες οὐδεμισθενομαι. The expression καλομαι... ἐστιν δὲ γὰρ διεκτόθεν, ... καλομαι δὲ δὲ is not where directly attested, but διεκτόθεν... καλομαι δὲ is not infrequent (2.178, 1.90).

40 Wilamowitz GliH II 199.

41 Dodds, Irr. 169 n. 80; Rathmann, PhW 54 (1934) 1181, artificially separates τοῦτον των ὀφθαλμών: "the rituals of these."
II. PYTHAGORAS IN THE EARLIEST TRADITION

natural in the longer text. On the other hand, it is hard to reach a definite conclusion as to the extent to which the variants ὁμολογεῖ and ὁμολογούσει fit in better with the assumption of interpolation or omission. 47

In any case, Herodotus states that there is a connection between Orphism and Pythagoreanism in the realm of ritual. In addition, the longer text contributes not only an indication that Orphism is connected with Dionysus, but a theory about its origin, namely that it comes from Pythagoras, who got his teachings from Egypt. There is of course a relation between this and the presentation of the doctrine of metempsychosis, and probably also other features such as the statement that the Egyptian priests abhor beans, which has not been confirmed by Egyptian evidence. 48 Thus Greeks in Egypt connected beliefs about the afterlife and religious customs with the name of Pythagoras in the same way as Greeks living in the area about the Hellespont and the Black Sea did with the beliefs of the Getae about immortality (Hdt. 4.95). The difference is that in the latter case the Greeks' feeling of superiority to the barbarians led them to make Zalmoxis the pupil of Pythagoras, while in Egypt their awe of the ancient foreign culture produced an opposite result, and Pythagoras became the pupil of the Egyptians.

In general, modern scholars have seen Orphism as the older and more comprehensive movement and considered Pythagoras to have been influenced by it, 49 but the reverse theory is not only in the longer version of Herodotus; Ion of Chios also made use of it. In his Πραγματικοὶ ὁρώσεις, whose authenticity is guaranteed by Isocrates, 50 he said that Pythagoras "had written some poems and attributed them to Orpheus" (ἐγένετο μήν σέ μονοι ὄρνιοι οὖν ὁ Ὀρφέας). 51 This report was unearthed by ancient scholars interested in the controversy over whether Pythagoras left any writings; but it is a misunderstanding to conclude from it that Ion knew works of Pythagoras, who referred in them to Orpheus. 52 Τῆς Ὀρφείᾳ (or Μουσαίου) ἀναφερόμενα is a common designation for the literature which circulated in the name of Orpheus or Musaeus. 53 What Ion meant was that the real author of certain poems circulating under the name of Orpheus was Pythagoras—that the Ὀρφεία were Ἡθολογίαι. It was not works of Pythagoras that Ion knew, but Orphic poems whose origin he was trying to determine. Herodotus says emphatically that the poets who were reputed to have lived before Homer were actually later (2.53); and this is aimed specifically at Orpheus. Clearly, to an educated person of the age of the Sophists it already seemed incredible that works had been preserved that were

44 Cf. Plut. De Is. et Os. 564e: ὁμολογεῖ δὲ καὶ ταῖς Τιτανικαὶ καὶ Νυκτελκοῖς τοῖς λεγομένοις . . . and Plut. Caes. 9 (on the Bona Dea festival) ἓν γανείας πολλὰ τῶν Ὀρφικῶν ὁμολογοῦσα δρᾶν λέγοντα. Rathmann, PhW 54 (1934) 1182f, emphasizes that ὁμολογεῖ is elsewhere used by Herodotus only of persons; but in the sense "agree" = "resemble," the accusative of specification ταῦτα would be odd, too. The verb is construed with οὖν and κατὰ τὰ 1.142, 2.18, 6.54.

45 Linforth argues acutely (40) that an interpolator would also have had to alter ὁμολογοῦσα to ὁμολογεῖ (a point missed by all who accept the textus receptus), whereas an abridgment would not have needed to alter ὁμολογεῖ to ὁμολογοῦσα, so that the discrepancy has been caused by interpolation. Yet we read, in the passage beginning in section 80, ἀναφέροντα δὲ καὶ τὰ ὅτε . . . τάτη . . . ἀναφέροντα, so that it was easy to alter a following ὁμολογεῖ δὲ ταῦτα, by assimilation, to ὁμολογοῦσα, and when the neuters became masculines the abbreviation was necessary.

46 Hdt. 2.37, with B. A. van Groningen's notes (Leiden, 1946).

47 A Ἰερός λόγος which Iamblichus read told of Pythagoras being initiated by the Orphic Aegaeomarm (i.e. Ψυχῆς 1.146). It was regarded as certain that Pythagoras got his doctrine of metempsychosis from Orphism by Rolfe (Physic 1.102, 336f, Eng. ed.), Zeller I 68f, 563, Rathmann, Guthrie (Orphic 216ff), Nilsson 170f, Kern (Die Religion der Griechen II [Berlin, 1915] 152). See also larger Thol. 157, 1.15 Eng. ed.; HDR 12 (1939) 135-147; R. S. Black, Plato's Meno (Cambridge, 1961) 61ff, 247ff. On the other

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written by a member of the Argonautic expedition. On the other hand, they had to account for the existence of the "Orphic" literature, and there were various ways to do this. One could construct a number of characters all named Orpheus, and this was done as early as the fifth century by Herodorus of Heraclea. Or, one could put the responsibility upon other writers—ancient epic poets, Onomacritus, or Pythagoras and various Pythagoreans. These methods also crossed over one another, and this is probably the explanation of the reports about an "Orpheus of Croton," or "Cercops the Pythagorean." Epigenes, especially, was prone to explain everything by Pythagoreanism, and Plato, too, shows that he knows Magna Graecia is a suitable place for such material to come from, when he attributes a myth about the underworld to a κομψός ἄνδρες, ἰθωνία Σικελίων τοις Ἰταλικοῖς.

Perhaps Heraclitus, who in one fragment (40) reproaches Pythagoras along with Hesiod for useless πολυμαθία, had already connected Pythagoras with Orphism: Πωθαγόρης Μνησάρχων λιστρόφι ἠγορασκεῖ μαίλστα πάνω καὶ ἐκλεξάμενος ταῖς τὰς συγγραφαῖς ἐπιστήμονα ἀδάματο σοφίας, πολυμαθίας, κακοτεχνίας. The genuineness of the fragment has been suspected, because its purpose in Diogenes Laertius is to prove that Pythagoras left writings. But since it does not prove this at all, because all it claims is that Pythagoras plundered the writings of others, and not that he wrote anything himself, its authenticity has been generally recognized since the discussions of Wilamowitz and Reinhardt. There remains what seemed to Wilamowitz the "extenuating difficulty" ("peinliche Schwierigkeit") of guessing what "writings" Heraclitus could have been thinking of. Surely not Homer and Hesiod, who were known to all Greeks. But aside from them, and Anaximander, there was scarcely anything in the purview of Greek culture that Pythagoras could have used for such a purpose, except Ὀρφεία and the like. Perhaps the word Heraclitus chose, κακοτεχνία, points in the direction of the φωτιζω καὶ μάθηται.

If this is right, Heraclitus saw the relation of Pythagoras and Orphism in the opposite way from Herodotus and Ion. Where they attribute Orphic doctrine to Pythagoras, he makes Pythagoras the borrower. The contradiction is not particularly surprising, for even at that early date no one could be precise about chronology, in matters of this kind. Each arranged the facts in the way that best suited his purpose: whoever wanted to find a tangible personality in the chaotic mass of Ὀρφεία hit upon Pythagoras, and those who wanted to cast doubt on his originality used Orphism for this purpose.

Thus the oldest sources show Pythagoras, unlike Orpheus, as a tangible personality of the historical period, but their doctrines as connected or even identical. There is no support in these sources for the modern attempts to discern a difference in doctrine between Greek and Ionian periods.

43 See also Arist. fr. 7.
44 Unlike the Pythagoreans, the Orphics committed their teachings to writing from the beginning, as is shown, aside from the evidence cited in n. 29 above, by the vase paintings showing a scribe standing before Orpheus' singing head, Linforth 122ff. (Incidentally, their use of writing gives us a historical terminus post quem.)
45 FGrHist 315F2.
46 Proclus, the composer of a Μύριας, in which a trip to Hades was described (Kinkel, EGP p. 215ff.), becomes the composer of the Orphic κατακεφαλία (Suda s.v. Orpheus = DK 1 A1; Wilamowitz, GtII 197 n. 3). On Cercops see n. 60 below.
47 On the role of Onomacritus see Nilsson, Op. II 643ff. It is certain that he collected and edited; but the idea that he composed whole poems (as Philop. De an. 186.26, Suda s.v. Orpheus) or "invented" the story of Zagreus (as Paus. 8.37.5), is philological conjecture.
48 Asclepiades ap. Suda s.v. Orpheus, where we also find an Orpheus from Camarina. Nilsson assumes (Op. II 644) that the poems in question actually came from Croton and Camarina.
49 The testimonia are in DK 15. A tradition transmitted by Aristotle names a Cercops as a rival of Hesiod (Arist. fr. 75 = D.L. 2.46), and the epic poem Αργείους was sometimes ascribed to Hesiod (fr. 294-301 M.-W.) and sometimes to Cercopes of Miletus (Hes. fr. 301 M.-W. = Ath. 11.503d; cf. also Schmid 1.254). It was natural to identify the rival of Hesiod with the composer of certain poems that stand in rivalry to Hesiod's Θεογονία—the Orphic theogonies; and, if then the Ὀρφεία were classified as Pythagorean, Cercops became straightway a Pythagorean. There is no need to distinguish two men named Cercopes (pote DK 1 106.6ff., Timpanaro Cardini 69 n.);
50 Epigenes ap. Clem. Al. Strom. 1.135.5 attributes the Καταπνευς and the Ἑραίος Λέγος to Cercops, the Πνεύμα and the Φωτεινός to Britomartis (DK 17).
51 Corp. 493a; cf. below, ch. III 2, n. 48. 
52 D.L. 8.66; it was branded spurious by Diels, AGR 1890, 451f., and included among the "dubious or forged" in DK. Zeller (SBBIn 1889, 988; 139 n. 5) wished to strike out at least the mention of writings: ἐκλεξάμενος ταῖς τὰς συγγραφαῖς.
53 For its genuineness, after Bywater: Burnet, EGP 134 n. 2; Reinhardt, Parn. 235;1; Wilamowitz, GtII 188 n. 1; Rathmann 39; Krauz, Hermes 1934, 227f.; Morrison, 14 1936, 136. Reinhardt called the reference to ιστορίας in ταῖς τὰς συγγραφαῖς "απὸ ἀθώτης ἐς ἐκλεξαμένη" (cf. fr. 5). There could have been something in the preceding context that would have made the word συγγραφάς easier to understand. (H. Gomperz, Hermes 18197 41, tries to show that fragments 40, 41, 129, and 81 followed each other, without a break, in that order; but this cannot be proven.)
54 See below, n. 226. Onomacritus worked ἐκλεξάμενος, according to Hdt. 7.6 (a reference supplied me by M. West). The reference is to Orpheus, according to Rathmann, ut. per contra, Krauz, Hermes 1934, 116, and Mondolfo in ZM 317 point out that, according to later tradition) Pythagoras collected Egyptian, Babylonian, and Persian traditions. Are we to suppose, however, that Heraclitus thought Pythagoras capable of doing research in foreign-language books? (To be sure, a certain Antiphon states that Pythagoras burned Egyptian, D.L. 8.3.)
55 Rathmann 43ff.
Orphism and early Pythagoreanism. It is only too easy for modern notions to intrude. If one believes, with Nietzsche, in a primal opposition of "Apollonian" and "Dionysian," then Pythagoras and Orphism must stand in the same polar relationship; and if, under the influence of later evidence, one regards the philosophy of number and the foundation of exact science as the essential ingredient of Pythagoreanism, the antithesis of Apollonian rationality and Dionysian mysticism fits in very nicely. We must bear in mind, however, that as the Greeks thought of them, Apollo and Dionysus were brothers; the supposed clear differentiation of Pythagoreanism from Orphism is simply not attested in the oldest sources.

In the historical and social realm, much more than in that of doctrine, we do seem to detect a certain difference between the two groups. Much of the evidence about the Orphics, as in general about the unofficial mystery cults, clearly reflects the activities of mendicant priests. Pythagoreans, however, both in Croton and probably elsewhere in southern Italy, held position of dominance in their cities, for a time. This means that something related to Orphism had emerged from the anonymity of back-alley ritual and become respectable; known persons, not apocryphal writings, are active in this movement—Πυθαγόρας, not Ὀρφεύς. Legend knew of royal seers like Melampus; the abnormal is sometimes rated higher and sometimes lower than the normal. While the cults based on the supposed writings of the mythical singer of antiquity remained suspect in the eyes of many, in the person of Pythagoras the ancient figure of the wandering purveyor of salvation had taken on a new radiance which cast its gleam as far as Plato's philosopher-king.

A general observation may advance our argument somewhat. Mystery cults offering the promise of salvation, even along with cosmogonic myths, are conceivable without a doctrine of metempsychosis; and the fact that Orpheus, but not transmigration, is often connected with Eleusis might be an indication that there was at least one branch of "Orphism" without metempsychosis. In that case, this doctrine must be an innovation upon a general Orphic background; for its introduction, Pythagoras is the only obvious candidate. There is a possibility, deserving serious consideration, that it came from India.

It is the Pythagorean variant of Orphism that is manifest in Pindar, Empædocles, Herodotus, and Plato, connected always with south Italian or Sicilian tradition. What details are to be attributed to Pythagoras himself remains an open question. Does every living creature have an immortal soul that migrates from one incarnation to another? Do plants have such souls? Do they only enter certain species of animals? Do only certain special individuals, even among men (nephates, Empædocles calls them) undergo this wondrous experience? To what extent does the doctrine imply clearly formulated}

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67. "'Απολλιναίσσα ἡ ὁρφισμός" is Cicero's characterization of the achievement of Pythagoras ("Orphismo et Pitoarismo nei loro rapporti politico-sociali," Atti R. Acc. di Arch., Lett. e Belle Arti n.s. 12 [Naples, 1931-1932] 209-223). At the same time, Cicero interprets Orphism as "democratic" and Pythagoreanism as "aristocratic." Linforth considers the conjunction of "Bacchic" and "Pythagorean" in the longer version of Hdt. 2.81 to be an argument for atheism. Maddalena (363 n. 98, 327 n. 25) would use atheism in this same passage to such effect as to remove completely any connection of Orphic and Pythagorean, and Delatte also (Litt. 4fr) is for carefully separating them. Cameron (3ff) assumes an independent development from a common source, and both Rohde (Q. 104) somewhat differently in Psyche II 107 = 316ff. Eng. ed.) and Dudds (fr. 143, 149 n. 95) also interpret the two movements as parallel developments. Frank (357 n. 168) and Boyancé (Muses 93ff, cf. REG 1941, 161 n. 2) stated that they were inextricably intertwined. Nock emphasizes that in contrast to Orpheus, Pythagoras is "a tangible figure" ("Herodotus 2.81," Studies Presented to F. L. Griffith [London, 1932] 248). Note the confident assertion in Herodotus 4.96 that Pythagoras lived many years later than Zalmon.

68. Cf. the anecdote of King Leotychidas and the Orphentrestes Philippus, Plut. La. apoll. 224f. It need not be true historically, but is good evidence for the general atmosphere. For the typical figure of the ἀφορύσης, which is in itself quite old, cf. Aesch. Ag. 127fr, Ar. Pau 105fr, Ar. 99ff, Demosth. 18.200, 19.240, 10.281 (on Aeschines' mother), Antiphanes fr. 139, Clearchus fr. 47 W.
beliefs, and how important is the word ψυχή? Was there present at its beginning the significant semantic innovation whereby the “soul,” as distinguished from the body and independently of it, is regarded as the “complete coalescence of life—soul and consciousness” —a world away from the Homeric conception—or is “soul” primarily a mysterious, meta-empirical Self, independent of consciousness, as some important witnesses seem to indicate? Is the soul newly incarnated immediately after the death of the old body, in which case Hades becomes unnecessary, or is there an intermediate phase, which would leave Hades there, as a way station? Is the process of palinogenesis the work of blind natural forces—a creature at the moment of birth sucks in the soul—or is it the execution of a penalty assessed in a judgment of the dead? Is there an endless, cyclical movement, or is there a fall at the beginning and a salvation at the end which is permanent—or perhaps has as its alternative an eternal damnation, in which case the concepts of Elysium and Tartarus again become relevant? Each of these questions is answered in more than one way in the ancient tradition, and there would be small prospect of success in an undertaking to crystallize out a sharply defined “doctrine of Pythagoras.” There is a veritable maze of conflicting tendencies—mythical tradition persisting along with newly developing concepts of the world, the tradition of the mystery cults, ethical demands, and a growing recognition of natural law. It is only too easy for the modern scholar, from the vantage point of his own rationalistic and systematic activities, to suppose that at the beginning there was a unified, carefully worked out, and firmly defined theory.

On such a topic as men’s conceptions of a future life, a topic bearing on matters so completely outside normal experience, a consistent and unified doctrine can only exist in the form of scripturally established dogma maintained by ecclesiastical authority. In oral tradition, or in the belief of a group no matter how closely knit, though agreement in essentials may persist, varying interpretations are bound to emerge. Conceptions of the afterlife are and have always been syncretistic. It is only theology, coming along rather late in the tradition, that is interested in smoothing out the differences. Thus it is not to be presupposed that Pythagoras would commit himself to answer all possible questions; the main point was not a well-rounded system, but the working power of the doctrine. If there was no established “Holy Word,” new interpretations were bound to develop among the disciples, on the ipse dixit, αὐτός ὁ ἐφασμόν principle. Only dead dogma is preserved without change; doctrine taken seriously is always being revised in the continuous process of reinterpretation.

The metempsychosis doctrine does present a new start, after all, even in the hybrid character that its various aspects reveal. It does not offer mythical narrative, a picturesque story which gives the interpretation of a ritual, but a general doctrine which claims to be immediately true. While even in Hesiod, no matter how carefully

77 Empedocles speaks of ἡμώνη. In Xenophanes fr. 7 it is not stated that the dog “has” a soul, it is “the soul of the friend, and “ cries out” (Fränkel DP 311).
78 Jaeger Theol. 9ff = Eng. ed. 83. See Long 2ff. It is beyond doubt that metempsychosis, or at any rate related religious concepts, played a significant role in the development of the notion of ψυχή (see Dodds, Irr. 140ff; on ψυχή in Heraclitus, B. Snell, Entdeckung des Geistes [Hamburg, 1953] 360ff = 179ff Eng. ed.), but they do not presuppose it.
79 Only exceptional persons like Pythagoras remember their previous incarnations. Pindar even sees a polar relationship between the soul and “consciousness”: The πάντως ἑβδομος, which comes from the gods, sleeps when one is awake, and is active while he sleeps (fr. 130; Aesch. Eum. 104; Hippoc. Insos. VI 640 L.; Arist. fr. 10). In relation to Indian ideas of metempsychosis, too, one finds it necessary to speak of a meta-empirical “self” (Long 10).
80 This is suggested in Hdt. 2.123 and Schol. BT II. 16.857. Empedocles has no underworld; for him earthly existence is Hades (as Rohde, Psyche II 179ff = 381 Eng. ed.; Dodds, Irr. 174 n. 114; wrongly rejected by Long 39ff).
81 Cf. Serv. Aen. 3.68. Some of the acusmata of Pindar (Arist. An. post. 94b33), and of dead persons under the earth (Ael. VH 4.17); Pindar and Plato insert a stay in Hades between reincarnations. The kataklysm stories involving an “underground dwelling” are rooted in the traditional conception of Hades. The bean taboo may also be relevant here (ch. II 4).
82 Hdt. 2.123: εἶ ἄλλα ᾿ ᾿ ἰ α ῾ αν καὶ γίγνεται ἑβδομος. Schol. T II. 16.857: Πυθαγόρας φησίν, ᾿ αν αναειρότα τ’ ἑβδόμην ἐν είναι γίγνεται [είν] ὡς ἐν γεννημένου σάμμας ἐν ηδόν κατάγεται (similarly Schol. II); the soul of Patroclus complaints of its fate, because it fears an unworthy rebirth. Cf. also Arist. De an. 407b22 (above n. 3). Stettner regards this amoral metempsychosis as the primary type (7ff, 20ff.) Kern, Orph. frag. 223 tries to compromise.
85 The version of Pindar and Plato (though Rep. 611a and Tim. 41f introduce the idea that the number of souls in the world must remain constant). The idea of a fall and a salvation is most prominent in Empedocles and in Plato’s Phaedrus. Reward and punishment in the next world are very important in Orphism (above nos. 30 and 68).
86 In Egypt, even the oldest pyramid texts show a good many mutually inconsistent ideas about the afterlife (see Kees).
87 This is true even if at first the metempsychosis doctrine did not apply to all living beings, or even to all men. The story of Euphorbus was intended not as an exceptional case but as an example. On the other hand, there do seem to be special cases in the accounts of Epimenides (below n. 166) and Aesopus (n. 170). Plato is thinking of metempsychosis in this way when he evaluates it as an effort λόγων διάδος (Men. 814a).
each detail of his exposition is planned, the relationships and the laws of the universe are only indirectly stated, in genealogies and mythical episodes, the doctrine of metempsychosis directly states a general law. It does not merely explain particular situations; past and present, pre-existence and life after death are comprehended in a single thought. And, insofar as animals and plants are included in it, the unity and homogeneity of the universe are grasped. In the thought of the pre-existence and immortality of the soul lies an attempt at consistent thinking, a groping for something like an eternal, imperishable Being—a foreshadowing of the ontology of Parmenides. To this extent, even leaving aside the ideas of science and philosophy, Pythagoras, as teacher of metempsychosis, is not simply the prophet of old-fashioned piety, but at the same time, in his own way, a thinker leading, through ancient forms, to a new level of consciousness.

A doctrine like that of metempsychosis, which transcends normal human ways of knowing, can find a guarantee only in supernatural experience, in the world of the divine or quasi-divine. If Pythagoras knew the facts about the fate of the soul in this life and the next, he must have had superhuman powers and faculties; the prophet must be able to refer to his own example. It is natural, then, that elements of the Pythagoras legend are, from the beginning, connected with the doctrine of metempsychosis.

It is often seen as matter for regret that miraculous tales have attached themselves to the figure of Pythagoras and make it difficult for the scholar to disentangle the thread of historicity from the web of legend and fiction. In the circumstances it is very tempting to use expressions like “neo-Pythagorean” or “late antiquity” to classify these tales. One feels confident, in any case, that they represent a secondary growth, layers that must be stripped off until what the scientific historian recognizes as “facts” can be seen. Only in a few cases has it been recognized that these miraculous stories do not conceal but reveal reality, that they give us a clue to the impression made on contemporaries by an actual person, and that they may even contain facts of a special character. The Pythagoras legend is the oldest available layer of the tradition on Pythagoras; it is attested earlier than any of the “historical” details of his life in Aristoxenus and Dicaearchus, and is presupposed by the Platonic reinterpretation of Pythagoras in the Old Academy.

As early as Empedocles, Pythagoras appears as a superhuman figure:

\[
\text{διε τις εν κείμενων ἄνδρα περιώσας εἶδος,}
\text{διὰ δὴ μηκοτὸν πραπτίων ἐκτίμασα πλοῦτον,}
\text{παυτοίων τε μάλιστα σοφῶν τ' ἐπείραμος ἀργῶν}
\text{οπρότε γὰρ πάσης ὅριασα πραπτίσεως,}
\text{μὴ δὲ γὰ τῶν ὅτις πάντων λεισσεαεικονύκεστον}
\text{kai τε δεκ' ἀνθρώπων καὶ τ' εἰκόνα αὐλόνζουσιν.}
\]

Thamus understood the words as applying to Pythagoras; others thought of Parmenides. This clearly shows that Empedocles did not give any name, but also that the anonymous figure appeared as the teacher and master of Empedocles. The praise of Epicurus by Lucretius, and the praise of Lucretius by Vergil, are in the same tradition; neither gives a name, any more than Ovid does when he introduces Pythagoras. Therefore, neither some anonymous Orphoeotelester nor a

88 In the saying ascribed to Thales, too, πάντα πληρόοθεος, the word πάντα points in this direction; there are not sacred and profane realms, but a unified world.
89 Cf. Kahn's judgment of Greek doctrines of metempsychosis (AGP 1960, 34): “by its rigor and its generality, such a doctrine is no longer primitive.” Nilsson, Op. III 51 sees the origin of the doctrine of metempsychosis in “pure logic”—an exaggeration, but it gets at an important aspect of the matter.
90 Cf. Empedocles' expression (fr. 117) ἕδε γάρ ποτ' ἐγώ, and also Epimenides, DK 3B2 (Orpheus and Musaeus are also, of course, sons of gods (Pl. Rep. 363c, Tim. 40d).
91 Zeller, Vortr. 30; Rathmann 17f: “fulbuls tamquam adventuerum.”
92 Zeller (I 367) says that a great part of the miraculous tales is “neo-Pythagorean” (cf. Vortr. 44f, and Schottlaender 342). Kerényi (13) speaks of “spatantik Pythagoras-legenden.” Rathmann (25ff) tries to show that all the stories of Pythagoras were derived from the legends of Epimenides, Altiris, Orpheus, and Zalmoxis; but this is refuted by the “golden thigh” theme. Maddalena (358) sees that everything essential is attested in 4th-century sources, but does not admit that it may go back further.
93 Bohle, in 1871 (Q 105), recognized the “ganz echten Mythencharakter.” See also Leypanum, Gigon, Ursprung 131, Dodds, Intr. 144f, with nn. 63–64. Levey thought the Pythagoras saga had, indirectly, even influenced the gospels (a view disputed, rightly, by LEIP. I.3 Langrange, Rev. bibl. 45 [1936] 481–511; 46 [1937] 5–28, and M. Goguel, RevPhil 44 [1938] 241–270). He tried to show that the historical tradition about Pythagoras, in Anaximenes and others, was a rationalization of the legend. Rostagni, too (Verbo 121 n. 1) speaks of the “gospel” of Pythagoras.
94 Fr. 129 (D.L. 8,54) — Timaeus FGrHist 566F14, and Nicomachus ap. Por. VP 4,1b (Im. VP 67).
95 Rathmann 45ff, Maddalena 341ff. For Pythagoras as teacher of Empedocles, see below, ch. III 3, n. 59; for Parmenides as his teacher (obvious from the fragments), Hofmann ap. D.L. 8,55 (Dox. 477f–18.
96 Im. 1,62ff, Verg. G. 2,490ff, Od. Met. 15,60ff (Rostagni, Verbo 266f, Long 18 n. 10). Rostagni mentions the report that Pythagoreans avoided pronouncing Pythagoras’ name (Verbo 231; Im. VP 88, probably from Aristotle; also Im. VP 150, 255).
97 Rathmann 138.
man of the Golden Age\textsuperscript{98} can be intended. The mental power of the great man extends over "ten and twenty human lifetimes","\textsuperscript{99} his mind is superior to the change of generations—is it his immortal soul that remembers "every detail"?

Heraclides has Pythagoras tell this tale.\textsuperscript{100} He had once been Aethalides, the son of Hermes, and received from the latter the gift of remembering everything, both in life and in death. Thus he knew that, as Euphorbus, he had been slain by Menelaus in the Trojan War, and that he had subsequently been Hermotimus, then Pyrrhus, a fisherman of Delos, and finally Pythagoras. Here we are on the shaky ground of Academic and Peripatetic controversy. Dicaearchus and Clearchus give two different lists of Pythagoras’ previous incarnations: Euphorbus, Pyrrhus, Aethalides, a beautiful prostitute named Alco, and Pythagoras.\textsuperscript{101} It looks as though each one treated the Pythagoras tradition as his whim or fantasy dictated.

Actually, it can be shown that there was an independent tradition involving Aethalides,\textsuperscript{102} Hermotimus,\textsuperscript{103} and the Delian fisherman,\textsuperscript{104} but not Pythagoras. Obviously Heraclides first put the two traditions together and the “beautiful prostitute” may be a sarcastic addition of Diogenes.\textsuperscript{105} Present throughout, however, is the assumption that Pythagoras had “proven” his theory of metempsychosis by recounting his previous incarnations; and also present throughout is the puzzling figure of “Euphorbus.”

In Antiphanes’ comedy \textit{Neothis},\textsuperscript{106} first presented not long after 442 B.C., appears a miser who gives his slaves nothing to eat:

\begin{quote}
\begin{center}
αὐθάρποσος ἀνώτερήλτος εἰς ποντάριαν, τιμώτας οὗς μηδὲν εἰς τῆς οἰκίας,
μηδὲν ἀνὸν Ὠδελαρέας ἑκένωσ ὄφθεν ἡ θαμακαρβίς, εἰσφέρειν ἑξω τῆς ὁδοῦ.
\end{center}
\end{quote}

He is alluding to Pythagoras’ ascetic precepts, but the decisive word for the interpretation is \textit{промышкептис}. \textit{Τρίς μικάρες Δωναίοι...} (Od. 5.306) would be a familiar phrase to every Greek, but \textit{промышкептис} is only used of one who has “gone to his reward,” that is, who is dead. The poet is punning; the “thrice blessed” Pythagoras is at the same time the “thrice dead.” One should not lay undue stress on the number 3, which has an intensive force, and is also explained by the conventional formula \textit{τρίς μικάρες}. But this much is certain: that it was well enough known in Athens, about 342 B.C., that Pythagoras had lived through several lives, that a comic poet could count on an allusion to the idea being understood.\textsuperscript{107}

Later testimonia often name no one but Euphorbus as an earlier incarnation of Pythagoras,\textsuperscript{108} and since this name is mentioned by both

\textsuperscript{98} Zeller, \textit{SBIIA} 1889, 990; I 396 n. 1, 584 n. 5. Against this interpretation Rostagni (\textit{Verbo 220ff}) cites fr. 132, which shows that in Empedocles’ view there are at all times “divine” men. The attribution to Pythagoras has been defended also by Krantz, \textit{Hermes} 1935, 112 n. 2; Mondillo in \textit{ZM} 339: Long 177ff: Timpanaro Cardini 176. Maddalena, too, concedes this (64.3), but contests the connection with metempsychosis, since Nicomachus interprets the words as referring to Pythagoras’ hearing the harmony of the spheres; but it may be that Nicomachus, or one of his predecessors, was trying to get rid of the metempsychosis theory.

\textsuperscript{99} ἀνθρώπον ἀνωτέρα δειον ἐν τοις ζωικοῖς μενεσθείων; DXK. Homer’s Calchas knows “what is, what will be, what was” (Il. 1.70), the sibyl, too, defines time (Heraclitus fr. 98); cf. also Parmenides fr. 4.1.

\textsuperscript{100} Fr. 89 W. = D.L. 8.4; repeated Hippol. Ref. 1.2.11, Por. VP 45 (Euphorbus at the beginning), Tert. An. 288 (Pyrrhus before Hermotimus), Schol. Eph. 62 = \textit{Suda} s.v. ἀναθέα (Πθῆςος Καράκης). Hes. \textit{In Rnf.} 3.40 (from Por. VP 45; Callicles instead of Aethalides), Schol. Ap. Rh. 1.664 (a double version, with Hermotimus once corrupted to \textit{Hephaistos} and placed after Pyrrhus, and the second time left out). See Heibl, \textit{Psyche II.} 417ff = 598ff Eng. ed., Rathmann 9 n. 67. Which book of Heracleides is the source is uncertain (\textit{Abaris} according to Corssen, \textit{RhM} 1912, 28; A. Rehm ibid. 423; Lévy, \textit{Sources} 40; Wehrli, hesitantly, suggests the \textit{dimois}).

\textsuperscript{101} Dicaearchus fr. 36, Clearchus fr. 10 = Gell. 4.11.14.

\textsuperscript{102} Pherecydes of Athens, \textit{FlGHist} 3F19 (Schol. Ap. Rh. 1.645. (See Jacoby’s note; the attribution to Pherecydes of Syros, DK 7B8, can scarcely be right.) Aethalides is a son of Hermes. He has the privilege that his soul may dwell part of the time on earth and part of the time in Hades; this is reminiscent of the Dioscuri, not of metempsychosis. Aethalides belongs to Leos, this may have something to do with the supposed Tyrrhenian origin of Pythagoras (ch. 2, n. 12).

\textsuperscript{103} Below, nn. 177, 178.

\textsuperscript{104} Pyrrhus is probably the same as Pyrrhus. The skill of a certain “Delian diver” was proverbial (D.L. 2.22, 9.12, Herondas 1.41). There is a swimmer near the ship \textit{Theta} as it lands in Delos, on the \textit{François} vase. Perhaps some Delian ritual lies in the background.

\textsuperscript{105} \textit{Ibid.}

\textsuperscript{106} \textit{Antiphanes} fr. 168 Kock = Ath. 4.1080c. The date of the \textit{Neothis} can be deduced from 1.59 Ath. 6.2232c, where allusion is made to Demosthenes’ position in the negotiations with Philip about Halonnessus in 342 B.C. (Ath. in his comment on the argument, hypothesis to ps.-Demosth. 7).

\textsuperscript{107} \textit{Ibid.} (The passage is missing in the index of Kaufel’s edition of Athenaeus, s.v. Pythagoras, and should be for this reason omitted in DK and Wein. Zeller cites it (II 2.93), but \textit{metempsychosis} is “previewsing” (LSJ has it right.) It is possible that \textit{Anagorapha} is dependent on Heracleides; he seems to allude to him in fr. 113 Kock (\textit{Hermes} 46:6).

\textsuperscript{108} I allow fr. 191.19 Pl., Dios. 10.6.1, Hor. Carm. 1.28.11 (where the word iterum \textit{affords further incarnations}), Nicom. (Por. VP 26 ian. VP 63), Hippol. Ref. 1.3.3. At 16 B 30 De Falcó, Androcles, Guduleides, Aristoxenus, Hippoboulus, and N. Bath is assumed, but it is impossible to make out how much may go back to \textit{Anagorapha} (fr. 12, with Wehrli’s comment p. 60). That Empedocles spoke of “Euphorbus” (Rostagni, \textit{Verbo} 220ff) is unlikely; ancient scholarship on Pythagoreanism would hardly have failed to mention such a fact.\textsuperscript{109}
Heraclides and Dicaearchus, the conclusion has been drawn, rightly, that this detail of the tradition is older than Heraclides. Before the beginning of scientific historiography, the historical consciousness of the Greeks relates mainly to two periods, the immediate past and the Homeric period known from the Iliad and the Odyssey; so that when Pythagoras spoke of his earlier lives, he had to establish his presence in the earlier period. But why did he choose precisely this not very distinguished participant in the Trojan War? Some suggested answers are based on the etymology of the name, but the most persuasive interpretation is that of Karl Kerényi, who found a clue to the riddle in the words of Homer. As Patroclus is dying he says to Hector,

109 Rohde, Psyche II 418 = 599 Eng. ed.; Corssen, RhM 1912, 420ff. (Contra, Lévy, Sources 34 n. 1: the naming of Euphorbus alone is a result of secondary abbreviation; but, if that is the case, why do Aetolidae, Hermotimus, and Pyrrha never get named by themselves as earlier incarnations of Pythagoras?)—Pythagoras recognized, in Argos, the shield of which Menelaus had despoiled the body of Euphorbus (Diod. 10.6.2, Ov. Met. 13.103f, Schol. T II 17.28). This happens in the Heraea near Mycenaean according to Nicom. ap. Por. VP 27 = Iam. VP 63; cf. Paus. 2.17.3. No place is given in Hor. Carm. 1.28ff, Hipl. Ref. 1.3.3. Maximus Tyrius (10.3) speaks, erroneously, of a temple of Athena. Herodotus (fr. 89) has Hermotimus find the shield, in the sanctuary of Apollo at Didyma, corrupted in Tert. An. 28 to "Delphi." This has the appearance of being secondary, and if so the story in the earliest version is at least older than Heraclides. Nicomachus finds the story silly: τὰ γὰρ ἤσοροικά περὶ τῆς ἁπάνταις παράτυχους καὶ πάνω δήμων.

110 This consideration would seem to indicate, again, that originally only one earlier incarnation, as Euphorbus, was mentioned. It may have been only later that people calculated a certain number of years. For instance, if the first reference to Euphorbus (D.L. 8.14 mentions 207 years, though this is surely to be corrected to 216, which is in itself a significant number (the cube of 6). This "writing" of Pythagoras, cited D.L. 8.14, is to be identified with the "tripartite" of D.L. 8.65, because of its Ionic dialect (Diels, AGP 1890, 450). See further Rohde, Psyche II 419 = 599 Eng. ed., Corssen, RhM 1912, 243, Delatte, Vie 181, Lévy, Sources 76. O. Skutsch, CP 54 (1959) 115, conjectures that Ennius' peacock originally belonged among the incarnations of Pythagoras; he would insert it between Pythagoras and Euphorbus. Scholars have mostly supposed that the source of the whole tradition was an old kataphasis poem (Rohde, Q 105; 1) and suppositione, 419 = 600 Eng. ed.; Norden is very positive: it was "an ancient Orphic Pythagorean poem of the sixth century" (Vergil VI, 5 and 21) which "was regarded as almost canonical," cf. Dieterich, Nekyia 129, Ganschinieta, RE X 2410. There is no reason to believe, however, that there ever was an ancient written source, or that, if the tradition was oral, it was transmitted in the form of a poem. What we have to reckon with is oral narrative and maxim (asument), not literary genres.

111 Delatte, Vie 157, interprets Euphorbus as the "good shepherd," and assumes the existence of a Phrygian cult of a "good shepherd" and an "Orphic" apocalyptic deriving from it; but this is pure hypothesis. Skutsch, CP 54 (1959) 114, understands Euphorbus as meaning "he who eats the right food," and connects him with Pythagorean dietary regulations. It may be that both these motifs, "shepherd mysteries" and "correct food," played a role; and generally it is well to keep in mind the possibility of multiple interpretation.

112 Kerényi 19. Rohde mentions (Psyche II 418 = 599 Eng. ed.) that Euphorbus' father was a priest of Apollo, but does not think this explains the matter adequately. Schottlaender (134f) cites the fact that his mother's name was Φιόφυτρα (II. 17.40), Corssen (RhM 1912, 22) calls the choice of Euphorbus incomprehensible; and Tert. An. 28 jeered at the irrationality of it.

3. Metempsychosis and "Shamanism"

Τοῦτο μὲν οὐδὲν ἀλήθεια καὶ Λαόθεος ἔκτενες νῦός,

τὴν δὲ Εὐφόρβου συν θεριονίων οἰκεῖος.

If we consider the arithmetic here, it seems as though Moira, Apollo, and Euphorbus only make up two, so that two of the three must be identical. The solution that Moira is not personified here, and thus not counted as one of the group, is by no means self-evident to the ancient scholars who busied themselves with the problem. "If someone wanted to say, 'I am perhaps Apollo,' he could, in Homeric terms, call himself Euphorbus," says Kerényi. The advantage of this interpretation is that it is entirely derived from the Homeric text. The name Euphorbus refers unmistakably to Homer, and the whole intellectual world of the archaic period takes its character from Homer. Innovation prevents itself in the guise of Homeric interpretation.

We are told in fact that in Croton Pythagoras was thought to be the "Hyperborean Apollo." For these aspects of the Pythagoras legend, Aristotle's book on the Pythagoreans is the important source. We have the miraculous stories in four versions, in the Historia mathematis of Apollonius,112 in Diogenes Laertius, Aelian, and Lamblichus.113 Aristotle records the following items:

(a) Pythagoras was called "Hyperborean Apollo" by the Croton-

(b) At the same hour on the same day he was seen both in Croton

113 We see the scholiast on the passage.

114 See above, ch. 12. The material is collected by Rose as fr. 191, discussed by Corssen, RhM 1912, 422ff, and Lévy, Sources 10ff. It is scarcely to be supposed that Aristotle failed to mention the Euphorbus story, since it was so well known he is not cited for it.

115 The immediate source of Apollonius is obviously Boethus, as is shown by the lemma θυσία at the beginning of the text (Diels, SBBh 1801, 393f; Wellmann, AbhBl 1921, 24, Lévy, Sources 14.4). Theopompus (FGrHist 116F70–71) is the source of ch. 5 (on Phoenicians) but not for ch. 6, as Diels wrongly supposes. All of it clearly comes from Aristotle, and Boethus is, at least here, compiler rather than forger. On Apollonius see W. Kroll, RE 51 suppl. 445ff: none of his sources is later than the second century B.C.

116 Also Plut. Numa 8, Ann. Marc. 22.16.21, and some echoes in Lucian. The wording of the principal source is often so closely similar that there may well have been a common immediate source (Boethus). Nicomachus, though taking departure from the same source, introduces variants that are mostly characteristic of him. He calls his authorities αἰσχορρατος ναι Σιμήθους (Por. VP 23 = Iam. VP 60).

117 In the following notes, a passage that names Aristotle is cited in bold face type.)

118 [In the following notes, a passage that names Aristotle is cited in bold face type.)

119 2.26, D.I. 8.11, Iam. VP 140 (on the text see Deubner, SBBh 1935, 677ff; other proposals in Helck 23, Corssen, RhM 1912, 37 n. 1; Lévy, Sources 14 n. 3.); cf. Iam. VP 30; Χριστ. orb. met. 20.3–Nicomachus combines this with the Abaris episode, Por. VP 10–Iam. VP 135ff.

119 2.26, 4.17, Ἀρ. Ιμ. 6. It is "Metamorphosis and Hecataemum" in Nicomachus (Iam. VP 29 = Iam. VP 134, cf. 176), "Thurii and Metamorphosis" in Philostr. VP 4.10. To the myth of Tyana he cites the fact that he was in Smyrna and Ephesus on the same day (Sextus, Loeb. cit.).
II. PYTHAGORAS IN THE Earliest TRADITION

(c) When Pythagoras stood up among the spectators at Olympia, people saw that one of his thighs was of gold.119
(d) He reminded Myllias of Croton that he had been King Midas.120
(e) He stroked a white eagle in Croton.121
(f) As Pythagoras was crossing the Casus River, the river hailed him in an audible voice, “Greetings, Pythagoras!”122
(g) As a ship was entering the harbor of Metapontum, he predicted that a dead man would be found in it.123
(h) In Caulonia, he correctly predicted the appearance of a white bear.124

119 Ap. Hm. 6. Ael. 2.26, 4.17, D.L. 8.11, Iam. VP 140, Plut. Numa 8. Cf. Lucian V. auct. 6, Gall. 18, Dial. mort. 20.3. Olympia is named in Ael. 4.17, Plut. Numa 8, and Amm. Marc. 22.16.21, and is probably to be supplied in Ael. 2.26, the other sources merely speak of an άγιον or θεατρόν. Lévy, Sources 20, assumes that Pythagoras appeared as an athlete (cf. above, ch. II 2, n. 5), but παρέβηση signifies an unintentional brief disclosure, not the nakedness of an athlete (see Plut. Proc. conj. 142).—Nicomachus mentions the showing of the thigh only in connection with the Abaris scene and dismisses the matter with the verb σφάιρισσα (Por VP 28 = Iam. VP 155, cf. 301).—Alexander of Abonutichus copied the golden thigh (Luc. Alex. 40).—On Schol. Luc. p. 124.66, see below n. 215.

120 Ael. 4.17, lam. VP 143; alluded to by Nicomachus ap. Por. VP 26 = Iam. VP 63 Maddalena (359 n. 86) contests the attribution of this to Aristotle, but since the same item occurs in two sources in the midst of Aristotelian material, it is at least probable.—Th name Myllias is used in Naeathes’ horror story at FGrHist 84 F1 = Iam. VP 189fl. Por. VP 61.

121 Ael. 4.17, lam. VP 142, Plut. Numa 8, Amm. Marc. 22.16.21. Nicomachus (Por VP 25 = Iam. VP 62) shows exact verbal agreements with Plutarch (οικοποιήσας ... καταγωγέας); but he makes the whole story a proof of the validity of divination in birds, and sets the scene in Olympia, while he excludes the “golden thigh” story from Olympia (above, n. 119).

122 Ael. 2.26, Ap. Hm. 6. The name of the river is Κόρας in Aelian, while Apollonius has the meaningless κατά άγιον (i.e. Κόρας?), and Diogenes (8.11) Νέος. In the Nicomachian tradition, Porphyrus (27) has Κοράς (meaningless; a mistake for iούντας? L. Bieler, WS 48 [1930] 201-205, tries to explain this last by assuming the loss of a line in Por.; but he overlooks the context in Nicom. and the parallel in Lam., which show that not more than one word can be wrong). Iamblichus (134) has Νέος (obviously substituted, from the “handbook source,” for Κόρας; cf. D.L., Lévy, Sources 104, 115). What we should expect is Κόρας, which is a river near Metapontum (Bacchyl. 11.119; Diels, Hermes 31 [1898] 334; Lévy, Sources 13). Can Νέος be a misunderstanding of Νέαδος, the river near Croton (as Aecius suggested)?


124 Ap. Hm. 6, with a gap which is filled from lam. VP 142. Nicomachus has a different version (Por. VP 23 = Iam. VP 60), according to which, in Iamia, Pythagoras converted a wild she-goat to vegetarianism, and a similar tale (Por. VP 24 = Iam. VP 61) about an ox in Tarentum which, on the urging of Pythagoras abstained for the rest of its life from eating beans.

(i) In Etruria, he bit a poisonous snake to death.125
(j) After predicting to the Pythagoreans the outbreak of civil strife (οινομαχία) he disappeared to Metapontum without anyone’s seeing him go.126

Two more reports are only found in Iamblichus, in context with the Aristotelian material. It is possible that they go back to Aristotle:
(k) Pythagoras took from Abaris, the priest of Apollo from the country of the Hyperboreans, the arrow with which he traveled, and thus established himself as the Hyperborean Apollo.127
(l) “They say of the man who bought Pythagoras’ house and tore it down, that he did not dare tell anyone what he saw, but that as a result of this crime he was convicted of sacrilege by the Crotonians and executed. For he was convicted of having stolen the golden chin which had fallen from the god’s statute.” Pythagoras’ house is iniolate, like a sanctuary of the mysteries; the transgressor dies the death of a scapegoat.128

These “miracles” are portents without interpretations, revelation and occultation at once. At a certain moment there is a glimpse of the divine—the gleam of the golden thigh, the greeting of the river god, the arrival of the Hyperborean. Superhuman powers are evident in Pythagoras’ prophecies, in his mastery of the animals, and in his control of space and time, as well as in the numinous dread that

125 Ap. Hm. 6. The tone is different in Iamblichus (142): τῶν μυκρῶν οὐκ, δὲ ἀπεκακώσε μαρτυρία. Rose corrects δῆλον to δῶν, but probably Iamblichus himself changed the wording.

126 Ap. Hm. 6. Lam. VP 140, excluded from the Aristotle fragment by Rose. Corsin, RHm 1912, 18, insists to show that Hermippus is its source, using the argument that Pythagoras, who simply takes his arrow away from “the unfortunate Abaris,” is presented in an unconvincing light; but, if Pythagoras is the “Hyperborean Apollo,” Abaris has reached his disillusionment and does not need the arrow any more (Lévy, Sources 18). Lévy, (Sources 141) discovered that the rare feminine η’ οἰνομαχία occurs not only in section 140 but also, perhaps, in Aristotle (Phys. 239b7). The excursus on Abaris, sec. 141, p. 79.18-23, has a close relationship to Ap. Hm. 4, and probably comes from a different source than the surrounding context. In that case, Iamblichus may well be the author of the transitional passage pp. 79.23 80.1. The expression οἰνομαχία ἐπιτίμησεν, the “confession” of Abaris, is a monument of the confession of Peter after the transfiguration of Christ, can scarcely be old. Perhaps it is not coincidence that in this passage (p. 79.23) οἰνομαχία is twice masculine and once feminine, in the manuscripts. (Deubner restores the feminine throughout.) The parallel version of the Abaris story at lam. VP 191fl has only the masculine.

II. PYTHAGORAS IN THE EARLIEST TRADITION

attaches to his house. There is always something enigmatic about the meaning of these miracles, which is apparently revealed to the insider but not explained to the unininitiated. The relation to Apollo is ambiguous, too. According to Aristotle, the Pythagoreans taught, ἐν τοῖς πάνω ἀπορρήτοις, that “among rational beings there is that which is god, that which is man, and that which is like Pythagoras.”

Here a phrase like λογικὸν τέλον betrays later terminology, though the idea of the intermediate would not contradict the identification with “Hyperborean Apollo”: the third kind of being could be gods in human form. But explicitness is avoided, just as the Euphorbus story does not go beyond a “perhaps” as to the hero’s divine status.

A second group of legends is also attested in fourth-century sources, but it was controversial even then whether they apply to Pythagoras or Pherecydes of Syros. In his Tripus, a book about the Seven Sages, Andron of Ephesus ascribed to Pythagoras what Theopompus ascribed to Pherecydes. Porphyry, who brings out the contradiction, brands Theopompus the “thief”—which prompts modern scholars to take the opposite view.

These are the stories:

(a) Either Pythagoras in Metapontum or Pherecydes in Syros took a drink of water from a well and predicted an impending earthquake.

(b) Pherecydes in Megara Hyblaea or Pherecydes in Samos predicted that a ship, sailing with a favorable wind, would sink, and this immediately happened.

(c) Pythagoras predicted the conquest of Sybaris or Pherecydes that of Messene, and in each case a friend was warned.

(d) Pherecydes or Pythagoras, on the authority of a dream in which Heracles appeared, told the Lacedaemonians not to honor gold or silver; and in the same night Heracles bade the kings to heed the words of Pherecydes or Pythagoras.

These miracles all belong to the realm of prophecy. Prediction of an earthquake is also ascribed to Anaximander; the addition of the “drink of water” suggests an origin in cult practices. Comparison of details yields no conclusive argument for the priority of either version, though there is a serious chronological difficulty in the juxtaposition of Pherecydes and the fall of Messenia. It is scarcely possible to judge the date or the reliability of Andron; but there is no doubt that Theopompus was hostile to the Academy and also to Pythagoras. In any case Andron attests the wide dissemination of Pythagoras legends in the fourth century. There is no reason to think of Aristotle as being dependent on Andron.

Later tradition adds little beyond the miraculous catch of fish, which does seem to be earlier than Nicomachus. What Aristotle reports is in tone, quite apart from the nature of its attestation, anything but characteristic of “late antiquity.” It was more likely to provoke later ages to mockery or protest. Alexander of Abonuteichus claimed

129 Arist. fr. 192 = lam. VP 31; cf. lam. VP 30, 143f, Schol. BT II. 1.340. Ps.-Apollo of Tyana, Ep. 50, counts Pythagoras ἐν γένει δαμάζων. Corssen (RhM 1912, 39) and Kerényi (18) emphasize that this is not incompatible with the equation Pythagoras-Apollo.

130 Porphyry, Φιλολογός απόκλεισεν ἀπ. Δυσεφ. Πραγμ. ενεπικ., 10.3.6 (DK 7.165) Theopompus’ version is also attested at D.L. 1.116 and Ap. Hm. 5 (PGrHist 115f70-71), Andron’s (earthquake and sinking ship) at lam. VP 136.

131 For priority of the Pherecydes version see Corssen, RhM 1912, 33ff, Ierermann 58f, Rathmann 28.

132 In Samos, according to Max. Tyr. 13.5. No location is named in Cic. Div. 1.111 and Plin. HN 2.191.

133 This story is only found in D.L. 1.117. Theopompus 171, with the addition ἢ δὲ Πολυδρόμου περατίων ταῦτα, But Porphyry mentions that Andron reported still more miracles of Pherecydes (καὶ ἔτερα τῶν ταύτων ἐπιμαχυών). Andron names Pherecydes as Pythagoras’ teacher (D.L. 1.110).

3. Metapsychosis and “Shamanism”
II. PYTHAGORAS IN THE Earliest Tradition

to be Pythagoras and displayed his golden thigh. In Lucian’s judgment he selected and even exceeded “all the vilest and most damaging slanders ever vented against Pythagoras.” The stories of miracles are widely regarded as attempts to discredit him and put down as slander. Nicomachus expresses candidly his distaste for the crude, “plebeian” miracles, and tells the stories in such a way that the inexplicable is played down or ignored, so that what remains is, essentially, clairvoyance and the doctrine of the kinship of all living beings, and the exhortation to ἡμερήσιον. The golden thigh is only mentioned in connection with the wonder-worker Abaris, and is quickly disposed of, and completely unsophisticated details like appearing simultaneously at two places or biting a snake to death are omitted. In their place appear the somewhat sentimental tales about the conversion of wild animals. But even much earlier than Nicomachus these reports had been transformed. “Epimenides, Eudoxus, and Xenocrates” are cited as saying that Pythagoras was the son of Apollo. This eliminates not only metempsychosis but identity with Apollo, and closeness to the gods is expressed in the customary form of divine ancestry.

The Pythagoras legend is attested for the fourth century B.C., and at least part of it was well known; what is more, it is distinct from the Platonic interpretation current in the Academy, which made Pythagoras a doublet of Plato himself. It antedates Antiphanes, Andron, Heracleides. It was recorded by Aristotle, perhaps not without a polemical glance toward the Academy’s modernizing interpretation. It was carried along, willy-nilly, by others, though assiduously reinterpreted or, sometimes, rejected as slanderous, and all the while made the target of ridicule by its enemies. Finally, then, with figures like Apollonius of Tyana and Alexander of Abonuteichus, the wonder-worker reappears in reality; such activity is to be sure “late antique,” but it revives preclassical patterns.

Oral tradition clearly is involved in the Pythagoras legend; we must expect alterations and distortions to have occurred. Still, for its origin no terminus post quem is set other than the historical Pythagoras, and faithful preservation of original traits cannot be excluded from the start. This is why we should try to understand the tradition before discarding it, even though, consciously or not, scholarship tends to proceed from the assumption that legend is always secondary, because the “historical kernel” must be an event or pattern of events amenable to common sense and not a “miracle.” What if the “facts,” as we see them, were experienced differently at that time? If the historical Pythagoras taught metempsychosis, this same historical Pythagoras must have claimed superhuman wisdom, he had to use his own life as an example and find himself in the Trojan War. And if he wanted to make this credible, he had to—perform miracles.

The katabasis of Pythagoras is very important in this context, but it is especially difficult to evaluate this tradition. Therefore we may first survey what there is of similar phenomena in archaic Greece, in the hope that this will bring various kinds of confirmation and illumination.

From ancient times Pythagoras, as miracle-worker, has been associated with figures like Aristeas, Abaris, Epimenides, Phormio, and Empedocles.

An epic poem entitled Arimaspeia, by Aristeas of Proconnesus,

In all cases of coincidence with the stories of Aristeas, Epimenides, or others, Rath


143 Lucian, Alex. 4 (cf. 40).
144 Above, no. 110, 119, 121, 127, 128.
145 Lam. VP 7 = Xenocrates fr. 22 H. Eudoxus fr. 86 Gisinger ... fr. 324 Lasserre. The authenticity of the Xenocrates fragment is doubted by Rohde (Q 128), Zeller (H 1:1023.5), and Lévy (Sources 9.3), that of the Eudoxus fragment by Schaarschmidt (44f)—without adequate grounds. Xenocrates had to reinterpret the Pythagoras legend (above, no. 19–20). Apollonius (Lam. VP 8 = Por. VP 2) quotes two lines from an elegy by a “Samian poet” in which Apollo is named as Pythagoras’ father. Lévy (Sources, 105 n. 4, Lég. 6 n. 4) thinks the citation of Epimenides, Xenocrates, and Eudoxus is an interpolation in the Apollonius passage; if correct, this would speak for its authenticity, for in that case it must have been taken from a “handbook” source.
146 The influence of the Pythagoras legend is traced by Lévy, Lég., cf. above, no. 93; on its effect on Athanasius’ life of St. Anthony, see R. Reitzenstein, SHStud 1914, Lévy Lég. 129ff. Lévy, however, strongly overstated the influence of Heracleides on the legend.
II. PYTHAGORAS IN THE Earliest Tradition

was in circulation in the early sixth century B.C. Aristeas told how, possessed by Apollo (δοξοθαλαπτος γενόμενος) he had traveled to the country of the Iseodones in the far north, and learned from them about the Arimaspi, the griffins, and the Hyperboreans who lived still further north. Herodotus adds a local legend from Proconnesus, to the effect that Aristeas died, and soon after was seen traveling abroad, while his body was found to have vanished. After seven years he appeared in town again, bringing his Arimaspeia, and then disappeared again. Herodotus also told, in Metapontum, that Aristeas had appeared there and bidden the natives to build an altar to Apollo.

"For (he said) Apollo had visited them alone among the Italians, and he himself had accompanied him in the form of a raven... And even now there stands in the agora, near the statue of Apollo, a statue inscribed with the name of Aristeas, and there are laurel bushes round about."

Herodotus calculates that the appearance of Aristeas in Metapontum occurred 240 years after his disappearance in Proconnesus. It is hard to account for this dating, though the incident in Metapontum must have been relatively late; one is tempted to think that the Metapontine coins might be relevant which, beginning to appear about 470 B.C., show Apollo with a branch of laurel.

The report from Proconnesus is inconsistent. First Aristeas dies, then his corpse disappears; possession and disappearance stand side by side.

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147 J. D. P. Bolton, Aristeas of Proconnesus (Oxford, 1962; cf. Gnomon 35 [1963] 235-240). The terminus ante quem is a representation of Arimaspi on a mirror from Kerameikos (Bolton, pl. i, ca. 575 B.C.), the terminus post quem the mention of the Cumans (311 B.C.). Pinard mentions Aristeas (fr. 271), and the Arimaspeia was used by Theocritos (Jacob’s notes on FGrHist 1 F393-4) and by Aeschylus in the Prometheus (Bolton 45-64).

The most important evidence: Hdt. 4.13-15.

148 Between Artace and Cyrus, according to Hdt. 4.14, "on the way to Croton" it Plut. Rom. 28, a contamination with the story which follows in Hdt. 4.14. At Ath. 13.664E, too, there is a mention of the dane γαλαη (δάφνας) II Hdt. 4.14-15; cf. Giannelli 63 n. 1 of Aristeas in the marketplace of Metapontum. (Perhaps from Theopompus: see Jacoby, FGrHist 115 F248 n.)


Aristeas also places Aristeas later than Homer (2.53; according to Strabo 14.5.35 "some" would reverse the relationship). Proconnesus was founded in the time of Gyges (Strabo 13.587, 590). If the connection with the coins is correct, and Herodotus thus learned in Metapontum, about 440, about an event of approximately a generation earlier the Arimaspeia would be dated about 710; but this does not fit the history of Proconnesus (see also Bolton [above, n. 147] 127). All the same, we must reject E. Schwyzer’s conjecture of συντηχεια for συντιχεια, which would show Herodotus dating his own visit to Metapontum rather than the appearance of Aristeas (FGrHist 42 [1923] 528; followed by Schmitt 1 1.301 n. 3). The point Herodotus is making is the time between Aristeas’ two apparitions (Meuli 134.2).

150 According to a late version (Ap. Hym. 2) Aristeas was seen on the same day in Proconnesus and Sicily. (For Pythagoreans, above, n. 118.)

151 The earliest mention of the Hyperboreans: Epigoni fr. 3, Hes. fr. 150.21 M.-W.; the arrival of Apollo in Delphi from the land of the Hyperboreans: Alcaeus fr. 307 προ, and a shield strap from Olympia (600-575 B.C.) in E. Kunze, Olympische Forschungen II (1919) 74; further references to the flying Apollo: Gnomon 35 (1963) 239 n. 1. The term ἀπορρήτος, or at least ἐν ἀπορρήτω, as meaning that Apollo had lived in Metapontum in the form of Pythagoras. The cult belongs with the legend; in both is reflected the activity of Pythagorean circles.

Abaris, too, is a priest of the Hyperborean Apollo, and even comes himself, from the land of the Hyperboreans. He brought gifts from

In it that “Herodotus has combined two versions of the legend.”

Later reports are unequivocal: the soul leaves the body and hovers about in the air "in the form of a bird." Perhaps the contradiction itself, the failure to smooth over difficulties, and the lack of a clear separation of body and soul, are signs of an archaic way of thinking.

Aristeas is connected with Pythagoras by more than the similarity of the legends in which they figured. Pythagoras was regarded by the initiated as the Hyperborean Apollo and died in Metapontum near the beginning of the fifth century. But the main authority on the Hyperboreans was Aristeas, and the god who flew away from the world, accompanied by his servant in the form of a raven, and finally alit in his special city, was the Hyperborean Apollo. If, about 470, a mysterious prophet brought it about, by his message, that an altar and a statue were set up, then the new cult was obviously dedicated to the Hyperborean Apollo, and for whatever Pythagoreans there were in Metapontum at that time we must assume that the proclamation of Aristeas was understood, at least ἐν ἀπορρήτω, as meaning that Apollo had lived in Metapontum in the form of Pythagoras. The cult belongs with the legend; in both is reflected the activity of Pythagorean circles.

152 Rohde II 921 329 n. 109 Eng. ed.; Corsen, RhM 1912. 44.


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158 Abaris, too, is a priest of the Hyperborean Apollo, and even comes himself, from the land of the Hyperboreans. He brought gifts from
there, according to Attic legend, to the Proerosia festival at Eleusis. Abaris has connections with other Greek cults, and there were charms and oracles attributed to him. Probably his meeting with Pythagoras had been recounted even before Aristotle’s time, though there were chronological difficulties.

According to Herodotus, without ever eating, Abaris carried Apollo’s arrow all over the world, but as early as Heraclides it was said that he flew on this arrow, and this version is regarded as the original one. It may be, though, that just as in the case of Aristeas, the tradition was self-contradictory from the beginning. There were alternative ways to report the activities of the miracle-worker; Abaris could not perhaps “actually” fly, but he could claim the ability, and even, in ecstatic ritual, act it out, as it were, as a shaman. Whoever was ready in his heart to believe, would speak of “flying;” those who were ready to discard the old-time magic would report the matter in the style of Herodotus.

Epimenides, the famous Cretan “purifier,” comes from a different environment. That he slept for decades in the cave of Zeus was known already to Xenophanes. This puts Epimenides in the main line of specifically Cretan cult and myth. King Minos visited his father Zeus in that cave every eight years. Epimenides called himself Aeacus—

3. Metempsychosis and “Shamanism”

thus making himself a brother to Minos—and claimed “that he had been reborn many times” (πολλάκις ἀναβεβαιωσέναι). The Cretans called him νέος Κάουρης, thus bringing him into close relationship with Zeus. A voice speaks to him from heaven, and the Cretans attribute to him as to a god.

Oracles and theognic poems were in circulation, bearing the name of Epimenides. Doubtless much was attributed to him by later men. Plato’s report that Epimenides prophesied the Persian War ten years before it occurred is explained as meaning that at that time a new book of oracles “by Epimenides” appeared. Legendary motifs were added, too. Nevertheless, the purification of Athens from the Cylonian curse in the time of Solon may be regarded as historical. There were “caves of Zeus” in Crete, sites of the initiatory ceremonies of wicca societies, who are reflected in myth as “Dactyls” or “Kouretes.” The initiation of the kαθαρσις by sleeping in the cave of Zeus is comprehensible from this point of view. It also represents death and resurrection, for in the meantime Epimenides was regarded as dead. The long fast, made possible by the magic food ἀλμωτ, as well as the tattooing, also are bound up with ritual. But given these facts, his being “reborn many times” and his identification with Aeacus are not necessarily “purely Pythagorean fabrication.” There is never any mention of a metempsychosis doctrine of Epimenides, only the unique character of the initiate. The explanation is rather to be found in a

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136 Lycurgus fr. 84–85 Blass, Hippostratus FGrHist 568F4; cf. the Hyperboreans in Delos, Hdt. 4.33.
137 Sparta, Kore Soteira (Paus. 3.13.2, Ap. Hm. 4, lam. VP 92, 141); Cnosus, (lam. VP 92); Palladium (Firm. Mat. Err. prof. rel. 15).
138 zeus, Pl. Charm. 158b, where Zalmoxis is also mentioned; Ap. Hm. 4, Schol. Ar. Eq. 720.
139 Cf. n. 127. On lam. VP 215, ch. II 1, n. 13. The source of the exposition in lam. VP 91–93 cannot be determined (Heraclides?). Nicomachus (lam. VP 135 = Por. VP 28) presupposes it—Abaris is given as a Pythagorean in lamblichus’ catalogue, VP p. 145.17.
141 Hdt. 4.36. Lycurgus, too, says that Abaris carried the arrow (fr. 85 Blass = Harpocrates s.v. Abaris), as does Aristotle (lam. VP 140; above, n. 127). For both linguistic and material reasons, we must reject the conjecture, repeatedly proposed (e.g. M. Maye RE XV 1537f., Roscher, Lex. I 2837) to substitute ὁ τὸν ( — τοῦ) διάστημα περιήρω for ὁ τὸν διάστημα περιήρω in the text of Herodotus.
142 Heraclides fr. 51c. He is αἴθροβάρσα in Nicomachus (Por. VP 29 = lam. VP 13 and in lam. VP 91).
143 Correspond, RhM 1912, 47. Medal 159f, Dodds, Irr. 161 n. 33 (contra Rohde, Psyche 91 n. 1 = 327 n. 108 Eng. ed.).
144 Xenophanes DK 21120 (Epimenides lived to the age of 154). Theopompus explicit mentions the Dictator case (FGrHist 114 167 69, Max. Tyr. 10.1 = cf. 37.1), though one might be more disposed to think of the one on Ida, mentioned by “Epimenides” (DK 21124: Rohde, Psyche 120 n. 1 = 108 n. 24 Eng. ed.).
146 1.1 114, cf. Procl. In Resp. II 113. When the Sida, s.v. Epimenides, says ὅτι ἔγιν αὐτὸν ἀνθρώπον ἄνθροπον καὶ πάλιν ἐμφανίζεται ἐν τῷ σώματι, the precise wording is no doubt secondary, perhaps taken over from Aristeas (as Dodds, Irr. 163.42); but it is hardly original in his case either.
148 Theopompus (FGrHist 115F6a, D.L. 1.113): as Epimenides was outfitting a sanctuary to the nymph, a voice commanded, μὴ Νησμόν ἄλλα Αἴγος. This Zeus was, then, worshiped in a grotto or cave.
149 1.1 114, 115.
151 Pl Leg. 642a1; Diels, SBBhn 1891, 395, and DK I 32 n.
154 Almous Theol. Hist. pl. 7.121, D.L. 1.114, Plut. Conv. sept. sap. 157D (DK 3A45). One of this is attributed to Heracleus by Herodorus (FGrHist 31F1), to Pythagoras by Herostratos by Phrynos Antoninus (Por. VP 34). See Rohde, Rom. 275 n., Hausleiter 797ff, Dodds, Irr. 161 n. 42 (Dieks regarded the “Orphic asceticism” of Epimenides as secondary, as Dodds, Irr. 161).
155 On tattooing, see Sida s.v. Epimenides (DK 3A25), Dodds, Irr. 165 no. 43 ff.
156 Dieks SBBhn 1891, 390 n. 1.
parallel development from common origins; Pythagoras too, as the legend says, sought initiation in the cave on Ida.176

A journey of the soul, in pure form, was attributed to Hermotimus of Clazomenae: His soul left his body and wandered about, while the body lay as though dead, until one day his enemies burned the body while the soul was absent.177 Hermotimus could predict future events; the Clazomenians built a sanctuary in his honor.178 Here too there is ritual to go with the legend.

The stories of Phormio and Leonymus take us to Croton. Phormio is mentioned as early as Cratinus, interestingly enough in the comedy Trophonius. Theopompus relates, among other things, that Phormio was wounded, in a battle, by divine opponents, the Dioscuri. He was told by an oracle to travel to Sparta—obviously to the house of the Dioscuri; and when he laid hand on the door, he found it was the door of his own house in Croton. He was home, safe and sound.179

The tale of Leonymus is almost like a doublet of this.180 In the battle on the Sagras he was wounded by Ajax, who was fighting in the ranks of the Locrians. To be healed he had to go, to the behest of the Delphic Oracle, to the “White Isle,” where he met Achilles and Ajax,

174 Por. VP 17 (Antonius Diogenes?), D.L. 8.3. The connection of Pythagoras and Epimenides is obviously secondary; Pythagoras is sometimes the teacher (Nicom., Por., VP 29 = Iam. VP 135; cf. Iam. VP 104, 221f) and sometimes the pupil (Apul. Flor. 15 p., 59, D.L. 8.3). For a purportive piece of evidence from Epimenides about Pythagoras, see above, n. 143.—A unique rebirth story was told of Aesop, as early as the fifth century B.C. (Plat. Com. f. 68, ca. 400 B.C.; Schmid I 4.145ff.; Hermippus ap. Plut. Solon 6; Ptolemaeus, son of Hephaestion ap. Phot. Bibl. 152b11). This is not “parody of Pythagoreans teaching” (as Hauerth says, RE VI 1710; cf. Schmid I 1.675 n. 2), but has its roots in ritual. The uncomfortable feeling about the killing of the scapegoat was counterbalanced by the purportive revival (A. Wiechers, Aesop in Delphi [München, 1901] 37ff).

175 Ap. H. m. 3 (perhaps from Theopompus, like sections 1 and 5; Rohde, Psyche II 95, 1 = 331 n. 112 Eng. ed.), Pline, HN 7.174, Plut. De gen. 592b-c (“Hermotaurus”), Tert. An. 44.


177 See Meinke, FCG II 1227ff; Diels, Parn. 177f; Cratinus f. 225 Kock; Theopompus FGHist 115 f. 392f – Suda s.v. Phormio (who adds an ecstatic journey to Cyrene). Clem. Al. Strom. 1.133.2 associates Phormio with Aristaeus and the rest (above, n. 146); Paul. J. f. 1f gives a different story of Phormio the Spartan and the house of the Dioscuri.

178 Paul. 3. 19.11–13. Somewhat differently Conon FGHist 261 f. 18, who has no epiphany on the island, only a sacrifice. Tert. An. 46.9 and Herm. Phldr. p. 75 Couvreur = Schol. Pl. Phot. 243a have him healed in his sleep. On the matter of the battle at the Sagras, above, ch. II 2, n. 40.

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175 Ap. H. m. 3 (perhaps from Theopompus, like sections 1 and 5; Rohde, Psyche II 95, 1 = 331 n. 112 Eng. ed.), Pline, HN 7.174, Plut. De gen. 592b-c (“Hermotaurus”), Tert. An. 44.


177 See Meinke, FCG II 1227ff; Diels, Parn. 177f; Cratinus f. 225 Kock; Theopompus FGHist 115 f. 392f – Suda s.v. Phormio (who adds an ecstatic journey to Cyrene). Clem. Al. Strom. 1.133.2 associates Phormio with Aristaeus and the rest (above, n. 146); Paul. J. f. 1f gives a different story of Phormio the Spartan and the house of the Dioscuri.

180 Paul. 3. 19.11–13. Somewhat differently Conon FGHist 261 f. 18, who has no epiphany on the island, only a sacrifice. Tert. An. 46.9 and Herm. Phldr. p. 75 Couvreur = Schol. Pl. Phot. 243a have him healed in his sleep. On the matter of the battle at the Sagras, above, ch. II 2, n. 40.

3. Metempsychosis and “Shamanism”

...and then returned home sound. The “White Isle,”181 later localized in the Black Sea, was originally identical with the “White Rock” in the underworld, and Leonymus was “the first” to visit this island. Thus in this case recovery is not possible without a regular journey to the nether world. Phormio obviously did his “traveling” in a trance state; and many thought that Leonymus had only been dreaming.

The legend of Stesichorus is bound up with the story of Leonymus: the latter brought from the White Isle Helen’s instructions to Stesichorus, that by composing his palinode he might regain his sight.

Both the Leonymus and the Stesichorus stories have been supposed to go back to a Pythagorean origin,182 and a certain amount of coincidence in place and time is not to be denied, though the epoch of Stesichorus is earlier than that of Pythagoras.183 There was a belief in southern Italy that healing could be won by an ecstatic journey into the world beyond, to the gods. And as they thought of their ancestral gods as quite literally fighting in the ranks,184 the healing legends too might be bound up with ritual activity.

In fifth-century Sicily, Empedocles could promise his pupils: “Thou shalt stay the power of the unweary winds which sweep upon the earth... and, if thou wish, bring back their breath again. After dark rain thou shalt cause a seasonable drought for men, and after summer’s thought bring on the streams that nourish the trees... Thou shalt bring back from Hades the strength of a man who has died.”185 The Empedocles legend reports the miracles as having been accomplished; as has been said, he was “the creator of his own legend.”186 But how could he have created it, how could he have called himself a god, if he was not able actually to perform, or at least to pretend to perform...
II. PYTHAGORAS IN THE Earliest Tradition

extraordinary and amazing feats? His saga must have its roots not in literature, but in reality; and indeed Gorgias testifies that “he himself had been present when Empedocles performed feats of magic.”

According to Timaeus he restrained the destructive north winds at Selinus by the use of bags made of ass’s hides—that is, by a secret sacrifice in the manner of the Hyperboreans.

A “journey into the underworld” was actually part of the ceremony at the oracle of Trophonius in Lebadea. The one who is to consult the oracle was borne by a wind into the depths, feet first, as the dead are carried. After the god has appeared to him, he returns to earth, often after a lapse of several days. Before the sanctuary were the springs of Lethe and of Mnemosyne, whom the initiate expected to see in the underworld, according to the evidence of the Gold Plates. Trophonius is a Zeus Xllobnos—Epimenides also knew Zeus in the bowels of the earth. But in Lebadea there can be no doubt that the main thing was ritual, not legend. We do not know whether the visitor to the oracle was put into a trance state—there were long ritual preliminaries, and not everyone was admitted—or whether perhaps machines may have been used, of the kind found in the katabasis of the Roman Bacchanalia, according to Livy. It is significant, however, that legend connected a Pythagorean, Parmicus, with the otherworld journey of Lebadea.

In many places there were subterranean installations which presented the underworld in physical form. The structures at Clarus are impressive, the adyton with the spring that gives out the oracle lies under the cela of the temple. And even more amazing are the subterranean passages at Baiae, near Cumae—if they really belong to the rites of the “Cimmerians” and not merely to the water supply. There is also the mysterious mundus at Rome, visited by boys. In connection with the cult of Demeter there were megara, subterranean rooms or caverns, into which offeratory gifts were lowered.

This last brings us back to Pythagoras. His house, says Timaeus, was made into a temple of Demeter; and vows to the uninitiated who entered it. In various segments of the tradition we have reports of subterranean chambers in which Pythagoras met with his disciples; and above all, we learn of a katabasis of Pythagoras himself, although the slightest allusions to it are in the distortions of ridicule and parody.

Herodotus of Rhodes says that Pythagoras descended to Hades and saw how the souls of Homer and Hesiod were atoning for what

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189 The principal source is Paus. 9.39: cf. Hdt. 8.134, IG VII 1055, 4136, Rohde, Psyche 1190ff = 229 Eng. ed., Radke, RE VII A 678-695. Nilsson (II 450) does not believe that the full development of the ritual came before imperial times. But Dicaearchus spoke of a katabasis (fr. 13ff), and said that “nescire ea melius esse quam scire” (fr. 17). Seneca (ForHist 366F10) speaks of one who consulted the oracle and lost his ability to laugh. Nilsson’s assertion that the “spring of Lethe” was an idea invented after Plato, and that the spring of Mnemosyne belonged to the Hellenistic age (II 226f, Op. III 85-92) has been refuted by the gold plate of Pharsalos, which has the phrase Μνημόσυνης Λήμνα (ca. 350 B.C.; Arch. eph. 1950-1951, 688f). U. E. Paolo calls attention to the significance of the fact that the visitor to the oracle is carried away feet first (Die Geschichte der Neuroi [Berlin, 1953] 43).

190 Livy 39.13.13: “raptae a displices dici, quos machinae illigantes ex conspectu in abdito specus abripiant.” On this, see Festugière, Mém. d’arch. et d’hist. 56 (1944) 49ff. (At p. 95, he conjectures that a mechanical contrivance may have been used in the Trophonius ritual.)

191 Seneca ForHist 366F16, and the Delian temple inventory of 279 B.C. (IG XI 2.161 B17, DK 20). The age of the legend is indeterminable; the older inventory lists are all fragmentary. It is possible that a dedicatory gift was ascribed later to Parmicus. (The form Παρμικός is found in the inscription, elsewhere often Παρμικός; cf. DK 20n) - Apollonius of Tyana, too, was a visitor to Trophonius (Philost. I 4.8.149).


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198 Herodotus of Rhodes says that Pythagoras descended to Hades and saw how the souls of Homer and Hesiod were atoning for what
they had said about the gods (this does not sound very archaic),\textsuperscript{109} and also how those were punished “who would not live with their wives; and this, of course, is why he was honored in Croton.” (The gibe in this last clause is unmistakable.) Much more influential was the account of Hermippus.\textsuperscript{200}

After arriving in Italy, he built a little underground room and instructed his mother to write down events, as they happened, on a tablet, recording the time,\textsuperscript{201} and to keep passing these notes down to him until he came back up. His mother did this, and after some time Pythagoras came up thin as a skeleton. He went into the assembly and announced that he had just returned from Hades. What is more, he read off to them an account of what had happened during his absence. Taken in by his words, they wept and moaned and were sure that he was some kind of divinity; so that they even entrusted their wives to him, thinking that they too would learn something from him. And they were called Πυθαγόρικα.

The mocking tone of this account of a “journey to the underworld” is of course unmistakable. The question remains, however: how did this sound when it was taken seriously; and how old is the story? The usual assumption is that Hermippus’ account is a simple transference to Pythagoras of what Herodotus and, after him, Hellanicus\textsuperscript{202} had reported of Zalmoxis, the Getic god; but it may be that the matter is more complicated than that.

\textsuperscript{109} The background of this detail is to be found in Xenophanes and Plato. On the Homeric poems in (late) Pythagorean tradition, see Delatte, \textit{Litt.} 190ff., and M. Detienne, \textit{Homère, Hésiode et Pythagore} (1962).

\textsuperscript{200} D.L. 8.41, Tert. \textit{An.} 28, Schol. Soph. \textit{El.} 62 = \textit{Suda} s.v. δαιμόνιος. Allusions at \textit{Lucian} \textit{Gall.} 18, Celus (Origen \textit{Cels.} 2.55), Euthydamus on \textit{Od.} 11.592, 24.264. See \textit{Delatte} \textit{Vie} 244ff., \textit{Lévy}, \textit{Sources} 37ff. It is open to question whether there is a relationship to Heracleids at this point (see ch. II 1, n. 32). The fact that Tertullian, the Sophocles scholium and the \textit{Suda} connect this passage with Heracleids fr. 89, does not prove that Hermippus himself had cited him (pace Corson, \textit{RHM} 1912, 23, and \textit{Lévy}, \textit{Sources} 40; Rohde, \textit{Psych} II 419 = 599 Eng. ed., is hesitant).

\textsuperscript{202} Το γυμνόν ἐξ ἔλεγον γράφειν σημειευμένον καὶ τῶν χρόνων. \textit{Lévy}, \textit{Sources} 38.4 \textit{Lég.} 130ff. translates “sealed letter,” and sees in the reading of a sealed letter a further miracle (cf. Philostr. \textit{VA} 3.16, Lucian \textit{Alex.} 19ff.). But this leaves the words τῶν χρόνων unaccounted for; and one would expect σημειευμένον. σημειευμένα means “Mark notes,” and the participle refers to τῇ μνήμῃ. (The change of case is not uncommon; ε Ευρ. Med. 57ff.)

\textsuperscript{200} Hellanicus, \textit{Βαρβαρικαί νόμιμα}, \textit{PGHist} 4173. Here Zalmoxis promises the “return of the dead and is therefore teaching a kind of metempsychosis, while Herodotus’ speaks of immortality. The complete dependence of this book of Hellanicus upon Herodotus (and Damiates of Sigum) is stressed by Porphyry (\textit{Euseb. \textit{Harp.} evam 10.3 \textit{PGHist} 4172), but it cannot be proven spurious.\textsuperscript{151})

The Greeks called the Getae \textit{Γεταί τοὺς δαιμονιζότας}, for this seemed particularly noteworthy to them. “They believe that they do not die, but that when someone succumbs he goes to the \textit{daimon} \textit{Zalmoxis}” (4.94). Every four years they send the god a “messenger,” in the form of a human sacrifice. “But as I learn from the Greeks who live on the Hellespont and the Black Sea, this Zalmoxis was a human being, a slave, in Samos, of Pythagoras the son of Mnesarchus” (4.95). Zalmoxis was set free and returned to Thrace a rich man. There, thanks to his Greek culture, he could easily impose on his fellows, especially since they were “a bit simple-minded” (ὑπάθρευτεροι), “being acquainted with the Ionian way of living, and with manners more polite than those of Thrace in that he had been familiar with Greeks, and with Pythagoras, who was not the meanest sage in Greece.”

Zalmoxis built a banqueting hall in which he entertained the most prominent citizens, and promised them that his guesses and their descendants would not die, but would live forever in enjoyment of everything good. In the meantime he had built an “underground chamber” to which he now withdrew for three years, mourned as dead. Then, in the fourth year, he reappeared; and now the Thracians believed in him, and believed in the conquest of death and in immortality. “Concerning this underground dwelling,” says Herodotus, “I am neither excessively doubtful nor excessively credulous. I do think, though, that this Zalmoxis lived many years before Pythagoras” (4.96).

It is not difficult to see the connection between the Thracians’ ritual and myth.\textsuperscript{203} Just as, once upon a time, Zalmoxis had disappeared for three years and reappeared in the fourth, so the god is summoned every four years, by the “messenger”—summoned to a ritual meal, which provides a guarantee of immortality.

Pride in Greek culture and Greek cleverness are obvious in the account the colonists gave Herodotus. They are far above Thracian barbarism; a slave among Greeks can be a god among Thracians and make them believe anything he chooses.

Hermippus’ story has been called a \textit{freche Ubertragung}.\textsuperscript{204} But the

\textsuperscript{109} On this see F. Pfister, “Zalmoxis,” \textit{Studies Presented to D. M. Robinson II} (St. Louis, 1901) 1131-1132 (p. 113 on the various forms of the name in Greek). There is now epigraphical attestation of “Zalmodegikos” as the name of a Thracian prince (D. M. Pippidi, \textit{Biblioth. 1} 1961, 53-66; \textit{S.E.G.} 18 (1962) no. 288).

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Greeks were often inclined to attribute to foreign peoples things that were really Greek, and Herodotus in particular mentions many things in his accounts of foreign peoples which he intentionally omits when writing about Greeks. The most striking example is the Egyptians' alleged doctrine of metempsychosis, but the situation seems to be the same with abstention from beans, and with the novella about Rhampsonitus' treasure house. Herodotus tells about werewolves among the Neuri of Scythia, but has nothing to say about similar phenomena in Arcadia.

The striking thing about the Getae is their belief in immortality, and this must have been the reason Herodotus' informants thought of Pythagoras. Ionian manners and Ionian cunning could have given Zalmoxis ideas in other realms, but Pythagoras was the authority in questions of the afterlife and of immortality, as we learn not only from Ion but from Herodotus himself. This reputation must have been familiar to the Greeks in plain and memorable terms. Herodotus assumes that one knows what he means when he says Pythagoras was "not the weakest" of the Greeks. Therefore, it will be well to consider whether specific details of the Pythagorean tradition are reflected in the Zalmoxis story.

It is doubtful whether the "subterranean chamber" really belongs in the Zalmoxis tradition. According to a report from the age of Caesar, there stood beside the king of Thrace a revered priest or shaman who was a successor of Zalmoxis—at that time a certain Decaenus, beside King Burebistas—and he dwelt, as Zalmoxis did, on the holy mountain Cogaenus. Given the strong tendency for religious traditions to cling to holy places, we may believe that in Herodotus' time, too, the Thracians thought of Zalmoxis as being on his holy mountain, and not in an underground dwelling somewhere. But if this is so, this is a Greek motif; and it may have been that the Greeks imputed to Zalmoxis rather a slavish imitation of Pythagoras.

Hermippus' account surely cannot, in all respects, be derived from Herodotus. The notes passed down into the underground room could be explained as an elaboration of the trick; Pythagoras must know who has died in order to describe his experiences in Hades in a credible way. But how does it happen that his mother is his confidante and assistant? It is highly unlikely that Pythagoras brought his mother with him to Corinth, and such an idea is never mentioned in the tradition. What we have, then, is a rationalizing version of something quite different. Pythagoras brings with him from Hades γῆς μητρός παραγγελματα (commands of "the mother"), a message from the divine Μήτρη—Demeter. Thus the "little dwelling" becomes a sanctuary of Demeter, as Timaeus says Pythagoras' house was (above, n. 196). In this case, however, Hermippus has an element of the story that does not derive from Herodotus, and whose significance is no longer understood—therefore something quite ancient, and belonging originally to the Pythagoras legend. Another feature of the story that makes the impression of being genuine and ancient is the fasting away of Pythagoras. Intensive fasting always forms a part of the routine of shamans and fakirs. This is not mentioned in Herodotus' account either, so that Hermippus' report has independent value as evidence alongside that of Herodotus. It shows Pythagoras in the role of a hierophant in the cult of Demeter.

The most remarkable detail of the Pythagoras legend, his golden thigh, points in the same direction. Antiquity understood this as a sign of divinity, but we find no explanation of just how this is so.

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205 Above, n. 48.
206 Hdt. 2.12.1. The cult legend of Trophonius and Agamedes is very like it, while on the other hand no Egyptian reference to it has been found (cf. Radke, RE VII A 680, Lévy, Lég. 187). In the Rhampsonitus story as with Trophonius, a katabasis is involved. According to one version (Schol. Ar. Nub. 568), Trophonius was starved in a subterranean chamber, as Pythagoras, too, died of starvation (Saturus ap. D.L.8.40, Dicacearchus fr. 35).
208 After Zeller (SBBlh 1889, 992), Rathmann reluctantly draws this conclusion from Hdt. 4.94ff. So does Maddalena (347ff), though he tries also to find that this is in contradiction to the doctrine of metempsychosis. Cf., however, the version of Helianicus, n. 202 above. On Ion fr. 44, and Hdt. 2.81, see above, nn. 13, 39.
209 Boyancé, Mauv 134, finds here the most ancient reference to the special significance of cult meals for the Pythagoreans. Morrison, CQ 1956, 137ff, goes further, trying to find a social and political significance: in the αὐτόν Pythagoras presents his doctrines to the assembled Ionian bourgeoisie, a σύνεσθε like Xenophon. But it seems that the essential point is in the following trick, without which not even the Thracians would believe in immortality. Strabo 7.297f.
Long ago Wilhelm Mannhardt made reference to myths of dismemberment and revival; but here again, the legend is based on the actuality of the initiation rite. It was merely imitation of Pythagoras when Alexander of Abonuteichus, at a mystery ritual, let a glimpse of his golden thigh be seen, but it is worth noting that this festival itself was organized on the pattern of the Eleusinian. More important is what Prudentius says in his most detailed description of the cult of the Mother, namely that persons dedicate themselves to the Mother and receive her “seal” (sphragitis), which is burned into them with red-hot needles; and at the burial of an initiate the “dedicated member” of the body was covered with a gold plate. Tattooing of the Galli is attested for the Hellenistic period. But, before all, the myths tell over and over of the favorite of the Great Mother being wounded in the thigh, as also of the thigh wounds of those who attempt to make their way into the underworld. Only he who bears the sign can descend into the pit with impunity. In the same way, Pythagoras’ golden thigh is the sign of the initiation which makes it possible for him to travel to Hades.

Are we to suppose, then, that Hermippus’ whole story, including the golden thigh, is literally true? In any case it contains reflections of ancient ritual practices which survived for a very long time. And though there is no direct testimony on the matter before Aristotle, Hieronymus, and Hermippus, we must take account of certain possible allusions in earlier literature. In the Electra, for example, Sophocles has Orestes say that he will not scruple to give a false report of his death.

The wise” win special honor by being regarded as dead for a time and then returning. The stories of Aristeas and Zalmoxis, as told by Herodotus, provide excellent examples of this; but would it be these persons that would most readily come to an Athenian spectator’s mind, or rather Pythagoras— “Ελληνικός οὐκ ὁ ἀνθρώπος σοφότερος (Hdt. 4.95)? He was, after all, Zalmoxis’ teacher; and it is to him that the ancient scholia refer.

Heracleitus calls Pythagoras the chief of swindlers, and accuses him of having made, from the books of others, his own sophia, πολυμεθα. Commentators have emphasized the words sophia and πολυμεθα, and paid much less attention to the climactic word κακοειδες. Χαριτια is not a doctrine, but an activity; κακοειδες is a technical term for the subordonation of perjury, and in general designates disingenuousness by which anyone attains an end. Thus Heracleitus is accusing Pythagoras of being a charlatan who, by ignoble deception, has attained to fame. The kind of procedure that Herodotus attributes to Zalmoxis, Hermippus to Pythagoras, and that the story of Aristeas in Metaphysics presupposes, is unquestionably a glaring example of κακοειδες; and it seems quite likely that Heracleitus, as well as Herodotus and Sophocles, had heard of a ritually enacted katabasis of Pythagoras.
Werner Jaeger wrote of Pythagoras, “The modern fashion of describing him as a sort of medicine-man has no claim to serious consideration.” But an examination of the most ancient evidence makes it difficult to forget about the wonder-worker. The concept that first made it possible to take this aspect of the tradition seriously was that of shamanism, introduced by Meuli and Dodds.

The word “shaman” comes from the language of the Tunguses of Siberia, and the phenomenon of “shamanism” was first studied in relation to certain Siberian tribes. The shaman has the ability, in an ecstatic state which is voluntarily induced by means of a definite technique, to make contact with gods and spirits, and in particular to travel to the Beyond, to heaven or to the underworld. Shamanism is the focus of these people's religious and intellectual life; the shaman's special task is to bring back health for the sick from another world and to conduct the souls of the dead to their new home. There are female as well as male shamans. The ability to achieve the shaman's ecstasy, and the technique necessary, are won, by those who have a special vocation, by a long ceremony of initiation, which includes the imparting of a certain mythical “knowledge,” comprehending earth, heaven, and underworld.

The ecstatic journeys of a Hermotimus, and the ecstatic healings of a Phormio or Leonymus are obviously analogous to this, and the apparent death and the journey of Aristeas seem easier to understand, like the “flying” of Abaris on his arrow. The connection of Aristeas and Abaris with central Asia is certain. Dodds speaks of the Greeks as coming into contact with shamanistic ideas from the north at the period of colonization of the Black Sea area; and he also brings Pythagoras into this context. Here we find, all together, association with gods and spirits, mastery of animals, disappearance, and simultaneous presence in different places.

To be sure, the katabasis of Pythagoras is not of a specifically shamanistic kind, any more than the miracles credited to Eumedes, Epimenides' visits to the cave, or the visit to the underworld from Lebadea.

The significance of the idea of shamanism for the history of philosophy lies in the conjecture that the new conception of the soul, which was to become the dominant one through the influence of Plato, is to be traced to this source. The independence of the soul from the body is immediately experienced and depicted in the shaman’s ecstasy; and this is the reason for the particular interest taken in these phenomena by the Platonist Heraclides and the Peripatetic Clearchus (frrs. 7–8). The Thracian Orpheus is not far from Scythia, and in this vicinity (geographical and religious) we are but a step away from the doctrine of metempsychosis. The belief that the spirit of a powerful revenant has entered a living body is common among shamans, and precisely this is attested for Epimenides. The inspired bard feels himself at one with the whole world: “I have been an eagle, I have been a sea coracle... I have been a sword in the hand, I have been a shield in battle, I have been a string in a harp.” It only needs a small stimulus, though its consequences are important, to make such ideas into a full-blown theory of metempsychosis.

The fact is that the very richness of the Greek tradition in stories of this type is somewhat embarrassing. At least some of them go back rather than the colonization of the Black Sea. Meuli attempted to derive Greek epic from shamanistic poetry, and Odysseus does have shamanistic traits. Then we must add Melampus’ learning the language of the animals, Teiresias’ changes of sex, Polyidus’ awakening

229 Jaeger, Paideia I 162 tr. Higbet (= I 221 Ger. ed.)—Meuli and Dodds first worked this out in detail, but Rohde had already referred on occasion to Eskins and Indians (Psyche II 97.1), and Diels had referred to shamanism (AGP 1807, 231ff., Parm. 146, NJB 1922, 239ff) as Lobeck, I 13 n. h., had already done.

230 The Airmaspi have their origin in central Asiatic mythology (A. Alfoldi, Gnomon 9 [1933] 566ff., Meuli 155ff). It has been recognized since the work of Meuli that the Scythians and Thracians had a genuine shamanism (Eliade 370ff); cf. the Thracian kanabáta (Posidonius FGrHist 87F104 Strabo 7.296) with the aithýbatḥa Abaris (above, n. 162).

231 Above, nn. 117–125, Eliade 101ff. Apollonios of Tyana also learned the language of animals (Philos. VA 1.20). It is a common motif that an eagle brought the power of shamanism from heaven (Eliade 79ff).

232 The Orpheus as a shaman see Dodds, I 147ff.

233 Dodds, I 144f., Eliade 91ff. Shamans also claim to be descended from the sky-god (Eliade 91) Kahn emphasizes (AGP 1960, 32ff) that a doctrine of the nature of the soul does not necessarily develop from such ideas.

234 Expressions of Irish bards, cited by Cornford (PrSep 122). “It is difficult,” he says, “to decide whether transmigration or metamorphosis is meant... But the difference between metamorphosis and metempsychosis is, after all, not great” (123). Cf. Empedocles 111, “I have been ere now both boy and girl, and bush, and bird, and mute fish in the waves” (fr. after Leonard). Dodds derives Indian doctrines of metempsychosis from shamanism (I I 172 n. 97, citing Acta orientalia 17 [1939] 164ff).

235 Mulier 144ff. E. Schwartz (Die Odyssee [Munich, 1924] 183ff) called attention to shamanism’s “features of the story of Odysseus—aside from his narrative of his journeys in the first person and his journey to the land of the dead, also his assumption of the role of a shaman, and especially his activity as an ágyṛm in Thesprotia.


Glaucus from the dead,238 the motif of the winged seer,239 and the magic quality of music, which can be ἐνεμνήσθη, an incantation with power over gods and spirits.240 Then there is the Messenian legend of the magician and king Periclemenes, who could change himself into any sort of creature,241 and of Aristomenes, who made his way back out of the chasm of Ceadas.242 In addition, if γόνα, a word that combines the magic of self-transformation with the mourning of the dead,243 originally meant something like “shaman,” this takes us very far back indeed.

It is a controversial question, however, how far we may go in assuming a general spread of the kind of shamanism found among the peoples of Siberia and among the Indians, and how this phenomenon is to be fitted in to the general history of mankind.244 It is especially difficult to be certain about the existence of an Indo-Iranian, or a specifically Iranian, shamanism.245 The mythical motifs concerned can be traced with certainty back to the early ages of Mesopotamian civilization—Inanna’s trip to the underworld, Gilgamesh’s search for immortality along the path of the sun beyond the great twin-peaked mountain.246 The pertinent ritual, the cult of the Great Goddess, probably goes beyond the Neolithic to the Paleolithic Age, and we may conjecture that from very early there were attached to it societies of men with their initiation ceremonies, bringing the renewal of life, possibly in wild, ecstatic orgies. The sloughing of a snake’s skin, attributed to the Gilgamesh epic to its having swallowed the plant of immortality, is described in an early Greek poem in the words, “Only the γόνα remains.”247 This complete separation of body and soul does not seem, however, to belong to the early Mesopotamian or early Mediterranean tradition, whereas in the Iranian tradition the dichotomy of “bony” (that is, corporeal) and “spiritual” life is firmly established.248 When Apian249 tells us that Pythagoras wore white clothing, a golden crown, and trousers, this last detail is a sure pointer toward the Iranian-Scythian area.250 And yet these northern nomads never developed a doctrine of metempsychosis, so that we are once more tempted to consider the possibility of a direct dependence of the Greeks on the Indians (above, n. 71).

It is impossible, at this late date, to search out and map every channel of historical influence. Whether or not the expanded conception of shamanism is recognized as legitimate is a terminological question for specialists; but it has in any case performed the useful function of taking the so-called myths and legends seriously and showing how they make sense as clues to actual cult practices.

In this is the picture of Pythagoras that emerges from the study of the most ancient testimony, not influenced by Plato. He is the hierophant of Great Mother mysteries with an Anatolian stamp, and has a new doctrine, probably influenced by Indo-Iranian sources, of immortality and of the triumph over death through successive rebirths. Epimenides and Empedocles were similar “shamans,” but the spiritual character of Pythagoras’ activity is seen above all in one fact—the continued existence of a society of Πυθαγόρευς.

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239 Museo has the gift of flight (Paus. 1.22.7). See further P. Wolters, “Der geflügelte Seher,” SBMitt 1928, 1.

240 Cf. Boyancé, Muses (though he avoids the word “shamanism”).

241 Hes. fr. 13 M.-W.

242 Paus. 4.18.5: An eagle, or the air resistance of the shield, bears Aristomenes unharmmed to the floor of the ravine, just as, at the Trophonius oracle, a wind bears the inquirer into the depths.—The Dioscuri take Aristomenes’ shield from him, and he finds it again in the sanctuary of Trophonius (Paus. 4.16.5-7). A 4th-century tradition connects Aristomenes with the mysteries of Andania (Paus. 4.26.8). Interestingly enough, there are close connections between Messenia and Magna Graecia; the colonists of Metapontum came mainly from Messenia (Kiechle 34f). Ganschinetz lists more kahabatien stories from Greece, RE X 2395ff. There are also relationships with Italian material: the Ausonians related of Mars, the “first man,” whose form was that of a centaur, that he lived 123 years, and τῆς ἀνθρώπου ἀνθρώπος τῆς (Ael. VH 9.16).


244 Elida regards shamanism in a quite general way as an ecstasy-inducing technique. L. Vajda writes against this generalization (Ural-Altaiische Jahrbücher 31 [1959] 456-485, with full bibliography): he would like to narrow it down as closely as possible—geographically, phenomenologically, and historically.


II. PYTHAGORAS IN THE EARLIEST TRADITION

4. ACUSMATA

The oldest form of transmission of the teachings of Pythagoras is represented by the acusmata, which are also called symbola, orally transmitted maxims and sayings. Our first tangible evidence about them goes back to about 400 B.C. Anaximander of Miletus (the younger), whom Xenophon names as one of those who could find the hidden meanings in Homer, also wrote an Explanation of Pythagorean Symbola. This shows that the tradition was pre-Platonic; and these symbola already needed explanation, or allegorical interpretation like the text of Homer, they must be much older, must go back in fact to pre-classical or archaic times.

It is a matter of question how much of the detail we have goes back as far as Anaximander’s book. Once again we find our most important evidence in Aristotle’s book on the Pythagoreans. In addition to fragments cited by name, there is a long passage in Iamblichus which

1 Hölk, a student of Rohde, first clarified the main lines of the tradition. Later, Boehm collected comparative material from folklore, which helps to interpret the various symbola. In the following pages his numbers are given, and these should be understood also as referring to his convenient collection of parallel references. The most extensive discussion is that of Delatte, Litt. 260ff.; the most important texts are in DK 86C.

2 See “Symbola Pythagoreorum” (Suda s.v. Anaximander = FGrHist 375 = DK 58C6). Corssen, RhM 1912, 246f, followed by Lévy, Sources 67, 3, maintained that the Suda confuses Anaximander of Miletus with Alexander (Polyhistor) of Miletus, who also wrote on the Pythagorean symbola. (FGrHist F354 = Lém. Al. Strom. 178.)

3 Jacoby, in his note on the passage, disagrees with Corssen. Xenophon, however (Symm. 3.6), refers to Anaximander’s method of allegorical interpretation, and we cannot hope for any other or more appropriate confirmation. The Suda dates Anaximander in the reign of Artaxerxes Mnemon (405–359). The dramatic date of the Symposium (422 B.C.) provides nothing, as Xenophon even introduces himself as a character (1.1; cf. Ath. 5.216a). The mention of Semeistromos together with Anaximander brings the latter closer to 480 than 580, to 360.

4 The Suda (s.v. Anaximander) cites three symbola: not to step over a yoke, not to put the fire with a knife, and not to eat from a whole loaf. The first two are favorite example of Pythagorean symbola (no. 30 and 33 in Boehm; cf. Por. VP 42), so that one is led to suspect that the Suda’s datum is not taken from Anaximander’s book but, for example from some handbook. On the other hand, the third example (no. 39 in Boehm) is not found elsewhere except in Heppol. Ref. 6.22.5, where the other two are also found (with σάρξ instead of ζύγον). Delatte (Litt. 286.2) supposes that Hepholius and the Suda are both dependent on Anaximander.

5 Cf. Hölk 21-40. The group of fragments collected by Rose (194–197) and printed without change by Ross must be revised: fr. 107 does not belong; Jerome copied Porphyry carelessly, and the name of Aristotle (Por. VP 41). Arist. fr. 198 got attached to a passage which Porphyry expressly distinguishes from the Aristotelian citation (Klotz, Q 139.4, Hölk 188f., Por. VP 42). Jerome belongs in the Androcydes tradition. Por. VP 193 is arbitrarily singled out by Rolde; D.I. 8.13 belongs to the Memoirs section (Hölk 27, 36ff., DK 6811a, FGrHist 272 (VP)). Fr. 192, first part (Dam. VP 10) is not an excerpt, but Iamblichus’ own composition.
instances of verbal agreement with Diogenes Laertius' citation from Aristotle,11 and a third passage agrees almost exactly with one in the Oeconomica attributed to Aristotle. Here a textual corruption in the latter may be removed by use of the lamblichus text.12 Some sentences in Aelian, quoted along with other material about Pythagoras,13 have points of contact both with fragments of Aristotle and with Lamblichus. The conclusion seems inevitable that not only the passage of Lamblichus14 but also that in Aelian go back to Aristotle.

The other two categories of acusmata in Lamblichus can also be traced to Aristotle. Porphyry cites, from Aristotle, some sentences of the same type as the τί ἐστι adages in Lamblichus. The pronouncement that Pythagoras was "the Hyperborean Apollo" is expressly attested as an acusmata;15 in the passage just mentioned, Aelian has some material that is closely related to what is attested by Porphyry;16 there is similar material in Diogenes Laertius,17 and also a remark of Aristotle himself.18

The traces of Aristotelian influence are still clear in the τί μᾶλλον says. "This is the same sort of wisdom as that attributed to the Seven Sages; for they too were trying to find not 'what is good?' but 'what most [good]?'" (Iam. VP 83). This is an excellent comment, and quite right from the point of view of the history of thought. Whatever the may be about the authenticity of the sayings of the "Seven Sages" (we have collections of adages in the τι μᾶλλον format),19 in the with for superlatives (τί ψάρτον, τί μᾶλλων) there lies a very fruitful kind of "wisdom," to be found not only in the Contest of Homer and Hesiod and in the Aesop legend, but also in Sappho (27 D.) at the beginning of Pindar's first Olympian. Now the designation of the Seven Sages in this passage as ἐπὶ τὰ σοφοτάτα, an expression already surviving obsolete in the fourth century, is attested precisely for Plato.20 What is more, in the passage we have cited twice above, than gives two τί μᾶλλον sayings, one of which agrees with Lamblichus,21 so that the similar ones in Diogenes Laertius must also be ascribed to Aristotle.22

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11 Iam. VP 84, p. 49.6 D.; D. 8.34 (Arist. fr. 195; Diogenes' version is more detailed). Iam. VP 86, p. 50.8 D.; D. 8.35 (ἀτομόν μὲ καταγγείλῃ). The last of the explanations given by D. (ἐπεὶ ἀπέ τοῦ τοῦτον ἔρχεται τὸ δέκα) must go with the sentence in lamblichus oí δὲ οὗτος ποιεῖται τοιοῦτον ἀρχέοντα καταγγέλλων καὶ συντριβότα, whether the cosmological interpretation proposed by Delatte (Vie 239) is wrong, or whether lamblichus misunderstood.

12 Iam. VP 84, p. 49.4 D.: γυαλικά γιὰ τοῦ διδάκτων την αὐτή; ἐπ᾿ Aar. Ο. 1344476 (DK 58C2): βιάζεται δὲν δοκεῖν διδάκτων. Instead of the usual reading δὲν [δοκεῖν] διδάκτω (as in DK) should be read δὲν διδάκτων [διδάκτω] (Deubner, SB 1935, 672). See also Iam. VP 48.

13 Ael. 4.17, καρδίας ἀπεκληθείς, cf. Arist. fr. 194; ἀπέκληθαι αὐτὸς θεοῦ λκοὶ τοῦ τιμωροῦν. Aelian. fr. 159. Iam. VP 84; μὲ χρήσεις βαλκανίζει μηδε βαλκανίζει τινος λελεκύνων, cf. Iam. VP 83. Thus the intervening sentence (ἀπεκληθείς) θεοτείνως also belongs to Aristotle (cf. H. 33; missing in Boehm).

14 Iam. VP 85, p. 49.9ff.: ἀγάμος οὐκ οὔϊν, αὐτὸ δὲ ὑδαίνῳ ἐκ παντὸς τρόπον κοινόν ἔρχεται τὸν ἀρχίσεως διδάσκων δισμόνης. As neatly pointed out by Arist. Protr. fr. 60 (Ael. 4.27): τοῦτο γὰρ δεῖ να εἰς αρχιστῆρας λέγων τὸ τάχα διδάσκα τὴν φύσιν τιμώρων καὶ ἰσορροπητικῶς μεγάλους τῶν ἀξιώσεως. (Cf. Cic. Consolat. fr. 8 M., perhaps from Ciceros: "hoc uerum sequerentur non aseci homines.") On Philolaus fr. 14, see ch. III 2 n. 47.—Death as a metaphor (Iam. VP 85, p. 50.2f.) is Platonic (Ap. 400, Phid. 1120) see Deubner on the lamblichus passage; but it is also understandable in the framework of the ancient doctrine of metametaphysic.

15 Por. VP 41 = Arist. fr. 196; also Plut. De Is. et Os. 32, Clem. Al. Strom. 5.50.1. Iam. VP 140; καὶ ἐν τοῖς τῶν ἀκομαίων ἔστι τίς τις, Πολυδάρα: οὕτω γαὶ Εἰναὶ Ἀπόλλων. Ἡ ἀρχή τῶν ἀρχηγῶν, cf. ch. II 3, n. 117.

16 Iam. VP 47.17; printed with Arist. fr. 196.

17 Iam. VP 83; γύρω καὶ πάντα τὸ μονώμα κόσμου νόμον ἐρχόμεθα καὶ κράτων ταύτων, ἀγάμω συμπεριφέρεται τοιοῦτον. This sentence, like the one preceding it (καὶ τῶν συμφωνάς τοῦ κόσμου σιμφώνα) in the strophe, τῶν ἐκ τούτων κόσμως is excluded by Rose from 195, but Delatte (Litt. 277, Vie 239) treats it as Aristotelian. The suspicious word ὁμοίω, in true pre-Socratic fashion, means nothing more than the shape of the body.

18 Arist. post 48333 (DK 58C1).
Among this group we can even distinguish a pre-Aristotelian line of tradition. Aristotle introduces a Pythagorean named Paron who is otherwise completely unknown (DK 26): οἱ μὲν σοφότατοι ἔλεγον (τῶν χρόνων), δὲ Πυθαγόρειοι Πάρων ἀμαθετάτων . . . (Phys. 222b17). But in Eudemos' more detailed account the story goes differently (fr. 90 W.): ἐν Ὀλυμπίᾳ Σιμωνίδης τῶν χρόνων ἐπαύσαντος ὡς σοφότατοι, εἴπερ ἐν αὐτῷ ἅ μαθήσεως γίνονται καὶ ἅ ἀναμνήσεις, παρόντα τά τῶν σοφῶν ἐπειπέ, τί δὲ οὐκ ἦσαν, σὺν ἐπιλαμβάνομεν μένουν ἐν τῷ χρόνῳ Ευδemos cannot be dependent on Aristotle, since he sets the scene more elaborately—Olympia, Simonides, etc.—but what is a proper name to Aristotle appears here as a participle: Πάρων-παρόν. We cannot however, follow Simplicius' proposal to correct Πάρων in Aristotle to Παρών, for without the scene as Eudemos has it, the word would be meaningless. Both are following the same source, a written source which, with the accents unmarked, could be understood in either of two ways, e.g. ΠΑΡΟΝ ΤΗΣ ΠΥΘΑΓΟΡΕΙΟΣ ΕΛΕΓΕ . . . Thus we have a pre-Aristotelian proof of the high valuation placed on wisdom by the Pythagoreans, and also of the Pythagoreans' use of proverbial wisdom in the context of the ancient sophia.

This shows that the entire section in Iamblichus is full of Aristotelian material. It may be that the division of maxims into three categories is likewise Aristotle's work. He was interested in logical distinctions, and in particular in the early stages of conceptual definition. From the lines of parallel transmission emerges quite an extensive body of material.

What are the Isles of the Blest? Sun and moon. What is the Oracle of Delphi? τετρακτύς: ἄπειρον ἢ ἀρμονία ἐν ἀρχήν. Pythagoras is the Hyperborean Apollo. An earthquake is a mass meeting of the dead. The purpose of thunder is to threaten those in Tartarus, so that they will be afraid. The rainbow is the reflected splendor of the sun.

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41 Diels, DK 217 n. "Eudem in folget auch einer derartiger Lesart oder Ueberlieferung." This evades the problem. The source is likely to be a Sophistic-rhetorical writing like Aleudama's Museum. Eudemos probably understood his source correctly, and the whole chapter, DK 26, should be eliminated (Wrocl 160).
42 Cf. his understanding of the Pythagorean number symbolism as attempts at definition (above, ch. 12, n. 63). The equations he mentions like, "marriage is 9," are at home among the acaimata.
43 Sam. VP 82; cf. above, n. 154.
44 Above, ch. II 3, n. 117.
45 Ael. 4.17. The idea behind this may be that of a battle or struggle (Nilsson 704f. referring to II 20.614f).
46 Arist. Av. post. 943b31.
47 Ael. 4.17; cf. Aristophanes A7, A18, and Xenophon fr. 32.
first, not travel by the main roads (λεωφόροι) (no. 41), not dip one's hand into holy water (no. 44), not use the public baths (no. 45), and not help a person to unload but only to load up. One should not have children by a woman who wears gold jewelry (no. 21), not speak in the dark (no. 51). One should pour libations over the handle of the cup, refrain from wearing rings with depictions of gods (no. 9), and not "pursue" one's own wife, since the husband, in receiving her as a supplant at the altar, has taken her under his protection. One should not sacrifice a white cock, because they are suppliants and sacred to the god Men. One should never give advice except with the best intent (for advice is sacred) (no. 63), nor make a detour on the way to the temple (no. 59). One should sacrifice and enter the temple barefoot (no. 3), and in battle hold one's place, so as to fall with wounds in the breast. Eat only the flesh of animals that may be sacrificed; abstain from beans; do not pick up food that falls from the table, for it belongs to the Heroes. Abstain from fish that are sacred (including τρίγλην, ἀκαλλήν, ἐρυθρίνως, μελάνωρος); do not break bread, put salt on the table as a symbol of righteousness. It is also forbidden to eat certain parts of the sacrificed animal, and there are many specific injunctions about the time and manner of sacrificing, about rites connected with death and burial. This is the context of the prohibition mentioned by Herodotus, of burying the dead in wooden garments.

There are other rules, of just the same kind, in the tradition stemming from Androcles. For example, one is not to stir the fire with a knife (no. 33), step over a yoke (no. 30), or sit on a bushel measure (no. 32). On rising one is to straighten the bedclothes and eliminate the traces of one's presence (no. 34), as well as destroying the marks of a pot in the ashes (no. 35). One ought not to clean a chair with a torch (no. 36) or to step, or make water, on nail parings (no. 48), and should point a sharp knife in the other direction (no. 37), and not look at oneself in a mirror with the help of artificial light (no. 52). On a journey do not turn around at the border.

Hermippus has some strange precepts: One should not pass by where an ass is lying, and should avoid "thirst-causing water." A coffin should not be made of cypress, for that is the material of Zeus' worship. The same sentence appears in a remarkable list of cult rules in Lamblichus (153-156); it combines quite trivial matters ("do not kill a house in the temple") with commandments as important as the prohibition of cremation ("in agreement with the Magi"). The prohibition of "roasting what has been boiled" is attributed in the pseudo-Aristotelian Problemata to "the mysteries."

What we have here is a very mixed collection of sayings and maxims. The threefold division, perhaps originated by Aristotle, is artificial and is not consistently followed. The rules and prohibitions regarding daily life attracted most attention in ancient times; and there were two opposite ways of interpreting them.
II. PYTHAGORAS IN THE EARLIEST TRADITION

The account in lamblichus, which goes back to Aristotle, leaves no doubt that the precepts are intended to be taken literally (lam. VP 86);

In some cases a statement is added as to why this is to be done (e.g., that one ought to have children in order to leave behind another to worship the gods in one’s stead), but in other cases no explanation is given. Some of the explanations added seem to be ideally suitable, but others are far-fetched ... The added, conjectural explanations are not Pythagorean, but originate with persons who introduced clever explanations from without, in an effort to give a plausible rationale.

The most prominent topic of the acusmata is sacrificial ritual; and the accounts of Herodotus, Isocrates, and Eudoxus speak of the special role of ritual ἄγνεῖα among Pythagoras and his followers.65

Nevertheless, the prevailing view in antiquity was that what was desired was not compliance to the letter but comprehension of the deeper meaning. As early as Anaximander allegorical interpretation was applied to this material as to Homer; and, when Aristotle used the word ἐπισφεξέομενοι, he surely had this method in mind.66 Our principal source for the later interpretation is Androclesy. Here the acusmata are regarded as ἀνίγνεια, riddles, which clothe a lofty wisdom in language unintelligible to the uninitiated.67 As Clement puts it, σφάλλεται μὲν ὀπέρος καὶ ἀμβής, καταλαμβάνει δέ ὁ γνωστῦς.68

It is obvious that Androclesy must have exercised a certain selectivity, for not everything would fit together consistently. The injunction καθόπιστα μὴ ἐσθίεις means that one ought not to worry,69 but its com-

63 ἐπιλέγεται ἄδια τί δεῖ. The insertion of ἄδια (a conjecture of Kiesling’s merely recorded in the apparatus by Deubner) is essential. The omission would be an easy one in a manuscript: ΕΠΙΛΕΓΕΤΑΙΑΙΔΙΕΙ.
64 ἐπισφεξέομενοι is attested from the corpus Hippocraticum (LSJ); but εἰκοσῳδος is not found in later texts (ps-Archytas p. 37.1 Thesleff).
65 Hdt. 2.81, Iloc. Bus. 28, Eudoxus fr. 36 Gisinger = 325 Lasserre = Por. VP 7.
66 Holk 21.
67 For the word ἀνίγνεια, see Tryphon, Demetrius, and Plutarch, n. 7 above. A certain Hippomedon asserted (lam. VP 87) that Pythagoras had given ἀγώνου καὶ ἐπισφεξέος for all his sayings, but that they had been lost through the carelessness of his successors. Plutarch joins in the game of “solving” these riddles (Quaest. conv. 8.7, where the words of παλαιοῦ in section 2 obviously refer to Androclesy; cf. D.L. 8.17). See also Philop. De an. 116.29, Olympiad. In Plfl. p. 8.22 Norvin.

66 Tryphon, Plutarch (above, n. 7), Arist. fr. 194. Androclesy may also have made additions; the proverbial ἀνιγνεῖα ἥλιος ἡμέρας (Poroeinogr. 2.770) only makes sense in a metaphorical interpretation.
67 Por. VP 53.
68 At D.L. 8.34, Ar. fr. 305 is quoted (above, n. 50; Rose unjustifiably omits the citation of Aristophanes from fr. 195).—καθάπερ ἐπί καὶ νῦν οἱ βασιλεῖς, D.L. 8.35 (cf. Lam. VP 96).
69 Hypomn. 33 (cf. Lam. VP 138). This excludes the “Androclesy” interpretation of the taboons on beans and that on blacktail (both mentioned in Hypomn. 33); and this also probably indicates that “Androclesy” should be dated later than the Hypomnemata (4th century B.C.). The “Androclesy” interpretation of the taboons on beans—that one ought not to partake in politics—is hardly pre-Hellenistic. (A different interpretation, based on the same reason, that the bean was used to vote with, is found in Arist. fr. 195. 8.14: ἐπὶ ἄκοντοις.)
70 Lam. VP 140. Elsewhere the word ἄκοντα has the sense “musical entertainment” (Syn. Mem. 2.1.31, Arist. Pol. 1336b4, Lam. VP 245, etc.).
71 Holk 21.
goes the designation of the members of the Pythagorean society as ὄμακος ὁ ὄμακων for their meeting house. On the other hand, the word σύμβολον carries with it the suggestion of a "symbolic" interpretation. The word is not a late addition, however, and carries another implication as well, prior to any "symbolic" exegesis. In the realm of mystery religion, σύμβολα are "passwords"—specified formulas, sayings, ἔπειδαι, which are given the initiate and which provide him assurance that by his fellows, and especially by the gods, his new, special status will be recognized. The acusmata or symbola still have this function as late as Lucian's parody: τούτων γὰρ ἀν μεθυμνάσων ἐπίδιως ἔχει τής εἰς τὴν νήσον ὁμίχλεος, says Rhadamanthys, in the True History, to Lucian as he leaves the Isle of the Blest. Aristoxenus mentions σύμβολα as "passwords" used by the Pythagoreans. The pentagram, too, was a symbolon in this sense; it was thought from the most ancient times to have a secret power and significance. But when once the allegorical and symbolic interpretation of the Pythagorean sayings had gained a foothold, it was inevitable that the word σύμβολον be understood in this sense, whereas ἄκουσμα had no such obvious meaning and fell out of use.

The riddle as a literary form (γράφος) is very old, and is used in the promulgation of oracles. There can be no doubt, though, that the acusmata are, rather than simple, commonplace wisdom in abstruse form, ancient magical-ritual commandments. It is not possible here to offer a complete analysis, but we may try to establish a few points of reference. There are obvious coincidences with rituals of Greek mystery cults, and they even share whole series of commandments. For example,

(a) The initiate at Eleusis must fast, avoid baths, abstain from domestic fowl (and the cock is especially named), as well as ἐνθέων καὶ κοιμῶν ῥοϊὰς τῆς καὶ μῆλῳ, καὶ ἐπ' ἑσόνις μελανώτατο τὸ τὸ λεοντος ἄφθονα καὶ τὸ θηρευτικῶν.

(b) At the Halaon, one was forbidden to eat ῥοϊὰ, μῆλῳ, ὅρθες κατοὐκίου, φά, as well as the fishes τρίγλη, ἐριδάνως, μελανώρος, κύριμοι, γαλαζ. 

(c) Preparation for the katabasis in the Trophonius cult included avoidance of warm baths, τρίγλη, τριγύλων, μελανώροις, and one must wear a linen garment.

(d) The μάγοι, καθαρτιά, ἀγάρη, ἀλαζῶνες against whom the Hippocratic work On the Sacred Disease speaks, forbid their patients baths, τρίγλη, μελανώροις, κυριμαῖ, ἐριδάνως, certain kinds of meat, various birds including the cock, and spices. They may not wear any black garment, lie on a goatskin, lay one foot or one hand over the other; and there are other such purificatory measures.

(c) In Delos, the worshipers of Zeus Cynthia approach him barefoot and clad in white. They abstain from sexual intercourse and from meat, and do not wear iron rings, keys, belts, purses, or weapons.
II. PYTHAGORAS IN THE Earliest Tradition

4. Acusmata

(f) The sacred law of Lycosura commands that one sacrifice barefoot, wear white clothing, and neither rings nor other gold ornaments.88

(g) In the Temple of Asclepius one may not pick up anything that falls to the floor (IAM. VP 126), and one who undergoes the sleep cure at Pergamum may not wear ring, belt, or gold ornament.89

It may be that in a few cases Pythagorean ritual was adopted in late times by various cults,90 but in general the latter are independent of Pythagoreanism and older than it. Some taboos are attested from an earlier date,91 and a good many are widely spread folk tradition.92 Above all, the form of such authoritatively prescribed commandments and prohibitions, which are not supposed to be understood but merely obeyed, is primeval; it is entwined in the very roots of religious ritual. In what one does and does not do is manifested the identity of the group, the membership of the members and the exclusion of outsiders. The more selective the society, the more careful are the “taboos.” Fasting, abstention from particular foods, and rules of sexual behavior93 play an important role. It is of first importance that the “wise man”—the priest, the hierophant, the shaman—who claims a special position in the social organization, gain and maintain, through a special ascetic regimen, the special powers that belong to him. To this extent, the acusmata also have their connection with the Pythagorean legend and its cultic bases. The saying that Pythagoras was the Hyperborean Apollo is expressly attested as an acusma (IAM. VP 146).

Pythagorean silence and secrecy should also be seen in the context of cult and ritual. To be sure, the secrecy of Pythagorean doctrine later was misused by forgers as license to “discover” more and more Pythagorean writings.95 But the testimony of Aristotle and Aristotle, which proves the existence of Pythagorean ἀκύσματα, cannot be ignored.96 Aristotle’s testimony has to do with the intermediate position of Pythagoras between man and god. All mysteries have secrets; the ritual is interpreted in a ἱερός λόγος which may not be disclosed to the uninitiate,97 and the initiate also learns secret passwords, σήματα, σπάνηματα. All kinds of societies that are bound together by cult have their esoteric aspect—even political clubs, trade guilds, and those of physicians.98 Among the Pythagoreans the practice of remaining silent for long periods—one of the most effective means of attaining inner composure—is to some extent a continuation of the practice of shamans and yogis.99 The ὅμακος is not supposed to speak but to hear, and as early as Isocrates and the comic poet Alexis Pythagorean silence was proverbial.100 A five-year period of silence as a test before acceptance in membership is attested by Timaeus.101 The practice was, also, to avoid using Pythagoras’ name.102

Aristoxenus mentions that Epaminondas called the Pythagorean Lysis, who was his teacher, “father.”103 Ludwig Edelstein brought this into connection with the precept of the Hippocratic Oath that the physician must regard his teacher as his father and the teacher’s sons as brothers.104 But the same usage is especially prevalent in the mysteries,105 whoever has led the candidate to be initiated becomes his father. The Pythagoreans, too, form a “brotherhood,” in accordance

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88 SIC 999 (Delatte, VIE 231f); similarly in Andania (SIC 736): no shoes, no gold, linen clothing; in Ialysus (SIC 338): no shoes as in the mysteries of Demeter (Callim. Cer. 124).
89 Sokolowski, 14. Cf. also L. Deubner, De inscriptione (Leipzig, 1900) 14ff.
90 The prohibition of beans is found in cult regulations of the imperial period: Rhodes (Sokolowski, supra, 108), along with the prohibition of ἄρωδες and καβριά; Smyrna (Sokolowski, 84) along with the prohibition of eggs. Perhaps the Titans were somehow brought into this context, but the text cannot be restored with confidence. Probably this is a case of neo-Pythagorean influence (Nock, HSCP 63 [1958] 415-421); an ancient ritual can of course be understood in late times as Pythagorean.
91 Hes. Op. 727 (above, n. 57); Hymn. Hom. Cer. 50 (above, n. 84); cf. also the ζήλοι διήνοτοις χάρισαται at Dodona, II. 16.234.
92 Cf. above, n. 43.
93 Complete celibacy is ascribed to Pythagoras at D.L. 8.19; but this is contradicted by the traditions that include his wife and children (above, ch. II.2). The acusma cited in n. 42 (above) implies the desirability of marriage, as do the sayings of Theano in D.L. 8.43 (cf. Hdt. 1.83; A. Rauhutseh, RhM 100 [1957] 139), and IAM. VP 112, Stob. 4.43-53. Pythagoras demands strict monogamy according to Philostr. I.A. 1.13, IAM. VP 48, 60, A.P. 5.43. Special regulations as to diet and regimen, in relation to sexual intercourse, are given by Aristox. fr. 19 W., D.L. 8.9, and Diod. 10.9.3ff. “Clibus,” a character in Plut. Quaest. conv. 644b, is negative on the matter.
94 Burkert, Philologus 1961; below, ch. III.1.
96 ἱερός is used here in the sense of ἀχιλλής λέξις, cf. Hdt. 2.61f.
98 Cf. also O. Casel, De philosophorum graecorum silento mystico (Giessen, 1919), G. Munching, Das heilige Schwören (Giessen, 1926).
100 FGHist 566F13 = D.L. 8.10, IAM. VP 72, etc. The technical terms ἐγκύρως and ἐγκύρων are found in Ath. 7.308d, Plat. Numa 8, Quaest. conv. 728e, Lucian Gall. 2, LAN. VP 94, ψευδοφρασια at IAM. VP 246.
102 Aristox. fr. 18 = IAM. VP 250, Diod. 10.11.2, Nep. Epam. 2.2, Plut. De gen. 583c.
103 CMGG I 1, p. 4.5ff; L. Edelstein, The Hippocratic Oath (Baltimore, 1943) 34ff.
104 A. Dieterich, Eine Mithrastudie, 1929, 146-149; 1 Cor. 4.15. For the adoption of Heracles and the Dioscuri in Eleusis, see Plut. These. 33. On “Chaldeus,” Diod. 2.29.4.
with the ancient custom of colleagues bound together in a cult. We are even told that Parmenides adopted Zeno.106

That the "Pythagorean life" developed from living custom, with all its complexity and paradox, rather than from clearly articulated doctrine, can be seen very clearly in the rules about abstinence from meat.107 The self-evident corollary of the doctrine of metempsychosis would have to be complete vegetarianism. Empedocles drew this conclusion, and according to Eudoxus, Pythagoras was not only a vegetarian but avoided any association with butchers and hunters.108 Aristoxenus, however, asserted that he only avoided eating plow oxen and rams, but was especially fond of the meat of tender young kids, sucking pigs, and cockerels.109 Hints of Athenian ways here are intended to lessen the absurdity of Pythagoreanism, for the benefit of the reading public. There are other remarkably persistent traditions that seem to know nothing of Pythagoras as vegetarian. The famous sacrifice of an ox to celebrate a geometrical discovery is attested by a (probably) fourth-century source,110 and the tradition is hardly later that it was precisely Pythagoras who introduced the meat diet for athletes.111 Nicomachus smoothed out the contradictions of the evidence by use of the reports about degrees of membership within the society: the ἂθρομος ἡ ἄκομαιμανται ate meat, though "seldom."112 But the very Pythagoreans whom we must suppose to be "acumatics," the "Pythagorists," Diodorus of Aspendus, and the Cynic-influenced Onesicritus, emphasize the radical formulation ἐμεῖς ἄστικοι, while Aristoxenus seems to be thinking rather of "theoretical" Pythagoreans, who still like meat.

To judge by Aristotle’s testimony, the acusmata did not contain any simple prohibition of the eating of meat, but various specific precepts: the heart and wombs of the animal, and perhaps also other similar parts,113 may not be eaten. Here, as also in the prohibition of killing a white cock or of eating the flesh of animals that have died a natural death,114 it is taken for granted that other kinds of meat will be eaten. Special treatment of certain parts of the slaughtered animal was part of sacrificial ritual from very early times. Karl Meuli has shown that the horror of death and the reverence for life manifest themselves in these usages and also express themselves in an attempt at recompense, that is, in the rebirth of the slain creature.115 The heart has a special role in Greek ritual.116 This is reflected in the Orphic myth in which Athena

106 Apollodoros FGrHist 244F30; παράγοι, Pl. Soph. 241d, De Vogel 240f.
108 Fr. 36 Gisinger = 325 Lasserre = Por. VP 7. (The "impurity" of hunters and butchers does not exclude the eating of meat, but is actually evidence for it; see Meuli, Opferbr. 228; Agatharchides GGM 154, on the Troglydtes.)—Vegetarianism is characteristic of the Πεθανούργια and of Diodorus of Aspendos (below, ch. II 5). See also Onesicritus FGrHist 134F17 (Strabo 15, p. 715), Callim. fr. 191d F. Pfeiffer. For the story that at Delos Pythagoras only worshipped at the altar of Apollo Genetor, where bloody offerings were not made, see, e.g., lam. VP 25, 35, Cic. Nat. d. 3,88; Clem. Al. Strom. 7,32; Macrobr. Sat. 3,6. Arist. fr. 459 = D.L. 8,13 and Timeaus FGrHist 566F147 = Censo. 2,3 mentioned the altar, but it is not certain that they brought Pythagoras into connection with it (Delatte, Vie 177). Among later writers, see Ov. Met. 15,75ff (with reminiscence of Empedocles) and Sotion ap. Sen. Ep. 168,17.
109 Fr. 25, 28, 29, D.L. 8,20, Diogenes Antonius ap. Por. VP 36, Lam. VP 150 (where cockerels are explicitly mentioned). Boyanez pointed out the connection of the sacrifice of rams and pigs with Attic ritual, REG 1939, 40ff. On the plow ox cf. Wehrli on Aristox. fr. 294, Hauserle 116,24, Meuli, Opferbr. 275ff. With Aristox. fr. 27 are connected D.L. 8,19 and Lam. VP 98 (Boyanez, loc. cit.); the former mentions woolen clothing. At Eleusis it was said to be the command of Triptolemus γονεῖς τιμᾶν, θεὸς καταφέρει αὐτόλεκτον, ζύλον μοι σύνεσθαι (Por. Abr. 4,22 = Xenocrates fr. 98), and Aristoxenus subscribes similar regulations to the Pythagoreans (D.L. 8,23, Por. VP 59, Lam. VP 96f).
110 The basic testimony is an epigram by a certain Apollodorus, who should perhaps be identified with the philosopher of Cynicus (DK 74) and dated earlier than Epicurus (below, ch. III 1, n. 51). The means chosen to avoid this testimony was to assert that Pythagoras had sacrificed an ox of dough (Diogenes Antonius ap. Por. VP 36; Greg. Naz. Ep. 198, Migne 37,344, where σταύρωμα is misread as πήλιον). This is obviously transferred to Pythagoras from Empedocles (Ath. 1,3c, Favorinus ap. D.L. 8,53, Suida s.v. Athenaios; cf. Philostr. V 1,1,2; Hauserle 162, Delatte, Vie 174). For similar substitute offerings, see Hdt. 2,47.
111 Por. Abr. 1,26 = Heraclides fr. 40 W. (though it is impossible to determine how much of the context goes back to Heraclides), Favorinus ap. D.L. 8,12, Por. VP 15. In order to explain away this evidence, some postulated a different Pythagoras: D.L. 8,13, 8,46 (here he is a Philisian!), Plin. HN 23,121, Lam. VP 25 ("son of Eratocles"): τοίουτος δὲ καὶ τὰ ἀληθῶς άνθρώπων άφθονον,  ὄλα ἄξια τῶν Πυθαγόρων τῶν Μηναύρων ἀνθρωπομοίων. There was in circulation, then, a coach’s handbook under the name of Pythagoras. The tradition must have been formed before Pythagorean vegetarianism was firmly established. Scholars have referred, in discussing this matter, to the famous athlete Milo of Croton, who was regarded as a Pythagorean (Hauserle 124f.; as Pythagorean, Lam. VP 104, Aristox. fr. 18 = Lam. VP 249, Strabo 6,261; as a heavy eater, Ath. 10,412e). Incus of Tarentum was a famous trainer (a Pythagorean according to Lam. VP p. 144,6; DK 25; Wülsemüller 566). The boundary between the "Pythagorean life" and rational diet fluctuates (cf. ch. III 3).
112 Lam. VP 1075, 150. (The allusion in 168 to legends about Pythagoras persuading wild animals not to eat meat points to Nicomachus as the source of the passage; above, ch. II 3, Rohde, Q 143f.) The differentiation according to classes of membership is accepted by Hauserle 116, Vlastos, Philos Q. 1952, 116 n. 66; von Fritz, STM 1966, 13; Eubulus, Περὶ τοῦ Μήδα ιστορία (ap. Por. Abr. 4,16) distinguishes three classes among the Magi; cf. below, ch. II 5.
113 Arist. fr. 194 (above, n. 69). Add συζεύκασθαι μη ἕστηκαν Lam. VP 109, Lam. Protr. 123,14, Plat. Quast. conv. 635c. There is a different list in Por. VP 43.
114 Ὑπερήφανος: Ael. 4,17, Hypomn. 33; cf. above, n. 13.
115 Meuli, Opferbr., esp. pp. 185ff.
116 It was cut out of the living animal and laid on the altar still beating (Gal. Plut. Hipp. et Plat. V 238 K; Ἰππ. παρὰ καρδιοσκέπασμα; Suid. s.v. καρδιοσκέπασμα).
II. PYTHAGORAS IN THE EARLIEST TRADITION

4. Acusmata

saves the heart of Dionysus Zagreus when he is torn to pieces by the Titans and hides it in a sacred chest,117 and also in the speculations of the physical philosophers, according to which the heart is the first of the organs to grow.118

Animal sacrifice was the focal point of the traditional religion, that is of the official cult of the polis, and to renounce it would have been more than religious reform. It would have meant a complete overturn of traditional ways. It is interesting that this is the goal of Zarathustra’s gospel.119 As far as we can judge, the Pythagoreans sought to compromise the matter; an acusma asks, “What is most just?” and answers, “To sacrifice.” An accommodation of the doctrine of metempsychosis and the traditional way was found, because it had to be found. “The only animals into which the souls of men do not enter are those which may, according to sacred law, be sacrificed. Therefore, those who are allowed to eat meat may eat only of those animals that may be sacrificed, but of no others.”120

In the late compromise, it seems that the ancient cult practice is still influential. Originally, and for a long time, abstinence was only a preparation for the sacred meal, so that omophagia and vegetarianism, different as they seem, are complementary.121 In the mysteries of Demeter and Dionysus the most important sacrificial animals are sucking pigs, cocks, and kids,122 the very animals of whose meat, according to Aristoxenus, Pythagoras was especially fond. Perhaps, then, this statement was not manufactured out of whole cloth, but was a rationalization of ritual. One report says of the Pythagoreans, “Throughout their lives they abstained from eating meat; and when, in their own stead, they made the prime-offering of an animal to the gods, they would just taste it, so that in truth they lived untouched by such things.”123

The famous, or notorious, taboo on beans is found in similar contexts. It is attested by Aristotle, Heracleides, Callimachus, and indirectly by Empedocles;124 only Aristoxenus denies it.125 This was a favorite target of those who would mock Pythagoreanism. As early as Aristotle several alternative explanations of the puzzling aspects of it were available. The interpretations that are most closely related to genuine Pythagoreanism are those which connect beans with the doctrine of metempsychosis. Along with Aristotle’s report that beans are like the “gates of Hades,” since they alone among plants do not have joints,126 we have a remarkable couplet in the scholia to Homer:127

ψυχής αλήθος βαίνει ἐξεµεναὶ ὡς ἀναβαθμον ἢς Ἀθαο δημον, ὅταν ἀνάγεται ἐστανίων.

It is through bean blossoms that souls return to earth for their reincarnation. Varro says that the souls of the dead are in the beans,128 and there are several variants of the conception that a soul emerges from a plant, or a bloom, to enter into a human body.129 This makes comprehensible the legend that Pythagoreans refused to walk through a

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118 Empedocles A 84, Plat. Quaest. conv. 635e; at 636d the Orphic prohibition of eggs is interpreted in the same way; it is attested for the Pythagoreans only at Hyppono. 33.

119 The fight against the sacrifice of the cow, Ysna. 44.20-92: cf. above, ch. II 2, n. 16.

120 Iam. VP 85, Por. Abs. 1.26; cf. also Boyancó, REG 1939, 52 n. 2. Though this interpretation may be a counsel of despair, it need not on that account be secondary, for the awkward situation was there from the beginning. There has, however, been an interpolation in the excerpt from Aristotle: καθαρσε... is not used impersonally by Aristotle.

— Another way out was to consider sacrifice as “justified execution” (Por. ap. Stob. I.49.59; cf. Pl. Lcg. 879c).


122 The cock and the mysteries of Demeter: Por. Abs. 4.16; sucking pigs sacrificed at Heliopolis: Ar. Pax 374, with scholia; kids: the ἐρυμος των γυδας of the Gold Plates; cf. also GRHN 7 (1960) 99.

123 Por. Abs. 2.28, which is not from Theophrastus according to J. Bernays, Theophrastos’ Schriften über Frömmligkeit (Berlin, 1866) 159, and W. Pötischer, Theophrastos Peri eireneias (Leiden, 1964) 176. Cf. Arist. fr. 194 (D. L. 8.19): Ἀριστοτέλης δὲ ἐφησυ καὶ μήτερος καὶ τριγύρων ἐνοικε. Here the word ἐνοικέω means “at certain times.” See also I. P 85: the acusmata are concerned, above all, περὶ την τιθεια καλῷ ἐκείστων τοῦ καιροῦ τῶν χρῶν ποιεῖται. Cf. Burnet, EGP 93.

124 Arist. fr. 195 = D. L. 8.34, Heracleides fr. 41 (surely to be attributed to the book Peri ton Pythagoreion, although Pythagoras is not named), Callim. fr. 553 Pfeiffer; also Emp. fr. 441, and the frequently cited verse ἔνα τούτου ἐνοικους τοὺς φιλεῖν καὶ ἐκδικεῖν τε τῶν ὀφθαλμῶν (references in Orph. frag. 291 Kern, Delatte, Faha 36 n. 2). The subject is fully canvassed by Boehm 14ff.; R. Wünsch, Das Frühlingsfest der Isis Malta (Leipzig, 1902) 11-46; Hausleiter 407ff.; Delatte, Faha; M. Marcovich, Philologus 108 (1964) 29-49. It is well known that the bean in question is a kind of European vetch (Vicia faba); the beans used as vegetables nowadays are of American origin.

125 Fr. 25 W.; cf. above, ch. II 1, n. 57.

126 Arist. fr. 195 = D. L. 8.34. On the word ἄγωνος see Por. De antr. nymph. 19 (Delatte, Litt. 36ff; Faha 36ff; R. D. Hicks is wrong in suggesting a lacuna before ἄγωνος, in his Loeb edition of D. L.)


II. PYTHAGORAS IN THE EARLIEST TRADITION

The bean field in bloom, but it scarcely brings us closer to understanding the real reason for the importance of beans.

We dare not take too lightly the rationalizing, physiological explanation, based on the difficulty of digesting beans. The seer or sage is very sensitive to small physical disturbances. It may have been seen by the ancients, even before the discovery of the chemistry of proteins, that of all vegetable diets one of beans is most like one of meat. A contributing factor in this recognition may have been that a certain amino acid, present in beans, can provoke strong allergic reactions in some persons. In any case the peculiarities of beans had obviously, from early times, been exploited in cult and myth. The most interesting of the explanations of the bean taboo claim amazing similarities between bean and man: the blossom of the plant, or the bean itself, are transformed through certain procedures into human form, or into the form of parts of the human body, they remind of the genitalia and smell like semen. At the origin of the world, bean and man emerged from the same primeval slime. And when we look for the origins of these prolific fancies, we are once more led to the mysteries. Demeter, says Pausanias, gave mankind all the products of the earth, with the exception of beans; and “whoever has witnessed an initiation at Eleusis or read the so-called Orphic writings, knows what I mean.” We are denied this knowledge. It may be that beans were eaten in a ritual meal, and if so, Aristoxenus could be acquitted, once more, of pure invention. In any case, the Pythagorean taboos are closely connected with ritual, either taken over from it or set up in opposition to it.

Side by side with the ritual material, the acusmata have rules and precepts that we would like to classify as rational, in the categories of ethics or physics. But the remarkable thing is the juxtaposition: the rainbow is a reflection of the sun, thunder is a noise to frighten souls in Tartarus, an earthquake is a mass meeting of the dead. Most just is sacrificing, wisest is number, strongest is intelligence, holiest a mallow leaf—taboo and proverbial wisdom jumbled together. Close examination shows an amazing, inextricable tangle of religious and rational ethics. A husband must not “persecute” his wife, for he took her under his protection at the sacred altar; one should only help his fellow to load up, never to unload, for we are in this life to be punished; good counsel is sacred—a moral commandment in religious form, like the exhortation to leave children behind for the sake of the gods. The command relating to behavior in battle, too (to fall, if one must, with wounds in the chest) probably has a religious background (see n. 49).

It is striking how constantly attention is oriented toward the world of the dead, the heroes, and the δαιμόνες. Earthquake and thunder, the clang of bronze and the ringing in one’s ear, the crumbs that fall from the table—are persons is always being moved and surrounded, even physically, by “stronger” powers. In the saying that the motes in the sunbeam are “souls,” this feeling is expressed in an almost disconcerting manner; wherever a sunbeam falls, it is swarming with souls. Another dictum that hints at “daemonic” forces is that attributed by Aristotle to Philolaus: εναὶ τῶν λόγων κρείττων ἡμῶν. The mallow, the “holiest” thing, is a plant of the dead, the αἰεωφάρον are the roads over which the dead are conveyed to their graves and (this is doubtless also the reason why one may not split wood in the road), and the τρίγλυθος was sacrificed to Hecate. Pythagorean silence also belongs in this context; the Greeks went past ἡμῶν in silence, in order not to disturb the κρείττων. In leaving home on a trip, one must not turn to look

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130 Mylius and Timycha; Neanthes FGrHist 84F31 = Iam. VP 191.
131 The ψωμίδες of beans is often emphasized, as in the joke used by Heniochus, fr. 4 (Ath. 9.409a), ἐναὶ τῶν λόγων ἡμῶν εἰκόνα τῇ μὲν γαστρόκαρπῳ ἄρχει τῇ δὲ πρὸς ὑγιήν. Cf. Hippoc. Vitt. 2.45; the collection of references in Delatte, Faba 54ff., and Boyancé, Muses 111.2. Delatte derives the taboo on beans entirely from their effects on digestion, but with too easy a transition from ψωμίδες to animism. The mythical account cannot be deduced from the physiological facts.—Amphiarous avoided beans for the sake of his art of divination (Geop. 2.358); cf. Hdt. 4.184, Cic. Div. 1.62.
132 Capparelli 1 187 n. 3, II 82ff., with references.
133 Heraclides fr. 41 (the bean in the coffin), Diogenes Antonius (Por. VP 44 = Lydos Mems. 4.42), Hippol. Ref. 1.2.15 (the buried bean blossom), also in a Demotic papyrus (T. Hopfner, Offenbarungszahmer 1 [Leipzig, 1921] 135).—Transformation into human blood: Lucian V. auct. 6. See also Marcovich, cited above, n. 124.
134 Arist. E. 195, Gell. 4.11.10.
135 Por. VP 44, Hippol. Ref. 1.2.14.
137 Boehm, nos. 61-69 (“Praecepta moralius”).
138 Arist. De an. 404a17.
139 EE 122530 = DK 44B16.
141 Boehm on nos. 41-42.
142 No. 42 Boehm.
143 Apollodoros Peri thèwv, FGrHist 244F109 = Ath. 7.325a-b.
II. PYTHAGORAS IN THE EARLIEST TRADITION

back “for the Erinyes are following.”\(^{146}\) The prohibition of bathing may also be related to demonology.\(^{146}\) No wonder that we have anecdotes pointing up how natural it seemed to the Pythagoreans to encounter spirits!\(^{147}\) Aristotle is our witness that the Pythagoreans “used to express great surprise if someone said he had never seen a daimon.”\(^{148}\)

The difference of the levels on which this “thought” operated does not seem to have been recognized; and here we see the continuation of a type of thinking that had already been left behind, in principle, by Anaximander and Anaximenes. Both of these sought to explain the earthquake on the basis of physical principles,\(^{149}\) and thunder as well.\(^{150}\) Recognition of the nature of the rainbow goes back to Anaximenes.\(^{151}\) But, whereas Anaximander asks, “What is the sun?... How big is it?” and answers, “A circle of fire... the same size as the earth,”\(^{152}\) the Pythagoreans ask, “What are the Isles of the Blest?” and are satisfied with the answer, “Sun and moon.”

The intimations of number theory are also part of this picture. “The wisest thing is number,”\(^{153}\) such a sentence can be understood without any esoteric doctrine and without any “Pythagorean mathematics.” More important is the tetractys, regarded as the epitome of Pythagorean wisdom. The Pythagoreans swore by Pythogoras as by “him who brought the tetractys to our generation,”

\[ού μᾶ τὸν ἄμετρα γενέα παραδόντα τετρακτύν, παγάν ἀπάντων φύσεως βίζωσι τ' ἐχουσαν.\] \(^{154}\)

The second verse of this couplet can scarcely be older than Empedocles.\(^{155}\) Possibly the first line stood by itself in the beginning; with its negative formulation, the oath probably applied primarily to the secrecy of Pythagorean doctrine. The τετρακτύς, a “tetrad” made up of unequal members, is a cryptic formula, only comprehensible to the initiated. The word inevitably reminds of τετράκτυς, the “trial” of different sacrificial animals. Is the sacrificial art of the seer, involving the shedding of blood, superseded by a “higher,” bloodless secret?\(^{2156}\)

The acusmata provide a hint toward an explanation: “What is the oracle of Delphi?” “The tetractys; that is, the harmony in which the Sirens sing” (Iam. VP 85). The later tradition is more explicit: The “tetrad” of the numbers 1, 2, 3, and 4, which add up to 10 (the “perfect triangle”), contains within itself at the same time the harmonic ratios of fourth, fifth, and octave. The Sirens produce the music of the spheres, the whole universe is harmony and number, ἄριθμος δὲ τε πάντως ἐπωνυμία.\(^{157}\) The tetractys has within it the secret of the world; and in this manner we can also understand the connection with Delphi, the seat of the highest and most secret wisdom.\(^{158}\) Perhaps Pythagorean speculation touched upon that focal point, or embodiment, of Delphic wisdom, the bronze tripod of Apollo. Later sources speak of its mysterious ringing,\(^{159}\) which must have been “daemonic” for Pythagoreans.\(^{160}\)

The meaning of the shibboleth or symbolon “tetractys” can only be explained in a tentative fashion. In place of that which was connected with it from the beginning, in the form of belief or experience, the later sources give us more and more rationalizations. Some way or

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\(^{145}\) Above, n. 58.

\(^{146}\) Abstention from baths and from sexual intercourse is important not only in cult but also in magic; see A. Aeb, Die Apologie des Apuleius von Madaura und die antike Zauberei (Giesen, 1908) 111ff, 114 nn. 5-6; C. Bonner, “Demons of the Bath,” in Studies Presented to F. L. Griffith (London, 1932) 203–208.


\(^{148}\) Fr. 193 = Apul. De deo Socr. 20.167.

\(^{149}\) Anaximander A 28, Anaximenes A 778, A 21.

\(^{150}\) Anaximander A 23, Anaximenes A 17.

\(^{151}\) A 28, A 18; above, n. 30.

\(^{152}\) Anaximander A 11, 21.

\(^{153}\) Cf. above, n. 22; Arch. Prom. 459, Trag. adelph. 470 N, Gorg. Pal. 30; also below, ch. VI 1; VI 3.

\(^{154}\) References above, ch. I 3, n. 120; Delatte, Litt. 249ff.

\(^{155}\) Kranz, Philologus 1918, 438; ἰδιώματα 166,1, παραδόθη 114,10. The general idea of φύσας is scarcely likely to have existed before the second half of the 5th century.

\(^{156}\) Cf. Iam. VP 93, 147: Pythagoras taught Abaris to use a bloodless form of divination, with numbers.

\(^{157}\) Connected in the tradition with the oath by the tetractys; above, ch. I 3, n. 126.

\(^{158}\) Boyancé, A. 1951, 421ff, cites the theory of the Delphians according to Plut. Quast. Quom. 745a (cf. De fato 368e), which connects the three Muses with three heavenly realms. Cf. Heinz 76ff, Dörrie Hermes 1954, 336ff.— Delatte, Litt. 260; refers to the Delphic Κηληδόνες, who were brought into connection with the Sirens (Pl. Phaed. 871; Snell, Hermes 90 [1962] 46, Ath. 7.290e, Paus. 10.3.12).


\(^{159}\) Eustath. p. 1067, 59: καὶ οἱ Πυθαγορακοὶ φασὶ τῶν χαλκῶν των συνήχει τιθέμενα πεζόμενα δι' καὶ τῶν Ἀπόλλων τίτασκεν ταυτάκειος ἀνώτατα. Cf. above, n. 14. For the ringing of the tripods at Dodona, see Demon FGrHist 327F20. Hach. sv. Ρέροφ δ ἐπί τῶν Πυθαγορακῶν ἐν δήλωμι τίτασκεν (cf. Por. VP 16) shows that Pythagoreans were concerned with the Delphic tripod. The statement of Aristoxenus (fr. 13) that Pythagoras got most of his doctrines from Themistocles the Pythagorean priestess, could well be a rationalizing interpretation of the connection suggested by the acusmata about the tetractys.

For speculation about the relation of the tripod to the number 3, see Plut. De 387c, Lobeck 386f.
other the secret of the world is to be found in number, but there is
danger in too much conjectural reconstruction. If this is a starting
point for Pythagorean science, the question remains, how far Pytha-
goras went in this direction. The pronouncement that number is
"the wisest thing" may be made on no other basis than naive wonder-
ment at its versatile usefulness and its indefeasible correctness, with
no foundation of sophisticated number theory or mathematical
philosophy. It does not take more than the amusement of an idle hour
to discover and establish that \(1 + 2 + 3 + 4 = 10\). Numerical relationships
in the cosmic order are to be found in primitive and mythical thought,
and in the same realm we can find the notion of the cosmic origin
and function of music, and it is not necessary to assume special astro-
nomical knowledge. The evidence about the tetractys, about num-
bers, and about music, is not necessarily on a different level from that
about other acusmata, such as that an earthquake is a rally of the dead,
that the rainbow is a reflection of the sun, or that the sun and moon are
the Isles of the Blest. To what extent the seeds thus planted had already
been developed in the direction of rational science and knowledge, is
something which the history of the exact sciences must try to
clarify.

The question of the date of this Pythagorean wisdom, thus far
postponed, cannot be answered in a completely satisfactory manner.
The latest terminus for the form of those that are sayings or precepts
without explanation is the allegorical treatment of Anaximander the
younger, about 400 B.C., and for the existence of a sizable, varied
collection it is the reports of Aristotle; but in the other direction there
can scarcely be any limit. In reckoning probabilities one must bear in
mind that in oral tradition such a loose aggregation of adages and
maxims is liable to constant change; some are lost, but others are
bound to take their place, especially when there begins to be a
certain competition to know "as many as possible" of them (Iam.
VP 82), and by the rule of ἀντικείμενον to ἐφανερώσαντο all are attributed to the Master.
Still we may be confident, in the light of the zeal to maintain the
discipline of the Master himself, that at least some original matter is
faithfully transmitted. It is like a gravel pile; there is no pebble of
which we can say that it must be primitive rock, but any single one may
be.

We can get a little beyond this non liquet by recognizing that not
only the content, in certain details, but certainly the form, as a whole,
is older than Pythagoras. There are taboo-precepts in all primitive
cultures; and the aphoristic formulation is attested for the Seven Sages.
Individual rules, and even the number symbolism are attested for an earlier period. And it is
unthinkable that ritual prohibitions like those of beans, heart, and
baths, widespread as they are, and variously modified, can have had
their origin in the doctrine of the historical Pythagoras. As befits the
role of hierophant which Pythagoras played according to the legend,
Pythagoreanism is attached to preexisting Greek cults.

The idea occasionally expressed, that the acusmata were simplistic
back formations from a developed, scientific doctrine of Pythagoras, is
thus in principle refuted. It is true that the history of human thought
shows relapses from physical science to magic, but vestiges of
science tend to be preserved. Insofar as Pythagoreanism agrees with
more ancient material in general Greek cults, it is only by petitio
principii that we could imagine a double, self-canceling development
—forward to science and back again to the starting point.

The only remaining question would be, then, whether the acusmata
can have worked their way into the tradition from outside, at a late
stage in its development. It is not impossible that this may be the case
with a few individual items, but for the entire collection it is utterly
improbable. There must have been a point of crystallization for any
possible secondary additions. In fact, even the later "mathematical"
Pythagoreans conceded that the acusmata came from Pythagoras. Thus the modern student too may, or rather must, conclude that a
nucleus of the collection goes back to Pythagoras and that the doctrine
of the historical Pythagoras was presented on the level of the acusmata
and passed on in this form.

For the pronouncements of the acusmata fit in with what we can
learn from the Pythagoras legend: the same universe of gods, daemons,
and souls interpenetrates the universe of man; we hear of Pythagoras
as divine; we are reminded of metempsychosis; and all of this has
its source in a "wisdom" that comprehends equally and without

161 Above, nn. 57, 84.
162 Below, ch. VI 3.
163 Providing that Anaximenes (A7, 18) is earlier than Pythagoras, as in the generally
accepted chronology.
164 Kri Schcke 33ff, and more recently Maddalena 362 (above, n. 82).
165 For example, astrology can be interpreted as a retrogressive development of Hellenis-
tic science (see Nilsson II p. 268). But it preserved, from its scientific background, the order
of the planets, the spherical shape of the earth, and advanced methods of calculation.
166 Below, ch. II 5; ἑωναί (sc. Pythagoras), Arist. fr. 196.
II. PYTHAGORAS IN THE EARLIEST TRADITION

differentiation the divine and the earthly, the rational and the religious—the lore of one who "knows more" than ordinary men.

Even if Pythagoras was adapting older material, there was naturally, in the process of choice, a certain λόγος, a kind of "reflection;" but this does not imply a rationally constructed system. Decisions may be taken in quite different strata of the psyche, as to what is appealing, or illuminating, or obvious. It is questionable to what extent one is justified in speaking of a "purification" or a "spiritualization" of religion. There is, in the form and function of the acusmata, a forward step; but this is on a different level.

The ritual prescriptions which made their way into the acusmata serve for unusual occasions. Their demands apply during the initiation ceremony, or incubation, or a journey to the underworld, or for the period of convalescence; afterward, one can live as he did before. The rhythmic alternation of holiday and nonholiday, of ἔρατος and ἡμέρας, is a mark of all primitive, naive religion. In such religion opposites dwell peaceably side by side—εὐθεία and αἰκατερινεία, chastity and license, earnest and jest, each in its proper time. The "sacred" animal is sacred just because one day it will be slaughtered and eaten. Vegetarianism and omophagia are not mutually exclusive. These rhythms respond to basic needs of the human psyche, and of society. But, when reason begins to reflect on man's beliefs and behavior and tries to remove discrepancies and to arrive at general, objective theses about gods and their laws, the ancient way of living must be reformed; whatever gods prescribe must be valid at any time.

If Pythagoras himself was a kind of hierophant, he found no successor; the Pythagoreans were left with their acusmata applying no longer to festivals but to normal life, which, as a consequence, seemed to others abnormal. Prohibitions like those of beans, heart, certain fishes, and baths are now absolute and must be observed at all times; and the Pythagorean always wears white clothing. He lives every day of his life as though he were preparing for initiation at Eleusis, for incubation at Asclepius' temple, or for the journey to Trophonius. He follows not the cult rules of a certain holy site, but those of a θεός which he has personally and consciously chosen. To be sure, as is clear from the example of vegetarianism, an open breach with ritual piety is avoided.

As metempsychosis changed from ritual and myth to a doctrine with a claim to truth, so here, ritual bound to certain conditions changed into unconditional, permanent rules of life. In both cases Orphism, or the Ορφείκης βίος, bears an embarrassing resemblance to Pythagoreanism. In Orphism, however, according to the testimony of Plato, the older practice of individual, magic rites did not die out; he makes the complaint against the Orphic ἀγίατα καὶ μάναι which they promised individuals and whole cities expiation for their sins, at the cost of a little sacrifice and a pleasant dinner. One can make use of the Ορφείκη without being an Orphic, but he who follows Pythagoras becomes a Πνευμονικός.

To take the acusmata seriously means an almost frightening constriction of one's freedom of action in daily life. Whether a Pythagorean gets up or goes to bed, puts on his shoes or cuts his nails, stirs the fire, puts on the pot, or eats, he always has a commandment to heed. He is always on trial and always in danger of doing something wrong. No more carefree irresponsibility! Everything he does is done consciously, almost anxiously. The mythical expression of this attitude to life is a world full of souls and daemons, which affect every moment of a person's life. Everywhere are rules, regulations, and an ascetic zeal for discipline; life is πόνος, which must be endured.

In his discussion of Greek shamanism, Dodds uses the word "puritanism," and by it he means the strongly felt tension between bodily needs and those of the soul, which is to be freed from the body. If our analysis is correct, the comparison with historical Puritanism can be seen in a still broader perspective. The Puritan and Pictist movements emerged as reactions against a Christianity relying on "primitive" religious forms, in which ritual and daily life, Church and world, holiday and workday ran along parallel. Their aim was to make the whole of life a service of God; every day was to be lived like Good Friday. The dangerous area of arbitrary human choice and of

147 Zeller I 411: "(We shall scarcely go wrong in believing) ... that he transformed the Dionysiac mysteries in the spirit of a more advanced moral teaching, and made them into an auxiliary of it." Perhaps the acusma about salt can be interpreted in this way (above, n. 52).
148 Later tradition constructed a list of "scholarchis," but there is not even agreement among the sources on the immediate successors of Pythagoras. Cf. ch. II 2, n. 33.
149 Cf. Pl. Rep. 600b. If Iam. VP 96-100 is mainly from Aristoxenus, it provides a very early report of a communal society of the monastic type.
150 Rep. 364b-c.
151 It. 139(6). A comparison has also been drawn, occasionally, between the Pythagorean life and that of Calvin's Geneva. On "täglicher Gottesdienst" see Nilsson HP 381 ff. H. Gomperz (PhSt 50ff) has good remarks on the "acusmatic" life.
II. PYTHAGORAS IN THE EARLIEST TRADITION

carefree joy in living was narrowed as much as possible. The ecclesiastical hierarchy was replaced by the democratic conventicle.

Thus the acusmata go beyond ritual piety, not in their content but in the way in which they regulate man’s life, and foreshadow a later development in Greek ethics, much in the way that the doctrine of metempsychosis foreshadows a later ontology. The significance of Pythagoras is to be sought not in the realm of philosophy proper but in the approaches to it, in his position as an intermediary between old and new.

§. ACUSMATICI AND MATHEMATICI

The tradition has contradictory reports about sects, hierarchical gradations, and schisms among the Pythagoreans. Very often a distinction is made between a lower and a higher degree of Pythagorean wisdom, and this goes back at least as far as Timaeus. He tells of a five-year period of probation, during which the new disciple must listen in silence, and did not even see Pythagoras face to face; the voice of the sage became from behind a curtain (σωκάς). Only after this period did the pupil become ἐσωτερικός. The distinction of Πυθαγόρασται and Πυθαγόρευοι is joined to that of “exoteric” and “esoteric,” and there is also a distinction between ἀκομαστικοῦ and μαθηματικοῦ. The mathematici are the “genuine,” or truly “philosophizing” members, whose goal is ἀκρόβεία, so that they correspond to the “esoteric” Pythagoreans. The position of the πολιτικὸς is confused; sometimes they are reckoned among the acusmatics, sometimes among those of the highest grade. There are some apparently artificial divisions into three groups, which seem likely to be secondary. The main point is always the existence of differences of rank or grade, but they are sometimes seen as steps in a continuous development and sometimes as names of separate groups. In this context the word ἀκομαστικός remains a puzzle. If the reference is simply to the probationary period of “hearing,” one would expect ἀκομαστικοῖ or ἀκροατικοῖ. What acusmata meant to the Pythagoreans we are not told in this connection.

Quite different is a notice that Lamblichus copied twice from the same source, without introducing a serious contradiction on one point. There were, he says, two kinds of Pythagorean philosophy, as there were also two kinds of Pythagoreans, ἀκομαστικοὶ and μαθηματικοὶ. In what follows, the text of the De communi mathematica scientia has preserved the original version.


8 Iam. VP 81, 87–89. (The account of the acusmata, which comes mainly from Aristotle, comes between these two passages, from 82–87, i.e. pp. 47.5–51.12 Deubner. On this, see above, ch. II.4. How mechanically the insertion is made is shown by the word τούτων, p. 51.13, which has as antecedent the ἀκομαστικοῖ named at pp. 46.25 and 47.1. The scholia-and-paste procedure is surely Lamblichus’ own.) This passage is nearly the same as Comm. math. sc. 25, pp. 76.16–78.8. The latter passage was written later, and has the error ἐξεχώρων for ἐπισταγμών at p. 77.20. It was not, however, copied from the Βία pythagorica, for it continues without a break, whereas the VP shows arbitrary alterations.

To set the crucial sections parallel to each other: Comm. math. sc. p. 76.19ff. τούτων δὲ οἱ μὲν ἀκομαστικοὶ ἄμολογοι Πυθαγόρευοι εἶναι ὑπὸ τῶν ἅτομων, τοὺς δὲ ἄκομαστικοὺς σύντομοι ὑπὸ ἀμολογοῦσιν, οὗτοι τὴν πραγματείαν αὐτῶν εἶναι Πυθαγόρευοι, ἀλλʼ ἔπισταγμον τούς τοὺς ἔπιστανοι οἱ μὲν Κριστοφάνην φασὶν, οἱ δὲ Μεταμορφίαν… Comm. math. sc. p. 71.12ff. οὐ δὲ τα μάθημα τῶν Πυθαγόρεων τούτων τοὺς τοῖς ἄμολογοι εἶναι Πυθαγόρευοι, καὶ αὐτοὶ φασὶν ἑτέρον μᾶλλον, καὶ τοῖς περὶ τα άμολογους αὐτοὶ ἄλληθεν εἶναι… Nothing can be changed in VP 87, because in the following passage the interpretation of the mathematici is given. Nauck and Höll (5) proposed simply extending 81 after the Comm. math. sc. passage, but this is impossible. See below, n. 9 (DK I 107 n.; Deubner, SBBh 1935, 620): Lamblichus’ source is correctly reproduced in Comm. math. sc. and this is the only one with historical value. The acusmata Hippasus is a conjecture of Lamblichus. (See further Delatte, Litt. 272ff; Frank, Logos 9 [1920–1921] 246 n. 1; Rey 228 n. 1; von Fritz, SBBh 1960, 21; Timpanaro Cardini has it wrong, 80.) The text used in DK should therefore be that of the Comm. math. sc.
II. PYTHAGORAS IN THE EARLIEST TRADITION

Of these, the acusmatici are recognized by the others as Pythagoreans, but they do not recognize the mathematici, saying that their philosophic activity stems not from Pythagoras but from Hippasus . . . But those of the Pythagoreans whose concern is with the μαθήματα recognize that the others are Pythagoreans, and say that they themselves are even more so, and that what they say is true.

According to this, the acusmatici are incontestably Pythagoreans, who refuse to recognize the mathematici, who, they say, are really followers of the innovator Hippasus. On the other hand, the mathematici maintain that they are the true successors of Pythagoras, “even more” than the acusmatici, and that the alleged innovation of Hippasus was nothing more than a plagiarism of doctrine taught much earlier by Pythagoras; “everything is due to ‘that man’”—Pythagoras.

In his De vita Pythagorica, Iamblichus seeks to maintain just the opposite of this: that the mathematici are the uncontestable Pythagoreans, who refuse to recognize the acusmatici on the ground that they are a sect founded by Hippasus. Thus Hippasus is an acusmaticus in this version, and in the other a mathematicus. But this is not maintained consistently in the Vita Pythagorica; after a digression on the acusmata follows the other version, agreeing with De communi mathematica scientia. The latter, which is consistent and complete in itself, is thus shown to be primary. The cause of the discrepancy is not a slip of the pen, for in two other passages Iamblichus calls Hippasus an acusmaticus.9 Actually, the account designed for the Vita Pythagorica is the only one conceivable to Iamblichus. For him the μαθήματα, the Wissenschaft, the incontrovertible proofs—so incontrovertible that it is not worthwhile to think them through again—10 belong irrevocably to the doctrine of Pythagoras. It seemed to him unthinkable that anyone could contest this, to say nothing of those doubters being acknowledged by their opponents as genuine Pythagoreans. Iamblichus knows the tradition that made the acusmatici the lower class, the “spurious,” the “many” who are not true philosophers. Here he can only believe that his eyes have deceived him, and quickly switch the two nouns.11

Pythagoras, they say, came from Ionia and Samos at the time of the tyranny of Polycrates, when the civilization of Italy was flourishing, and the first men in the cities became his trusted associates. The older of these he addressed in simple style, since they had little leisure, being occupied with political affairs, and he saw that it was difficult to speak to them in terms of μαθήματα and proofs. He thought they would be better off for knowing how to act, even without knowing the reasons, just as persons under medical care get well even though they are not told the reason for every detail of their treatment. The younger men, however, who had time to put in the effort of learning, he addressed with proofs and μαθήματα. They themselves, then, the mathematici, are the successors of the latter group, and the acusmatici of the former.

Hippasus, they said, only published, for his own aggrandizement, things that Pythagoras had taught long before.

When we approach the problem of the source, and therefore also the value, of this report, we are struck immediately with the impartiality with which these two contradictory versions are presented; the author does not start with a preconceived answer. In each of the other reports of lower and higher levels of Pythagorean wisdom, only one side is given; they follow the version of the mathematici, who distinguish themselves, as “more genuine” Pythagoreans, from others who

9 Iam. In Nic. 10.20 (DK 18.11), Iam. De anima ap. Stob. 1.49.32.Syrianus derives his material from Iamblichus (Met. 123.7ff, 142.21ff, with the same apocryphal citation of Hippasus). Thus Iamblichus is the only source for the tradition of Hippasus as an acusmaticus.
10 Cf. Iam. VP 157: τὰ γραφήτα ὑπὸ τῶν Πυθαγορείων ἐπιστήμην, περὶ πάνων ἐξωτικὰ τὰς ἀλλήλας . . . καὶ πραγμάτων ἐγχώριαν καὶ ἀνακαλυπτών ὡς δίδακται μετὰ μετὰ ἀποδείξεως ἐπιστημονικῆς καὶ πλήρους, τὰ λεγόμενα, συλλογούμενα . . .
11 At Iam. VP 187ff, in the version of the mathematici, this exchange produces an impossible result, and Iamblichus should have dropped the whole text. But it suited his predilections so well (cf. VP 80, 90ff) that he could not let it go.
II. PYTHAGORAS IN THE EARLIEST TRADITION

proffect a simpler form of Pythagorean doctrine. From the time of Timaeus, the account of the mathematici holds the field, and all that remains of the other side is the unexplained term acusmatici. Iamblichus' account of the schism, which gives the other side a chance to be heard along with, and even before the mathematici, cannot possibly be derived from the one-sided version that prevailed from Timaeus' day, and therefore must be, in essentials, earlier than Timaeus.

Now the term acusmatica is only found in the exposition of Iamblichus, which goes back to Aristotle, while the non-Aristotelian tradition uses the word symbola. This suggests that the report of the schism also stems from Aristotle, and that Iamblichus has, in a way quite consistent with his usual compilatory method of writing, artificially rearranged two excerpts that come, ultimately, from the same source. This conjecture is strongly corroborated by the introductory sentence in De communi mathematica scientia: διὸ δὲ ἐστὶ τῆς Ἰταλώνης φιλοσοφίας ἐδώ καλομείγνης δὲ Πυθαγορικῆς (p. 76.16f). This corresponds very closely, without being a direct copy, to expressions in Aristotle, though elsewhere, understandably enough, Iamblichus does not use the expression καλομείγνης Πυθαγορικῆς. In addition, the language of the passage that follows in De communi mathematica scientia shows close kinship, without being a quotation, with expressions in the Metaphysics. Thus we have a piece of evidence traceable to Aristotle, which in its very wording has preserved marks of its origin.

Iamblichus' account of the division among the Pythagoreans, unique in content, takes on tremendous significance because of the authority of its source. In fact, the modern controversies over Pythagoras and Pythagoreanism are basically nothing more than the continuation of the ancient quarrel between acusmatici and mathematici. Is there nothing more in the doctrine of Pythagoras than what is indicated by the acusmatica, with which the Pythagoras legend and the theory of metempsychosis are of course closely connected? Or was there from the beginning, behind these religious and mythical features, whose existence cannot be denied by the modern scholar any more than it could by the mathematici, a new, scientific approach to philosophy, mathematics, and the study of the world's nature?

Before pronouncing on this question, we must discuss the report in the light of other testimony from fourth-century sources, so as to confirm its Aristotelian origin and at the same time get from reports to historical facts. Aristotle recognizes among the Pythagoreans a twofold πραγματεία: on the one hand Πυθαγορικοῖ μνήμονα, metempsychosis, the Pythagoras legend, and the acusmatica, and on the other a philosophy of number connected with mathematics, astronomy, and music, which he never tries to trace back to Pythagoras himself and whose chronology he leaves in abeyance. In this he dissociates himself from the Platonists, who attribute to Pythagoras himself a more sophisticated version of the Pythagorean number theory. Plato and his pupils thus stand in the tradition of the mathematici, and it is not surprising that their version carried the day. The doxographical tradition, too, was dominated by the Platonic interpretation of Pythagoreanism.

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12 Notice the expression ἄρτι μᾶθειν and the emphasis on ἄποδειξει αὐτῷ at Iam. VP 87, taken along with the expressions collected in no. 2-6 above.
13 This is perceived by Hölk sf.
14 This solution is favored by Delatte, Litt. 273ff, though he too readily takes the passage on the acusmatha as a unity with the surrounding material about the division of the society (cf. ch. II 4). Bertramann assigned the whole to Androcycles, because in his superficial way he failed completely to notice the difference between the conception of the acusmatha held by Aristotle from that of Androcycles.
15 Mete. 342b30: τῶν Ἰταλώνων μνήμων καὶ καλομείγνων Πυθαγορικῶν; Calcl. 293b20: οἱ περὶ τῆς Ἰταλῶν, καλομείγνων δὲ Πυθαγορικῶν. The adjective Πυθαγορικὸς is found at De an. 407b2a.
16 Above, ch. I 2, n. 112.
17 It is hard to say whether some material from other sources may be interspersed with the Aristotelian. It is perhaps doubtful whether Iam. VP 86ff (pp. 50.18-51.6, and 51.7-12) or VP 89 = Comm. math. sc. pp. 77.14-78.5 (discussed below in ch. VI 3) come from Aristotle. The concluding sentence, Comm. math. sc. p. 78.6, is Aristotelian in its phrasing (περὶ μὲν οὖν τῆς διαφοράς ἐκκαθάρισε τῆς πραγμάτειας καὶ περὶ τῶν μαθημάτων σχεδόν ταύτα τι καὶ τωάτα ἦσσι τὰ συμβαρύνοντα).
18 The account of the division into groups is taken seriously by Rohde, Q 107ff; Hölk 5f; Delatte, Litt. 273ff, 275ff, Pol. 275ff; Jaeger, SBBh 1928, 416 = "On the Origin and Development of the Philosophic Ideal of Life," tr. R. Robinson, in Aristotle, 2nd Eng. ed. (Oxford, 1948), 455ff; Frank 69ff; Burnet, EGP 94; Minar 34ff; Guthrie 192f; and others. On the other hand, Zeller (I 411 n. 1) rejected the whole tradition as a late construct. It would not be unthinkable that Aristotle, having observed differences and worked out the idea of a split in a theoretical way, then came to think of this as a historical reality; but the explicit statements as to the extent to which each group recognized the other cannot be accounted for in this way.
19 Frank finds the exposition of the mathematici simply ustinius. Speaking for the acusmatici as the original group are Rohde, Q 107ff, Hölk 4, (with reserve) Delatte, Litt. 311ff; on the other side, among others, is Jaeger (Paideia 1 221 = Eng. ed. l 162). Even von Fritz (SBBh 1966, 14, 18ff) is overcome by the weight of the later tradition and seeks to weaken the force of the report. The acusmatha, he thinks, were only concerned to reject certain developments of doctrine, whereas the mathematici wished to emphasize the "truth" of their findings and did not attribute these to Pythagoras. But the words of the text are unambiguous: εἶπε δὲ πάντα ἀκούειν τοῦ ἀδρός (Iam. VP 88). Guthrie seems to forget the claim of the acusmatici (I 192).
20 Cf. above, ch. I 2, 3, and 4.
We can identify certain Pythagoreans who may be classed among the *mathematici*: first of all Archytas with his pupils and also some of his predecessors, among whom he himself mentions the name of Eurytus; doubtless also Simmias and Cebes, the "auditors" of Philolaus; and finally Echecrates of Phlius, with his friends. It is this same circle to whom Aristoxenus refers:

> For the last of the Pythagoreans, whom Aristoxenus himself saw, were Xenophilus of Chalcidice in Thrace, Phanto of Phlius, and Echecrates, Diocles, and Polymnestus, also Phliasians. They were pupils of Phylla and Eurytus of Tarentum.

Aristoxenus named Archytas as a friend of his father Spintharus. Whatever its basis may have been, Aristoxenus' assertion that Pythagoras was especially fond of beans obviously has a polemical point; and it is now clear that it is aimed at *acusmatici*. Equally clearly, his statements about Pythagoras eating meat are to be explained along the same lines; and perhaps the main purpose of the *Ποθωγορική ἀποφάσεως* was to drive out of currency the enigmatic or ridiculous *acusma*.

When we start looking for *acusmatici*, we think first of the *Ποθωγορική* who appear in leading or secondary roles in the Middle Comedy. It is repeated again and again that they eat *οὐδὲν ἔμφυχον*, not even the meat of sacrificial animals, only a lot of wretched vegetables. Sometimes they scarcely eat anything at all; they drink plain water, attract attention with their silence and their *στοιχεῖα*, they wear a ragged *τρίβλα*, go about barefoot, and are stiff with dirt since it is against their principles to bathe. Shabbiness turns into arrogance; they are typical ἄλαζόνες.

The comic poets mock these mendicant Pythagoreans as living so ascetically because this was the best they could do; but a fragment of Aristophanes reveals another motive: "He said that he had gone down to visit those below in their daily life, and he had seen all of them, and that the *Ποθωγορική* had far the best lot among the dead. For Pluto dined with them alone, because of their piety." The interlocutor then remarks that Pluto must be a very easy-going god, to associate with such dirty vagabonds; but in spite of the element of caricature, the goal of Pythagorean asceticism is clearly apparent: a favored life in the next world and intimate acquaintance with the gods. The reference to the "thrice blessed" Pythagorists, in Antiphanes, shows that the "Pythagorists" were also concerned with metempsychosis.

These Pythagoreans are placed in the south Italian scene by the very title, *Tarentini*. In a play of Alexis, who himself came from Thurii, the happy-go-lucky life of Athens is described to one of the characters; obviously the play contrasted the sober habits Pythagorean and the easy-going Athenian. At the time of the play's performance—

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22 Above, ch. 14, n. 40. A woman from Phlius belonged to the Academy (D.L. 4.2).
23 Aristox. fr. 19 = D.L. 8.46; the same names are given in fr. 18 = Iam. VP 251.
24 Above, ch. II 1, n. 50.
25 Above, ch. II 4. Aristoxenus gives the same historical account as the *mathematici*: Pythagoras comes to Italy in the time of Polycrates (fr. 16), and acquires influence over the political leaders of the city (frs. 43, 17, 18).
26 Cf. Zeller Ill 293f, Weihcr 55-68, Mecaits 10ff. The most important texts, from D.L. 8.37 and Ath. 4.166f, are printed in DK 58f. The fragments in question are the following (Kock's numbers, which are followed by Edmonds): Antiphanes Corpus fr. 135 = Ath. 4.161a, Mnemata fr. 160 = Ath. 4.161a, Neetis fr. 168 = Ath. 4.168-1 (above, ch. II 3, n. 106), inc. fáb. fr. 226 = Ath. 2.60f; Mnesarchus Almenon fr. 1 = D.L. 8.37; Alexis Pythagorica fr. 196 = Ath. 4.161c-d; fr. 198 = Ath. 3.122f; fr. 199 = Gell. 13.8, Tarentini fr. 239 = Ath. 11.1463d-e (see Weihcr 666f), frs. 220-221 = Ath. 4.166b-c, fr. 222 = Ath. 4.1342-a; Aristophanes Pythogorics fr. 9 = Ath. 4.161a, fr. 10 = Ath. 6.238c-d (see Mecaits 12ff), frs. 12-13 = D.L. 8.38; Cratinus the younger Pythagorica fr. 6 = D.L. 8.37, Tarentini fr. 7 = D.L. 8.37. There is no way of knowing whether Naevius' Tarentilla belongs in this context. Delate conjectures (Weihcr 241) that the plays of Alexis and Cratinus entitled Tarentinn and those entitled Pythagorica were identical, in each case, and that their authorship was contested; but here again there is no basis for a firm answer. The difference in the appellations *Ποθωγορική* (Aristophanes: title and frs. 9, 12), *Ποθωγορίκη* (Antiphanes fr. 165), *Ποθωγορίκη* (Alexis fr. 196), and *Ποθωγορίκη* (Alexis fr. 220) is not a matter of technical terminology (cf. Weihcr 56). The dietary regulations are in any case based on the authority of Pythagoras himself (Antiphanes fr. 168).
probably in the middle third of the fourth century, "Pythagorists" must have been a familiar sight in Athens.

Clearly, Aristoxenus' veiled polemic has the purpose of destroying this image of Pythagoreans as a group of low-class tatterdemalions, addicted to superstitious abstinences. There is special point in his insistence that the Pythagoreans he knew personally were "the last";

"They held to their original way of life (δύνατη), and their science... until, not ignobly, they died out" (fr. 18 = Iam. VP 251). Nothing about abstinence from beans or meat—only μαθήματα; nothing about poverty, dirt, or lice—only noblesse, to the very end.42 Diodorus, in a passage certainly based on Aristoxenus, places these "last" Pythagoreans about 366/365 B.C.,43 but the comic allusions would take us a few decades later; and there was a Pythagorean Lycon who wrote after Aristotle.44 This Aristoxenus can ignore; such persons, he implies, are not really Pythagoreans.

All the same, it is not so easy to equate the Pythagoreans of Aristoxenus with the mathematici and those of comedy with the acusmatici. According to Aristotle's report, the mathematici recognize the Pythagoreanism of the acusmatici, whereas Aristoxenus ignores conceptions deviant from his own. The acusmata themselves do not enjoin complete vegetarianism, but complicated rules of dietary detail; but in the comedies absolute abstinence from meat seems to be prescribed. In addition, there is an occasional mention of rhetorical, dialectical writings, or speeches, as though they were regular Sophists.45

Neither Aristoxenus nor Middle Comedy provides a wholly undistorted reflection of a particular type of Pythagoreanism. The former is contradicted by Eudoxus, who, as a personal pupil of Archytas and himself one of the most brilliant Greek mathematicians, must have been as close as anyone to the tradition of the mathematici,

and who states that Pythagoras abstained from ἐφύσει and avoided association with hunters and butchers.46 He recognizes, or perhaps rather exaggerates, the abstinences. Thus Aristoxenus appears to be dissociating himself from Eudoxus and the mathematici, insofar as he denounces the abstinences and does his best to present Pythagoras' way of life as normal.

The comedies, on the other hand, make everything cruder than it was. At least one hint—not conclusive, to be sure—suggests that more precise regulations were mentioned, in the matter of abstinence from meat.47 The main point here is, though, that comedy is not the place for hairline distinctions. Its effect comes from mixing up the incompatible. It is tempting to conjecture that there were Pythagoreans, in the first half of the fourth century, emigrating from southern Italy to Athens,48 although there are certain difficulties in this.49 This much is certain, though, that Plato, who had achieved extraordinary fame inside and outside of Athens, and who was also often alluded to in the comedies,50 traveled several times to Sicily and Magna Graecia, and had an especially close relationship with the Pythagoreans of Tarentum. Also, his disciples equated their own philosophy with that of Pythagoras and in various books presented a new picture of the Samian sage.

In this extent, Pythagoras was a live topic of conversation in the Athens of the fourth century, and it seems likely that the mockery of the "Pythagorists" was aimed at the Academy as well. More precisely, when the comic poets identified the loftiest philosophical attempts of the Platonists with the common, mendicant Pythagoreanism, they were ridiculing one group as well as the other.51

The distinction between acusmatici and mathematici, drawn in the reports of the schism, is not directly reflected in the contradictions of...
the pictures of Pythagoreanism drawn by Aristothenes and by the comic poets; but it does seem to be clear that alongside of the scientific Pythagoreanism with which Plato and his pupils allied themselves, and which Aristothenes stubbornly claimed to know best, there was another quite different type of Pythagoreanism whose adherents were mendicant "teetotalers" with special hopes about the next life.

The first evidence for this tendency is the Telages of Aescharines of Sphettus. Dittmar has elicited the fundamental facts about it from the exiguous fragments.58 Telages was depicted as a representative of the Pythagorean life, characterized by a shabby poverty proudly displayed, decked out with a κύβον and beggar's scrip.

We can get a clear picture of Diodorus of Aspendus, as a historical person of the first half of the fourth century B.C.58 A mocking verse by the musician Stratonicus claims that with a "crazy garment of skins" and "arrogant presumption (δίδημος)" he gathered followers.59 Archestratus, in his Ηνωμένα, recommends shark meat as a delicacy, and says that anyone who will not eat the flesh of a creature that itself eats human beings should restrict himself to vegetables, should seek out Diodorus the Sage, and live the ascetic Pythagorean life with him.55 Here Diodorus is the well-known Pythagorean, famous for his vegetarianism. Timaeus mentions the striking clothing he wore, and Socrates describes it more precisely; long hair, long beard, folded τριβόν, wallet and staff, "though the Pythagoreans before him wore shining bright clothes, bathed and anointed themselves, and had their hair cut according to the fashion."56 Athenaeus, without naming a source, refers to him as barefoot and dirty.

This picture of Diodorus is the conventional picture of a Cynic, so

59 The most important testimonia, from Stratonicus, Archestratus, Timaeus, and Socrates, are in Ath. 4.161c; C. Tannery, Msc VII 201-210. Stratonicus died about 350 B.C. (Wilamowitz, Ind. Schol. Göt. 1893/1894, 16.1, Maas, RE IV A 326f). Timaeus cites him (FGrHist 566F16). Archestratus wrote about 330 B.C. (Zeller 426 n. 1) confused the citation of Archestratus with a nearby citation of Timon, and therefore mistakenly dated Diodorus in the 3rd century. W. Crômer, RhM 62 (1907) 311f (dubiously Powell, Coll. Alex. p. 212), suggested attributing the stratifying verses to Cercidas; but he could scarcely have been cited by Timaeus, to say nothing of Archestratus.
60 Timaeus FGrHist 566F16 = Ath. 4.163c-e: τῷ περὶ θρησκείας μανίας δίδημος τε περσάτσµον σταύρον ἔχων Πυθαγόρας πελάτη.
61 Fr. 23.18ff Brandt = Ath. 4.163d-c; διὸν πρέπει καθαρὸς ἢπόσιος τάξις μυρολύγοντος τοῦ λοιχοῦς προσαγεῖ καὶ πρὸς Διδώνων ἅλλην τὸν σφάδα ἔγκυκλον μετὰ οἰκίῶν πυθαγόρειων.
62 Ath. 4.163f and D.L. 6.13. This account of Socrates, who seems to use Aristoxenus (Ch. 3, p. 249), is in turn the basis of Schol. Theor. 14.5 (DK 1478.37f, above, n. 6), on the differentiation between Πυθαγόρας and Πυθαγοραῖος. Long hair is also found in the Pythagorean legend: ἀνεχθόν κυρίτρη, Iam. VP 111, with the parallel passages.

that the usual thing is to dispose of him quickly as one who managed "to represent his Cynicism as Pythagorean philosophy."57 Or we may read that "the ascetic trend caused many Pythagoreans, after the rise of Cynicism, to assume the Cynic garb and way of life."58 Thus Diodorus would be a Pythagorizing Cynic or a Cynizicing Pythagorean—in any case some kind of hybrid. But this ignores the chronology. The testimony of Stratonicus, who died about 350 B.C., shows that we should regard Diodorus as in any case contemporary with, but more probably, earlier than, Diogenes. According to the tradition, Diodorus' rivalry for the honor of having invented the Cynic costume was not Diogenes but Antisthenes.59 If, as modern scholars rightly believe,60 Antisthenes was not yet really a cynic, then Diodorus' garb was not Cynic either; rather, the Cynicism of Diogenes is in a way a continuation of Pythagoreanism of the "acustatic" stripe. There are unmistakable coincidences—the praise of πῶς and contempt for ἐνδομην,61 and in general the choice of a special ὅς in contrast with the "normal" hit-or-miss life-style of the ordinary man. Just as the Pythagorean feels himself a stranger on the earth, so the Cynic tries to free himself from all ties. Threads lead from here to the Stoas as well; both Zeno and Chrysippus were interested in aspects of Pythagoreanism.62

But Timaeus emphasizes that Diodorus "pretended" to have associated with Pythagoreans, and Socrates declares that the appearance of Diodorus was a novelty and that before him the Pythagoreans had lived a more normal life. Finally, there is a statement in Iamblichus, probably derived from Timaeus,63 that after the catastrophe of the school Diodorus of Aspendus was taken into the Pythagorean society "because of the need for members," and that, returning to Greece, he published the "Pythagorean sayings"—obviously meaning the acustatas.64 The tendency of these reports is obviously to deny real Pythagoreanism to

57 Zeller 1 462 (where Diodorus is wrongly dated; see above, n. 53).
58 Ueberweg-Paechtner 64. Cf. Rostagni, StrMin II 1.41.
59 D.L. 6.13 (Diocles and Neanes name Antisthenes, Socrates names Diodorus of Aspendus; above, n. 56).
61 Above, ch. II 4, n. 14; for the comparison of ψοχή and ἀρμοσία see D.L. 6.27, 65.
62 Zeno wrote Πυθαγορεύει (D.L. 7.4); Chrysippus cites a Pythagorean verse (Gell. 7.2.2; SVF II 204). Cass. Dio, an. 54.
63 Iam. VP 246; cf. Delatte, Musée Bégal 1920, Rostagni, StrMin II 1.41.
64 Delatte (Πυθαγορεύει τοις πυθαγόρειοις φωνῇ; ὑπερὶ φημῆς, "saying," see II. Plut. 341b: τῶν Σιμωνίδου φωνῆς . . . , Epictetus Fr. 1.36.)
Diodorus, though Iamblichus admits that he had been a member of the school. But the two oldest witnesses, Stratonicus and Archestratus, call him a Pythagorean, without qualification. Sosicrates, however, is dependent on Aristoxenus (above, n. 56), and Timaeus is in the same line. Thus what first was generally regarded as Pythagorean in the fourth century is later branded as “alleged” Pythagoreanism, and contrasted with that which is “genuine.”

With Diodorus doubtless belongs Lycon, who criticized Aristotel’s extravagant way of living. His ideal must have been something in the nature of Cynic self-sufficiency. Aristotle introduces him as one “who called himself a Pythagorean,” showing the same sort of reserve about his Pythagoreanism as Timaeus did about that of Diodorus. There is no reason not to identify him with the Lycon of Iasus who wrote on the Pythagorean life, emphasizing the “moderate” regime of Pythagoras. After Lycon, who must have been approximately contemporary with Aristotle, there are no more Pythagoreans of this type to be found. An echo of such activity can still be heard in Onesicritus, who named Pythagoras as one of the Greeks who, before Socrates and Diogenes, had taught doctrines like those of the Indian Gymnosophists. But Onesicritus is regarded as a Cynic. The tendency in Pythagoreanism represented by Diodorus of Aspendus was absorbed by Cynicism, which took shape as the form of the self-sufficient, world-despising bios which suited the demands of the age. Meanwhile the spiritual power of Pythagoreanism found, through the interpretation of the Platonists, a new vehicle adaptable to changing times.

In the fourth century, then, alongside those Pythagoreans with whom the Academy felt akin in their philosophical and scientific endeavors, there were others of a quite different type, exemplified by Diodorus of Aspendus but presupposed also by Aeschines and the comic poets. Their characteristic mark is not μαθήματα but a bios. They remind us of the acusmata (alluded to in the tradition: above, n. 64) by their avoidance of baths and of shoes, and by their vegetarianism, though the evidence, almost entirely satirical and negative, oversimplifies and distorts the picture. Allusions in comedy permit us to attribute to them a belief in metempsychosis and hopes for a better life to come. Aristotle acts as though this kind of Pythagorean did not exist, though his portrait of Pythagoras is specifically intended to correct the impressions they made. From Timaeus on, these “alleged” Pythagoreans, distinguished from “genuine” ones; and from Timaeus on the tradition becomes canonical that the real teaching of Pythagoras was esoteric, as distinguished from the imperfect, exoteric preliminary stage.

Aristoxenus is corrected by Eudoxus in one point, and in a way this corroborates Iamblichus’ account of the division. This report reveals details that were obscured from the time of Aristoxenus and Timaeus, for tendentious reasons. This confirms its Aristotelian origin, and it must be taken seriously as an expression of historical facts. The mathematici, whose successors Plato and his disciples thought of themselves as being, tried to regain the ancient wisdom of Pythagoras by scientific studies of their own, while acusmatici like Diodorus of Aspendus wished only to live a straitlaced life in accordance with the ancient precepts. The mathematici, followed in this by Eudoxus, did not attack the ritual observances taught by Pythagoras, but the acusmatici saw a defection from Pythagoras in the further development of scientific study. But neither tendency could endure except in altered form: “mathematical” Pythagoreanism in the reinterpretation of the Platonists, and the “acusmatic” way of life, rationalized and secularized, in Cynicism. And, since the “mathematical” tradition, in its Platonic metamorphosis, became completely dominant in the literary realm, the contention of the mathematici also won out, that the acusmatici were not genuine, but only imperfect, Pythagoreans.

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66 Ath. 10.418E (DK 57.3): Λύκος ὁ Ἰασεύς εἰς τὸν περὶ Πυθαγόρειον βλέπ. Capelle doubts the identification (RE XIII 230 Ef, s.v. Lycon n. 13). Ism. VP 267 (the catalogue of Pythagoreans) lists a Lycon as Tarentine, D.L. 5.69 knows only one Πυθαγόρειος Λύκος. This Lycon is naturally to be distinguished from the historian Lycon of Rhegium (earlier than Timaeus: FGrHist 570), and Jacoby in Ilb. Komm. 597 ff, also Laquerre, RE s.v. Lykos, no. 50—who, considering his interest in miracles, asceticism, and the like (fr. 5, 6, 7), certainly must have mentioned Pythagoras. —Por. VP 5 cites Λύκος ὁ τῆς Πυθαγορείας ἱστορικ. The emendation Λύκος is tempting, but the second person σοι is not very likely in a work of history. Jacoby classes the passage among the doubtful fragments of Lycon (FGrHist 566 F15) and is inclined to assign it to Lycon (Ilb. Komm. 601). The ιστορικαί cannot be the same as the περὶ Πυθαγόρειον βλέπ, since it only takes up the origin of Pythagoras in the fourth book. There can be no certainty about the attribution in the case of the botanical data in DK 57.5 (Lycon) and 57.2 (where the MSS have lykus; there were also physicians named Lykos, Re s.v., nos. 51 and 52).

67 FGrHist 134 F17 — Strabo 15.716 (Dittrich saw the relevance of this to the “Pythagorean” context, Alciphron 211f). Ιασεύς οὗτος δὲ ἦσαν αὐτὸς among the rules of Atoka: Journ. Asiat. 246 (1958) 1f.

68 On the question of the continuity of the Pythagorean tradition, see Burkert, Philologus 1961. In the Hellenistic period, or, say, in the 3rd and 2nd centuries B.C., there do not seem to have been people calling themselves Πυθαγόρειος, nor is there evidence for the survival of Pythagorean cult, ἔτυμα of the acusmatici, which are commonly assumed to have continued. It does seem, however, that there was a good deal of interest in Pythagoreanism: and this was part of the reason for the apocryphal literature. The Πυθαγόρεια from Athens in Theocritus (14.5), barnes, pale, and hungry, may be a figure from comedy (Wilamowitz, Platon II 84).
II. PYTHAGORAS IN THE EARLIEST TRADITION

Hippasus, who plays a crucial role in the versions of both groups, is mentioned in certain early reports. Aristotle and Theophrastus attribute to him, as well as to Heraclitus, the doctrine that fire is the ἀρχή. Ancient scholars drew a chronological conclusion from the fact that his name comes first, but this is hardly justified. Aristotle credits him with an acoustic experiment and brackets him with Glaucus of Rhegium. Iamblichus names him three times, along with Archytas, as discoverer of the “harmonic mean,” again a connection with music theory. According to the report of the schism, Hippasus claimed to have discovered the dodecahedron and was drowned at sea. He wrote no books. The evidence seems to point toward the first half of the fifth century—including an apocryphal report that he was the teacher of Empedocles. His home city was Metapontum, and there is a slight trace of a connection with Phlius.

Thus Hippasus is the oldest Pythagorean we know of who worked at mathematics and music theory, and also had something to say in the realm of natural philosophy—though, to be sure, not in terms of a theory of number, or of a philosophy of “limit,” “unlimited,” and “harmony.” The independent reports of Aristotle and Aristotheles confirm the account of the division into sects—in the version of De communi mathematica scientia—according to which Hippasus was one of the mathematici. This report shows the position of Hippasus between the fronts which were later formed. For the acusmatici his activity was something new, and subversive; but the mathematici, in order to find a firm basis for new doctrines, were forced to abandon Hippasus and brand him a plagiarist. Since he became in this way a scapegoat for both sides, it was easy for further charges to emerge, such as that he had traduced Pythagoras.

It is impossible to determine whether the breach between Hippasus and the other Pythagoreans came before or after the political catastrophe about 450 B.C.; it is tempting to suppose that there was a connection between the inner and outer crises of Pythagoreanism. But it is a more important question, which of the two opposed conceptions of Pythagoras should be considered correct. The name μαθηματικοῖ, interesting as it is, scarcely helps; we must weigh the

78 According to Apollonius (lam. VP 227f), the name Hippasus was one of those who rebelled against Pythagoreanism. Nonnus produced a λόγος ἱερός, παντοκράτορ καὶ γεγογγός οὗ ἡ μὲν ἀρχή ἡ ἀρχον ἀνάγκης διαβάλλεται (lam. VP 238); and there must be some connection between this and the remark of Heracleides Lembus, in his enumeration of Pythagorean writings, τὸ δὲ Μονακότος λόγος Ἰππακός . . . εἶναι, γεγογγόν καὶ διαβάλλει Πεδαγόρου. Either Heracleides is dependent on the story which later appears in Apollonius, only that Hippasus, the apostate, is substituted for Nonnus; or the narrative is late and there was an actual λόγος ἱερός which was regarded as overly primitive and therefore discreditable, so that it was branded a forgery and a libel and put down to the account of the wicked Hippasus. In this case, however, Hippasus would not be an acusmati (as Timpanaro Cardini thinks), but a mathematici who wished to conceal his plagiarism and characterize Pythagoras as a primitive—as scandalous letters of Epiphanes were forged not by Epiphanes but by a Stoic (D.L. 10.3). Cf. Frank 70.

79 Tannery (Géom. 85f), relying on the account of Apollonius in lam. VP 227f, thinks that the internal division of the school started by Hippasus finally led to civil war and to the final catastrophe. Tannery speculates (MSC VII 209f, in Diodorus of Aspendus) that after the collapse of their political strength the Pythagoreans restricted themselves to their religious and supersticious activities; against this is the fact that the mathematici whom Plato knew were all active after the political catastrophe.

5. Acusmatici and Mathematici

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5. Acusmatici and Mathematici

The word μαθηματικοῖς “concerned with the subjects of learning,” came to have a narrower sense “mathematician” precisely among the Pythagoreans (von Fritz, SMBM 1960, 20f) or only in the Academy (below, ch. VI 1), the history of the word cannot decide whether the mathematici were descended from Pythagoras or Hippasus. Perhaps the terms μαθηματικοῖ and διδασκαλοί do not go back to the original schism, but were only later applied to the rival groups. Formations in -οίς become frequent from Plato on. (A. Amman, -οίς bei Platon, Diss. Fribourg, 1953; the earliest occurrence of μαθηματικοῖς is Pl. Soph. 2140). The primary contrast is between διδασκάλους and μαθηματίς. It was in the educational movement of the 5th century that the word μαθηματίς acquired its specialized sense. Parmenides (frs. 1.13, 8.52) and Empedocles (fr. 17.14) call for μαθηματίς, and Pythagoras wrote Περὶ τῶν μαθημάτων (D.L. 9.33; the sense is “On the Branches of Learning,” and it is a mistake to apply it to mathematics, as in DK 8017; cf. Pl. Prot. 115d and 313d, Lachcs 179c, 182d). In the Clouds the μαθηματικὸς is an established type. In Pythagoreanism, “hearing” plays a prominent part, and this too suggests that διδασκάλος is the older idea (above, ch. II 4, n. 76). If, as time went on, certain Pythagoreans emphasized the importance of μαθηματίς as contrasted with διδασκάλος, they were integrating themselves into the intellectual development of the 5th century: they do not want “hearsay” but the comprehension of truth.
content of the two versions. The mathematici claimed that their studies were no more than explication of the doctrine of Pythagoras, and that "everything came from him." This principle, which seems to have been accepted in the day of Xenocrates, emanates from a completely unhistorical point of view, close to that of myth. What seems important and desirable, takes the form of a quasi-historical assertion: in the beginning, there was Pythagoras.

On the side of the acusmatici is the fact that the acusmata, the ideas of metempsychosis, and the legend of Pythagoras are early attested and primitive in character. The mathematici, in their effort to get free of this annoyance, resorted to the suspicious expedient of secrecy and esoterism.

If, in spite of this, efforts continue to be made to show that the acusmatici were not entirely in the right, that Pythagoras was "not merely" a kind of shaman, and that science too was present in Pythagoreanism from the beginning, at least in embryonic form, at the base of these efforts lies not only the force of the dominant tradition, but also the seemingly ineluctable idea of the law of development, the postulate that the later Pythagoreanism must in some way be foreshadowed in the earlier. Yet one may well ask just how much that is coherent and stable is necessary to explain the development. The question of what is scientific depends more upon form, method, and proof than upon the content or the practical function. May it not be that the conceptual and scientific impulse simply provides a new form for an ancient and pre-scientific lore or attitude? The "wisdom" of the acusmata, the "wisdom" of a shaman-like "divine man" can stand without the prop of science, and did so in the activity of the acusmatici down into the fourth century. Greek science, including Greek mathematics, may well have another and non-Pythagorean origin.

6. EARLY EVIDENCE FOR PYTHAGORAS AS A SCIENTIST?

In discussing Pythagoras scholars have always, with varying degrees of uneasiness, taken cognizance of his doctrine of metempsychosis and the accounts of his miracles. But the principal endeavor of the historians of philosophy, in this area, has been to show that this aspect of the man is not the only one, and not even the most important one; they have looked to find at least the "germ" of the mathematics, natural science, and philosophy that were current, in later times, under the name of Pythagoras. Often a simple "not only—but also" has seemed enough; he was not only a "medicine man" but also a thinker. But may not even a "shaman" perhaps accomplish intellectual feats, without necessarily clothing them in strictly rational or conceptual form? More penetrating are the efforts to discern an inner connection between apparently heterogeneous things.

The point of departure for these must always be the little group of testimonia which, in praise or in irony, speak of the preeminent, comprehensive knowledge of Pythagoras. First comes Heraclitus: πολυμαθής νόμον έχει ου διδάσκει, ήσιοι δέν γάρ άν έδιδαξε και Πυθαγόρην αυτός τε τε Σεσοφάνεια τε και Ἐκατατό (fr. 40). Πυθαγόρης Μνησάρχου ἱστορίην ἠσκεσαν ἀνθρώπων μάλιστα πάντων καὶ ἐκλεξάμενοι ταῖσας συγγραφάς ἐποίησαν ἔτι οὐσίαν, πολυμαθήν, κακοτεχνήν (fr. 129). Then Empedocles (fr. 129):

... ἄνὴρ περίποτα εἶδος,
δὲ δὴ μῆκιν οἱ προσίδων ἐκτισματο πλουτόν,
παντοῖον τε μάλιστα σοφῶν <τε> ἐπίθρανος ἔργων ...

And Ion of Chios:

ἐσερ Πυθαγόρης ἐτύμως σοφός, δὲ περὶ πάντων ἀνθρώπων γνώμας εἶδε καὶ ἐξέμαθεν.

Herodotus tells of Zalmoxis, who was cleverer than the Thracians, ὁ τὰς Ἑλλήνες ὑπάρχουσαν καὶ Ἑλλήνην οὐ τῷ ἀσβεστάτῳ σοφοτήτι Πυθαγόρη (4:95). The key terms are, then, πολυμαθής, ἱστορίης, κακοτεχνής.

From the accusation of polymathy, "we may infer that the later 'so-called Pythagoreans' . . . were right in naming Pythagoras as the founder of their peculiar science." More must be involved than religious revelation, for the name of Pythagoras stands beside that of Xenophanes. Surely, however, the idea of polymathy must apply to all four of the men named in the fragment, and especially Hesiod, who

1 Jaeger asks (Paideia I 221 — 1 162 Eng. ed.), "But what connexion has all this [number theory, geometry, music theory, and astronomy] with the doctrine of transmigration . . . ?"

2 Cf. ch. II 3, n. 13.

3 Jaeger, Paideia I 221 — 1 162 Eng. ed. Rohde, too (Q 105; Pyche II 159 — 374 Eng. ed.), finds in this word the seed of later science. Cf. Mondello in ZM 316f; G. Vlastos, Philo Q 2 (1952) 111 n. 64, K. Freeman, The Pre-Socratic Philosophers (Oxford, 1949) 76 (with this word Pythagoras was classified among scientists by Heraclitus). Against these attempts to extract "science" from polymathia, see Reinhardt, Parm. 212ff, Frank 356 n. 166, W. J. Verdenius, Mnuosyne 3.13 (1947) 280ff.
II. PYTHAGORAS IN THE EARLIEST TRADITION

is the first mentioned; and in fact Hesiod’s Theogony and the Catalogues are polythany in the truest sense of the word. But Hecataeus and Xenophanes are distinctly separated from the other pair by the word αἰσθ., suggesting that Pythagoras belongs more closely with Hesiod than with them. This can scarcely be accounted for as a chronological indication; the difference in time between Hesiod and Pythagoras is greater than that between Pythagoras and Xenophanes or Hecataeus. Something in the nature of the subject matter, then, must be what connects Pythagoras with Hesiod. Hecataeus and Xenophanes have in common their emphatically modern and polemical attitude, critical of the traditional mythology; Pythagoras does not belong in their company. His “much knowledge” spans the entire world of man and, above all, the world of the supernatural, both this life and the next. This is clear from Empedocles. He stands beside Hesiod as the representative of Orpheus, so to speak—the representative of non-Hesiodic mythical teaching about the world and its gods.

‘Ιστορία is rightly regarded as a key word for the open-minded kind of inquiry based on observation, pursued by the Ionians; so some have seen Heraclitus as testifying to Pythagoras’ “scientific research.” If it only were not for those awkward συγγραφαί which sound very much like “Orphic” writings, and the word κακοστρέφων, which means “skullduggery”; when one considers the formulation by Hermesi-anax of Colophon (third century B.C.), ‘Ησίοδον δέοις ἄρανον ἱστορίαν (fr. 222 D.), one realizes the insubstantiality of all the inferences drawn from the passage of Heraclitus. ‘Ιστορίαι does not necessarily imply anything more than the Hesiodic type of πολυμαθής, and does not make any definite allusion to rationale Wissenschaft, “science.”

The word σοφιστής is translated “scientist,” too; and it has even been asserted that the word proves that Herodotus “knows nothing of Pythagoras as a miracle-worker.” Here again the perspective is distorted. In the fifth century it is poets, more than anyone else, that are designated σοφιστής, and even for Isocrates Homer is ὁ μεγάλες ἐπὶ σωθήνη δόξαν εἰρήνη (Soph. 2). For Herodotus, along with Pythagoras, successors of Melampus and Solon are σοφιστής. Melampus, who according to Herodotus introduced the worship of Dionysus from Egypt, is without a doubt a Wundermann par excellence; and even if one supposed that Pythagoras was closer to Solon than to him, he is still far from science. The new thing that was introduced by those whom Plato called Sophists is not at all suggested by the word. “Wisdom” is an advantage admired and sought from time immemorial, but there are utterly different conceptions, from time to time, as to what it consists of, what it pertains to, and how it manifests itself. Were it certain that Pythagoras was a scientist, these words could be understood in this sense. As long, however, as only his “shamanistic” activities are early attested, our conclusion must be that no statement about the “wisdom” of Pythagoras carries us further than these activities. Quite the contrary, in fact: the Hippocratic book On the Sacred Disease attacks the itinerant “medicine men,” who expect to heal the sick with magic rites, μάγου καὶ αὐθάραται καὶ καθαραὶ καὶ αὐλακώνει (VI 354 L.), and adds, προσποιοῦμαι...πλέον τι τι οἴδας. The claim to know “more,” beyond the limits set for ordinary men, is the mark of the shaman; τεκμηρίζεσθαι is that of the scientist (Alcmaeon fr. 1).

The efforts to discover a unity of scientific and religious thinking in Pythagoras, and thus a step beyond the merely ritual and mythical realm, center about the ideas of κάθαρος and διάμισθης. In both cases the question of Plato’s position is difficult; that is, did he merely take over these ideas, or reinterpret them in an independent manner? Since the time of Döring it has been thought that the concept of κάθαρος holds the key to understanding the connection of religion and science in early Pythagoreanism. Rigorous scientific work, especially mathematics, is supposed to release the soul from its close tie to the body and in this way becomes the principal agent of “purification,” upon which

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4 This interpretation is attempted by Lévy, Sources 2 8; H. Thesleff, On Dating Xenophanes (Helsinki: Soc. Scient. Fenn., Comm. Hum. Litt. 23.3, 1957) 7 (dating Xenophanes late). H. Gomperz, Hermes 1923, 36 n. 1, and M. Mercovich, Philologus 108 (1964) 40, speculate that Pythagoras, unlike Xenophanes, was already dead, and therefore classed with Hesiod.


6 Above, ch. II 3, nn. 64, 228.

7 Burnet, EGP 85; “scientific man,” 97: “does imply scientific ideas.” Contra (rightly), Rathmann 47, Vlastos, Philo Q 2 (1922) 111 n. 64.

8 Gomperz, RH 1913, 45.

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9 Pi. Isthm. 5.36, with schol., Soph. fr. 820 N., Hes. I. 153 οὐ σοφίστης, G. B. Kerferd, CR 64 (1980) 8–10. This removes the necessity of interpreting the special position of the “wise man” in the metempsychosis doctrines of Pindar (fr. 133 Schr.) and Empedocles (fr. 146) as related to the “scientific” character of Pythagoreanism (Mondolfo in ZM 124f).

10 In Hdt. 1.29, 2.49. In Eur. (7) Rhes. 949 Orpheus is σοφιστής, and Menander seems to have dubbed Epimenides σοφιστής (K taxiōn usum fr. 2 Krömer fr. 308 Kock).

11 Döring, AGP 892, 955, and, following him, Burnet (EGP 97f), through whom, especially, the idea became current. Cf., for example, Délatté, Pol. 5, Rouger 101f, Robin, Pensée 65, Mondolfo in ZM 646, Cornford, PrSup 110f.
depends the well-being of the soul in this life and the next. This is clearly stated in the *Phaedo*, which introduces Pythagoreans as interlocutors and has a good deal to say about Pythagorean matters.\textsuperscript{18} Separation of soul and body, “dying,” is the goal of all true philosophy, so that pure truth may be apprehended by the soul, in its pure state (643a)—καθάρασις through φιλοσοφία (67cd). And in the *Republic* it is the sciences of arithmetic, geometry, astronomy, and music that lead from the world of appearances to the ideas and effect the necessary περιπατητήριον of the soul (521c). But here we must move carefully; this line of thought is so closely bound up with the theory of ideas that without this, it becomes meaningless. Without Plato’s χωρίσμος, without the view that there are incorporeal, nonsensible objects of knowledge and science, all science must remain tied to the visible world, and this commitment becomes deeper and deeper. And Aristotle attests that the Pythagoreans were in this very situation.\textsuperscript{19} Among these Pythagoreans there is not a trace of the idea of science as an escape from the world; it may not be a coincidence that at the point in the argument of the *Phaedo* where Socrates first speaks of dying as the goal of life, Plato has Simmias laugh (643a).

In fact, the idea of purification through science was apparently not ascribed to Pythagoras before the time of Iamblichus.\textsuperscript{20} To be sure, an important and widely cited sentence of Aristoxenus says οἱ Πυθαγόρειοι . . . καθάρεις ἔχοντο τοῦ μὲν σώματος διὰ τῆς λατρείας, τῆς δὲ ψυχῆς διὰ τῆς μουσικῆς, and a number of similar reports may be found in Iamblichus and others,\textsuperscript{21} but the topic here is music in the proper sense of the word, which is understood in the light of magic and is bound about by ritual. Science is neither involved in the origin of these usages nor a necessary consequence of them—though by means of a new interpretation of music it may naturalize itself in this realm.\textsuperscript{22} There is, however, no testimony to the occurrence of such a transposition before Plato. Festugière derived the idea of purification, in Plato, from general-Greek cult practices;\textsuperscript{23} Boyancé replied with the significant comment that the question was not about a single, ephemeral ritual act, but about a lasting style of life, and that in this respect the Pythagoreans seem to have been Plato’s predecessors.\textsuperscript{24} But scientific activity is not necessarily comprised in the Πυθαγόρειος τρόπος τοῦ βιου, other.

The situation is scarcely different with recollection (ανάμνησις) and related ideas. Here, too, there is a discernible Pythagorean background: Empedocles (fr. 129) says that Pythagoras, when he put all his spiritual power to work, could survey ten and twenty generations. Surely this assumes that he recalled his earlier incarnations. The later tradition tells of a system of memory training among the Pythagoreans. They tried in the morning, or in the evening, to recall all the events of the past day, and even of the day before.\textsuperscript{25} In the world of the dead are the springs of Lethe and of Mnemosyne, and the initiate is warned of the former and directed toward the latter.\textsuperscript{26} So the attractive conjecture has been made that the original goal of Pythagorean exercises in concentration was to enhance the strength of the soul so that, following the example of Pythagoras, it may avoid the spring of Lethe.\textsuperscript{27} Perhaps it was also customary practice in the mysteries to awaken in the initiate a “memory” of his divine descent.

The concept of reminiscence is basic to Plato’s epistemology. All true knowledge, as knowledge of an idea, is a process of reminding oneself of something that the soul has seen before its entry into the body. To make this clear, Plato makes use of the doctrines of metempsychosis, and he demonstrates the existence of anamnesis, with the help of mathematics, in the famous passage in the *Menex* (80dff).

Because of the tendency to regard everything mathematical as eo ipso

\textsuperscript{18} On Simmias, Ceber, and Echecrates, above, ch. I 4, nn. 39-40; on the doctrine of ψυχή-μυσία, below, ch. III 2.
\textsuperscript{19} Met. 989b29; cf. ch. I 2, n. 15.
\textsuperscript{20} Theo Sm. 14ff is only concerned with Plato. The Lysis letter (Hercber, *Episto. gr.* p. 602.12ff = lam. VP p. 44.1ff, closely dependent on Pl. *Rep.* 429d) uses katharsis in a different sense, namely moral preparation for admission to the esoteric teaching.—In detail, lam. *Comm. math.* 22, esp. p. 69.6ff, pp. 53ff, p. 84.12.
\textsuperscript{21} Aristox. fr. 26; below, ch. V 1. The sharp distinction of soul from body reminds of Plato (Sophr. 227c); later sources represent music as also acting on the body (lam. VP 110f, etc.), which doubtless corresponds more closely to the original magical practice.
\textsuperscript{22} Socrates in Pl. *Phld.* 61a: ὃν ἄλλην μὲν οὖσαν μεγάλην μουσικῆς.
\textsuperscript{24} *RIG* 1043, 164 n. 3; above, ch. II 4.
\textsuperscript{25} *B. E. D. D. D.* 105.3.1, lam. VP 165 (in the morning), Cic. *Srm.* 38. The verses cited at Por. IVP 40 are the same as *Carm. aur.* 40ff; one also cited at D.L. 8.22 (evening). The source of this tradition cannot be determined with certainty (on Delatte’s reconstructed Ἰἐρος ἀνάμνησις see below, ch. III 1). See also Apollonius ap. lam. VP 256. According to Th. ar. 81.17, the number 10 is Μεθύμ and 1 is Μηθύμοισιν; below, ch. III 2, n. 48; above, ch. II 4, n. 24.
\textsuperscript{26} Cf. *B. E. D. D.* 113.5, n. 189.
Pythagorean, all of this is commonly thought of as Pythagorean, and if true this would show a basic correlation of metempsychosis, genuinely philosophical epistemology, and exact mathematics. But a closer look reveals that the connection of Pythagorean with Plato, in relation to *anamnesis*, is scarcely more than an equivocation. In Plato, what is “discovered” is that group of elements of human knowledge, beyond the empirical and not discoverable in experience, which are called in modern philosophy the a priori. They can be shown most clearly in the realm of mathematics or in expressions of relationship like “equality.” Therefore, in Plato mathematics is a means of proof, and reminiscence as well as metempsychosis is a myth-like clarification of facts of the greatest philosophical relevance. Plato makes use of mathematics because it is necessary to his doctrine, not because of any partiality toward Pythagoreans or mystical attraction toward number. The intended goal of his argument is the theory of ideas, for it is in the ideas that Plato finds that knowledge, beyond the empirical. The object is not the extension of human experience to “ten and twenty human lifetimes,” nor the projection of this life into another, very real, next life, but a vision of a quite different kind. The knowledge of the ideas is supramundane, and cannot be attained by living through even a large number of earthly lives. This is made clear in the myth of the *Phaedrus*, and in the *Meno* hints at it. Any slave can “recall,” because what is at issue is not just any experience, but that which is generally valid; it is explicitly stated that this slave must have attained this knowledge “when he was not a human being” (*Meno* 86a). Here we are in a realm which, according to Aristotle, was foreign to the Pythagoreans: διαλέγονται μέντα καί πραγματεύονται περὶ φύσεως πάντα (*Met.* 989b33). It is impossible to credit Pythagorean eschatological doctrine with Plato’s mathematics and his theory of ideas. The development of the concept of *anamnesis* into epistemology and the proof of it by mathematical means are seen to be typical “Platonic transposition.” What is Pythagorean is practice in concentration and memory, which have no necessary connection with mathematical science, but belong to the world of “shamanism.”

Generally speaking, the argument from silence is not very persuasive. And yet one is impressed by the negative consensus of the ancient sources, down to Aristotle, in the matter of Pythagoras the philosopher and scientist. Herodotus mentions him in connection with Zalmoxis, and mentions Pythagoreanism in speaking of the rituals of Egypt; but, when he deals with the rise of geometry in Egypt (2.109), Pythagoras’ name does not occur. Democritus was acquainted with Pythagoreans, and wrote a book entitled *Πυθαγόρη*, but it is classified, in the catalogue of Democritus’ writings, among the ethical tetralogies, along with Περὶ τῶν ἐν Ἀιδῶν. The *Diássoi logos*, too, speak of Pythagoreans, along with followers of Anaxagoras, but in relation to the teachability of *σοφία*. Plato names Pythagoras just once, as ἰδρύμα τινῶν πυθαγόρεως and creator of Πυθαγόρεως τρόπος τοῦ βιοῦ. It is the Society’s way of life, and not any particular knowledge, that makes his fame. The doxographical sketch in the *Sophist* alludes to Pherecydes, Alcmaeon, the Eleatics, Heraclitus, and Empedocles, but the “Eleatic stranger” obviously does not think of Pythagoras when he is listing the attempts τι καὶ διαρκείας to πάσα τε καὶ ποιών ἔστειν (242c; cf. *Thet.* 132c). Alcidamas stated that Empedocles had studied with Pythagoras and Anaxagoras, καὶ τοῦ μὲν τὴν σεμνότητα ἐξεύρεσα τοῦ τε βίου καὶ τοῦ σχήματος, τοῦ δὲ τῆς ἐνθουλογίας. This is very doubtful as a historical statement, but it

22 Burnet, *THPl* 43: “I see no difficulty in referring this doctrine in its mathematical application to Pythagoras himself.” He is more reserved in the edition of the *Phaedo* (72), since, as represented by Plato, Simias and Cece do not seem familiar with the doctrine; therefore the doctrine is attributed to Socrates: “he applied the old religious doctrine of ἀναμνήσεις to science.” Here the difference is seen; we only need to replace “Socrates” with “Plato.”–The matter is most fully discussed by Cameron, who has the twofold thesis that ἀναμνήσεις in Pythagoras comprehended “a theory of knowledge” and that “divine knowledge was Number” (20). Neither of these points is provable (cf. Chrennis, *AFPhil* 1940, 361), and in particular there is no evidence that for the Pythagoreans “recollection” was bound with number. (Heracleides fr. 44 is surely thinking along Platonic lines; cf. ch. II 3, n. 78.) Long (68f) emphasizes that the doctrine of transmigration as taught in the *Meno* is dependent on Pindar, but finds, in the nearby mathematical discussion “additional proof” that Pindar, too, had a “Pythagorean background”; mathematical, ergo Pythagorean. (On this see ch. VI.)


25 This is attested by his contemporary Glaucus of Rhegium (D.L. 9.38). Apollodorus of Cyzicus mentioned Philaclus (ibid.). Cf. also ch. II 2, n. 2.

26 *DK* 681Boa. Against its genuineness, Zeller *SBBh* 1889, 996, Wellmann *AbhBlh* 1921, 12f, and Dicte (with hesitation); but cf. above, ch. II 3, n. 146; Frank 67; Gigon, *Urngrup* 128f.—It is tempting to juxtapose *Parm. aur.* 12 with fr. 84 (cf. also frs. 244 and 254). Democritus praised Pythagoras (Thrasyllos ap. D.L. 9.38).

27 6,8 (DK II 414.13).


29 D.L. 8,36, from the book *philosokhia*; Husniska, *REI* 1358. *Livy*, Sources 1 n. 6, considers the possibility of forgery; and Burnet, *FGrHist* 202, would substitute *Hedonomiai* for *Hedomiai*; but if even Timaeus (*FGrHist* 506f14) can make Empedocles a direct pupil of Pythagoras, surely a rhetor like Alcidamas could have done so.
gives an insight into the common view of Pythagoras; the "natural science" (φυσικὰ χεῖρα) comes from Anaxagoras, not Pythagoras. But the solemn, pompous mein which Pythagoras and Empedocles have in common is precisely the manner of the shaman—Empedocles the "deathless god" and Pythagoras the "Hyperborean Apollo."

Isocrates in the Busiris says in a general way that Pythagoras "was the first to introduce all the other philosophy to the Greeks," but specifically that he "more conspicuously than others attended to sacrifices and temple rites." It is only in the context of φιλοσύφις that Isocrates thinks of Pythagoras, not along with ἀστρολογία και λογομολ και γεωμετρία (23).

In his accounts of the history of philosophy, Aristotle consistently avoids the name of Pythagoras. In the Protrepticus he describes Greek natural philosophy as τὴν περὶ φύσεως τε καὶ τῆς τουατζός ἀληθεῖας φύσεως, οἷᾶς ἐκ τῆς Ἀναξαγόρας καὶ Παρμενίδης εἴσηγησαν. These are the living names in the field of physical philosophy; Pythagoras is not included. Theophrastus names Anaximander and Xenophanes as the teachers of Parmenides, and Parmenides as the teacher of Empedocles, where others speak of Pythagoras or Pythagoreans. If we take all these passages together, their silence about Pythagoras the philosopher and mathematician is an extremely remarkable coincidence—if it is that.

The other tradition sets in at just about the time of Aristotle. Aristippus dealt with Pythagoras in a book entitled Περὶ φυσικῶν. Hecataeus of Abdera and Anticyllus, who wrote about Alexander the Great, represent Pythagoras as introducing geometry from Egypt. By this time a conception of Pythagoras had become dominant which was obviously quite unknown to the pre-Platonic writers.

30 Bus. 28. It is clear from Herodotus that there was some connection between Pythagoras and Egypt (2.81, 123), and for him too the link was in religious and cultic peculiarities. The Busiris was probably influenced by Plato (Gnomon 33 [1961] 332).

31 Arist. fr. 32 = Iam. Comm. math. 79.13f. The very fact that Thales and Pythagoras are not named in this passage is an indication of the Aristotelian origin of the wording.

Jamblichus seems to have preserved something of this tradition in another passage (VP 166, perhaps from Timaeus; cf. above, ch. II 1, n. 34): Thanks to Pythagoras, he says, the expression Magna Graecia was invented, philosophers, poets, and jugglers flourished there, rhetoric was born, the Gnomes of Epicharmus were much celebrated, καὶ περὶ τῶν φυσικῶν θεῶν τυχά μεταξύ πεποίηται, πρὸς τῶν ἐπικεφάλεως καὶ Παρμενίδην τὴν ἐν Ελευθερίᾳ προφερόμενου τυχόνοισιν. Pythagoras himself is not named as a physical philosopher.

32 Theophr. Phys. op. frt. 6, 63. 3 (on which see Dox. 477); cf. below, ch. III 3.

33 D.L. 8.21 = Al.2 Giannantoni.

34 Diod. 1.98.2 = FGrHist 264F25.

35 D.L. 8.11 = FGrHist 140Ft; cf. below, ch. VI 1.

Every age has its own ideal of Wisdom; and there came a time when the ideal of the Wise Man, who by his own innate powers has achieved a commanding spiritual position and insight, became embodied in the persons of certain great men who seemed to fulfill the highest conceptions of wisdom and power that were attributed to the ecstatic seer and priest of purification. We cannot call them philosophers—not even the forerunners of Greek philosophy. More often their point of view was one which the real philosophic impulse toward self-determination and the freedom of the soul consciously and decisively rejected, and continued to reject, though not indeed without occasional wavering and backsliding.

No wrote Erwin Rohde, in reference to figures like Epimenides and Albinus, without including Pythagoras. But the most ancient evidence indicates that it is precisely in this perspective that we must see Pythagoreanism. To a later age it seemed natural to retroject their own notion of "wisdom" upon the great figures of the past and to impute to them that which from a modern point of view is "science." Only by such reinterpretation could the ancient remain acceptable. This process of the renewal of ancient wisdom took place, apparently, within the Pythagorean tradition, with many a false start and new beginning. For what was most characteristic of Pythagoreanism, from the beginning, was not so much the new as the exotic and archaic. No ancient witness appears to testify that Pythagoras was a scientist; in studying the shadow he cast on later generations, all we can be sure of is the cryptic νόημα τα εἰσεῖαν of the "shaman" and hierophant.
I. THE SPECIAL POSITION OF PHILOLAUS’ BOOK IN THE PYTHAGOREAN TRADITION

It has long been an established principle in the study of the pre-Socratic philosophers that only their original words, so far as they have been preserved, can provide an adequate foundation for interpretation and reconstruction. In any paraphrase lurk potential errors, for alteration of the form is bound to affect the content, whether because of adaptation to modern ways of thinking, or of a polemical bent which results in a reluctance to see the true sense, or of both. The special difficulty in the study of Pythagoreanism comes from the fact that this principle cannot be applied; whether an item of the tradition may be regarded as an authentic pronouncement of a Pythagorean must in each case be decided first on the basis of the indirect testimony. For the mass of writing that was forged and attributed to Pythagoras and his pupils was so vast that, contrary to ordinary methodological principles, in the case of any text purportedly composed by an early Pythagorean, the burden of proof lies with anyone who wishes to maintain its authenticity.

Even in ancient times there was controversy as to whether Pythagoras had written anything. From the third century on, ostensible works of Pythagoras kept appearing in the market, but none of them could hold its place. The opinion was widespread that no such book had been preserved, or even that Pythagoras avoided the written word on principle and recorded his teachings only in the minds of his disciples. There is a Platonic coloration here, and one may suspect that this emphatic declaration served as a pretext for discarding certain Pythagorean writings that were felt to be an embarrassment because of their old-fashioned character. Delatte attempted to refer the various lines of Pythagorean verse, one of which was cited early as Chrysippus, and most of which eventually found their way into the late compilation called Carmen aureum, to a ἵππος λόγος of the fifth century. Rostagni, still more boldly, claimed to have found the “Word of Pythagoras” itself. Heraclides Lembus lists a ἵππος λόγος in hexameters in second position in his catalogue of Pythagoras’ writings, and, in the manner of an ancient librarian, quotes the first line (D.L. 8.7). The older evidence for this secret document is, however, very fragile. Hecataeus of Abdera reports that Pythagoras brought “the ἵππος λόγος,” along with mathematics and the doctrine of transmigration, from Egypt. This does not mean, however, that he had seen a book entitled ἵππος λόγος. Like the statements about mathematics and transmigration, that about the ἵππος λόγος seems to come from Herodotus, who does use this term in connection with Pythagoreanism. But the context, along with the parallel passages, shows that what he is talking about is the mythical explanation given by the Egyptians, not a “Holy Word” of Pythagoras.

1 To what extent the expression “forgery” is legimate here is a question we need not go into. In any case, we are talking about writings which, in an unhistorical manner, the Suda, as well as popular philosophical thought. Cf. Burkert, Philolaus 1964, and, for a collection of material, Theissen, Texts.
4 Plut. Numa 22, Nicom. ap. Por. VP 57: ὅστε γὰρ ἀετὸς Πυθαγόρου σύγγραμμα ἔγρα. (This sentence is not included in the parallel report, lnam. VP 252. Rohde, Q 140f, assumes that Porphyry has inserted it. But the passage reads smoothly in Porphyry, whereas there is unevenness in lambichus: the words πλῆν ὅλγον πάνω, p. 152.22, do not go with what immediately precedes them, but with συνεπενδύεισθαι, line 19; and it is just here that Porphyry has the sentence quoted. In lambichus there are hints both earlier [ἐπεφυλακαν...ἐν τοσι στήθοιν, lines 19f] and later [p. 116.8] that there were no written works of Pythagoras; but, as lambichus repeatedly cites works of Pythagoras, he has every motive to rush this up. Cf. above, ch. II 1, n. 15.—David In Porph. Is., CAG XVIII 2 p. 25.28ff, adds Plato’s warning against written texts (Phdr. 277f).
5 Apollonius ap. lnam. VP 258, D.L. 8.7, on the ἵππος of Μουσικός λόγος. See Gigon, Ursprung 124ff.
6 Delatte, Litt. 1ff; contra, Theiler, Gnomen 2 (1926) 147ff. The material is in Theissen, Texts 158-163. Chrysippus, SVF II no. 1000 = Carn. ait. 54.
7 Il verbo di Pitagora. He relies especially on Οv. Met. 15; but the principal source of that is Empedocles.
8 FGrHist 264F5 = Diod. 1.98.2 (cf. Jacoby Ilia 75f). Pythagoras is named among the sages who visited Egypt at 1.98.4 and 1.96.2. In the latter passage occurs a mention of the ἀντὶ ταῦτα ἐν ταύτῃ ἰπποσ ποιηθέντων which is regarded as a strong clue that Hecataeus is the source. There are probably “occasional and not extensive” additions by Diodorus (Jacoby 78.24); but the naming of Pythagoras is the ἵππος λόγος is firmly rooted in the context. Hecataeus could not avoid mentioning Pythagoras (cf. also above, ch. II 2, n. 15).
9 Htd. 2.81; cf. above, ch. II 3.—Geometry, Htd. 2.109, metempsychosis 2.123.
10 Pace Guthrie I 160; Herodotus describes the custom, names the Greek parallels, and remarks in closing that he is not going to reveal the explanation. Cf. ἵππος λόγος, 2.62; parallel αὐτῷ διάνω βασιλέως, 2.61; similarly 2.171, 2.51.
Philolaus' Book in Pythagorean Tradition

Philolaus fragments have been a matter of controversy for many decades. For A. Boeckh, who devoted a fundamental study to them, they were “in the labyrinthine maze of the traditions about Pythagorean wisdom and the Pythagorean society... a point of light whose radiance could, perhaps, brighten this night a little.”5 The authority of Zeller and Diels won acceptance for this view for a good while, though with an important modification.6 This was true especially in the German-speaking world; among speakers of English, Bywater’s condemnation of the fragments, followed, with additional arguments, by Burnet, held sway.7 Then, since Frank published his elaborate attack against the fragments, the dominant mood has been uncertainty, though scholarly caution has somewhat tipped the balance toward the negative.

In spite of the mountainous bibliography, there does not yet exist a full interpretative study of the Philolaus fragments. But since the days of Bywater the situation has somewhat changed; there are new perspectives, refined methods, and, above all, new source material. A papyrus published in 1893, with excerpts from the history of medicine written by Aristotle’s pupil Menon, now makes it certain that there was a book by Philolaus extant in the middle of the fourth century B.C.8

The fact that the controversy lives on, in the case of Philolaus, is in itself significant. Writings like those of “Ocellus,” “Timaeus of Locri,” and a great deal of the material attributed to Archytas have

matically. Archytas could determine \(\sqrt{2}\) (used in doubling the cube) geometrically, but not arithmetically. Fr. 4 is comprehensible from the point of view of Plato’s doctrine of ideal numbers: according to the system of derivation, number is primary as compared to geometrical magnitudes (δρομητική διαβαστα γωμητίας, Arist. Met. 982a26ff., etc.). Cf. also below, ch. V 1.9

13 Boeckh 3.

14 The aethesis of fr. 21 (below, ch. III 2).

15 For authenticity: Zeller 186ff; Diels, DK 44; Reinhardt, Parm. 65 (and elsewhere); for Plutarch, Hermes 153ff (for the genuineness of fr. 20); Gomperz, Hermes 1934, 155ff (for the genuineness of fr. 28).—Origen, C. 30, 2ff. (for the genuineness of fr. 22).—Sibyl, C. 31, 2ff. (for the genuineness of fr. 22).—Sibyl, C. 32, 1ff. (for the genuineness of fr. 22).—Sibyl, C. 33, 1ff. (for the genuineness of fr. 22).

16 The aethesis of fr. 21 (below, ch. III 2).

17 For authenticity: Zeller 186ff; Diels, DK 44; Reinhardt, Parm. 65 (and elsewhere); for Plutarch, Hermes 153ff (for the genuineness of fr. 20); Gomperz, Hermes 1934, 155ff (for the genuineness of fr. 28).—Origen, C. 30, 2ff. (for the genuineness of fr. 22).—Sibyl, C. 31, 2ff. (for the genuineness of fr. 22).—Sibyl, C. 32, 1ff. (for the genuineness of fr. 22).—Sibyl, C. 33, 1ff. (for the genuineness of fr. 22).
been convicted as spurious and set aside. The "forger" betrays himself by letting results, or concepts, or terminology of later philosophers thought creep in. (An important criterion here is the evidence of Aristotle that among the Pythagoreans the distinctions had not yet been fully developed between form and matter, sensible and super- or insensible, corporeal and incorporeal.\textsuperscript{19} Another sign of forgery is too close an adherence to the wording of a passage of Plato or Aristotle; even when Plato is dependent on Pythagoreans, he does not copy word for word. According to these criteria the metaphysical fragments of "Archytas," for example, are unquestionably apocryphal,\textsuperscript{20} but the case of Philolaus cannot be settled in this way.

It would be convenient if external, linguistic factors could be decisive. Burnet tried to show that a Pythagorean of the fifth century, even in south Italy, would have had to write not Doric but Ionic Greek, the dialect that had become standard for philosophical and scientific writing. The physicians of Cos and Cnidus wrote Ionic, as did Antiochus of Syracuse\textsuperscript{21} and Herodotus, deserting in each case the dialects of their home cities.\textsuperscript{22} Burnet will admit that Archytas wrote Doric,\textsuperscript{23} as Thucydides chose Attic. But the author of the pseudo-Xenophontic Constitution of the Athenians was already writing Attic; why would Philolaus have had to restrict himself to Ionic? The word περακτός perhaps points in the direction of Doric.\textsuperscript{24} But, above all, we are told that the physician Acron of Acrasias, whose floruit was about 430 B.C., wrote in Doric;\textsuperscript{25} and it is beyond question that Corax and Tiasios spoke and wrote in Doric. All this shows that prose literature in Doric began to appear at least a generation before Archytas. The individual dialect forms, however, are so often inconsistently reproduced in the manuscript tradition that even obviously false forms do not provide a criterion of spuriousness. Hyperdorisms are to be found in Sophron, Theocritus, and Archimedes, and may even have penetrated into popular speech.\textsuperscript{26}

Generally speaking, definite proof cannot be offered for the authenticity, but only for the spuriousness, of a book. But the defender of a book's genuineness is not confined to destroying the opponent's arguments, knowing the while that new ones may always appear. What one can and must do is to ask where, in the transmission, the sources of error lie. "Forgeries" are usually pieces of quasi-historical reconstruction; the Pythagorean pseudepigrapha, for example, show what people wanted to be regarded as Pythagorean. We can detect in them a certain tendency of interpretation, a general purpose which is also discernible in the distortions of the doxographical tradition. A genuine fragment must show itself so by standing aloof from this tendency and not being deducible from it. In this way one can establish the presumption that something is probably genuine.

First let us look into the external evidence for Philolaus' book. In various ways, Philolaus' name is connected with the beginnings of written Pythagorean literature.\textsuperscript{27} According to a tradition that goes back at least as far as Satyrus, Plato in a letter to Dion commissioned him to buy "from Philolaus" three "Pythagorean books" for the

\textsuperscript{19} Above, ch. 1.2.
\textsuperscript{20} E.g., there is Aristotelian terminology in Peri árkhon, p. 19.19 Thesleff; á μὲν μορφον οὗτος ἥν αὐτία γενοι τῷ τῷ ἔμεν, δὲ οὐκ οὗτος ὡς ἡμεῖς, παραδεχόμενος τὸν μορφον. Peri nóu kai aíthéras (39.3-25 Thesleff) is mostly copied word for word from Plato Rep. 559b et seq. The logical writings of ps.-Archytas (pp. 15-19, 21-32 Thesleff) are so obviously Aristotelian that even Themistius supposed they were written not by the ancient Pythagorean but by a later Peripatetic named Archytas (Boeot. In Cat. I, Migne 64.162, p. 22.2 Thesleff). Archytas, οὐ μάλιστα καὶ γνώση λέγεται εἶναι τὰ συγγράμματα (the opinion of Por. In Pol. p. 36) and since he is somewhat later than Philolaus, is treated as more reliable, especially by Frank. It is easy to overlook the fact that more was forged in his name than in that of any other Pythagorean, since the rather extensive fragments of these works are not printed in DK.

\textsuperscript{21} FGrHis 555F2. In Alcmeon's fr. 1 are not only a Doric ἣθος but an Ionic Κρατικός. Burnet (EG2 282 n. 5) and Thesleff (Intr. 80) believe that he wrote in Ionic.

\textsuperscript{22} The "three books" attributed to Pythagoras are Ionic (D.L. 8.6ff), as are "Androcles" (ch. II 4) and "Perictiones" (Stob. 4.25.50, 4.28.19; pp. 142-145 Thesleff). It is not at all certain that, as many assume, the so-called Hypoimmunita were Ionic; see Burkert, Philologiae 1961, 27 n. 3.

\textsuperscript{23} EGP 283.

\textsuperscript{24} The καίρα in περακτός is surprising, compared with Ionic χείλασις and Lesbian χελάσιος, but not simply "Doric." Schwzyer (I 597) suggests analogy with τρικτός, cf. ἀντροκτός in Delian inscriptions, τρίκτος in Sophron fr. 3 Kaibel (cf. I 119), τρίκτος in H 1126, 2 fr. from Delphi. It is doubtful whether the form ἄφροις πόροις (Simp. Cad. 212.12 - Arist. fr. 204, Proct. In Iul. 101.17, v. L. Zuppi) can be counted as Doric; see Schwzyer I 577 n. 4. Schwzyer, Dial. 696 (Chius).

\textsuperscript{25} Suda s.v. Ἀκρον: ἄμφιβλητος. There is no reason to think of a pseudepigraphon (pace Thesleff, Texts 1). The doctors in comedy speak Doric, from Crates (fr. 41 Kock) onward, see Euphron fr. 3, Alexis fr. 142, Menander Aspis 439-464: one of them is said to come Σωκλῆς ἄποι γῆς, Epictetes fr. 11.27. The cookbook of Mithaecus (Pl. Gorg. 518b) was also Doric (Ath. 7.235f).


\textsuperscript{27} Wiestmann correctly analyzed the tradition, Mimmausy 142, 23f. The misunderstanding about the "three books of Philolaus" hung on, from the time of Boeckh, until it made its way into 1DK, in spite of the express testimony τρικτός πόροις τε (1.1. 8.85). Bywater (40ff) and Schaarschmidt (74ff) capitalized on the contradiction.
stupendous sum of 100 minas; Philolaus had "made these books known." This alleged letter of Plato must actually have existed, to judge by the form of citation, and its purpose is not hard to guess. Ancient publishers frequently preface apocryphal publications with a letter that gives an account of the origin and importance of the writing. There was in circulation, at the time of Satyrus, about 200 B.C., a set of three pseudonymous Pythagorean books, introduced by this letter from "Plato," which named Philolaus, though not as its author. The word ἔσχηγγε is used of someone who "brings out" or publishes a writing previously unknown, perhaps secret, but in any case already in existence for some time; but it is not used of the author himself. We must get rid of the misconception, which goes back to Boecch, that Philolaus had written the "three books," and only then will we be able to identify them.

28 D.L. 3.9: λέγοντα δὲ τυχεὶς, ὦν ἐκαί καὶ Σάτυρος, ὅτι Διὸς ἐπέστειλε (Πλατόν) εἰς Σκιλλᾶν ὀνόμασθαι τρία βιβλία Πυθαγορικά παρὰ Φιλόλαου μνὸν εκάτον.

D.L. 8.15: μέχρι τοὺς Φιλόλαους εἴη για τὴ νύν Πυθαγορικὸς εὐγένειας ὑπομνήματα περιπετευσιῶν πρὸς τὴν Φιλολάου ἡμέραν Ἐκλείπτων τρία βιβλία δὲ μένεινον ἔσχηγγε τὰ σωματικά τοῖς τρία βιβλία τὸ λέγεται Διὸς ὁ Συμφόρος ἐκάτον μνὸν πρίονος Πλατόνων κελεύσας.

D.L. 8.84: παρὰ τοῦτον (Φιλόλαου) Πλατόν εὐγένειας τὰ βιβλία τὰ Πυθαγορικὰ Διὸς γράφει, Χελ. 3.17.1: "Πλατόνος" τρις Φιλολασικοὶ ἱστορικοὶ δεκεμβρίου διαμελισμα δεκαμήνιον. (Δεκαμήνιο = δραχμα, so that 10,000 δεκαμία = 100 minas.) Cic. Rep. 1.16: "(Platonem) Philolaii commentarios esse nuncut." Tzetzes ingeniously combined the testimony of Satyrus, Hermippus, and the letter of Lyco to produce a version in which, with Dion's help, Plato bought a book of Philolaus, for 100 minas, from impoverished Pythagorean widows (Chel. 10.790ff; 11.399). Cf. also the anonymous Prolegomena to Plato 27, p. 201 Herrmann.

29 γράφει D.L. 8.84. Theiler is doubtful about this (Gnomon 2 [1926] 878), as is Beulter (RE 2:213). Beulter's argument, that the letter would have been famous if it had existed at all, is not persuasive; even the χρυσόμονα τρία βιβλία perished almost without a trace.—The ancient sources give various, often contradictory, guesses as to where Plato got so much money. (Onetor ap. D.L. 3.9 says Dionysius gave it to him; Chelius 3.15.1 says it was Dion, according to "certain persons." Also, how could a Pythagorean be such a clever business man? (Ian. VP 199 says Philolaus was in need; certain έτεροι in D.L. 8.85 said the book was sold to Plato by certain Pythagoreans in gratitude for his having procured the release from jail of one of their number.) The letter was there; imaginary circumstances were devised to fit the "fact." 30 If Burket, Philolaos 1961. Examples are the letter of Lyco and the Hypomnemata (Delatte, Litt. 1.103f), the correspondence between Plato and Archytas and Ocellus (Harder 4.17f), 2 Macce. Diogenes Antinus, and Dictys Cretensis.

31 έσχηγγε is used of Hippasus in the version of the mathematici, and in the same passage Φιλολασίες is used of geometry (Ian. VP 88 Comm. math. se. p. 77.19). The abbreviated expression "Philolai libros, commentarium" in Cicero and Chelius is irrelevant to this question.

"Pythagoras wrote three books: On Education, On Statesmanship, and On Nature." This so-called tripartium was available to Heraclides Lembus, who also used Satyrus, and also to the author who put together the account which is at the base of the lives of Pythagoras in Diogenes Laertius and Hesychius. For a time it was regarded as just what it claimed to be: the authoritative and genuine work of Pythagoras.

It was natural enough to invent a correspondence between Plato and Dion about Pythagorean writings, just as the author of the "Ocellus" book represented Archytas as corresponding with Plato. The remarkable thing is that Philolaus was included in the plot, in spite of the chronological difficulties involved. Apparently this was the result of the belief, attested at least since Neanthes, that before Philolaus there were no Pythagorean writings known. No anecdotal or legendary features characterized the publication of Philolaus' book. We have only the simple datum: Philolaus and the first appearance of Pythagorean writings belong together.

We have from Hermippus another story, older than that of the "triptium": "He (Philolaus) wrote one book, which according to Hermippus some writer said Plato the philosopher... bought from the relatives of Philolaus for forty Alexandrine minas of silver, and from which he copied his Timaeus" (D.L. 8.85). In spite of similarities,
III. PHILOLAUS

This report is plainly different from those previously mentioned. Only one book is mentioned, it was written by Philolaus himself, and the price is 40 minas. There is no mention of Dion or a letter from Plato, and the purpose is not the introduction of a forgery, but a charge against Plato, that the *Timeus* was a plagiarism from Philolaus.

Hermippus, a pupil of Callimachus, is to be dated somewhat earlier than Satyrus, and himself cites a still earlier source; consideration of the content also shows that this story is earlier than the "tripartitum" legend. The latter presupposes, without explanation offered, that one could get Pythagorean books from Philolaus. In this story, a direct connection is alleged between Plato and Philolaus' book, and this very theme is present in a still older piece of evidence. Timon, the sillogographer, addressed Plato with the reproach,

Much silver hast thou spent for one small book,
From which thou then *Timeus* learnt'st to write.\(^{39}\)

This is not to be taken as meaning that Timon was the inventor of the story,\(^{40}\) for charges of Platonic plagiarism are plentiful and even earlier than this. Alcimus found Plato's philosophy preexisting in Empedocles; according to Theopompus, Plato imitated Aristippus and Bryson in his

\(^{37}\) The expression "Alexandrian minas" is an appalling historical blunder (Boeckh 21, Schaarschmidt 77); it is incredible that an Alexandrian writer did not know that his city was founded by Alexander the Great. Perhaps a conversion computation was included and has been lost. (Alexandrian money was famous: D.L. 7,18; Paton and Hicks, *Inscriptions of Cor 34*.)

\(^{38}\) The "tripartitum" legend drove the other one out: three books instead of one, 10 minas instead of 40, Pythagoras himself the author. Though the mention of the *Timeus* hints at a work of physical philosophy, in the "tripartitum" ethics is in the foreground, in true Hellenistic fashion (Diels, *APG* 1890, 462).

\(^{39}\) πολλὰν δ' ἄργυραν ἀξίωμαν ἥλαξαν βιβλίον, ἐδεῖν ἀπαρχόμενοι τιμαιογραφεῖν ἐξίδιδόντας,

Timon fr. 54 (*PPF*) = DK 44A8 = Gell. 3,17,6; also Procl. *In Tim.* 1,1,11 et saep.

\(^{40}\) As did Rohde, *O. 161*. Timon cannot be the *συγγραφέας* mentioned by Hermippus, either, for this word specifically designates a prose writer (PL. *Phdr.* 278E, cf. 235E), pace Schaarschmidt 77. Aristoxenus is regarded as the originator of the allegation (Burnet, *EGP* 279f.; Wilamowitz, *Plato II* 87; Geffcken [below, n. 41] 94; Wiersma, *Muemosyne* 1942, 24; Raven in KR 308; Harder 41). Wahrld (ARISTOC. 67) does not distinguish between the Hermippus tradition and that from Satyrus, and arbitrarily attributes D.L. 8,15 to Aristoxenus fr. 43, as though the latter could have known the διαβολὴ τρὶς βιβλίων. The fact that a citation from Aristoxenus follows proves nothing, considering Diogenes' mosaic-like methods of composition. Lam. *VP* 199 (DK 14,177) stands between Aristoxenus passages (Lam. *VP* 87f. Aristox. fr. 30; Lam. *VP* 205ff. Aristox. fr. 38; but the citation probably begins at lam. *VP* 210f.; cf. DK 58,108). Wilamowitz pointed this out (loc. cit.), but fr. 30 (followed by 11) belongs to the book *Περὶ Πεδιγραφικῆς βίου*, and fr. 38 to the *Πεδιγραφικὸς άποφασις*. Lamblichus uses a similar technique, too. There is some likelihood that the author may be Aristoxenus, but nothing approaching certainty.

dialogues; Aristoxenus stated that the *Republic* was contained ἀρχεῖον ὧν ἐν τῇ Ἀριστολογικῇ of Protagoras,\(^{41}\) and anyone who wanted to go back still further could maintain that Plato "stole" the doctrine of immortality from Homer.\(^{42}\)

It is a mistake to suppose that the originals of Plato's alleged plagiarisms were ad hoc forgeries. Not only the Homeric poems but the books of Protagoras, Aristippus, and Bryson were readily available; and even the Epicharmus fragments cited by Alcimus may be genuine.\(^{43}\) The charge of plagiarism is like a philological discovery, a clever inspiration that is all the more effective if the similarity is not apparent on the surface. The keen-witted *Kritiker* detects the "theft" which completely deceives the man in the street. Therefore we must suppose that the book of Philolaus, too, upon which the *Timeus* was said to be based, really did exist, but not that it showed so thorough an agreement as does the "Timeus Locrus" book. It is not necessary for the similarities to be much closer than those between Homer and Plato, for the malicious assertion to be made.\(^{44}\)

The conclusion will be, then, that before Timon, in the fourth century, there was in existence a book of Philolaus which could plausibly be brought into connection with the *Timeus*. The later tradition, and especially its use in the story of the "Tripartitum," shows that this book was regarded as the oldest book by a Pythagorean. What is more, the Menon papyrus proves the availability of a book of Philolaus in the fourth century.\(^{45}\) According to this witness Philolaus' book contained discussion of medical matters; and such topics take up considerable space in the *Timeus*.

Scholars have assumed that this book, though relatively early, cannot have been written by Philolaus, but must have been composed by someone else and only later given the prestige of his famous name.\(^{46}\) But this leads immediately to the question, how famous he was and

\(^{41}\) On these accusations of plagiarism, see Zeller II 1, 429 n. 7; E. Stempfle, *Das Plagiat in der griechischen Literatur* (Leipzig, 1912) 23ff.; J. Geffcken, "Antiplatonia," *Antiphoniten*, Hermes 64 (1929) 87-109; Alcimus: D.L. 3,9-17; *EGHist* 560f. 10K 231f. 6; *Athen. 55c* 14f.; *APG* 1890 (DK 58,108); *Plato II* 87.

\(^{42}\) Athen. 11,597e (from Herodicus).

\(^{43}\) Alcimus forged the Epicharmus quotations he used, according to the view of A. Covotti, *I Presocratici* (Naples, 1934) 144. Bywater (26.1) cast doubt on Protagoras' book, though it is in Diogenes' catalogue of his works (955).

\(^{44}\) Therefore it is not necessary to assume that Timon had the "Timeus Locrus" book or the like, as Proclus thought (above, n. 39).

\(^{45}\) A27-28. "It must be based on something in writing," says Wilamowitz (*Plato II* 88).

\(^{46}\) Below, n. 86.
what he was famous for. Practically nothing is known of his life.\textsuperscript{47} His home was Croton, or maybe Tarentum, and he spent some time in Thebes—all the rest, what little there is of it, is demonstrable embellishment or simple misunderstanding.\textsuperscript{48} The mention of Philolaus in the \textit{Phaedo} provides one fixed point chronologically: he must have been in Thebes before 399, and was therefore a contemporary of Socrates, though perhaps somewhat younger. Another fixed point can be found in the statement of Aristoxenus\textsuperscript{49} that the “last Pythagoreans,” with whom he was acquainted, had been pupils of Philolaus and Eurytus. Archytas spoke of Eurytus; and Philolaus, Eurytus, and Archytas are frequently associated in the tradition.\textsuperscript{60} Apollodorus of

\textsuperscript{47} Cf. Wilamowitz, \textit{Plato} II 86f; Frank 294 n. 1 (hypercritical: “timeless, mythical person”); Wulsemieier 566ff. The statement of Raven (KR 312), that “there is abundant information concerning Philolaus in the works of several later writers,” is at best misleading.

\textsuperscript{48} Croton: D. L. 8.84, Menon (DK 44A27). Tarentum: Iam. VP p. 144,11, \textit{In Nic.} 119,1, Vitruvius 1.1.16 (DK 44A6), Claud. Mam. \textit{De statu an.} 2.3 (DK 44B22), and especially Aristoxenus fr. 19 (D.L. 8.46). Wulsemieier (567) tries for a compromise: he was born in Croton, and lived after the catastrophe in Tarentum.—Plut. \textit{De gen.} 13,583a, in the dramatic framework of the dialogue, represents a Pythagorean named Theanor as having come to Thebes for the purpose of making offerings in honor of Lysis. He relates that Lysis, in his day, had escaped the great catastrophe of the Pythagorean society along with Philolaus. As against the report of Aristoxenus, who calls Lysis’ companion Archippus (fr. 18, followed by Neanthes \textit{FGrHist} 84F30), Plutarch’s novelistic treatment has no value; he has simply put Philolaus in place of the completely unknown Archippus. Olympiodorus (\textit{In Phld.} A 13, p. 9 Norvin, followed by Schol. Pl. \textit{Phd.} 616 = DK 44A1a) has Philolaus himself traveling to Thebes and says that he escaped along with Hipparchus (= Archippus (?). He has apparently combined elements from Plato, Plutarch, Aristoxenus, and the Lysis letter.—The statements about the poverty of Philolaus are inventions to fill out the story (Iam. VP 199; above, nn. 29, 36).—According to Syennesus (\textit{De duo astr.} 2, p. 134 Terraghi; not in DK; followed by Theophractus, DK 4419.19) Philolaus and Archytas were generals. This seems to go back to a source known to Cicero (\textit{De or.} 3.139) which asserted that Philolaus the philosopher was the teacher of Archytas the general.—The report that Philolaus was killed because he was believed to be aiming at tyranny is based on a misunderstanding of his source by Diogenes Laertius (8.84; Olivier 30; Wulsemieier 567; Maddalena 335); the source referred to Dion, who was mentioned just before.—The account of a meeting between Plato and Philolaus (D.L. 3.6) could be an invention based on the story of Plato’s plagiarism.—Aelian remarks, on the injustice of fame (\textit{VF} 1.123), ἐν τοῖς ἔλεγοι τοῖς πάλαι μακρὰ τῇ δόξῃ διέσχεσε Γρηγόριας ἀπὸ Λευκίμμονος καὶ Πυθαγόρας Δημοκρίτου, τῷ δὲ σοφίᾳ γονός ἡμῖν ἡ δόξα, δαόν ἄψων παῖε. This simply means that Philolaus’ fame came late—because of his book.

\textsuperscript{49} Fr. 19 = D.L. 8.46; above, ch. II 5.

\textsuperscript{50} In this order in the catalogue Iam. VP 267, p. 144,11; as immediate pupils of Pythagoras, ibid. n. 104 (see n. 51); Philolaus and Eurytus in connection with Plato, D.L. 3.6; Eurytus as a pupil of Philolaus in the legend, Iam. VP 139, 148 (related to Arist. fr. 193 and to what Hermippus tells of Pythagoras and Calliphon, Joseph. Ap. 1.164 — DK 19,2); Philolaus as teacher of Archytas, Cíc. \textit{De or.} 3.139 (the report of Phil.-Demosth. Prot. 46 only mentions Archytas; cf. Härder 43 n.1); Vitr. 1.1.16, Synesius (above, n. 48).

Cyzicus, who must be dated earlier than Epicurus, said that Philolaus was teacher of Democritus.\textsuperscript{49} Philolaus is, then, a name that appears in accounts of teacher-student relationships, which is connected with the Pythagoreans of Archytas’ circle, of Phlius, and of Thebes, but which is not associated with a body of legend or anecdote. In the \textit{Phaedo}, Plato represents him as having spoken about the prohibition of suicide, but as not having said “anything precise” about the reason for it.\textsuperscript{52} Aristotle says a saying of his,\textsuperscript{53} but it has rightly been emphasized that nothing in Plato or Aristotle would give reason to think that he had written a philosophical book,\textsuperscript{54} and apparently the ancient commentators on the \textit{Phaedo} could find nothing relevant to their purpose in a book of Philolaus.\textsuperscript{55} The meaning of all this is that the tradition of the special role of Philolaus as the first Pythagorean who published writings, and in particular of his special relation to the \textit{Timaeus}, cannot have been spun out of Plato and Aristotle,\textsuperscript{56} but must have been based on a fact independent of them—on the existence of a book by Philolaus in the fourth century B.C.; and we know from Menon that there was one. Philolaus’ special position resulted from the fact that this book was the first—and for a certain period, perhaps, the only—written exposition of Pythagorean speculation on nature and on number. Naturally, we shall attribute to this book, and to Theophrastus’ use of it, the doxographical reports pertaining to Philolaus.

\textsuperscript{51} D.L. 9.38 = DK 74.2 = 44A2. Apollodorus (v.l. Apollodotus) is dated from his connection with Hecataeus of Abdera and Nausiphanes, who is reported to have been teacher of Epicurus (Clem. Al. \textit{Strom.} 2.130; DK 75.74, 75)—A letter purporting to be addressed by Pythagoras’ son Telages to Philolaus, and mentioning Empedocles, was already pronounced spurious by Neanthes (\textit{FGrHist} 84F26 = D.L. 8.55, cf. 8.53, 8.74). Cf. Nicom. \textit{Ench.} 9, p. 242.13: Φιλόλαος δέ Πυθαγόρας διαβούς.

\textsuperscript{52} Pl. \textit{Phd.} 616e. Suicide was regarded as dishonorable in Thebes (Arist. fr. 502).

\textsuperscript{53} E.E. 1225a30 = DK 44B16; cf. above, ch. II 4, n. 139.

\textsuperscript{54} Frank 294, Raven in KR 310, 312.

\textsuperscript{55} To what extent that which follows in Plato—the \textit{phronematos} and the thought that a human being is ἐν τῶι κτήματι τοῖς ἰδέασι is to be attributed to Philolaus is uncertain. In any case Xenocrates, in his statement that the \textit{phronematos} is \textit{Tησαυρός καὶ έις Δίανοις ἀποκομβοθαί}, is independent on Orphism (Pl. \textit{Crat.} 400c) and not on Philolaus. Nor are the other commentators, led by Olympiodorus (\textit{In Phld.} p. 84f Norvin); therefore there was nothing on this subject in Philolaus’ book.—In \textit{Phd.} p. 113, pp. 87f Norvin) shows that he is thinking of them as allegorical, and therefore he is drawing on a late tradition, not Philolaus. Cf. above, ch. II 4.

\textsuperscript{56} To be sure, a book was attributed to Cebes, too—\textit{Cebetis Tabula} (see T. Sinko, \textit{”De lineamentis platonicos in Cebetis q.v. tabula,”} Isis 45 [1951] 3–31; he dates it in the second century A.D.). And a whole series of writings, of the most dubious authenticity, are attributed to Simmias (D.L. 2.124f; cf. Hobein, RE III A 144–155).
III. PHILOLAUS

From the time of Speusippus, Xenocrates, and Heraclides, a Platonizing interpretation took the place of what Aristotle gives us as the Pythagorean tradition; and in the succeeding era it held the field almost exclusively. Pythagoreanism no longer seemed meaningful except in this form, so that it was propagated by the Pythagorean pseudopigrapha and taken over by the neo-Pythagoreans. Only on certain special questions did one consult Aristotle; in general, the Platonic “system of derivation” was supreme, and there was even polemic, direct and indirect, against Aristotle. One exception stood contrary to this powerful and, for the most part, unified tradition—Philo1aus.

According to the explicit testimony of Aristotle, Plato replaced the opposition of πέρας and ἀπειρὸν which had figured in Pythagorean speculation, with the pair ἐν-ἀόρατος δόξα, so that the place of older cosmological ideas is taken by a system of ontological derivation, whose leading ideas are in the realm of the categories form-matter and active-passive. This system Speusippus regarded as Pythagorean, and in Aëtius too it is presented under the lemma “Pythagoras.” Nothing is said of πέρας and ἀπειρὸν. In the enumeration of opinions about the ἀρχαί, however, we read: Φιλόλαος ὁ Πυθαγόρειος τὸ πέρα καὶ τὸ ἀπειρόν. When, later on, the opposites πέρας and ἀπειρὸν enter the scene once more, the earliest reference, Nicomachus, adds a quotation from Philolaus. In this fundamentally important line of thought, in which, from the time of Theophratus, Platonic terminology and content had replaced the pre-Platonic Pythagoreanism that Plato and Aristotle themselves discussed, the name of Philolaus appears as the sole representative of the original Pythagoreanism.

Nor does this observation stand alone. The most fundamental difference between Pythagoreanism and Platonism, according to Aristotle, is χαρασμὸς, the discovery and definition of a supersensible, incorporeal realm of being, of which the Pythagoreans knew nothing. “Theano,” Syrianus, and Proclus contradict Aristotle on this point.60

Along with the Platonic development of a hierarchy of being went a terminological differentiation between ἐν and μονάς, though in two different ways. While Speusippus always called the highest principle, beyond being, ἐν and distinguished it from the μονάς in the realm of numbers,62 the “Pythagorean” tradition set up the μονάς as the basic principle and relegated the ἐν to the realm of number and of sense perception. All shared the objective of establishing clear terminological distinction between the realms of existence. But Theo adds, in the passage in which he deals with the distinction of ἐν and μονάς, Ἀρχαῖας δὲ καὶ Φιλόλαος ἀπειρῶν τὸ ἐν καὶ μονάδα καλοῦσι καὶ τὴν μονάδα ἐν. In Philolaus and Archytas he could find no warrant for the separation of levels of being as the Platonists thought of them. Actually, according to the Platonist the Pythagoreans uttered the words μονάς καὶ ἐν in a single breath. Once more we see Philolaus, with Archytas, standing apart from the Platonizing line of interpretation.

Even more striking is a similar situation in the tradition about the movement of the earth. According to Aristotle, the Pythagoreans believed there was a “central fire,” about which revolved, first, an invisible “counter-earth,” and then our earth, “as one of the stars,” and then in successive paths the moon, the sun, the five planets, and the fixed stars. This astronomical system, which has been celebrated in modern times as an anticipation of Copernicus, disappeared, except for a few traces, in the later Pythagorean tradition. (The ancient debate over the word ἄλλως, which Plato uses of the earth in the Timaeus, kept alive the idea of the earth’s moving.) According to the Hymnometria (25) the cosmos has the earth “in the middle,” and the same thing is presupposed by the biography of Phiotius63 and by

62 Cf. above, ch. I 3, n. 62, Iamb. Com. math. st. p. 17.15 (for the attribution to Speusippus, see Merlan, Philo, 96-128). Xenocrates, according to Aëtius 1.7.30 (fr. 15 H.), spoke of the Μονᾶς as the highest god, and according to Favorinus 5.7, of the ἐν (αιων) as the highest principle (fr. 16 H). Perhaps it was not until after Speusippus and Xenocrates that terminological exactitude came to be highly valued.

63 Theo Sm. 20.19ff. = DK 44150ff = 45780f.

64 Arist. fr. 203 (p. 139 Ross) = Alex. Met. 39.15: τὸν νόον μονάδα τε καὶ ἐν δεινοῖς.

65 Below, ch. IV 2, n. 16 (Tim. 40b).—The counter-earth is used in the interpretation of the Timaeus by Chalced. In Tim. 122 ff, further, Plut. Numa 11, De or. procr. 1028b. Por. VP 31 is unclear (ὑπερβολῇ... ἄριστον). Nicomach. Th. ar. 59.8ff, 82.3, 8 (ὑπερβολή, the moon the third from the bottom), Lydus Men. 4.51 p. 108.5.

66 Anon. Phot. 439b17ff under the 8 spheres of the heavenly bodies, the spheres of the 4 elements fire, air, water, and earth (cf. the Stoic teaching, D.L. 7.137, 155, ps-Arist. De mundo 392b2ff). Asclepius (Met. 35.19ff) counts the elements below the moon as one sphere and adds the counter-earth as a tenth, in an attempt to reconcile the “10 spheres” with the geocentric system. (Perhaps the same motivation is present in Por. VP 32 and Th. ar. 59.8ff, 82.3, 8; cf. also Origen, Comm. on John, 13.40.260).
Aétius.68 Theo quotes as Pythagorean doctrine the verses of Alexander of Ephesus on the harmony of the spheres, in which the earth is expressly said to be at rest in the center of things,69 and Timaeus Locus simply replaces Plato’s worrisome ἀλλεσφορικήν with the word ἠθρομένη.70

There is even outright polemic against Aristotle’s report. In the ancients’ way of thinking, diametrically opposed to the modern in this point, the idea of the earth’s moving was a false notion which had been scientifically refuted;71 and this forced upon them the task of showing that the Pythagoreans had not taught any such absurdity, but what was “correct.” Thus Simplicius, in explanation of the Aristotelian account of the Pythagorean system of the world, writes as follows (and quite similar words may be found in Asclepius and in an anonymous scholium to Aristotle):72

This is the way he understood the Pythagoreans’ theory himself. But those of them with more genuine knowledge understand by “central fire” the creative force which, from its mid position, produces life over the whole earth, and keeps warm the parts of it that tend to cool off . . . And they used to call the earth a “star” because it too, like the stars, is a creator of time; for it is the cause of days and nights. The part of it which is shone upon by the sun makes day, and that which is in the cone produced by its shadow makes night. The Pythagoreans called the moon “counter-earth,” as though to call it “the ethereal earth,” and because it intercepts the sun’s light, which belongs characteristically to the earth.

The expression about “more genuine” Pythagoreans here has been eagerly seized upon, because it seemed reasonable that there should have been a geocentric system as a precursor to the more complicated system described by Aristotle. Some have even asserted, mistakenly, that Simplicius is citing Aristotle,73 though the contrast is clearly drawn between the “more genuine” Pythagoreans and Aristotle “himself.” Also, there is no question here of the earth’s rotating.74 The “more genuine Pythagoreans” are represented as explaining three expressions: ἥπει τοῦ μέσου πῦρ, the earth as σταύρος, and the ἀντίκήρων. These are the distinctive ideas in Aristotle’s exposition of the Pythagorean system, where they appear in precisely this order,75 and the counter claim being made is that these words do not mean what they suggest to the unprejudiced reader. The “central fire,” they suggest, is a figurative expression for the life-giving force which, emanating from the center, permeates the universe; the earth is called a “star” because it is the instrument of time—a simple quotation from the Timaeus76—because the shadow of the earth makes night.77 Finally, the moon is called “counter-earth” as being ἀντίκήρων γῆ— an expression rather frequent in the late tradition, bound up with the notion of astral immortality and the moon-Hades.78 In a word, the intent of the “more genuine” Pythagoreans is to show that Aristotle basically misunderstood Pythagorean astronomy, that it did not include a moving earth, and that it did not imply anything else than the “normal,” geocentric cosmology which had been dominant since the Hellenistic age. Their method is a thoroughgoing allegorical interpretation of Aristotle’s words. Just as Aristotle’s exposition of the Pythagorean number theory was countered with the “original” words of Theano,79 so here the attack on Aristotle is armed with the claim of “more genuine” doctrine.

In this way, then, an effort is made to expunge a Pythagorean doctrine of terrestrial movement which was felt as an embarrassment.80

68 Aét. 3.13.1: οἱ μὲν ἄλλοι μὲνον τὴν γῆν, in contrast to Philolaus, Heraclides, Ephorus, and Democritus; Pythagoras is obviously included among “the others.” If, according to Aétius (2.12.1, 1.14.1; cf. below, ch. IV 1), Pythagoras divides heaven and earth similarly into 5 zones, this too presupposes a geocentric system. Aét. 2.29.4 ascribes the counter-earth to “the Pythagoreans,” citing Aristotle (above, ch. I 3, n. 26).


70 cf. p. 121.7 Theod. Proclus cites this in his interpretation of the Timaeus (III 130f).

71 For refutation of the idea that the earth moves, see Arist. Car. 2.14, and esp. Procl. Synt. 1.5, 7.

72 Simp. Car. 512a69, 1DK 58c37; more briefly Ascl. 35.24f; Schol. Carcd. p. 504b42f Brandis. Cfr. ch. IV 1.

73 Duhelm 20f; Frank 257; Cornford, Tim. 128; Klein, RHMI 1957, 124. Contra, Chernis, Plato 502, who suspects Lamblichus is the source (relying to Simp. Carc. 507.12f), Zeller judged the matter correctly, I 120f.

74 Schol. Coisl. 509a36; on the earth: πάντο ἐκ τοῦ σταύρου φέρομεν κάθε καὶ τὴν θερμάνσαν πώσα . . . was taken as evidence of a moving-earth theory by Boeckh, KauSy 96; Duhelm 89f; van der Waarden Astr. 38f (Heath, Aristarchus 250, thinks the scholium is based on a misunderstanding). The basis is merely Arist. Car. 293a2, reinterpreted.

75 Carc. 293a2, 22, 24.

76 Tim. 38c, 40c, 42d. Plutarch declares (Quaest. Plat. 1006b) that the passage does not imply any movement of the earth.

77 Cf. Emp. fr. 48 (though he is not consistent; cf. A30), and the μαθηματικοῖ of Porphyry (Stob. I 49.61; i.e. professional astronomers; above, ch. I 2, n. 76).

78 Cf. Cumont, Symb. 187 n. 6. The moon as ἀντίκήρων γῆ: Por. ap. Procl. In Tim. I 147 (presented as a teaching of the Egyptians; at II 48.17 further systematized) Macrobius, Somm. St. 1.11.7, 1.19.10 (a doctrine of “physici”). Ὀλυμπία γῆ: Plut. De def. or. 13.4.6: ὀφθαλμὸς ὡς ἀντίκήρων: Clem. Al. Strom. 3.139. Ps.-Arist. fr. 245 (from a collection of problematica whose origin is doubtful) gives a different interpretation of the doctrine that the earth is a star and the moon is an earth, namely that moon and earth consist of the same elements. Only a part of the moon is ἀντίκήρων, according to Plut. De fac. 29.564c: in Cic. Tus. 1.68, the inhabited southern zone of the earth is ἀντίκήρων.

79 Above, ch. I 3, n. 52.

80 Proclus sharply criticizes Aristotle for understanding the word ἀλλεσφορικήν in the Timaeus as denoting a rotation of the earth (in Tim. I 137.7f).
or the book was forged on the basis of Aristotle’s accounts. In the first case the book must be regarded as genuine, that is, composed by the Pythagorean Philolaus about 400 B.C. or a little earlier. There is no basis on which to argue that the book Aristotle had was a forgery, or that an authentic and ancient Pythagorean book had been attributed to Philolaus. Naturally, the question would still remain, whether all the fragments we have come from this original work, or whether some or all may come from a revised edition, an imitation, or even a forgery made to supply its loss. Nevertheless, the Philolaus tradition, however troubled, would even in this situation bear witness to a source of tremendous importance.

Aristotle never names Philolaus, except when he cites that memorable apophthegm in the Endean Ethcs, and critics have made this their principal argument against the authenticity of the fragments. If Aristotle used this book, they think, his silence is quite “inconceivable.” There is a prior question to answer, however: what were Aristotle’s sources for Pythagorean teachings? Original Pythagorean writings? Oral tradition? Writings of the Platonists?

This last possibility is excluded for most of Aristotle’s reports, for, quite unlike the Platonists, he makes a sharp distinction between Pythagoreanism and Platonism. The role of oral tradition ought not to be underestimated, though here too Platonists were the intermediaries.

84 So Bywater (60), following Schaarschmidt (55), and recently esp. Raven, PyEl 98ff, CR 90ff, Ch. III, 2, n. 91.
85 So Frank 390f, 327f, and passim. He tries to show that the forger was Speusippus (311ff); similarly Bollinger 44f. Against this, above, ch. 1.
86 Wilamowitz, Platon II 93, thinks of an original Pythagorean writing, later than Aristotle, which was then later fathered on Philolaus; but this neglects the close connection with the account of Aristotle.
87 The Philolaus fragments are sometimes cited in this spirit, as a “forgery” which nevertheless includes valuable material; e.g., Ross, PTI 160f.
88 1253a20 = DK 44B16; above, ch. II, 4, n. 139.
89 The word “inconceivable” is Burnet’s (EGP 284 n. 2); “almost inconceivable” in Raven in KR 310, cf. PyEl 100; Schaarschmidt 14, Tannery Mê IX 323f; Wilamowitz, Platon II 88, 93 (“Aristoteles und Aristoxenos kannten es [Philolaus’ book] nicht”); Cherniss, Pres. 37 n. 140 (“Aristotle’s silence certainly implies that he had not seen the book of Philolaus”), 386f. For a different answer, see Zeller (KISch I 136–144) and A. Burnes, C&G 25 (1964) 91–128.
90 Tannery, Mê IX 234, thought that Heracleides was a principal source of Aristotle; Frank, MS IX 235, compared with other Platonists (258f, 390f, 327f); similarly Frank nominated Speusippus, along with other Platonists (358f, 390f, 327f); Speusippus may be the source for the “table of opposites” (above, ch. I, 2).
91 What Archytas told about Eurytas may have been passed on, as a curiosity, in the Academy and the Peripatos (above, ch. I, 2, n. 69). On the Ἀρχαῖοι ἀκροατζεῖς mentioned in connection with Speusippus (Th. 821fr), see below, ch. III 2.
through whom he knew Pythagoreans, so that he is hardly likely to have derived from this source the decisive points that he was able to make against the Platonists. He expresses himself in very definite terms: οὐ μὲν οὖν Πυθαγόρειοι τὸτερον οὐ ποιοῦσιν ἡ ποιοῦσι γένεσιν οὐδὲν δεὶ διστάζειν φανερῶς γάρ λέγουσιν ώς... Aristotel is citing the "clear words" of the Pythagoreans in the discussion as to the sense in which one may speak of a genesis of number and thus of the whole; and his polemic is aimed against a Platonizing reinterpretation.92 The fact that he can insist on the exact wording in this way shows that he has a written source, for oral reports are not amenable to such exactitude. The use of written sources is also suggested by the way in which Aristotle cites Pythagorean technical terms, distinguishes their statements from their "assumptions," and sometimes even states that this or that question remains unanswered.93 We cannot suppose, however, that the book or books he read claimed to be written by "so-called Pythagoreans" (καλούμενοι Πυθαγόρειοι). The name of an individual must have been attached to each. To put it bluntly, even if we disregard the problem of Philolaus, the "inconceivable" is just what happened. Aristotle based his account of Pythagoreanism on at least one book, whose author he never named.

The Anonymus is not Archytas;94 what Aristotle cites from him indicates an advanced degree of abstraction, though the evidence is far from plentiful. In addition to his books on the Pythagoreans, Aristotle wrote three περὶ τῆς Ἀρχιτεχνὸς φιλοσοφίας, and this may be taken to show that he distinguished clearly between Archytas and general Pythagoreanism.95 Archytas on occasion mentioned earlier Pythagoreans,96 but he was an independent scholar and surely was not concerned to write as detailed a doxography as would have been necessary if Aristotle were to get all his facts about Pythagorean philosophy from it.

The only name that is closely connected, in Aristotle's reports, with the philosophy of the "so-called Pythagoreans," is that of Eurytus; but he seems himself to have essayed an extension of the number theory. Now, from Aristoxenus on down, the tradition constantly associates Philolaus and Eurytus.97 Philolaus was the teacher of the Pythagoreans from Thebes and Phlius whom Plato knew. So we see the skimpy biographical evidence on Philolaus converging with what can be deduced from the doxography about his book; namely, that it was to Aristotle, in a way, an authoritative exposition of Pythagorean number theory.

Philolaus does not appear as a rebel against the traditional obligation which Pythagoreans and, to a certain extent, Platonists felt, to recognize that all doctrine originating in their own minds was only extension and confirmation of the ancient wisdom of Pythagoras.98 At least his pupils, and Plato too, must have understood Philolaus' work in this way. It is not only from this point of view that we can understand Aristotle's strange uncertainty about the chronology of the Pythagoreans, as well as his silence about the originator of the system. Subsequently the Platonic view of Pythagoras became dominant, and that book was no longer an accurate presentation of Pythagoreanism, as it was now understood. Its doctrines now appeared to be the private δόξα of the author, Philolaus.

The thesis, on the other hand, that the book of Philolaus was a forgery that took its materials from Aristotle takes no account of the general development of the Pythagorean tradition. It is simply not true that the picture of Pythagoreanism standard in late antiquity was derived in its essentials from Aristotle's exposition.99 The Platonic

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92 Met. 1094a17ff., above, ch. 1, n. 41. Zeller pointed out the significance of the passage (Klehr I 138ff.). Schaarschmidt (82ff) denied that Aristotle had written sources for Pythagoreanism.

93 Χρον., Μεευ. 492a50 (above, ch. 1, n. 96), μημικες, Met. 987b11 (above ch. 1, n. 82), ουδεν γαρ δοκεται και λεγοναι... Met. 990a14, 15, οποιεν διακης, Met. 1038b20.

94 P. Frank 361: "Archytas seems to have been the only Pythagorean from whose writings Aristotle learned about Pythagorean philosophy." This is repeated by Wieldenmier, 577. Cf. also Frank 135ff, with n. 107, and pp. 97, 106. Aristotle's citations of Archytas, though, do not yield much: Pol. 1340b26 (DK 47A10) Ἀριστοτέλης πλαταγή, a child's toy (Frank, 339, rightly compares Pl. Leg. 653d); Rhet. 1412a12 (DK 47A12) τοιαύτα δεν διανικήσαν καὶ βιαζόν, an apothegm that is reminiscent of the auscultator. The "definitions" of Archytas (above, ch. 1, n. 104) take us a bit further. At Arist. fr. 199: Theogn. 22. 30, συμφέρει δε τοιαύτας καὶ Ἀρχίτεχνος (DK 47A21), is obviously an addition by Theogn (who on another occasion connects Archytas with Philolaus, above, n. 64). Frank states this, though with hesitation (257 n. 2), only to affirm later (361) that Aristotle "cites as the special doctrine of Archytas (fr. 199 Roze) a view which in another passage (Met. 996a19) he presents as general Pythagorean." Philosophically interesting are Archytas Λ2 (ch. 1, n. 106), Λ23 (1 J, n. 150), and Λ24 (Eudemus fr. 65).

95 Cf. P. Moraux, Les listes anciennes des ouvrages d'Aristote (Louvain, 1951) 106, 201, 301 (DK 47A12). Frank (n. 335) considers referring to this book Philop. A. mund. p. 522. 20 (on the πειπτον αυλαμα). Some kind of connection between Archytas and the Timaeus is indicated by the title Εκ τον Τιμαίου καὶ τον Ἀρχιτεχνον (no. 94 in D.L. 5.25, no. 83 in the anonymous catalogue); cf. above, ch. 1, n. 4.5, 1 J, n. 166.

96 Cf. fr. 1 and the mention of Eurytus (above ch. 1, n. 69).

97 Above, n. 50.

98 Above, ch. 1, n. 36.

99 The view of Schaarschmidt, p. 15.
interpretation is dominant, and the evidence of Aristotle was ignored, distorted, or rejected. The “forgeries” were not intended to deceive philologers, but to provide a cachet of age-old wisdom for a doctrine adapted to the needs of the writers’ own times. This is the reason why the pseudo-Pythagorean writings take Platonism as their basis. If the Philolaus reports alone, along with some material from Archytas, form an exception to this general trend, then their agreement with Aristotle is not a reason to suspect them, but quite the opposite: an indirect proof of their authenticity. Of course, only a keen and attentive study of all the details, especially in the directly quoted fragments, can enable us to decide, in each case, what reason there may be to believe that they are authentic or spurious. But the external tradition itself shows that the situation of Philolaus is different from that of the great mass of would-be Pythagorean authors. A book made him famous; it was still in existence in the fourth century; and it showed traces, at least, of ancient, pre-Platonic Pythagoreanism.

2. THE SPURIOUS AND THE GENUINE IN THE PHILOLAUS FRAGMENTS

If any genuine fragments of Philolaus’ book have been preserved, they are the most important, because the only original, documents of early Pythagorean philosophy. This statement can easily lead to a double misunderstanding. One expects to find bedrock, so to speak, forgetting that Philolaus was a contemporary of Socrates and Democritus, Gorgias and Diogenes of Apollonia, and that he wrote and taught not only after Parmenides, but later than Zeno, Melissus, Anaxagoras, Empedocles, and Protagoras. Again, one is likely to expect to find thoughts of striking and unique originality, overlooking the fact that in the second half of the fifth century, thinkers were much more concerned with assessing the many original but contradictory assever-

ations of their predecessors—comparing, reconciling, adapting. It was no longer a day of lonely prophets but one of far-reaching debate. A statesman like Melissus of Samos might take part in it, or a poet like Ion of Chios; physicians, too, were beginning to formulate in written terms the scientific basis of their art. A book by a Pythagorean, in this period, cannot have been so much like an erratic boulder as a link in a long chain of tradition.

We are inclined to see in Philolaus nothing but the transmitter of older Pythagorean teachings, and in fact all indications are that the Pythagoreans themselves saw nothing more in his book than an expression of the wisdom of Pythagoras. But before Philolaus there was no written exposition of Pythagoreanism, and the transition from oral to written teaching is much more than a matter of externals; it means a fundamental transformation, of content as well as of form. An orally transmitted doctrine, in spite of all the special training of the memory in ancient times, notably among the Pythagoreans, must always be enmeshed in the very fabric of life, always exposed to psychological forces that can mold and transform it. No matter how persistently one holds to a traditional knowledge, even if the basic attitude and intentions remain unaltered, changes of detail will appear. As needs change, from time to time, different aspects will move into prominence. Oral teaching is always directed toward specific learners, and therefore its nature is determined by the listener as well as the speaker. In particular, the form of oral transmission will alter, because it must each time be impressive—except that poetic form guarantees a certain consistency. On the other hand, a written work is free from the restraint of a specific situation of speaking and hearing. It makes a claim to be valid in itself, independent of any special καὐμός, and not needing any further intervention by the writer. A book in prose, furthermore, renounces the embellishment of verse, because its message is supposed to be dependent entirely on the subject matter, and not at all on any predetermined formal structure. The first prose book to prefigure and exemplify this development was that of Anaximander. It is only when linguistic expression attains this kind of objectivity that scientific or philosophical discourse really becomes possible; from the time of Anaximander, Greek science and philosophy

10 It was modern critical method, especially the work of Zeller, that first set Aristotle’s testimony, clearly distinguished from the Platonizing and neo-Pythagorean conception of Pythagoras, in a position of authority which, as is easy to forget, it certainly did not enjoy in the ancient world.

1 Diels, Hermes 1893, 417ff, even considered the etymology φιλέω-φιλέων in Menon’s excerpt (A27-28) as borrowed from Prodicus (fr. 4); cf., per contra, Fredrich 37 n. 1, Olivieri 29f, 45f—Frank (304) states it is impossible to suppose that Plato harked back, in the Philobus, to a book “almost 100 years old”; but at the time of the composition of the Philobus Philolaus’ book was scarcely more than 50 years old.

2 Bywater (29) does not see in the fragments an “original effort of mind,” and athesizes them on the basis of this “test of excellence.” But maybe what was original in Pythagoreanism was not philosophical expression.

3 Above, ch. III 1.

4 DK 12A7; cf. A1. Placeredex of Syros, who is also named as the first writer of prose (Suda s.v. — DK 7A2), may be dependent on Anaximander (von Fritz, RT XIX 2030f).
presents itself in this form, of written exposition. This means—though the fact has scarcely ever been clearly recognized—that, if Pythagorean doctrine was not committed to writing before Philolaus, then there did not exist, before Philolaus, any Pythagorean philosophy, in the Greek sense of the word, but only a different kind of thing: a lore or "wisdom" consisting of disconnected teachings about the world, gods, and human beings, having its foundation in a specific way of life and transmitted in individual maxims. This "wisdom" and way of life were variable in detail and lacked logical foundation or systematic and conceptual coherence; in fact they consisted in our familiar aeismata, the doctrine of transmigration, and the bios Pythagoreos in which they were rooted.

If a Pythagorean, in a situation like this, undertook to write peri phusis, this was a metafasis eis allo gevo. It meant the adoption of a manner of exposition foreign to the Pythagorean tradition, the product of a different kind of development. The tools for such an enterprise—concepts, definitions, modes of argument—come from a non-Pythagorean background. What we should expect to find then, would not be a unitary and original product, but a hybrid eclecticism, a conglomeration of Pythagorean attitudes and borrowed conceptualization. For there is clearly nothing in the Pythagorean teaching itself that would demand written consolidation, or formulation in philosophical and scientific terms.

August Boeckh decided, with relation to the Philolaus testimonia as a group, that "the only solution is to recognize all we have as genuine or to reject all as spurious." Those who doubt its authenticity have been happy to cite this sentence, for it opens up the possibility of affecting the whole by an attack on a part—if they could loosen one column, the whole structure would collapse. But this is a hasty oversimplification, especially since Boeckh's conciliatory assumption that Philolaus wrote a single work in three books has turned out to be based on a misunderstanding. Hermippus speaks of "one book" of Philolaus, and Demetrius of Magnesia gives it its title, peri physeon, and cites the opening line, an established bibliographical custom. This citation is closely related to the long excerpt in Stoebus, to which he gives the heading peri kosmon, but from which Nicomachus gives a fragment with the label Philolaos . . . en tō πρώτῳ φαινήσας. Stoebus' evidence for the title is not important, for his own section, within which Philolaus is cited, bears the heading peri kosmon. Nicomachus in the Theologumenon has a further fragment of a peri physeon, and this title is also mentioned by Theo Smyrnaeus. Proclus and Boethius are both doublets dependent on Nicomachus. The doxographical

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9 Above, ch. III 1.
10 D.L. 8.85 = DK 44B1: tovov phos Deinitos en 'Omphamous prwton ekdoxan tovov Pythagorov <biblia kai epigrapha> peri physeon, de arxh. Dei's supplement can scarcely be right, and his reference to D.L. 3.9 is certainly a mistake (above, ch. III 1, n. 28). Perhaps there is no lacuna, and the object of ekdoxan is the title (cf. Strabo 1, p. 15: peri tov prwton ekdoxan τον αυτον πραγματευτ). Plut. Rom. 8 [P. Pappas: Καλάς de δικαιωσιμον ekdoxan Ρώμης κητώς]. The indefinite plural de περί necessitating an interpretation of D.L. 3.9 as peri kosmon, as suggested by most other commentators, is not necessarily mean that more than one book is intended. After all, only one "beginning" is being quoted. In a similar way, D.L. cites the incipit of books of Diogenes of Apollonia (5.7, 6.81), Pherecydes (1.19), Alcmaeon (8.83). At bottom lies the practice of the library at Alexandria (U. von Wilamowitz-Moellendorff, Antigons von Karytos [Berlin, 1881] 323; Wiesers, Mhntopos 1942, 23).
11 Stob. 1.21.7 = DK 44B2, 4.7. There was another excerpt in the lost prologue of Stoebus; the only trace remaining is the marginal note, Philolaus (Stob. 1 p. 154 AAPP.). Nicom. Ench. 9, p. 221.11 (the title cited); pp. 252.17–253.3 = DK I 409.10–410.3. Evidently the short, very corrupt citation in the Hagiopolites is from Nicomachus (M. A. J. H. Vincent, Notices et extraits des manuscrits XVI 2 [Paris, 1847] 268 = DK 44B6, I 409.105: πνευματικον φαιλοας, en tivn poimati au'tov, προς των γυναικων Πυθαγορος ekdoxhamenos. See also Tzetzes, cited ch. III 1, n. 28.) Further, Nicom. Ench. 12, p. 264.3 (= fr. 6, DK 1 410.11); Ar. 2.18.4 (= B3, DK 1 407.21) (translated by Boeckh in Ar. 2.32, p. 126.7ff.); Nicom. Ar. 2.26.2 (= A24). See next note.
12 Th. ar. 25.17 (= fr. 15); on 74.10 (= A12), see below, nn. 41–44.
13 106.10 (DK ad fr. 11); below, n. 169.
14 Procl. In Tim. I 14.4, 117e.28ff. (ἐπειρασμών καὶ άπειρων = fr. 1.2–12; cf. Nicom. Ar. 2.18.5, 1.11, p. 132.7, p. 132.7). Probable the expressions διέσχειν and άπειρον are from Nicomachus (Procl. In Tim. II 168.28f; the source is given as τοις παλαιον ταις); I 190.f (cf. A26, B6); perhaps also the report about the gods of various angles (A14; cf. below, ch. IV 3); cf. Theol. Pl. 1.4, 9.9 Portus: καὶ γαλ τοις άπειροις άνασε τοις θεοις καὶ τα σχήματα, καθαρόν λογουν απα τα εκεινα ισοτεροσ σπουδαος. — Damascius is probably drawing directly on Proclus at Prin. 111.12 Ruelle (ἐπειρασμών καὶ άπειρων), cf. I 103.5: τοις peri φαιλοας. Also, Damascius may have drawn information about the dedication of certain geometric figures to various gods (A14) from an account of Proclus which has been lost.
15 Above, n. 11. Boeckh surely derived even details of music theory from Nicomachus, whose musical writings are only preserved in fragments (Mus. 3.5 = A26; 3.4 = n. on B6). Lamblichus, too, can have taken his Philolaos citation from Nicomachus In Nic. 7.24 = fr. 3; repeated by Syrianus, at Met. 147.17).
reports in Aetius\footnote{A9, 15–21. The “indirect” proof of authenticity applies for these accounts (above, ch. III 1).—Also from the doxographical tradition are the notes in Cens. 18.8 (A22), Macrobius Somn. Sc. 1.14.19 (A23), Sext. Emp. Math. 7.92 (A29; below, n. 55).} cohere closely with these citations, suggesting the probability that this book Περὶ φύσεως from which Demetrius of Magnesia (time of Cicero), Nicomachus, and Stobaeus quoted was the one that Theophrastus read.

Caution must be the watchword, however. Nicomachus knew of more than one book attributed to Philolaus, if he could cite the “first,” Stobaeus has a fragment from a book Περὶ ψυχῆς,\footnote{Stob. 1.2.3 – fr. 21.} and both Stobaeus and Proclus mention a title Φιλολάου Βάκχαι.\footnote{Frr. 17–19; below, nn. 140–148.} Claudianus Mamertus speaks of “many books” that Philolaus wrote, and cites a “third book Περὶ ρυθμῶν καὶ μέτρων.”\footnote{Frr. 22; below, n. 45.} And in addition we have a long passage given without title by Stobaeus,\footnote{Fr. 11–12; below, nn. 169–185.} and various isolated citations.

What the tradition offers us, then, under the name of Philolaus, is not a unified picture, but a great variety. The first task that suggests itself is to demonstrate its homogeneity, but there seems hardly any prospect for success in that, if only because of the open contradiction between the “one book” of the older tradition and the plurality of books in the later. It is true, though, that from the beginning a certain nucleus seems to be discernible: fragments 1, 2, and 4–7, along with the accounts of Aetius and Boethius, dealing with ontology, astronomy, and music theory.

We may take as the starting point for a closer examination of the tradition the fact that one of the longer fragments is certainly spurious, namely the paragraph on the world soul (21). Since Zeller pronounced it spurious, no one has seriously defended it, and it is unnecessary to repeat all the arguments.\footnote{Zeller I 169–3, 476.1, Kλήρος I 143ff; Schaarschmidt 24ff; Bywater 30ff; Frank 382ff; Moreau, AMc 145ff; Thesleff, Testos 150ff. A few points may be emphasized, beginning with the Aristotelian doctrine of the eternity of the world ζὲ αἰώνας εἰς αἰώνα, and the presence of technical terms like ἀνάρτησις (DK I 418.6), τὰ γεννημένα πατέρα καὶ δημιουργοῖ (I 418.11; cf. PL Tim. 37c; here even Boeckh is forced to admit the intrusion of late terminology; cf. Frank 290 n. 1), φύσει διαπνεύμονοι (I 417.14) like the Stoic πνεῦμα δύναμις (for such Stoic colouration in later Pythagoreanism, see Cic. Nat. d. 1.27, Sext. Emp. Math. 9.127). At I 417.14, the meaningless εἰς ἀρχήν of the MS tradition should be replaced by ἐξ ἀρχῆς (as in the inscription cited ad loc. in DK, CICG 5235.2, SIGR 712.1). It is an adjective, formed after εἰς ἀρχῆς as ζωφόροις after ζωφόροις, and is predicative with ἀρχῆς in line 13. On the question of the ἀρχη κατακεκλωμένη see Arist. Cat. 289b6ff.—Macrobius cites as the doctrine of a secta of Platonici sentences that show extensive verbal agreement with Philolaus fr. 21 (Somn. Sc. 1.11.5f; noted by Capelle, DE lua 9).—Frank, in accordance with his general theory, must credit this fragment to Speusippos, although this compels him to consider the possibility of “spätere Retusen” (290). Moreau (AMc), attempting to find intermediate steps between Plato and the cosmology of the Stoics, would like to date the fragment in the age of Aristotle (149), making it approximately contemporary with the DE cade; but he cannot convincingly demonstrate parallels to φύσει διαπνεύμονοι (pp. 136ff).—Rostagni (Verbo 53) maintains that at least in content the fragment preserves genuine material, in spite of its late style; one wonders if it is not rather the case that Rostagni’s version of Pythagoreanism shows Platonist features.} This passage is an example of Hellenistic

“cosmic piety,” the resultant of the hymn-like eloquence of the Timaeus and Aristotle’s doctrine of the eternity of the universe. Significantly, the fragment has a good deal of wording like that of Ocellus. If Harder is right that it is Ocellus who is derivative,\footnote{Above, ch. I 3, n. 118.} then this fragment must have been composed, at the latest, in the second century B.C., and the one certainly spurious fragment would be the earliest attested.

The decisive question is whether this fragment is closely connected with the rest. Frank tried to prove this,\footnote{A18; cf. ch. III 1, n. 82.} but his arguments do not suffice. There are points of contact with some elements, contradictions of others. In contrast to the hymn-like celebration here of the eternity of the world, ζὲ αἰώνας καὶ εἰς αἰώνα, where the “creator,” as in the Academic tradition, is to be understood as a didactic concept, not a reality,\footnote{Frr. 1, 6, 7, 17; cf. Arist. Met. 1091a13ff, below, ch. I 3, n. 119.—Frank (288f) equates the eternity of the world in fr. 21 with the eternity of being in fr. 6; cf. below nn. 86–87.} one of the other doxographical notes speaks about the φθορά τοῦ κόσμου,\footnote{Stob. 1.22.1d Aet. 2.7.7 (missing in ps-Plut.) 1D 44A16.} and expressions like ἀρμόχθη, συνέστα, ἥρετο γίγνεσθαι, and ἀρμονία ἐπεγένετο in the word-for-word citations\footnote{285ff. Contra, Mondolfo in ZM 376f. It is admitted that there are points of agreement with the second part of A16 (cf. below, n. 37), A17 (below, n. 38), and fr. 22. But it is wrong to interpret ἀρχη κατακεκλωμένη in fr. 13 as “world soul”; Ψωφία (= “life”) is explicitly distinguished from it. The emphasis on the one world (fr. 17; presupposed in fr. 11, 2, and 6) does not prove anything; this is an old controversy (Aet. 2.1.2–3), and Aristotle ascribes the doctrine that there is but one world expressly to the Pythagoreans (fr. 201); cf. below, n. 26.} presuppose an origin for the “order” of our world. The contradiction is evident.

Under the rubric Περὶ τάξεως τοῦ κόσμου, Stobaeus\footnote{Above, ch. I 3, n. 118.} first ascribes to Philolaus the cosmic system known from Aristotle: the order is
the second part was genuine. But the stronger arguments are on the other side. The Pythagorean origin of the ideas of the second part is only a conjecture; it is through Platonism that they had their influence. On the other hand, the cosmology of the first part is guaranteed Pythagorean by the testimony of Aristotle, while later Pythagoreans tried to get rid of it. The idea that the highest heaven consists of the ἐλεκτρίνα τῶν στοιχείων comes from interpretation of the Timaeus. The threefold division of the cosmos corresponds to ideas of the Platonists, and the expression φασματίδος γένεσις shows in both style and content a relationship to the fragment on the world soul. Thus the second part bears the mark of Platonism, while the first corresponds to the Pythagorean doctrines attested by Aristotle, so that there is no question of the second part being authentic; this question rises only in the case of the first. Finally, it is likely a priori that if one part is a

23 Mnemon 1942, 25. The first part is explained as the product of an error by Theophrastus, who, he thinks, worked the later system into his interpretation of Philolaus.

24 Above, ch. III 1.


26 The Timaeus separates terrestrial γένεσις from the regularity of the world of the stars just as it does the ποικίλλα of the planets from the παράστημα of the fixed stars. The resulting tripartition was emphasized by the ancient commentators (e.g. Aratus' ap. Theo Sm. 148 13ff. Athen. Phot. 39ββ7ff. also gives a special status to the ἀπόλος. The翔 ὁρισμός is used by the Platonists, region of region, region of stars, or heaven, and highest threefold division of the world (sublunar region, region of stars, or heaven, or what is beyond the heaven) in connection with divine powers and psychic functions (e.g. Ph. 651f.); Heracleides (fr. 95) divides the cosmos into the realm of Zeus, that of Ἀνεβοῦ (39ββ7), and that of Ἑδῆς (745; 745f.)

27 On this point even Boeckh admits post-Platonic terminology (100). Cf. fr. 21 (I 418.3), where the words γενεσίων καὶ μεταβολῶν are used to characterize the sublunar sphere.
Further, Nicomachus ascribes to Philolaus a number system which represents a development of being in numerical stages: 1. point; 2, line; 3, plane surface; 4, solid; 5, ποιότης καὶ χρώμα; 6, animation; 7, νόης καὶ γνώσει καὶ τὸ ὑπ’ αὐτῷ λεγόμενον φῶς; 8, ἀριθμός, φύλλα, μέγες, ἐπίφανες. This scheme contradicts the report of Aristotle, according to which the Pythagoreans called the plane surface χρώμα, that is, they were not able to distinguish, even in terminology, between “surface” and “color.” In general, this gradation of being, and in particular the order of geometrical forms, is Platonic and not Pythagorean. In fact, Plutarch cites the very system here attributed to Philolaus, as Platonic. Nicomachus and Proclus made abundant use of this “Pythagorean” scheme, which may also have been included in a ἤτοι λόγος attributed to Pythagoras.

Cladiusian Mamertus’ report of a doctrine on the immateriality of the soul comes from a neo-Pythagorean source, where it was right next to a spurious Archytas fragment, the contradiction of Philolaus’ fragment 14 (in one σώμα-σήμα, in the other “love” of the soul for the body) proves that they cannot both be genuine. 

34 Th. ar. 74.10 = DK 44A12. 
35 Sens. 430b20: cf. above, ch. 1, n. 95. 
36 Quaest. Plut. 3.1.1002a: cf. above, ch. 3, n. 66. Somewhat different is the account in De E (390 c-d) where 5 5 is equated with animation. Philo must have known a similar system, since he equates 5 with αἰθήματος (Qu. in Exod. 2.9). 
Individual apophthegms, in the tradition, are always problematical. The sentence cited by Clement about the punishment and burial of the soul in the body is based on παλαιοὶ θεολόγοι τε καὶ μάντες, but this, and the close correspondence with Aristotle’s Protrepticus, are in themselves cause for suspicion. It was only by a misunderstanding that Boeckh thought this saying of Philolaus was cited in Plato’s Gorgias. It seemed so natural to reconstruct Philolaus’ doctrine from the passage in the Phaedo that names him, that nothing more than this seems to lie behind the explicit statement of Athenagoras about the φρονήσεως. The saying of “Philolaus” in praise of number (fr. 23) is as spurious as its neighbor attributed to Hippasus. Through a misunderstanding of Philo, Philolaus is credited with a seemingly monotheistic saying which Lydus ascribes to Onetor. Philolaus equated the number 7 with Athena, the “motherless,” and this agrees with Aristotle. The thought that the number 2 is the “consort of Cronus” (Rhea) may belong to the same tradition. There is no reason to suspect the authenticity of the apophthegm passed along by Plutarch, according to which geometry is the ἀρχή καὶ μητρότος τῶν ἄλλων μαθημάτων, or the thought that the κρύστηρα is the λόγος ἀπὸ τῶν μαθημάτων περιγυμνάμενος, though the terminology is influenced by the more abstract thought of later times, reflected in the doxographers. There is no occasion to attribute to Philolaus the definition of ἀρμονία as πολυμεγέντως ἐννοιας καὶ δίκας φρονεώντων συμφρόνησις.

We cannot regard the word σοφός (cf. above, ch. I 3, n. 157) or the use of allegorical interpretation as a sure indication of Pythagoreanism. There are, however, some indications of Pythagorean influence: “Hades” is located in man’s lifetime, as in Empedocles (cf. ch. II 3, n. 80; IV 4); ἀφθονος and σιγή are thought to hold together the forces of the ψυχή (cf. ch. II 4). The background to be assumed is a writing similar to the one introduced to us by the papyrus from Derveni.

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47 The word θεολογία is first attested at Pl. Rep. 379a; θεολόγος is common in Aristotle. Cf. Jaeger, Theol. 4; Vlastos, PhiloQ. 1952, 102 n. 22. The latter points out that parallel formations like metereologus, epimelados, μνημολόγος are common, and that ἰδοὺ θεολόγος occurs in Emp. fr. 131. V. Goldschmidt, “Theologion,” REG 63 (1925) 20–42, ignores the Philolaus fragment. With fr. 14, μαρτυρεῖται δέ μοι οἱ παλαιοὶ θεολόγοι τε καὶ μάντες, compare Philo Op. 100: μαρτυρεῖ δέ μοι τὸ λόγον Φιλόλαου, and esp. Arist. fr. 60: καθιστήσαντι οἷς τὸ τελείον λόγον... τοῖσον γάρ θεοί οἱ ἁρχαίοντεσ θεοί (lamb. Protr. 42.73), or in Latin, “ut interdum veteres illi sive vates in sacris initis tradendis divinæ mentis interpretés... alicui vidisse videantur” (Cic. Hortens. fr. 95). Likewise, with fr. 14: ὅδε δίδα τις τιμορίας, Arist.: “ob aliquo scelera” (Cic.) or διδάσκαι τὴν φυσὶν τιμοριῶν καὶ κινήματος καὶ καταλείψοντες τῃ θεολογίᾳ (lamb. Protr. 47.25f); fr. 14: αἱ φυσὶ τῷ σώματι συνέλεξασκε... Arist.: η σύνεξε... πρὸς τῷ σῶμα τῆς φύσεως (lamb. Protr. 48.25f). By “priests of the mysteries” Aristotle clearly means the Orphics. But at Pl. Crat. 400c the expression σώμα-σύμμα is distinguished from their view (Wilamowitz, Oldg. II 199; Thomas 51; Dodds, Iatr. 169 n. 87); and we may suppose that, if it is not Orphic, it is likely to be Pythagorean. The ostensible Philolaus, however, contaminates the two and leaves us to guess whether he regards Pythagoras as an “ancient seer.” The σώμα-σύμμα idea was familiar in later times (Cic. Rep. 6.14.71, Macrobr. Somm. Sc. 1.109f, Serv. Aen. 6.127f, 439) — Clearchus, fr. 38, quotes a similar expression from the Pythagorean Euthydemus; but we do not know whether this was a historical character (Wellmann, RE VI 1539; Wehrli 59).

48 At Pl. Gorg. 493b–d, Socrates–Plato cites a σοφὸς (ὡς ὁ πρὸς ἐμαί λέγως, 493b) who teaches that life is death and the body a grave, and bases this doctrine on allegorical interpretation of what μαθηματών κοιμὸς ἄνεος, ὡς Ἰακεμίως τυψή Ἡσαύριος had said. The “mythologist” and the exegete must be distinguished from each other, though this is often forgotten. (The wrong interpretation is in Boeckh 183f, Bywater 478f, Schaarredendorf 6f, Carcopino, Bab. 285f, Thomas 53f, Long 74f. Even Wilamowitz, Platon II 89, does not draw the line sharply enough between the “mythologist” and the interpretation. For the correct solution, Frank 298ff, Willemse, CHP, UCPCP 1944, Dodds, Iatr. 225 n. 5, Gorg. 296ff.) The “mythologist” spoke of Hades and the punishment of the uninitiated, of carrying water in a sieve—a teaching of the mysteries widely known in the 5th century (Polygnotus’ painting in the leche of the Cnidians at Delphi: Paus. 10.31.9, 11; south Italian vases, Mecitus 78f; A. Rumpf, Misc. Att. Ber. II 2 [1930] 41f; cf. Willemse, CHP, 532; an allusion at Pl. Rep. 363c). The myth may have formed part of an Orphic Katalasis; that the author was perhaps a Sicilian or an Italiote is phrased like a conjecture, but alludes to the Pythagoreans (cf. above, ch. II 3; Ἰακεμίως τυψή ἄνεος is an allusion to Timotheus fr. 17 [Wilamowitz, Platon II 80]). That the one who offered the interpretation was himself a Sicilian or an Italiote (as Frank seems to show, 90, 299, n. 210) does not follow; so that it is hard to support the conjecture that Philolaus is intended. Carcopino, Bab. 287f, finds the same “tics d’expression” here as in fr. 17 and At 17 Th. ar. 81.15; but the basis for such an assertion is too small.
The decisive group of fragments, in which Philolaus speaks about being which is “limiting” and “unlimited,” as well as about number and harmony, has received very little attention; and, in particular, the opponents of their authenticity have not attempted any thorough analysis. Yet only such an analysis can determine whether what we have is a post-Aristotelian “forgery” or a pre-Socratic composition. In the former case, it should be relatively easy to explain it in the categories of Platonic, Aristotelian, and post-Aristotelian thought; in the latter, we should expect that the familiar concepts and formulae of later times would prove inadequate, and that an approach would be possible only through the special characteristics of other pre-Socratic thinkers.

'Α φύσις δ’ εν τῷ κόσμῳ ἀρμόδηθη εξ' ἀπειρῶν τε καὶ περαινόντων, καὶ δολος κ’ ἄρμος καὶ τὰ ἐν αὐτῷ πάντα (B 1).

'Ανάγκη τὰ ἐντὸν εἶμεν πάντα ἡ περαινόντα ἡ ἀπειρα ἡ περαινόντα τε καὶ ἀπειρα' ἀπειρα δὲ μόνον ἡ περαινόντα μόνον; αὐτά καὶ ἐτέρω τοιάδυτα φαίνεται οὔτ’ ἐκ περαινόντων πάντων ἐντόνον οὔτ’ ἐκ ἀπειρῶν πάντων, διὸ δεῖ τὸ ἐκ περαινόντων τε καὶ ἀπειρῶν δ’ τ’ ἐν κόσμῳ καὶ τὰ ἐν αὐτῷ συναιρέομεθη ... (B 2).

Περὶ δὲ φύσεως καὶ ἀρμονίας ἄρ’ ἀπειραθεὶς, ἀ μέν ἐντὸν τῶν πραγμάτων ἀδιάφορον ἀσῦν κατά μὲν αὐτόν καὶ ἀρμόδηθη τ’ ἐναντίον τῶν ἑνυπ'% ἐμοίων καὶ γενενοκωμένων οὐ’ ἀμοίω.

The most extensive is that of Rothenbùchter (60ff); but his conclusion, that the whole complex is “absurd and not Pythagorean,” reveals his failure to understand it. By-water only cites some verbal reminiscences of pseudo-Pythagorica. Frank (30ff) paraphrases Scorr (Grph 133ff) treats the matter more fully; see also A. Burns, CEM 25 (1964) 92–128.

Heidel’s conjecture of ἀ πειραθή δ’ τ’ ἐν κόσμῳ (JCP 1907, 79) is mistaken; cf. Anaxagoras fr. W. ἐν τῷ ἐν κόσμῳ; Diogenes of Apollonia fr. 2: τὰ ἐν τῷ κόσμῳ ἐντόνον (similarly Hippocr. Nat. hom. 7, VI 50 L); Heid: 1: "mündi forma sein omnis omne erat cumvulnymo quae sunt singulnum" (τοῦ κόσμου ἡ ἔνθ’ ἐν κόσμω τῶν ἐν αὐτῶν ἑντόνων φύσεων, Pfeiffer, Stirngl. 31.4); ἄ φυσις τ’ ἐν κόσμῳ would be itself suspicious (the other way around in Eur. fr. 910: ἐδεναινόν ἀεισίσ φύσεως κόσμῳ ...); cf. also Kahn 228ff. On φύσις see D. Holwerda, Commentatio de vocis que est φύσις vel atque usu praesertim in Graecia Aristotelica anteior (Diss. Groningen, 1955). The use of φύσις to mean the totality of ἑντὸν is common in the history of Philolaus (as Eupides in the passage cited above).—On ἀρμόδηθη cf. Emp. fr. 107: ε’ ἀπειρῶν γαρ πάντα πεπήγας ἀρμοσθήναι, and the Dermeri papyrus, col. 17.

68 Cf. Hippocr. Med. 15: τὰ γενενοκωμένα τὰ γενενοκωμένα are equivalent to the known facts of the world of experience; Democritus fr. 155: ἀφιμενοὶ ἐνιάντ’ ἐναντίον εἰς ἐντόνον εἰς ἐναντίον ... (B).

The MS reading could be understood by comparison with Leucippus (cf. below, no. 101), but all the editors except Heidel (JCP 1907, 78) proposed ἀπειραθή, which was accepted by DK, but this word is late attested elsewhere. From actual pre-Socratic vocabulary a possibility is ἀποκαθίστα (Meineke). Cfr. below, no. 74.

The MS reading is τὰ τοῦ ἐν κόσμῳ ἀρμονίαν αὐτοκεκλείσθη, ἠ重要意义. The usual reading is τὰ τοῦ ἐν κόσμῳ ἀρμονίαν, and Meineke further altered ἡ εἰ τ’ ἐν κόσμῳ and added ἀ νυπότος refers to a characteristic previously mentioned or known (see Kührer–Geth 118.3 630ff), rather than to a following φοιν. Anyway, "such a harmony" would be an odd phrase; the author is not talking about any particular organization. Thus it seems best to keep τὰ τοῦ ἐν κόσμῳ, which can refer to ἀνυπάτος etc., and to punctuate before ἀ νυπότος, even though this creates an asyndeton. (A sentence is frequently introduced by ἀ νυπότος, as Democ. fr. 253, 277, Antiphon fr. 44 [DK II 354, col. I line 15], Zeno fr. 1, Melissus fr. 7, Philolaus fr. 2.)—For αὐτοκεκλείσθη see Emp. fr. 27: ἀρμονίας πείραι κρύφαι ἀνυπότος.
The author's basic idea is the division of ἐννα or πράγματα into ἀπειρα and περαιοντα; the breach which this opens up in the cosmos is healed by a ἄρμονία. There are obvious similarities with what Aristotle reports, but also characteristic differences. In the first place we have περαιοντα ("limiting [things]," or "that which limits") instead of πέρας ("the limit") or τὸ πεπερασμένον ("the limited"). Though it has a model in the not infrequent expression περαιον ὑπὸ σοφίας, this still the absolute use is notable. But it is comprehensible, or even necessary; a "limited" presupposes something else which is "limiting" or "limit-setting." The necessary complement to what is unlimited is the existence of "things" that "make limits," which bear the principle of limitation as an active power. Featuring Pythagoras passage with the perfection of musical harmony, he uses the phrase ἡ περαιονον τῆς φύσις for the nature of an odd number. Hesychius с.ν. περαιον, i.e. περαιον, says ὁ Πυθαγόρειος τοὺς περαιον τῶν ἄρμονίων, and formulations like this are found in Nicomachus. The subject of this passage of Philolaus is not primarily numbers, but "things," and the antithesis περαιοντα καὶ ἀπειρα is used in a broader and more nearly original sense than in the definition given by Hesychius.

Significantly, the "principles," in spite of the occurrence of the suspicious word ἀραξια, are never designated by an abstract expression like τὸ πέρας, τὸ πεπερασμένον, τὸ ἀπειρον, or, as in the formulation of Aristotle cited above, ἡ περαιονον τῆς φύσις—or, as in the Philebus.

29 πράγμα in the general sense is equivalent to ὅπως elsewhere: Democrit. fr. 164 (cf. Hdt. 2.52), Gorg. 17.18, Pol. 24, Epicharm. fr. 3, Euenus fr. 8 Diehl (cf. Theog. 472); obviously ἄρμονια is the older expression, and is always used by Anaxagoras and Melissus (Aet. 1.3-4 gives the gloss Anaxagoras fr. 1, ἄρμονια λέγων τὰ πράγματα): cf. Diehl, SBBln 1884, 350 n. 1.

70 This is emphasized by Reinhardt, Parm. 65 n. 1, and Scoon, Grth 140ff; Melissus fr. 5: περαιον ὑπὸ τῆς ἐννας ἀπειρον, ἑπταγηνον της περαιοντας, Nicom. Ar. 2.18,3 (where the citation of Philolaus fr. 2 follows in the next section).

71 A similar plural Hippocr. Med. VII 542, 582, 609 L. (Kees 121ff); cf. also P. Nem. 1.8. It is significant that in Philolaus a past tense is used (differently from ps-Arachthos 1.8. In philoloys a past tense is used (differently from ps-Arachthos 1.8. It is significant that in Philolaus a past tense is used (differently from ps-Arachthos 1.8. It is significant that in Philolaus a past tense is used (differently from ps-Arachthos 1.8. It is significant that in Philolaus a past tense is used (differently from ps-Arachthos 1.8.

26 Frank 305: "This sterile repetition of the same points, while the argument makes no progress at all, this continual manipulation of the same phrases, is intolerable; it betrays a second-rate mind."

46 Diehl, SBBln 1891, 580 n. 2, on Anaxagoras fr. 6; cf. Diehl, Parm. 23ff on naive iteration (referring to Parm. fr. 8.3ff); also Melissus fr. 2, Diog. Apol. fr. 25, Democrit. fr. 191.

58 Her. fr. 1, Ion fr. 1, ps—Xen. Ath. Pol. 1, Hippocr. Mid. 1, Nat. mul. 1, Fossa exc. 1, Septim. 1, Glaud. 1, Xen. Ap. 1, Oor. 1.1, Critias fr. 46. First came the author and title (e.g., Ἡφαίστεις Κριτικάται περὶ φύσις ὑπὸ λέγειν). In a continuous text ὑπὸ can be followed by a clause introduced by ἀρα ως Hippocr. Arr. 13. Cf. K. Gelzer, Die Sicht von der Stätte der Athene (Berlin, 1877) 100ff. Böckh mistakenly concluded from the ó περὶ that fr. 1 could not be the beginning of the book (43f). This usage, it is true, was often imitated in later times, e.g. by Pollux (E. Bethe, Hermes 72 [1937] 240).

58 Wilamowitz, Plato II 90.
III. PHILOLAUS

η τοῦ ἀπέρου φόσος, though this is the regular practice in the later tradition.76 “Limit” and “unlimitedness” are not isolated as entities in themselves, concealed into an abstract substantive or hypothetized as intangible substance, but they are thought of as scattered or deployed, so to speak, in individual things, περαίνων or ἀπερα. This is a basic difference between the thought of the pre-Socratics and that of the Platonic and Aristotelian schools, a difference brought out especially by Cherniss.77 By a word like θερμόν or ψυχρόν, for example, a pre-Socratic thinker does not mean an abstract quality, or an οὐδεία, but quite concretely the sum of particular things characterized by the word. Only in Plato’s dialectic was the foundation laid for separating qualities and quantities from objects and regarding them αὐτὰ καθ’ αὑτὰ. For later ages these distinctions came to seem self-evident. By saying not ἄναγκα διό ἀρχας εἶμεν τῶν ἐντόνων, τό τε πέρας καὶ τό ἀπέρα,78 but ἄναγκα τά ἐντόνα εἶμεν πάντα ἡ περαίνοντα ἡ ἀπερα ἡ περαλαίνοντα τε καὶ ἀπερα, Philolaus shows his affinity, in expression and thought, with the pre-Socratics and his difference from all that is Platonic and post-Platonic. From this point of view a forgery is unlikely, since what is involved is not the kind of technical terms or verbal flourishes that are easy to imitate, but the way in which the mind goes about trying to understand reality.

The division of “things” into “limiting” and “unlimited” is of course more abstract than an analysis in terms like “warm,” “cold,” “dry,” and “moist,” where reference is made to areas of immediate sense perception. There is no indication what kind of specific experience is implied in the idea of “limit” and “unlimited.” The author does give what looks like a hint, in the continuation of fragment 2: δηλοὶ δὲ καὶ τὰ ἐν τοῖς ἐργα, τὰ μὲν γὰρ αὐτῶν ἕκερ περαλαίνὼν περαλαίνι, τὰ δὲ ἐκ περαλαίνων τε καὶ ἄπερων περαλαίνι τε καὶ οὐ περαλαίνων, τὰ δὲ τὰ ἐπερῳκά ἄπερα φανέραν. But this sentence is couched in quite general terms, and it is impossible to tell what specific sense the word ἐργα has.79 All we learn is that the “limiting” and the “nonlimiting”

that “things” do is dependent on the “limit-forming” and “unlimited” constituents that go into their makeup.

It is very easy to understand “limit” as a formative principle, and “unlimitedness” as a material principle. But in considering a question of authenticity we must differentiate between philosophical interpretation, which seeks to understand the author better than he understood himself, and philologial interpretation, whose first duty is to understand and place historically what the author put down. The thought that god gives form to formless matter, by imposing “limits,” περατοὶ τῶν ἐντόνων ἄπερων, is characteristic of the Platonists,80 that it does not go back to the Pythagoreans, and that it was not an accomplishment of theirs, as is often thought, to conceive for the first time, in explicit terms, of a formal principle, follows from the exposition of Aristotle, according to which the form-matter dichotomy is not applicable to their number theory. If the situation were different in the Philolaus fragments, this would be a serious cause for suspicion; but any such interpretation is impossible. Boeckh equated the “limit” with the One, the “unlimited” with the Indefinite Dyad, so that the difference between Philolaus and the Platonic system disappeared.81 On this basis, Frank was able to maintain82 that Philolaus agreed with Plato, against the testimony of Aristotle. But the text of Philolaus is against this. The One is itself a ἀμφότερον, and is therefore not simply περατοὶ.83 After all, “limiting (things)” (περατοὶ) and “unlimited (things)” (ἀπερα) stand side by side, both in the plural, and there is no indication that in the process of “harmonization” (ἀμφότερον) the “unlimited” disappears;84 nor is there any suggestion of the one affecting the other, in the way that “form” must affect “matter.”

The ideas of ἑστώ (“being”) and ἄμφωνia compound the matter

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76 Ἀετ. 1.1.10 (DK 44A9), Damasc. Princ. I 101.3 Ruelle, Arist. passim.
77 Pres. 375ff, IH 342ff.
78 Cf. ps.-Archytas 19.5 Theseff (above, n. 74).
79 Boeckh (50) translated “Iawuwerke,” Diels (Vorsokratiker) “Acker,” Scoon (GrPh 138) advanced the idea that ἀπερα are mountain meadows without definite boundaries. Newboll (1865) essayed a geometrical answer, Leckey (QSt 4, 154f) an arithmetical (ἦργον is the product; “odd” x “odd” = “odd,” etc.). Heidel (AJPh 1907, 77; Proc. 1913, 770) has “nothing but the bald assertion that observation or knowledge confirms.” Similarly, W. Nestle (Philologus 67 [1908] 544f). Cf. Anaxagoras 117: μὲν εἰδέαα . . . μὲτε λόγω μὲμε ἐργα. Frank (307 n. 1) points out that expressions like δηλοὶ τοὶ ἐργα occur in

80 Aristotle (Met. 349b35, cf. Met. 1086ba); but they are also in use earlier, as Hippoc. Plat. 5: πρὸς αὐτὰ τὰ ἐργα . . . περατοὶ ἄμφωνεα . . . On the idea, cf. Hippoc. Nat. pur. VII 348 L. : δὲ ἀπὸ πυκνῶν ἐνέργεια πυκνὰ ἐντὸς καὶ διὰ ἀπὸ ἐνίκει ἐνεργ. 81 Plat. Quest. conv. 8.2.3, 710c. Cf. the construction of elements in the Timaeus and in general the system of the One and the Indefinite Dyad.
82 Boeckh 54f, followed by DK I 407 n.: “Grenze (Form) und Unbegrenztheit (Stoff)”; I 409, in the translation, “diese Prinzipien (1 und 2),” Contra, above, ch. I 2.
83 268, 308 n. 1, 309 n. 1; accepted by Cherniss, Pres. 391. The fact that the octave, which is also called ἀμφωνεα, shows the ratio 2:1, has nothing to do with the Indefinite Dyad.
84 Fr. 7: τὰ πρῶτα ἀμφωνεα, τὰ ἐν ἐν τῶ ἡμείς τὰς σφαιρὰς ἑστία καλεῖται. Though τὸ ἐν looks suspiciously like ditography, the combination of the πρῶτον ἀμφωνεα with the πρῶτον ἐν ὑστερα in Aristotle (Met. 348b20, cf. 1091a12ff) still stands — quite apart from the problem of authenticity.
85 As in Plutarch (above, n. 83) and in Scoon’s interpretation (below, n. 84).
still further. We cannot simplify it by equating ἔστω with ἀπειρόν and ἀρμονία with περαιτέρων. The ἔστω τῶν πραγμάτων... καὶ τῶν περαιτέρων καὶ τῶν ἀπειρών is prior to the differentiation of “limiting” and “unlimited,” and belongs to one of these as much as the other, and “harmony,” when it “comes,” applies to both of the “beginnings, which are not alike nor of the same kind.” Nor can ἔστω be thought of as ὁλόθ or ἀρμονία as formal cause. Ἐστω is not ἀποικος ὁλόθ, but already differentiated into “limiting” and “unlimited,” and “harmony” comes along later; it would not be necessary at all, if it had not been for the antithesis of “limiting” and “unlimited.” There is no hint here of a division of being into form and matter, which must always be thought of in relation to each other—nothing that goes beyond the pre-Socratic picture of the cosmos as the proper ordering of something previously chaotic.

The word ἔστω is not to be understood as a material principle in the Platonic and Aristotelian manner. The only thing that is said of it is that ὅποτερες. Being is. This is the fundamental position of the Eleatics, and it is from this point of view, rather than that of later, abstract conceptualization that the sense comes clear. Since Parmenides, all philosophers had spoken of an eternal being, and the Sophist Antiphon used the word ἄειστάτο. This concept of being had been the subject of heated controversy since the time of Zeno, Melissus, and Gorgias.

88 As by Scoon, CP 1922, 354, Crith 138ff, esp. 145 n. 30, who equates the unknowable ἔστω with ἀπειρόν and πέρας with number, the principle of knowability, and the Āpērē, he thinks, are rudiments, which the ἀρμονία has not yet fully grasped. Howald was able to cite Scoon in order to bolster his argument that the fragments are infected with Platonism (p. 67; cf. Burnet, EGP 285 n. 3). A suspicious resemblance between the unknowable ἔστω and the Aristotelian ἀποικος ὁλόθ, ἀνωτάτος καθ’ ἀπώτατα, is seen by Schaar-schmidt 66, Bywater 34f, Frank 308, and Raven in KR 310f. But for Plato and Aristotle ἀποικος is unknowable in principle, and not the object of some higher, divine knowledge.—In ps-Archytas p. 19.19 Thesleff, ὁποτε ἐστώ is certainly the Aristotelian material principle, contrasted with ὁμοφύλος.—Rostagni, Verbo 409f, interprets ἐστώ as “essentia,” ideal Being, supernatural Unity. This, too, is conceived from a Platonic point of view. The contradictions among the interpretations show that Platonic principles are not applicable to Philolaus 6.


90 Antiphon fr. 22 = Harpocr. s.v. (It is explained as τῷ ἀδιάτητα καὶ τῷ ἐπὶ τῶν ἀτόμων διὰ τῶν ἐστών.) On the formation of the word, cf. εἰστιν, Asch. Sept. 187, Ag. 647, 929, Hdt. 1.85, Democrit fr. 2c; ἀποικος, Hdt. 9.85; συνειδησία, Hdt. 6.128. It is obviously an Ionic formation; Pl. Crat. 401c shows that the Doric is ὁποτε or ὁποία. Lign. 162 cites ἐστώ as one of Pythagoras' neologisms, along with κόσμος, φιλοσοφία, περαιτέρων. Ps.-Archytas, pp. 19.4-20.7 Thesleff, used ἐστώ alternatively with ὁποία;—On αὐτά μὲν ἄ φιλος in Philolaus fr. 6, cf. κατ’ αὐτόν in Parmenides fr. 8.58; similarly, Emp. fr. 17.34.


There was no agreement as to what attributes it had, whether it existed at all, or whether it was expressible. But Philolaus pushes this controversy aside, entrusting its solution to a divine intelligence. He is clear that there are things that are known to us (ὕποκως καὶ γεγνωσκόμενα), and that their origin would have been impossible if—as Gorgias thought—“nothing” existed. We cannot go beyond this; nevertheless the world, as it is given us, can be understood, as a harmony of opposites.

Like the question of being, that of the origin of harmony and its “arrival” is avoided, with the explicit phrase ὧν τούτων ἔστω σχέδιον ἐγένετο. It is simply a necessary prerequisite for the cosmos, just as ἔστω is prerequisite to the πράγματα determined by the opposites. Raven thinks he sees here the clearest proof that the fragments are a post-Aristotelian forgery: “It is surprising... to find the author... expressing perplexity about what seems to have been the most important constituent in his whole cosmology.” Aristotle speaks of the ἄπορον of the Pythagoreans in a very similar context, in relation to the formation of the first One. Raven’s argument is that if the Pythagoreans had themselves discovered the gap (“omission”) in their system, they would have filled it up; if it is noticed and left, a forger mindlessly copied Aristotle. The assumption here is that it is a primary concern of a philosopher to have pat answers to all possible questions, and a fully consistent system. In fact, this is rather the concern of the doxographer, but in a real effort to get at truth there is always an awkward “remainder” that will not fit in, or perhaps an unfilled gap. Scoon aptly mentions that the Νος of Anaxagoras...
presents an analogous problem. According to the commentator of the Derveni papyrus (col. 17), Zeus, the divine Aēr, brought ἀρμονία. Philo does not mention the divine. How harmony came to the world is passed over just like the problem of ἐστί; the world is there, and it is in order.

The question of ἀπειρός and πέρας was much canvassed in Greek philosophy. Anaximander takes the ἀπειρός as his starting point, Parmenides declared that "Being" was perfect and therefore limited (fr. 8.42ff), Zeno dealt with the antinomy of πειραμένων and ἀπειρός under the assumption of the existence of plurality (fr. 3), and Melissus decided that being is ἀπειρός (fr. 2–4). Philolaus takes a stand on the question, saying that πειραμένοι and ἀπειρός exist side by side, brought together by the agency of harmony. As others, faced with the multiplicity of the world, picked certain leading ideas as central—finding the basic outline of reality in pairs of opposites like warm and cold, wet and dry, or like Parmenides analyzing the whole world of appearance as the mutual influence of light and night—in the same way Philolaus takes the one feature of limitation as having determined the nature of the cosmos.

But how can one speak of a plurality of ἀπειρός? The air can be called unlimited, or space, or Melissus' Being, but none of these has a plural. One can speak of some things as "unlimited in number" (ἀπειρός τὸ πλήθος), but πειραμένοι is not the opposite of this. Now Anaxagoras, following Zeno's lead, spoke of an ἀπειρός of the small, of a never ending process of division. Divisibility, dependent on the "empty space in the middle," plays a role in the Pythagorean equation of even number with unlimitedness. If the ἀπειρός πάργαμα are thought of in the context of endless divisibility, the plural is compre-

99 Von Fritz, Philosophie und sprachlicher Ausdruck (New York, 1938) 15; Arist. Cael. 303a8; above, ch. I 2, n. 70.
98 KR. 408.
97 Arist. De an. 404a1 et seq.; above, ch. I 3, n. 130.
96 Arist. De an. 404a1 et seq.; above, ch. I 3, n. 130.
95 D.L. 9.32 = Leucippus A1; above, ch. I 2, n. 51; Democrit. 40a.
94 Anaxagoras fr. 2.5, Melissus fr. 6.
93 Anaxagoras fr. 1, Zeno fr. 3.
92 Soon, GrPh 143f.
91 Anaxagoras fr. 1, Melissus fr. 6.
90 Anaxagoras fr. 1, Zeno fr. 3.
89 Anaxagoras fr. 1 and 3 expressed the idea of the infinitely small; Zeno fr. 3 (cf. fr. 1) used the ἄνάλογος ἀπειρός which result from the assumption of plurality, polemically, as a reductio ad absurdum.
88 Above, ch. I 2, n. 31.—Aristotle argues, at Phys. 204a8ff, against the idea of the ἀπειρός being Ὑνων. An Ὑνων as such cannot be divisible into a plurality of ἀπειρός, πολλά ἀπειρός εἶναι τὸ σῶμα ἀπάντων (204a25f), and yet it is a πολλά. Ὑνων ἀπάντων ἀπειρός ἀποφαίνεται ὑπό ἀριθμοῦ τῶν Ὑνων ἀπειροὶ τῶν Ὑνων ὑπὸ ἀπειροὶ καὶ μέρους (204a32ff). In this connection, Aristotle associates ἀπειρός with ἀπάντων (204a31). Ross relates Aristotle's criticism of the Pythagorean identification of ἀπειρός with even, i.e. divisible number, but the criticism is more telling, couched in general terms, if the Pythagoreans actually spoke of a plurality of ἀπειρός—rather than Philolaus did.

On the other hand, Democritus denied that there could ever be φύσιν μίαν ε' ἕκεννα (sc. τῶν ἄρχων), καμάθη γὰρ ἑπετές ἐναι τὸ δίο δῦν τὰ πλούσια γεννάθη ἀν ποτὲ ἐν. Arist. fr. 208 = Democritus A37.

90 D.L. 9.38: πᾶντος μέντοι τῶν Πυθαγορικῶν τῶν ἑκάστος ἑκάστης ἄρτος ἡ Ῥημικα. Duris (EigHist 76F2) said his teacher was Pythagoras' son Artimeus; Apollodorus of Cyzicus (above, ch. I 3, n. 51) mentioned Philolaus. For Leucippus and Philolaus as pupils of Pythagoras, see Mos. VP 104. On Democritus' Πυθαγορικόν see above, ch. II 6, n. 26; on the Pythagoreans coins of Abdera, above, ch. II 2, n. 3. Frank argues for the dependence of the Pythagorean philosophy on Democritus; but the relationship may be a whole generation earlier. Surely there was another atomist influence on some later Pythagoreans. (On Echphantus, see above ch. I 2, n. 74; W. Kranz in Conzum: Festschr. K. Ziegler [Stuttgart, 1954] 26f. There could be a relationship between Echphantus and the cosmology of the papyrus from Derveni; see Burkert, A&A 1968, 99.)

100 That the question of the Democritus and the atomists, no matter how surprising it seems from the point of view of the Platonic interpretation of Pythagoreanism, is actually corroborated by both internal and external evidence. It is not only that Leucippus, like Philolaus, belongs in the Eleatic tradition and scarcely can be outdone by the Pythagoreans' number theory in “forcible abstraction.” Aristotle remarked that the atomists, too, in a certain way made things out to be numbers. The “void” of the atomists is not a single, endless space, but the plurality of interstices which make divisibility and plurality possible. The cosmos grows by taking in material from outside. It is obvious that the Pythagoreans who inspired the atomists to see in the atoms in a sunbeam an indication of the nature of the soul-atoms. We even have the testimony of a contemporary that Democritus studied with a Pythagorean; thus Philolaus and Leucippus are thrown close together. To be sure, Philolaus maintains a distance from the atomists; he does not speculate further about being, but looks for relationships in our given, familiar world and finds them in the ordering function of number.

Before that, though, he seeks to prove his basic doctrine; and here Frank thought he had found an infallible sign of post-Platonic forgery. But the refutation of this was already at hand; Reinhardt had shown that
especially in the *Epinomis*. Raven thinks it very important finding that Aristotle gives no hint of this indispensable epistemological underpinning of the Pythagorean number theory. But in a passage of Iamblichus derived from Aristotle we read that “everything is revealed through numbers,” and Alexander, using Aristotle’s book on the Pythagoreans, says that in the Pythagorean view, μήτε γάρ δύναι τι των ὀντων χαρις ἁμαμος εἶναι μήτε γνωριζεσθαι ἄλλος, τοῦ δὲ ἁμαμος καὶ χαρις των ἀλλων γνωριζεσθαι. It is clear that the basic idea of the Platonic “system of derivation” had an influence on Alexander’s wording, that is, the distinction of gradations of being according to whether one can be thought of without the other. But this does not necessarily imply the idea of “knowability”; and Alexander adds the words μήτε γνωριζεσθαι as an afterthought. Thus it seems that Aristotle did not completely ignore the Pythagoreans’ theory of knowledge, even if, not being much interested in epistemology, he failed to mention it in the *Metaphysics*. The question of knowledge, once again, stems from the Eleatics: οὐ γάρ ἂν γνώσῃ τὸ γε μὴ ἕνωσεν οὐ γάρ οὕσειν ἂν ἔστην ὑπὸ οὐκ ἔστη. So said Parmenides (fr. 2.7, 8.8); and from his time on, the problem of knowledge was a frequent topic of discussion in the fifth century. The thought, then, that knowledge is only possible through number, and that it is not only ἀπερα that exist, but that they must be joined together with περαιων through the agency of a harmony, is far from impossible in this century.

“Everything that is known has number.” An example of this is the numerical relationship of the musical intervals: “The extent of a harmony is a fourth and a fifth... The fourth is one and one third (3:4), the fifth one and a half (2:3), the octave a doubling (1:2)”

104 Raven, Phyl. 40, 89. 137: δύναι το ἁμαμον (in relation to the theory of rhythm, where Pythagorean influence is a possibility; cf. Pl. Phyl. 170). For a later formulation, see Simp. Cal. 608. 25f. εἰ ἄρα δύναι ἂν παστίαν ἄρα δύναι το neglect διάκρισιν καὶ παραιτο το γνωσθή. In Philolaus, however, knowledge is not entirely on the side of Limit, but is rather a matter of the harmonious coexistence of ἄραμα and περαιων (fr. 6; fr. 11 is different).—In *Ps.-Pl. Epin. 976d* et seq. number is praised as the basis of all knowledge; reference is made to the τόνως (977d-6), music (978a), celestial movements (977a-b), and φύσιν in general (977d et seq. in obscure hints that obviously relate to the “system of derivation”). It is unlikely that Philolaus fr. 4 was derived from the *Epinomis* passage, as Thiel’s thought (Gommon 1931, 325)—the simple and jealous from the nuanced and stylish!


106 Se Melinus fr. 8 on sense perception, Gorg. fr. 3 on the unknowableness of the ὑπέρον. Anaxagoras fr. 12 and Diogenes of Apollonia (fr. 8) emphasize that ὑπέρον or ὑπέρ “knows everything.”
III. PHILOLAUS

(f. 6). "Harmony" here (ἀρμονία) has the special musical sense "scale an octave long," but the essence of musical harmony is the same as of that harmony which holds the world together. Of the directly quoted fragments the ordering of the cosmos by number is only alluded to in the sentence about the One in the middle of the sphere of the universe (f. 7); but we may infer from Aristotle's account that there was also application of numerical and musical relationships to the cosmos.

The Hippocratic writings illustrate how some thought of the numerical and musical ratios as bearing on the life of man. For example Regimen defines its basic question as the determination of the right relation between nourishment and activity:

If indeed . . . it were possible to discover for the constitution of each individual a due proportion of food to exercise, with no inaccuracy either of excess or of defect, this would mean, precisely, the key to health for human beings. 113

Health, in "precise" terms, is a numerical ratio; whoever knows the numbers has found all he needs.

The numerical ratios have a more special role in embryology. The growth and health of the embryo depend on whether he finds the right harmony—and this is expressed in musical terms:

If, on changing position, they achieve a correct attunement, which has three harmonic intervals, the fourth, the fifth, and the octave, they live and grow. . . . But if they do not achieve the attunement, and the low do not harmonize with the high in the interval of the fourth, of the fifth, or in the octave, then the failure of one makes the whole scale of no value. . . . 114

The treatise On Seven-Month Babies is more precise still: the same numbers determine the course of the embryo's development and the course of illnesses: θεορεῖν δὲ χρῆ οὕτως τρίασσε τε καὶ τετράσσε, ταῖς μὲν τριάσις συμμετέχειν, δὸν δὲ παρὰ δὸυ διεξουσίμεναι. 115 All the odd numbers are important, and of the even ones, the 14th, 28th, and 42nd days. οὖσα γὰρ ὁ δρόμος τῆς τῆς ἁρμονίας λόγω πρὸς τινών καὶ τὸ ἀριθμός116 την τὲ κάλεσις ἁρμονίας· δε' ἢ δὲ αὐτὴν, μακρότερον ἄν ἐντῇ ἐπι τοῦ παρόντος ἀκολουθεῖν. From this it appears that this book is based on some specific source. In what follows, the doctrine is applied to seven-month and nine-month children; the fact that eight-month babies do not live is thought to be due to the imperfection inherent in an even number.

Similar doctrine is found in the treatise Περὶ σαρκῶν: τὸ παιδίον ἐπτάμηνων [γόνων] γενόμενον λόγῳ γεγένηται καὶ καὶ λόγῳ ἔχει τοιοῦτο καὶ άριθμοῦ ἀπέρακε ἀτ ἐνδομάδας . . . ἐν ἔνα οὖν καὶ δέκα ἄμερον γόνω γίνεται καὶ καὶ ἐγείρει τὸν άριθμοῦ ἀπέρακε ἔτος ἐνδομάδας. 117 The periods calculated are 280 days (= 40 x 7) and 210 days (= 30 x 7). Other books as well118 go into the difference of even and odd numbers; even numbers are the weaker, more likely to bring misfortune.

The beginnings of numerology, including the special role of the odd numbers, are older than Pythagoras; 119 but, when significant numbers are thought of specifically as λόγοι ἁρμονίας and also defined in musical terms, the Pythagorean theory of the numerical relation of musical notes must be part of the background. In addition, we find the same archaic expressions for the "fourth" and the "fifth" as in Philolaus, namely συλλαβη and δε' ἀριθμοὶ, as well as his expression ἀριθμοῦ ἔχειν. There seems to be a direct connection here; and the Hippocratic treatises, by their forms of expression, are clearly dependent on a more detailed treatment of these matters than their own. Lucian attests that Philolaus called the tetractys άγείας ἀρχῆς, 120 and this is to be taken quite

113 Hippocr. Septim. 9, VII 448 L. The MSS have συνεξεγέρεις, but there is obviously a reference to the musical terms συμμετέχειν and διεξούσιμαι. "Continued" triads make up the series 1, 3, 4; 3, 4, 5; 5, 6, 7 . . . Thus he means the odd numbers 1, 3, 5, 7, etc. Correspondingly the arrangement of "tetrad" gives the series 1, 4, 7, 10, 13, 16, 19, 22, etc. (Delatte, Méli. Thomas 165ff).

114 "Perfect" (wrongly "even") LS]. cf. ἀπαρίστος, Ἰδ. 7, 29, Pllli. 1220, Harnoc. s.v. = Lisias fr. 28 Baier-Saupe, ἄτι τοῦ ἄριθμου περίον καὶ πληρός ἀριθμοῦ.


117 Below, ch. VI 4.

118 Hippol. 1 = Lucian Laps. 5. It is not clear, either from the Philolaus testimonia or from the hints of the Hippocratic books, precisely what calculations they made. According to late sources, by virtue of the tetractys, from the one tetrad of harmonious numbers 6, 8, 9, 12, or from the other one 6, 9, 12, 18—whose sums were, respectively, 35 and 45—came to us, by multiplying these figures by the "perfect" number 6, at the total of 210 days for the partis minor (7 months) and 270 for the partis major (9 months).

(Cf. Varro ap. Cens. 9, Th. ar. 51, 63, Athenius p. 8 Heiberg, Aristid. Quint. 3 p. 142, Macrob. Somm. Sc. 1 6.15ff, Procli In Rem. II 34, etc.) Delatte (Méli. Thomas 171) believes
suggested in the words περαίωντα καὶ ἀπειρα, signifies from the beginning that "everything has number." Things "have" even number, insofar as they contain ἀπειρα; they "have" odd, insofar as they consist of περαίωντα. Both elements are bound together in "harmony," just as the harmony of music always consists in the connection of even and odd numbers.

Aristotle's usual formulation is that, according to Pythagorean doctrine, things "are numbers" or "consist of numbers," while Philolaus speaks of them as "having number" (ἀριθμὸν ἔχειν). But Aristotle also uses this expression in a passage clearly under Pythagorean influence, at the beginning of the book On the Heavens: "For, as the Pythagoreans say, the whole and everything in it is comprehended in the number three; for end and middle and beginning [taken together] have the number of the whole (τῶν ἁρμὸν ἔχειν τῶν τῶν πατρών), and this is that of the triad." This is a remarkable place to find the word "number." One would expect something like λόγος, and only after that, the introduction of the concept of "number." Beginning, middle, and end, considered together, "have" or "contain" or "conspire" the idea of a "whole" or "totality," and since they are three in number they "have" three. A point must be raised here that has been well made by others: that the Greek word ἁρμὸν is not completely equivalent to the modern concept of "number." ἁρμὸν is always a whole number, and tied up with the actual procedure of counting. Thus it is closely connected with things, and in fact it is itself a thing, or at least an ordering of things. ἁρμὸν means a numerically arranged system, or its parts. Isocrates represents the mythical king Busiris as establishing the castes of priest, artisan, and warrior in Egypt: ἂν τοὺς ἁρμὸν περιάβαλς, ἐξ ἀριθμῶν ἂν τὸ κοινὸ διακόσμησο, ἢ τὸν τότε τὰς αὐτίς πράξεις μεταχειρίζονται προσέθη εἰσ...
Busiris took “all numbers,” that is all the classes, in their state as numbered and ordered groups that would be useful in the government of society. ἀριθμὸς stands for that which is counted. A sentence in the Hippocratic treatise *On Generation* is to be understood in the same sense: Injured or lame parents may have healthy offspring, ἔχει γὰρ τὸν ἄριθμον πάντα τὸ παιδιομένον τὸ ἄγετα. He is alluding to the τέσσαρες ἔννεια of the Hippocratic doctrine of humors; if these four principles are present in full strength, the child gets what it needs. The crippled has the whole number,” that is, all that is necessary, in its numerical order, “as well as the healthy.” One may also speak, in the same sense, of τὰ μέρη ἔχειν πάντα. A significant aspect is the “aristocratic” sound of the word ἀριθμὸς. It is only what is important that “counts;” only the fully competent, effective warrior is ἐναριθμός, μετ’ ἄνδρῶν ἀριθμῶν. In contrast to the δῆμος ἀπείρων (II. 24.776). To ask about the “number” means to ask about the essential. Each of the pre-Socratic philosophers, in bold simplification, selects a certain phenomenon or a single aspect of the world’s multifarious reality—water, air, fire, the warm and the cold—and each thinks he has got hold of the one most significant thing. The categories of “having” and “being” are not yet strictly separated. The “is” is not taken as indicating a logical classification, but as a statement of what is essential. The book *On the Nature of Man* develops the doctrine of four humors with reference to the views of predecessors who saw the essence of life in one of the humors—blood or phlegm or bile. And the theory is expressed in the form, “Man is blood” (or phlegm, or bile); they proceed from the observation that at the time of death the body loses its blood, or phlegm, or bile. “Each man is, they thought, that which they saw him lose as he died.” On the other hand, one may say that the soul “has” a mixture of fire and water, and this means nothing else than that the soul “is” this mixture.

In a similar way, Philolaus tries to get at what is fundamental in the cosmos by pointing out the antithesis of “Limiting” and “Unlimited,” and the presence of number and harmony. “Everything has

number” means about the same as “everything is, basically, number.” Aristotle, in the context of his effort to build a systematic ontology, would naturally choose this kind of formulation; at the same time, under the influence of the Platonic theory of numbers, the role of number would naturally be stressed, and ideas of limit, unlimited, and harmony would recede into the background. Philolaus, however, explicitly refused to make any pronouncement about “Being” (ἐστι) and is for that very reason more free to follow up the many relationships of numbers—the meaning of ἀριθμὸν ἔχειν.

The correct approach to the Philolaus fragments is not through the categories of Aristotelian or Platonic thought. To be sure, they agree, as far as content is concerned, with the accounts of Aristotle; but the manner of thinking and the style point rather to the pre-Socratic period, or, to put it more precisely, to that era, in the second half of the fifth century, of the coexistence of Eleatic dialectic and Ionic philosophy. Thus the idea of a post-Aristotelian forgery is out of the question, and the fragments may be regarded as remnants of that book which the Pythagorean Philolaus wrote not very long before 400 B.C.

The concepts with which he operates—έκτος, κόσμος, ἄπειρον, as well as φύσει καὶ ὀν νόμω (fr. 9)—are borrowed from contemporary philosophy. It is not in this area that one would find the originality of the Pythagorean who rises to take his part in this discussion, which has been going on for so long. Most important, the question of Being is bracketed out, and the effort is to establish relationships, by means of the ideas of number and harmony. This step could have led from natural philosophy to natural science and was a necessary step, if one was to make any progress, in avoiding the Eleatic ἄπορία about Being. But Philolaus clearly did not recognize its significance. If his goal had been exact science, he would have been investigating the many facets of the concept of number, and specific problems would have taken the place of generalizations. For Philolaus, philosophical ideas and specific items of scientific knowledge seem to have been no more than a means of expressing and illuminating a preexisting picture, of a world consisting of a pair of basic opposites, informed by harmony and defined by number. In Aristotle’s exposition the mythical background becomes clearer: the antithesis of “limiting” and “unlimited,”


124 Hippoc. Vict. 1.7.

125 II. 2.202, Od. 11.449, etc., cited by Plato at Phlb. 179e; cf. also Eur. fr. 519, and the famous oracular response to Aegiam or Megara (Demiistr FGrHist 306F8 = A. P. 14.73).

126 Hippoc. Nat. hom. 6: τούτο δὲ ἐκαστὸν αὐτῶν ἐνόμισεν ἐναὶ τὸν ἄθροισιν, ᾧ τα ἐκαστάρμοντον ἐναὶ αὐτῶν ἀποδοχάκοιται.

127 Hippoc. Vict. 1.7: ψυχὴ πυρὸν καὶ ἱδατος σύγκρητα ἔχεισα.

128 Cf. above, ch. 1.2.

129 His pupil Archytas seems to have been much concerned with special problems in mathematics and acoustics.
of “odd” and “even,” is also that of “male” and “female,” and their conjunction is “marriage” (γάμος). The whole range of observation and experience is compressed, with resolute abbreviation, into a formula which singles out as essential only something which is not observable. What is to be shown is not what the world is, but that the world is “orderly,” and in all its parts and aspects ruled by harmonious combination—a postulate that is present also in pre-scientific interpretations of the world.

So that everything may fit neatly into the cosmic pattern, the fire which envelops the world must have as its counterpart the “central fire” alluded to in the expressions τὸ πρῶτον ἀρμοσθεν, τῷ ἔπει, ἐν τῷ μέσῳ τάς σφαιράς ἐστίν καλεῖται (fr. 7). Whether or not the related idea of the movement of the earth is possible in the fifth century is a question to be studied in the light of the history of astronomy.  

Stobaeus has a passage on the structure of the cosmos, in the chapter he entitles Περὶ σχημάτων, purporting to come from a book called Βάσικας. It lacks the usual tinge of dialect; and this is as surprising as its mysterious title. ὁ κόσμος εἰς ἔστιν, ὁ ἤρεστος δὲ γίγνεσθαι ἄρχει τοῦ μέσου καὶ ἀπὸ τοῦ μέσου εἰς τὸ ἀνώ διὰ τῶν αὐτῶν τὸι κάτω, ἦτα τὰ ἀνώ τοῦ μέσου ὑπενικέον κείμενα τοῖς κάτω, τοῖς γὰρ κάτω τό κατωτάτῳ μέρος ἐστὶ διπλάσιον ἄνωτά καὶ τὰ ἄλλα ὑπενικέον πρὸς γάρ τὸ μέσον κατὰ ταυτά ἐστιν ἐκάτερα, ὡς μὲ μετεχείνειται. The cosmos develops from the center out, in each direction equally.

139 Ανδρέας; cf. above, ch. 2; 2, 7, 2; 27.  
140 Below, ch. IV 3. Anatolius has a noteworthy comment about the Pythagoreans (p. 30 = Th. ar. 6.11f): περὶ τὸ μέσον τῶν γεωμετρίας στοιχείων κεῖσθαι τινα ἡθικώς διάπνοον κύρος, οὔ τιν τυσίτας τῆς θέσεως καὶ ὁ Ὀμηρός εἰδέναι λέγοντα… (II. 8.16). The point of view is geocentric; but Philolaus too identified the ἐστί in the middle with the ἐν and the cube with γεωμετρικὴ ἀρμονία, ἀπὸ τοῦ κατὰ τὰ τρία διαστήματα ἤρμοσθαι ἑαυτὴν ἑαυτίκα (Ανδ Ανδρέας = Nicon, ar. 2.26.2). Was the central fire, the πρῶτον ἀρμοσθεθήν, thought of as a cube?  
141 Fr. 17 = Stob. 1.15.7.  
142 Arist. fr. 202 τῶν μὲν οἰκετῶν εἴναι ἔνα…  
143 The conjecture ἄρχει for the difficult ἄρχει, usually accepted since the time of Meineke, brings it with an awkward repetition. Perhaps ἄρχει could be understood in relation to a primary phase of cosmic development, “as far as the middle,” i.e. until the middle is formed (τὸ πρῶτον ἀρμοσθεθήν).  
144 άρχει Wachsmuth, ἤτεροι (γάρ) Diels.  
145 μέγα MSS, μέρος Wachsmuth n.). Diels reads τῶν γὰρ κατωτάτῳ τὰ μέρα ἐστὶν ἄστερ t. 4, which does away completely with the reversal of direction (μετεχείνειται). On μέρος, cf. Pl. Phd. 1122e—In the next clause, the principal manuscript, F, has τῷ ἄνωτάτῳ, Diels τὰ ἀνωτάτῳ.

“...What is above is that which is over against the middle, from the point of view of those that are below; for to those below the lowest part is like the highest, and similarly for the rest; for both (upper and lower) have the same relationship to the middle, except insofar as their positions are reversed.” The author is trying to express the idea of the relativity of “above” and “below” in the world, but keeps using these terms because he cannot free himself from the idea of an absolute up and down. Plato dealt with this problem in a much more sophisticated way, in the Timaeus; and the doxographers record that “Pythagoras, Plato, and Aristotle recognize no up and down in the cosmos.” The only other laborious exposition like this is in the On Sevens of the Hippocratic corpus—a fifth-century document: ἢν ὁ ἄρχοντα ἄρμοι τὴν ἴδεν οἱ ἀπὸ τῇ ἴδει ἀνάφηκον τοῖς οὖσ᾽ ἀνέπηργησι τοῖς κάτω τὰ ἀνώ κάτω εἶναι, τὰ ἀνώ κάτω ἀνώ. Clearly the Philolaus fragment is to be classed with this pre-Platonic passage rather than with the easy mastery of thought and expression in the Timaeus; and therefore, even if the dialect difference has been smoothed out in the course of transmission, it may be regarded as authentic. And incidentally, the idea of the gradual development of the universe is another feature which would comport ill with post-Aristotelian forgery.

Another fragment cited by Nicomachus from the book περὶ φύσεως also gives the impression of being old. καθαρὰ δὲ φυσικὰ καὶ ἀληθινά, ὄμφαλος δὲ βεβαίως καὶ ἀναφοράς τῶν πρῶτων, αὐθοίνεον ὀπὸ στῆρεματος καταβολας το θεομυσίας ἐκκέφαλος δὲ σαμαίνει τὰν αὐθαυτοῦ ἀρχήν, καθαρὰ δὲ τὸν ἣν, ὀμφάλος δὲ τὸν φυσικόν, αὐθοίνεον ὀπὸ τὰν ἐναπάντων πάντα γὰρ ἀπὸ στῆρεματος καὶ θάλασσαι καὶ βλαστάνειν. Of course, scholars have attempted to identify portions of this with

145 For ἀναφοράς with the genitive, cf. Hdt. 3.80, 7.153. τὸι κάτω is dative of relation, as in the next clause. In the translation of DK, the construction of the genitive τοῦ μέσου is not clear: “denn was oben liegt von der Mitte aus, verhält sich zu dem, was unten liegt, entgegengesetzt.”  
147 Hdt. 2; on its date, see below, ch. III 3, n. 63. The MS reading (according to Roscher) is … τοῖς κάτω τὰ τὸ μέσον ἀνώ κάτω εἶναι (corr. Boll.).  
148 Of the other citations of the Bichar, fr. 18 (where only the subject heading περὶ φύσεως is preserved) could be brought into connection with A19 (below, ch. IV 3), and fr. 19 (a mere allusion) with A14 (both are cited by Proclus). Can Βάσικα be a late, “romanticizing” substitute for the title Περὶ φύσεως?  
149 Fr. 13 = Nicon, in Th. ar. 25.17.
Plato’s three parts of the soul, and their localization in the *Timaeus*, as well as the Aristotelian hierarchy of ἀρετής ἰδιότητων, ἀληθητικών, ὄργανων. But such an interpretation demands an incredibly stupid forgery with a remarkably profound conception of nature. What are we to make of *ψυχή καὶ αἰσθήσεις* in the heart? The words “soul” and “perception” are obviously not used in their broader signification; and this very fact suggests a pre-Socratic milieu. *Ψυχή* means nothing more than “life,” and *αισθήσις* goes with it, not as sense perception through the various specialized sense organs, but as the apprehension of stimuli, the faculty of being influenced and reacting. Critics wrote that this kind of *αισθάνεσθαι* was closely connected with the *ψυχή* that is, to life, what is dead is *ἀναίσθησιν*.

Democritus uses metaphors from botany in speaking of the importance of the navel; the expression *πτέρυγμα καταβολά* occurs in the passage about Philolaus in Menon’s history of medicine (A27); both Empedocles and Diogenes of Apollonia know the hierarchy of plant-animal-man, and the distinction between man and beast by the

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180 At Pl. Tim. 69b–70c, the three parts of the soul are localized in the head, the breast, and the belly. Bywater (44ff) thought that the Aristotelian tripartition of the soul and its functions was mechanically grafted onto the scheme of the *Timaeus*; but this involves the oddity of assigning the *αισθήσεις*, since it is the middle member, to the heart. And how does the *ψυχή* come to be associated with it there?—Olerud, too, finds Plato’s tripartite soul in Philolaus (74f). Can anyone equate *ψυχή* and *αισθήσεις* with *θυμόν*?

181 Scharrer (2002) tries by arbitrary combinations to establish a connection between this and A12 (ψυχήν ἐν ἐπίκλισι—cf. above, n. 41). The gradation seen in fr. 13 is inconsistent with the scheme of derivation in A12.


183 If this were the case, the natural thing would be to localize sense perception in the head. Cf. Arist. fr. 95.

184 On the pre-Socratic use of *αισθήσις*, see H. Langerbeck, *Δίκαιος ἐπιστήμων* (Berlin, 1935) 44ff. It means “sensation [Empfindung],” but is not used as a general term for the aggregate of the five senses (cf. Democfr. fr. 11, Diogenes of Apollonia fr. 5). In non-philosophical language *αισθάνεσθαι* means “perceive” or “comprehend” (Soph. *Afr*. 553; Thrasymachus fr. 1), or “perceive” and “feel” (a misfortune, for example, as at Soph. *OT* 424). Plants, too (Protagoras ap. Pl. *Tht.* 167c), and even lifeless things have *αισθάνεσθαι* (cf. Hippoc. *Morb.* VI 386 L., *Vet. med.* 13). According to Hippoc. *Morb.* VI 392 L., heart and diaphragm are organs of thinking, and yet they *αισθάνονται μικτά* i.e. “feel” [emotions] most poignantly. “The philosophical concept of *αισθάνεσθαι* is worked out in Plato’s *Theaetetus* as something new (152c, 160c, 180d–e).


2. Spurious and Genuine in the Philolaus Fragments

criterion of νοὸς may be derived from Alcmaeon. The tendency to think in terms of parallels and antitheses, which appears here in Philolaus, is unquestionably ancient. The thought of microcosm and macrocosm is also relevant in this context, insofar as the types of living beings are brought into relationship with the organs of the body. In these correspondences, one more, we find the orderly arrangement and “harmony” of the cosmos, here divided into four and to that extent defined by number. Thus this fragment fits into the general point of view shown in the others, even though there is nothing in it about Limit and Unlimited.

There is perhaps even less of this in the best-attested passage, that from Menon (A27–28). Here the subjects are heat and cold, the causes of disease, blood, bile, and phlegm—all treated quite in the manner of a physician writing in the tradition of natural philosophy. Living beings, we learn, develop from “the warm,” for the semen is warm as well as the uterus. After birth the infant draws in air and expels it again “as though it were a loan.” This is how it cools itself—*ψυχή* is explained from *ψύχης* as *φλεγμα* is derived from *φλέγω*. Whether or not this is dependent on Prodicus, a relation which would give us a terminus post quem, is matter for controversy. The closest relationship of the passage seems to be, once again, with Diogenes of Apollonia. The necessarily eclectic nature of Philolaus’ book is especially clear in this instance. To be sure, a relationship to Pythagorean cosmology has been alleged here. Just as, according to Aristotle, the cosmos breathes in the “unlimited” void, as the One, after its origin, sinks in the Unlimited, so the newly born, warm living creature breathes in the cool air; here again microcosm and macrocosm are set parallel. Still, in order for the analogy to be really striking, the general subject would have to be embryology, whereas here we have a completely formed infant making its first contact with the air. Perhaps even to a Pythagorean it was more important to have knowledge of a broad range of things than to have a perfect system.

Olerud 72ff.

In a fourfold division the power of the tetractys is of course at work (above, ch. 3; II 4; above, n. 120).


188 Diogenes of Apollonia A28: γενομεθανεία μὲν τῇ βρέφει ἱππυρα, ἐν θερμοτητί δὲ ἐθνός τῷ ἐμπροσθὸν ἔθνος προσκεκλήτου τοῦ βρέφους τῷ πεθανόντων ἐθνίτι τῷ πατρίου ἀφέσειται. This is like Philolaus *A27*: the living creature arises from the birth in the warm, and μετὰ τῷ ἐκτείνειται ἐθνίτι ἐπιδικόν τῷ ἐκτείνειται ἐθνίτι τῷ ἀπεθάνοντα πετρίτι. Hippo. *Afr.* also has something quite similar.

189 Above, ch. 1, n. 47. The connection with Philolaus *A27* was seen by Frank (1275f), Mondscher in *ZM* (180), Olerud (42ff), and Raven (in *KR* 134).
There remain several passages in praise of number, cited by Stobaeus in his prologue without indication of the title of the book they are taken from. First comes a paean on the number 1, then one on number in general, which makes possible all our knowledge, is at work everywhere, and admits no ψευδός. Throughout there is manifested an animation that is almost poetic, and a definitely rhetorical conformation of style—in the choice of words like παντελῆς καὶ παντοκράτος, σύμφωνον τὰ τὸ ἄρμαν γνεῖς, in placement of words (πάνω παντά, οὐδείν οὐδέν), in the use of parallelism and chiasmus. The content is simple, sometimes even trivial. Only one sentence, bearing on the theory of knowledge, is obscure, though not incomprehensible. The idea “in the soul, in harmony with sense perception, makes everything knowable and mutually agreeable, working like a carpenter’s square,” fixing and loosing the proportions of things, each for itself separately, those that are unlimited and those that are limiting.”

Both in content and in expression there appear many points of contact with pseudepigraphic Pythagorean writings, and especially pseudo-Archytas. Verbal agreement with an expression of Plato’s, in the context of the “nuptial number,” is suspicious; for even though in this matter Plato is “Pythagorizing,” what that means, in his case, is not copying a striking phrase but following a line of thought. The author of the fragment uses twice the phrase ἡ οὐσία τὸ ἄρμαν, three
times φύσεως τῶν ἄρμιδων,174 and once τῶν ἀπειρῶν καὶ ἀνοητῶν καὶ ἀληθείας φύσεως. Many parallels for all these can be cited from Pythagorean pseudopigrapha, and in particular the heaping up of various concepts to clarify two opposite φύσεις:174 but both form and content are unexamined in pre-Socratic or pre-Platonic philosophy. Here φύσεις is not, as in fragment 1 and fragment 6, an expression for “all that exists,” and also not the existence, in accord with natural law, of a particular thing; it is a realm or grade of Being. Precisely this kind of division or gradation of Being was unknown to the Pythagoreans, according to the testimony of Aristotle; they clung to the single, perceptible world and did not speak of immaterial οὐσίαι.175 Along with this goes the high poetic style of compounds like παντοτερόγονος καὶ

παντελῆς, favorites among the imitators of the Τιμαῖος,176 and the propensity for formations in -ικός,177 which seems to show a direct connection with Aristotle.178

On the other hand, there are clear contradictions with the other group of fragments, in spite of their having in common the phrase τῶν πραγμάτων τῶν τῶν ἀπειρῶν καὶ τῶν περιφρόνων—something that might very easily have been copied.179 To be sure, fragments 1–7 have an έστιν τῶν πραγμάτων, but this is general, comprehensive “being”; number does not have a peculiar mode of being, and Limiting and Unlimited are not characterized as two separate principles of φύσεως. Harmony, in those fragments, stands over the pair of opposites which first made it necessary, while here the “nature and harmony of number” is set over against the “nature of the unlimited and unintelligent and irrational.” “Soul,” ψυχή, is here, unequivocally, the comprehensive notion; knowledge takes place “in it,” “in agreement with sense perception,” in the fully developed sense. It is placed in a relationship of harmonious tension with thought, that is, with number.180

Thus fragment 11 shows so many suspicious features that it cannot have been composed in the fifth century B.C., but only after Plato and Aristotle and along with the rest of the Pythagorean pseudopigrapha.181 But the status of fragments 1–7 is not affected by the rejection of fragment 11, for aside from one easily imitated phrase there is no apparent connection between them, but in fact a definite difference. There is nothing in the manner of transmission to suggest that they should be taken together; in Stobaeus, fragments 11–12 are introduced between “Τιμαῖος of Locri” and Aristotle, Platonically interpreted, on one side, and pseudo-Archytas on the other.182

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174 Cf. above, ch. I 2, n. 159, esp. ps.-Archytas fr. 1 p. 20.15 Thesleff: τῶν ἀρβημάτων καὶ λόγων ἔχοντα, τῶν ἀρμιδῶν καὶ ἀρρήτων. There are similar phrases in Aristotle, above, nn. 72–73. Cf. also Arist. Met. 1010a: ἡ τῶν ἀρίστων φύσις, “Eurosusus” p. 88.11 Thesleff: δόδοι φύσες εἰς τὸ εἴδω τάδε... ἢ μὲν ἄρτη καὶ τέταρτη καὶ λόγων ἔχον, ἢ δ’ ἄρρητος καὶ ἀτάκτος καὶ λόγος καὶ σφυροκοπ人心 ἑνών, καὶ ἀνακοπηκτικῶν ἑνών, καὶ above all the Ἱερὸς λόγος of “Pythagoras,” Iam. VP 164 = 164.9 Thesleff: τῶν ἀρρητῶν ὁδοντῶν ἔχον ἄρμιδον προσανατότατος τῶν παντῶν ὁμολογιών καί γάς καὶ τάς μετὰ λογίων, ὁτ’ δὲ καὶ πεθανόντων (MS with, corp. Festugière, Rév. I 338 n. 3) καὶ θεοῦ καὶ θυμῶν διαμόνων διάμονως ἔχον (Philo in 11: θείων καὶ θερμούει πάνω καὶ ἀνακοπηκτικῶς ἄρχα...).

175 Mondolfo, in ZM 337f., sees the suspicious nature of the language, but would like to evade the difficulty by understanding οὐσία as “wealth” and pointing to the pre-Socratic use of έξορρογία (Parmenides fr. 10, etc.). Cf. however, the wording of Arist. Crit. 298a28 (Frank 311 n. 1): τῶν φυσικῶν λογικῶν ταύτα ἄρδη οὐσία, τά δ’ ἐργα καὶ πάθη τούτων... with the alleged Philolaus, τά ἐργα καὶ ταύτα οὐσίαι τῶν ἄρμιδων.

176 Above, ch. I 2.
III. PHILOLAUS

Along with fragment 11, no doubt, we must give up fragment 12, and its "vertefultes Lastsich." This in spite of the fact that the four elements are neither called by the Platonic term στρογγύλα nor derived from polyhedra. Though it is quite thinkable that Philolaus took over his theory of the elements from Empedocles, the mention of a "fifth" raises doubts. Our conclusion must be that here once more we have a popularizing enumeration of the elements by someone familiar with Platonic and Aristotelian teachings.

Although some dubious material and some that is unquestionably spurious has been transmitted under the name of Philolaus, there is a group of fragments—1–7, 13, and 19—which can only be interpreted on the basis of pre-Socratic ways of thinking, not those of Plato and Aristotle and their followers. Here the proof of authenticity drawn in, in an indirect way, from the doxographical tradition, is directly confirmed; there have been preserved for us remains of a book composed by Philolaus in the pre-Platonic period, including both word-for-word fragments and doxographical reports, which advocates that philosophy of Limit and Unlimited, of number and harmony, to which

183 Wilamowitz, Plato II 91. The principal opponents of the authenticity of this fragment have been Howald (56f), Theler (Gnomon 7 [1931] 35f), and Festugière (REG 1945, 16 n. 4). Sachs (41ff) was able to treat it as genuine because it does not have to do with the regular polyhedra (cf. above, ch. I 3, n. 116; the inclusion of the reference in Aet. 2.6.5, attributed to Pyth. or Phylus, as Philolaus A15 is of course a slender reed).——καὶ ὁ τάς σφαῖρας ὄλκοι, πέμπτων—the impressive image of the ship is hard to give up. But there is no meaningful way to connect this with τρόπως διεσκε 

184 From Archytas (who was the teacher of Eudoxus) and in the philosophical reinterpretation of Plato did Pythagoreanism attain to a form in which its real influence could develop.

We shall have to test the result reached here by considering the musicological and astronomical doctrines of Philolaus in the framework of the history of those sciences. Here too we shall find that the Philolaus fragments represent a stage before Archytas, Eudoxus, and Plato, and that one cannot, without serious qualification, speak of a specifically Pythagorean science before Philolaus.

3. REFLECTIONS OF PYTHAGOREAN PHILOSOPHY IN THE FIFTH CENTURY B.C.?

If the earliest book of Pythagorean philosophy was not composed until the latter part of the fifth century B.C., the gap between it and the historical Pythagoras is uncomfortably wide. In order to close it, or at least to narrow it, many attempts have been made to date individual doctrines, in the realm of science or natural philosophy, in the early period; in this way Pythagoras himself could be assured of the credit of providing the "germ," or at least the initial impulse, for the later development. Philolaus himself probably thought of his doctrines as merely an extension of the wisdom of Pythagoras. If that were the case, it would affect our interpretation of the Philolaus fragments: we should expect more genuine Pythagorean tradition and less eclectic. But when the Philolaus fragments are treated as spurious, the search for chronological clues becomes really crucial. There is, in fact, not a single piece of direct evidence to be found. The later tradition, though it uses the name of Pythagoras, is contaminated with Platonism and must be ruled out of court. And the older, reliable

185 Aside from Archytas, though he seems to have devoted himself more to specific problems, and from Euphanes, who took his own way.

1 Above, ch. III 1, n. 68; I 4, n. 36.

2 Above, ch. I 3.
evidence has to do with the Pythagorean legend and the acusmata. It reveals the picture of a shaman-like sage and a biot lived in accord with his precepts, a life in which philosophy or science, as the Greeks understood it, does not necessarily have any part at all. The acusmatici, later on, did not recognize the mathematici as Pythagoreans.

If there is no direct evidence, can indirect testimony be found, for example reflections of Pythagorean teachings in the works of other philosophers? They may have taken over Pythagorean material, or entered into polemic against it. Since the day of Tannery scholars have been treading this path, with growing confidence. They attempt to discover doctrines of Pythagoras from their influence, as an astronomer sometimes infers the existence of a hitherto unknown star from irregularities in the course of known planets. In this way a tempting chapter of the history of philosophy may be built; erratic boulders and unidentifiable gravel coalesce into a comprehensive structure. The suspected interaction of the Eleatics and Pythagoreans, in particular, becomes a lively dialogue. Parmenides, the apostate Pythagorean, sets up his own system in opposition to that of the school; in response, the Pythagoreans revise their theories, only to be subjected to new attacks, by Zeno; this forces them to undertake further revision . . .

This structure, however, rests on a shaky foundation. It is true that there are many points of contact between what Aristotle or the later tradition attributes to Pythagoreanism, and pre-Socratic doctrine; but to conclude immediately that these come from a Pythagorean origin would be to presuppose what ought first to be proved. It is also possible in each case that a later Pythagorean like Philolaus, in an eclectic spirit, borrowed material from others, or that the later tradition is wrongly ascribing to Pythagoras material from a foreign source. Again, what

appears both in Pythagorean and in other sources may go back to a common source rather than to the result of mutual influence. In each case, proof of an "irregularity in orbit" would be of decisive importance in the evaluation of the indirect source, proof of a change of direction or a distortion of the course of thought and the system which could only be explained on the basis of external influences. But this is a very difficult thing to prove, even in the case of Plato, and seems to lead to nothing but further controversy; for the pre-Socratics, preserved only in sorry fragments, it is practically hopeless.

Unknown quantities keep multiplying, for the nature and characteristics of Pythagoreanism, whose influence and diffusion one is trying to determine, are far from being clearly understood. In order to get any kind of start, one has to take something or other as presupposed, "given." Scholars have frequently regarded it as almost self-evident that the Pythagorean doctrine of Limit, Unlimited, and number must have existed from the day of Pythagoras in some form or other, which in any case was abstract and philosophical; in this way, the only question is to decide what aspects of it Parmenides and Zeno presuppose. But this is the very thing—the existence of these doctrines before the book of Philolaus—which ought to be proven first; and particular attention should be paid to clarifying the nature of a philosophy without written documents. Other scholars, whether consciously or not, proceed on the assumption that Pythagoreanism had certain general characteristics: it was a powerful religious movement, the source of all non-Homeric elements of Greek religiosity; or it is the great synthesis of religion and science, featuring the cura animarum as the special concern of philosophy, and celebrating the harmony of man and cosmos. Or, finally, it was the source of exact science among the Greeks, responsible for all significant attainments, especially in the realm of astronomy and mathematics. Each of these hypotheses

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3 Above, ch. II 3–5.
4 Tannery, HSdH 125, on Pythagoras: "en l'absence de documents authentiques directs . . . nous ne pouvons deviner ses opinions particulières sur le monde que par les traces qu'elles ont pu laisser dans les écrits de ses contemporains ou des penseurs de la génération suivante."—Cherniss (Pres. 387, 157 n. 68) considers the "number atomism" deduced from Zeno so certain that he uses it as a basis from which to criticize Aristotle (396; cf. ch. 1 3, n. 66). Raven (KR 236, PyEl passim) considers that their relationship to the Eleatics provides our only opportunity to learn any detail or to arrive at any chronological determination for the Pythagoreans; he knows, though, that this method is "hazardously conjectural."
5 The comparison is made by J. Stenzel, Metaphysik des Altertums (Munich, 1931) 46, and Gigon, Ursprung 126.
6 Consider, e.g., Philolaus' theory of the sun (below, ch. IV 3), and the astronomical theories of Alcmaeon (below, ch. IV).
7 E.g., the doctrine of the 4 elements (above, ch. I 3, n. 113), or the division of the earth into 5 zones (below, ch. IV 1).

8 Cf. above, ch. III 2, n. 31, on "Odygates, below, ch. VI 2, on even and odd.
9 Raven proceeds, tacitly, from this premise.
10 To carry out this idea consistently, Homer himself must, it is thought, have been revised in the light of Pythagorean ideas. The second part of the Nekyia is thought to be Pythagorean (Od. 11.565–632: Delatte, Litt. 135; Od. 11.661ff, and also 576–602: Carnady, Symb. 169 n. 5, Luc 190); also the second Nekyia and the "Gate of dreams" (Od. 14.10ff, Symb. 169 n. 5, Luc 190) and also the second Nekyia and the "Gate of dreams" (Od. 14.10ff, Symb. 169 n. 5, Luc 190); also the second Nekyia and the "Gate of dreams" (Od. 14.10ff, Symb. 169 n. 5, Luc 190); also the second Nekyia and the "Gate of dreams" (Od. 14.10ff, Symb. 169 n. 5, Luc 190).
11 Cf., e.g., Gigon, Ursprung 12: "... dass mit Xenophanes ein neues Element in der Philosophie auftrat..." Und wir wissen, dass Xenophanes... Pythagorais gekannt hat, nennen wir dieses Element Pythagoreisch."
12 Chs. IV VI.
contains something that is attractive, and perhaps a measure of truth; but as long as they are not defined and supported by direct evidence, they will not serve as the foundation for reconstruction of the system.

An epoch in which a unique development in the history of thought took place, like the period of something more than a hundred years between Pythagoras and Plato, surely saw inner transformations even in apparently stable traditions. In the relation of religion and rationalism the center of gravity must have shifted. If we take the legend of Pythagoras, the acusmata, and the accounts of the acusmatici as seriously as they deserve, we realize that within his school it was only after the day of Pythagoras that the movement from myth to science took place. We also run the danger, influenced by Plato’s evaluation, of seeing the φαντασία of Anaxagoras, for example, in too rational and nonreligious a light, and disregarding mythical and religious forces which were at work in this area as well.

It is a reasonable guess that thinkers from Magna Graecia would show Pythagorean influences; but only meticulous study of the internal and external evidence can raise this possibility to a probability—to say nothing of certainty.

Most important of the relationships between Pythagoreans and other groups are those with the Eleatics. Geographically these were close to the south Italian centers of the Pythagoreans, and the abstract, inmaterial character of their philosophy naturally results in coincidences with a philosophy of number. In addition, the ancient tradition makes Parmenides and Zeno Pythagoreans, or at least pupils of Pythagoreans.

Even in Xenophanes Tannery saw polemic against a Pythagorean teaching. Diogenes Laertius paraphrases his idea of god, δολον δε όραν

13 The statement of Sotion about the hero shrine that Parmenides dedicated to his teacher Ameinias, the son of Diochetaeus, may be derived from a genuine inscription (D.L. 9.21), though it would still be doubtful whether Ameinias was called a Pythagorean. For Parmenides and Zeno as Pythagoreans, see Anon. Phot. 439435, Strabo 6, p. 352, Procl. In Parm. 619.4 Cousin (DK 28A4); Proclus cites Nicomachus as his source, but there is a variant reading “Callimachus,” which is taken into the text by Cousin and Stallbaum, and later accepted by Zeller 1680.1; the Latin translation makes the reading “Nicomachus” certain: R. Klibansky, Plato Latius III [London, 1953] xxxii; Callim. 822 Pfeiffer, is among the “delenda”). Cebes Tab. 2, l. 190 VP 165: Parmenides and Melissus in the catalogue of Pythagoreans; cf. schol. p. 150 Deubner, l. 190 VP 165; cf. ch. II 4, n. 160. Down to imperial times there was a guild of physicians in Elea which traced its history back to Parmenides. The president had the title φαλαχρος, which is reminiscent of the Pythagorean ἀρχας (above, ch. II 4, n. 109). Cf. P. Ehren, Rassegna storica Salentina 23 (1962) 447ff. M. Gigante, Parola del passato 19 (1964) 135–137, 450–452; P. Merlan, AJP 48 (1966) 267–276; H. Jucker, MH 25 (1968) 181–185. The feud between Pythagoreans and Eleatics, which most scholars have believed in since Tannery, is not hinted at in the ancient sources, as Capparelli emphasizes (I 267ff).

14 But according to the Pythagoreans the cosmos breathes in the Void, or Unlimited. Xenophanes, Tannery concludes, is rejecting this idea, thus presupposing that it had been stated. If this were right, an essential motif of Pythagorean cosmology would be older than Xenophanes, perhaps suggesting, in the opposition of κόσμος and πνεῦμα the theory of ἀνασάς and ἀναπνέων in general.

But this conclusion is anything but certain. Xenophanes is not talking about the cosmos, but about god, and it is a much mooted question in the interpretation of Xenophanes whether his god is to be equated with the universe. But even if this question were to be answered in the affirmative, it would be just as reasonable to suspect an allusion to Anaximenes as one to Pythagoras; in fact it is impossible to prove that he is voicing criticism of any particular person. Xenophanes is reacting against the naive, anthropomorphic conception of the gods. The principal criteria of a living being, along with the ability to see, hear, and apprehend psychologically (νοεῖ), are breath and motion. The god, “as a whole,” exercises the former functions, but not the latter. There is no more reason to suppose the denial that he breathes is directed polemically against contemporary philosophers than the denial that he moves about.

Parmenides has been exploited much more as a source for Pythagorean philosophy, though in a different way. Tannery maintained that the doxa section of his poem, with the specific statement that its teachings are “deceptive” (fr. 8.52), was a doxography, from a hostile point of view, of Pythagorean cosmology. Later writers claimed that

15 This much is preserved as a directly quoted fragment (24).
17 Above, ch. I 2, n. 46.
18 Tannery, HSCH 125ff, followed by Burnet, EGP 108; DK I 113.26 n.; Rostagni, Verbo 25; Rey 134, 207ff; Mondolfo, ZM 314ff. Contra, Rithmann 37ff.
19 The case for a panteistic interpretation is argued by Zeller, 165ff, and is presupposed by Tannery, HSCH 125. Reinhardt (Parm. 116ff) believed that world and god were separate for Xenophanes. Aristotle’s comments in Met. 980b10 are of course not conclusive.
20 Burnet, EGP 108, refers to Anaximenes fr. 2. The Pythagorean doctrine discussed by Aristotle can be brought into connection with Diogenes of Apollonia (A9, fr. 5), and thus with Pyhagoras (above, ch. III 2, n. 161). Pl. Tim. 31c argues against this theory.
21 Fr. 26. This is of course directed against the Homeric tales of the gods; though it goes without saying that Homer’s gods breathe. Cf. Aesch. Eum. 651, on Zeus: οδέοις ἀσπασμῶν μὲνε.
in Parmenides’ particular manner of developing his argument there could be seen a reflection of Pythagorean mathematics. Then, finally, Raven sought to explain the Parmenidean predicates of Being as a polemical expression against the Pythagorean doctrine of opposites.

The world of doxa is, in Parmenides’ poem, the result of opposition and interprenetration of the contrary powers of Fire and Night. The attempt to bring this into relation with the Pythagorean opposition of Limit and Unlimited has led to contradictory results. There is not a single detail that can be shown to be exclusively Pythagorean; there is nothing in Parmenides about number and mathematics, nothing in the realm of doxa about Limit and Unlimited, and nothing about the harmony which unites the opposites. Pairs of opposites played a part in Ionian physical philosophy from the time of Anaximander, and also in Alcmaeon. Their reduction into a single opposition is a result of Parmenides’ fundamental thesis; as he comprehends Being as a unity, the multiplicity of the world turns out to rest on one basic contradiction. Thus the doxa section of the poem is for this very reason a personal achievement of Parmenides—a fact generally accepted since

22 Gomperz, GrD 126; Ree, 189ff; Cornford, PLParm 29, PrSap 117. Contra: Szabó, AA 1955, 67ff; 1956, 109ff; cf. below, ch. VI 1.

23 Raven, PyEl 21ff, KR 274. Similarly, K.-H. Ilting, ABG 9 (1964) 103-131, tried to show that πέρας-ἀπέραντον in Parmenides presupposes a Pythagorean doctrine.

24 Tannery, HSH 207 and 213, equated Night with earth (with Aristotle, Met. 986b34 = A2a) and with Limit, and Light with Unlimited. On the other hand, Burnet (EGP 129, 186f, followed by Rey 208, 273, 327ff) equated Night with πέραμα and Unlimited, but Light with form and Limit. (This is in harmony with the placement of Light and Darkness in the table of opposites, and fits in with the fire doctrine of Hipparcus.) Esh 327, Inf. 324ff, assumes a reversal of positions: the Unlimited was originally a dark πέραμα until, after Philolaus, it becomes the fiery aether (περιέχον, A16): Timpanaro Cardini argues against this (1946, 328ff).

25 The astronomical system of σφεδρίνα (A37, fr. 12; below, ch. IV 1), carried on by Plato’s system of σφεδρίνα (Rep. 616c; cf. Morrison, JHS 1955), comes from the fiery wheels of Anaxiander. On the relation of Anaximander and Parmenides, cf. Fränkel, WP 186ff. Why Pythagoras is thought to be the intermediary is hard to see (Tannery, HSH 237).—On Ὄλογονως, above, ch. III 2, n. 31. Ἀωνεύος, absolute necessity conceived of by thought, is a discovery of Parmenides. (It is of course a misleading convention to spell the word with a small alpha at fr. 8,16 and a capital at 8.30 and 10.6.) The later tradition has no indication that ἀωνευος is specifically Pythagorean (Tannery, HSH 242ff, above, ch. I 3, nn. 143-150. At Emp. fr. 115 Ἀωνεύος decides the fate of the soul, but in Pfr. 113 it is Persephone, and in the Gold Plates, Moira.)—There remains the “right-left theory” of his embryology in fr. 17 (cf. E. Lesky, AhhMainz 39ff). The association of right with male and left with female appears in the table of opposites; but this leads nowhere except into a realm of pre-philosophical musings about analogies and “order.” See G. E. R. Lloyd, JHS 82 (1962) 56-66. On the astronomical discoveries that are ascribed to Parmenides, rivaling those of Pythagoras, see ch. IV 1.

26 Reinhardt, in particular, has worked out the argument for the origin of the doctrine of opposites in the thought of Parmenides (Parm. 71ff, 236ff).

27 There is nothing to impel us to insert, between the Ionians and the philosopher of Elea, a specifically Pythagorean doctrine of opposites.

According to the view of Raven, Parmenides arrives at his pronouncements on the ὅλος as an alternative to the Pythagorean doctrine of opposites. The ὅλος is limited, single, indivisible, and cannot come into being or perish, while in Pythagoreanism Limit stands over against the Unlimited, and in a cosmogonic process the One develops into a Many. Raven himself, however, acknowledges that Parmenides takes his stand, on principle, against any kind of cosmogony, and this takes the force from Raven’s argument. For there is no way of showing that Parmenides is aiming at a specific Pythagorean system. To be sure, his preference for πέραμα is significant, and to a modern the solution of Melissus seems more natural. But this is merely a matter of pre-philosophical evaluation; the connection of πέραμα and τέλος is deeply rooted in Greek language and thought. He explicitly rejects the notion of condensation and rarefaction advanced by Anaximenes.

This shows that Pythagoreans were not, at least, his only opponents. In fact, there is nowhere any unequivocal indication that Parmenides’ poem presupposes any Pythagorean science or philosophy.

There may be relationships on a different level. The prologue, in which Parmenides depicts, in the present tense, a journey in a horse-drawn chariot beyond the great gate, to a meeting with a divinity and the revelation of truth, is, as long been recognized, equivalent to a
journey into the next world. We may compare it with Aristeas or Epimenides, as well as the shamanistic healing of illness and the journey of the dead into the other world found in southern Italian sources, especially the katabasis of Pythagoras. Parmenides puts his knowledge into competition with older wisdom. Light and Night represent the realms of life and death. The goddess "sends the souls, now from the visible into the invisible, now back again." This means that the existence of the soul is antecedent to the cycle of life and death, and implies a kind of transmigration. Further, we learn of the Δαίμονος,

πάντα γάρ ὦ στυγγερόσι τόκου καὶ μίξεις ἄρχει.

"Hateful birth and intercourse"—such a phrase cannot be a simple, formulaic allusion to the pains of childbirth; the attitude of disgust and revulsion expressed in the word στυγγερός, στυγγές is contrary to normal feeling and betrays a remarkable degree of alienation from ordinary ways of thinking.

κλαίσαι τε καὶ κώκυσα ἰδὼν ἀπονήθητα χῶρον

—this is Empedocles' reaction, as expressed in the Katharmoi, to entrance into human life; and a Pythagorean asuuma calls our birth a punishment. Such a "puritanical" attitude to life, which sees our existence mainly as a burden and a punishment, can scarcely be called anything but Pythagorean, especially in southern Italy. This with-

drawal from the world of the senses, and concentration on the solitary "Is," has its prototype in yoga-like exercises in concentration, and in "shamanistic" ecstasy. Parmenides' doctrine of Being is, to a degree, a transposition of the theory of immortality to an entire new plane. It is not from Pythagorean science or philosophy that this man takes his departure, who himself initiated a new era in Greek philosophy, but from that complex of "shamanistic" prophecy, unusual beliefs about the soul, and puritanical outlook on life to which the story of Pythagoras and the aecumata lead us.

Some have thought they discovered in Zeno's polemical argumentation still more specific references to Pythagorean philosophy and mathematics. In Plato's view he was defending Parmenides against those who made mockery of his doctrine of the one Being, by proving that the assumption of plurality was even more absurd, and it has been thought that the partisans of plurality could only be Pythagoreans. The complex of fundamental problems that makes its first appearance in the paradoxes of Zeno is still live in modern logic and mathematics, and this gives an additional incentive to understand Zeno's arguments as fully as possible and to identify his opponents. To be sure, even the most determined advocate of his relationship to the

36 Cf. Fränkel DPh 417-420, and above, ch. II 3.
37 Pl. Parm. 128c, not of course a direct quotation from Zeno about his book, but Plato's view of the matter.
38 Tannery, HSH 258ff. and Burnet, EGP 316f, who argue that, since the atomists and Empedocles were later than Zeno, only the Pythagoreans could fit the circumstances and that Zeno was not likely to be attacking ordinary common sense, because that is inexplicable. Plato, however, speaks of others "making fun" of him (κουμανδίων), and that does not require a formal philosophical system. Tannery's thesis was developed further by Cornford, CQ 1923, 7f. PL Harm 36f, who was supported by others, notably Lee. Cf. Stenzel, Metaphysis des Alteuropa (Berlin, 1929), 45 f; Chrysopas, Pres. 41 n. 165, 95 n. 401; Mendelsohn, Inf. 238ff, on the mathematical side, Hase-Scholz. It was contradicted by Zeller I 722 n. 1; Junge, Symb. 232ff; Heidel, AJP 1945, 21f; Vlastos, Gnomon 1953, 1f; Frankel, WP 234 n. 1; G. E. L. Owen, Proc. of the Arist. Soc. 58 (1957-1958) 199-222; D. J. Furley, Two Studies in the Greek Atomists (Princeton, 1967) 44-56: above, ch. 1, n. 106-74. For the specifically mathematical aspects, van der Waerden, MtAm 1943-1948: doubts: Booth, Phronesis 2 (1957) 1ff, 99ff: mediating: Maupin, Inf.—Plato and Simplicius speak of a single book of Zeno (Heidel, AJP 1945, 22). According to the Suda (A2) Zeno wrote, along with other books, one with the title Πρὸς τοὺς φιλοσόφους, and this is taken as a reference to Pythagoreans (Burnet, EGP 312 n. 2; Lee 8; Joly 31f; cf. Burkert, Hermes 1960, 170). Zeno was regarded as the ancestor of skepticism (D. L. 9.72; cf. 9.99); one might say that, just as Sextus Empiricus divided philosophy into logic, physics, and ethics, and wrote books πρὸς λογικοῖς, πρὸς φυσικοῖς, πρὸς θεολογοῖς (Math. 7-11), so Zeno wrote πρὸς τοὺς φιλοσόφους, but this is an indication of content, which became a title only in the Suda: it has nothing to do with Pythagoreans.

38 See, for example, Bertrand Russell, Mysticism and Logic (London, 1917, repr. 1950) 48ff.
Pythagoreans admits that his arguments against movement and against the idea of space are expressed so generally that it is impossible to relate them to any historical persons. It is only the arguments against plurality that are referred to specifically Pythagorean doctrines, a mathematics of infinitesimals or a "number atomism."

In the directly quoted fragments, Zeno formulates the thesis to be refuted as generally as possible: ει πολλα ἐστιν (frr. 1, 3). This is interpreted, "there is a plurality of concrete things ... each of these concrete bodies is a number, or plurality of units." This way of understanding the phrase is supported by the fact that some sources designate the "plurality," against which Zeno is speaking, more specifically as a πλῆθος ἐνάδων; but, thanks to the thoroughness of Simplicius, we can see precisely how this tradition came to be, and can show that it represents a diversion from Zeno's own thoughts and manner of speaking.

In the second book of the Metaphysics, Aristotle discusses, along with other problems, those of εν and ὅν, in the tradition of Plato's Parmenides. Here he mentions the view that εν is identical with the point and remarks that the latter would be, κατά το Ζήρωνος ἀξίωμα, "nothing." This is a reference to fragment 2, which we have in Zeno's own words, and nothing else. It is employed in the context of Platonist discussion; and the definition of a point as μονας θεων ἔχουσα too, belongs to Plato and not to Zeno. Eudemus, also following along Platonist lines, develops the antonym of ἐν and πολλα, which he regards as solved by the Aristotelian distinction between δυναμει and ἐνεργεια. Then, with a façade, he adds a dictum of Zeno's: καὶ Ζήρωνα φαιν ἐγενεν εἰ τις αὐθεν τὸ ἐν ἀποδοθῇ τι...
The idea that Zeno conceived the plurality that he wanted to refute as a "plurality of units" is a secondary product of an exposition of Alexander, who combined the historical Zeno with an argument of Eudemos' which was not meant as a historical statement at all. Aristotle and Eudemos, in discussing the equation of point and ēk, cited a passage of Zeno which had not been written for this purpose. So there is no justification for using this passage as a basis for inferences about a Pythagorean "number atomism."

The origin, meaning, and intention of Zeno's philosophy will remain controversial, because the problems he raised are of so fundamental a nature; therefore we may leave unsettled the question, to what extent Zeno's ἀπολλύματα are intended to prove a positive doctrine, like for example the idea of a continuum. The connection of Zeno and Parmenides is crucial; Plato himself made it clear that Zeno's arguments represented the polemical "reverse" to the "obverse" of Parmenides' philosophy of Being. Zeno's target is the naive worldview of "sound common sense" in general. An additional consideration here is that any polemic reshapes its own opponent. The same realization brings recognition of the correct and rejection of the false; the antithesis is determined by the thesis. If one is, rightly, mistrustful of Aristotle even where his aim is "merely" to report the opinions of others, because interpretation, as well as polemic, will always introduce some distortion, then it must be quite a hopeless undertaking to reconstruct an opposition from Zeno's polemic, where an opponent is not even named. It cannot be proved that there existed a Pythagorean philosophy or science before Parmenides and Zeno.

The possibility of Pythagorean influence should be considered in relation to many thinkers of the fifth century. Ancient tradition brings Alcmaeon, Epicharmus, and especially Empedocles into connection.

On the form of the name, see Dk I 495.39. For Alcmaeon as a Pythagorean, D.L. 8.83, Iam. VP 104, 267, Philop. De an. 88.11, Sophonis De an. 14.31, schol. Pl. Alc. 121b, Simp. De an. 2.3. Simplicius emphasizes that Aristotle does not call him a Pythagorean: at Met. 986a27ff (on the text, above, ch. 2, n. 6) a relation between Alcmaeon and the Pythagoreans is seen, in their doctrines of opposites; but this very point sets them up as different from one another. Brotin, Leon, and Bathyllus, to whom Alcmaeon's book is dedicated, are regarded as Pythagoreans. (On Brotin, above, ch. II 2; the catalogue, Ian. VP p. 144.2 D., names Leon, and at 145.10, "Bathyllas" as Pythagoreans.) Modern writers emphasize sometimes the originality of Alcmaeon (Heidel, Medicine 43; R. A. Sedla, "Importanza di Alcmeone nella storia del pensiero greco," R. A. I. A., 336 [1935] 233-247; Guthrie I 341-350, and sometimes his dependence on the Pythagoreans (Rostagni, Verbo 356; Mondolfo, ZM 60off; Timpanaro Cardini 118ff, with refs.). Zeller tries to compromise between the two positions (I 596f).—If, as Favorinus says, Alcmaeon was the first to write προπλωσις (D.L. 8.83 = DK A1, cf. A2), he is obviously to be dated earlier than Parmenides.

Epicharmus as a Pythagorean: D.L. 8.78, Iam. VP 266; with the support of apocryphal writings, D.L. 8.7 (cf. 78), Plut. Numa 8 (DK 21B65), Iam. VP 241ff. Ennius, in his Epicharmus, deals with Pythagorean doctrines of transmigration (DK 23B47-47). Rostagni, more than anyone else, used Epicharmus in the attempt to reconstruct early Pythagoreanism (Verbo 71ff; cf. Mondolfo, ZM 318ff; Zeller had already taken the opposite position, I 60ff). Timpanaro Cardini does not include Epicharmus in his collection. Reinhardt (Parr. 118ff) sets out the relations to the Eleatics. Epicharmus cited Xenophanes by name (Arist. Met. 1010a45 = DK 21B5). In considering the fragments in detail, one is faced at every step with the problem of authenticity. What Alcmaeon quotes may be accepted, tentatively, as genuine (III 1, 43; cf. M. Gigante "Epicarnio e Platone," Parola del passato 8 [1953] 161-175). What looks like a theory of ideas we are sure, from the testimony of Aristotle, is not Pythagorean (fr. 3, spurious according to Diels, DK I 193, and according to Schmid I 644 n. 6, comprehensible "only if Pythagorean"). It is not, however, fundamentally Platonic (see Zeller I 608 n. 6); we can only guess at the comic context from which it may have come. Is it the contrast between ἡμῖν ἀναπληρωμή and ἀναπληρωμή as an unattainable πρόγια; in that case the fragment might be genuine. On even and odd numbers in fr. 2, see below, ch. VI 2.—Among the entirely or partly apocryphal works, the Politia (DK 23B6-57) was composed, according to Aristoxenus (fr. 45 = DK 23A10) by Chrysogonus the flute player (a contemporary of Alcibiades; cf. Ath. 12.315d). What Clement quotes from it has a Pythagorean ring, but also shows affinity with Platonism. Fr. 57.1: ὁ λόγος ἀνθρώπου κυβέρνει κατά τρόπον ἀρίθμων. No. 2. φαίην ὄνομα καὶ λογισμῷ τετραγωνῷ σύνθεσιν ἐποίησεν. Cfr. Epic. 956θη: θεὸς ἐν ἰσοτρίῳ (ἀρίθμων) ἐν μένῳ ἀρίθμῳ ἕκατον. See also the Derveni papyrus, col. 20, on the ἀρίθμῳ of the winds and seasons. On the date of Empedocles, see Schmid I 638f before 488/487, going by Arist. Poet. 14.813.)

On fr. 129, see above, ch. II 3; Alcidas ap. D.L. 8.86 (above, ch. II 2); Timaeus FgrHist 566F14 = D.L. 8.54; Neaesthos FgrHist 845E6 = D.L. 8.55 (quoting the "Telauges" letter); Hermippus, D.L. 8.86. A spurious line of Empedocles names Telauses as his teacher (fr. 155; cf. Euseb. Praep. evang. 10.14.15; Theodoret 2.23, Suda s.v. Empedocles); Alcidas and Timaeus name Pythagoras himself, and the Telauses letter names Hippasus and Empidocles. Theophrastus mentioned only his relation to Parmenides, but
tion with Pythagoras himself, but also Leucippus and Democritus. Hipparus and probably also Hippocrates were called Pythagoreans. Modern scholars have seen Pythagoreanism in the book On Seven, and in ancient medical writings generally, as well as in Polycrates.
do go together, and in Empedocles the two are still combined. 78 A kind of "knowledge" about the arrangement of the cosmos, which is partly a matter of number symbolism, forms part of this amalgam, closely bound up with ritual in the medical prescriptions and procedures which are supposed to lead to recovery. In particular, the transition from the βίος Πυθαγόρειος (or 'Orphicus') to a rationally based regimen is only a new approach to the same thing; in each case there is a system of injunctions to abstinence, whose goal is to concentrate, to enhance, and to control the powers of the individual. This is how the trainer Icetus of Tarentum became famous, even before Herodicus of Selymbria; 79 and before him the tradition calls Pythagoras the inventor of a new kind of regimen, 80 and sometimes even calls him a physician. 81

Of course, one can hardly think of Pythagoras as the only, or even the most important, originator of these trends. The oldest of the famous physicians of Croton, Democedes, who was active in the courts of Polycrates and Darius, is likely to have been approximately contemporary with Pythagoras. His father, Callipho, came from Cnidus, which shows that the connections between Ionia and Croton were not all due to Pythagoras. 82 Empedocles drew many inspirations from the medical tradition, and in particular from Alcmaeon, 83 and in turn

78 As Hippoc. Mor. 1–2 attacks shamanistic charlatans, Vet. med. 20 attacks the physiology of Empedocles. Democedes functioned as a seer in the court of Darius (Hdt. 3.132). Cf. also F. Wehrli, MH 8 (1951) 306.
79 DK 25. Also Ael. NA 6.11, VH 11.3, Lucian Hist. cont. 35 (Wuilleumier 366). The dating before Herodicus of Selymbria comes from Pl. Prot. 316d: "ἰσχευτικοί δὲ ταραντίνοι καὶ ἐν τῷ ἔνω ἰώνων ἀδιάφοροι ἄρθρον αὐθεντής ἥμαρυκιον ἐν Σελῆμαβριον." See above, ch. II 4, for Pythagoras as the alleged reformer of the regimen of athletes.
80 On the bean taboo, Diogenes Antonius (who sometimes used good sources) says, speaking of Pythagoras (Por. VP 35), αὐτῷ καὶ τῷ οἴκῳ ἢσπερ ἐπὶ αὐτὸν ἐπὶ τὴν αὐτὴν ἐξεισέλασθι αὐτῷ ἔστιν ἔκφρασις, οὗ πολεῖ μεν ὡνόμων, πολεῖ δὲ νοησοῖν, ὥσεὶ αὐτῷ μεν πιασμοίναι καὶ αὐξάνομεν, πολεῖ δὲ λεπτομέναι καὶ ἱστανομέναι. Similar, Aristo. ap. Iam. VP 196c.
81 Ael. 4.17 (from Aristotel? cf. above, ch. II 3): ὁποῖος ὑπεκαύομεν δὲ τότε τὸν ἀνθόν, ὑπέρέχειος λόγος οὗ Πυθαγόριος ὁδύκειν νοτιῶς ἀλλὰ ἵσταντοι. Ὑπὲρ υγιείας σαυροτετρυχοῦς. [Por. VP 35], ἵσταντοι τοῦ πάντων παρ' ἐμοῖς ἀριθμίως.
Did the background of these thinkers include a Pythagorean doctrine of Limit and Unlimited? In considering this, let us remember that it is hardly possible to arrange in parallel the opposites named by Alcmeon, which are taken from everyday observation. Does not this fact, in itself, show that his idea is pre-Parmenidean? The "table of opposites," on the other hand, is probably later, and Alcmeon takes up ideas about opposites which had been a part of the Ionians' thought since the time of Anaximander. Hippasus the Pythagorean appears in the doxographical tradition as a plain monist. Hippo studies moisture and dryness, warm and cold, in the same way as the Ionian physiologers; and even in Philolaus the warm and cold of his physiology do not seem to be integrated into the concept of the Limiting and Unlimited of his cosmology. There is nothing to show that this theory of opposites was already there at the beginning of the fifth century. If we restrict our attention to the evidence, it seems much more likely to be a position taken on the question that rose, in reaction to the work of Parmenides and Melissus, about the limitedness or unlimitedness of Being. Number symbolism is to some extent older, but there are no traces of it in Parmenides, and at the most, very slight traces in Alcmeon and Empedocles. Before Philolaus, Hippasus seems to be the only one to have dealt with music theory.

94 According to Aristotle Met. 963a22 (and elsewhere), Alcmeon mentioned white and black, sweet and bitter, good and bad, large and small. Aëtius (S. 30.1 = fr. 4) names moist and dry, cold and warm; but these are so common that we cannot rely on this report (cf. Pl. Soph. 242d; D occult). These ideas appear also, however, at A 5 and A 9. Timpanaro Cardini also thinks that the table of opposites is later than Alcmeon (120; cf. above, ch. I 5, n. 120). Mondolfo, ZFM 321, posits another, more primitive duality in Pythagoreanism as the source of Alcmeon; but why should we need to insert an intermediary between the Ionians and Alcmeon? Warm and cold, dry and moist are already there in Anaximander (A 9, 11 §86f, 172, 27, 1); cf. Anaximenes A 21, Heraclitus fr. 126, Kahn 100ff, 160ff.

95 Above, ch. II 5. Burnet (EGP 199) tried to bolster his version of Parmenides' Light with a reference to Hippasus (cf. above, n. 24), but there is no mention of an opposing principle in Hippasus, and ever since Aristotle he has had his place beside Heraclitus.

101 DK 38A111, cf. A 10. At the same time, the life-giving "moist" is for him the real ὀξύς.

102 Above, ch. III 2, on A 27. According to the evidence of Menon, our bodies "consist of the warm," and the cold air is something foreign, borrowed, so that alongside the idea of harmony a monistic thought seems to find a place.

103 Cf. Alcmeon A 15. On Empedocles' doctrine of the elements, below n. 101. On ἄμετανοι Emp. fr. 27.3, 96.4, 23.4. Empedocles speaks, in fr. 96, of a numerically determined proportion in the mixture of the elements, and Mondolfo (ZM) decides that this is an indication of Pythagorean influence. Aristotle, however (Met. 1192b6ff), differentiates between number as the ἄμετανόι μέτωποι, and the Pythagorean and Platonic concept of number (Zeller I 1026).

84 Cf. the polemic at Hippoc. Vet. med. 20.

86 Above, n. 63. One is reminded of Pythagoreanism by remarks on the healing power of music (ch. 30); but the "vowels" (ch. 9) were not yet known in Magna Graecia in the 5th century. In the expression ὁλίγμος κόσμος, see above, ch. III 2, n. 31. In place of ἄρατος κόσμος for the outermost heaven sphere, Roscher suggested ἄρατος, which Pfeiffer (Stengel 335) combined with Philolaus A 16 (see above ch. III 2, n. 35); but ἄρατος is guaranteed by the equation of inseparabiles soliditatis with ἄρατος πάσος in ch. 6 (a gloss of Galen, VIII 637.1 L.; Kranz, NGS 1938, 124 n.). ἄρατος is "endless," Parm. fr. 6.7, Crit. fr. 19.4. The idea of macrocosm and microcosm unites On Sevens with Regimen (see esp. Kranz, NGS 1938), and, more remotely, with Philolaus fr. 13. An Iranian origin for this concept was suggested by A. Götze, "Persische Weisheit in griechischem Gewande," Zs. f. Indol. u. Iran. 2 (1932) 60–98, 167–177. See Olerud, passim; J. Duchesne-Guillemin, in Problemi attuali di scienza e di cultura 76 (Rome, 1965) 627, has withdrawn the objections he offered in HTR 49 (1956) 115–122. Götze (86ff) believed that the Pythagoreans formed an intermediary link between Persia and Greece, as did Olerud (212; at p. 220, however, he envisages the tradition as passing from Persia to the Pythagoreans via Cnidus.

85 Cf. above, ch. II 4, n. 104. The most striking parallel is the prohibition of suicide, but this makes perfectly good sense in the medical tradition; the physician as preserver of life may not promote death. Classification of the Oath as a Pythagorean document by Edelstein outruns the evidence.

86 J. Schumacher, Antike Medizin (Berlin, 1965) 46ff, 81ff, traced the scientific basis of Greek medicine to Pythagoras (the idea of the regularity of nature, the concept of health as a norm and as harmony, the responsibility of the sick person to himself, the importance of the daily regimen). This means, of course, simply deciding a priori that what is scientific is Pythagorean. Jones, Ph. Med. 1ff, thinks that at the least the combination of philosophy and medicine is attributable to Pythagoras, though the unity of medicine and cosmological ideas was present very early in the context of shamanism, and the only question is, what Pythagoras contributed that could be called scientific.
Alcmaeon had things to say about astronomy, and in particular the "contrary movement" of the planets, not a topic in which a physician would obviously be interested. He also tried to prove the immortality of the soul by its relation to the stars: both were, he thought, in eternal motion. This has led to "Pythagorean" a ring that scholars have often suspected a post-Platonic forgery. Still, the astronomical views can be derived from the Ionians, and the proof of immortality, which is not identical with that in Plato's *Phaedrus*, seems to be Alcmaeon's original contribution. If this is so, what Alcmaeon did was to translate pre-scientific, and Pythagorean, material into scientific language; nothing is presupposed that is Pythagorean except the theory of metempsychosis and a connection of immortality with the heavenly bodies, which appears in the *acussmata*.

Empedocles has a perennial interest because of his intermixture of rational explanation of natural phenomena and religious pronouncements like those of a prophet, both emanating from an eloquent and passionate personality. We cannot maintain a neat correlation of the two sides of his nature with his two poems, *On Nature and Purifications*; and it has been rightly emphasized that this very fact shows

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98 Wilamowitz, *Platon I* 461, 717; Frank vii; Moreau, *Anm 155 n. 6*. The difficult point about this thesis is that it requires us to suppose Aristotle let himself be deceived by a post-Platonic forgery.

99 Below, ch. IV 2.

100 Arist. *De an. 450a20 = Alcmaeon A12*, cf. At, as well as the expression in fr. 2, "People perish because they cannot join the beginning with the end" (with an allusion, no doubt, to formulae of the mysteries about beginning and end; cf. Pl. fr. 137a, Her. fr. 103, Hippoc. *Vet. 1.19, Nutr. 9, 24, Loc. hom. 1, Ocelus 1.14*). The equation of *psyche* and movement makes the foundation for the proof of immortality: when life ceases, the creature stops moving. That which is a conclusion from analogy in Alcmaeon becomes in Plato, through the idea of self-movement, a dialectical proof (*Phed* 245b).

101 Cf. above, ch. II 4, and below, ch. IV 4.—Whether Alcmaeon taught transmigration is as controversial as the relationship between the doctrine of immortality and his physiology. (Rostagni, *Verbo 102aff* distinguishes between an "anima supranaturale" and a "natura corpora," which were confused, through a "grosso equivoque"; cf. Kerényi 288ff, Stella 278ff [above, n. 57]. [Guthrie I 155f].)

102 This is emphasized by Long 66ff. A development "vom Mythos von Logos" is accepted, after J. Bidez, *La biographie d'Empédocle* (Ghent, 1894). 159ff, by Kränz, Hermes 70 (1913) 111, *Emped. passim*. On the opposite side, for dating the *Purifications* after the *On Nature* (favored by fr. 131) are Diels (SBBl 1898, 396ff), Wilamowitz (SBBl 1920, 626ff), and especially Reinhardt (CP 45 [1950] 170ff). For the inner unity of the two poems (e.g., fr. 115 has the elements and Strife; fr. 23.11 represents Empedocles as god or nearly so, fr. 15 has immortality, and in the phrase "life, as men call it" an allusion to doctrines of release or salvation), Nestle (*Philologus 65 [1906] 545ff*), Rostagni (Verbo 262ff), would like to discover a Pythagorean doctrine of elements; his references (including Philolaus fr. 12) are all under the influence of the *Timeas* (above, ch. I 3). To be sure, there is a pre-philosophical conception of "elements" especially in the Indo-Iranian area, in which the cosmos is divided into regions of fire (the stars), air, water, and earth. (Cf. J. Przybuski, "L'Influence iranienne en Grèce et dans l'Inde," Rev. de l'Univ. de Bruxelles 37 [1932] 283-294; Olerud 136ff. We cannot go into the question here, to what extent something of the kind existed in Greece (Kränz, Hermes 70 [1935] 113ff, mentions Orphism, but this is a very uncertain factor).

103 Above, n. 93; ch. III 2, n. 56.

104 When Empedocles compares the shape of the world to an egg, we cannot ignore the relationship of this to Orphic cosmogony (Asō, *Ar. Av. 693ff*).
harmony of opposites in a single world. This is comparable with Alcmaeon’s ιονομοία of opposites in the organism; but in Parmenides the opposites are separate and irreconcilable. If we postulate a philosophy of form, or of an advanced degree of abstraction, we might set Philolaus alongside the Eleatics; but the atomists come much closer to belonging in this company than the sensuous world of Empedocles, to say nothing of Hippasus and Hippo. There is no single characteristic trait to unite the astronomical views of Alcmaeon, Parmenides, Empedocles, and Philolaus; in fact there is one point in which the latter agrees rather with the Ionians than with Parmenides and Plato.¹

If there is a sense in which Pythagoreanism forms part of the background for all this, it is not Pythagoreanism as a fully formed scientific and philosophical structure, but as an inspiration or stimulus of a prephilosophical sort, in the area of mythic lore and religious aspiration, which each, in his own way and in accordance with his own personality, allowed to influence him in the formulation of his thoughts. This is true not only of Alcmaeon, Parmenides, and Empedocles, but also of Philolaus, whose thoughts of the “limiting,” the “unlimited,” and of number were helped by the efforts of Plato to a career whose scope could hardly have been foreseen at the beginning.

There is no law of reciprocal interaction in the field of thought, which could make it possible for us, by inference, to fill adequately the gaps in our tradition, as the law of gravitation enables astronomers to calculate the position or movement of an unknown star. In fact, the suspicion persists that the lacuna in the tradition about early Pythagoreanism is not an accident. If we cannot get a clear idea of the philosophy and science of Pythagoras, it is because Plato and Aristotle did not consider him a philosopher. If we cannot find a clue to the philosophy of Limit and Unlimited and their harmony achieved through number, before the day of Philolaus, it is because this doctrine, in this abstract form, was first created as Philolaus worked to formulate anew, with the help of fifth-century φυσιολογία, a view of the world that came to him, somehow, from Pythagoras.

¹ In the assumption of a material περίπτερον outside the heaven (Kahn 234 n. 4).

I. THE STRUCTURE OF THE WORLD AND THE PLANETARY SYSTEM

The Greeks acknowledged, almost too eagerly, that Greek astronomy was based on the accomplishments of the East.¹ Modern scholarship, studying the original Babylonian and Egyptian sources, has made clear how much of the final achievement is due to the Greeks themselves.² It is true that Greek astronomers used the observational data which by various routes made their way from the Orient, and especially from Babylonia,³ but they made an original contribution, without precedent in the East, in the development of a conception of the world’s structure from the insights of Greek mathematics—the famous Ptolemaic system, in which the planets circle about the spherical earth, at various distances, enclosed in turn by the sphere of the fixed stars. The risings and settings of the stars were thought of as related to the geometry of the sphere, and the irregularities in the paths of the planets were explained by the combination of mathematically perfect circular movements. The problem of cosmic distances was taken up too, and not without success.⁴ From Eudoxus through Hipparchus to Ptolemy, the development of this Greek scientific enterprise can be followed fairly easily, but what came before the first great epoch is, as so often, difficult to make out clearly.

¹ See, for example, (Pl.) Epin. 986c, Arist. Cael. 292a28, Diod. 2.30.
² See esp. Neugebauer, ExSt 156. It is important that, contrary to a widespread belief, the Greek planetary system cannot be shown to have been Babylonian; cf. Boll, RI VII 256ff.
³ According to Sen. QNat 7.3.2, Eudoxus was the first to bring from Egypt exact data about the movements of the planets (T13; cf. T12–20 Lasserre). About 500 B.C. Babylonian astronomy was exerting an influence in Egypt, as is shown by a new discovery: R. A. Parker, A Vienna Demotic Papyrus on Eclipse- and Lunar Omen (Providence, 1959); van der Waerden, Anf. 131–133. Also, cf. the reference to “Egyptian” observations in Arist. Met. 343b10, 28; cf. Cael. 292a8; Chaeremon FGHist 618F7. During Alexander’s expedition, Callisthenes is supposed to have sent Babylonian data to Greece directly (Simpl. Cael. 506.11 = FGHist 124T3; cf. Hipparchus ap. Ptol. Syn. 4.11 p. 340 Heiberg). Suspicion is aroused by the late attestation, its novelistic presentation, and the absurd claim that observations had been carried on for 31,000 years in Babylon (Neugebauer, ExSt 157).
⁴ Hipparchus came fairly close to establishing the distance of the moon correctly, putting it at 33 times the diameter of the earth (RI VIII 1676).
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also of Philolaus, whose thoughts of the “limiting,” the “unlimited,”
and of number were helped by the efforts of Plato to a career whose
scope could hardly have been foreseen at the beginning.

There is no law of reciprocal interaction in the field of thought,
which could make it possible for us, by inference, to fill adequately
the gaps in our tradition, as the law of gravitation enables astronomers
to calculate the position or movement of an unknown star. In fact,
the suspicion persists that the lacuna in the tradition about early Pytag-
oreanism is not an accident. If we cannot get a clear idea of the philosophy
and science of Pythagoras, it is because Plato and Aristotle did not
consider him a philosopher. If we cannot find a clue to the philosophy
of Limit and Unlimited and their harmony achieved through number,
before the day of Philolaus, it is because this doctrine, in this abstract
form, was first created as Philolaus worked to formulate anew, with
the help of fifth-century φυσιολογία, a view of the world that came
to him, somehow, from Pythagoras.

104 In the assumption of a material περίχων outside the heaven (Kahn 234 n. 4).

I. THE STRUCTURE OF THE WORLD AND THE PLANETARY SYSTEM

The Greeks acknowledged, almost too eagerly, that Greek astronomy
was based on the accomplishments of the East.1 Modern scholarship,
studying the original Babylonian and Egyptian sources, has made
clear how much of the final achievement is due to the Greeks them-
selves.2 It is true that Greek astronomers used the observational data
which by various routes made their way from the Orient, and especially
from Babylonia;3 but they made an original contribution, without
precedent in the East, in the development of a conception of the
world’s structure from the insights of Greek mathematics—the famous
Ptolemaic system, in which the planets circle about the spherical earth,
avarious distances, enclosed in turn by the sphere of the fixed stars.
The risings and settings of the stars were thought of as related to the
geometry of the sphere, and the irregularities in the paths of the planets
were explained by the combination of mathematically perfect circular
movements. The problem of cosmic distances was taken up too, and
not without success.4 From Eudoxus through Hipparchus to Ptolemy,
the development of this Greek scientific enterprise can be followed
fairly easily, but what came before the first great epoch is, as so often,
difficult to make out clearly.

1 See, for example, (PL) Epin. 986a, Arist. Cael. 292a8, Diod. 2.30.
2 See esp. Neugebauer, ExSc 156. It is important that, contrary to a widespread belief,
the Greek planetary system cannot be shown to have been Babylonian; cf. Boll, RE VII
2561 ff.
3 According to Sen. QNat 7.1.2, Eudoxus was the first to bring from Egypt exact
data about the movements of the planets (T15; cf. T12–20 Lasserre). About 500 B.C.
Babylonian astronomy was exerting an influence in Egypt, as is shown by a new discovery:
R. A. Parker, A Vienna Demotic Papyrus on Eclipse- and Lunar Omen (Providence, 1959);
von der Waerden, Abh. 131–133. Also, cf. the reference to “Egyptian” observations
in Arist. Meteor. 348b10, 28; cf. Cael. 292b8; Claeissens FGrHist 618F7. During Alexander’s
expedition, Callisthenes is supposed to have sent Babylonian data to Greece directly
Heiberg). Suspicion is aroused by the late attestation, its novelistic presentation, and
the absurd claim that observations had been carried on for 31,000 years in Babylon (Ne-
gebauer, ExSc 151).
4 Hipparchus came fairly close to establishing the distance of the moon correctly,
putting it at 33 3/ times the diameter of the earth (RE VIII 1676).
The earliest connected discussions of astronomical matters are found in the works of Plato, and it is not merely a coincidence that almost all the important astronomers of later times were Platonists. The Greek idea of the general structure of the world is set forth here in all its essential features: the earth is spherical and rests, free of support, at the center of the sphere of the fixed stars; the planets are stationed in concentric paths at varying distances; and their apparent irregularities are explained by mathematical principles. The order of the planets, from the earth at the center, is moon, sun, Venus, Mercury, Mars, Jupiter, Saturn—an order retained by Eudoxus, Callippus, Aristotele, and even Eratosthenes. It is "correct," insofar as the planets are arranged according to how long it takes them to make a circuit through the zodiac, with a longer time corresponding to a greater distance. Since the inner planets Venus and Mercury are, from the geocentric point of view, "isodromous" with the sun, that is, like it they make the circuit of the zodiac in a year, their position in relation to the sun cannot be determined on this principle, and this is what led to the ancient controversies about the order of the planets.

It is known that the facts about the planets came to Greece from Babylon. They were known centuries earlier there, and it is from there and not from Greek mythology that we have the association of the planets with individual gods, which still provides their names. The discovery of the most important data on planetary movement is due to the Babylonians, and in particular the time of their orbits. At least in the case of Saturn, whose orbit takes 294 years, this would require several generations. The discovery of the planet Mercury, which is difficult to observe, also belongs here; in fact, a fund of exact knowledge about the planets is doubtless the most obvious debt to oriental knowledge in the pre-Platonic period.

Along with knowledge of the planets, the recognition of the spherical shape of the earth and the postulate of perfect circular movements make up the world picture of Greek astronomy, as it is presented by Plato. Ever since the ancient commentators on the Timaeus it has been thought that "the" Pythagoreans were the source to which Plato owed his astronomical interest and knowledge, and this is not the least important cause of the high place accorded them in the history of science.

Aristotle, however, records, along with the famous ideas of the harmony of the spheres and the special theories of "some" Pythagoreans about comets and the Milky Way, another peculiar Pythagorean system. Here the earth is one of the planets, and circles, along with a "counter-earth" which is invisible to us, about a "central fire." The doxographers attributed this system to Philolaus, and we may justifiably use the well-established expression, "the system of Philolaus," without prejudice to the question of its date or its relation to the Pythagorean named in the Phaedo. In fact this relation has come into question among modern scholars. The attribution to Philolaus of a system described by Aristotle suggested the reconstruction of an older, more simple, geocentric system, supposedly developed by Pythagoras himself or the early Pythagoreans; the materials for this enterprise were some late references, but, above all,

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6 The Platonism of Eratosthenes and Ptolemy is well known; but Hipparchus, too, shows himself to be in the influence of the Timaeus when he teaches "cognitionem cum homine siderum animasque nostras partem esse caeli" (Plin. HN 2.95).

7 Perspicuously described at Rep. 616e et seq.; Tim. 38d emphasizes the number 7 and names Venus and Mercury, but when the idea is taken up again he reverses the order and names Mercury first. This is why Aet. 2.15.4 has Plato putting Venus over Mercury (followed, no doubt, by Eratosthenes in his Hermes; Chalcid. 73). Epin. 987b has a simple enumeration; here it is also stated that aside from οὐσίωροι the planets are nameless, though ἐπωνύμως ἐπιθύμησαν θεοί. Here and later the style of the pl. Ἀφροδίσις ζῶσης ... the names Ἀφροδίσις, Σελήνη, Πυρὸς, Φαῦσις, Φαεῖαν, Φαῖων do not occur before the Hellenistic period (Heraclides fr. 66 is of doubtful value; cf. Wehrli 82). Not before the first century B.C. is a planet called simply Aphrodite or the like (Cicumont, AC 1915, 5-43). On Plato's astronomy see the commentaries on the Timaeus; Duhem 28ff; Heath, Aristarchus 134-180; Math. 310ff.

8 Eudoxus: Procl. In Tim. III 63 (cf. Eudox. Ars astr. 23) = D9 Lassere; Xenocrates fr. 17. Callippus-Aristotele: Met. 1071b17ff, Procl. loc. cit., De mundo 392a23ff. Chrysippus: SYP II no. 527. Eratosthenes Hermes: Adrastus ap. Theo Sm. 142.7 = Chalcid. 73. "Pythagoras" Anon. Phot. 439b17ff. Cic. Nat. d. 2.52f (Stoic praise of cosmic order); IG XII 1.913 (100 b.c.); Pap. Aberdeen 17 (Aug. 1, A.D. 187). This order of the planets is also to be assumed for Heracleides Ponticus fr. 95 that the sun is not central, as Wehrli 92f supposes; the spheres assigned to Pluto are, aside from that of the moon, those of the elements; cf. Anon. Phot. 410b25 and Heibl. 1).--On the whole subject, Immsch 60ff, and esp. Boll, RE VII 256dff.

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9 See Boll, RE VII 256dff; Gundel, RE XX 2025, 2029; Meissner 405ff; van der Waerden, astr. 105, 108, 172. The Babylonian gods are Ishtar, Naba, Nergal, Marduk, and Ninurta (Aphrodite, Hermes, Ares, Zeus, Kronus). The designation of Saturn as θηλείν ὁ στηρύς is Babylonian (Pl. Epin. 987c); the text is emended even by Burnet; see against this, Eudox. Ars astr. col. V, Diod. 2.30, Simp. Carl. 495.28, J. Bidez, REP 29 [1903] 319ff, Meissner 254f. The colors attributed to the planets in Pl. Rep. 616e go back to Babylonia (Bidez, Eos app. 1), in spite of the objections of W. J. W. Kosker (Le mythe de Platon, de Zarathoustre et les Chaldéens [Leiden, 1951] 66ff), who thinks this is based on independent observation. For example, Jupiter, called θηλείν θύρας by Plato, is simply called "the white star" (Molobabar) in Babylon (Meissner 404).
conjectures based on Parmenides and Plato. Then Frank declared the Philolaic system too bold and advanced even for the end of the fifth century and dated it in the years of Plato’s old age. For him, the history of astronomy was one of the most striking proofs of the spuriousness of the Philolaus fragments. These arguments, however, must be reexamined. From the point of view of the history of science, the most important points are the discovery of the spherical shape of the earth, the recognition of the five planets, and the explanation of the apparent irregularities of their courses by means of circular movements.

The history of the exact sciences is of special importance in the history of thought, because, as O. Neugebauer has put it, “the inherent accuracy of the mathematical sciences will penetrate to some extent into purely historical problems.” For this reason it is a special methodological advantage that the investigation upon which we are now embarking can be conducted independently, in all important respects, of the conclusions reached in earlier chapters. But it is very important to establish at the outset the extent, and thus also the limits, of the mathematical exactitude to be attained in the history of science.

With great confidence, Frank reconstructs the main stages of Greek astronomy, in an apparently necessary order: development of the understanding of space, of solid geometry and perspective, by Anaxagoras and Democritus; discovery of the sphericity of the earth and the “true movements of the planets” in the form of “geometrically perfect orbits” (28) by the Pythagoreans of Archytas’ circle; first mathematical explanation of the movement of the planets, by Eudoxus; discovery of the rotation of the earth on its axis, and finally the “Copernican view of the world” in the system of Philolaus (35)—which thus represents the second step beyond Eudoxus. Similarly, van der Waerden sees in “the logical development of astronomy” the “strongest proof” that the system of Philolaus was preceded by that geocentric, Pythagorean system which must be assumed. Still and all, in the history of science logical necessity and historical sequence are not always identical. Of course, every forward step in this field depends on a certain group of preliminary studies, but it is just as obvious that there do occur backward steps, significant enough to cause correct answers already found to be given up. And intuitive anticipations occur, too, based on inadequate foundation, so that, in such a case, true progress consists in giving up results which, to hindsight, will be seen as correct. Perhaps the allegedly “Copernican” system of Philolaus will fit into this category.

Here, as always, that evidence must be decisive which gives us a clue to the purely fortuitous aspects of the development; for if we were to rely on logical calculation of the probabilities, we should find a good many alternative possibilities. Above all, an inference about preconditions, though it have almost the certainty of mathematical demonstration, can never, at the same time, prove who is responsible for these earlier achievements. That the astronomy which preceded the Philolaus system was Pythagorean, or that Pythagoras set up an astronomical system of the same or even a higher order than those devised by Anaximander or Parmenides, is nothing but a historical hypothesis, which cannot be corroborated or refuted by the inner logic of the history of science, but must be known from external testimony. For the most part, this question is not even considered; under the influence of the Platonic tradition all mathematical science in early Greece is called Pythagorean. To put the question at all means to remove one of the main supports from under most of the reconstructions.

Late sources attribute to Pythagoras a decisive role in the formation of the Greek view of the world’s structure, but these reports are strikingly contradicted by statements about Parmenides and the latter has the oldest authorities on his side. The matter in question is the

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10 Gruppe, KohnSyst. 50; Boeckh, KohnSyst 89; esp. Martin, Pyth. Döring already was calling the theory “generally recognized” in AGP 1892, 508; cf. i.a., Burnet EGP 10ff.; Berger, Erdkunde 185f.; Heath, Aristarchus 48ff.; van der Waerden, Astr. 20ff.; Guthrie I 280ff.
11 Below, ch. IV 3.
12 ExSc 1.
13 What we shall presuppose, from previous discussion, is a fundamental skepticism of the post-Aristotelian tradition, strongly influenced by Platonism, as well as the relative priority of the axioms—and both of these points are confirmed again and again. We may add a few points of detail, like the dependence of the so-called “more genuine” Pythagorean on Aristotle (ch. III 1) and the (related) analysis of Philolaus A16 (ch. III 2).
14 Frank 10ff.; cf. 184ff.
15 As compared with Anaximander, the astronomical views of Xenophanes and Heraclitus are regressive (Heath, Aristarchus 49ff.); but so are those of Anaximenes, who does not dare to let the earth hang free in space. On the relationship of Aristarchus, Ptolemy, and Copernicus, see below, ch. IV 2, n. 1; ch. IV 3, n. 1.
16 If Aristarchus had not happened to mention the Philolaic system, modern scholars would confidently date it in the era of Aristarchus of Samos.
17 Even Frank clings to the Pythagorean origin of Greek mathematical science, only he dates it in the time of Archytas.
sphearicity of the earth and the identity of evening star with morning star, which means the beginning of a true understanding of the planets.

Diogenes Laertius writes of Pythagoras: τὸν οὕτων πρῶτον ὄνομασα κόσμον καὶ τὴν γῆν στρογγύλην ὅν ἐκ Θεόφραστος, Παμμηνίδην.19 In another passage, which in all likelihood goes back to Theophrastus, he likewise says of Parmenides, πρῶτος δὲ οὗτος τὴν γῆν ἀπέφανε σφαιροειδή καὶ ἐν μέσῳ κεῖθαι.20 The spherical earth appears again as a doctrine of Pythagoras in the Hypomnemata (reported by Diogenes Laertius, 8.25).

At first, the natural assumption seemed to be that Parmenides had learned this fact from Pythagoras and published it,21 but Frank maintained vigorously that Parmenides could not have known that the earth was spherical; it could be seen from Plato's Phaedo, he thought, that at the time of that dialogue this was a quite new discovery. Parmenides had, according to Theophrastus' evidence, merely been the first to use the word στρογγύλως for a disc-shaped earth.22 And recently it has even been contested whether Plato does describe the earth as a sphere in the Phaedo.23

But the Theophrastus passage cannot be eliminated so easily.24 We cannot, in fact, ascertain what word Theophrastus used, but it seems sure that in the context he was thinking of a spherical rather than a discoid body,25 so that the dubious inference from the Phaedo cannot be used against the evidence of Theophrastus.26

18 D. L. 8.48 = Theophr. Phys. op. fr. 17, Dox. 492 = DK 28A44.
19 D. L. 9.21 = Theophr. Phys. op. fr. 6a, Dox. 482; on the attribution to Theophrastus, Dox. 166ff.
20 Tannery, HScH 336.
21 Frank 184ff, 188ff; Heidel, Maps 70ff, 83ff; Morrison, JHS 1955, 64.
22 T. G. Rosenmeyer, CQ 50 (1950) 193-197; W. M. Calder, Phronesis 3 (1958) 121-125; Rosenmeyer, ibid. 4 (1959) 71-72; Morrison, ibid. 101-110. The myth may preserve a bit of older cosmology, but Phid. 112e seems to refer to the middle of a sphere. Theophrastus' testimony on Parmenides helps to decide the issue.
23 Against Frank, P. Friedlaender, Plato 1 (New York, 1958) 386; Mondolfo, “La prima affermazione della sfericità della terra,” Accad. d. scienze di Bologna 1932 (not available to me; see ZM 319ff); Rehm–Vogel 12; Kuhn 11iff; cf. Thomson 110ff.
24 The word στρογγύλως is used with reference to Pythagoras, and here surely means “spherical.” The parallel passage D. L. 9.21 has σφαιροειδῆς, which is of course not reliable. (At D. L. 2.1, the earth of Anaximander, the “column drum,” is called σφαιροειδῆς, as Frank remarks, 200.) Στρογγύλως can mean circular or spherical; it is used of hallstones at Aristoph. Nub. 1127. It is not certain what shape is attributed to the earth by Diogenes of Apollonia; it is called στρογγύλη in D. L. 9.37.
25 At Pl. Phd. 108e, the word περιφερέτης (στρογγύλως, 97e) is used quite casually; the main thing is the assurance that the earth does not need the support of air or the like: Plato is here using another doctrine to hark back, against Anaxagoras (A8), Diogenes of Apollonia (A16A; cf. C2 – Eur. Tyr. 884), and Democritus (Arist. Car. 294b14ff; also Anaximenes A6, 20), to Anaximander and Parmenides.

Not only was the Democritean Bion of Abdera acquainted, about 400, with the mathematical consequences of the spherical shape of the earth,27 but about 430 B.C. Hippocrates of Chios had projected the celestial circles onto the earth, obviously presupposing its sphericity.28 A spherical earth is but an easy step from the thesis that it is located in the middle of the universe and “because of equality” did not fall in one direction or the other. This is a thesis Parmenides took over from Anaximander; and if the description of the world of doxa is an independent creation of Parmenides, it is not at all far fetched to suppose that, having in mind the perfect “sphere” of Being, he was the first to call the earth on which we live a sphere.29 Empedocles seems to have followed him.30 That this theory did not win out, and that Democritus clung to his peculiar idea of a bowl-shaped earth,31 is one of those retrograde steps that sometimes occur in the history of the natural sciences.

Posidonius found that Parmenides was the ἄρχων of the division of the earth into five zones, but objected that he had made the torrid zone too broad, so that it extended beyond the tropics.32 Most scholars have been very skeptical of this datum,33 but it fits quite well into the Parmenidean dichotomy of Fire and Night: just as, in the heavens, Fire and Night occur both pure and mixed, so on the surface of the

27 DK 77.1 = D.L. 4.58: οὗτος πρῶτος ἐγενεῖται διὰ ταύτα, ἐν τῇ πολλᾳ ἐξαιρέσει, ἐνθα γίνεται ἐξ ἡμῶν τὴν ἡμέραν, ἔχουσα τὴν σκόπους, ἐνθα γίνεται τὸ ὁμόνωμον τῆν ἑαυτῇ καὶ ἐν ἡμέρᾳ. Strabo calls him ἀστρολόγος (1, p. 29 = DK 77.2). Wellmann (RE III 486) dates Dion, rightly, before Eudoxus. Dion wrote Ironic and Attic (DK 77.1). We need not decide whether older myths may have contained reminiscences of the long polar nights (cf. the controversies over Od. 10.82ff and Hdt. 4.25).
28 DK 42.5 = Arist. Met. 343a8: the torrid zone between the tropics. On Hippocrates’ date, see below, n. 77.
29 The earth at the center: A44, following Anaximander (below, n. 44); Being compared to a sphere: fr. 8.43; cf. Theophr. Phys. op. fr. 6, Dox. 482. See Gigon, Uebertragung 806, 275.
30 Kranz, Emped. 50: “die Erde nach damaliger, althethagoreischer Vorstellung als Kugel im Mittelpunkt” (but cf. below, n. 44). Actually, the idea (A56) that the sun is a reflection of the “round” earth (εὐκορφητής) implies sphericity, since the sun is not stationed directly over our section of the earth, and the reflection of a disc would appear distorted rather than perfectly round. But it is naturally an open question to what extent Empedocles was preoccupied with such geometrical considerations. He did give up the idea of Hades as a region under the earth (below, ch. IV 4); and this would be consistent with a spherical earth.
31 A94; Frank 187ff. Oriental tradition may be behind this. According to Babylonian teaching the earth is σφαιροειδὴς καὶ κυλη (Diod. 2.31; Meissner 107ff). Herodotus ignores the sphericity of the earth (e.g., 3.104). On Heb. 2, above, ch. III 3, n. 63.
32 Posidonius ap. Strabo 2, p. 94 = DK 28A44; derived from this, Ach. Is. p. 67.27 Maas, Aet. 3.11.4 (where, in contradiction of Posidonius, a connection with the γώνιος τροπική is claimed—a simplification).
33 Reinhardt, Parm. 147 n. 1, Kosmos 361 n. 2; Heidel, Maps 76, 80, 91; cf. Rehm–Vogel 11f.
earth there is a mixed zone between the extremes; and, as the “circles” in the heavens were arranged in some kind of symmetry, the southern hemisphere is a symmetrical reverse of the northern. And the very fact that, according to Posidonius, no mathematical astronomy was associated with this, or any projection of the tropics onto the earth, is an encouraging sign. On the other hand, the doxographers ascribe to Pythagoras the precise astronomical division of the earth into five zones, bounded by the tropics and the polar circles, and trace the corresponding division of the celestial sphere to Thalès Πυθαγόρας καὶ οἱ ἄντροι, and the determination of the obliquity of the ecliptic, upon which the location of the tropics of course depends, to Pythagoras himself, adding Ἱπποκράτης ὁ Χῖος ὁ δὲ ἐπίσης σφαίρης. We have a parallel for this last sentence in the words of Aristotle’s pupil Eudemus, the best authority for the history of Greek science before the Hellenistic period: Οὐκ οὖσα ἐπίσης σφαίρης. The later tradition maintains that this is a plagiarism from Pythagoras. For both the sphericity of the earth and the division of it into zones, Pythagoras’ name stands in competition with Parmenides; in the second case the more advanced, “correct” formulation of the theory is ascribed to Pythagoras; and in both cases the earlier evidence—that of Theophrastus and Eudemus—has no mention of Pythagoras.

Favorinus records the same rivalry for the discovery that the evening and morning stars are the same. This planet—did he already call it after Aphrodite?—had a special meaning for Parmenides, and he has it circling in the pure αὐθήριον above the sun. This double-track tradition about Parmenides and Pythagoras can be understood in the light of a statement of Diogenes Laërtius about Parmenides: “He seems to have been the first to discover that the evening star and the morning star are the same, as Favorinus says . . . but some attribute this to Pythagoras; but Callimachus denies that he is the author of the poem.” Callimachus denied Pythagoras’ authorship of a certain poem which dealt, among other things, with the planet Venus. Actually, Heracleides Lembus, in his list of the writings of Pythagoras, gives first place to a work περὶ τοῦ διαβασμοῦ (D.L. 8.7). It is likely that this poem dealt not only with the heavenly bodies but with the spherical shape of the earth and its division into zones, borrowing from Parmenides, and certainly also from Empedocles. No one will claim that there was a didactic poem actually written by Pythagoras. Callimachus was right in rejecting the poem, and in doing so joins Theophrastus and Eudemus as a third witness against the “Pythagoras” version of this tradition. We need not be concerned whether the doxography took its reports from the apocryphal poem or perhaps from older statements by Platonists, for

34 On the problems of Parmenides’ celestial system, see below, n. 40. Frank (200 n. 1) wrongly states that Posidonius “attributed to Parmenides the projection of the tropics from the celestial sphere onto the concentric spherical earth.” Posidonius’ words were (Παρμενίδου) σχεδόν τι διαπέλατον ἀποτελέον τοῦ πλάτους τῆς διευκολύνου... ἑκατέρους ἑκατέρων τῶν τροπικῶν ἔρι τό ἔρικτο... According to this the width of the torrid zone would be not 46° 54’ 6” but about 90°, i.e., for Parmenides it is a “ring” whose width is about a fourth of a circle; and this has nothing to do with astronomy or tropics (differently Hippocrates, above, n. 28).

35 Aet. 3.14.1; cf. Mart. Cap. 6.609 (MSS phytkaros or pythagoras, wrongly corrected to phytkaros in Dick’s edition).

36 Aet. 2.12.1; cf. 2.23.6: Πλατάνος Πυθαγόρας Αριστοτέλης (on the obliquity of the ecliptic), where the position of the name Pythagoras is a giveaway.

37 Aet. 2.12.2.

38 Eudemus fr. 145 = Theo Sm. 198.14f. Diels (DK 41.7) conjectured λόγοιν for διαβασμοῦ. The topic of discussion is the measurement of the angle of the ecliptic; the fact of its obliquity—i.e. of the zodiac—was known since Anaximander (A5) and Cleosthenes (DK 612b2); see von Fritz, RE XVII 2260f. Of course, Theo gives only a much abbreviated excerpt from Eudemus; but, if his original had corresponded to the report of Aetius, the name Oenopides would have fallen out rather than Pythagoras.—Aet. 2.32.2 ascribes the 59-year cycle to “Oenopides and Pythagoras,” while Ael. V.H 10.7 and Cens. 19.2 speak only of Oenopides. The mention of Pythagoras is to be explained either from the reproach of plagiarism or as a conclusion from Philolaus A22.

39 D. L. 9.23 (below, n. 41): Parmenides also Aet. 2.15.7; Pythagoras, Apollodorus FGrHist 244F91 and Plin. HN 2.37 (dated 612 B.C. as a discovery of Pythagoras). It is almost incredible that so elementary an astronomical fact remained so long unknown to the Greeks; about 1580 B.C. the Babylonians already knew the time of an orbit of Venus (van der Waerden Adv. 49). Wilamowitz, Hermes 18 (1883) 416-423, found the identity of morning and evening star referred to in mythology; and Ibycus of Rhegium (approximately a contemporary of Pythagoras; Burnet, EGP 191 n. 3) spoke of it (fr. 331 Page).

40 Aet. 4.5. In general, it is regarded as an almost hopeless task to reconstruct the celestial system of Parmenides (fr. 12: A37). See Raven, KR 284f; De Vogel, GP 1 41ff; among older writers, Tannery, HSCH 238ff; Zeller I 708ff; Burnet, EGP 187f; Gigon, Ursprung 276ff; a quite different answer by Morrison, JHS 1955, 60ff. Reinhardt (Parn. 11ff) is probably right that it was primarily a cosmogonical process (which would be understandable in connection with Anaximander A10 and Empedocles A30, 49, 50); but the result must be our world.

41 D. L. 9.23, Callim. fr. 442 Pfeiffer; cf. Wilamowitz, Platon II 85.2. Pfeiffer thinks of a poem whose authorship was contested between partisans of the two men; but there was only one book by Parmenides (D.L. 1.16 = DK 28A13). Diels (DK I 225, n. 4) thought that this one didactic poem of Parmenides was circulated under Pythagoras’ name, but it is more likely that there was an apocryphal revision labeled “Pythagoras.” —D. L. 8.14; προτέρως τοῦ Ἐπικουρού καὶ Φιλόσφοντος τῶν ἀκονίων ἐπικούρου (Πυθαγόρας) ὁδοιπορίας. This has been emended from the time of Casaubon to τοῦ ἑαυτοῦ ἐπικούρου. Diels, DK 28A40a considers ὁδοιπορίας (καὶ) ἔπος Παρθενίουs. Burnet, EGP 191 n. 3, defends the MS text. But, if Parmenides had mentioned Pythagoras or even made a clear allusion to him, the ancient scholars, in their search for evidence about Pythagoras would have preserved the verse as they did Empedocles fr. 129.
example in the commentaries on the *Timaeus,* 48 in neither case are the statements reliable.

For there is no corroboration in the astronomical field, more than anywhere else, that Parmenides got his philosophy of nature from a Pythagorean source. The later tradition ascribes to Pythagoras a much more advanced system than Parmenides had. 49 In his astronomy, Parmenides holds quite closely to Anaximander, and Empedocles closely follows him; 49 there is no reason to insert a Pythagorean science.

Commenting on a passage in which Aristotle refers to the professional astronomers for the question of the order and the distances of the stars, Simplicius says, "Anaximander was the first to discover the inquiry into the sizes and distances (of the stars), as Eudemus reports, adding that the Pythagoreans were the first to give their order." 50 Here is ascribed to the Pythagoreans—though not to Pythagoras, 46—a not unimportant contribution to the development of the world model found in Plato. Eudemus is trying to discover, in good Aristotelian fashion, the nature of the progress in science that led to the situation as he knew it; results are what he wants to record. 47 In pursuit of this aim, he attributed to the Pythagoreans the order of the planets known to Plato, Aristotle, and Eudoxus. But in order to estimate the period to which this takes us back, and to understand the relation to the datum about Anaximander, 48 we must survey the theories of the pre-Socratics on the order of the planets. 49

50. Tannery, *HSCH* 90ff; Zeller 1 297ff; Burnet, *EGP* 62ff; Diels, *AGP* 1897, 228ff; Heath, *Aristarchus* 31ff; Gigon, *Ursprung* 84ff; Raven, KR 111ff; Kahn 75ff.

51. A26, A11 §3. Heidel, *Maps* 68ff, 151, doubts Aristotle's statement, on the ground that Anaximander's cosmos was not a sphere (below, n. 54). But the argument applies also to circles, rings, or wheels. Aristotle (Cael. 293b11ff) is dealing expressly with the *eirion* for the *muon* of the earth. Anaximander must have stated a reason, so that this cannot be simply a false deduction of Aristotle.

52. A11 §3; A18, A21, A22.

53. "Roots of the earth," *Hes.* Op. 19, Xenophanes fr. 28; the return of Helios, Mennennus fr. 10, Stenichorus fr. 185 P.; Anaximenes fr. A7 §6, A14 returns to the older idea. Xenophanes (A33, A41) and Heraclitus (fr. 6) are not interested in mathematical astronomy.

54. It is hard to decide whether a solid shell of the universe is presupposed—a perfect sphere (as Gigon believes, *Ursprung* 85)—or whether we look out between the "wheels" into the *ariou* (as Burnet, *EGP* 69). The solid shell is part of the mythic background (υακνος χωρος) and is also found in Anaximenes (κρυσταλλοειδες), Empedocles (A30, 51), Hebr. 6 (διπλος παγος), Hippocr. *V. i.* 10 (δε περιψω παγος). Cf. also *Epim.* mag. s.v. *βραδ*.

55. A11 §5 (incomplete), A21, A22. The numbers for the stars have been reconstructed since Tannery (*HSCH* 94f). The two sets of figures are generally interpreted as an indication of the "thickness" of the rings—which must correspond to the size of the earth, if the sun is the same size as the earth, so that one time the outer, and another time the inner, diameter would be intended, though this involves an error in computation (Raven, KR 136 n. 1).

56. Theog. 722ff (cf. the *Odyssey* διιος διιος). See Nestle, *ZN* 301 n.; Kranz, *Kosmos* 13f, etc. (9 is an augmentation of 3, which is itself a symbol of plurality; below, ch. VI 4). Diels (AGP 1897, 230ff) mentions similar ideas among the shamans. Kahn (94-97) is hardly right in denying these mythical elements in Anaximander.

57. R. Eisler, *Weltenmantel und Himmelszelt* 1 (Munich, 1910) 90 n. 3; cf. Boll, *RE* VII 2565; Kranz, *NGG* 1918, 156; Cumm. *Lac.* 143; Burkh. *RHE* 1963, 97-134; *Died.* 2.30.6 says that the doctrine is Babylonian; probably it is a matter of Babylonian-Iranian syncretism. (The Babylonians had a different doctrine of 3 heavens over one another: Meissner 108.) Kahn (90) follows Diels (AGP 1897, 225ff) in believing that Anaximander thought the matter out independently, concluding that the largest fire is the highest.
the stars, as distinguished from the sun, do not provide us heat, and are therefore further away (ὅτι τὰ μῆκος τῆς ἀποστάσεως). 58

This makes Eudemos' report on Anaximander comprehensible. The arrangement of the celestial bodies, ἡ τῆς θέσεως τάξις, is wrong, in his account, but still he "discovered" a fundamentally important fact. Not only did he consider the question of the sizes and distances of the heavenly bodies, but, in spite of the arbitrariness of his hypotheses and the incorrectness of his results, he did find the right path toward the answer. The sun is the same size as the earth, although it looks to us about a foot wide, 59 because it is tremendously far off and apparent size decreases as distance increases. True size, apparent size, and distance stand in a definite mathematical relationship. Thus the basic ideas of geometrical proportion and optics have been "discovered," and are then boldly applied to cosmic magnitudes and distances, where any kind of verification is impossible. This is the λόγος which, according to Eudemus, Anaximander "was the first to discover."

What the Pythagoreans could contribute in addition to this was, after the correction made by Anaximenes, the correct order of the planets. The planets do not seem to have been discussed by Anaximander. 60 Of course, morning and evening star had been known for a long time, and people must have noticed long before this that some other bright stars are not always associated with specific constellations, but "wander about." Thus the word itself, πλαγής (ἀστήρες) may be old. 61 But the further details—that there are five planets, that they have definite orbital periods and thus also regular courses, and that in the character of their movements they are to be compared rather with the sun and moon than with the fixed stars—here this came to the Greeks from Babylon, and later than Anaximander, though before Plato.

Detailed knowledge of the planets cannot be proven for any of the

60 Arist. Mete. 343a 31; cf. Aratus 454). It was a scientific achievement not to be underrated to go against appearances, separate the five planets from the fixed stars, and classify them with the sun and moon, so that there are seven planets: ἐν τῇ κόσμῳ. Pl. Tim. 36d, 38c-d; oi ἐν τῇ ἀστήρες, Schol. Arat. p. 429.12, 478.8 Mass.; oi ἀστήρες oi ἐντά, Dio Cassius 37.18.1; τὰ ἐντά ἀστήρα, Hippol. Ref. 1.2.2; "septem sidera," Manil. i.308; Plin. HN 2.12; cf. Hymn. Hom. Ars 7, Cic. Rep. 6.17, etc. Of course this is correct, from the geocentric point of view.

61 A7 = Hippol. Ref. 1.7.4: ἕλων καὶ σελήνη καὶ τὰ ἄλλα ἄστρα πάντα πώρα ὄντα ἐποικοδομέα τῷ ἀέρι διὰ τὴν πλάνης. Cf. A15, but A14 = Aét. 2.14.3: ἕλων δὲ τῆς καταπηνηῡτικὴς τὰ ἄστρα τῷ κρύσταλλῳκεῖ. The contradiction can be removed by understanding the first of the sentences quoted of the planets, and the second of the fixed stars. A slight alteration in the text of the next clause in Aétus (2.14.4) would yield this sense: ἄνω (MSS ἄνω) δὲ πέταλα εἶναι πώρα ὄντας ἄξοναβάλχουσα (Hestheus, Aristarchus 42). Ach. Is. p. 40.20 Mass., however, has τέινας ἐκ τῆς corresponding clause; the source of the trouble must be older. Cf. Gundel, RE XX 2042; Guthrie I 135–137.

62 A7 = Aét. 2.163 may originally applied only to the sun, since Alcmaeon's name appears only as an afterthought. In A12 ἀστήρες are named along with the ὀφθαλόου, but it is not really necessary to interpret this as indicating a distinction between planets and fixed stars. In any case, Alcmaeon would not have been the first to speak of the planets (as Gigon says, Usteriung 150), but following Anaximenes, with whom the doctrine of the "flat" sun also brings him into contact (A4; cf. Anaximenes A13). 

63 A4: τοῖς μὲν ἀστήρες ἀστήρας συνεδρίασε τῷ κρύσταλλῳ, τοὺς δὲ πλαγίας ἀνικόσσα. The word πλαγίας cannot refer to the sun and moon alone; Empedocles had special theories about them.

64 A10: τοῖς δὲ κομματαῖς σύνοδον πλαγίας ὄφθαλμον. This sounds as though he were thinking of a regular collision; in that case, Anaxagoras would have had to station all the planets in the same region. The comet's tail was explained as "flame," with the help of the analogy of frost. Aristote (Mete. 342b25) speaks of σύμφασα, "apparent conjunction"; but, since he includes Anaxagoras and Democritus together (cf. below, n. 74), he may have obscured the difference between them.—Anaxagoras was the first to make neighbors of the sun and moon (Eudemus fr. 147; cf. above, n. 58).—It may remain open argument whether the sentence in Hcd. 2, lines 64ff, τοῖς ἀστήρας τῷ ὀφθαλμῷ ἐντὰ σάλλα τάξις ἔχει τῶν ὀρίων ἕκαθη, refers to the 7 planets, as Boll contended (Nhb 1913 = KSil 220ff), following the commentary of Pl.-Galen (CMC XI 2.1. p. 35). Krantz agreed with him (Nagl 1938, 142, Kornos 12 n. 8) as did Gundel (RE XX 2040) and Roscher (Hed. p. 134 n. 191; but he rejects it SBLPZ 1919,5, 65ff). The dating of Hcd. depends precisely on the question whether it shows knowledge of the planets and the spherical shape of the earth. In my opinion the thesis that the planets were known to the author falls with correct interpretation of the phrase ὀρίων ἕκαθη, which must mean "the succession of the seasons" (cf. Hed. 4, 14). The constellations named in the next paragraphe are connected with this (Arcurus, the Pleiades, the Hyades, Orion, and the Dog), but the 7 planets are not (except, of course,
also in Democritus. He has been severely criticized for rejecting the sphericity of the earth, and consequently his entire astronomical system has been called backward and almost primitive.67 But a book title like Ἐκπεραίαμα should make us prick up our ears; if Diels is right about its meaning, the subject was a projection of the armillary sphere on a plane surface, or in other words, mathematical astronomy.68 Democritus was well informed in both mathematics and astronomy and wrote a whole book περὶ τῶν πλανήτων.69 His series of heavenly bodies, from the earth as center, was moon, Venus, sun, planets, fixed stars; and the planets themselves were put at various distances from the earth.70 The reason for this arrangement is that in the “whirl” the stars nearer the earth lose ground relatively to the fixed stars. Thus the planets are integrated into the world system, with different distances according to their different “speed.” Democritus knows that there are a number of planets, and that they have definite orbital periods, in some cases longer than those of the sun. The special position of Venus points to influence from Babylon, where the trinity of sun, moon, and Venus is attested very early.71 It is incredible that Democritus should not have known the five familiar planets.

the sun). Perhaps the words ὑπὸ ὑδατα ἐπιλαθα to the fact that certain constellations have 7 stars (the Bear, Pleiades, Orion, and, according to Pheres, the Hyades; FGrHist 3F90, Hippias DK 86B13); Varro in his Hebdomades (Cell. 3.10.2) mentioned the seven-group of the Septentriones and the Pleiades even before the planets. (For seven-groups of fixed stars in Babylon, see Meissner 407f). Zevi ably αἰθορκος (Heph. 6.2, line 17) cannot mean the planet Jupiter (as Kranz thinks, NGG 1938, 125 n. 2; cf. Kosmos 32 n. 8), since this manner of referring to the planets is only attested for a much later period (above, n. 6; Cumont's conclusion could only be said to be “corrected”—Kranz, NGG 1938, 126—if there were no other possible interpretation). There remains Boll's idea that it means the sun, because mention is made of its “change of color,” i.e. the alternation of day-sky and night-sky (AbhMu 1938, 25; cf. below, ch. IV 3, nn. 27, 64).72 The “childish character” of Democritus' astronomy is spoken of by Burnet, EGP 339; i.e., he was not a Pythagorean! (Cf. above, n. 31).

67 DK 68B17c; also the ii 141, with ref. to Ptol. Geogr. 7.7.

68 Fr. 5b. In comparing Democritus with the allegedly more progressive “Pythagorean” astronomy of Plato, one must bear in mind that for Plato we have detailed expositions in his surviving works, but for Democritus only scattered and incomplete citations. If one matches the doxographical accounts of Plato with those of Democritus, the latter comes out fully as well (compare l. 4.11). AET. 2.13/4, 2.15/3/4, 2.20/5/7, 2.25/6/9, 2.20/6/30/3); an outstanding difference is that he gives physical causes rather than mathematical descriptions (cf. 2.16/7, 2.23/6/7). In his explanation of the Milky Way, Democritus (unlike Aristotle) has the right anser (AET. 3.1.6).

69 A86, A40 84; Lucr. 5.621ff = Democritus A88.

70 Shamash, Shin, Ishtar, represented as a group as early as the Narumtini stelae; see Cumont, AC 1935, 10 n. 7; Meissner 188, 402ff. It is entirely impossible that Parmenides is subject to oriental influence in the important position he gives to Venus (cf. above, n. 40); but Democritus is certainly not following Parmenides. In fact, he differs from Leucippus as to the position to be assigned the sun (above, n. 58).

Seneca, though, says that Democritus did not “yet” have an adequate knowledge of the course of the five planets. “Democritus quoque... suspicari se ait pluris stellas esse, quae currunt, sed nec numerum illarum posuit nec nominat, nondum comprehensis quinque siderum cursoribus.”73 Yet this does not mean that Democritus “knew neither the number nor the names of the planets.”74 According to Seneca's report, Democritus expressed the suspicion that there were “more” planets, which surely means “more” in comparison to those he did know and describe. Seneca, or his source, sees in this belief—in which Democritus was correct, as we now know—a deficiency or shortcoming; the astronomy of later antiquity thought it had attained final results, and no longer reckoned with undiscovered stars.75

Eudemus must have found Democritus' arrangement of the planets incorrect, too, because of the special position of Venus. The “correct” one was, however, included in the system of Philolaus; for the sequence of the ten “divine bodies” was, in the unanimous testimony of Aristotle and the doxographers,76 central fire, counter-earth, earth, moon, sun, five planets, heaven of the fixed stars. If we consider only the portion between earth and heaven, this is the order accepted by Eudoxus, Plato, and Aristotle. In addition, this is the system which Aristotle ascribes simply to “the Pythagoreans;” so nothing seems in the way of the assumption that Eudemus meant the same Pythagoreans, and his report that “the Pythagoreans” had established the order of the planets referred precisely to the system of Philolaus.

One would hardly suggest that Philolaus was dependent on Democritus;77 rather, there must have been a common source, someone who, in the interval between Anaxagoras and the time of Philolaus and Democritus, introduced to Greece, from Babylon, detailed knowledge about the planets. In fact, this very epoch is that of the efflorescence of professional Greek mathematics and astronomy. The floruit of

72 QNat 7.3.2 = Democritus Ap2.

73 Frank 202, followed by Gundel, RE XX 2040; more cautiously Cumont, AC 1935, 9f: Did Democritus intentionally ignore the divine names of the planets?


75 Arist. fr. 203, Philolaus A16. Gundel (RE XX 2106f) erroneously takes the system described at Plut. De an. proc. 1028b as that of Philolaus.

Hippocrates of Chios was about 430 B.C. He is true that our only direct information about his planetary theories concerns an exceptional case, that of the comet; but in this is implied that he discussed the behavior of the planets in detail. They move within the tropics, that is, in the area of the ecliptic; and they “stay behind” the fixed stars, each along his peculiar orbit (ὑπολειμμένον ὀλον τῶν ἰσιωτίκων κύκλων, Arist. Mete. 343a6), that is, they have definite periods. Heavenly and terrestrial events affect each other—the comet sucks up moisture. But the spherical plan of the celestial movements is already there, in some detail: the parallel circles, oblique to the horizon and cut by it at various angles. Hippocrates probably was influenced by his countryman Oenopides, who had much to say about “circles” and “inclinations.”

June 27, 432 B.C. is the date of the summer solstice observed by Meton, who was also “astronomer and geometer.” We know that he had some contact with Babylon, however it was established, for his nineteen-year cycle had been used there since 499. Meton is named several times along with Eudoxus. He seems to have been familiar not only with the idea, but with the graduation, of the zodiac. In the Babylonian manner, he designated the ascension of various fixed stars as weather signs, and Democritus followed him in this. Tzetzes ascribes to Meton a doctrine of the “Great Year” and of the destruction of the world when all the planets meet in the sign of Aquarius. This may be merely a confusion of Meton’s nineteen-year cycle with the World Year, and from a single rather indefinite and general note we cannot tell for certain in what sense Meton dealt with the “distances” of the stars. The fact remains that at the epoch of Meton and Hippocrates we can ascertain the presence of not only empirical astronomical data borrowed from Babylon, but also a precise, geometrical conception of a spherical universe. From this time on, the general public was interested in astronomical topics. This suggests that by then a fairly accurate
IV. ASTRONOMY AND PYTHAGOREANISM

The opinion is widespread that Pythagoras himself, who is supposed to have traveled in the East, brought this astronomical knowledge back to Greece with him and passed it on through his school. In fact, he is thought of as the most important link in the transmission of oriental science to the Greeks. More cautious scholars are more likely to speak not of Pythagoras, but of the early Pythagoreans, who are supposed to be the only Greeks before Philolaus to have any advanced astronomical knowledge.

This opinion leaves unanswered the question why we can detect no influence of such knowledge, even in relation to the planets. In Parmenides and Empedocles, at least, one would have expected to find some traces; and if Oenopides “stole” the determination of the obliquity of the ecliptic from Pythagoras, why did he not also take his knowledge of the planets? Since there is no apparent reason why the doxography should report the views of Democritus in more detail than those of earlier thinkers, we are driven to the assumption that the planets did not really play so important a role with them. The Pythagoreans’ astronomical erudition would be, then, a secret doctrine with no effect on others—a buried treasure. For Democritus’ astronomy cannot in any case be derived from Pythagoreanism. There is not a trace in the Pythagorean tradition of the special position of Venus in his system, which points unmistakably toward Babylon. The conventional view involves a remarkable double development; what the Pythagoreans had long ago brought from Babylon had to be fetched, a second time, from the same source.

But there is no good reason to assume a mysterious, secret pre-Philolaus astronomy, belonging to Pythagoras or the Pythagoreans, aside from one dubious inference from Parmenides and some even more dubious late reports. Scholars have seized upon the assertion of the “more genuine” Pythagoreans, that the central force is a force in the interior of the earth, named Ἑῴα. In fact, this epithet is applied to the earth a few times in the fifth century, and Empedocles spoke of fires beneath the earth. Here, it is thought, we have traces of a geocentric system belonging to the early Pythagoreans, which we should postulate anyway and which displays a suitable mixture of myth and science. Nevertheless, the basis of the reconstruction, the report of the “genuine Pythagoreans,” is an artificial reinterpretation of the reports of Aristotle, and without independent value as a source. The only other point is the name of Ἑ.forChild, but this is comprehensible as an expression of the central location of the earth, without the idea of a central fire. And since we may not regard every point of contact between mythology and φυσιολογία as a priori Pythagorean—tradition even associates Euripides and Anaxagoras as among those who called the earth Ἑ.forChild—the conclusion that Pythagoreans are behind this

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80 Above, ch. I 81. 72. The idea of a central fire within the earth was developed, as the original Pythagorean theory, by H. Richardson (CQ 20 [1926] 113–133), following Burnet, EGP 297ff, then also by Wiersma, Mnemosyne 2024, 32ff. One piece of evidence is Th. ar. 6.11ff (cf. above, ch. III 2, n. 139). A theory of fire in the earth is well attested for the Stoic Archelaus (S/P 3264).


82 Fr. 52, 62. This is natural enough for a Sicilian, with Etina before his eyes. At Parmenides A37, a conjecture of Diels introduces the idea of fire beneath the earth; but, uncertain as the whole world system of Parmenides is, this must remain in doubt (above, n. 40).

83 Anap. p. 30 = Th. ar. 6.115f. . . . κατηγορηθήσατο τοῦ Πυθαγορείου ἐν τῷ περὶ Ἑ��δοκέλα καὶ Παρμενίδην καὶ σχεδόν αἰτιολόγησε τῶν πάλαι συστάματος τῆς μονοδυνής φύσις ἑῴας πρῶτον ἐν μέσῳ ἱδρύθη . . . Here the subject of discussion is the central position of the earth, in Parmenides and Empedocles, not a fiery center for the earth, such as is attributed to the Pythagoreans. The hearth is the center of domestic life, and the newborn child is carried around it (Ἀμφιφόροις; cf. Nilsson 101. 195 n. 8; Hesiod μέσῳ ὀνείρῳ Πυθ. Not. 30). Fire burns on the hearth, not in it.

development is quite shaky. And there is no proof of special astronomical knowledge, going back before Anaxagoras.

At the same time, the effort continues to attribute to the Pythagoreans before Plato, and even before Philolaus, a different, geocentric planetary system which became dominant in the later Hellenistic period. In this, the sun is in the middle of the seven planets, flanked on each side by three of them—Venus, Mercury, and the moon in the direction of the central earth, and Mars, Jupiter, and Saturn in the direction of the heaven of the fixed stars. This arrangement of the planets attained a canonical position in astrology, and still determines the order of the days of the week. It cannot be documented earlier than Archimedes, yet it was attributed to Pythagoras and connected with a system of the harmony of the spheres, in which the sun, both in its position and its function, is μέσον. This coincidence, so gratifying to every Pythagorean heart, and in general the "solar theology" that pervades the theory, insured that the age and the originality of the system would not go without defenders. An independent testimony seems to corroborate this. Ptolemy says that the "older mathematicians" (παλαιότεροι) put the sun in the middle, whereas "some later ones" (πρὸς τῶν μετὰ τήρα) put all the planets above the sun. Since this is precisely the conception of Plato, Eudoxus, and Aristotle, the conclusion is drawn that the "older" mathematicians in question are pre-Eudoxian, and therefore Pythagorean, scientists.

The evidence of Eudemos makes considerable difficulty for this view; he can only be referring to the arrangement of the planets accepted by Aristotle, whose discovery he traces back to the Pythagoreans, for Simplicius is citing Eudemos merely to help explain Aristotle. If there had been two different Pythagorean planetary systems, Eudemos would surely have mentioned the fact; and he could not have spoken of a discovery, but of a dilemma, since both systems are worth about the same from a geocentric point of view. As it is, there is not a trace of the second system, mentioned above, before Archimedes; and it seems clearly to be later in origin. Beginning with the naive idea of "sun, moon, and stars," one's first step in advance is to recognize five of the stars as a special group—the planets—to distinguish them from the fixed stars and class them, to a certain extent, along with the sun and moon. One sees the expression of such a development in the systems of Philolaus, Plato, and Eudoxus: moon, sun, five planets, fixed stars. Only when the close connection of the "seven" has come to seem a matter of course, would one appreciate the symmetry of the other arrangement, in which the insignificant planet Saturn counterbalances the old familiar moon.

The conclusion drawn from Ptolemy does not hold up. When he compares "old" and "newer" observations, he means by "old" astronomers those from the third century on, as far as Hipparchus, ca. 150 B.C. (who himself figures as the most important of the "ancients"), by contrast with the "moderns" of the last hundred years before Ptolemy. Eudoxus and Callippus are never cited in the Almagest; the astronomy of the fourth century has already sunk from sight. Aristarchus, be sure, assigned the sun to a central position; but Plato's great prestige kept his own system alive, even among professional astronomers. The expression "some later ones," then, refers to astronomers later than Hipparchus, who, doubtless because of their orthodoxy Platonism, championed the older system. Even the
brilliant Eudoxus had fallen into oblivion, there can be very small profit in looking to Ptolemy for evidence on pre-Platonic Pythagoreanism.

Nor can internal indications of "solar theology" or the harmony of the spheres prove that this late-attested system belonged to the early Pythagoreans. Boyancé is able to cite a number of rather early references to the identification of Apollo and the sun, and to the connection between the sun and the harmony of the cosmos. But the interrelation of myth and the interpretation of the natural world is not Pythagorean alone; all Greek cosmology had to make its peace with the existing myths; and the thought of cosmic harmony is by no means tied to any particular planetary system. We shall not have to alter the conclusion drawn from the passage of Eudemus: the arrangement of the planets found in Plato and in the Philolaic system, and no other, was known to Pythagoreans before Plato. We may conclude that this went back to the acceptance of Babylonian information in the time of Meton, just before that of Philolaus, and not to a more ancient Pythagorean tradition.

The *acusma* which says that the planets are the "hounds of Persephone" may well be older. Most nearly comparable to this is the Babylonian designation of the planets as "rams." But we need not suppose a direct dependence; the background is that of naive observation of the skies: the stars that catch attention by their independent movements are thought of as living beings. These Pythagoreans, therefore, are looking for the realm of Persephone in the skies; it is also known that the sun and moon are the "Isles of the Blest." As well as this fits in, it does show that sun and moon were not reckoned among the planets. When one notices that certain stars "wander about" in the skies, the idea of mathematical arrangement and regularity does not immediately spring to mind. There is here a kind of "astronomy," or we might better say, certain notions about the heavenly bodies which prevailed among Pythagoreans and were peculiar to them—for there is nothing else in Greece like this—but they had nothing to do with scientific Greek astronomy. There was a pre-scientific area within Pythagoreanism itself. And this confirms that there was not a continuous transmission of advanced astronomical knowledge from the day of Pythagoras, but that scientific elements were added only later on, in the course of the general development of Greek science.

There is only a single passage in Aristotle to indicate that the Philolaic system was not the only one in vogue among the Pythagoreans, stating that "some" of them believed, as did Hippocrates of Chios and his pupil Aeschylus, that "the" comet was a planet; a view also shared by Diogenes of Apollonia. In the system of Philolaus, the importance of the perfect number 10 leaves no opportunity for any such intrusive planet. It is difficult to interpret chronologically the "similarity" of the Pythagorean theory to that of Hippocrates. The latter gives a complicated explanation of the origin of the comet's tail—a phenomenon of reflection, he thinks, which happens only under certain conditions, which is why the comet is so seldom visible. The Pythagoreans do not go into the question of the tail; they only say that this "planet" is only seldom visible and does not rise far above the horizon. One might conjecture that the simpler theory is the older; and in fact Aristotle mentions it first. But this makes it difficult to see why the comet is not accounted for in the system of Philolaus. It would also be possible that the Pythagoreans in question should be dated later than Philolaus, and later than the refutation of Hippocrates' theory by the comet of 427/426 b.c. (above, n. 77). Their concern was not to set up a bold new scientific hypothesis, but merely to suggest a possibility.

With relation to the Milky Way, too, Aristotle knows of different theories held by Pythagoreans. Some see in it the path of a star that was driven from its course in the catastrophe caused by Phaethon, others,
a route once followed by the sun. Philolaus (A18) spoke of fire flowing from the sky. A doxographical entry attributes the second answer to Oenopides and adds that the sun changed its course out of disgust at the meal of Thysestes. The conclusion has been drawn from this that Aristotle thought of Oenopides as a Pythagorean; but in that case Democritus' pupil Metrodorus of Chios, who held the same view, would also have to be a Pythagorean. A "good idea," once expressed, keeps getting repeated. It is entirely possible that Metrodorus and the Pythagoreans were both dependent on Oenopides; we know that Philolaus took over the idea of "the Great Year" from him. In any case, there is no trace, before Philolaus and Oenopides, of a highly developed Pythagorean astronomy.

2. THE THEORY OF PLANETARY MOVEMENTS

From Plato to Kepler, astronomy was dominated by the assumption that all movements of celestial bodies were to be explained by the combination of "perfect," uniform, circular movements. On the basis of this assumption there are two approximately correct explanations of celestial movements, a heliocentric explanation like that of Aristarchus or Copernicus, and a geocentric system like that of Ptolemy, with epicycles and eccentrics. From a mathematical or descriptive point of view, the two systems are simply equivalent; the decisive advance was Kepler's recognition that the orbits in question were ellipses rather than circles.

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On the origin of the idea of the Milky Way, see Tannery, Astr. 10; Neugebauer, ExSc 121ff. For Copernicus, see Scop. 129; Huygens, Sacc. 96, REG 1052, 348.

Arist. Met. 343a1ff, and Aet. 3.1.2 (DK 58B37c). The latter also includes under the lemma Ποταμος καθαρος the explanation of the Milky Way as a phenomenon of reflection which goes back to Hippocrates (Arist. Met. 343b10ff, DK 42a5); and he does the same thing, at 3.2.1, with Hippocrates' theory of comets. The error doubtless comes from the fact that a mathematician was automatically regarded as a Pythagorean (cf. above, n. 77.)

Ach. Is. 55.18 Maass = DK 41.10. Elsewhere it is stated that the stars reversed their courses because of the meal of Thysestes; i.e., they formerly rose in the west and set in the east (Eur. El. 726ff, Or. 1001ff, Pl. Pol. 268e).

Eudoxus' theory of proportion (Euclid, book 5) is highly esteemed by modern mathematicians; see Hasse--Scholz 13ff; Becker, MD 104ff; van der Waerden, S. A. 187-189.

2. The standard study is G. V. Schiaparelli, Le scienze antiche di Eudosso, di Callippo e di Aristotele (Milan, 1875; Ger. tr. by W. Horn: Leipzig, 1877). See esp. Heath, Aristarchus 190ff, and for an attempt to reconstruct the method of Eudoxus himself, Becker, MD 80ff. For Menelaechmus and Callippos, see Dercyllides ap. Theo Sm. 201.25.

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8. The only controversy was whether we should employ eccentrics or epicycles; and Apollonius had already shown that these amounts to the same thing mathematically; above, n. 3). Iamblichus, on the other hand (Procl. In Tim. III 65.7), and Proclus following him (In Tim. III 96.27ff, 146.14ff) reject the complicated tangle of circles altogether; they are only interested in the metaphysical answers, not in a mathematical description of phenomena.

2. A recent champion of this view is van der Waerden, Astr. 377ff; it is extended by Becker, RPhM 97 (1954) 89-92. The main text is Tim. 38 c--d: the god created sun, moon, and planets as "instruments of time": ἔθυρεν ὡς τὰς περιφέρεις, ὡς τὰ θρόνον τῶν πλανητῶν.

postulate, completely regardless of observed phenomena, to be ascribed to the Pythagoreans.

For to suggest, as Frank does, that the "discovery of the true movements of the planets" was due to the Pythagoreans of Archytas' circle,¹⁷ is in itself absurd. This discovery is supposed to consist of the thesis that the planets "describe geometrically perfect circular orbits according to strictly mathematical laws" (28; cf. 201). What gave the

¹² Rep. 330a-b: (ὅ τοι διὰ ἁγίων ἔννοιας) οὐκ ἄστος, οὐδὲ, ἀκόντιον, τῷ νόμῳ τεύχει τὰ πράγματα ὑπὸ ἵππον συνέχεια παραπληκτών, σάμη τῇ ἔρευνα καὶ ἀφρόδεμα...

¹³ πλανήτης... συμπληρωμένος διὰ θαυμάσιον... τὸν πλανήτη καὶ τὸν πρωτότοκον ἱππότην (40b); but in the Laws (824d) the "normal" conception of the planet is a βασιλική... Simplicius, e.g., noticed the difference between Timaeus and Laws (Cael. 489.4f). To be sure, the expressions ἐναύλῳ δύομαι (Tim. 18d) and ἐναύλουκλησις καὶ προστάσεως (Tim. 400) could be applied to the Eudoxian system (above, n. 8; Cornford, Tim. 135f).

¹⁴ Frank's argument (202) is correct, up to this point.

¹⁵ Rotation of the earth was seen at this point by Schiaparelli, Voi. 39ff, Burnet, EGP 355 n. 2, and Heiberg 51.9, among others. Van der Waerden's nominee is the Philoic system (Astr. 55). But neither of these systems explains the one essential question, that of the motion of the earth. Therefore Gruppe supposed that Plato knew the heliocentric system (Kosm.Syst. 1283d; more cautiously Burnet, ThPl 348; C. Ritter, SBHeid 1919 no. 19, pp. 124f.), a supposition neither provable nor likely. Certain other passages have been cited in this connection. The word ἰδέμενον at Tim. 408b was taken by Aristotle (Cael. 293b30ff, 296a26ff; perhaps following Heraclides, as Cherrnis thinks, Plato 554ff) as referring to the revolution of the earth on its axis (cf. Taylor, Tim. 226ff; Heath, Aristarchus 174ff). The most ingenious solution is that of Cornford, that the earth turns in a direction opposite to that of the fixed stars, thus canceling out its movement and appearing at rest (Tim. 120ff; Cherniss agrees, Plato 554ff, with further elaboration;...
planets' their name, what drew the attention of observers to them at all, was the fact that they do not, like the fixed stars, follow perfect, uniform circular orbits, but that, compared to the latter, they "wander about." It would also have seemed noteworthy that they do not, like the sun, move regularly from west to east through the zodiac (the sun is not originally a "planet" but the paradigm of cosmic order, ἵ ὁ πρώτος χρόνος τάξις). The first important lesson learned in Greece about the planets, namely the identity of morning and evening star, means simply that Venus stands now to the right and now to the left of the sun, moves away from it for a certain distance and then back, overtakes the sun and is overtaken in its turn. The stoppings, the retrograde movements, and the alterations in velocity were inherent in the very discovery of the planets; they were known from the beginning. A theory of the planets that takes no account of these matters explains nothing at all.

Now Plato makes it a matter of reproach against the Pythagoreans that, at least in music theory, they value empirical data too highly, and he surely gives no indication that the ideal astronomy demanded in the Republic had actually been developed among the Pythagoreans. Aristotle says the Pythagoreans do not go beyond what is subject to sense perception. This is not true, it does not seem likely that they would have set up a postulate that contradicted all the evidence of the senses, without seeing any possibility of solution.

Geminus, on whom Proclus is obviously dependent, writes:

"The Pythagoreans were the first to approach such questions, and they assumed that the motions of the sun, moon, and planets are circular and uniform." (Is. 1.19, tr. Heath). But the "stationary points" of the planets are also explicitly mentioned, and in new formulation the problem of the Pythagoreans is, προέκειται ὁ κόσμος, πῶς ἕκαστον καὶ ὁμάδας καὶ ὁμάδας ἀποδοθεῖ τὰ μαθηματικῶς (Is. 1.21). Thus the Pythagoreans seek not arbitrary postulates, but the explanation of phenomena; Geminus undoubtedly, like Adrastus and later writers, considers the system of epicycles and eccenters to be Pythagorean.

Simplicius has a different account:

Eudoxus of Cnidus, as Eudemus recounts in the second book of his History of Astronomy and as Sosigenes repeats on the authority of Eudemus, is said to have been the first of the Greeks to deal with this type of hypothesis. For Plato, Sosigenes says, set this problem for students of astronomy: "By the assumption of what uniform and ordered motions can the apparent motions of the planets be accounted for?"

Whether the statement about Plato forms part of the citation from Eudemus and deserves any credence is a controversial question; the anecdotal style and the fact that Simplicius names Sosigenes twice may well rouse suspicion. But the sentence about Eudoxus, which is certainly taken from Eudemus, is itself enough to refute the assertion of Geminus. For what the latter attributes to the Pythagoreans—that they "were the first to devote themselves to such inquiries and based their account on uniform circular movements"—is attributed by Eudemus, in almost the same words, to Eudoxus. He was "the first to make use of such hypotheses," namely "what uniform, orderly, circular movements one must assume, in order to save the phenomena."

What doubt can there be that Geminus, in diverging from Aristotle's pupil Eudemus, is citing a late fiction, just as the epicycle theory he cites is later in origin than the theory of Eudoxus? The explanation of the movements of the planets, and thus the outstanding achievement of mathematical astronomy, begins not with the Pythagoreans but with Eudoxus.

But it is new knowledge of which Plato speaks in the Laws, not a

26 Namely (Simp. Cael. 488.16), τῶν ὑποτεθέντων δὲ ὁμάδας καὶ τετραγόνου καὶ ἑκατέρους καὶ ἑκατέρους διαστάσεις τὰ μαθηματικῶς.
27 The sentence about Plato was deleted by Hultsch (RE VI 939) as a conjecture of Sosigenes; it is defended by Friedlaender, Plato I 353 n. 15.
postulate without a solution. Just as, in the Timaeus, he mentions the "overtaking and being overtaken," the "retrogradations and advances" of the planets, he cannot, in his later book, regard them as either non-existent or unimportant; instead, he emphasizes the "exactitude" with which the heavenly bodies follow the "calculations" (967b); this is "not easy to understand, but not extremely hard, either" (821e). Since the matter is "proven," it is to be incorporated into the educational system (822c). Μαθηματα are "prerequisite" to instruction in astronomy (967e). The mathematical theory of the planets which Plato knows in the Laws can therefore only be that of Eudoxus. There is no chronological impediment to this conclusion; in content the allusions in the Laws are compatible with Eudoxus’ system; and there is plenty of evidence for a relationship between Eudoxus and Plato. The astronomical conception which became the basis of the world view of Plato and Aristotlce and indeed dominated people’s ideas of the world and their attitude to it until the time of Galileo—the differentiation of the eternal and inalterable order of the heavens from the chance and confusion of terrestrial events—comes not from Pythagoreans but from Eudoxus. Plato did not, in a dogmatic spirit, adopt a certain early Pythagorean system; he was au courant with the scientific activity of his time, and was capable of appropriating its most recent results for his own use.

Eudoxus is listed as a pupil of Archytas, in geometry, and is included by Diogenes Laertius among the Pythagoreans. With respect to the indisputably original achievements for which Eudoxus became famous in his lifetime, his supposed membership in the school has little meaning. We know little about Archytas’ astronomy. Circular movement was for him the "natural movement," which contains the "proportion of equality"; this is why circles and round bodies occur in nature—and this is applied, in the passage we have, to organic nature. But Archytas was concerned with the analysis of curves produced by a moving body, and it is tempting to suspect the influence of Archytas, the “inventor of mechanics” (D.L. 8.83), in the celebration of the marbles of the circle in the Aristotelian Mechanics (847b15—848a19), and the reduction of mechanical problems to the relationships of larger and smaller circles. He had exerted an influence on Plato’s thought as early as the Gorgias. Eudoxus went on from there; but the admiration of the perfect circle takes us back to a much earlier period. Alcmaeon spoke of the imperfect circle, of the failure to join beginning and end which is the cause of a man’s death (fr. 2).

28 Above, n. 14. Also, the word μαθηματα (Tim. 40b) obviously refers to a mechanical model to illustrate the matter (Rivaud, Rev. hist. philos. 2 [1928] 4ff; Taylor, Tim. 2.444f).
29 Cf. also Epin. 990a-c.
30 That Plato was referring to Eudoxus seems self-evident to Wilamowitz. Plato, I 502; Bidez, Eso. 150; W. Schadewaldt, “Eudoxos von Knidos und die Lehre vom unbewegten Bewegung,” Satura: Festscr. Weinrech (Baden-Baden, 1952) 105 n. 5; not to Lasserre (183—182, 19.70).
31 On the chronology of Eudoxus, see Jacoby, Apollodor 314ff; von Fritz, Philologus 85 (1930) 478—481; G. de Santillana, Isis 32 (1940) 248—262; Merlin, Studies in Epicurus and Aristotle (Wiesbaden, 1966), 59ff; Lasserre 137—139. Even with the generally accepted later dating (ca. 395—342 b.c.), not only Lec. 821 but the Timaeus could well be referring to Eudoxus (above, n. 8; see also O. Ritter, Philologus 111 [1967] 33). Lasserre’s assertion that “ein System kreisförmiger, regelmäßiger Plantenhähnchen nicht erst von Eudoxos ... ausgearbeitet wurde” (290) is not based on any evidence and contradicts Eudemus.
32 Plato’s statement that each planet traverses “the same path, not in many, but in one only” (822a) has been understood to mean either that the model of Tim. 39a (the overlapping of daily movement and revolution in relation to the zodiac) or also the system of Eudoxus, in which 3 or 4 circular movements overlap (Burnet, Thpl 347; Taylor, Tim. 216f, 21f; van der Waerden, Astr. 55, and others; this is the reason for the efforts to prove that Plato believed in rotation of the earth or a heliocentric system; see above, n. 26). Of course the theory of epicycles and eccenters would be affected in just the same way; and, since the Copernican system does not enter into the question, we would have to renounce even speculating what Plato meant precisely (Taylor, Tim. 233). The tradition of antiquity thought of both the epicycle system and that of Eudoxus as responsive to the Pythagorean postulate. Theo. Smyrnaius says (181a4f), τα ἐν ἀεικαῖς (ταῖς σφαιραῖς) ἀστρα τοῖς τοῦν ἀπλή καὶ ἀμφιθετήρας σφαιράμονας κατά συμβολὴν αὐτή δοκεῖ συνθέτων καὶ ἀναπλασμὸς καὶ συγκλάισε τοῖς πολέμοις φορεῖς. This evaluation is of course correct. The naive conception which Plato rejects is characterized by formu- lations like οἱ πλανημείρηκας τὴν περιστάσαν (821b), οἱ βηθέντες ἀποκαλοῦσα ἀπὸ τῶν αὐτῶν δρόμων ἀκλὴ πλαγμών (821c), πλαγμέναι παντα (822a), πολλαί (δύοι δεξιότερα) (822a). This is the original idea, that the planets do not have fixed courses, but move at random; they are "wandering" stars, "rams," "hounds of Persephone." In Eudoxus’ system, as with the epicycles, a planet does not move faster or slower to/from time to time, nor sometimes forward and sometimes back, but with a uniform velocity and direction, in which a circle itself is moving, but just as regularly and in a perfectly circular course. Thus the planet does have a specific, uniform path, governed by a uniform mathematical law and repeating itself with mathematical regularity. This is what Plato means.
as well as of the eternal circular movement of the divine stars (A12). A Pythagorean austoma says that circle and sphere are the most beautiful shapes.\(^{41}\) But even before Pythagoras, in Anaximander, the marvelous properties of the circle keep the earth in equilibrium; the cycles of day and year are even older ideas; and Homer himself speaks of the "sacred circle."\(^{42}\) Thus in the postulate of uniform circular movement, which formed Eudoxus' point of departure for the solution of his problem, there is a reminiscence of more ancient speculations. Thinking of the austoma and the evidence of Archytas, we may be justified in seeing a Pythagorean inspiration here. But Eudemus' testimony precludes us from supposing that Pythagoreans earlier than Eudoxus had succeeded in applying this concept to the apparent irregularities in the planetary orbits and thus become the founders of mathematical astronomy. Pre-scientific inspiration is in a sense the matrix of science, but the two should be neither confused nor equated.

Except for certain stationary points and retrogradations of the planets, the sun, the moon, and the planets seem to make the circuit of the zodiac, each in its respective period, traveling from west to cast. This difference from the uniform east-to-west movement of the fixed stars was interpreted in antiquity in two ways. Some of the older natural philosophers spoke of the planets as "getting left behind" (υπολειπομένων) by the fixed stars in the all-embracing cosmic revolution—for example, Anaxagoras, Hippocrates of Chios, and Democritus.\(^{43}\) Others thought of them as having a contrary movement of their own from west to east, in spite of which they are carried along with the general movement of the heavens, like ants crawling the wrong way on a potter's wheel.\(^{44}\) This theory of contrary movement is found in Alcmaeon,\(^{45}\) Oenopides,\(^{46}\) and Euripides.\(^{47}\) Plato rated this theory very high and thought it was only the one that explained correctly the velocities of the planets: Saturn is not the most rapid of them because he is left behind least by the fixed stars, but the slowest, because he is least able to free himself from their influence.\(^{48}\)

The theory of contrary movement is regarded as specifically Pythagorean and as a great advance over the Ionian theory. This is inferred from Plato and Alcmaeon, whose evidence seems to take us back to Pythagoras himself.\(^{49}\) But the relation of the two theories is more complex.

Oenopides was an Ionian, a pupil of Anaxagoras;\(^{50}\) so that we hardly ought to speak of a fundamental opposition between Pythagorean and Ionian astronomy. And, if Euripides could allude in a play to the "contrary course" of the stars, we may be sure that the theory was also known to Democritus; he may have lost no occasion to adopt this "advanced" view.

Actually there is no difference, from a purely descriptive point of view, whether one speaks of "getting left behind" or of "contrary movement," as long as attention is confined to explaining the apparent fact, that is the seeming displacement in the zodiac, which is inclined with relation to the celestial equator. Hippocrates of Chios,\(^{51}\) and thus doubtless the other advocates of the retardation theory, did not maintain that this μπολειπομένων took place on parallel circles to the celestial equator. No one can deny that the sun moves further to the north in the summer and to the south in the winter. Naturally Anaxagoras and Democritus knew about the movement of the sun—and the planets—in the zodiac.\(^{52}\)

The discovery that was crucial for the description of the movements of the planets was that of the zodiac. According to reliable tradition its twelve signs were introduced to Greece by Cleostratus of Tenedos, a pupil of Anaximander,\(^{53}\) after Anaximander himself had spoken of the

\(^{41}\) Above, ch. II 4, nn. 18, 23. Cf. Pl. Tim. 33b, Arist. Cael. 280b10ff, Hypornn. 25, Ocelus 1.15, etc.; also Empedocles fr. 274 = 28.2.

\(^{42}\) Il. 18.504. Cf. the circle of slabs around the shaft graves at Mycenae. Also, above ch. I 3, nn. 144-145.

\(^{43}\) Anaxagoras A78, Hippocr. DK 42.5 = Arist. Mete. 343a5ff, Democritus: Aet. 2.16.1 and A88 — Lucr. 5.621ff; Geminus Is. 12.14ff.

\(^{44}\) This comparison is made by Cleomedes (1.3) and others.

\(^{45}\) Aet. 2.16.3. "Contrary movement" is given as the doctrine of the mathematici, meaning professional astronomers, with the later addition, συνομολογεῖται καὶ Ἀλκαίων (above, ch. IV 1, n. 64).

\(^{46}\) DK 43.7 = Macrobi. Sat. 1.17.31. Here the reference is only to the sun.

\(^{47}\) Fr. 861: δεισάς γὰρ διηνον τὴν ἐννοιαν ὀδον . . . (perhaps words of Atreus from the Thyestes).

\(^{48}\) Tim. 38c, Leg. 822a et seq.; cf. Rep. 617a, Epin. 987b.

\(^{49}\) Tannery, HSCH 214; Burnet, EGPy 110f (Pythagoras himself); Heath, Aristarchus 50; van der Waarden, Astr. 27.

\(^{50}\) Above, ch. IV 1, n. 79.

\(^{51}\) Arist. Mete. 343a8 (about the comet in Hippocrates' theory): ὑπολειπευμένη δ' αὐτῶν καὶ πρὸς ἀρετὸς καὶ πρὸς νότον . . . Geminus Is. 12.19 objects, against this theory, that the υπολειπομένων would have to take place in parallel circles.

\(^{52}\) For the solstice there had to be a secondary cause, along with the vortex. Anaxagoras found it in winds (A72), following Anaximander (A27) and Anaximenes (A15). Democritus' answer to this problem is not clearly reported (A89).

oblique wheel of the sun. The retardation of the planets answers the attempt to replace arbitrary movement with movement according to natural necessity. The difference between the two theories of, then, retardation and of contrary movement, comes to a difference between physical explanation and mere description, whether naive or mathematical. The transition from one to the other is not simply a forward step; rather, the line of development turns back, on a higher level, to an earlier stage. The rash conjectures of the Ionians on physical matters led into a blind alley. The peculiarities of the planetary movements could not be explained by the force of the cosmic whirl, and still, as people were learning, the phenomena in question were regular and periodic. The only way open to progress in astronomy was to abandon physical explanations based on the necessary laws of movement and to adopt purely mathematical description. The result was the Greek mathematical theory of planetary motion, a tremendous achievement. It was not possible, however, to find one's way back from its complexities to simple physical laws; so that, from Aristotle's time on, the two-world theory was dominant, regarding the realm of the heavens as wholly different from and foreign to ours. Only with Galileo and Newton did astronomy once more, from the heliocentric standpoint, align itself with physics. Plato thought it was an inescapable conclusion that the orderly movement of the stars is due to beings with souls; it is a voluntary, chosen order. Here sophisticated Greek science harks back to the pre-scientific way of thinking and comes to rest in it.

If Alcmaeon thought the stars were divine, he must have ascribed to them a movement of their own; there is no advanced astronomical theory involved here, as against Anaximander or Cleorstratus—in general it is obvious that Alcmaeon's astronomical views are dependent on the Ionians. Thus Alcmaeon's statement is no evidence for scientific advances attributable to Pythagoras. It was of course possible for Eudoxus to take his departure from the idea of contrary movement, and, following out the principle in a mathematical way, to lay the basis of the descriptive, though no longer physical, astronomy of the Greeks.

The oldest Babylonian text yet known that refers to the "signs" of the zodiac, not to constellations, is a horoscope from the year 410 B.C. Van der Waerden takes expressions like "at the end of Pisces," which occur some decades earlier, as evidence for the introduction of the twelve signs before that date, in place of the ancient names of constellations. For even centuries earlier than this, people had marked out the "path of the moon" in the sky and already discovered that the sun, too, and the five planets, travel the same route. In expressions about the "path" of the sun or moon, it reflected the conception that is natural for the unsophisticated observer, that the sun and moon, and basically also the planets, make their way from constellation to constellation in an eastward direction. One disregards, for the moment, the daily movement of the fixed stars—which is easy if one's observations are made at approximately the same time of night—and discovers the peculiar movements of the other stars. They seem like living creatures of a higher kind than ours, which move freely through the heavens—whether one calls them "rams," "hounds of Persephone," or gods.

The conception of the independent movement of sun, moon, and planets, their heavenly journey from west to east, is accordingly earlier than the vortex theory of Anaxagoras and Democritus and is rooted in naive observation and interpretation of the skies. Anaxagoras, Hippocrates, and Democritus do not give less than this; what they want to do is to contribute more. In place of unexplained, spontaneous movement they posit a universal force, which works with the power of necessity. As the cosmos is divested of its anthropomorphic features, the stars too must become things, governed by the ἀνάγκη φόροις.

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84 Anaximander A 5, 22. Cf. Tannery, HS CH 96ff. On the relation of this to the report on Oenopides, see above, ch. IV, n. 38.
86 A. Sachs, J. of Canaan Stud. 6 (1952) 49; Neugebauer, ExSt. 187; van der Waerden, Auf. 247.
87 Auf. 124–125 (differently Neugebauer, ExSt. 140); cf. 98–99, 132, on texts of the 6th century which still are concerned with stars rather than "signs." Van der Waerden dates the Babylonian "system A" of planetary calculation to the time of Diocles I (Archive for the Hist. of the Exact Sciences 5 [1968] 70–78; cf. Auf. 171; above, n. 55.)
88 The series Mul–Apin, van der Waerden, Auf. 77–79. 
Alexander has a report, in the context of discussion of the harmony of the spheres, about the Pythagoreans' doctrine of the planets. This is taken from Aristotle's special book on the Pythagoreans. They thought, he says, that the distances between the planets, and therefore their velocities and the tones they make, are according to harmonic ratios, and that those which move at the greatest distance move the most rapidly (καθότατα δὲ τὰ γάμιστα μεν τὰ τὸ μέγεστον διάστημα κινοῦμενα). Thus for these Pythagoreans Saturn is fastest, next to the sphere of the fixed stars, and the moon is the slowest—the very view which Plato ridicules as an absurd blunder. We cannot assume that Alexander has made a mistake, for he repeats his statement with the notation, ὡς προείρηκε (Ἀριστοτέλης). And in fact this conception fits the idea of harmony of the spheres better than the one Plato favors; in the latter the sphere of the fixed stars, which moves fastest, is followed by Saturn, the "slowest" of the heavenly bodies, whereas in the former, retardation provides for a progressive gradation—a necessity for the connection with the musical scale. Far from simply taking over a Pythagorean system, Plato hits the Pythagoreans, as well as Democritus, with his criticism. There is one way out, as always: to posit a Pythagorean astronomy anterior to or foreign to Philolaus, and claim that Plato is dependent on this; but there is no foundation for such an assumption. In any case, the idea of contrary motion is not unanimously held, even among the Pythagoreans.

Plato's astronomy is not a copy of any Pythagorean system. To be sure, the idea of the harmony of the spheres is Pythagorean, as is, according to Eudemus, his planetary order, though here the only difference from Democritus is in the position of Venus. There is probably also a reflection of Pythagoreanism in his admiration of the circle. Beyond this, Plato takes his place in the lively discussion of his time, which could look back to almost a century and a half of Greek astronomical speculation—an Ionian tradition, Anaximander to Cleostratus and Anaxagoras to Oenopides. Plato takes a polemical stance toward Anaxagoras and Democritus, because for him not physical causality but logico-mathematical relationship is decisive. The earth's ability to hang free in space, and also the movement of the planets, are comprehensible on grounds of mathematical order,

without any extraneous causation involved. Perhaps on this point Plato learned from Archytas. With regard to planetary movement, what he cared about was the mathematical theory, and this accomplishment must be credited to Eudoxus.

3. THE COSMOS OF PHILOLAUS

Copernicus says that he got the stimulus for his revolutionary cosmological system from ancient sources, and in this connection he twice names Philolaus. And, since the time was far from past when everything new was taken as a rediscovery of something ancient, the Copernican system was current for a time under the name of astronomia Pythagorica or Philolaica—an error long ago laid to rest. Nevertheless, the association with Copernicus still exerts a dominant influence in the discussion of the Philolaus fragments; so subtly developed an astronomical system cannot have been devised in the fifth century, it is thought, and certainly not by the Philolaus who, as Plato said, provided no "clear" rationale for his teachings.

The doctrine in question, according to the consistent testimony of Aristotle and the doxographers, was that our earth is "one of the

1 Ed. Soc. Cop. Thorunensis (Thorn, 1878) 17, 6ff ("Philolaus Pythagoricus, mathematicus non vulgaris"), and in a passage deleted from the definitive edition, p. 34 n. See also the letter of dedication to Ptolemy III, 6.6ff. His sources were "Plutarch," i.e. Ætius 3.13.1-3 (DK 44A21) and the report about "Nicetus," i.e. Hecata, in Cic. Acad. pr. 2.123 (DK 50.1).


3 Van der Waerden, Astra. 45ff, 54. Before him Frank, especially, had sought to prove the imposibility of the system of Philolaus in the fifth century B.C. (35, 207ff.) He was followed by Rehms-Vogel (47), Gundel (RE XX 2096ff), and others. Wiersma, Memorie 1942, 23ff, tried to prove that Philolaus did not advocate this 'system of Philolaus' (above, ch. III 2, n. 3). Its origin is attributed conjecturally to Hecata (DK 50) by Wiersma and van der Waerden.

4 Arist. Caed. 293a 18ff, 9866ff, frs. 203, 204; Aet. 2.29.4 = Arist. fr. 16 Walzer; also Aet. 2.7.7, 3.11.3, 3.13.1 = Philolaus A16, 17, 21. On the relation of the "Pythagoreans" of Aristotle to Philolaus, above, ch. III 1. Aristotle and Aetius agree not only in substance but in actual wording, so that it is not only permissible but mandatory to integrate the material included in one line of tradition but not the other into the total picture. (The "plus" in Aristotle: the creation of night and day, Caed. 293a22; the proportionality of distances and velocities, fr. 203, Caed. 290b21; the absence of parallaxes, Caed. 293b23ff; the counter-earth and lunar eclipses, Caed. 293b23, Aet. 2.29.4. The "plus" in the Philolaus doxography: fire as περίπτον, A16; the sun as a burning-glass, A19; the moon in inhabited, A20; movement of the earth in the same direction as sun and moon, in an oblique circle; world catastrophes, A.18. Mondolfo, Inf. 286, 140ff, sees a difference in the fact that in Philolaus the world is surrounded by fire, but in Aristotle's report by ἀνερχομένη, κενόν [above, ch. I 2]; but this is a contradiction only if, like Mondolfo, one supposes that the surrounding fire is infinite in extent, and there is no good reason to do so.)
and the moon, the sun, five planets, and an invisible "counter-earth," revolved about a "central fire." The earth a planet! This seems to anticipate Copernicus' momentous discovery, and one involuntarily regards the Philothen system as an attempt to explain, in as clear a way as possible, certain specific astronomical observations.

Nevertheless, this interpretation leads from one difficulty to another. The system of Philolaus would be truly Copernican if it could provide an explanation for the apparent irregularities in the orbits of the planets. Frank thinks it does: "In this 'Pythagorean' or 'Philothen' system . . . the apparent retrograde movements and pauses find their explanation . . . in quite the same way as in our modern heliocentric system." For this reason, he thinks, the system must be later than Eudoxus and Heraclides Ponticus. The fact remains, however, that it was not the sun which the Pythagoreans thought occupied the central position, but the central fire; and this, like the counter-earth, is always invisible to us "because the earth is in the way." It follows that the earth's circular orbit and its rotation on its own axis must be combined in such a way that the earth always exposes to the central fire the side opposite the one on which we live. This is inconsistent with the main point of the system of Aristarchus or Copernicus; for in the latter, the earth's rotation on its axis explains the daily rising and setting of the stars, and its annual revolution about the sun explains the changing seasons and the convolutions of the planetary orbits. If these two movements are inseparably connected with each other in the system of Philolaus, they can only explain and astronomical fact. Aristotle says explicitly that the earth "by its circular movement about the center creates day and night" (Cael. 293b22ff). Thus the period of revolution about the central fire, but relatively to the sun, is one day. This excludes the "Copernican" idea of explaining the distortions in the orbits of the planets as distortions of perspective due to the movement of the earth. The planets would have to turn retrograde in the course of a single night—unless there are in fact no appreciable parallaxes! This is the point of view taken by the Pythagoreans, according to Aristotle. Even on the geocentric hypothesis, they thought, the astronomer is not making his observation from the center, but is one earth radius away from it; "there is no reason not to suppose, they think, that celestial phenomena are the same, even though we are not at the center," but on the earth which revolves about the central fire (Cael. 293b25ff). In other words, this Pythagorean system, which expressly denies a parallax as the result of the earth's movement, cannot provide any theory of planetary movement and has no intention of doing so. In this respect it is to be classified with pre-Eudoxian astronomy, in which the capricious prancings of the planets were simply taken to be inexplicable.

Frank does not hesitate to assert that the central fire and the daily revolution of the earth around it are "speculative and mythical reinterceptions" of a truly Copernican system, made by the "philosophers of the Academy," and he therefore explains the traditional account as the corruption of an unattested "Pythagorean system of scientific astronomy," thus himself abandoning the realm of historical reality in favor of arbitrary speculation.

The effect of the earth's movement in Philolaus' system is the same as that of simple axial rotation. One might see in this a magnificently bold and fruitful idea: the daily rising and setting of sun, moon, and stars, central facts of human life, become illusion, caused by an unobserved movement of the earth, though it always must appear to be fixed and at rest—a triumph of thought over mere appearance. The earth moves each day, from west to east, "in the same way" as the sun and moon, in an "oblique circle." Its path is therefore in the plane of the equator, that of the planets in that of the ecliptic. The planets, including the sun and moon, likewise move from west to east, but much more slowly; their angular velocity is less in proportion to their distance from the central fire. If one can forget about the unexplained irregularities in the courses of the planets, this makes a system of impressive symmetry. Each celestial body has one and only one circular

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7 207, 208.1. For the central fire Frank substitutes "den (ideellen) Mittelpunkt des ganzen Planetensystems" (35; cf. 207), about which the earth circles "in einem Jahr" (208). This last item is taken over by Rehm—Vogel (47), but without Frank's quotation mark. Frank's citation of Heraclides misses its mark (below, n. 19; Wachtl 97), and his reconstruction is unsatisfactory astronomically. As long as the course of the sun remains outside that of the earth (see Frank's drawing, p. 30), there can be no adequate explanation of the phenomena. On the "ideal center" Heath says (Aristarchus 278), speaking of Heraclides, "It is inadmissible to suppose that, in Heraclides' time, any one could have assumed that the place in the centre of the universe was occupied by nothing and that both the sun and the earth revolved about an ideal point."

8 ἑκοταρανόω, Philolaus A21. Elsewhere the expression "oblique circle" is always used of the ecliptic (Andriani 7f).

9 Schiaparelli, Vorr. 14; Martin, Philolaus 155.
course to complete—provided, that is, that the sphere of the fixed stars is stationary. But this very proviso cannot be allowed, according to the explicit testimony of the sources! Ten bodies “dance a roundelay” about the sky; nothing is stationary but the central fire, the ἐστία in their midst. However the movement of the fixed stars is imagined, they must carry the rest of the stars with them, in order to conform to observed appearances. Thus the astronomical significance and special advantages of the Philolalic system have vanished. None of the attempts to devise a precise interpretation of the movement of the heaven of the fixed stars is successful. As a matter of fact the confusion becomes complete only when we include the testimony of Alexander, so often overlooked, that the stars move with a velocity proportional to their distances from each other—the further away, the faster they move. This means that the sphere of the fixed stars is the fastest of all, and there is no difference from the “whirl” of Democritus. There is no point in discussing the various possibilities of this system; one can only analyze motion in relation to something stable; and the point of reference in the system of Philolalus is the eternally invisible central fire. Proceeding in this fashion, one can conjecturally attribute any kind of motions to the heavenly bodies, so long as they all participate in them in such a way that their relative displacements conform to observed facts; but this is not what is usually meant by astronomy.

Things are different with the astronomical doctrines ascribed to Hicetas, Euphantus, and Heraclides Ponticus. Though there is controversy about many details, this much is clear, that Hicetas and Heraclides believed that the outer heaven was at rest and emphasized that from this hypothesis the phenomena could be explained just as well as if it were thought of as rotating. Heraclides stated explicitly that the apparent position of a planet with relation to the heaven was determined by the straight line from the earth, that is, the eye of the observer, to the planet. Thus he was dealing with the basic ideas of perspective and of projective geometry, the differentiation of true and apparent position, or true and apparent movement—in other words, this is mathematical astronomy. It is relevant here that Philip of

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10 On the idea of an unnoticeable, undetectable motion, cf. Martin, Philolalus 140ff., 155, who refers to Prot. Synt. 1.7. p. 24 Heiberg; Schiaparelli, Vols. 13, 18. The end result is about the same in the more complicated solution of Andristi (who is followed by Timpanaro Cardini 1946, 325). He posits two motions of the earth, which run counter to each other and cancel each other out in their astronomical effect, (cf. above, ch. IV 2, n. 16, on Tim. 40b).

11 Hicetas, DK 50. (According to Heath, Aristarchus 189, he is to be identified with Hicetas of Syracuse, the tyrant of Leonis, friend of Dion and rival of Dionysius II. Cf. Plat. Dion 38, Timoleon 31–32.) Euphantus, DK 31. Heraclides, frs. 104–110. We cannot enter into the controversies about the astronomy of Heraclides (see above, ch. IV 2, n. 11). In Tannery’s view (MSS VII 140ff., IX 232ff., 237ff.) Euphantus and Heraclides are both characters in a dialogue of Heraclides (agreeing: Frank 138 and nn. 40ff.; disagreeing: DK I 441 n.; Mondolfo, ZM 349; van der Waerden, Atr. 55; cf. Wehrli 96). The conjecture is based on the naming of Euphantus along with Heraclides in Aet. 3.12.3 (= DK 31.5 = Heraclides fr. 104) and the similarity of some kind of atomic theory (DK 31.1–4, Heraclides frs. 118–121). Theophrastus’ reference to Hicetas (Phys. op. fr. 18 — DK 50.1) is not a positive disproof of this (contra DK; cf. above, ch. I 3); but the counter-earth, advocated by Hicetas, is never mentioned by Heraclides.—The system of Hicetas, as described by Cicero, is impossible. Not only the fixed stars but also sun and moon are to be stationary (DK 50.1). Tannery thinks Cicero made a mistake (MS IX 234f.), as does Duhem (23). Frank thinks Hicetas exaggerated for didactic purposes (n. 402); van der Waerden tries for a solution by slightly altering the translation (55). See also Wehrli, Heraclides 95.

12 Hicetas, DK 50.1: “cadem effici omnia quaerit Si stante terra caelum movetur.” Heraclides fr. 108: σάν γε τια θαυμάζειν τοις μαγούς και τοις ἄστροις ἡμέρανται, τῆς ἐκ γῆς . . . κοινωνίας. Rotation of the earth on its axis is asserted for Euphantus and Heraclides (DK 31.5, Heraclides frs. 104–108); on the connection with Arist. Cat. 291b10ff., 296a26ff., above, ch. IV 2, n. 16—Schiaparelli, Vols. 501, and van der Waerden, Atrr. 58f, are wrong in concluding from Schol. Cosm. Arist. p. 5054 that the “more genuine” Pythagoreans taught that the earth rotates; the sentence in question is a citation of Arist. Cat. 293a222; cf. above, ch. III 1, n. 74.

13 Fr. 109: “. . . solem et lunam et luciferum et omnes planetas, ubi uestra quiescit, una linea a puncto terrae per punctum stellae exspecta demonstrat . . .”
Opus, who was mainly interested in astronomy, wrote a book entitled Οὐρανός.20

There is nothing of this sort in Philolaus, only an invisible central fire, an equally invisible counter-earth, unobservable movements of the earth and the stars—mythology in scientific clothing, rather than an effort, in accord with scientific method, to "save the phenomena." This is just what Aristotle says of the Pythagoreans: "not seeking accounts and reasons to expain the phenomena, but forcing the phenomena and trying to fit them into arguments and opinions of their own."21 The system of Philolaus is not a scientific astronomy,22 and there is no call to set it late chronologically on grounds of its sophisticated and advanced nature.

There are details among the astronomical teachings of Philolaus that point toward the fifth century, and in particular his remarkable theory of the sun. The sun is a "glass-like" body which receives light and warmth from the "etherereal fire" which it then "strains through" certain "narrow interstices," so as to shine on the earth.23 His basic idea is obviously that of the burning-glass, which became familiar precisely in the late fifth century; Gorgias speaks of its πῦρ, through which fire penetrates.24 Diogenes of Apollonia seems to have had a quite similar idea: the sun is a body "like pumice stone," on which "rays from the aether concentrate."25 Here again, the aether is the source of the sun's fire, the visible sun only a transmitter and amplifier, and the passages or "pores" of the pumice correspond to the "narrow interstices" of Philolaus. There are also other points of agreement between Diogenes and Philolaus.26 But Empedocles as well had a peculiar theory of the sun, and, in spite of various contradictions in the evidence, it is clear that it was similar to that of Philolaus.27 Circling about the earth, he thought, were a fiery and a dark hemisphere. The fiery one is the source of the sun's light and heat, whereas the sun we see is a phenomenon of reflection, or maybe a burning-lens. Though it is only by conjecture, at best, that a chronological order can be seen in this complex of solar theories,28 it is at least certain that they all belong closely together. Obviously the realization that the moon has no light of its own29 was exerting an influence in impelling people

Aétius and Achilles supplement one another. The "mirror" mentioned by Aétius must be an error, as δίστρας shows; so that the report of Achilles is the more reliable (Brockel 127; Martin, Philolaus 135f; cf. also Tannery, HSCh 327f; Heath, Aristarchus 117). For πῦρ περὶ διαβάζει see Philolaus A 16. Cf. also Guthrie I 285f.

20 Suda s. v. φιλόσοφος.

21 Carl. 209a25, tr. Guthrie. Speculation and concentration on the world apprehended by the senses are two interpenetrating tendencies in pre-Socratic philosophy. It was only after Plato had made clear the dichotomy between immaterial and material that mathematics, become independent, could set about from a new basis to "save the phenomena."

22 Wilamowitz, Platon II 93: "Ich fürchte, es ist von Astronomie dabei wenig zu rühmen."

23 Aét. 2.20.12 = DK 44A19. Also Ach. Is. 19, a passage that Diels did not include in the Vorsokratiker because he thought Achilles was directly dependent on P-S-Plutarch (Dox. 22ff.). This thesis is not tenable, and it was withdrawn (orally) by Diels himself; see Pasquale, GGN 1910, 221f. Achilles draws from a more detailed source, as the following parallels show:

Aét. 2.20.12 (from Stob. and Plut., supplementing each other; DK 44A19):

υπολογίζει τὸν ἄνθρωπον ἑνάδε ἑυθείων ἑνάδε ἑυθείων

Ach. Is. 19 p. 46.13ff Maass:

(τὸν ἄνθρωπον) τὸ πυρός καὶ διασκεδάζει

(τὸν ἄνθρωπον) τὸ πυρός καὶ διασκεδάζει

..


25 Diogenes of Apollonia A 13 = Aét. 2.20.10: καθοδεύει τὸν ἄνθρωπον, καὶ ἐνδοδρόμησα ἐν τῷ ἑνάδε ἑυθείων.

26 See above, ch. IV 1, n. 112; below, n. 36.

27 Fr. 54. A 30, A 60; cf. Burnet, EGP 231f; G. Kafka, Philologus 78 (1921) 212ff (p. 213: the introduction of glass in this period); Kranz, Hermes 73 (1948) 101, and Emped. 50 A 36 is very close to A 30 in the description of the two hemispheres, but seems to say that the sun is in the dark half of the sky. Kafka and Kranz mention the burning-glass, but it is striking that all the testimonia speak of the role of the earth in the ἀνέπαυσα phenomenon. Can Empedocles be thinking of reflection, after all—the sun as a sort of mirror-image of earth in the sky?

28 The facts that the doctrine of Philolaus is relatively clearest is due to Achilles' omission of the corresponding theories of Empedocles and Diogenes. Philolaus was probably younger than Empedocles. Burnet (EGP 298) and Heath (Aristarchus 90f) believed that Empedocles' theory of the sun was earlier, Zeller (1982, 1027) the reverse.

29 See above, ch. IV 1, n. 112; below, n. 36.

30 Aenxines A 16 = Eudemus fr. 145 (Tannery, HSCh 216ff, can hardly be right in questioning the authenticity of this; cf. Gigon, Urgeschichte 108), Anaxagoras A 76, Empedocles fr. 43, 46, A 30; on Parmenides (fr. 15, A 42) see Jaeger, RhM 100 (1957) 42ff.
to look for foreign sources of the sunlight and starlight as well.\textsuperscript{30} Even if it were not certain that the historical Philolaus lived in the late fifth century, we should have had to assign the theory of the sun in the Philolaic system to that period.

The report of Aristotle and Philip of Opus, that the higher frequency of lunar than of solar eclipses was explained by the presence of the counter-earth, and perhaps also other earth-like bodies in space, takes us into a similar context.\textsuperscript{31} Sometimes the earth, and sometimes the counter-earth, shuts off the sun’s light from the moon.\textsuperscript{32} This astronomical use of the counter-earth has often been played off against Aristotle’s ironical statement that the counter-earth was only invented to bring up the number of celestial bodies to ten;\textsuperscript{33} but from an astronomical point of view, this explanation of lunar eclipses is unsatisfactory and betrays a lack of exact information.\textsuperscript{34} In any case it is not Pythagorean in origin. Anaxagoras had invisible, dark bodies circling the earth below the moon, responsible, along with the earth’s shadow, for eclipses.\textsuperscript{35} Diogenes of Apollonia followed him, and, earlier, even Anaximenes had assumed such “earth-like” bodies.\textsuperscript{36} Once more a feature of the Philolaic system leads us to the ambit of fifth-century φυσιολογία; it is impossible to separate “Ionian” and “Pythagorean” astronomy.

Even the most exciting idea of the Philolaic system, the movement of the earth, may not be unexamined. Leucippus declared: \( τῇ \, γῇ \)

\textsuperscript{30} “Men saw reflected light everywhere,” says Burnet (\textit{EGP} 239, writing on Empedocles); cf. Metrodorus of Chios, \( \text{DK70a9} \) (the fixed stars illuminated by the sun; also \( \text{τῶν} \, \text{δαίμων} \, \text{ἀνταγωγῶν}, \text{Hebd.} \, \text{1.2} \), and Hippocrates (above, ch. IV 1, nn. 113, 115).

\textsuperscript{31} Solar eclipses are equally numerous, but always visible from only part of the earth’s surface.

\textsuperscript{32} Philip of Opus (from his book \textit{Περὶ \, ἐκδηλώσεως \, σελήνης}; cf. Suda s.v. φιλαστόφος; Martin, \textit{Philolaus} 150) and Aristotle (fr. 16 W., not in Rose) = \textit{Aet.} 2.29.4: \( \text{ἄνωθεν \, \text{φως} \, \text{τῆς} \, \text{γης} \), \text{τούτου} \, \text{δὲ} \, \text{τῆς} \, \text{ἀντίγραφης}; \), cf. \textit{Arist. Cael.} 293b24.

\textsuperscript{33} Against Aristotle (\textit{Met.} 986b8), Burnet (\textit{EGP} 305) alleges that the counter-earth was “a hypothesis intended to account for the phenomena,” and Chersinis (\textit{Pres.} 199) says that Aristotle in another passage shows that “he is aware of the baseless captiousness of his former criticism.”

\textsuperscript{34} Emphasized by Martin, \textit{Philolaus} 150. It has been conjectured that the “dark bodies” are introduced to explain the rare phenomenon of the sun being visible during an eclipse of the moon, on the opposite horizon (Cleomedes 2.6; Heath, \textit{Aristarchus} 79f; Boll, \textit{RE VI} 2351; actually, it is a phenomenon of refraction). But in that case the body casting the shadow would have to be above the earth, not below it as the Pythagorean counter-earth is; and the Pythagoreans would have spoilt the special point of their hypothesis. That they had this very uncommon phenomenon in mind is, however, unlikely.

\textsuperscript{35} Anaxagoras \textit{A} 55\textsuperscript{a}, 9, \textit{A} 77 = \textit{Theophr. Phys. op.} fr. 19, \textit{Dox.} 493.

\textsuperscript{36} Diogenes of Apollonia \textit{A} 12.

\textsuperscript{37} Anaximenes \textit{A} 7, 14.
No one can give us information about the inhabitants of the moon and the stars except someone who is one of them or who can make contact with them; and the idea of a "journey through the skies" brings us back once more into the world of "shamanism." Certain details seem to confirm that this is a real connection: the fact that the other world is "greater and more beautiful" is part of the experience of ecstasy, and freedom from excretory function suggests an existence apart from the body.

But the counter-earth as well fits into this picture. The same Herodorus asserted that vultures are not native to our earth, εἶναι τὸς γύρος αὐτῶν ἐκ τῆς ἄρσης τῆς ἀνάπτυξις. This cannot mean the moon, which is anything but invisible. But the Pythagorean counter-earth is a γῆ ἁλημ, ἡμῖν ἀνάπτυκα, so that the ἀνακύκλωσις surprisingly turns up in fifth-century literature, in a purely mythical context. The vultures of which Herodorus speaks are rationalization of the griffins—γυραί—whose lives along the road to the land of the Hyperboreans, at the entrance to the world beyond, "Grapes Hyperborei, quos... generum mundatus alter." Theopompos represents Silenus as telling of an infinitely great "continent outside this world of ours," whose inhabitants are twice as large and twice as long-lived as we. All their laws are exactly opposite to ours. And, when these creatures once wished to come and visit us, they only got as far as the Hyperboreans and turned back. The theme of a "counter-world" where everything...
is the opposite of what we know, is widespread in folklore and has its
effect on travel literature as well; it is especially common to represent
the realm of the dead in this manner. Thus the Pythagorean counter-
earth, which is naturally thought of as inhabited, has a real meaning
in the world of myth, and its position in the astronomical system is an
expression of its character. Clearly there is a background for this, in
the tradition of shamanistic narrative, similar to that which lies behind
the story of the inhabited moon. The legend of Pythagoras and the
doctrine of transmigration, as well as the acusma about the moon and
sun as the Isles of the Blest, presuppose just such a shamanistic
outlook, so that we are probably justified in calling the whole
complex Pythagorean—perhaps various features of it actually
originated with Pythagoras himself, and perhaps he was just the
most conspicuous member in a more comprehensive chain of
tradition.

The system of Philolaus, which we found intractable to analysis
as an expression of scientific astronomy, now takes its place beside,
perhaps even before, Herodorus of Heraclea, that is, in the second half
of the fifth century B.C.—precisely the time of the Pythagorean
Philolaus of whom Plato speaks. It emerges from the same kind of
concern as that of Herodorus, the interpreter of myth: ancient lore,
transmitted ἐν μύθοις σχήματι, is newly formulated in the terms of
contemporary natural philosophy or φυσιολογία. Thus shamanistic
myths take concrete form as specific components of the world, and the
earth is relegated to equality with them as “one of the stars.” An
“eccentric” attitude toward things, so to speak, a devalution of
earthly existence in comparison with “purer” worlds, is doubtless
as strong a motive force here as the requirement of symmetry which
makes correlates of Ὀλίμπος and Ζαυς φυλακτό, the “Limiting,”
which is united in harmony with the “Unlimited.”

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Proclus and, after him, Damascus report that Philolaus "dedicated" certain geometrical figures to particular gods—the angle of the triangle to Cronus, Hades, Ares, and Dionysus, the angle of the square to Rhea, Demeter, and Hestia, and the angle of the dodecagon to Zeus. Damascus adds that the semicircle was sacred to the Dioscuri. One would quickly reject this late testimony, if it were not corroborated by a very ancient piece of evidence. Eudoxus mentions that in the Pythagorean doctrine the angle of a triangle belongs to Hades, Dionysus, and Ares, and that of the square to Rhea, Aphrodite, Demeter, Hestia, and Hera, that of the dodecagon to Zeus, and that of the 56-sided figure (the ἕκκαπεντεκονταγώνιον) to the beneficent Typhon. This remarkable doctrine is thus attested for pre-Platonic Pythagoreans by a contemporary of Plato. Scholars from Boeckh to Zeller scarcely knew what to make of it, till Tannery, Newbold, Boll, and Olivieri pointed out the connection with astrology. According to an astrological procedure often repeated, with certain variations, triangles and squares are inscribed in the zodiac and are then associated with elements and planets. There are four τρίγωνα and three τετράγωνα; a triangle spans four signs, a square three. This seems to explain the striking connection of three goddesses with the square, and of four gods with the triangle, in Philolaus (even though this precise correspondence is not attested in the Eudoxus passage). The dodecagon, which corresponds to Zeus, is the whole zodiac with its twelve signs. Half the

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60 Philolaus A14 = Procl. In Enul. 130.8, 166.25, 173.11, 174.12. Proclus alludes elsewhere to the Pythagorean connection of σχήμα and gods (Theol. Pl. 1.4 p. 97 In Parm. p. 647, 924 Cousin [Paris, 1864]), so that Damascus (Princ. II 127.7 Ruelle) may have derived his additional data about the circle and the semicircle from a more detailed exposition by Proclus which is now lost. In any case the sentence καί μέφος ("perhaps") ὥς καθένα εἶναι τὸ μὲν παραπάνω κοσμόν σχήμα ἐστὶ πάντων τῶν νοερῶν θεῶν νοερό, τὰ δὲ εὐθυγραμμα ὅσα ἐκάστως is Neoplatonic interpretation, probably by Damascus himself.

61 Eudoxus fr. 84 Gisinger = 293 Lasserre = Plut. De Is. et Os. 30.163a. Schaarschmidt (43ff) maintained, consistently with his general argument, that the Eudoxus fragment was spurious. But Boll (Sphaera 472-478) traced other similar indications of astral theology to Eudoxus.

62 Boeckh 152ff, Zeller I 499 n. 1.

63 Tannery, AGR 2 (1899) 379-386 = MS. VII 131-139; Newbold 198ff; Boll unfortunately never worked out in detail the hints he gave at Nb 1908, 119 (= Kelscher 196, repeated 328). Olivieri sff. The τρίγωνα are associated with the 4 seasons and the 4 elements, though the goddesses named by Philolaus cannot be related with them directly, nor with the zodiacal signs. Olivieri tried to carry the interpretation out in detail. Boll pointed out the connection of the planet Jupiter with the dodecagon. Newbold hazard an astronomical interpretation of the connection between Typhon and the 56-sided figure (207ff). Cf. the 28-day month (Plut. De Is. et Os. 367ff).
signs of the zodiac become visible in a single night, and this suggests the semicircle, which is sacred to the Dioscuri. These sons of Zeus live and die on alternate days.\textsuperscript{64} The orbital period of Jupiter is almost exactly twelve years (11.86), so that in a single year it traverses, approximately, one sign. \textit{Dodeketerides} built on this are ascribed to Zoroaster and Orpheus; van der Waerden relates them to the “primitive zodiacal astrology” developed in Babylon in the sixth century B.C.\textsuperscript{65} Frank insisted that no traces of astrology can be detected in Greece before the time of Plato’s old age;\textsuperscript{66} yet not only Eudoxus\textsuperscript{67} but Ctesias as well speak of the Chaldeans’ divination by the stars,\textsuperscript{68} and reflections of astrological ideas have been detected in the Hippocratic \textit{Regimen}.\textsuperscript{69} If the zodiac was introduced to the Greeks from Babylon before 500,\textsuperscript{70} and the planets about 440, one would infer the presence of the related astrological conceptions even without specific evidence. In the mélange of myth and \textit{phusia oloia}, which Philolaus’ astronomy proves to be, we also find the first traces of astrology; though here the Babylonian tables are replaced by the idea of the angle—the graphic, geometrical, that is to say, the specifically “Greek” element.

4. HARMONY OF THE SPHERES AND ASTRAL IMMORTALITY

A very famous and impressive image, uniting the disciplines of music and astronomy, is that of the “music of the spheres.” It is

\textsuperscript{64} Od. 11.303ff; cf. Delatte, \textit{Litt. 131f}; Carcopino, \textit{Bac. 358f}, and esp. Cumont, \textit{Symb. 72ff}, \textit{Lux. 192ff}; (Pl.) \textit{Achid. 371}; Jan. \textit{VP 155}; Eust. p. 1686.30, 410.18. At Sext. Emp. \textit{Math. 9.37} the Dioscuri are related to the hemispheres of day and night (cf. Empedocles, above, n. 27). The doctrine of the 2 halves of the heaven was ascribed to Pythagoras by Lact. \textit{Plac. Theb. 4.527}.—The sky is called Zeus (Empedocles fr. 142.1, Democritus fr. 30; cf. Cirtitas fr. 25 and later Aratus 18ff, Ennius fr. sc. 345 Vahlen, Macrob. \textit{Somn. 1.17.14}). He is so as fire (Empedocles fr. 6.2), air (Diogenes of Apollonia A8, Eur. \textit{Tro. 886}), \textit{atos ailep}, Hdt. 7.89, Eur. fr. 839; also see \textit{Hebd. 6}. above, ch. IV 1, n. 66.

\textsuperscript{65} Auf. 243.

\textsuperscript{66} Frank 281ff.

\textsuperscript{67} Fr. 341 Lasserre = \textit{Cic. Div. 2.87}: “Chaldeus in praedictione et in notatione cuminum vitat ex natali die minime esse credendum.” Reports of dubious authenticity are given by \textit{ps.-Arist. fr. 32 = D. 2.45 and Gell. NA 15.20.2}.

\textsuperscript{68} FGHLit 688ff: §§23-25.

\textsuperscript{69} W. Capelle, “Alteste Spuren der Astrologie bei den Griechen,” \textit{Hermes} 60 (1925) 373-395.

\textsuperscript{70} Above, ch. IV 2, nn. 53-55 R. Bükner, \textit{RE XXIII 876}, on the ground of a dubious conjecture in \textit{Ph.} \textit{HN 2.31}, ascribes to Cleostratus the zodiacal trigonos.

\textsuperscript{1} \textit{Cael. 290b10ff} (“the Pythagoreans” named at 292a8), fr. 203. Archytas fr. 1, \textit{Plut. M. 1147a}. The expression “harmony of the spheres” is inappropriate, strictly speaking, as applied to the time before Eudoxus, for then one spoke of bodies, wheels, rings, circles in the sky, but not yet of spheres (Burnet, \textit{EGR} 110).

\textsuperscript{2} \textit{Rep. 510d, Cat. 205c}; also \textit{Rep. 617b}, in the myth of Er. On the \textit{Timaeus}, see below, n. 19.

\textsuperscript{3} On the \textit{acmea} of the tetracts, see above, ch. II 4, n. 154. \textit{Pl. Rep. 617b}. On the Siren idea, see E. Buschor, \textit{Die Mienen des Jenseits} (Munich, 1944); on the Pythagorean interpretation, Delatte, \textit{Litt. 132f}, 260f. In the Temple of Del at Parnax (early Empire) the Sirens are depicted next to the planetary gods; this is interpreted by L. Curtius, \textit{RM 50} (1915) 349-353, as having to do with the music of the spheres. On the other hand, later Pythagorean tradition spoke of the Muses in relation to the harmony of the spheres and thought of the Sirens as representing sensuous, worldly music; see Clem. \textit{Al. Strom. 1.148.6}, Por. \textit{VP 39}, Demophilus 23 (Mullach I 480), and P. Courcelle, \textit{R/A 46} (1944) 73ff (vs. Cumont, \textit{Symb. 329f}).—“Pythagoras” himself says, Schol. \textit{Od. 1.371} p. 172.6 Thesleff, \textit{Hed. 65} τόδε οἰκομένης τοῦ αἰώνα αἰκία τὸν ἐμβρύακον άμιμα; and Nicomachus also records that Pythagoras was able to hear the harmony of the spheres (Por. \textit{VP 30}—lamb. \textit{VP 66f}, with a mistaken interpretation of Empedocles fr. 120).

\textsuperscript{4} Martin, \textit{Pyth. 110ff}; Tannery, \textit{Astr. 327}; Zeller I 540 n. 2; van der Waerden, \textit{Astr. 29}. Tannery (cf. \textit{MSc. VII 158f}) puts the harmony of the spheres later than Philolaus, though usually the Philolaic system is thought of as a later development. It is not important that the surviving fragments of Philolaus do not refer unambiguously to the cosmic music; the doxographers would hardly have had occasion to use this under the rubrics \textit{περὶ τάξεως αστρών} (A16), \textit{περὶ θεῶν} (A19), \textit{περὶ γῆς} (A21). We do find the expression \textit{χορεύειν} in A16; and Aristid. Quant. p. 145 M. = 119.27 W.-I. finds in the words \textit{αστρών} φρόνος an allusion to the τῶν πλανήτων ἠμελή κίνησιν. Though before the time of Timotheus, Hittacus of Colophon had introduced a 10-string lyre (Nicom. \textit{Exc. 274.4 Jan}).


\textsuperscript{6} Varro Atacinus fr. 14 Morel (Gramm. Lat. VI 60), Philo Op. 126, Nicom. \textit{Ench. 3} (cf. \textit{Th. ar. 71.13ff}), Lucian \textit{Astrol. 10}, “Orpheus” \textit{Orph. frag. 578a}. \textit{Serv. Am. 6.645}. Dio Cass. 37.18, Lydus \textit{Mens. 2.3}. There do not seem to be any older references than these on the number 7 as a link between music and astronomy. Both are omitted from \textit{On Seven}, as Roscher emphasizes. The 7 analogy as point of departure for the whole theory of cosmic music is accepted, among others, by Jan, Philosophus 1893, 15f; Wilamowitz, “Die Harmonie der Sphären,” \textit{Reden aus der Kriegzeit} III 6 (Berlin, 1915) 15; and Gundel, \textit{R/I XX 2056}.}
Harmony of the Spheres and Astral Immortality

created by the desire to build cosmic measurements into it.\textsuperscript{14} An attempt in Cicero's \textit{Nommium Scipionis}\textsuperscript{15} to combine eight spheres with seven notes is a patchwork compromise. Nicomachus is at least more consistent, in eliminating the movement of the fixed stars and basing the system on the individual movements of the planets in the zodiac, so that the moon being fastest, is credited with the highest note; but in both respects he contradicts the reports of Aristotle.\textsuperscript{16} The most lucid and consistent system of cosmic harmony, employing only \textit{φθορος}\textsuperscript{17} \textit{έτερες}, makes no claim to be derived from the ancient tradition.

Alexander intimates that Aristotle himself knew no detailed exposition of the Pythagorean cosmic harmony. The distances of the heavenly bodies from one another ("intervals"), Alexander explains, have a certain \textit{άναλογία}; their velocities correspond to these distances, and the tones to the velocities. Thus the tones themselves have a "harmonic" relationship, determined by number: "Thus, the

\textsuperscript{14} Burkert, \textit{Philotheus} 1961, 31ff.
\textsuperscript{15} Cic. Rep. 6.18. Venus and Mercury seem to have the same note ("in quibus eadem vis est duorum"), as also in Ptolemy (below, n. 17); cf. Macrobr. Somm. Sc. 2.4.0 (Boethius is wrong, \textit{Mus.} 1.27). Boyancé, \textit{Sorin} 111ff, interprets this expression as meaning that the moon and the fixed stars make an octave (cf. Por. \textit{In Ptole.} 104.7, on the octave: \textit{δυσθεὶς} \textit{έπιος} \textit{κατα\ ι}εν; similarly, \textit{Aris.} Pr. 19.14, 19.18; contradicted by O. Seel, \textit{PhW} 58 (1938) 49ff. There remains in any case the question the sun can have a different tone from its two \textit{ιδάδους}. At Plut. \textit{De an prot.} 1029b, sun, Venus, and Mercury are obviously taken together, so that there remain only \textit{νέρες} \textit{διησεμα}.

Nicom. \textit{Ench.} 5 p. 241 Jan, and following him Boeth. \textit{Mus.} 1.20, 27. This is held to be the original view by Jan, \textit{Philotheus} 1893, 17ff, and van der Waerden, \textit{Astr.} 36ff. Frank, too (31), thinks the moon had the fastest movement. An apparent argument for this is that here Hypate, taken literally, is the "highest" note (Saturn); but the names of the notes derived from the way the instrument was held. In defense of his position that the ancient Pythagoreans derived the harmony of the spheres from the independent movement of the planets in the zodiac, van der Waerden (\textit{Astr.} 36ff) cites \textit{Aris.} fr. 203; but he quotes only a part of it and overlooks the fact that, later on, more rapid movement is attributed to the more distant planets. Against Nicomachus' statement that the \textit{synmemnon} system was original, see below, ch. V 2. The \textit{Excerpta ex Nicomachus} emphasize that, differently from Nicomachus, the "first" (οἷον \textit{δι} \textit{πρώτος}) made Saturn the Nete and the moon Hypate (3 p. 272.9 Jan).

\textsuperscript{16} Mentioned at Plut. \textit{De an prot.} 32.1029b (ibid. 1029a-b, a somewhat different system based on the 5 tetrachords); Ptolemy, \textit{Canopus} inscrip. (\textit{Op. min. ed.} Heiberg, II 134f; authenticity contested by Jan, \textit{Philotheus} 1893, 35ff; Hoog, \textit{Cornon} 6 [1930] 657ff; defended by Düring 1934, 280ff; cf. van der Waerden, \textit{RE} XXIII 1818-1823; the corresponding exposition in the \textit{Harmonics} of Ptolemy is lost); \textit{Th. ar.} 75.8ff = \textit{Exe. Neap.} 2, p. 412 Jan (\textit{Mus. sc. gr.}), with the superscription \textit{Πτόλεμου μουσεύμα}, \textit{Anon. Bellern.} 884. The notes are, transcribed, A B c a b d e' a' b', corresponding to the series 8 9 12 16 18 21 24 32 36. Since 241 (found in Ptolemy, \textit{Canopus} inscrip.) is not a whole number, and much have simply substituted 21, which is musically wrong. The \textit{Canopus} inscription distinguishes earth (8 = A) from air (9 = B), and in compensation gives Venus and Mercury only a single note (16 = 2); the others count 8 celestial spheres—Since this system is older than Plutarch, it could go back to the mathematician Hypsicles, who, according to Ach. Is. 16 p. 43.9 Maas, wrote on the harmony of the spheres.

\textsuperscript{8} Arist. Cael. 290b21, fr. 203.
\textsuperscript{9} Cf. Frank 30ff; van der Waerden, \textit{Astr.} 26, 29ff; Junge, \textit{C&G} 1947-1948, 183ff.
\textsuperscript{10} Alex. Met. 41.2ff = Arist. fr. 203 expressly relates the harmony of the spheres to the "moving bodies."
\textsuperscript{11} According to Tannery, \textit{Astr.} 328, the harmony of the spheres was deduced from (or "followed," \textit{découle}) the idea of the kinship of the sciences; according to Junge, \textit{C&G} 1947-1948, 183ff, from the discovery that the planets have fixed orbital periods.
\textsuperscript{12} Pl. \textit{Rep.} 617b, Eratothenes fr. 14 Powell = Theo Sm. 105.15, Anat. p. 38 (\textit{Th. ar.} 75.6); cf. Theo Sm. 142.7ff = Chalcid. 73.
\textsuperscript{13} Arist. Cael. 290b18: τὸ πλῆθος ἀστρων... φερομένων, cited by Zeller I 541.2; Heath, \textit{Aristarchus} 108.
distance of the sun from the earth, being, say, double the distance of the moon...they considered that there was some arithmetical ratio in the case of the other planets as well..."18 Alexander introduces his exposition with φῶτον ἐκεῖνον, that is, "supposing," or "say, for example." The figures given belong to a hypothetical case, cited for clarity's sake, and do not belong to a traditional account. If Aristotle had credited the Pythagoreans with an unambiguously described scale, Alexander would not have used a fictitious example.

Nor does the structure of the world soul in Plato's 'Timaeus' signify a system of cosmic harmony, in which each planet had a note corresponding to its "distance." The seven planetary orbits are derived by sixfold division of the inner sphere of the heavens, which is dominated by the nature of the "other." This division is made "according to" three powers each of the numbers 2 and 3, which had previously been developed; but the game he plays with 6 and 7 seems to suggest that one ought not simply to identify 7 numbers with the 7 planets and 7 notes.19 To be sure, musical theory is involved,20 but Plato says expressly that the movement of the world soul takes place ἀνευ φθόγγου καὶ ἀκοῆς (37b); the mythic image of the Sirens, from the 'Republic', is translated into the realm of the immaterial and abstract. At the same time the naive thought of the Pythagoreans by reported by Aristotle,21 that the tremendous bodies of the stars could not move without making a noise, is rejected. In the 'Republic' Plato demanded, in explicit polemic against the Pythagoreans, that the harmony of numbers be regarded in itself, and without reference to audible sounds, and he carries out this program himself in the 'Timaeus'.22 Thus the relationship between astronomy and music is traced to its ultimate, immaterial principle. The numbers in their order are the basic principle of the cosmos, which is at the same time "beauty" and reflection of "the Good." Therefore the basic constituents of matter, the elements, are also derived from the process of geometrical arrangement. All this, as Aristotle and Plato make clear, is not Pythagorean but Platonic.

The contradictory nature of the later tradition, too, sometimes using seven,23 sometimes eight,24 and occasionally three musical notes,25 and even connecting the nine Muses with the sphere of the All,26 shows that the idea of cosmic music is not bound to any particular astronomical system. This concept has nothing to do with mathematical or musical theory, but comes from a deeper root; and this is why it was able to outlive even the Ptolemaic cosmology. Of course this carries the implication that any conclusion about certain astronomical theories of the Pythagoreans, on the basis of the harmony of the spheres, is hazardous in principle; we need not suppose that any detailed system formed its basis.

There are also other conceptions of cosmic music, quite different though related. One need not speak of planets, or of a many-storied universe. No matter how things are divided, the correctly attuned ear will hear music.27 The ancient lyre had only four strings, "in imitation

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18 Alex. Met. 40.3ff = Arist. fr. 203, tr. Heath. Heath saw the significance of φῶτον ἐκεῖνον (Timaeus 111 n. 2). Tannery, too (Astr. 327) believed that all detailed expositions of the celestial music were late.

19 Tim. 56d: σύνοψα ἡγάκε αὐτά κώδων ἄνων κατὰ τὴν τοῦ διπλασίου καὶ τριπλασίου διάτομα, εἰκόνι, ἐκατέρων τρίων, depending on 35b-c. On the contradictory ancient attempts to relate the 7 numbers of the 'Timaeus' with magnitudes, velocities, and intervals of the planets, cf. Plut. De an. proo. 31.1028a, Chalc. 96, Macrob. Somn. Sc. 2.3.14f, Procl. In Tim. II 212, Taylor. Tim. 161ff. Like Zeller (II 1.779 n.), Boeckh (KisSch III 168), and Duhen (53), Taylor comes to the conclusion that "Timaeus is giving us an estimate of the radio of the planetary orbits in terms of the diameter of the moon's orbit" (165). Heath (Arithetus 163) and Cornford (Tim. 79) are rightly dubious of this. Like all musical relationships (cf. 356-58), all numerical relationships are ultimately derivable from the numbers given; the basic rules of the "system of derivation" (above ch. I 17) are presupposed. Junge (CfM 1947-1948, 183ff) attempts an interpretation that makes sense astronomically: we have "an expression of the orbital periods of all the planets by means of the numbers 2 and 3 and their multiples." For example, the moon takes 27 days, the sun 365 (≈ 277) half-days, Saturn 27 years, Mars 2, and Jupiter 12 years. It is likely that Plato's thoughts were running in this direction (Junge refers to Rep. 588a); but the basic idea is more important than any specific result—the problem is more important than the solution.

20 This was denied by Rivaud, Rev. hist. philos. 3 (1929) 16ff (so that he found it necessary to brand not only the Philolaus fragments but those of Archytas post-Platonic forgeries; he seems to overlook, on pp. 6f, that Plato himself, at Rep. 530, expressly mentions Pythagorean musical theory). The harmony of the spheres, he thinks, is merely crude interpretation of the 'Timaeus'. (Similarly Moreau, Æne 55.)

21 Cael. 290b15ff.
of the cosmic music, which consists of four elements." The four seasons correspond in turn to the elements, and stand in concord with one another: spring makes a fourth along with fall, a fifth with winter, an octave with summer, "as they say Pythagoras taught." All the sources for this are late, and the part about the elements is consciously derived from the Timaeus. But, if Scythinus and Cleanthes called the sun the "plectrum" of the cosmos, this fits better with the rhythm of the seasons, which does depend on the course of the sun, than with the planetary scale. A ἀρμονία of the seasons is spoken of by Euripides, and then also by Plato. Such ideas are traced to oriental beginnings. The idea of a cosmic music, and specifically one connected with the changing seasons, is widespread; for example, the five notes of Chinese music correspond to five elements and to the seasons. Here one should not think of direct influence, one way or the other, but of a parallel development due to similar psychological impulses; and for this very reason one seems here to be closer to the root of the idea of cosmic music than in the context of the planetary scale.

Before the beginning of reflective thought, man feels, in various contexts, an involvement. He unconsciously arranges the multiplicity of phenomena into a restricted number of schemata. It is the business of reflection, when it begins, to raise these transitory insights into the realm of consciousness, to name them, and to assimilate them to one another. This is how the world becomes comprehensible. In myth and ritual man tries to make these realizations present and clear, to assure himself that, in spite of all confusion and all the immediate threats of his environment, everything is "in order." It is in such a pre-scientific conception of order that the idea of cosmic music has its roots; and number speculation springs from the same soil.

But relationships that usually have their effect unconsciously, or only enter consciousness as the result of slow and patient reflection, become immediate, overwhelming experience in ecstasy. The soul that in ecstasy, or dream, or trance, travels to heaven, hears there the music of the universe, and its mysterious structure immediately becomes clear to him. The incomparable and supernatural sound is part of the same thing as the incomparable beauty and colorfulness of other worlds. If Pythagoras was something like a shaman, who in ecstasy made contact with worlds "beyond," then the tradition that he personally heard the heavenly music surely preserves something of truth. When we look beyond the facade of analysis and the explication of the harmony of the spheres, what we find is neither empirical nor mathematical science, but eschatology. In the religion of Zarathustra, the paradise to which the soul ascends is called qara demana, "House of Psalms." It was related of Pythagoras that in his dying hour he asked that the monochord be played: "Souls cannot ascend without music."
all around and limited in size, there is no longer any place for the subterranean Hades of Homer or for the “Isles of the Blest” in the distant Occident. In general, the idea finally prevailed that the Beyond is in the realm of the stars, that man’s soul came from the skies and will return there some day. With many variations in detail, these concepts are dominant from the late Hellenistic period on, both in works of literature and in the art and the inscriptions found on gravestones.39

Looking for the decisive turning point in this development, modern scholarship, following ancient tradition, has seen oriental influences at work—Babylonian and Iranian—and, notably, the influence of Pythagoreanism.40

To be sure, Rougier’s attempt to establish the belief in an immortality among the stars as a deduction from an astronomical discovery of Pythagoras, making the religious idea a logical conclusion from scientific knowledge, is not convincing in the light of the history of either religion or astronomy.41 It is a dangerous oversimplification to represent “the Homeric religion” as followed by a “Pythagorean revolution.” In people’s beliefs about the afterlife, there are present from the very beginning a large number of overlapping and contradictory themes: even in Homer, the idea of the muddy “House of Hades,” hateful to the gods, coexists with that of Elysium and with the apotheosis of Heracles.42 The mystery cults bring man the hope of escaping death and joining the gods; and it is an easy step from this to the doctrine that man is of divine descent and returns at death to his place of origin.43 The association of gods and sky is primeval and seems self-evident.44 Artistic representations from the archaic period show the journey of the deified dead into the Beyond. A team of winged horses may provide escort “to Heaven,”45 whereas sea creatures— Tritons, Nereids, dolphins—point to the Isles of the Blest, beyond the vague suggestion that the Pythagoreans may have tried the same kind of explanation for the other planets as for the sun (22); but here he fails to notice that the passage of Theo he cites (150.12ff.) anachronistically introduces the epicycles. Rougier overlooked an important piece of evidence, the aitia about the planets as “hounds of Persephone,” which does provide a connection between Pythagorean teaching and the oriental ideas alleged to be so radically different (above, ch. IV 1, n. 109). For argument against Rougier, see also Cumm. Symph. 116 n. 3, Lux 147 n. 1. —Rougier republished his 1933 monograph almost unchanged in 1959 (La religion astrale des Pythagoriciens), but omitted the references, so that the older work has been cited here.

41 A Maori can depict the dead as descending into the underworld, while at the same time maintaining that they live in the tenth heaven, says L. Lévy-Brühl, The “Soul” of the Primitive 175. Thus in the Trophonius ritual Timarchus travels into the depths and is by virtue of this translated to the stars (Plut. De gen. 22.359b; cf. Vergil Aen. 6, and below, n. 46.)

42 The Elysium idea itself shows the blending of lightning worship and the myth of the Isles of the Blest (Buckert, Glotta 39 [1961] 208ff.). The apotheosis of Heracles is doublet very old, though the verses about it at Od. 11.601ff. and Hel. 25.26-33 M.-W. were atheitized in antiquity.

43 For τον κοινον χρόνον μετὰ θεών διάκεισθαι as a goal in the mysteries, see Pl. Phed. 81a. Also the Gold Plates: θεοὶ ἐγὼν καὶ ἐσπαθήματος (DK 11818, 20) and καὶ γὰρ ἐγών ἥνοι κείμενον δίδονος εὐπνεόνοις εἰς (DK 11813, 19-3; cf. 17.7, 17.6, 1731.)

44 Θείαι σύμμακα, cf. the relationship of root and idea in Zeus, deus, divus. On Olympos, above, ch. III 2, n. 31. Dionysus, too, is brought into connection with the stars: τοῖς πανανθρώποις κυριάρχει ἀσπίδα (Soph. Ant. 1147), cf. Diod. 1.11: Εὐλογημένος μέν ἐν ταις βασιλείαις ἔπειτα θρόνον ἀναφέρθη ἀνάμεσα ἐν ἀνδροφυίᾳ παρασκ. Musaeus, the father of Iomilus, is a son of Selene (above, ch. IV 3, n. 48).

45 Etruscan gems (Furtwänger, Gemmen I 7, nos. 1, 2, 3, ca. 600 b.c.) were compared with the Phaedrus myth by Delette, Litt. 74f.; the iconographic motif of the team of winged animals was, however, taken from the Orient, and originally does not have sepulchral connotations; see the examples published by J. Boardman, Antike Kunst 10 (1967) 21; but it acquired these by the 6th century B.C.; see, on the bronze chariot of Monteleone in New York, R. Hampe and E. Simon, Griechische Statuen in der frihen etruskischen Kunst (Mainz, 1964) 53-67; generally, Meuli, Bacchofen VII 498f.; F. Matt, Griech. 33 (1961) 63-65. On the throne of Amynaclea, Hyacinth and Polyboea were represented “going to heaven” (time of Ctesias: Paus. 3.19.4), although at the same time Hyacinth receives his death offerings through a door in the altar (Paus. 3.19.3). Cf. also Wulfenius 357, 547ff.
the Ocean. The journey to the Beyond is outside of all geography. In any case, the notion of divine origin and of return to heaven was widespread, at least in germ, before the time of Pythagoras. Another element present is the divinity of the stars, which inevitably gained in significance through contact with Babylon. Then there is the popular belief in a connection between man and star, 48 which had taken on a special form in Egypt, 49 and the general belief in ghosts—that is, in the spirits of the dead and their proclivity to roam about through the atmosphere, especially at night, so that in a mysterious way they have a special relationship to the moon. 50 And, finally, there is the Iranian conception of the soul’s making a journey to heaven. 51 The position of Pythagoras and the Pythagoreans in this complex tangle is a question that cannot be settled by general considerations, but only by analysis of the evidence.

The idea of astral immortality is attested a good number of times for the fifth century B.C. Aristophanes alludes to the doctrine that people become stars after death; 52 Alcmacon put the eternal motion of the “divine” stars together with that of the soul to make a proof of immortality; 53 and Euripides represents Helen as translated to the “palace of Zeus” beyond the starry sky. 54 The idea is often expressed that man consists of body and soul, and in death the body returns to the earth, and the soul to the heavenly aether whence it came; in the epigram for those who fell at Potidaea (432 B.C.) this view acquires a quasi-official recognition. 55 It occurs not only in Euripides, but in Epicurus; and this might seem to point directly toward a Pythagorean origin. But in this respect as in others Euripides is connected closely, in the tradition, with Anaxagoras. 56 The latter spoke of an aetherius calor in the embryo—life comes from the “warm” aether, then. 57 For Diogenes of Apollonia the soul was a part of the “divine” air and thus a μορίον θεοῦ. 58 Anaxagoras, too, regarded the soul as immortal, 59 and that is why these two are ranged with Pythagoras. Medical writers reflect the same belief, 60 but even in Leucippos and

48 Cf. Wilamowitz, Glw I 253ff = I 248ff, 2nd ed.; Nilsson, Op. III 31–39. Aristotle says (Met. 1074b1) that the ἄρχατος καὶ παράπλης taught the divinity of the stars ἐν ἡθοπληκτὶ, but worship of the stars was generally taken as a sign of a barbarian (Ar. Pax 406, Pl. Crat. 397d). For a certain example of oriental influence on the Pythagoreans, see above, ch. II 4, n. 47. Aeschylus calls the stars (Ag. 6) ἄμαρτος δυνάστη ἐμπερενούσε αἰθήρ.

49 Examples from American Indians to Australian aborigines in J. G. Frazer, The Golden Bough IV (London, 1911) 64ff; Capelle 19ff. There are two versions. Either the soul of the dead becomes a star, and a shooting star is a soul hastening to its rebirth, or the living person “has” his star, which at his death is extinguished in the form of a shooting star. The former version is still influential in Plato’s Phaedrus, as Stenzel showed (Hellchr 1ff; he compares Rep. 621b and Plut. De gen. 22.591c–d). The latter is the opinion of the σολογος according to Plin. Hz 2.28; cf. Euseb. Migne 86.1433, Lucian’s Lynchopolis, Ver. hist. 1.29, Boll, ZNTW 18 (1917) 40ff, Eur. fr. 971. The metamorphosis of the dead into a star is often mentioned in episcopal verse (not in prose inscriptions; poetic form allows more liberty): Peck, nos. 1097, 1829, 1776, 648; Arch. EPh. 1923–1934, 2, 290–296.

49 The dead person lives on in the sky, in the retinue of the sun god (Plut. De Is. et Or. 21: Kees 42, 87ff).—Greek catasterisms go back to a very early period; this is attested at least for the Bear and Orion (Her. 148, 161), though these are exceptional cases.

50 Cf. Capelle 3ff; Cumont, Symb. chs. II–III. It is not clear how far back the identification of Hecate and the moon-goddess goes (Norden, Vergil VI 23f, and Kerényi 79f find it as early as Hymn. Hom. Conv. 52), or that of the Gorgonion with the moon (Meuli, Bachofen VII 497f).

51 Above, ch. IV 4, n. 47.

52 Ar. Pax 832–837: the slave asks Trygaeus, after his return from the sky, whether it is true that the dead become stars, as people say. Trygaeus replies that it is indeed true, and that Ion of Chios, who composed a poem on the morning star, was, after his death, greeted in the sky as δίκαιος ἄνθροπος. This need not be based on anything more than Ion’s poem; and for the claim that Ion was a “Pythagorean poet” (Cumont, After-Life 95: Capelle 24ff), the fact that he mentions Pythagoras in 2 of the 119 fragments (in Hellenistic’s collection) is a slight basis (above, ch. II 3, nn. 13, 51).
Democritus the soul is “fiery,” like the sun and the moon, so that Lucretius, carrying on their materialist tradition, could borrow expressions from Euripides. The soul was “fire” according to Parmenides and Hippasus, but Heraclitus too is said to have called it scintilla stellae essentiae. And, as early as Anaximenes, the φως was related to the “divine” air, which surrounds and supports the cosmos.

That the human soul has a very close relationship to the sky and the stars, and even that it comes from heaven and returns to it, is thus a generally held belief in Ionian φυσιολογία, at least from the time of Heraclitus and Anaxagoras. In the garb of φυσιολογία, and in “materialist” phraseology, what starts as a μεθος continues to exert its influence sometimes with more emphasis on salvation of the soul, and sometimes with more on the general thought of microcosm and macrocosm: man is made of portions of the cosmos, and in death like returns to like.

If this whole development were to be traced back to Pythagoras, one would have to ascribe to his teaching a maximum of influence and a minimum of definite content; for the testimonia are extremely disparate. The divinity of the stars, which was known to Epicharmus, was called sun, sometimes θερμός, sometimes νόστος. From the circle of Diogenes come expressions like that of Ar. Nub. 223ff; and Pl. Crat. 396e (the Muse Odyssea is ἀναφέρει τὸ ἄνω, διὰν δὲ καὶ φως...καὶ καθαροῖς νοῦς παραγίνεσθαι τοῖς μεταμορφοῖς) is not simply “Pythagorean,” either (pace Bouyére, REG 1941, 196ff; Joly 196f).—A “divine” θερμός as φως δύναμις, ἀνέλοχος ὀθόνη τῶν ἀκρωτών στοιχείων, is recognized by Aristotle (Gen. an. 736b29ff, on which see F. Solmsen, JHS 77 [1957] 119-123; H. A. T. Reiche, Empedocles’ Mixture, Endoxan, Astronomy, and Aristotle’s Conmata Pneuma [Amsterdam, 1960] 97ff; Arist. Περὶ φιλοσοφίας fr. 27 W. = Cic. Acad. 1.26).


Lucr. 2.99ff, after Eur. fr. 89.

DK 18.9, 28A45; cf. 46A-b.

Heraclitus A15: the νόστος is related to the περίκλεια; A16 612ff, cf. A17: the return of the soul πρὸς τὸ ἰσογενές, Stoic interpretation and systematization seems to be involved here; Heraclitus spoke of the “death” of the soul (fr. 36), or at least of certain souls (cf. Kirk in KR 20Aff).

Anaximenes fr. 2, and J. Longrigg, Phronesis 9 (1964) 1-4. A θεῖον περίκλεια (the ἰστορία) is already present in Anaximander.

“"There is nothing Pythagorean here," says Nestle (ZN 161 no. 1) of the Epicharmus fragments concerned. On the other hand, Pfeffer (Stengel 114) avers that Heraclitus owed "wichtig Grundzüge seiner Weltanschauung" to Pythagoras. It seems to be Plato's adverse criticism that prevents scholars from advancing the same thesis in relation to Anaxagoras. Already in the Rig Veda; see H. v. Glasenapp, Die Religionen Indiens (Stuttgart, 1943) 84; Olerud 153. Plato, Tim. 63e, purposely restricts this "theory of dissolution", to the material.

Mendender fr. 614 Körte: ὡς μὲν ἐπιφανείας τοὺς θεοὺς εἶναι ἀλήθες ἀλήθεια ζωῆς πάντων ἀστερῶν. To be sure, this sounds less like ancient piety than incipient skepticism; since one no longer believes in the mythical gods, one restricts himself to what is visible.

and the analogical conclusion of Alcmaeon are as far from the idea that the soul is made of heavenly fire as is the uncomplicated notion that souls become stars. To be sure, these various contradictory ideas are all at one time or another ascribed to Pythagoras. But in Pindar’s exposition of metempsychosis there is no more trace of astral motifs than in Empedocles’ theory of the fallen daimon exiled to earth, into the “cave” (Hades he placed in the realm of air). In the monuments of southern Italian eschatology, there is no reliable, early evidence for belief in astral immortality.

There remains to consider the aëstas which asks the question, “What are the Isles of the Blest?” and answers “The sun and moon.” This places the Beyond in the orderly cosmos; it represents the same desire for stability that forges a theology of the soul out of myths about the soul and puts ritual taboos together into a “way of life.”

4 Hypomn. 27: The sun, month, and stars are gods by virtue of the θείαι working in them. The soul is an ἀνώσωσια ἄθροισι, immortal (28) and invisible (30). After death it floats in the air until Hermes conducts the pure souls ἐν τῷ ὑποστήριῳ (on this expression see Cumont, Or. rel. 58, 273 n. 93; RE s.v. Hypsistos; Cook, Zeus II 876ff; Zeller III 2.106. n. 2, is hardly right in seeing Jewish influence here).—Varro in Comm. Benn. Luc. 9.9 p. 291 Usener: “Pythagoras dixit amans in cellis converti viorum fortunis” (Varro Antiq. rer. div. fr. 25b Agath; but see Reinhardt, RE XXII 593).—Plutarch gives an exposition of astral immortality in the dialogue on the daimones of Socrates, whose setting is a Pythagorean circle in Thebes (above, n. 39).—On the moon as abode of souls see above, n. 75; on the Milky Way and the gates of heaven, below, n. 90, 94.

69 Cf. above, n. 38, and ch. II 3, n. 80. ἄστορ, Emp. fr. 120; in fr. 6 Ἀστικοῦς is air, Hera earth; cf. the Derveni papyrus, col. 18.7. On the reinterpretation of “Hades,” Cumont, Symb. 33ff, LXX 180ff, Nilson II 228ff. Possible solutions: Hades could be the night half of the heavenly hemisphere (above, ch. IV 3, n. 64); it could be the moon and the sublimary region (below, n. 89, 93); or it could be life on earth (Lucr. 3-978-1023; cf. the full treatment of Carcopino, Bas. 264ff).—The differentiation of sublimary from sublunar regions is ascribed to Empedocles (A 62), as well as Alcmaeon and Heraclitus (cf. above, ch. III 2, n. 32), but we cannot assume that Empedocles held the later doctrine of the eternal, perfect order of the celestial realm; this is perishable, destined to enter into the perfect unity of the “Sphere.”

71 The Brindisi disc, which depicts the journey of the heroised dead in the zodiac (Wullemier, 544ff, pl. 45, with refs.), is hard to date. Wullemier decides on a period before the Roman conquest, i.e. the 3rd century B.C. at latest. It shares with other discs from Tarentum a good many motifs, but not the zodiac.—The vases of Canosa (3rd century B.C.) which Bachofen made famous have no unambiguously astral symbolism (Meuli, Bachofen VII 439ff).—The traces of astral immortality seen in the gold plates are uncertain (ἄνωθεν ἐπὶ γῆς ψάθος, DB 1817.7, from Petalae, ἄνωθεν ἐπὶ γῆς ψάθος, DB 18217.8, from Thrana, see Carcopino, Bas. 34ff, and Diels, DK I 16 n. 1; A. Dieterich, Kl. Schr. 95 compared Orph. Arg. 761, but in the next verse Persephone is called ἡθοι διαβάλλει)—The puzzling conclusion, ἐπειδὴ εἰς γῆς ὑπέρτας, was interpreted as an allusion to the Milky Way by A. Dieterich, De hypomn. Orphics (Marburg, 1891) 35ff = Kl. Schr. 95ff; more emphatically by Carcopino, Bas. 31ff, Wullemier 547ff. Contra, S. Reimach, Cultes, mythes et religions III (Paris, 1909) 123ff; K. Wyss, Die Mithil im Kultus der Griechen und Römer (Giessen, 1914) 53ff. The word ἐπειδὴ points to Dionysus (cf. above, n. 45).

72 Ian VP 82 (above, ch. II 4).
(Hes.). Height and depth, fall and ascent do not become dominant ideas in the theology of the soul until the realm of the stars is taken in to become part of the picture. Related ideas are that the Great and Little Bear are the hands of Rhea, the Pleiades the lyre of the Muses, the planets the hounds of Persephone, an earthquake is a conspiratorial meeting of the dead, and that the purpose of thunder is to frighten those being punished in Tartarus. In this context it is taken for granted that Hades is beneath the earth, and all the allusions to a Pythagorean katabasis give the same impression. The conclusion that astronomy makes an underground Hades impossible was not drawn by the Pythagoreans. The Isles of the Blest are not a part of Hades; they are far removed from the land of the dead. The soul goes there, finally, as to its last resting-place, where it listens, no doubt, to the “harmony” of the “Sirens.” This separation of the Isles of the Blest from the realm of the dead is doubtless ancient, and presumably the earliest form of the school’s doctrine, and therefore the total picture, along with the acusmata, makes consistent sense. Of course it is wholly pre-astronomical, as is shown even by the simple association of “sun and moon,” as though they were islands in the same sea. There is no hint of the multi-storied universe which is standard in the later tradition.

Here, once more, we find that the acusmata represent a strand of tradition independent of the later tradition, and also of Empedocles and Plato, and evince a Pythagoreanism still innocent of the scientific view of the world.

Even in Plato’s eschatological myths, in which, according to the usual opinion, he does most “Pythagorizing,” astronomical motifs only gradually make their appearance, showing that he is not reproducing an already completed system. In the Gorgias, the Beyond is not brought into any kind of relation to the structure of the cosmos; the only question is the ethical one, of “what kind of man one ought to be.” The myth of the Phaedo interprets our life as an existence in the deeps, by contrast with that much more precious and wondrous life on high; but in carrying this out, it only describes “the earth,” in its amazing size and remarkable structure.77 The image of the Cave expresses the same conception of life, but it does not have to do with the cosmos, but with the τοῖς τόπος beyond it.78 In the concluding myth of the Republic, the image of the spindle with its complicated spindle whorl revolving in the lap of Necessity, the system of the planets is depicted, but it has no relationship to the previously described “paths of the souls.” When this description presents us with two doorways, in sky and earth, with the good souls traveling through the heaven and the bad ones through the earth, the general conception presupposed is the pre-scientific one of the flat earth, the hemispherical heavenly shell, and the dread Underworld. The souls making their way along the heavenly path naturally find themselves in the company of the stars, and many have the impression that the stars enter and leave the sky by two doors.79 An indication of Pythagorean influence on Plato is the identification of the road “up” with that “to the right.”80 But the adjustment of scientific and mythical views of the world was not yet attained.

It is in the Phaedrus that the fate of the soul and the movement of the heavens are for the first time brought into a really intimate connection. The winged soul-horses, following the gods in their journey to the heavens, striving, at the zenith of the heavenly vault, for the vision of the realm beyond the heavens, are borne along in the revolution of the universe, and brought back, by a circular course, to their starting point.81 The fall and reascent of the soul are given their most grandiose

78 Above, ch. II 3. Carcopino, Bas. 272 n. 4, sees here a backsliding into the primitive, by the acusmatici.
79 Above, nn. 3, 37. There may be some significance in the fact that Homer (Od. 12.52, 167) speaks unmistakably of two Sirens.
80 The moon is very often called the Isle of the Blest (references in Capelle 100ff; Cumont, Symb. 177ff; Nock, AJA 1946, 142f; e.g. Castor of Rhodes (1st century B.C., ForHist 250fFed Plut. Quaest. Rom. 76.282a, Plut. De fac. 29, Por. ap. Stob. 1.49.61, Serv. Aen. 6.640, 887). However, the moon is usually merely an intermediate stop, and the sun is the next higher stage (Plut. De fac. 29.944c, Amat. 20.766b, Por. ap. Stob. 1.49.55). Only in Comm. Bern. Luc. 9.9 (Posidonius according to Reinhardt, RE XXII 389f, 780) are sun and moon on an equal basis: the soul returns “in suam sedem, hoc est in solis globum ac lunae.” Reinhardt (Kosmos 312 n. 2) doubted the antiquity of the Pythagorean acusma.
81 It is difficult to be sure about the relation of the acusma to Alcmene, Empedocles, and Philolaus; one gets the impression that they are harking back to still more ancient strata—Alcmene to the divinity of the stars, Empedocles to the noncosmological myth of salvation, Philolaus to shamanistic lore about the inhabited moon.

77 Pl. Phd. 108d.
78 Pl. Rep. 517b. We need not raise the question here of how much myth and ritual may lie at the base of the simile of the Cave. (Empedocles has the word ἄνθρωπον in fr. 120.)
79 On the idea of the two heavenly gates, see Cumont, Symb. 40ff; Meissner 110. On Egyptian details (the sun bark, ships of the dead, the gate between upper and lower world), see Kees 64f, 67ff, 84f.
81 Pl. Phdr. 246b: πάντα οὖν ὀρθώτατα περιεῖρον, 246c: μετακινομένι καὶ πάντα τοῦ κόσμου διόπτερα, 247a: μακρὰ χλωμα καὶ κατὶ ἐκ τοῦ οὐσίου οὐσίας (Σφεδραῖος is the technical term for the periodic celestial movements: Hdt. 2.24, Eur. And. 1086; Hdt. 1. had ξεδραῖος), 247b: ἐκὰ πτωπεια καὶ πάντα τῶν ὀφθαλμῶν ὀφθαλμοῖς πρός ἀνώτατον, 247c: αὐτὸς περιείρον ἡ περιφέρεια, 247d: ἵνα ἔν τε ὀνείρο ἡ περιφέρεια εἰς τούτων περιείρον ἐν ἐν τῇ περιφέρεια...
expression in the *Phaedrus*. Then comes the *Timaeus*: the Demiurge creates as many souls as there are stars, for each star a soul, puts each into its star “as in a wagon,” and thus shows it “the nature of the universe.” Then it must leave its star, to be incarnated on earth; then after a period of trial on earth or another planet it may claim the promised return to its ἀναγεννητόν ἀνήρ.²⁸ Though we hear echoes of folklore motifs,³⁸ at the same time the relation of soul and celestial movement has become very close indeed: the form of the world soul is the moving principle of the cosmos (36b). The same basic ideas are developed in the *Laws*, without any mythical garb and with an earnest claim to embody the truth. The soul, as the principle of self-motion, is primary as against any kind of corporeality; soul reveals itself in the celestial movements, and the scientifically proven regularity and perfection of these circular movements shows that the soul of the universe is intelligent and good.³⁴ Then the *Epinomis* repeats the same line of thought, in a systematizing way: the stars are gods, and astronomy is worship.³⁸

Plato’s students interpreted and systematized their master’s myths, and seem, in this process, to have developed the final form of “astral immortality.” Little is known about Xenocrates in this connection.³⁸ It is a tempting conjecture, though not directly attested, that Crantor, in his influential book *On Grief*, presented the thought in a popular way.³⁷ We can, however, get some impression of the exposition of Heraclides Ponticus, who also worked on special problems of astronomy.³⁸ He related the vision of a certain Empedocles.³⁸ Pluto and Persephone appeared to him, freed him from the usual restrictions of human eyesight, and revealed to him the truth about the nature and fate of the soul. The cosmos, he saw, is divided into three realms. The spheres of the moon and the elements below it belong to Pluto and comprise the heavenly Hades. The Milky Way, conceived, as in Aristotle, as an atmospheric phenomenon below the region of the stars,³⁹ is the pathway for these souls; from there they sink to earth, and later they return to it. The soul is a “light,” ἀιθέριον, οὐράνιον σῶμα. The heavenly Hades is not a place to stay forever; above it are the spheres of the planets, the realm of Poseidon, and highest of all is the heaven of the fixed stars, which belongs to Zeus. There must be a connection between the three realms and the three “gates” which Empedocles sees in the region of the zodiac. One is the entrance Heracles used at his deification.⁴¹ Surely, for every soul the ultimate goal is to follow this road into the company of the gods.

The decisive influence was that of the myths of Plato, Heraclides, and the other Platonists;⁴² and especially important were the concepts of the sublunary Hades³⁹ and the Milky Way as dwellings of the souls.⁴³

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⁸⁰ Boyancé, REG 1952, 335 n. 7. The sun is thought of as being below the planets, not in their midst (above, ch. IV 1, n. 7). The Milky Way was known to the Babylonians as a road between earth and heaven (Meissner 111).

⁸¹ Fr. 94. The gates are in the sign of Scorpio, between Leo and Cancer, and between Aquarius and Pisces. Wehrli thinks (92) that this detail is hardly correct, because “it is hard to imagine the zodiac as a path to the sublunar region.” But the zodiac is not a hollow ring; it pervades all the spheres, and even the moon travels along it. The road toward incarnation passes through Cancer to Leo, according to Macrobius (*Somn. N. N. 1.12.4, 7*), with the sun moving downward, and the first gate is between these two signs. Pisces and Aquarius are the suitable signs for the realm of Poseidon; and the third gate, that of Zeus, lies directly opposite, in the middle of the sign itself—no longer a transitional stage. The journey depicted on the disc of Brindisi (above, n. 71) goes to Scorpio. The division of the circle into portions of 105°, 105°, and 150° (7:7:10) may have seemed especially “harmonic” to Heraclides (7 is “the rational diameter of 5” in Rep. 546c).

⁸² This has also been put forth by Boyancé, REG 1952, 321ff.


⁸⁴ Numerus and Cronius ap. Per. *De anfr. nymph.* 28,Macrobi. *Somn. *1.11.4, Procl. In Remp. *1.120, Schol. *13.103 (the discrepancy between this and Porphyry is probably a mistake of the scholar rather than a reflection of independent tradition, as Delatte assumes, *Litt.* 130). The testimonia are collected by Capelle 19f and Leeman 147f. See also Delatte, *Litt.* 120ff. Where the Zodiac crosses the Milky Way, he says, in the sign of Cancer and Capricorn, there are two gates, one “for men” in the north, through which souls descend for rebirth, and one “for gods” in the south, through which souls return to their place of origin. Below the Milky Way begins the realm of Hades; the exposition is ostensibly an explanation of the Homeric lines about the gate of the sun (*Od. 24.112f*) and about the cave of the Nymphs (*Od. 13.102ff.*). The only part of this that is explicitly attributed to Pythagoras is that the souls which are punished with reincarnation gather in the Milky Way; and this is brought into connection with the fact that the infant’s first food is milk. On the Milky Way as a habitat of souls see also Cic. *Rep.* 6.13, 16; Gundel, *RI VII* 561–566; cf. above, n. 90.
IV. ASTRONOMY AND PYTHAGOREANISM

Probably Heraclides mentioned Pythagoras, too, in his Empedotimus, but a simple inference from the Timaeus may be sufficient cause why the details of astral immortality in later tradition were attributed to Pythagoras. These testimonia do not have pre-Platonic content. Of course the interpretation of the Milky Way could be combined with the naive identification of soul and star, but the Pythagoreans of Aristotle’s accounts had a different explanation of the Milky Way. The idea of two heavenly gates is simpler and therefore earlier than the picture drawn by Heraclides, but they belong primarily to a different realm.

Thus the idea of astral immortality only gradually developed into a system apparently built on a scientific basis. At the inception stands shamanistic “knowledge” of cosmos and soul, in the most ancient stratum of the Pythagorean tradition, the aeusmata. Later on, scientific discoveries became current. The knowledge that the earth is spherical did away with the subterranean Hades—from the time of Empedocles. Recognition of the orderly character of the movements of the planets confirmed—from the time of Eudoxus—the contrast of celestial order and earthly imperfection. This was the path that led to that synthesis of astronomy and religion which we find in the later Plato, in Heraclides, Aristotle, and Xenocrates; we cannot simply call it “Pythagorean.” This doctrine then, taking its departure from Plato and Aristotle, finally became canonical. The agreement of science and religion, emphasized by the Stoics, obviously made a tremendous impression on the Romans. That it was, basically, a hasty oversimplification remained unnoticed until, from the time of the Renaissance, Greek natural science was carried forward by new methods.

V. Pythagorean Musical Theory

1. SPECULATION, EXPERIMENTATION, AND FICTION

It is a striking paradox that music, which is the most spontaneous expression of psychic activity, at the same time admits, or rather even challenges, the most rigorous mathematical analysis. There are two systems by which the division of the tonal continuum may be described, and the distinction between natural and tempered tuning follows from them. One can think of the interval in a spatial metaphor, and equal intervals as representing equal distances or lines; then greater intervals are made up of the sums of smaller ones. The standard unit is the “tone” (or “step”), the difference between a fourth and a fifth; it can be subdivided at will. A fourth, then, comprehends \( 2 \frac{3}{5} \) whole tones, a fifth \( 3 \frac{3}{4} \), and an octave six. The usual divisions of the tetrachord in classical Greek music are, in the diatonic genus or scale, semitone, whole tone, whole tone; in the chromatic, semitone, semitone, tone and a half; in the enharmonic, quarter-tone, quarter-tone, ditone. The image of a line and its divisions is especially natural for us, because of our familiarity with the piano keyboard, and of our system of musical notation; but the Greeks used this image, too, as is shown even by the word they use for “interval,” διαστήμα.

A different system results from the recognition that the harmonic intervals can be expressed as simple numerical ratios. These can easily be illustrated by the length of vibrating strings or sounding pipes. It has been known for a long time that pitch depends on the rate of vibration; and simple, whole-number ratios of frequencies result in the musical concords. The ratio for the octave is \( 2 : 1 \); for the fifth, \( 3 : 2 \); for the fourth, \( 4 : 3 \). Addition of intervals results in multiplication, and subtraction results in division, of the numerical ratios; to halve an interval means the extraction of a square root. In the terminology of modern mathematics, the intervals, thought of as lengths of line,

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86 Julian writes (ap. Suda s.v. ᾿Εμπεδότιμος, Heraclides fr. 92): ἡμεῖς δὴ ᾿Εμπεδότιμῳ καὶ Πλατάνῳ πιστεύουσα σὺς τε ἓκεθεν λαβὼν Ἡρακλείδης Ὀ Πεντεκόσιος ἡμῖ... But Julian is not citing at first hand (Wehrli 31). Wilamowitz says of “Pythagoras” teaching about the Milky Way, “Pythagoras has taken the credit here that belongs to Heraclides” (Gild H 1135 n. 1).

87 Above, n. 48. The conception of the Milky Way as a collection of many single stars does not seem to be documented, however, before Democritus (Ap. Ap).

88 Above, ch. IV 1, n. 115.

89 Above, n. 79. Perhaps the two gates were assigned to the zodiac (Cancer and Capricorn) even before Heraclides, as Wehrli supposes (92); it is (fr. 86 ff) thinks of Eudoxus in this connection.

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1 According to modern usage, the scales are here considered as ascending series. In the following we shall employ the usual transcription, related to the modern system of notation (the absolute pitch being optional). Thus the tetrachords are e f g a (diatonic), e f g flat a (chromatic), and e c' f' (g double-flat) a (enharmonic).
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of imprecision, and were accused, in their turn, of using arbitrary hypotheses and contradicting the clear testimony of the musical ear.

In the one passage where he explicitly names of Πυθαγόρας, Plato credits the Pythagoreans with a mathematical theory of music; this is one of the few fixed points in the reconstruction of Pythagoreanism before Plato. As early as Xenocrates, the crucial discovery was attributed to Pythagoras himself; and though this testimony is treated with great reserve, still it is generally regarded as established that the first natural law to be formulated mathematically—the relation between pitch and the length of a vibrating string—was a discovery of the Pythagorean school. Let us try, however, to answer somewhat more precisely the question, to what extent, and from what time, there was anything in this area truly analogous to modern science. For in this field, as in others, it may be that a different kind of speculation preceded real science.

In the Republic, when he discusses the necessary subjects for the education of the “Guardians,” Plato takes up music after astronomy; they are “sister sciences, as the Pythagoreans say and we agree”: ἀδιάλειπτος τῶν οὐσιών . . . ὥς οἱ τῇ Πυθαγορικῇ φασὶ καὶ άρτι . . . συγχρονοίς (530d). Socrates will follow them, he says, but they do not fulfill adequately the basic requirement, to push forward beyond the sensible world to true Being and thus to really exact knowledge: “they waste their time like the astronomers, measuring audible concords and sounds against one another” (531a).

Glaucun, the interlocutor, follows this up with a depiction of the activities of the musicologists, bending over their strings and arguing whether it is possible to detect still another difference of pitch, or whether the “smallest interval, the unit of measurement” has been reached. These “worth men,” says Socrates mockingly, subject the strings to a painful inquisition, stretching them on the rack, with pegs. But, he continues, these are not the musical theorists he meant, but...
"those of whom we were just saying that we would consult them about harmony"—that is, the Pythagoreans (531b). They, too, however, fall into an error similar to that of the astronomers:

They try to find the numerical properties hidden in these audible consonances, but they do not rise to the level of formulating problems and investigate which numbers are consonant and which not, and why.\footnote{531b. προβλήμαta is a technical term in mathematics; see Oenopides DK 41.12 (τούς δόντας τούς δόντας), Pl. Rep. 530b: προβλήματας...χρυσοίς ὁσπέρ γυμναίραν ὄστα καὶ ἀποθηρευμένοι μέτωπες.}

Thus Plato distinguishes two schools of musical theorists. One group try to find the smallest interval, as a unit of measurement, proceed in a purely empirical way, and rely on their musical sense. They must conceive of the tone continuum as a line, since they are seeking the basic unit, and are therefore the predecessors of Aristoxenus.\footnote{Among non-Pythagorean music theorists we can name the following: Laos of Hermione, the teacher of Pindar, was the first to write περί μουσικῆς (Suda s.v., Mart. Cap. 9.936); see Schmid–Stählin I 1, 544f. Epigonus (Aristox. Harm. 1 p. 3.21, Philochorus PhilHist 328f23 = Ath. 14.631f, Ath. 4.183d, Por. In Prot. 3.4) constructed an instrument with 40 strings (Pollux 4.59). Stratoniceus, according to Phaeas fr. 32 W., was the first to make a διαδρομή (on his chronology, see above, ch. II 5, n. 53). Then there are also Eratocles (Aristox. 1 p. 54f, Por. In Prot. 3.5), Agonos (Aristox. Harm. 2 p. 37, Isoc. Ep. 8.1, Por. In Prot. 3.5), and Pythagoras of Zacyntus (Ath. 14.637bff, Aristox. Harm. 2 p. 36f). In all these the combination of practical and theoretical music is characteristic. On the other hand, Damon (DK 37) is only cited for the ethical and pedagogical value of music. That Aristocles, too, knew two schools of musical theorists is shown by his expressions οἱ κατὰ τοὺς ἀριθμοὺς ἀρμονικοὶ (Top. 107a15) and οἱ ἐν τοῖς μαθηματικῶς ἀρμονικοῖ (Met. 997b21).}

Their method of study is, to Plato, not even worth discussing; but for the Pythagoreans, who are looking for the "numbers" in the musical concords, he has appreciation to mix with his criticism. He will follow them in detail, though they are still too closely wedded to the empirical. What Plato desiderates is not an analysis of audible music but pure number theory, above and beyond experience. In the Timaeus, Plato carried out this program, at least by way of suggestion, using a series of numbers derived from the ultimate principles, which arrayed themselves in a scale without audible sound, the numerically harmonic structural pattern of the world, the "world soul."\footnote{531b. πρόβλημα is a technical term in mathematics; see Oenopides DK 41.12 (τούς δόντας τούς δόντας), Pl. Rep. 530b: προβλήματας...χρυσοίς ὁσπέρ γυμναίραν ὄστα καὶ ἀποθηρευμένοι μέτωπες.}

Succeeding ages regarded the construction of the world soul in the Timaeus as one of the most illustrious examples of Plato's "Pythagorean" wisdom; but his own words, in the Republic, show that he went beyond the teachings of the Pythagoreans in an independent way.

Frank tries to determine the character and chronological position of Pythagorean musical theory from Plato's pronouncement: he "rejects their comparative measurements as the most contemptible empiricism.\footnote{Frank 132, cf. 13; 161, 172.} "What these scholars are striving for is, in a word, natural science and physics on a mathematical basis, quite in our modern sense" (172). It was Plato who first introduced a priori speculations into music theory; and this is another reason, he thinks, to regard the Philolaus fragments as spurious. The time of origin of Pythagorean music theory is "established beyond question" (159) by Plato: Glaucos's misunderstanding showed (Frank thought) that the Pythagorean theory was practically unknown in Athens in Socrates' time, and had therefore only emerged about 400 B.C., in the circle of Archytas. Of course, the numerical ratios that make the basic concordant intervals must have been known, on an empirical basis, for a long time—"every maker of instruments had to know these numerical formulae" (11); but it was only the theory of proportion, on one hand,\footnote{Frank 136, cf. 129, 172; 176.} and on the other the recognition of the nature of sound as air vibrations, that made possible the origin of Pythagorean musical theory.

Van der Waerden paints a different picture, on the basis of a much more even-handed study of the sources. Pythagoras himself, he thinks, may be credited with the arithmetical manipulation of the basic harmonic ratios, which had been known for a long time, and also with the recognition that sound is derived from movements of air.\footnote{Van der Waerden, p. 65, 154, 155 ff.} But even in Archytas, in his view, considerations of number theory, rather than the construction of the world soul in the Timaeus as one of the most illustrious examples of Plato's "Pythagorean" wisdom; but his own words, in the Republic, show that he went beyond the teachings of the Pythagoreans in an independent way.

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Frank 152, cf. 13; 161, 172.

Frank's statement that "the proportion theory of the intervals was first worked out by Eudoxus" (160) is not supported by Theo Sm. 61.11ff (= Archytas A194); Eudoxus made the theory of proportion applicable to irrational relationships (cf., e.g., Becker, MD 101ff; van der Waerden, EW 109ff – SA 187ff) and this has nothing to do with music theory (above, n. 4); the bases of calculation by ratios are much older; cf. ch. VI 1.

Hermes 1943, 179, 192. Van der Waerden rightly emphasizes the age of the acusmata tradition. (Above, ch. II 4, n. 157 on the tetractys.)
theory can be demonstrated with any approach to exactitude, is an artificial device for experimentation, the time of whose invention is controversial.²⁸

An extensive and richly attested tradition makes Pythagoras himself the discoverer of the numerical ratios; but the observations and experiments attributed to him are impossible, physically. One story is that in passing a smithy he recognized, to his surprise, that the sounds made by the hammers exemplified the intervals of fourth, fifth, and octave. He ascertained that the only difference among the hammers was their weight, and found that their weights were related in the ratios 4:3, 3:2, and 2:1. The law presupposed here, that the vibration and sound of a metallic body are directly proportional to their volume and weight, is false.²⁸ The story goes on that Pythagoras hung weights corresponding to these hammers from equally long strings, and found, on plucking them, that the same intervals were produced.

²² The crucial point is the Sectio canonis (Κανονική κανών), transmitted under the name of Euclid (ed. H. Menge). Euclid's authorship is contested (Tannery, MSc III 213ff.; Menge xxxii & 177), and in addition Tannery claimed that the only passage to mention the κανών (props. 19-20) is an addition made in the time of Eratosthenes. In these propositions the diatonic scale is constructed, but in prop. 17 the λοχος is 2 whole tones below the Mese, which presupposes the enharmonic scale. But aside from the fact that 19-20 may have displaced an older, enharmonic sectio canonis, the juxtaposition of enharmonic and diatonic is comprehensible. The enharmonic is basic to musical practice, and hence comes the name λοχος = f, but as the basis for μεθοδολογία one needs the diatonic ὅμοιον σύμπληκτον, which is built up in props. 19-20, and here we do not find the word λοχος but only δόνους = g. That the illustrative figure gives λοχος means nothing, in view of the uncertain transmission of such drawings. Van der Waerden (Hermes 1943, 173ff., 177) therefore puts the invention of the κανών after 300 B.C.; but, since neither Tannery's analysis nor the conclusion drawn from it is certain, we may go back further than that. Aristotle never mentions the κανών; for him music belongs to arithmetic, just as optics does to geometry (Op. Post. 755b14ff., 765a9ff., 22ff., Met. 1078a14ff.), while the κανών presents a combination of music and geometrical line-division. We should probably be justified in taking this as a terminus post quem. H. Koller dates the invention of the κανών in the 5th century (Clelia 18 (1959) 60ff.), but his derivation of logic and epistemology from music theory is pure construction. Duris speaks of the κανών (FGrHist 76F3); see also Philodemus (Mus. p. 100 K.) and Varro (Gell. 16.18.4). Its invention is attributed to Pythagoras by D.L. 8.12, Gaudentius 11 p. 341.12ff, Jan, Both. Mus. 1.11; cf. Aristid. Quint. 3 p. 116 M., Procl. In Tim. II 174.23, Por. In Ptol. 120.12ff. We can disregard here the other experimental devices that Ptolemy describes (Harm. 2.2, 2.12ff., 3.1). See also H. Oppel, KANON (Leipzig, 1937; Philologus Supp. 30.4). Late texts on the "Pythagorean kaiou" have been edited by A. Stammbach, Tres canones harmonici (Dis. Strassburg, 1881).

²³ The physical impossibility of the alleged experiments of Pythagoras was shown by M. Mersema, Questions harmoniques (Paris, 1634) 166 (Schuhl, Essai 262.2; Capparelli II 627). See also van der Waerden, Hermes 1943, 170ff; H. Oppermann, "Eine Pythagoras-legende," Bonn. Jb. 130 (1925) 284-307. For the nomenclature, see Nicon. Hist. 6 p. 245ff.; Iam. V 1338ff., abbreviated 1am. In Nic. 121.13ff., most vividly Macrobius, Somn. 1.21ff., also Gaudenius 11 p. 340 Jan, both. Mus. 1.10; Isid. Et. 3.16.1 (cf. Zeller 1 508.1).
But again, the proposition that the frequency of vibration of a string is proportional to its tension is false. 24

Our oldest attestation for this tradition is that of Nicomachus and Adrastus. It is definitely earlier than Ptolemy, who rejects the weight experiment, since it could only bring διαβολή to the correct theory. 25 Thus the inauthenticity of the experiments had already been recognized and used in polemic against the Pythagorean musical theory. It is unlikely that Adrastus and Ptolemy are dependent on Nicomachus, 26 for the excesses of the Timaeus reflected in Adrastus and Macrobius is based on earlier material. In fact, Xenocrates attributed the discovery of the musical ratios to Pythagoras himself (above, n. 8); and it is as unlikely that he omitted to mention the way the discovery was made as it is that he mentioned the κανών (above, n. 22). Should we suppose that here again we have a falsified tradition about Pythagoras emerging from the Old Academy?

In any case the legend, in spite of its physical impossibilities, does make a certain kind of sense. The mythical inventors of smithcraft, the Idaean Dactyls, were regarded not only as wizards and founders of mystic rites, but also as the inventors of music. They were mentioned along with the Curetes and Corybantes, but also with Orpheus, and even with Pythagoras. 27 Music is found throughout the mystery cults 28 and takes on a special character among the Pythagoreans. Because of this ritual and magical background, we should take seriously Aristoxenus’ reports about the Pythagoreans’ musical καλλαμάρια 29 and Plato’s conception of music belongs in this context. 30 The acousticus 31 which states that the sound of bronze when struck is the

voice of a daimon 32 makes the transition, in the Pythagorean milieu, between music and metal-working. The claim that Pythagoras discovered the basic law of acoustics in a smithy is a rationalization—physically false—of the tradition that Pythagoras knew the secret of magical music which was discovered by the mythical blacksmiths. 33

A report about Hippasus, based on Aristoxenus, is of a different kind: “Hippasus prepared four bronze discs in such a way that their diameters were equal, while the thickness of one was 1/3 that of the second, 1/3 that of the third, and double that of the fourth; when struck, they made concordant intervals.” 34 This experiment is “correct,” in terms of the physical principles involved. With free-swinging circular metal plates of the same diameter, the vibration frequencies are directly proportional to their thickness. 35 Therefore we must regard as authentic the statement that Hippasus knew and studied the numerical ratios of the basic concords. 36

Another report associates Lasus of Hermione with Hippasus:

Λάος δὲ ὁ Ἑρμωνεύς, ὁς φασὶ, καὶ οἱ περὶ τῶν Μεταποτῶν Ἰππασοῦ Πυθαγορικῶν ἀνδρῶν συνεπέσει τῶν κινήσεων τὰ τάχη καὶ τὰ βραδυτήτας, δὲ νῦν αἱ συμφωνίαι . . . εἴν αἴρθων ἀρχύστων λογίων τουσών εὖλαμβανεν ἐπ' ἀγγείους. ἦσαν γὰρ διητῶν καὶ ὁμοίων πάντων τῶν ἀγγείων τὸ μὲν κενὸν ἔσσα, τὸ δὲ ἦσαν ἄγρο διηπότα πληρώσας ἑρόταε κεκατέρω, καὶ αὕτη ἡ διὰ πατῶν ἀπεδείκτη τοίνυν . . .

The text can scarcely be sound, 36 but Lasus seems to be the subject throughout, as well as in the succeeding passage about the subdivision 

24 Nicomachus, Iamblichus, Gaudentius, Macrobius, Boethius; as cited in the preceding note; without the story of the hammers, Adrastus ap. Theo Sm. 57.4 = Chalcid. 45, Cens. 10 (Varro?), Por. In Iol. 119.29ff, cf. Theo Sm. 60.7f; 66.21ff; Aristid. Quint. 3 p. 113 M. For an attempt to explain why such an experiment fails to work, see Ptol. Harm. 1.8 p. 17.7ff. In reality, the pitch or frequency is proportional to the square root of the tension; e.g., 4 times as much weight produces an octave; but this law does not seem to have been discovered in ancient times (Tannery, MSL III 440). The κανών is mentioned last in Porphyry, Gaudentius, and Boethius; and, in Nicom. Ench. 10, it is treated without any mention of the discovery of musical ratios.


26 Adrastus was about contemporary with Nicomachus. He does not tell the impressive story of the smithy.

27 See Kern, REV IV 2018ff; B. Hemberg, Ermos 50 (1952) 41–59; esp. Euphorus FGrHist 70F104 = Diod. 5.64, Plut. Mus. 5, Clem. Al. Strom. 1.73.1, Solinus 11.6. Terpander was regarded as the descendant of the Dactyls, Schol. T II 22.39.1, Pythagoras as an initiate of the Dactyls, Por. VP 17.

28 Boyancé, Muses passtum.

29 Aristox. fr. 26, 121; also Iam. VP 66ff, 110ff, Aristid. Quint. 2 p. 110 M., Schol. T II 22.39.1, Por. VP 30, 32ff, Iam. VP 163ff, 224. See also Zeller I 406f; Boyancé, Muses 91ff, Restagni, StrMtn I 133ff. The music of the monochord accompanied the death of Pythagoras, Aristid. Quint. 2 p. 116 M.

30 Tim. 47d; Boyancé, Muses 173.

31 Above, ch. II 4, n. 34.

32 Burnet, EGP 106ff, said of these stories, “Their absurdity is their chief merit. They are not stories which any Greek mathematician could possibly have invented, but popular tales bearing witness to the existence of a real tradition that Pythagoras was the author of this momentous discovery.” But the content of the tradition changed as it was rationalized.

33 Aristox. fr. 90 = Schol. Pl. Phd. 1038 = DK 18.121; on the text, see above, ch. II 5, n. 71. Also Euseb., Migne 24.268, Zenobius II 91, etc.; cf. O. Crusius, Philologus 52 (1893) 51ff. The ratio of the discs’ thickness was therefore 12:9:8:6. Experiments with discs are mentioned by Theo Sm. 57.7, Nicom. Ench. 6 p. 248f, 15, Por. In Iol. 120.13ff, and (critically) Ptol. Harm. p. 17.18. According to Aristoxenus the musician Glaucon of Rhegium played on the discs of Hippasus. There were also other comparable cymbal-like instruments. A Diocles, who may be the Pythagorean of Philus (Aristox. fr. 10), in the 5th century discovered τὴν τοῦ δεξιοῦ ἀρμονίαν ἐν ὀπίσθωσι ἀγγείοις (Suda s.v. Diokles; cf. Crusius, cited above).

34 Handbuch der Physik 8 (Berlin, 1927) 232; H. Gomperz, PhSt 57f; van der Waerden, RE XXIV 279.

35 It is uncertain how much of what Eubulides reports about Hippasus goes back to Hippasus himself (DK 18.14).

36 Theo Sm. 57.7ff. The source is neither Thrasyllus, who is copied in 47.18–49.5, nor Adrastus, whose report (cf. 49.6, f.90 in 50.4, 50.5–12 = Por. In Iol. 7.24–8.5, 50.22–51.4 = Por. In Pet. 96.2–6) apparently extends at least as far as 57.10 (= Chalcid. 45) and possibly to 59.3, and is then taken up again at 61.17: ἕπαθενθον ἐν ἐν ἕνω...
There is no necessary connection between the discovery of the musical ratios and the knowledge of the nature of sound, of vibration or wave movement of the air. A nearly correct description is found in the *Seccio canons* of Euclid, the Aristotelian *Problemata*, and in the *De audibilius*.\(^{41}\) Sound is a very rapid sequence of πληγαί ἄφες, which are disseminated as a result of the condensation and rarefaction of the air; the idea of the sound wave is attested as early as the Stoa.\(^{42}\) A closer succession of πληγαί makes a higher tone; and the concords are correctly explained by the simple coincidence of πληγαί, where the ratios are simple.\(^{43}\) There was no attempt to measure the frequencies of vibration experimentally.\(^{44}\)

A much less fully developed theory is found in Plato and Aristotle. Here too we find πληγαί and φορά, and rapid movement makes higher pitch; but rapidity of propagation is confused with frequency, so that higher tones are said to come to the hearer sooner than lower ones.\(^{45}\) This conception is found in the Archytas fragments.\(^{46}\) Clearly there

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\(^{41}\) See, in general, Schürer 26-40 (E. Graf, *Die Theorie der Akustik im griechischen Altertum*, Progr. Gumbinnen, 1891, is unsatisfactory). On the authorship of *De audibilius* (possibly Strato), see Zeller, II, 295f.; Düring 1934, 169f.; Wehrli, *Strato* 71f.; *See* *ilium proem.*: πάνες οἱ ἄφθονος γίνονται πληγή τῶν γονήτων... οἱ μὲν πεντάχρονοι (κινήσεις) δέχεται ποιέον τῶν φόρων, ἵνα διὰ τῶν ἀριθμῶν βαθύτερον, (Arist.) *Αναφ. 803b4: τὸν ἄφρον κυκλιότερον... συσταλόχρονος καὶ εὐθείωνος...* 803b5: διὰ γάρ ἐν ἄφροις ἀθν. τῶν ἐκείνων καὶ... *Πρ. 11.6.89b9: οὐ μὲν γὰρ φωνή γίνεται οὐ μὲν ἄφροις οὐδὲν ἄφροις...* Ait. 419a4: ἐσπέει δὲ πληγή (ἐν ἄφροις), κυκλιότερον στὰ κύκλων ὀρθός (the comparison is with a wave in the water), καὶ αὐτὸ μὲν κυκλιότερον κυκλιότερον, δὲ ἀθν. αἰσθησιών.


\(^{43}\) There is no way of verifying Capparelli's idea (II 659) that "the ἀρχότον plastos with a contrivance to measure vibration frequencies (DK 47A10) could have been a kind of ratchet wheel, with a flexible pawl engaging a toothed wheel.

\(^{44}\) *ἔφαρμος* φορά, *Rep. 533d*, Tim. 47f. At Tim. 67f. we find the correlation between faster movement and higher tone, as between slower movement and lower tone. At Tim. 803b there is an attempt to explain why in spite of this we hear concords (the slower tone finally catches up with the faster: καταλιμπάνει). Arist. *Seccio* 4481a11 sets forth it as a problem, λέγοντες... ὡς αὐτὸ μὲν ἀδύνατον ὡς οὐκ. This is cautiously denied *De an. 402a11f* and more determinedly, from the point of view of the audibility of the sound, *Theophr. fr. 88*, Por. *In. Pol. 63.19f.* cf. also Arist. *Πρ. 11.6, 16, 20, 21, 19.37, 39, 42.

\(^{45}\) Archytas and Eudoxus, *Theop. 51.11f* DK 47A12.1 Duhm *Lassen*... τοῖς μὲν πενταχόροις ἄφθονοι ἡλίκιον ἔτει πληγαίων συνεχείς καὶ υδράτων κυκλών τῶν ἄφροις, τὸν δὲ βραχίονα βαρύτερον... *Πρ. 11.39* says of Archytas: μείζονον τῶν Πυθαγορίκων ἑπιμετρών μεταφερεῖ. The calculation of the types of scale made by Archytas (A16) and the proof from the properties of number transmitted by Boethius (A19) may be regarded as authentic; but there are grave doubts about fr. 1 (cf. also *Πρ. 11.3, 14, 20*). Here Archytas first praises his predecessors (like Hippoc...
was no thought of vibration numbers, for Archytas apparently assigned the smaller number to the high tone and the larger to the low.\textsuperscript{47}

A certain Heracleides attributes the most advanced, and relatively most correct theory of acoustics, the idea of tone as a multiplicity of

\textsuperscript{47} V. PYTHAGOREAN MUSICAL THEORY

Vict. 1.1, Hedh. 53); half of the fragment is taken up with their findings, in indirect discourse. Thus Archytas appears in the role of a mere transmitter of Pythagorean wisdom, as in the Ocellus fiction (D.L. 880). The word μαθήματα is used as a technical term, and goes beyond the usage of Plato; τοί περὶ τὰ μαθήματα seem to have fulfilled the demands of the Epinomis. It is odd that alongside the quadrivium—περὶ γεωμετρίας καὶ ἀρίθμων καὶ σφαιρικῶν καὶ ὀψιῶν μαθημάτων—there stands, independently, περὶ τὸ δῆμος τῶν ἀναπόλεον τιμῶν καὶ ἐπιτηδείων καὶ δοσολογίων. What is this if it is not the content of σφαιρικῶν? One sentence, DK 1 432.75, corresponds exactly to Pl. Rep. 350d, the single passage in which Plato cites Pythagoreans by name. But a stroke of luck, if the source of this particular passage has been preserved! But Plato is speaking here (and cf. Crat. 495a) only of the relation of astronomy and music (harmony of the spheres), whereas Archytas obviously is intending to include all the μαθήματα. Especially suspicious is περὶ γὰρ ἀκολούθησα τὰ τῶν οὐσιῶν πρῶτα δύο εἶδα τῶν ἀναπόλεον ἔχει (DK 1 432.81). This apparently means πλῆθος (θνητῶν) καὶ μέγεθος (πτυχλοικόν), exactly corresponding to the scheme from which Nicomachus derives the quadrivium (Ar. 1 12f; διὸ εἰς ἵδη, ibid. p. 4.20).

This kind of diachrony of being is not even attested for the late Plato and Aristotle. Archytas, however, speaks so briefly and allusively that one has to rely on Nicomachus as commentary. On the other hand, not only does the enumeration of σημεία in the second part make a good impression, but precisely the lack of clarity about the concept of speed (rapidity of propagation or frequency of vibration) leads to the thought that Theophrastus's polemic was directed against Archytas (fr. 89). Archytas combines the idea of "high" with that of "strong" tone, which "can be heard further" (πάνωθεν κ' ἀκολούθησα, DK 434.13; cf. 431.13ff, 434.4f, but also Arist. Pr. 19.37, 920b25ff) against which Theophrastus maintains that an equal amount, though a different kind, of force is necessary for a lower tone (Por. In Plol. 63.12f, 63.20: εἰς γὰρ καὶ πολλοτέρως ἀκολούθησα δύο δεύτερων φθόγγον τῶν παραμυθῶν διὰ τὴν τῆς καθαρότητας ἑνήδε τεκνίαν). Still, Theophrastus says of his opponents that they judged music not as a science but were only "ηθικοί λόγοι (Por. In Plol. 62.2f), while the Pythagoreans are too empiricist for Plato (Rep. 316c). Cf. Aristoxenus's comment on his opponents' musical theory: τὸν μὲν αὐθηναίων ἔκκλησιν... καὶ δὲ κατακελεύουσας αὐτάς, Harm. p. 32 M. This makes it seem that Theophras is arguing against Pythagorean musical theory formed from a Platonic point of view, or on the basis of a Platonic treatment. In fact the vagueness of the Timaeus on the concept of velocity exercised a continuing influence in the commentaries on the dialogue; see Adrastus ap. Theo Sm. 50.5ff = Por. In Plol. 7.24ff, and Ael. ap. Por. In Plol. 33.1ff. In any case Frank's statement is wrong, that Archytas "understood the proportions of the intervals... as the relation of the vibration-numbers" (12, cf. 174ff).

\textsuperscript{47} At Archytas Α16 the greater number is associated with the lower tone. To be sure, Ptolemy always proceeds in such a way, in accordance with his canon experiments; he could have converted the table of Archytas to correspond to his own system. In the Timaeus there is no unambiguous indication how numbers and tones are connected (cf. Kytzhler, Hermes 87 [1959] 395ff); thus the problem of the authenticity of Philolaus Α26 is not affected. But, since most experimental observations must lead to the association of low tone and large number, this may—in line with the Archytas passage in Ptolemy—be regarded as original (otherwise, Düring 1934, 162 n. 2, who cites Schönberger; cf. the next note). On the other hand, Arist. fr. 47 (Plut. Mus. 1139b) assigns the number 12 to the Neto, 6 to the Hypate (Pr. 19.23, 910b1; διαλείπει ἡ ἀριθμός ἡ ἀποκρία), and Theophr. fr. 89 says of the higher tone, πλευρωσὶ ἀρμοδίως κατακελεύουσα (Por. In Plol. 62.14ff, 63.19f). Similarly, Pr. 19.35, cf. also Plut. De ai. proc. 1021e; in the contrary sense, Pr. 19.12, 23, 50; detailed treatment of the opposed principles of arrangement, Nicom. Euth. 10, P. 254.

\textsuperscript{48} Thus the whole is given as the reasoning of Pythagoras; Xenocrates is cited at the beginning as a worthy authority, but scarcely more than the one sentence can come from him.\textsuperscript{50} This account has no more

\textsuperscript{48} Ἡρακλείδου ἐν τῇ μουσικῇ ἐσάγαγη, Por. In Plol. 30.2-31.21, 32.23-33.4. Zeller vacillated (I.1 1306 n. 1 for the identification, but 1 309 n., against it), and Heinze (6 n. 2) opposed the identification; but Jan, Musici scriptores graeci 135ff, Schönberger 113ff, and Düring 1934, 154ff argued in detail for it. Jan and Düring see in Heracleides Ponticus the discoverer of tonal vibration, Schönberger (113ff) and van der Waerden (Hermes 1943, 192) that Pythagoras is being cited here.

\textsuperscript{50} So expressly Schönberger 88f.

\textsuperscript{51} Wehrli, Heracleides 113, gives the following arguments for a negative verdict: (1) Contradiction of Heracleides fr. 122. (2) Heracleides Ponticus can hardly be citing Xenocrates, who was more probably younger than he (already in Heinze); Heracleides did not hesitate to attribute important material to Pythagoras, on his own responsibility (cf. Burkert, Hermes 1960, 159ff). (3) “The detailed, systematic structure of the theory” goes even beyond Aristotle. We may add: (4) καταλαμβάνεσθαι, “grasp,” “recognize” (Por. In Plol. 32.34), and δεκαάκρυπτος (31.19), “unknowable,” are technical terms devised by the Stoic Zeno (SVF I 160). (5) A title in the form εἰσαγωγὴ cannot be cited before Chrysippus; and we may suppose that the pretentious Heracleides of Pontus did not busy himself with schoolbooks. But this Heracleides’s εἰσαγωγὴ is pedantic, long-winded, and characterless. Compare the passages (Arist. Aud. 80b32ff, ἐπὶ τὴν ἀκολούθουσαν... τὰ πολλὰ καὶ κακοχρηματεύοντα, διὰ δὲ μικρότητα τοῦ ἑνοετοῦ τῆς ἀκολούθος ἑνιακάποισα τὰ πολλὰ, ἐκεῖνον ἄνω ἀμαρτίαν γίνεται. (33.7) καὶ οὕτως ἂν ἔστησε...

\textsuperscript{53} The φοῖνικ that keeps recurring in the Heracleides fragment is not referring to Pythagoras (as Heinze 7), but to Heracleides; it is a sign that we have an epitome, and serves the function of a new quotation mark. Düring 1934, 155ff, thinks Didymus was the intermediary source.
historical value than the doxographical paragraph, in which Pythagoras is put alongside Plato and Aristotle in the explanation of the nature of sound,34 or the report of Adrastus, who attributes to Pythagoreans the Aristotelian theory of sound, using Aristotelian terms.35

There were theories of sound and acoustics among the pre-Socratics quite outside the Pythagorean ambit. A connection of air movement, resistance, and tone was obvious to anyone who considered the human voice or a wind instrument. Alcmaeon discovered the auditory canal and the cardrum, and spoke of the “echo” in the interior of the ear, though instead of air he spoke of the “void.”36 Empedocles built onto what Alcmaeon had achieved, speaking of the “movement” and the “blows” of the air.37 This has been taken as an indication of the Pythagorean origin of this doctrine,38 but scholars have overlooked the same explanation of sound from movement of air and πλήξις in Anaxagoras,39 and no attention has been paid to a statement about Archelaus, the pupil of Anaxagoras, who was active in Athens about 440 B.C.: πρῶτος δὲ εἶπεν φωνῆς γένεσιν τὴν τοῦ αέρος πλήξιν.40 This report, about one of the less prominent pre-Socratics, is important for that very reason. In his case, the doxography apparently found represented for the first time a formulation that was later considered correct. It follows that Archelaus must have gone beyond Anaxagoras and Empedocles, in a way not made completely clear; but it can hardly mean anything more than that he gave more exact information about the connection of

“blow” and tone, and accordingly the influence of the velocity and strength of the “blow” on the pitch and intensity of the tone.

The acoustical theory found in Archytas, Eudoxus, and Plato is therefore not a Pythagorean theory, but belongs in the general context of Ionian φωνολογία. Aristophanes, too, shows that speculation about the origin of tones was in the air in the fifth century, when in the Clouds he represents Socrates as investigating scientifically the buzzing of a gnat and explaining it, ingeniously, by the principle of the trumpet.41

If neither the recognition of the simplest numerical laws of music nor physical theory on the nature of sound is exclusively Pythagorean, then there only remains, as that which, in Plato’s eyes, distinguished the Pythagoreans, the fundamental emphasis on number as such, which led to development of the mathematical theory of music quite beyond the requirements of actual practice.42 Later presentations of Pythagorean musical theory tried to derive as much as possible from a priori considerations, and to refer as seldom as possible to experience and experiment. Even the basic facts are—apparently—derived from speculation, and everything else is derived from calculation of ratios. The basic principle, at least from the time of the Sectio canmis and the Aristotelian Problematas, was that musical intervals are expressed in the form of “superparticular” or “multiple” proportions.43 The reasons for the preferred position of the superparticular proportions are not immediately obvious. It is based partly on the fact that all these

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34 Αἰτ. 4.20.1: Πεθαγόρας Ὁδότις ἀσώματος (τὴν φωνήν).
35 Αδράστος ἤρ. Θεοκ. Σμ. 50.66 = Πολ. 7.228f: φῶς ὁ δὲ πλῆξις ἀέρος κεκυκλώματι βρισκόμεθα, cf. Αριστ. De an. 419a/b, esp. b10ff, pl. 102a5 ἐγενέτο βραχεία ἡ κεραιά (ὁ ἄρχοντας), ὁ τοῦτον κύκλον φῶς φωνῆς. What is worked out in detail in Aristotle becomes a pithy definition for the “Pythagoreans.”
36 Αλκμαῖος 45: ἀκόσιον ἄθεον... τοῖς ὁσίοις, διότι καθοῦν εἰς ἑαυτὸν ἐνήχοις τόσον γὰρ ἤχους, ἐφ. Αθ. 46.
37 Εἰμπ. ΑΦ. = Theophr. Serm. 9: τὸν ὁ ἀκόσιον ἀπὸ τῶν ἀχών ἐκείσαθα φῶς ὅταν γὰρ ἀκόσιον ἐπὶ τῆς φωνῆς κυνηγῆς ἤχους ἄνωτε... κυνηγομένου δὲ παίσαν τὸν ἀέρα πρὸς τὰ στερεά καὶ ποιοῦν ἤχους.
38 Schönbberger 315f, van der Waerden, Hermes 1943, 192.
39 Αναξαγόρας ΑΙΤ.: τὴν φωνήν γένεσιν πνεύματος ἀντίκειται στερεάμενον ἀέρα, τῇ ὀπτικομαθῇ τῆς πληρότητος μέχρι τῶν ἀκών προσνοεθέντος. Diogenes οἱ Ἀπολλώνιοι also follows Empedocles, ΑΣΤ.: τὴν ἀκοήν γένεσιν τοῦ ἐν τῇ κεραυνοῦ ἄερα ἀπὸ τῆς φωνῆς τυπομείκτου καὶ κυνηγομένων. According to Theophrastus (Serm. 59), the explanation of φωνῆς as κύκλος τοῦ ἀέρος is the answer of “earlier” thinkers generally. He names Empedocles, Anaxagoras, and Democritus. Democritus’ theory of sound was complicated by atomistic ideas (Α135), but in any case, τὴν φωνήν εἶναι πνεύματος τοῦ ἀέρος καὶ μετὰ βίων εἰσόντος (§5).
40 1DK 601a1 = D.L. 2.17.—Theo (59.9) ascribes to Laus and Hippasus the explanation of sound by “rapidity and slowness of movement” (above, n. 36); but this may be the interpretation of the source, as the emphasis is on ἐν ἄρθροις.
41 Ar. Nub. 150ff: κοῖλον πρὸς στενοὶ προσκειόμενον τῶν προκοτῶν ἤχους ἀπὸ βίων τῶν πνεύματος. This carries a verbal reminiscence of Democritus A 135, above, n. 59: στενοὶ—πνεύματος, βίως. Perhaps the ultimate source is Leucippus, from whom Diogenes also borrowed (Theophr. Phys. op. fr. 2).
42 Performing musicians work, according to Pl. Phlb. 56a, ὃς μέγας ἄλλης μελητής στοιχεῶς. Arist. fr. 52 = Lam. Comm. math. sc. p. 80.1ff: ὃς μὲν γὰρ τὰς ἀποδείκτες καὶ τοὺς συναρμολογητές ξειρατιστεί, περὶ συναρμολογήσεως τῶν ἀκών ἂν εἰπὲν ὁ πόθεν τῶν ἀκών τῆς τούτων ἄρθρου, ἃν καὶ τοῖς ἑαυτρίζοντας αὐτῷ ἀκούσται ναμίδησι τῆς μελαφρίας, ὅταν ἀνάπηρον τὰς ἀποδείκτες, ὅπως ἑπιστέειν εἰσὶν αὐτά ἡμείς πρόκειται.
43 Σπηλαίων ὁ λόγος are proportions of the type (n + 1): n; in Greek they are called ἐπιτέρτοι, ἐπιτέρτοντα, and so on. Like the λόγος πολλαπλάσιος (δεκαπλάσιος, τριπλάσιος, etc.), they are therefore “expressed in one word,” εἰς ἄρθροι πρὸς ἄλλης λεγόμενοι, STOR. CAP. p. 160.11ff, and this is regarded as adequate basis for the postulate. Cf. Arist. Pr. 19.34, 41. The arbitrariness of the principle is obvious in the problem of the eleventh (octave: 4, fourth = 8:3), which belongs among the concords but is not recognized as such by the Pythagoreans. This was seen by Ptolemy ( Harm. 1.6), who therefore gave the whole postulate: Actually, the important thing with the concords is small whole numbers rather than a single name: see also Lam. In Nic. 120.18ff, Boeth. Mth. 1.18-20, 5,8-9.
proportions were designated in Greek by a single word,⁶⁴ but at the same
time every ἐνθέματα λόγος represents the connection of an odd
and an even number, and thus exemplifies the harmony of Limit and
Limited. So Pythagorean musical theory is intimately related to
numerical cosmology, and the importance of superparticular propor-
tion comes from its relation to number speculation in general.

The only empirical observations presupposed by the Sectio canonis
(10–12) for the derivation of the basic concords are that an octave
consists of a fourth and a fifth, and that, while a double octave is
consonant, a double fifth or double fourth is not. From these facts are
derived the mathematical proportions 2 : 1, 3 : 2, and 4 : 3. A proposi-
tion first proved by Archytas plays a central part in this,⁶⁵ so that the
whole procedure is credited to him, though Ptolemy, who briefly
recapitulates the proof, speaks of “the Pythagoreans” generally.⁶⁶

In any case Archytas “devoted most attention, among the Pytha-
goreans, to music” (Ptol. Harm. p. 30.9). Frank tried to show that
“there is not a trace, in Archytas, of the kind of a priori numerical
speculation” that was introduced into music theory by Plato (166).
In fact, only seven of the nine calculations of Archytas (A16) show
superparticular ratios; and in one point Winnington-Ingram has
shown an interesting connection between Archytas’ calculation and
musical practice.⁶⁷ Tannery had argued that the postulate of “super-
particular or multiple proportions” was not early Pythagorean,⁶⁸ and
Frank maintained that Archytas’ only concern was “to determine by
exact measurement the string lengths corresponding to the tones in
question.”⁶⁹

It is hard to estimate, at this late date, how close Archytas was to
actual musical practice. That he assigns the same pitch to the Pythpary
in all three genera corresponds to the uniformity of the name, and
comports with the method of notation.⁷⁰ But it contradicts the evidence
of the senses, according to Ptolemy;⁷¹ and Aristoxenus is witness for
Ptolemy against Archytas in this point. Was Archytas depending more
on nomenclature than on the ear? In any case, proportions like
243 : 224, or even 32 : 27 must have been derived by calculation rather
than exact measurement; and he must have based his calculations on
certain postulates. Even the fundamental idea that intervals are expres-
sible in proportions of whole numbers cannot be verified by measure-
ment. And quite exact measurement on the monochord (if Archytas
used this instrument at all; see above, n. 22) is required to decide
whether a double tone is 81 : 64 or 80 : 64 = 5 : 4.

Ptolemy says of Archytas that he begins with the postulate of super-
particular proportions, but then abandons his own principle.⁷² Of
course it would be possible that Ptolemy ascribed to Archytas, wrongly,
the postulate which later came to be regarded as self-evident in
Pythagorean theory; but Archytas’ tenet that the superparticular propor-
tion is indivisible (A19) shows that this proportion played an
essential part in his music theory, for it is only important there. It
means that the octave, fifth, fourth, and whole tone cannot be divided
into equal parts, so that the arithmetic and harmonic mean take the
place of the geometric in music theory.⁷³ And we have evidence that
the problem of means was important to Archytas.⁷⁴ Thus it is not a
coincidence that in Archytas’ table seven of nine proportions are
superparticular. How the exceptions came about remains a question.

Van der Waerden is right, then, in seeing Archytas’ work as

⁶⁴ Above, n. 63; cf. ch. VI 1.
⁶⁵ A19; cf. Sect. can.; cf. ch. VI 2.
⁶⁶ Ptol. Harm. 15, p. 12.8–27, referred to Archytas by van der Waerden, Hermes 1943, 170; Rl. XXIV 279. Ptolemy advertises (pp. 11.20–12.7) a “more speculative” (διαγωνίσιον) derivation of the basic concords. The “best” ratio, 2 : 1, corresponds to the “most beautiful” concord, and the “first two” superparticular proportions correspond to the “first two” concords, the fourth and the fifth. Van der Waerden (Hermes 1943, 168, 169) extracts from this an axiomatic theory which he attributes to the Pythagoreans before Archytas.
⁶⁷ CQ 26 (1923) 195–208; cf. Düring 1934, 251f; van der Waerden, Hermes 1943, 185. The interval Parhypate–Hypate (28 : 27) of Archytas, with the Hyperhypate, which is one whole tone lower (9 : 8), yields the pleasing interval of the diminished third (7 : 6). This very interval, Parhypate–Hyperhypate frequently occurs in the fragment of the music for Euripides’ Orestes; so Archytas apparently took as his point of departure the practical application of enharmonics.
⁶⁸ MS III 76f. He stressed that it was not held to by either Archytas or Plato.
⁶⁹ Frank 266. At p. 157, Frank cites the διαμετροῖς in Rep. 531a; but against this van der Waerden correctly emphasizes (Hermes 1943, 176f) that in Plato’s view διάμετροι and συμφωνίαι are measured against another (ἀλλαγῆς), which sounds more like the calculation of ratios than empirical measurement of the length of strings. One number “measures” the other.

⁷⁰ Düring 1934, 253; Winnington-Ingram, CQ 50 (1956) 179; Vogel I 84. Vogel ascribes to Archytas’ calculations a basic significance in Greek music (47–57, 83–91). But the oldest music theorists, who were only concerned with the enharmonic genus (Aristox. Harm. p. 2 M.), must have been active before the day of Archytas, since he takes all the 3 genera into consideration.
⁷¹ Ptol. Harm. p. 30.13 (cf. 32.1ff): ἀπὸ δὲ σαφῶς τῶν ἀκτυρίων ἡ ηῆς αὐτῶν συστάθη τὸ … χρωματικὸν πατρίκορθος.
⁷² Ptol. Harm. p. 32.1ff (cf. 30.13ff): παρὰ μὲν δὲ τῆς πρόθεσις … αὐτὸς συστάθη τὸ … χρωματικὸν πατρίκορθος.
⁷³ Düring the octave according to the arithmetical and harmonic means gives the famous series 6 8 9 12 (ε ὡς ε’; cf. Arist. fr. 47), the ρυθμολογία διανομή which van der Waerden (Hermes 118.19ff tells us was brought by Pythagoras from Babylon.
⁷⁴ On the “harmonic mean” see Hippasus and Archytas, OK 18.15, Philolaos A24, and below, ch. VI 2. Archytas’ fr. 2 offers no impediment. For the application of the means in music, see also Tim. 36a, Eup. 190e, which points back to Archytas again. For the attempt to derive the table of Archytas (A16) from the doctrine of means, see van der Waerden, Hermes 1943, 184ff (following Tannery, MS III 105); Vogel I 52–53.
V. PYTHAGOREAN MUSICAL THEORY

2. NUMBER SYMBOLISM AND CALCULATION OF PROPORTIONS IN PHILOLAUS

"Entaché d’absurdités mathématiques et d’erreurs pratiquement énormes"—this is the judgment of such an expert as Tannery1 on the musical system transmitted to us under the name of Philolaus. Nevertheless, Tannery, whose thesis was later developed more fully by Frank,2 relied mainly on a different argument to prove the spuriousness of the fragments in question. This was, that the calculation of a diatonic scale to which Plato refers in the Timaeus was Plato’s own work, not something borrowed from earlier Pythagorean calculations.3 From this it followed, in his view, that since the Philolaus fragments,

3 Van der Waerden, Hermes 1943, 166ff.
4 Cf. below, ch. VI 2.
5 Archytas A17 = Por. In Pol. 107.15ff, the intermediate source is Didymus. Ptolemy ( Harm. 1.6, p. 14.1ff) argues against this method, without naming names. The idea of ἔνατον and δύομα reminds of the ἕκαστον καὶ ἀκούσμενος of Pl. Rep. 546b (the "nuptial number"), where the concept of πιθανόν also occurs.
6 MTr III 223.
7 Frank 263–275.
8 Tannery concludes, from the contradiction between Plato and Archytas, that there had not been in existence any earlier Pythagorean calculations, especially since Archytas (fr. 1) says he is following—not correcting—his predecessors (above, ch. V 1, n. 46). Frank develops the idea of the contradiction between Pythagorean "empiricism" and Platonic "apriorism" (above, ch. V 1, n. 13).

unlike Archytas, agree with the Timaeus, they must be a post-Platonic forgery.4

The scale of the Timaeus, which has only equal whole-tone intervals in the ratio 9 : 8, and therefore pure fourths and fifths, but no thirds,5 is in Frank’s opinion a "purely speculative fragment," which has "scarcely anything in common with the scales of real Greek music."6 Ptolemy is differently minded. He introduces the διάδοτον διατόμενον explicitly in relation to practicing musicians, who find it especially "useful": it is quite common in the tuning of the lyre.7 Actually such tuning is most easily accomplished by the use of fourths and fifths.8 Ptolemy adds that musicians, without regard to accurate theory, speak of the "half-tone" between Hypate and Parhypate;9 and this corresponds exactly to the conception of Aristoxenus, who certainly did not take his description of the diatonic genus from the Timaeus. In fact the word "diatonic," διάδοτον, means precisely that this scale is constructed διὰ τῶν, tone by tone; the alphabetic series of note designations, too, follows the diatonic series. The conception that the diatonic tetrachord consists of two equal tones plus a "remainder" is therefore the earliest, rooted in the practice of musical performers.10 Any

4 The following is a table, for ready comparison, of the various calculations of the diatonic tetrachord:

<table>
<thead>
<tr>
<th>Mese (a)</th>
<th>Philolaus A26, fr. 6</th>
<th>Archytas A16</th>
<th>Tim. 36a–b</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt; 9 : 8</td>
<td>9 : 8</td>
<td>9 : 8</td>
</tr>
<tr>
<td>Lichanos (g)</td>
<td>&gt; 9 : 8</td>
<td>8 : 7</td>
<td>9 : 8</td>
</tr>
<tr>
<td>Parhypate (f)</td>
<td>&gt; 256 : 243</td>
<td>28 : 27</td>
<td>256 : 243</td>
</tr>
</tbody>
</table>

Hypate (e)

5 The scale of the Timaeus passed, by way of Boethius, into the musical theory of the Middle Ages. Archytas knows the major third 5 : 4 in the enharmonic genus (but not in the diatonic). This is called by Frank "the decisive step in acoustics, upon which the entire physical analysis of the scale depends to the present day" (167). The arrangement of our scale is 9 : 8, 10 : 9, 16 : 15, and here (9 : 8) × (10 : 9) = (5 : 4); but, since Greek music does not use triads, the third scarcely plays any part in it.
6 Frank 13; cf. 181ff, 268.
8 "Our piano tuners still follow this procedure," says van der Waerden (Hermes 1943, 190), rightly referring to Aristox. Harm. p. 55 M.; also see Sect. can. 17.
10 Van der Waerden, Hermes 1943, 190.3, refers also to Aristox. Quint. p. 22 M. This passage was traced to Damon by R. Schäfer, Aristides Quintilianus von der Musik (Berlin, 1937); but the attribution proves to be untenable (R. Wagner, Arch. f. Musikforsch. 4 [1939] 316ff.—Ptol. Harm. p. 39.18ff emphasizes that the διάδοτον διατόμενον was not significantly different from his own values for the diatonic genus (9 : 8, 10 : 9, 16 : 15); on the other hand, Frank maintains (166) that the ditone (9 : 8) × (9 : 8) gives "a dissonance quite intolerable to our ear." It seems to me more likely that Ptolemy tested this out than that Frank did; or are we to suppose that Ptolemy was tone-deaf?
calculation taking its departure from here must necessarily lead to the figures found in the Timaeus, and only the search for "superparticular" ratios, along perhaps with keener observation, could discover a difference between the two "whole" tones. This in itself is enough to destroy the basis for Frank’s rejection of these fragments.

We can get further by consideration of the inconsistency in the system of Archytas which Ptolemy mentions. According to Archytas, the pattern of the chromatic genus is,

Mese (a)  
\[ > 32 : 27 \]

Lichanos (g flat)  
\[ > 243 : 224 \]

Parhypate (f)  
\[ > 28 : 27 \]

Hypate (e)

In the effort to explain these remarkable figures, scholars have pointed out that the two lower intervals together make a whole tone, or that the interval Lichanos–Paramese is a pure fourth. But the rationale offered by Ptolemy has not been much noticed: “Archytas obtains the second tone in the chromatic genus (g flat) … with the help of the tone that occupies the same position in the diatonic genus (g); for, he says, the second highest tone in the chromatic genus stands in the ratio of 236 : 243 to the corresponding tone in the diatonic genus.” This explanation is so odd that we can not attribute it to an intermediary source or to Ptolemy himself, even aside from the plain ἀρχάρα (Archáras). Who could have got the idea, instead of using the obvious relationships pointed out by modern scholars, of introducing a calculation so complicated and based upon a different genus, the diatonic? We have no alternative but to recognize the derivation Ptolemy gives as that of Archytas. He found the highest interval in the chromatic tetrachord not by harmonic division and not by reference to the natural concords, but by the extrinsic addition of two previously known values, that of the diatonic whole tone (9 : 8) and the ratio 236 : 243—the “remainder” when two whole tones are sub-

tracted from a fourth. Thus Archytas is presupposing two things: a music theory which builds its scale by the addition and subtraction of intervals, and a calculation of the diatonic scale by the numerical values found in the Timaeus.

This music theory may be identified with the one ironically rejected by Plato, which sought to identify the smallest interval, as a standard of measurement; and we may conclude from Plato, as Frank does, that this theory was better known than the Pythagorean. The Aristoxenian conception of the tonal continuum is in any case primary; both the language of professional musicians and the beginnings of musical theory are couched in its terms. According to this approach, the diatonic tetrachord has the structure tone, tone, semitone; the chromatic, tone and a half, semitone, semitone; and the enharmonic, ditone, diesis, disess.

But beyond this, Archytas presupposes a calculation of the scale by ratios—a diatonic tetrachord with the intervals 9 : 8, 9 : 8, and 256 : 243. Since his own picture of the diatonic tetrachord is different from this, it can hardly be his own invention; but, since musical theory based on numerical ratio is a hallmark of Pythagoreanism, Archytas must have borrowed the value 256 : 243 from a Pythagorean predecessor. This much we can infer from Archytas quite without reference to the Philolaus problem; but since the ratio 256 : 243 does appear among the Philolaus testimonia, Archytas must be regarded as the most important witness to the authenticity of the Philolaus fragments dealing with music.

This is really the point where difficult problems begin. First of all, the remarkable terminology of the directly quoted fragment (fr. 6)

18 Van der Waerden, Hermes 1943, 184f.
commentaries that the interval 230 : 243, usually called λέιμα, had been called διευθεία by the Pythagoreans. These are, then, ancient technical terms of professional musicians whose use by Pythagoreans is early attested. Still, the question might be raised, whether Theophrastus was thinking of Philolaus’ book, or some other work, in which case some later forger made sophisticated use of the lexicographical glosses.

The situation of Philolaus’ Trite is more complicated, and indeed leads to problems of the origin of the Greek musical system which are mainly unsolved, and, given the present state of the source material, insoluble. Boeckh saw here a point “so deeply recondite that it cannot be attributed to any ordinary composer of pseudopigrapha” (70), while Tannery thought, “Il est à craindre que l’auteur n’ait trop arachié”. and Frank speaks of a “commonplace which can be found in almost any popular writer on music of late antiquity” (275). The problem is that of the seven-stringed lyre.

According to the ordinary system of Greek music, the basic tetrachord (c–a) is continued by another, either synemmenon (a–d) or diezeugmenon (b–c). In one case the Trite is b flat, in the other c, but in Philolaus it is b, a whole tone from the Mese and a fourth away from the Nete. It is no wonder that, according to Nicomachus, many accused Philolaus of an error. This very fact, however, makes it improbable that some forger has injected an artificial archaisms; the purpose of an artificial patina is to arouse confidence, not mistrust. So, before deciding that this is a mistake stemming from sheer stupidity, we should try to interpret the name in a way that makes sense.

It is certain that the lyre, for a long time, had seven strings, and that the number of strings was gradually increased in the fifth and fourth centuries B.C., but next to nothing is known about how these seven strings were tuned. From the time of the Aristotelian Problematia the theory is attested that the seven-stringed lyre embodied the synemmenon system, and that that of the diezeugmenon was introduced later; Nicomachus attributes this step to Pythagoras. To this extent we can

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39 Tannery, MSc: III 240.
37 Nicom. Euch. p. 233.12: οδί ο ο (i.e., the explanation of Nicomachus, below, n. 30) μυ συμπεινέται αυτός ο ο ο δυνατόν ο νυθρίον λόγον είναι τρίν κατά υγίης.
36 Cf. above, ch. IV 4, p. 6.
35 It is presupposed in the way the question is put at Ar. Pr. 19.7, 47 that the Nete was missing in the ancient heptachord (on the text, see below, n. 10): cf. also section 25. Nicom. Euch. 3 p. 241: 5 p. 244.14ff, cf. 7 p. 249.20ff, 11 p. 257.17ff, Boeth. Mis. 1.20.
34 The “ancient” heptachord synemmenon (e f g a b flat c’ d) is clearly described in
accede to Frank’s use of the word “commonplace”; but this does not explain Philolaus’ Trite. In addition, the whole theory seems suspicious. It is hardly credible that the most impressive of the musical intervals, the octave, was not used in the early systems. The expression διὰ πάσων says in itself that “all the strings” together make an octave, and not a seventh. An eight-stringed lyre is scarcely mentioned; it follows from this that the seven-stringed lyre comprehended an octave, and this means that one note of the scale was missing.

Nicomachus makes a twofold, and very tortuous, explanation of Philolaus’ enigmatic Trite.30

By Trite he means here the Paramese in the heptachord, before the insertion of the dividing tone in the octachord. (1) This [i.e. the Trite = Paramese] was separated from the Paramete (d) by an undivided interval of one and a half tones.31 The inserted string took away a whole tone from this, and the remaining semitone was left in the diezeugmenon between the Paramese and the Trite. Understandably, then, the old Trite was separated by a fourth from the Nete (b-c’), an interval which now is closed by the Paramese (b) instead of the other . . . (2) Others say, quite persuasively, that the inserted tone was not between Mese and Trite but between Trite and Paramete, and that it was now called Trite, while the old Trite became Paramese in the diezeugmenon.

In both the interpretations that Nicomachus gives, the scale presupposed is obviously e f g a b d’ e’.32 The only difficulty rises from Nicomachus, and the transition to the octachord (e f g a b c’ d’ e’) is described in the words ἔναν τοῖς διόρθωσαν ὁμός τῶν δύο ἑξών μεταξὺ μὴν τὰ παραμένον, p. 244.22f; μεταξὺ μέσης καὶ τρίτης, p. 253.15; τῶν δύον δύον . . . μεταξὺ μήσης καὶ τῆς ἀρχαίας τρίτης παραμένον, p. 257.18f. It is not specifically mentioned that the whole upper tetrachord is thus changed in pitch by one tone. The names of the notes remain the same, except for the differentiation of Trite and Paramese; perhaps Nicomachus’ ideas come more from calculation than from hearing. Another interpretation is proposed by J. Chailley, “L’hexatone grec d’après Nicomacque,” REG 69 (1950) 73–100. He postulates an original defective heptachord e f g a c’ (Paramese = Trite) d’ e’, (pp. 73ff), and finds this in Nicomachus (77ff). He thinks Nicomachus kept jumping back and forth between a (never described) defective heptachord and the diezeugmenon heptachord which he described in detail: “In order to preserve the octave and so that the Mese might not be distant by a fourth from the two ends,” Pythagoras inserted the new note between Mese and Paramese.

30 Nicom. Ench. 9 p. 253.4ff.
31 The MSS have ἡμιτόνον. Meibom emended to τριτομέτων. This expansion is necessary, because it is essential to what follows (one tone is taken away and a half tone remains over), and also to the epithet διαδώκειν, which is incomprehensible if applied to a semitone (especially since only the diatonic is in question here). Vogel (II 771.) rejects the correction, and ignores the succeeding context.
32 Cf. Jan, Musici scriptores graeci 81 n.

Nicomachus’ dependence on the “insertion of the dividing tone” as a transition from the synhemmenon to the diezeugmenon, which can contribute nothing but confusion, in this context.33 The clue to the correct interpretation lies in recognizing the existence of an earlier, defective scale with only seven notes but the range of an octave, in which one note was missing in the upper tetrachord, the later Trite, c’. Now we have, in fact, further traces of such a scale. The question is posed in the Aristotelian Problematen, referring to the synhemmenon: “Why did the ancients, in their seven-stringed scales (ἀρχαίας) retain the Hypate, and not the Nete?”34 But the answer offers the consideration that both notes, Hypate and Nete, were present, and that it was rather the Trite which was omitted, taking us back to the scale e f g a b d’ e’.35 The same interpretation is to be given the statement that Terpander “removed the Trite and added the Nete”;36 and the στοιχεῖα τῶν τρίσος of Olympus, in which the Trite is supposed to have been lacking, belongs in this context.37

The history of Greek scales is to a great extent still obscure, and in

33 The first explanation is very puzzling. Nicomachus, as the second explanation shows, is thinking of an interpolation between Mese and Paramese (= Trite), which comports with his own system (above, n. 29); but it is hard to see how he could say that the insertion of an interval could “take away” anything from the interval Paramese-Paramete.
34 (Arist.) Pr. 19.7; the names of the notes are given as in the σύστημα τέλειον. Nete (= c’) is lacking in the synhemmenon system.
35 Jan, Musici scriptores graeci 81 n., and Chailley (n. 29 above) 96 believe that the synhemmenon Trite (b flat) was named as lacking, so that the series was e f g a c’ d’ e’; but since the Nete named surely belongs to the diezeugmenon system, we are dealing, in the question as well as in the answer (ἀρχαίας γράμματα καταλήψις, τῆς δὲ τρίτης ἡμιτόνον), with diezeugmenon notes, i.e., the Trite diezeugmenon (c’) is missing.
36 (Arist.) Pr. 19.32: κεκεφισθεὶς γὰρ τὸν θρόνον, τὴν νῆσον προσεφέρετε. The synhemmenon system is again taken as primary (to ἁρχαῖα), but the name Nete is taken from the diezeugmenon system, and therefore also the Trite; once more, c’ is omitted, and the series is e f g a b d’ e’. Platarch (Mus. 28.1140f) also speaks of the invention of the “‘Doric Nete’ (c’)” by Terpander.—Another definitive system is presupposed at Pr. 19.47, in which were lacking “what is now called the Paramese and the whole-tone interval,” that is, b. Chailley (loc. cit. above, n. 35) puts this down as evidence for his scale e f g a c’ d’ e’, but apparently without reading the following context: θιάσθαι δὲ μέσης τὴν ἑσκετήσιν ἡμιτόνον τοῦ ἄρχοντος, i.e. next above the Mese (a) came a πυκνόν, certainly not a τριτομέτων διαδώκειν as in Chailley’s theory. The initial sentence, at 92b25, ὅπως Ἰωάννης ἢ τὸ τῆς ἔρχεται . . . ἡμιτόνον, is obviously corrupt; it could be corrected with Gevaert to ὅπως τῆς νῆσος . . . (i.e. perhaps e f g flat a b flat c’ flat e’), or to ὅπως τῆς νῆσος μὲν . . . (i.e. the synhemmenon system = e f g a b d’ c’), with Jan.
37 Plat. Mns. 19, looking back to 11 (= Aristox. fr. 83). The reconstruction of the στοιχεῖα τῶν τρίσος is especially complicated, since there seems to be a tone assumed as standing between c’ and d’. Cf. Tannery, MS 3 III 290f; R. Winnington-Ingram, “The Spondean Scale,” CQ 22 (1928) 83–91 (he accepts e f g a b c’, as also for Philolaus, maintaining that e f g a b d’ e’ is later reconstruction intended to explain Philolaus; p. 87). See also Vogel II 9–38.
V. PYTHAGOREAN MUSICAL THEORY

We still have to consider the “mathematical absurdities” reported by Boethius. This seems on the face of it a very late source; but since Boethius is virtually translating Nicomachus, and since the latter quotes directly Philolaus’ fragment 2 and a part of fragment 6, one must suppose that the long word-for-word quotation, fragment 6, and the details reported by Boethius come from the same line of tradition and stand or fall together. We then have that of the finer division of the tone. Philolaus divides the whole tone into two unequal parts, dēsis, 256:243, and apotome; the difference between the two is called komma. According to this, the apotome would be 2187:2048, and the komma 531441:524288—pure frivolity. Philolaus’ treatment is different. He establishes as the basis of tone the number which first makes the cube of the first odd number and was highly honored among the Pythagoreans [i.e., 27]... a number which is separated by a whole tone from 24. From this, then, Philolaus makes two parts, one, which is larger than half, and which he calls apotome, and another, which is smaller than half, which he calls dēsis; later it was called “the smaller semitone.” The difference between these parts he calls komma. And, first, he thinks that the dēsis consists of 13 units, because this is the difference between 256 and 243, and because this same number, i.e., 13, is made up of a 9, a 3, and a 1. Of these the one assumes its station as the point, the three that of the first odd-number line, and the nine that of the first odd square number. Thus when he has for these reasons established 13 as the dēsis, the so-called semitone, he concludes that the remainder of 27, which consists of 14 units, is the apotome. But since the difference between 13 and 14 is 1, he decides that 1 is to be established as the komma. But he makes the whole tone consist of 27 units because 27 is the difference between 216 and 243, which are distant from each other by a whole tone.

Philolaus finds a special meaning in the numbers 1, 3, 9, 13, and 27, to the detriment of their mathematical sense. Intervals are actually determined by numerical ratios, and here a single number is meaningless; but Philolaus is deciding on the inner meaning of single numbers. He seems to have calculated a diatonic tetrachord correctly, working from the ratios 4:3 for the fourth and 9:8 for the whole tone to reach the numbers 192 (Mese), 216 (Lichanos), 243 (Parhypate), and 256 (Hypate). But then, for him, the dēsis (Parhypate–Hypate)

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88 It is certain that the player on the lyre (kithara) did not have a special string for each tone he could play. (The principal evidence is that of Pherecrates ap. Plut. Mor. 1114d; cf. Düring, Branos 43 [1945] 176–197; Winnington-Ingram, Lustrum 3 [1958] 266, cites further literature.) It is not clear, however, how they produced the alteration of tone; the kithara has no fingerboard. Taking account of special characteristics of the Greek musical notation, C. Sachs, and, following him, O. Gombosi, Tonarten und Stimmungen der antiken Musik (Copenhagen, 1939), have tried to reconstruct the string arrangement of the ancient lyre. According to them the 5-stringed lyre (common in artistic representations till the 5th century B.C.) had the strings a e b d’ e’; here, according to Gombosi (42.8), is where Philolaus fits into the picture. The 7-stringed lyre had, according to Gombosi (43.8) e g a b d’ e’; or rather, including the Hyperhypate, d e g a b d’ e’ (Düring, Branos 1945, 1926). But see Winnington-Ingram, CQ 50 (1956) 169–168, and Lustrum 3 (1958) 15. The epigram on Ion (fr. 6 Diehl, cf. above, n. 28) speaks of intervals of a fourth on the 7-stringed lyre.

89 Ancient theory, from the time of Aristoxenus, denied that there had been any such development; “the ancestors,” they said, had known all the notes, and only refrained, on aesthetic grounds, from using some of them (Plut. Mor. 139, harping back to 11 = Aristox. fr. 83: o παλαιοι οι δε’ αγγειαν απειχουσα τη τριτη... ἀλλα δήλων δε’ τ’ τον ἁθων κάλλος...).

90 Above, ch. III 2.
"consists of" the number 13, and the whole tone of 27, because these are the differences between the numbers occurring in the respective ratios; and the fact that a whole tone may also be expressed by the ratio 27:24 (= 9:8) seems to him a confirmation—in spite of the totally different role played by 27 in this case. Philolaus is not thinking of the mathematical uses of numbers, but they are to him entities in themselves, and he combines them by addition and subtraction, where the proper procedure is the multiplication and division of ratios. Of course this leads him out of the realm of commonly understandable numerical relationships. He seems to assume that, because 256:243 is a diatessaron and 243:216 a whole tone, the diatessaron also extends from 243 to 230 or from 229 to 216, and the apotome from 243 to 229 or 230 to 216, as though 256:243:243:230:229:216 were the same ratio!

Sawney thought, quite understandably, that one could scarcely ascribe such mathematical absurdities even to a composer of pseudographa. According to Frank (271) this kind of thing cannot have been written "any mathematician, and least of all by a Pythagorean; their principal achievement was in the discovery of the mathematical principles which Philolaus so disgracefully betrays." All the same, Frank does not hesitate to attribute all this to Suseipus, so that one cannot help wondering how the latter could gain admission to the Academy so áγεωμέτρητος.

Boeckh's expedient, to suggest that Boethius was responsible for the errors, would not work; of all extant authors, Boethius has the most detailed—and correct—exposition of the apotome and komma. Nor can the error be removed by the correction of one at another of the numbers, though this be madness, yet there is method in it. The basic flaw is that again and again difference takes the place of

propotion; in place of the calculation of proportions, the idea of addable and subtractable lines takes the center of attention. This impression is strengthened also by the report that Philolaus used the expression ὑποστήριγμα with relation to all the intervals.

Similar numerical games are to be found in the discussions of the Timaeus by Plutarch and Adrastus. The number 13 is the essence of the lemma, only the number 27 is the tone; and here Plutarch cites the Pythagoreans as his authority. There is doubtless a close connection between this and the fragment of Philolaus, and the reliability of Boethius is confirmed; the only question is, whether the commentators on the Timaeus had an authentic book by Philolaus or whether the Philolaus book was written, pseudepigraphously, on the basis of the Timaeus commentaries.

But Philolaus goes even further, as Boethius elsewhere reports. After the diatessaron is defined as the measure by which a fourth exceeds two whole tones, and the komma as the excess of the whole tone over the sum of two diatessarons, comes the statement, "sich auf dem dimidium commissi, diachisma vero dimidium diatessaros." The

43 Tannery, M. S. III 232: "... qu'on peut à peine attribuer même à un faussaire."
46 Boeckh 79f. He refers to Proclus, who names Philolaus in the context of the scale of the Timaeus. Procl. In Tim. II 180f proves that the lemma is not a full semitone and therefore calculates the apotome—with the correct figures; above, n. 41.—In the calculation of the scale of the Timeus Proclus finds 34 ἀριθμός, and Timaeus Locius 36, since he inserts two apotomes. Proclus rejects this, but carries through the calculation, concluding, ἄριστα ἄριστα μὲν τῶν ἐκ τῶν Φιλολαύν οὖν ἐν τῆς ἑπτάδος τῶν παρά τοῖς Τιμαίων γραφήσεως ἄριθμος (II 190.8). It was not Proclus himself, then, but earlier commentators on the Timaeus, who inserted apotomes with citation of the authority of Philolaus. It cannot be determined whether they found them in the correct calculation, or perhaps in the concept of apotome (the difference of whole tone and diatessaron). In any case, Proclus did not find his calculation in Philolaus; he had worked out correctly in advance.
47 Diels' sentence, DK I 405 n., "The apotome has 17½ units, not 14" (based on a misunderstanding of Boeckh, p. 79), gives an "edemation" worthy of Philolaus. But, aside from that, fractions are not allowed in Greek calculation of proportions (Procl. In Tim. II 184.6 excuses himself for taking this liberty).
shock of this is less in the hairsplitting procedure of dividing even the komma once more than in the nonchalance with which this process of bisection is introduced. If the komma can be bisected, surely a whole tone may; but it is a basic tenet of Pythagorean musical theory that neither the whole tone, the octave, nor in general any of the "super-particular ratios" can be divided into two equal parts.\textsuperscript{54} To be sure, Philolaus realized that the diesis 256 : 243 is smaller than a semitone; but when the calculation of proportions is abandoned, bisection comes in by the back door. In Tannery's opinion, to allow this is to disqualify oneself as a Pythagorean.\textsuperscript{55}

Once more, however, there is a significance underlying the error. The apotome is necessary for the construction of the chromatic tetrachord; in this case the series semitone, semitone, tone and a half is defined more precisely as diesis, apotome, diesis plus a whole tone; and this formulation is obviously presupposed by Archytas.\textsuperscript{56} The bisection of the diesis, however, has to do with the enharmonic genus, whose structure, rather than quarter-tone, quarter-tone, ditone become diachisma, diachisma, ditone.\textsuperscript{57} This makes it seem that Philolaus dealt with all three genera, taking the usual, "Aristoxenian" conception as his point of departure. It is to some extent restated in terms of proportion, and correspondingly corrected, but this is not carried through, and the numbers are treated as addable magnitudes. The idea of lines and distances replaces the theory of proportion, just as number symbolism smoothes mathematics.

The coincidences with commentaries on the Timaeus, which were to be found in the faulty determination of the diesis (above, n. 50), do not extend to the further subdivision of the diesis and komma. There was no necessity, from the point of view of the Timaeus, to go into the chromatic and enharmonic genera; but the Timaeus commentaries never fail to state that the whole tone cannot be bisected—\textsuperscript{88}—not even by subterfuge. Thus we are precluded from deriving the Philolaus testomina from the Timaeus tradition; and Philolaus stands quite alone in the later tradition with his statements about the schisma and diachisma. But Archytas was the first to give a proof of the basic tenet of music theory which Philolaus violates. If we consider this in connection with the fact that Archytas presupposes the calculation of the diesis at 256 : 243, and the structure of the chromatic tetrachord in the manner of Philolaus, it follows that the material introduced by Boethius must have belonged to Pythagorean musicology before Archytas. The very inadequacy of the mathematics of Philolaus' system, which takes no account of the accomplishments of Archytas, becomes a proof of its authenticity. Thus the results of our examination of the philosophical fragments and the astronomical system are corroborated in a third area: there have been preserved, from a book of the Pythagorean Philolaus, written toward the end of the fifth century B.C., some authentic fragments, partly in the original wording and partly as reported by others.

If these Philolaus testomina are genuine, the conception of Pythagoreanism which Frank and Tannery take as their point of departure will at any rate have to be corrected. If a Pythagorean of the fifth century B.C. could fall into mathematical inconsistencies, and even commit gross errors, then the nature of pre-Platonic, or rather, perhaps, pre-Archytan Pythagoreanism cannot have lain in exact mathematics or in natural science, but in the interpretation of the world with the help of numbers thought of as symbols. In fact the practical significance of Pythagorean musical theory is minimal. Only the basic facts can be established by observation, and it was impossible to transpose the values established by calculation back into audible tones. From this point of view it is not surprising that Philolaus' results for the apotome and komma were mistaken. The attraction and the significance of this theory lie not in the theory itself but in the orderly, rational pattern that it reveals. Order and pattern, however, which the human spirit craves, are to be found not only in the form of conceptual rigor and neatly logical structure, but, at an earlier level, in richness of mutual allusiveness and interconnection, where things fit together "symbolically." Thus the interrelation of number and music can be conceived, earlier than any mathematically oriented natural science and quite apart from it, as an aspect of the universal orderliness of the cosmos. In the Chinese culture, where the recognition of basic musical relationships is developed into an ingenious and intricately varied numerical
structure, mathematical precision is purposely avoided.\textsuperscript{59} Not from Philolaus alone does it become clear that the important thing in Pythagorean musical theory was not the function of the proportion but the meaningful numbers. Van der Waerden draws attention to the tetractys, which has its roots in the ancient stratum of the acusmata tradition.\textsuperscript{60} The “Fourness” which is the “harmony” in which the Sirens sing, suggests the numbers 1, 2, 3, 4, which group themselves into the fundamental concords 2 : 1, 3 : 2, and 4 : 3, and thus comprehend the orderliness not only of music but of the universe; and the sum of these four numbers is 10, the “perfect” number. The tradition of the acusmata is independent of Philolaus,\textsuperscript{61} and leads back, past him, to the oldest stratum of Pythagoreanism; and the idea of the music of the cosmos is also of great antiquity. According to a report of Eudemos, the Pythagoreans emphasized that the fourth, the fifth, and the octave are comprised in the number 9, because 2 + 3 + 4 = 9; and here, too, it is clearly number as such, not proportion, that is the significant thing.\textsuperscript{62} Not only Hippasus but Archytas as well classified the intervals by use of individual numbers.\textsuperscript{63} The earliest Pythagorean musical theory is not founded on mathematics or on experimental physics, but on “reverence” for certain numbers in their roles in music and cosmology; and this situation is never completely abandoned. On this basis, according to our information, Hippasus made certain experiments, and Philolaus, in his effort to express Pythagorean lore in the form of Ionian σωματολογία, made individual statements about the numerical structure of ordinary music, showing a truly remarkable mixture of calculation and numerical symbolism, in which its “sense” is more important than its accuracy. Only in Archytas does a real mathematical number theory grow out of this, and, in its application to music, a certain sort of natural science—though this is nearly incapable of further development or progress, since its effort is to discern unalterable order in what already exists.

\textsuperscript{59} Cf. below, ch. VI 4.
\textsuperscript{60} Hurnes 1941, 178f.
\textsuperscript{61} On astral immortality, above, ch. IV 4.
\textsuperscript{62} Eudemos fr. 142 = DK 58B18.
\textsuperscript{63} Hippasus DK 18.14 (but see ch. V 1, n. 35), Archytas A17 (ch. V 1, n. 77).

1. DID THE PYTHAGOREANS LAY THE FOUNDATIONS OF GREEK MATHEMATICS?

As pre-Greek mathematics has been rediscovered in Egyptian papyri and Babylonian clay tablets, a clearer light has been thrown on the outstanding achievement of the Greeks in the development of pure mathematics. The Babylonians, in particular, had made considerable progress in the accumulation of detailed knowledge, in practical calculation, and in the solution of even rather complicated problems in arithmetic; beyond question, the Greeks had much to learn from them. But it was always single problems they were concerned with, making use of certain “recipes,” without any theoretical explanation or even an attempt at proof; “we cannot even be certain that the Babylonians formulated theorems in general terms.”\textsuperscript{1} Some of the “recipes” or formulas are inexact,\textsuperscript{2} but this did not matter as long as they provided a practically useful approximation. Only with the advent of Greek geometry do we find the demand for generalized and stringent proof, for a deductive system based on axioms and postulates.\textsuperscript{3} This is the system presented to us in the Elements of Euclid, a model which until the nineteenth century seemed not to require any essential improvement. All later achievements, including those of the Indians and the Arabs,\textsuperscript{4} build on the foundations laid by the Greeks.

The very importance and influence of Euclid, however, makes the study of early Greek mathematics more difficult. What is preserved for us is, as so often happens, the final accomplishment, and by its very greatness it obstructs our vision of the earlier development. It

\textsuperscript{1} Becker, Grdl. 22; cf. MD 11: “Sammlung von Rezepten”; Neugebauer, ExSc 48: “Babylonian mathematics never transgressed the threshold of prescientific thought”; 146: “Greek mathematics of the Euclidean style is a strictly Greek development.”
\textsuperscript{2} For example, the Babylonian formulas for the volume of a frustum of cone and pyramid are wrong (though the former is still used today by woodmen for calculating the volume of a tree trunk): van der Waerden, SA 75f.
\textsuperscript{3} On this see the important paper of Kurt von Fritz, ABG 1955.
\textsuperscript{4} Neugebauer, ExSc 166ff.
VI. PYTHAGOREAN NUMBER THEORY

The two small, extant astronomical writings of Autolycus of Pitane (ed. J. Mogenet, Louvain 1950; see also Neugebauer, *Eck Sc 257* f) are somewhat earlier than Euclid.


Jr. 133-142, along with the fragment preserved in Arabic, below, ch. VI 2, n. 8.

Fr. 133 = Procl. In *Eucl. 64, 64-64.4*. The most important indication of its derivation from Eudemus is Proclus' remark (68.4), oí mén òn òs òstóris òn anagraphtes méthre toú tòv òttos proódoûs tòv òs òstóris tòv òttos telexnos (i.e., until the time of Plato and Philippus of Opus); Eudemus wrote before Euclid.

Above, ch. IV 1, n. 77.

Eudemus fr. 140; edited with commentary by E. Rudio, *Der Bericht des Simplicius über die Quadratein des Antihon und des Hippokrates* (Leipzig, 1907). On the importance of Hippocrates, von Fritz, *Philologus* 1912, 47ff. Hippocrates knew, for example, the Pythagorean theorems in the generalized form applying to scalene triangles. There are 5 lunes that can be squared, and Hippocrates squared 3 of them (Heath, *Math. I 1960*).

Aristotle accuses him of a fallacy (Phys. 185a16, Soph. el. 171b12), which consisted, according to Simplicius (Phys. 67.4ff, 69a2ff), in supposing that he had squared "every" lune because he had succeeded in doing so in the cases in which the outer arc is greater than, equal to, or less than a semicircle, and that, further, he had hereby also solved the problem of squaring the circle, because in another special case he could square the sum of a lune and a circle. Historians of mathematics hesitate to attribute such a fallacy to Hippocrates, in view of the sophistication of his mathematical procedure (Heath, *Math. I 1959*, 196 n. 1; von Fritz, *Philologus* 1932, 40ff; *ABG* 1955, 91 n. 174; Wehrli, *Eudemus 117*). The goal of Hippocrates, however, was surely the squaring of the circle. A difficult question, hardly ever asked, is how much Eudemus is modernizing in his report and making Hippocrates' proofs seem more sophisticated than they were (as a modern historian of mathematics unhesitatingly uses algebraic notation even in discussing the ancient texts).

10 See Burkert, *Philologus* 1959, 193ff; below, n. 98.
Hippocrates, and is dated, at least by implication, in the first half of the fifth century B.C.

To be sure, the general belief in “Pythagorean mathematics” has not gone without criticism. Tannery more than once expressed himself skeptically on the matter, and the mathematicians Junge and Vogt subjected the “geometry of Pythagoras” to a critical examination which had the notable result of redating the discovery of incommensurability to the end of the fifth century. Though this chronological revision was disputed, the attribution of this discovery to Pythagoras himself, made by Proclus, has not since been seriously defended. Eva Sachs’s book had as its objective to “get rid of the myth of Pythagoras the mathematician” (p. VI); and she was able to show, with relation to the special problem of the regular polyhedra, how the tradition of Pythagoras’ treatment of it had been derived from the Timeus, obscuring the contribution of Theaetetus. This result has been widely accepted, though not enough attention has been paid to the more general implications, and in particular those resulting from analysis of the “catalogue of geometers.” Somewhat later Heidel made a fresh start on the whole question of the Pythagoreans and Greek mathematics and worked out in a more adequate way the development of non-Pythagorean, Ionian mathematics.

No other branch of history offers such temptations to conjectural reconstruction as does the history of mathematics. In mathematics every detail has its fixed and unalterable place in a nexus of relations, so that it is often possible, on the basis of a brief and casual remark, to reconstruct a complicated theory. It is not surprising, then, that the gap in the history of mathematics which was opened up by critical study of the evidence about Pythagoras has been filled by a whole succession of conjectural supplements. They have been based mainly on the analysis of Euclid’s Elements, regarded increasingly as a collection rather than an original work, and, secondly, on conclusions drawn from the argumentation of the Eleatics, especially Zeno, in which scholars have found a specifically mathematical sense. Tannery was a pioneer in both these lines of research, and the most influential reconstructions have been those of Becker and van der Waerden. Becker’s results, especially, have come to be looked upon as a fixed point in the history of Pythagorean science.

We have already mentioned the general problems involved in this kind of reconstruction, in the history of science. The fact of a logical inference can never determine precisely the time or place of the inference, or the person who made it; no matter how exactly it may be determined what propositions Hippocrates of Chios assumes as proven, this will not increase the likelihood that Pythagoras or certain Pythagoreans discovered and proved these propositions. This is the point at which the critical analysis of historical evidence must supplant logical inference. Another consideration cuts even deeper: a mathematically impeccable reconstruction is valid only to the extent that it can be known with certainty that mathematical logic was at work in the original formulation. In dealing with the beginnings of mathematics, when mathematical logic was first being developed, it cannot be regarded as certain a priori that a mathematician respected the presuppositions and drew the conclusions that mathematical logic would have dictated. Of course logic has been inherent in human thinking from time immemorial, and perhaps especially in Greek thinking. But abstract mathematics, making use of proof, was an invention of the Greeks, and not part of their original mental equipment. The manner of its development cannot be determined by logical inference, since this presupposes the decisive point. In the history of science the old maxim is still valid, that we must not ascribe to thinkers, especially those of early times, “either the principles of their consequences or the consequences of their principles.”

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17 HSCh 381ff, MS II 206ff.
20 The standard example is the reconstruction of the planetary system of Eudoxus (above, ch. IV 2, n. 2).
21 For the argument based on the Eleatics, see above, ch. III 3. In his evaluation of “Pythagorean arithmetic,” Tannery renounced his earlier skepticism when he became aware of Archytas A 19 (1905: MS III 244ff), and decided for “l’existence, dès le temps d’Archytas, d’Élémentsarithmétiques développés sous la forme que nous noms nous euclidienne” (249).
22 QS T 3; cf. below, VI 2, n. 46ff.
24 Thus Reidemeister (16ff) pushes all the evidence to one side and finds the true “pythagoräische (sic) Arithmitik” in Becker’s reconstruction.
25 Above, ch. IV 1, n. 14-16.
26 “Ni les principes de leurs conséquences ni les conséquences de leurs principes,” as formulated in the 18th century by Charles Batteux (Mondolfo, Inf. 306), and cited repeatedly by Tannery.
What is the origin of the firmly rooted conviction that Pythagoreanism was the source of Greek mathematics? This question is easy to answer: it came from the educational tradition. Everyone comes upon the name of Pythagoras for the first time in school mathematics; and this has been true from the earliest stages of the Western cultural tradition. None of the ancient textbooks which formed the basis of the medieval curriculum forgets Pythagoras. He is the companion of Arithmetica in Martianus Capella; and according to Isidore he was the first, among the Greeks, to sketch out the doctrine of number, which was then set forth in detail by Nicomachus. This takes us back to the origin of this tradition; Nicomachus, who is himself called a Pythagorean, begins his Arithmetica, which was much used as a school-book, with praise of the Master. Boethius' Arithmetica, drawn largely from Nicomachus, also names Pythagoras in its first line. Likewise, Gerbert of Aurillac mentions the name of Pythagoras several times in his geometry, and among the patrons of their Ars geometriae medieval Freemasons include Pythagoras. The Ars geometriae bearing the name of Boethius, though obviously not composed before the High Middle Ages, even presents an early version of the Arabic numerals as an invention of the "Pythagorici," and describes the method of calculating with these apices on an abacus, called mensa Pythagorea—perhaps the most striking of the anachronisms in which the Pythagorean tradition is so rich. Finally, the early modern period derived the astronomy of Copernicus and Galileo from Pythagoras.

The general belief in the Pythagorean origin of mathematics thus stems from the Neoplatonic and neo-Pythagorean scholastic tradition of late antiquity. In evaluating this it is worth bearing in mind that according to an earnestly meant statement of Iamblichus, even the problem of squaring the circle was solved by Pythagoreans. The earlier tradition is much more sparse, but it does take us back to the threshold of the Helleneistic period. It is repeatedly stated that geometry was imported from Egypt; Herodotus speaks of the Nile floods and the continuing necessity to resurvey the land, and Aristotle of the leisure (σεληνίου) of the Egyptian priests, which enabled them to engage in speculation (θεωρία). Now, as early as Hecataeus of Abdera, Pythagoras is represented as bringing to Greece from Egypt, along with the doctrine of metempsychosis, τὰ κατὰ γεωμετρίαν ψευδήματα καὶ τὰ περὶ τῶν ἀρίθμων; and not much later Antichides the historian of Alexander wrote that Pythagoras brought geometry to perfection, after it had been invented by the Egyptian king Moeris. Hermes of Colophon regards Pythagoras as primarily the corypheus of geometrical astronomy, and Callimachus alludes, though in an ambiguous way, to Pythagoras' discoveries in geometry. To be sure, the notable development of Egyptian mathematics is legendary, and it is remarkable that so remote a writer as Antichides should be concerned to testify to the mathematical achievements of Pythagoras.

27 Mart. Cpr. 7.729, 8.803.
28 Isid. Et. 3.2.
29 Heath, Math. I 666.
30 M. Ghika, Le nombre d'or, rites et rithymes pythagoriciens dans le développement de la civilisation occidentale I (Paris, 1931) 66.
31 Boeth. Geom. p. 396f Friedlein. The numerals in question are the so-called Gobar numerals (1 to 9, but no zero). See K. Menninger, Zahlwort und Ziffer II (Göttingen, 1958) 231ff and 132ff. The problem of the Boethian geometry, and especially the mensa Pythagorea, has not been completely solved; see M. Maniti, Lat. Lit. d. Mittelalters I (Munich, 1911) 371; Heiberg 49; Schanz-Hosius, Röm. Lit.-Gesch. IV 2 (Munich, 1920) 153f. The literature is cited by Capparelli (I.5ff), who may well be the only one to believe that Pythagoras himself invented the Arabic numerals. See M. Folkerts, "Boethius," Geometrie II: Ein mathematisches Lehrbuch des Mittelalters (Wiesbaden, 1969).
32 Above, ch. IV 3, n. 2.
33 Heath, Math. I 121f.
34 Arist. Met. 911b23.
36 D.L. 8.11 = FGHist 140F1 (Müller, FGHist I 212 had mistakenly taken this clause with the preceding citation of Timaeus, FGHist 566F17); καὶ γεωμετρίας ἐπὶ πέρας ἀγάλματα (οἱ Πεθαγόρας). Μοιρώθη κὼς εὐρέως τὰς Δρυς τῶν στοιχείων αὐτῆς ὡς θητήν Ἀιτηθῆσαι ἐν δεσμῷ περὶ Αἴαλκυθοῦ. The whole sentence must come from Antichides, for Diogenes Laertius has no other reason to cite him except for his mention of Pythagoras.
37 Hermesianax fr. 2.83ff Diehl:

... Πεθαγόρας ἐλέους κομψὰ γεωμετρίας
δύον καὶ κύκλων, δουν περίβλεπτην αὐτήν,
βασανίν καὶ ἀφαίρετον ἀποκαλύφθων.

(So Powell: ἀντισώζομεν MSS, ἀποκαλύφθων Hemsterhuis, Diehl, ἀποκαλύπτων Kaibel.)
38 Below, n. 106.
39 Modern scholarship rates the Babylonian influence on Greek mathematics much higher (see Heath, Math. I 122ff; Becker, MD q; van der Waerden, Sa 13f, 35f). On the other hand, the Babylonian tradition is scarcely mentioned in the Greek literature (aside from Iam. In NIC 118.23; below, ch. VI 2, n. 89). But we learn that Pythagoras learned τὰ περὶ ἀρίθμων τὰ καὶ λογισμῶν from the Phoenixians (Por. VP 6, Iam. VP 158).
Neither Herodotus nor Isocrates sees any occasion to mention Pythagoras in connection with Egyptian geometry, though they do so in speaking of Egyptian burial rites and ἀγαπεῖα. Was there a significant change in the image of Pythagoras between Isocrates and the epoch of Hecataeus of Abdera and Anticleides—that is, in the period of the Old Academy? The way the tradition about Pythagoras expanded can be seen in a sentence of Aetius, appended to his report of Hipparchus’ theory of vision: “Some also credit Pythagoras with a share in this doctrine, as being the chief authority in mathematics (βεβαιωμέν οί μαθηματικοί).” Here tradition is not being transmitted, but manufactured, on the basis of the dogma that it was Pythagoras who established the mathematical sciences.

Modern scholarship bases its judgment about Pythagoras the mathematician on certain other pieces of evidence that appear to be more reliable. Tannery’s point of departure was the crucial question of when “Pythagorean geometry” was committed to writing. In Iamblichus we read, ἐκαλεῖτο δὲ ἡ γεωμετρία πρὸς Πυθαγόρου ἱστορία. and Tannery translated this, “geometry was called ‘the tradition according to Pythagoras.’” In consideration of the context in Iamblichus, he interpreted this to mean that before Hippocrates of Chios there was published a treatise on geometry with the title The Tradition according to Pythagoras. This Pythagorean textbook was the cornerstone of Tannery’s reconstruction, and it has continued to play a part right down to the present day. It owes its existence, however, to an obvious mistake in translation; it is impossible to take πρὸς Πυθαγόρου ἱστορία together, and the meaning must be “geometry was called ἱστορία by Pythagoras.” Thus the topic under discussion is Pythagoras’ use of words, and not anything about a book. Hölk long ago found the surprising explanation of this report. Heraclitus wrote, “Pythagoras son of Mnesarchus practiced inquiry (ἱστορία) most of all men” (fr. 129). This sentence is given by Diogenes Laertius as proof that Pythagoras left writings, and thus it has been a link in the Pythagorean tradition. It was only natural for a later Pythagorean to draw the conclusion: Pythagoras was neither a historian nor a geographer, but βεβαιωτέων τῶν μαθημάτων; therefore Heraclitus must mean γεωμετρία. Thus the whole sentence is an erroneous philological inference from a sentence of Heraclitus. Even if this explanation were not allowed as more than a possibility, there remains no firm basis for the belief that Pythagoras was a geometer, and in any case no attestation of his having written anything.

The chief testimony for Pythagoras as a mathematician, always cited in the literature, is in the “catalogue of geometers” given by Proclus, whose principal source is rightly thought to be Eudemus. “Pythagoras turned its (geometry’s) philosophy into a form of liberal education, seeking its first principles (ἀρχές) from a higher source (ἀνωθεν) and hunting out its laws by a nonmaterialistic and intellectual procedure (ἀνάλογος καὶ νοερός) . . .” The weight of this pronouncement is enhanced by the prestige of Eudemus as a pupil of Aristotle, as well as by the undeniable fact that the special character of Greek mathematics consists precisely in its theoretical structure, as distinguished from the oriental “recipes.” To be sure, the passage that follows, ascribing to Pythagoras the discovery of irrationality and of the “cosmic bodies,” is less often accepted; but even in the sentence quoted there are suspicious features. Does the phrase ἀνάλογος καὶ νοερός seem more like the phrase of an early Peripatetic, or like a favorite theme of all Neoplatonists, and especially Proclus? And does Aristotle not say expressly, of the Pythagoreans, “they apply their propositions to bodies”—bringing out the distinction, in this regard, between them and all genuine

43 Cf. above, ch. II 3.
42 Astr. 4:13.10 (cf. above, ch. I 2, n. 76).
41 Iamb. VP 89 = Comm. math. sc. p. 78.5; on the connection between the two parallel versions, see above, ch. II 5. The sentence cited here is obviously an interpolation in the Aristotelian context; cf. below ch. VI 3, n. 59.
45 Reyi 217f; Michel 81, 174f; van der Waerden, SA 116f; Szabó, Maia 10 (1987) 106ff. Heath had the correct translation (Math. I 160), as did Frenkian, Maia 11 (1959) 243-245.
46 Hölk 81f. Thus it is not so that “it is utterly inconceivable how . . . anyone could have thought of inventing anything of the sort” (von Fritz, SBM 1960, 20).—Pr.-Hippoc. Ep. 22 uses the phrases ἱστορία γεωμετρία (1) and ἡ ὣς γεωμετρία ἱστορία (2), along with the word ἀρίθμησις; thus he knows the connection of γεωμετρία and ἱστορία, and uses it as an element of ionic coloring.—Von Fritz (SBM 1960, 20) interprets the word ἱστορία as reflecting the fact that Pythagoras collected specific pieces of mathematical knowledge from oriental sources; and Frenkian’s interpretation is similar.
47 E.g. Heath, Aristarchus 46, Math. I 141; Rey, 216f; Michel passim, esp. 160ff; Morrison, CQ 1956, 153; Becker, Grd. 22, MD 12. A cautious attitude is expressed by von der Waerden (SA 90f; but cf. 100), by Wehrli (Eudemus 114; but at 135 the sentence is interpolated as deriving from Eudemus), by Reidemeister 18f, Heidel (AJP 1940, 16f), and von Fritz (RE XXIV 198—no mention is made that the sentence comes from Iamblichus). Timpanaro Cardini (I 30 n.) cites Sachs (whom Frank follows, 163 n. 209), but without reason given concludes, “ritengo che sia da sostenere la derivazione da Eudemo”; Iamb. Comm. math. sc. is ignored.
48 Procl. In Eucl. 65.16 = Eudemus fr. 133 (cf. above, n. 8) = DK 14.6a.
49 Above, nn. 18-19.
50 ἀνάλογος is attested once by Aristotle, νοερός twice, but neither as adverb. Cf. Procl. In Eucl. 9.13, 63-3.137.27, et saep.
Platonists? And does not Eudemus, as far as we know from other fragments, always speak of Pythagoreans, never of Pythagoras, just as Aristotle himself does in philosophical or scientific contexts? A lucky coincidence turns suspicion into certainty: the sentence in question is taken word for word from Iamblichus' *De communi mathematica scientia*—a work that Proclus copies sometimes by the page in his commentary on Euclid. Iamblichus is concerned with "Pythagorean mathematics." He mentions (ch. 21) that the origins of mathematics lie before Pythagoras, in the work of Egyptians, Assyrians, and Chaldeans. The distinctive aspect of Pythagoras' work is not only in new discoveries, but above all in the "purity, subtlety, and exactitude" of his method, and in the way it purifies the soul and leads on to the highest principles and a realm of pure, immaterial Being. Iamblichus admits that Pythagoras and his pupils did not write any of this down, and therefore it is necessary to reconstruct, with considerable effort, "what they would probably have said if one of them could have taught his doctrine publicly." But Iamblichus has no doubts about his Neoplatonic theme: "If we are to pursue mathematics in the Pythagorean manner, we must follow its upward path, full of divinity, which brings purification and perfection." So, too, in the *Ancient Wisdom"* (τὴν πέρι τὰ μαθήματα φιλοσοφίαν εἰς σχῆμα παιδείας ελευθερίων μετέταξα, καὶ τῷ τε πλήθει τῶν δεινωμένων προῆγεν αὐτά καὶ τῷ τῶν ἀποδεξίων ἀκριβείας, τῇ τε ἀναγκαίᾳ χρήσει πρὸς τὸν βίον πεπροτότων αὐτά ἦκησαν, ἐπείδη σώματα κατακαθιεῖν. This is the beginning of a new chapter, in which the significance of mathematics is discussed, both for practical life and in itself. This may be based in part on Aristotle, but, if anything in it is original with Iamblichus, it is the chapter division and the transitional formulae. In the succeeding passage there is nothing about Pythagoras, neither about the Pythagoreans, except Iamblichus' introduction and his concluding sentence, "It was natural, then, that for all these reasons the Pythagoreans honored the study of mathematics."

Therefore the often cited sentence about Pythagoras in the "catalogue of geometers" is not from Eudemus, but is a formulation of Iamblichus, as had been recognized over sixty years ago. Thus its "authority" is precisely reversed; if, in a context whose significant parts are obviously derived from Eudemus, the passage dealing specifically with Pythagoras has been supplemented with material from Iamblichus, this is an indication that there had been a gap to fill, and that Eudemus did not give enough information about Pythagoras or even none at all. As far as concerns the specific discoveries attributed to Pythagoras, the "cosmic bodies" are regarded by Proclus as the apex, or the quintessence, of all geometry, and the discovery of irrationality had, long before Proclus, been interpreted in a Platonic sense and bound up with the thrilling story about mathematical secrecy and its betrayal and the ensuing divine punishment. Proclus has merely attributed to

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84 Arist. Met. 1031a18; on the other hand, Plato Rep. 521d, Plut. Quest. conv. 8.2.1. For the "Pythagorean" saying σχῆμα καὶ βάση, see Procl. In. Eucl. 84.15ff., lam. in the text above, and cf. above, ch. 12. Plato's direct criticism of the Pythagoreans is also to be considered; above, ch. V 1.

85 Eudemus fr. 60, 88, 136, 147, 142, 146. For Aristotle, ch. 1. 2.

86 See the references in N. Festa's edition of *Comm. math. sc.* (Teubner 1891).

87 *Comm. math. sc.* p. 66-67: ἐκεί δὲ τῆς Πυθαγορακίας μὲν μαθηματικῆς προφορομένων ἀναγκαίαμεν...

88 *Comm. math. sc.* p. 67.6f. (cf. the table of contents 6.21f.): τὴν... ἰδιότητα αὐτοῦ τῆς μαθηματικῆς...

89 *Comm. math. sc.* p. 67.22f.: καὶ μὴ ἀποδείξεως γε καθαρότητι λεπτότητι τε καὶ ἀκριβεία...

90 *Comm. math. sc.* p. 68.1ff.: ἐπὶ τούδε τις ποιεῖν ἀπὸ συμπλήρωμα ἀρχήμενος αἰσθημάτων ὁμοιοποιύντος τίτι καὶ τοῦπα καὶ συνακέρσεις εἰς ὁράμα τε αὐτὰ ἀνάγκης τῆς προφορομένης καὶ τᾶς παραδείγματι ἀναπληρωμῆς στοιχείων τὰ κατὰ τὸ δυνατό τῆς ἑκείνων γνώμης, τὰ δὲ ἐπίσπευδα ἀπὸ αὐτῶν διδάσκασιν...

91 *Comm. math. sc.* p. 23. p. 70.1ff.: ἐλευθερία παιδείας points to the succeeding context (p. 70.15ff.), προῆγεν καὶ ἀκριβεία to the preceding context (p. 67.3ff.). Thus it is out of the question that Iamblichus just happened to quote Eudemus in the transitional sentence.


93 Deubner showed that the division into chapters and the prefixed summaries were the work of Iamblichus himself (SBBl 1935, 680f). Merlan (PINeoP 142) is inclined to attribute the mention of Pythagoras to Aristotle, though he admits (126) the likelihood of some reworking by Iamblichus, precisely at the beginning and end; Festugière (140; see preceding note) also expresses doubts. In the parallel passage to which Merlan refers, however (Arist. Protre. fr. 11 W. = Iamb. Protre. p. 51 ff), the mention of Pythagoras is another addition by Iamblichus (cf. Burkert, *Hermes* 1960, 160ff).

94 *Comm. math. sc.* p. 73.17: the passage between (pp. 70.7-73.17) is mostly put in the first plural.

95 Vogel, *Bibl. math.* 1908-1909, 31f; Sachs 30ff (1917). A refutation of this has never been attempted and could scarcely succeed; the general denial by Friedländer (Platon I Berlin, 1928) 180 n. 3) was omitted from the second edition (F 1954) 331 n. 15; Eng. ed. 353 n. 15.

96 Procl. In. Eucl. 70.24: πέρι τῶν κοιμημάτων σχημάτων ἐν τῷ σώματι τῆς γεωμετρῆς λόγοι... τελευτῶν... ἐν τῷ ποιλήμα της τούτων σωσάσεως (cf. 65.20: τῶν κοιμημάτων σχημάτων σώσασα)—The terminology is therefore that of Proclus. Naturally the latter was affected in this judgment by the *Timaeus*, which for him shared with the Chaldean Oracles the honor of being the most important book in the world (Marinus V. Procl. 38).

97 Cf. below, ch. VI 3. There is no occasion to reject the reading ἄλογα in favor of the weakly attested å̄lò̄gα, as von Fritz does (MarkAnt 1945, 244 n. 14). (Friedländer refers not to any manuscript but to "αλυα," named by E. E. August in his edition of Euclid (Berlin, 1824).)
Pythagoras the two most famous and, from the Platonic point of view, most significant achievements of geometry, supplying an illustration to accompany the generally phrased sentence from Iamblichus. Nothing is left of the supposed testimony of Eudemus to the achievement of Pythagoras in the foundation of mathematics.

Another testimony, supposedly coming from Aristotle himself, is cited, scarcely less often, as evidence for Pythagoras as a mathematician. It is given as follows in the collections of Aristotle’s fragments: “After these men, chronologically, Pythagoras, the son of Mnesarchus, first worked at mathematics and numbers, but at some later period he also indulged in miracle-mongering like that of Pherecydes.”64 This comes from the Historiae mirabiles of Apollonius, who has just finished dealing, successively, with the miracles performed by Epi- menides, Aristeas, Hermotimus, Abaris, and Pherecydes. For Pherecydes, Theopompus is his source.66 The legendary material which follows, about Pythagoras, is certainly derived from Aristotle; but naturally the transitional sentence between Pherecydes and Pythagoras, which also separates the two sources Theopompus and Aristotle, is supplied by the compiler, either Apollonius or his source Bolus. It corresponds to the Hellenistic conception of Pythagoras as bēbaivōi τῶν μαθημάτων, but does not give us any information about Aristotle’s view of the matter; in his extant works he never connects Pythagoras with μαθηματικα. It would be superfluous to set this out in such detail, if it were not that the supposed fragment of Aristotle has been cited, even in very recent studies, as a basis for the assessment of Pythagoras’ contribution.67

The decisive passage is the introduction to Aristotle’s chapter on the Pythagoreans in the Metaphysics: “Contemporaneously with these philosophers (Leucippus and Democritus) and before them, the so-called Pythagoreans were the first to take up mathematics; they advanced this study, and having been brought up in it they thought its principles were the principles of all things.”68 Taken in isolation, this passage could mean that the foundations of mathematics were laid by the Pythagoreans, if they were the first to “take up and advance” it. Still there is a chronological problem: What about Thales? Aristotle is speaking of “Pythagoreans,” and he dates them in relation to the atomists.69 Thales, however, who is, in Aristotle’s view, the founder of philosophy (Met. 983b20), was regarded as the patron saint of mathematics even in the fifth century,70 and Eudemus ascribes to him some special discoveries.

In order to understand why Aristotle passed over Thales in the passage cited, one must consider the function of the passage in its context. The objective of the first book of the Metaphysics is to demonstrate that there are four first principles. The discoveries of earlier thinkers, in spite of their many imperfections in detail, are all seen as moving, in the developing history of thought, in the direction of this conclusion. Whoever was “first” to bring forth a new view is carefully recorded. The “first” of the Pythagoreans is to be seen from this angle: they “first took up” mathematics; this means that they were first to see the relevance of the “principles of mathematics” to the general question of “principles.” The question is not who invented mathematics, but who connected mathematics with philosophy. From this point of view, there was no occasion to name Thales; his mathematics had no connection with his doctrine about water.

Inextricably entwined with this, in the passage of Aristotle, is the psychological question of how the Pythagoreans came to their system, which seemed as odd to him as it did to others.72 His answer is that they devoted themselves so intensively to mathematics and became so closely identified with it that they saw nothing in the whole

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64 Arist. fr. 191 = Ap. Hm. 6 = DK 14.7
65 Phil. Hist. 155F10; cf. ch. II.
66 Heath, Math. I 66; Burnet, EGP 97; Ciaceri II 94 (who ingeniously combines this with Heracleit. fr. 129; first Ionian ἱστορίᾳ, then It. καταιγίδιον); Sarton, Hist. 203f; Cameron 25; Raven, PyEl 2; Mondolfo in ZM 354; van der Waerden, Aatr. 7; Morrison, CQ 1936, 158; De Vogel, GP 170, no. 22; Stapleton, Osiris 1958, 44. The correct interpretation: Heidel, AJP 1940, 8; Philip 23.
67 Arist. Met. 983b22ff = DK B84. The reading πρῶτον, which Christ adopted from recentiores, has against it the authority of the tradition (πρῶτον) is also found in the second version of Alexander of Aphrodisias, on p. 37.12 Hayduck and Ascl. Met. 35.30; also Bekker, Diels, Ross, Jaeger.—πρῶτον goes with ἄρχομαι: cf. ἐκ ἀρχῆς ἄρχομαι, 984a28; Gen. corr. 350b34.—The sentence is cited, for example, by Zeller I 495 n. 2; Heath, Math. I 66; van der Waerden, Hermes 193, 163, and often. We may add a passage from a lost writing of Aristotle (fam. Comm. math. sc. p. 78.8ff; cf. above, ch. I 2, n. 112, and on the text, below, ch. VI 3, n. 1): οἱ δὲ Πυθαγόρειοι διαστρέφονται ἐν τοῖς μαθηματικαὶ τὰ τα ἀρχαιότατα τῶν λόγων ὁμολογουμένων, ὅτι μόνον καθ᾿ ἀποδείξεως ἢ μεταχείριστον ἀναθρόπως, καὶ ἀμφοτέρως ἀριθμοῖς τὰ περὶ τῆς ἀρκοῦν ἂν δὴ ἀριθμοῖς, ὅπερ ἔστα τῶν ἂν ταῦτα ὑπήρχαν ἐννθαὶ καὶ τάς τοιούτων ἀρχὰς . . .
68 Above, ch. I 2, nn. 99–100.
69 Below, n. 83.
70 Cf. the constantly repeated πρῶτον and similar expressions, Met. 983b6, b20, b29, 984a27, b18ff, b23, 985a8, a29f, 985b21.
71 Cf. Met. 986b24, on Xenophanes, and especially 987a20ff, on Plato (with the same ingressive aorist ὑπήρχαν 987b19, 988b6). On ἄρχομαι, cf. Pl. Tim. 19d. The balance of the sentence, projected in the use of ταύτα is disturbed by the intrusion of the psychological point of view and the interpolation of ἄρχομαι: . . . instead of, for example, something like ταὐτα τὰ προήγουσαν καὶ τᾶς τοιούτων ἀρχὰς τῶν ἀρκοῦν ἂν ἂσκεον.
world but numbers. Thus for Aristotle the Pythagoreans’ number theory is a by-product of their mathematical studies. This view has found adherents, but is by no means self-evident. There is plenty of number speculation which does not presuppose mathematics; and we know from Aristotle’s own words that nonmathematical associations were influential in forming the Pythagorean numerical cosmology. That it was mathematics of the deductive type which preceded and produced the number philosophy is a psychological conjecture of Aristotle, which the historian is not obliged to accept. We may credit Aristotle’s statement that there were certain Pythagoreans who achieved success in mathematics and that there did exist a Pythagorean philosophy of number. But as far as our information from Aristotle goes, it remains an open question when the mathematical achievements of these Pythagoreans occurred and how they are to be placed in the general development of Greek mathematics. His concern was the history, not of mathematics, but of philosophy.

A statement by Aristotle’s pupil Aristoxenus is obviously influenced by his master: “Pythagoras seems to have honored, most of all, the study of numbers, and to have advanced it in withdrawing it from the use of merchants and tradesmen, likening all things to numbers...” The “advancement” of mathematics, and number philosophy, is here moved back from the Pythagoreans to Pythagoras, and also seen in the light of the Platonic demand that λογιστική should not be carried on

79 The same basic idea is expressed in a somewhat more friendly spirit in the parallel passage cited above, n. 68: the Pythagoreans like the exactness of mathematics, see its application in music, and by this route arrive at their number theory. Thus Zeller, Vorr. u. Abb. 39: “From these mathematical and scientific studies they proceeded to develop a set of beliefs about the nature of things in general”; Sarton, Hist. 304: “In order to develop a mystical theory of numbers it was necessary first of all to obtain a sufficient knowledge of them”—as though elementary calculation did not provide plenty of material for numerical speculation, especially considering the existence of Babylonian techniques of computation.

80 Met. 983b29: ὅτι τὸ μὲν τοιοῦτον τῶν ἀριθμῶν πάθος δικαίωσε, τὸ δὲ τοιοῦτον ψυχὴ καὶ νοῦς, ἔτερον δὲ καυρός... See below, ch. VI 4.

81 This is confirmed by the details Eudemus gives; and Hippasus may also be recalled in this connection (below, ch. VI 3).

82 Aristox. fr. 23 = DK 588b2, according to Stob. 1 proem. 6 ἐκ τῶν Ἀριστοτέλους περὶ ἀριθμητικῆς. Wehrli (54) doubts the title—Stobaeanus’ whole introduction is περὶ ἀριθμητικῆς—and opines that “in the existing formulation, of an elementary introduction to numerical concepts, fr. 23 is not from Aristoxenus.” One has the impression of an abbreviated excerpt; but the introductory sentence, with its meticulous formulation, looks like an exact quotation. The name of Pythagoras is cited with an air of introducing a new element. Wehrli’s assignment of the fragment to the book Περὶ Πολυμαθείας καὶ τῶν γνώμην ὕποσται is conjectural. Iloth Frank (266 n. 1) and Wehrli (54) point out the relation to Aristotle, as well as the connection between Aristoxenus’ further statement, οὐ δὲ ἐκ τῶν θεῶν περαιφών εὑρισκόμεθα (sc. φασίν τῶν ἀριθμῶν) with Επιμ. 978c.

83 Pl. Rep. 525c.


85 Palamedes: Aesch. fr. 303 M., Soph. fr. 432 P., Gorg. Pal. 30; Pheidon of Argos: RE XIX 1943; Prometheus: Aesch. PV 459, and also Pl. Phil. 16c, where the inventor of the philosophy of number (above, ch. 14) is called “Prometheus.”

86 See Tannery, HSiH 54ff, Géom. 80ff, Heath, Math. 1 122ff, van der Warden, A 8ff, esp. von Fritz, ABG 1955, 77ff; and Becker, MD 31ff, 217ff; von Fritz, Becker, and Wehrli (Eudemus 115) have returned to a more positive evaluation of the tradition about Thales, while the extreme skepticism of Burnet, EGP 45ff, and Schuh (Essai 175ff) has been taken up again by D. R. Dicks, “Thales,” CQ 53 (1959) 294-309. Dicks, however, ignores the arguments of von Fritz and Becker, as well as the passages in Aristophanes. On Thales’ alleged prediction of an eclipse of the sun, see Neugebauer ExSt 142.

87 See B. Snell, “Die Nachrichten über die Lehren des Thales und die Anfänge der griechischen Philosophie und Literaturgeschichte,” Philologus 96 (1944) 170-182; he determined that the intermediary source was Hippasus (D.L. 1.24 = DK 8617).
shape to serve as a pair of compasses (διαφάνεια), meaning of course that he was going to make a geometrical diagram—and under cover of this activity he steals a coat. "Why do we go on admiring old Thales?" cries Strepsiades (τι δήν ἐκείνω τῶν Θαληθῶνα). This shows that for the Athenian public Thales and geometry belong together; and this is also evident from another passage in Aristophanes. Meton arrives in Cloudcuckooland; "I want to survey the air for you—γεωμετρῆσαι βουλώμα τούτος ὑμῖν." He will apply his ruler, "so that the circle may become a square," which will look like a star... All Pitharchus can say is "The man's a Thales" (Ἄνθρωπος Θαλῆς). Eudemus gives detailed reports about mathematical propositions, proofs, and constructions that Thales was supposed to have discovered, he distinguishes between tradition and deductions of his own, even recording an archaic locution used by Thales. This implies that there was a book, available to him or his authority, ascribed to Thales. The book in question must be that On the Solstice and the Equinox. Whatever the situation may be with regard to authenticity, there obviously existed, in the sixth century, Ionic technical writings on problems of astronomy and the calendar, already with them, geometrical concepts—circles and angles—seem to take the place of

Babylonian calculation. The Babylonians do not use the concept of the angle. Becker has shown that all of the propositions attributed to Thales can be derived simply from considerations of symmetry, and von Fritz points out that the method assumed, that of superposition (επιστροφή), was consciously avoided in later Greek geometry. With Thales the point is still a graphic or perceptible "showing" (δεικνύων). But in this perspicuity itself there is a new element by contrast with the Babylonian "recipes." It is in the perceptible figure that mathematical propositions become clear in all their generality and necessity: Greek geometry begins to take form.

Thales' name was the only one to remain familiar; aside from him we can discern the specifically Greek innovation in geometrical thought only through its effects. The world of Anaximander "is constructed on severely mathematical ratios"; it is "essentially geometrical." Cosmology is subject to mathematical logic; the earth must hang free in the middle because it is equidistant, in every direction, from the "wheels" of the stars. His statements about the size and distance of the heavenly bodies, in which geometry extends its purview to the whole universe, presuppose at least some knowledge about geometrical proportion in the correlation of distance with true and apparent magnitudes. Anaximander relies on the accuracy of geometry in matters beyond the range of any kind of verification—in its application

Babylonian mathematics has a measure of inclination (ἰσοχών, corresponding to the cotangent function) but not the concept of an angle (Becker, Erkl. 27, MD 10, 37).

In 517 B.C.


It is hard to make out anything about the brother of the son of Eudoxus and the son of Steichorios as a geometer by Hippias (Proc. In Enol. 65.12 = DK 86.12 = Eudemus fr. 133, the source of Hero Diff. 161.1 p. 108.12 Heiberg and Suda s.v. Steichorios). The correct form of the name is probably Μαρκάριος (as Hero; the Suda has Μαρκάριος, the MSS of Proclus Apol., apogr. Αριστομαχος s.v. Μαρκάριος; Μαρκάριος is the name of a city in Brutium near Rhigium (Oldfather, RE XIV 952). Thus combinations with Marmakos the alleged father (D.L. 8.1) or Marmakos the son of Pythagoras (Plut. Alex. Paul. 1, Numa 8; Festus p. 23 L.; M. Detienne RHR 152 [1957] 142) are built on sand, especially since Steichorios is generally agreed to have lived in the first half of the 6th century.

Jäger Pädicis I 157 (Eng. tr.).

Heidel AJP 1940, 30; Kahn 81: "Anaximander is ancestor...also to the geometric philosophy usually associated with the name of Pythagoras"; cf. 79ff, 92ff. The assertion in the Suda that "in general, (Anaximander) drew up the blueprint for geometry" (s.v. Anaximandros = D.K 12.12...), is unworthy εἰς and τρισχειά and φύσις... γνωμών τε εἰσόχων καὶ δῶς γεωμετρῶν εὐτύπων οἴκος is a cruder version of Evagoras ap. D.L. 3.17: εἰς... καὶ γνώμων... τρισχειά αὐτεχέας καὶ γῆς καὶ κλίσεως περίμετρος πρώτον ἐγγραφεῖν.

Above ch. IV 1.
to cosmic proportions and also in contradiction to appearance, which suggests that the sun is about as large in diameter as the width of a human foot.

The concept of geometrical similarity is also the precondition for Anaximander’s attempt to construct a map of the world. And geometrical thought is still in the saddle in the work of Hecataeus, mocked by Herodotus for representing the earth as circular (quartered by the Nile and the Danube) and depicting less well known countries like Libya as built of rectangles and squares. As early as Thucydides and Simonides we find these concepts or words used as symbolic of impeccable truthfulness and accuracy. Is this elevation of ideas, from the realm of craftsmanship to the plane of the symbolic, Pythagorean? Pythagoras of Samos came from a milieu in which technological and geometrical thinking was in its heyday. It is unthinkable that he took no notice of it; but there is no way of knowing whether he made any contribution to it himself. The career of Greek geometry began before Pythagoras’ time, and there is no warrant for supposing that every trace of mathematical and geometrical thought in early Greece is co ipso Pythagorean.

108 γωνία used of buildings, Hdt. 1.51 et seq.; IG XII 7.1; γωνίασις ἄθος and the like in building accounts, IG I 372.19, 161, 313-86, 373.80. See A. Debrunner, ἸΣ 60 (1949) 38-46. It is not entirely clear whether the vowel in γωνία shows compensatory lengthening of the lengthened grade (as Schwzyzer thinks, I 358). If it is compensatory lengthening, Debrunner would like to refer to the Pythagoreans of southern Italy, though he must admit that for this area the compensatory lengthening to γωνία is not firmly attested. We can regard it as certain that architecture before Pythagoras did not get along without the term πετράγωνας (πετράγωνας is only the brick mold) and perhaps one ought not to dismiss as readily as Debrunner Hesychius’ gloss γώνια. Δισκεύσας—According to Proclus, In Remp. II 26.18 and Hero, Diff. 15, the Pythagoreans called the angle not γωνία but γωνίας (elsewhere used of the feathering of an arrow: τρίγωνας ἀλοχῆς, II. 5.393, 11.507, Soph. Trach. 681). There are examples of τρίγωνας, “3-cornered,” Pi. fr. 322, Callim. fr. 1.36; of τρίγωνας, “3-cornered,” Leonidas A. P. 6.343-3; and an expression ἀντίγωνας τρίγωνας, Nom. Dion. 6.23. This evidence would seem to show that γωνία is a poetical term, used as a substitute for the unmetrical γωνία. If ancient Pythagoreans had in fact used it, the whole development of Greek geometry was unaffected: Procl. In Eucl. 130.8 etc. = Philolaus A14 uses γωνία as casually as Eudemus fr. 141 = Archytas A14.

109 The restriction of this word’s sense—from “quadrangle” to “rectangle,” and usually “square”—is a development resulting from the usage of masons. For τρέχωνων in the context of architecture, see Hdt. 1.179, 181, 2.124; IG I 313.101.

110 According to Plin. HN 7.198, the inventions of Theodorus of Samos included “norma, libella, torus, clavis.” Technologists in particular had connections in the Orient. Not only did they build bridges of ships for Darius and Xerxes, but there were Greek masons at the court of Cyrus I (G. M. A. Richter, AJA 50 [1946] 15-39).

111 Theog. 805f: τόσον καὶ στάθησαν καὶ γωνίων ἄδυσθαν αὐτοῖς ἔστησεν χέρι (ἐχείρ), Κύρη, φαλασάδων (cf. 543); Simonides 542-3 Page: δυσφάθεν μὲν ἀλλαθείος γενεάς χαλέπους, χερόν τε καὶ σταίνων καὶ νῦν τετράγωνον, ἅνω ψάγει τετράγωνον (like a building stone with which the supervisor or foreman can find no fault. Cf. the “corner-stone” of Ps. 117.22, Isa. 28.16, Matt. 21.42, Acts 4.11). A dedicatory inscription from the neighborhood of Sybaris (Bull. Epig. 1957, no. 697, 6th century B.C.) has a noteworthy expression about a votive offering Πάντοις (μακουσ) τε πόρος τε (a cubit). A pot by the Cage Painter (Louvre G 318; E. Portier, Vases antiques du Louvier III [1922] pl. 135; ART 1 2nd 348, 3; dated 460-470 B.C.) has a scene representing instruction in geometry: a boy with a pair of compasses, and a γωνία hanging on the wall.

112 Fränkel drew attention to the thought pattern of the “geometric mean” in Heraclitus—man is to god as ape is to man, etc.—and thinks there may be Pythagorean influence here (AJP 29 [1918] 309-337, in German W 253-283, esp. p. 265 n. 5; similarly Minar, CP 34 [1939] 337-340). The idea of proportion, however, antedates Pythagoras.
VI. PYTHAGOREAN NUMBER THEORY

Callimachus makes play with the crossing of the traditions about Thales and Pythagoras in the well-known passage from the Iambi, in which an Arcadian is to give a bowl “to the wisest.” First he comes to Thales, who is “scraping the earth and drawing the figure which the Phrygian Euphorbus discovered, the first of men to draw triangles and oblique figures and the curved spiral, who taught us to abstain from animal food…”

Thales is concerned with geometrical propositions discovered by “Phrygian Euphorbus,” which naturally means Pythagoras. This has been misunderstood to mean that some kind of tradition actually ascribed such discoveries to Euphorbus, or that Pythagoras, as a youthful prodigy, founded Greek mathematics during Thales’ lifetime. This makes the chronological inconsistency, to which Callimachus so wittily alludes, disappear. He consciously ascribes to Thales a knowledge that he could not have had, chronologically, but then, with the help of the doctrine of metempsychosis, makes the impossible possible. Pythagoras introduced geometry from Egypt to Greece and brought it to perfection; yet before Pythagoras’ time Thales was already famous as a geometer. Therefore Pythagoras must have made his discoveries in an earlier incarnation.

From about the middle of the fifth century, it is clear that mathematics is a center of intellectual interest. Almost all the important thinkers are concerned with mathematical questions. Anaxagoras, Hippas, Antiphon, and Hippocrates of Chios worked on the squaring of the circle; this was a problem so widely known that it could be used in comedy. Hippocrates of Chios was pioneer on the line of thought which Archytas followed up in solving the problem of doubling the cube. Protagoras tried to refute the geometers on principle by maintaining that their postulates never fit reality: there is no visible line touching a circle at only one point. His disciple Theodorus dealt with irrationality. Democritus did a good deal of work on mathematical problems. The statues of Geometry and Astronomy at the entrance of Socrates “Thinking-shop” are a clear indication that before the end of the fifth century the branches of mathematics, as such, had a firm place in the curriculum of the Sophists’ program of higher education. When Xenophon represents Socrates as criticizing excessive enthusiasm about geometry, astronomy, logismos, and medicine, when they are carried beyond the needs of practicality, this is not simply an anachronistic criticism of Plato. Hippas, as Plato tells us more than once, taught logismos kai astrophomai kai geometrias kai moussikef. And all these special fields are also attested for Democritus. It follows that the quadrivium is not merely a Pythagorean import brought by Plato from Italy. If, as in the later tradition, it is to be regarded as Pythagorean, its influence must have had its discoveries in an earlier incarnation.

106 Callim. fr. 901.58-62 Pfeiffer. The words τα σχήματα (“the figures”) may hold an allusion to the Pythagorean theorem (Pfeiffer and Howald in the Artemis edition, 1935, p. 320f). τρίγωνα και σκελλῆνα (“triangles and oblique figures”) reminds of Eudemus fr. 136-137 (below, ch. VI 3). In the first part of verse 61 (καὶ κύκλος ἐκ[ων] Diodorus (10.4.6) has the unmetrical κύκλος ἑπτασίμης, POS. VII 1011 (p. 32; cf. p. 71), Hunt reads ἐτὸς with ἕ τε written above π; according to Pfeiffer ἐτὸς is “satis certa.” Pfeiffer considers ἕτος ἐπί ἡμέραν. Μαας suggests ἐπίσας. Diel’s conjecture ἐπίκεια (DK) has an important point in its favor: ἐπί is a technical term for the apparently “spiral” movement of the planets which results from the overlapping of the revolution of the earth and the planets’ own movement (Pl. Tim. 39a, Hermesianus fr. 2.86f Diehl, Pho. A. P. 9.577); and the description of the zodiac was ascribed to Pythagoras, in rivalry with Onopodes. As the word ντεμπουκαί shows, Callimachus is not speaking of mathematics alone, but is giving, in a coin form, a general characterization of Pythagoras’ teaching—and therefore could certainly not leave out astronomy (cf. lines 54f, on the astronomy of Thales). On the text of line 62, see H. Lloyd-Jones, CR 17 (1967) 125-127.

107 Above, ch. II 3.

108 Delattre, Vitr. 157f, makes Euphorbus, “Good-Shepherd,” a Phrygian culture hero; Rostagni agrees, Verto 120 n. 1, 240 n. 1.

109 Lévy, Socrates 41f.
extended to Hippas, Democritus, and the Aristophanic Socrates. It was the cultural influence of the Academy that brought the system of the “four fields” to their position of special prominence; the only unequivocally Pythagorean element is the arithmetization of music theory and, to a degree, the elevation of number theory (“arithmetic”) to an independent branch alongside geometry. All the rest had been generally known and could assume their place in the scheme of higher education without the help of any esoteric or Pythagorean influence. This is the way Isocrates is looking at the matter when he distinguishes between the traditional pair, μαθηματική and μονική, and the subjects that had emerged in his own time, namely γεωμετρία, αστρολογία, and διάλογον ἐρωτικοῖς. Here he is alluding to the educational system of the Academy.

Along with the appearance of the quadrivium comes the semantic development, in which the word μαθηματα, “fields of study,” is reduced to the branches of “mathematical” study, while μαθηματική is restricted to mathematics proper (including astronomy). This usage is not consistently followed by Plato and Isocrates until their later works to be taken up then in the Epinomis and in the works of Aristotle. In general Plato still uses μάθημα in its original, broader sense. Thus it seems that the word “mathematics” became fixed only in the time of the Old Academy. The question of the extent to which Pythagoreans anticipated Plato, in the treatment of the four branches and the development of the concept of the μαθηματα, depends on the problem of the genuineness of the long fragment of Archytas.

In any case Greek geometry assumed its final form in the context of the Old Academy. After Plato had placed an especially high valuation on mathematics, and had fixed its position as a discipline of pure thought (so that Protagoras objections became irrelevant), there came an unprecedented development of these studies. Aristotle writes, “Those who concern themselves with geometry and calculation and the other sciences have from small beginnings made by now so much progress in a very short time as no other field has made in any of the arts.” But even in the age of Anaxagoras, Protagoras, and Hippocrates of Chios, we notice that mathematicians are no longer concerned merely with individual problems, but are working on fundamental ones. The problems of squaring the circle and doubling the cube are not soluble by ordinary geometrical means, with the use of ruler and compass; and the fact of irrationality can only be deduced, or “shown” in a logical argument, never made immediately perceptible. Here geometry freed itself from its bondage to the needs of practicality. For the needs of everyday life, there were already plenty of approximative values to use in the calculations involving roots and the dimensions of the circle. In the geometry of the fifth century, however, it was obviously recognized that these were merely approximations, and that there is a basic difference between these and any exact solution resting on proof; it was understood that the task of expressing a magnitude like \( \sqrt{2} \) in whole numbers, or expressing the length of the diagonal in terms of that of the sides of a square, was not only “not yet” solved, but in principle insoluble. Mathematical logic and deductive proof go beyond what is perceptible, and this is what carried Greek geometry far beyond its predecessors, no matter how suggestive, in the oriental cultures.

If a basic influence was exerted by Pythagoras or by Pythagoreans on Greek mathematics, this would have had to take place in the period between Anaximander, Cleonides, and Hecataeus, on the one hand, and Anaxagoras, Oenopides, and Hippocrates of Chios on the other. What ensued was, in the first place, the development of detail and, in the second, a metaphysical and logical undergirding in which the

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180 Plato treats the four μαθηματα, already adding solid geometry, in Rep. 524b ff; see also Lg. 747a, Epin. 990e ff, 991c. On the development from Plato on, see Merlan, Phil. 1937. He does not go into the pre-Platonic period.
181 Below, ch. IV.
183 ib. Lg. 817f, Isoc. Arist. 261, 10, Panath. 27.

185 Acad. Ind. Hcr. p. 15ff Meilr (cf. below, ch. VI 3, n. 86); Procl. In Eucl. 66,8 = Eudemus fr. 133, where no fewer than 11 mathematicians between Plato and Euclid are named—and we may add Polemarhus and Helicon of Cyticus, Amphilochus, Bion of Abdera, Aristocles, Autolycus, and a certain Aristaeus. Even if Plato was not a professional mathematician, his philosophy seems to have provided the decisive breakthrough in the establishment of mathematics, and at the same time have directed the attention of philosophers, even more than had been the case, toward mathematics. See above n. 6. On the motto δενομετρητη μαθηματικον, see H. D. Saffrey, REG 81 (1960) 67–87.
186 Fr. 53 = Iam. Comm. 98. 312ff (with Aristotelian origin guaranteed by Cic. Tus. 3.69): τον but δε νομυπληθάντων ἐν μικράς ἀφορμῶν ἐν ἐλάχιστῳ χρόνῳ ἵπτοντος αἰ τε περὶ τὴν γεωμετρίαν καὶ τὸν λόγον καὶ τὰς ἄλλας παιδείας, δοκεῖ ἐκεῖνος ἔρχετο ἐν ὑδέμνι ἕνας τῶν τεχνῶν.
Pythagoreans, according to the testimony of Aristotle, had no part: τὸ γονὸν τεωρήματα προκάτοχον τῆς πάμαν ὡς ἢ ἔκεινον οὐκ ἔντον τῶν ἀρχαίων.\textsuperscript{127} In that early period there does seem to be a certain gap, coinciding with the turbulent period of the Persian Wars. There were, nevertheless, direct connections between the older Ionic school and Anaxagoras, and, as Anaximander’s pupil Cleostratus was an astronomer, so was Oenopides of Chios, the pupil of Anaxagoras and competitor of Hippocrates. According to Eudemus, Oenopides worked out, for the first time, certain elementary geometrical constructions “since he believed that this would be useful for astronomy.”\textsuperscript{128} The significance of these apparently simple accomplishments seems to lie in the fact that in them geometry consciously restricts itself to the tools of compass and ruler.\textsuperscript{129} Oenopides is still not doing mathematics for its own sake; but in the demand for precise construction, and the concept of the πράξις,\textsuperscript{130} he represents a definite forward step in the direction of “pure” theory. Hippocrates, too, was concerned with astronomy,\textsuperscript{131} but he also wrote a book called Στοιχεῖα. If there were more reliable evidence about the discovery of pure theory by Pythagoras than the philosophia anecdote of Heraclides and the supposed Eudemus,\textsuperscript{132} then one would have to recognize the true accomplishment of the Pythagorean school through its influence here in Oenopides and Hippocrates, and one might speak with perfect justification of the Pythagorean basis of Greek mathematics. But there is another solution.

The attempt at purely logical argumentation, a systematic progression from one thought to another, and the advancement of proofs and conclusions in conscious contradiction to the evidence of the senses make their first appearance in Parmenides. Kurt von Fritz has shown how νοεῖν, which previously meant an intuitive comprehension, first became logical “thinking” in Parmenides.\textsuperscript{133} “They were led to transcend sense perception and to disregard it on the ground that ‘one ought to follow the argument’”—this is Aristotle’s characterization of the Eleatics,\textsuperscript{134} and ἀκολουθεῖν τῷ λόγῳ is the distinctive feature of Greek mathematics as early as Hippocrates. “La mathématique grecque restera plus ou moins éloïtique jusqu’au bout” is Abel Rey’s way of putting the matter (148), and more recently Arpad Szabo, in a number of publications, has shown the dependence of Greek mathematics on the thought of the Eleatics.\textsuperscript{135} The connection of geometry, and especially that of Hippocrates of Chios, with the logic of the Eleatics is obvious. The Eleatic thought pattern of the differentiation of cases dominates Hippocrates’ treatment of the problem of the quadrature of lunes; the exterior arc of the lune is either greater or less than a semicircle, or equal to it, so that all possible cases are exhausted.\textsuperscript{136} Zeno’s methods of proof, the reductio ad absurdum and the regressus in infinitum, are basic to all the proofs about irrationality.\textsuperscript{137} In the logic of the Eleatics we find the factor that brought about the advancement of mathematics, in the hiatus between Anaximander and Anaxagoras; there is a similar development in natural philosophy between the early Ionians and the post-Parmenidean thinkers. Anaxagoras, Empedocles, and Leucippus, each in his own fashion, achieved an advance by the combination of Ionian beginnings with Eleatic ontology and patterns of thought. There followed then the playful intellectual diversions of the Sophists. That which had emerged from everyday activity and bold speculation in the sixth century, was discussed, sifted, and gradually brought into a logical system. In the process, natural philosophy drifted into a dilemma between eclecticism and skepticism, but geometry took on its typical Greek form.

A contrary interpretation would have Parmenides deriving his style of argumentation from Pythagorean mathematics: “The method of

\textsuperscript{127} Arist. Met. 1083b18.

\textsuperscript{128} Oenopides, DK 41.13 = Procl. In Eucl. 283.4 (constructing a perpendicular to a straight line through a given point); DK 41.14 = Procl. In Eucl. 333.1 = Eudemus fr. 138 (to construct an angle equal to a given angle at a given point on a straight line); the first notice also obviously comes from Eudemus.—CCAG VIII 3. p. 95.12: πρὸ ὧν ἦν ἀφελομένας μὲθοδοὺς ἔχειν γεγαγομένοις για τὸ γνώμονα. Oenopides in fact uses the expression κατά γνώμονα for “vertical” (DK 41.13), but he constructs the angle without mechanical help.

\textsuperscript{129} DK 41.12.

\textsuperscript{130} Cf. above, ch. IV 1, n. 77. On στοιχεῖα, Burkert, Philologus 1959.

\textsuperscript{131} On Heraclides, Burkert, Hermes 1960, 519ff; on Eudemus fr. 133, above, n. 62.

\textsuperscript{132} Von Fritz, RE XVII 2266. Every surveyor has a simple instrument for measuring right angles (basically the γνώμονα). Oenopides in fact uses the expression κατά γνώμονα for “vertical” (DK 41.13), but he constructs the angle without mechanical help.

\textsuperscript{133} Von Fritz, CP 40 (1945) 223-242, 41 (1946) 12-34, esp. 1945, 241.

\textsuperscript{134} Cfr. corr. 325a13; cf. Caec. 298b22: the Eleatics were the first to believe in a Being uncreated and immutable, though not (as in Plato) distinguished from the perceptible.

\textsuperscript{135} AA 1955, 67ff: “Wie ist die Mathematik zu einer deduktiven Wissenschaft geworden?” AA 1956, 109-112: “θεωρηματικος als mathematischer Terminus für ‘beweisen’,” Maia 10 (1958) 117ff: “Die Grundlagen in der frühgriechischen Mathematik,” Studii 39 (1958) 1: “the historically necessary prerequisite for the earliest Greek mathematical science” is “the Eleatic philosophy.” All the same, Szabo does not free himself from the communis opinio on Pythagorean mathematics, but accepts Becker’s “doctrine of even and odd,” and places the Pythagoreans, as “the first representatives of deductive mathematics” (Maia 1948, 130), between Parmenides and Hippocrates. Were the Pythagoreans the only ones who could learn mathematical logic from Parmenides?

\textsuperscript{136} Above, n. 10; cf. Reinhardt, Parm. 35ff, 64ff; above, ch. III 2, n. 104.

\textsuperscript{137} Cfr. below, ch. VI 2, n. 47; VI 3, n. 81.
reasoning he imported into philosophy is the method of geometry.” Consequently, but does Parmenides “import” someone else’s results? The price at which the Pythagorean origin of mathematics is saved, in this interpretation, is tremendous. Are we to suppose that Parmenides merely applied a previously developed method to a new concept, τοῦ δείκτου? Only if Parmenides himself gave some indication of the fact, or if other testimony gave a clear indication of the existence of Pythagorean mathematics before Parmenides, would this thesis be acceptable. It would greatly diminish the originality and the basic importance of Parmenides in the growth of Greek philosophy, which has been generally recognized since the work of Reinhardt. In Parmenides himself there is not a word that points toward the field of mathematics—what a contrast with Plato and Aristotle! Further, if it can be shown that Zeno, in his manner of argument, holds close to the ideas of his teacher,138 and if Zeno also, in his manner of expression, shows no dependence on mathematics, then there is no occasion to interpolate a “Pythagorean mathematics” between the two.139 In discussing Being, Parmenides discovered the independence of thought; and deductive mathematics as well as logic took rise from this beginning; from the point of view of the development of thought, ontology is prior to the formal schematism.

Greek mathematics did not emerge from the revelation of a Wise Man, and not in the secret precinct of a sect founded for the purpose, but in close connection with the development of the rational Greek view of the world. Taking its departure from “measurement,” geometry becomes a component of natural philosophy, φυσιλογία. In one of these realms as well as the other, Thales and Anaximander, Parmenides and Zeno are the important names, and even Oenopides and Hippocrates are φιλοσοψος in their astronomy. Then, earlier than geometry and astronomy become the domain of spe-

cialists, because the increasing complexity demanded a specifically mathematical talent, which is not dependent on one’s origin or one’s membership in any kind of school. At the same time, the Sophists brought about a division, since the exactness of mathematical results was more and more obviously in contrast with the uncertainty of φιλοσοψος. Thus in Plato’s time mathematics was already the model science, which even skepticism had to take seriously. Individual Pythagoreans had some part in this development, but in its “essence” mathematics is not Pythagorean but Greek.

2. PYTHAGOREAN ARITHMETIC

That Pythagoreans were much concerned with numbers is established fact, but it is a question, in what sense this activity can be called mathematics; for the paradigmatic form of Greek mathematics, as a deductive system based on axioms, is geometry. On the other hand, there is one remarkable type of arithmetic that appears exclusively in the Pythagorean tradition, in which numbers are represented by figures made with counters or pebbles, ψήφου. Aristotle knows of “triangular numbers” 1 and the “perfect” number 10, in its deployment in the form of the “tetractys,” was certainly presented as a triangular number long before Aristotle. And what at first seems merely a game does lead to arithmetical combinations that are by no means trivial. For example, if the odd numbers, when added successively in a pebble figure, make a square each time, this means discovery of the rule for the series of square numbers; and, if in the ἕτερα ἑν ἕνος, constructed in similar fashion from the even numbers, one recognizes the triangular numbers, doubled, then he has the formula for the sum of triangular numbers, a special case in the arithmetic series.4

1 Above, ch. 1, n. 27. Ψήφου provide the simplest tool for practical calculation, in the abacus (on which see M. Lang, Hesperia 26 [1957] 271-287). It may be mentioned also that numbers have been indicated on dice, from very early times, by point figures (see Durenberg-Saglio s.v. tessera). These figures, which make the difference between gain and loss, are naturally looked upon with a certain emotion and come to appear as independent beings. In the substantive τὸ ᾑδονής is preserved the Indo-European root for “one”; and the substantive τὸ μανᾶς was also used in dice games (Pollux 7.204).

2 Met. 1092b11: ὁ τὸν ἄγαθόν δίοντες εἰς τὰ σχήματα τριγώνων καὶ τετραγώνων.

3 Above, ch. 1, n. 120.

4 ἕτερα ἑν ἕνος are numbers of the form n × (n + 1) (see Theo Sm. 26.21ff, Nicom. Ar. 1.19-19, 2-17, 1, 1. In Nic. 74.19ff). From the figure it can be seen that 1 + 2 + 3 + 4 + ... + n = \( \frac{n(n + 1)}{2} \) + n. In itself the word ἕτερα ἑν must mean “having unequal sides” in general (cf. ἐπὶ κλόους, ἐπὶ κλώνως, ἐπὶ κλωςκόλυ, ἐπὶ κλωςκος, ἐπὶ κλωςκεν).
From the πρόγλυμνος diagram can also be derived immediately the formula "of Pythagoras," which can be traced back as far as Anaxilus, for calculating the rational sides of a right triangle, beginning with an odd number, and this very fact has been rightly regarded as showing that this is an element of ancient tradition. This of course presupposes the knowledge of the "Pythagorean theorem," that most famous of all features of Pythagorean mathematics, connected in the tradition with the proverbial sacrifice of an ox. There has been much controversy over this tradition. Proclus, in his curiously twisted sentence, does not in any case cite Eudemus. The principal testimony consists of two verses, cited several times but not known in their original context, from a certain Apollodorus, known as ἀρχαῖος or ἀρχαῖος, who may be the same as the Democritean from Cyzicus, and in that case would be datable to the fourth century B.C. For him, already, the γράμματα of Pythagoras and his offering of cattle are "famous." The fact that the offering of cattle featured in the story is in flagrant contradic-

τερόπλασμα. The restriction of its sense (cf. Iam. and Nicom. in the passages cited) indicates, as is the case with τετράγωνον (above, ch. VI 1, n. 103), that the term is rather old (Tannery, MS II 97). The general term for "rectangular" in later times was παράκερατος, coined, according to D.L. 3.4, by Plato (at Thet. 147e it is used interchangeably with τετράγωνον). See further Rep. 546c, Tim. 54a; τετράγωνον in the general sense: Xen. Eq. 7.14, Arist. De an. 413a17, Cat. 11.11, Eucl. 1, def. 22.

1 Procl. In Eucl. 428-78; Hero Geom. 9 Heiberg IV p. 216; both are cited in the scholion to Eucl. p. 235-37; cf. 235.59f. On the relation of the formula to the "gnomon" arrangement of pebbles, see Heath, Math. 81; Heath, Eucl. I 358ff; Becker, MD 53; von Fritz, AnnMath 1945, 352. The corresponding formula for even numbers, which is ascribed to Plato, is hard to derive from the pebble diagram.

1 The fullest treatment of the problem is Heath, Eucl. I 350ff. Proclus' sentence (In Eucl. 426.6 = DK 580219) runs: τῶν μὲν ἱστορεῖ τὰ ἀρχαῖα βουλεύοντες ἀκούσαν ἡθεῖα τὸ 

1 Ψηφιδάρχων τῶν περικελλεῖ εὐρέος γράμμα κεῖν ἐν δ' ἐν τοιν πλευράς ἂν ἄνωθεν βουλεύει. D.L. 8.12, 1.25 (ὁ λογικός), Ath. 10.418d (ὁ ἀρχαῖος; in line 2, κείνος ἐν τῷ λαμπρῷ ἐνεῖν). Probably from D.L., A.P. 7.115; RE s.v. Apollonius (68). On Apollonius of Cyzicus, see above, ch. III 1, n. 51; DK 74. Eudoxus had founded his school in Cyzicus, and his pupils Helicon and Polenarchus came from there; but the Democritean Bion of Abdera (DK 77) was also a mathematical astronomer, so that the Democritean Apollonius might well have been an ἀρχαῖος. The verses do not say which "famous" figure of Pythagoras is in question, but D.L. and Ath. call it "the Pythagorean theorem," and Plutarch presupposes the same interpretation in his varying version (on the application of planes, see the passage cited and Quaest. conv. 8.2.4).

2 Pythagorean Arithmetic

tion with Pythagorean vegetarianism ought rather to be considered an indication of antiquity than the reverse. 8

What Neugebauer first suggested as a possibility in 1928 has since then grown into a certainty—namely, that the "Pythagorean theorem" had been used routinely for centuries in Babylon, and was therefore obviously not a discovery of the Greeks. 9 It must have been introduced as a piece of Babylonian arithmetical technique. It is possible that Pythagoras was the intermediary; but, in view of the multiplicity of contacts between Greece and the Orient in the sixth century, the "fame" of Pythagoras can hardly be explained on this one ground. There is no testimony that he gave a strict proof of the theorem, and this cannot be made to seem probable. 10 The suspicion remains that the theorem had more than a mathematical significance in Pythagoras' school, and that the numbers involved seemed in a cryptic way meaningful. The formula of "Pythagoras" points in this direction, as it belongs to the context of the pebble figures, like the form of the tradition that only mentions the triangle with the sides 3, 4, and 5. 11 In fact, this fits especially well with the kind of number speculation we learn of from Aristotle, where 3 is male, 4 is female, and 5, which mysteriously unites them in the Pythagorean triangle, is "marriage." 12 Plato's "nuptial number" obviously presupposes this interpretation. 13

What we are considering here, then, is not Pythagorean geometry, but arithmetic, developed by speculative interpretation of Babylonian formulas.

8 Above, ch. II 4, n. 110. Cic. Nat. d. 3.88 is dubious about the sacrifice of an ox; he passes over the geometrical problem with the neutral phrase "in geometria quidam." One cannot simply brand the tradition "impossible," as van der Waerden does (NA 100).—Pamphila, ap. D.L. 1.24, seems to be transferring the Pythagoras story to Thales.

9 Neugebauer, NGG, math.-ph. Kl., 1928, 46-48; QSF 3 (1936) 257; Eucl. 351f; cf. Becker, MD 50, 53; van der Waerden, SA 76ff; Stapleton, Osiris 1965, 1ff. A table ("Plimpton 322") with "Pythagorean numbers" was published by O. Neugebauer and A. Sachs, Mathematical Cuneiform Texts (1945) 38-41. The "Pythagorean theorem" was also known in India (Apastamba-Sulva-Sutra; cf. Heath, Eucl. I 352ff, Math. I 145ff; Becker, MD 55ff)—though Greek influence is not impossible here—and in China (Becker, MD 56; cf. below ch. VI 4, n. 38).

10 The classic "windmill proof" comes from Euclid, but more primitive proofs are possible; see Heath, Eucl. I 352ff, Math. I 147ff; Becker, MD 55ff. It is pure guesswork to suggest that Pythagoreans tried anything of the sort.

11 Vitr. 9 praef. (mentioning the sacrifice of an ox), Nicom. ap. Th. ar. 50.21ff, Iam. VP 130f, ch. 179 (and Delatte, Pd. 59ff), Alex. Met. 75.27ff (from Plato?).


13 ἔπειτα παραφαίνει τὸν ἀριθμὸν τοῦ ἔργου, Rep. 546c; brought into connection with the "Pythagorean triangle" in Iamblichus, Alexander (above, n. 11), Arist. Quint. 3 p. 151ff M. Cf. also below, ch. VI 4.
Based on the "Pythagorean theorem," there appear the Pythagoreans' "side numbers" and "diagonal numbers," a series that provides increasingly accurate approximations to the value of $\sqrt{2}$. Plato apparently knew about this calculation. The manner of constructing the series can be perceived from a simple jigsaw-puzzle procedure, and the theoretical basis for the procedure is given in the tradition in the form, "The unit, as the origin of all things, is both the side and the diagonal of the square." This does not in any way presuppose a concept of irrationality.

Further reconstruction of Pythagorean arithmetic depends on the question, to what extent the detailed expositions in Theo, Nicomachus, and Iamblichus may be regarded as evidence for early Pythagorean mathematics. They are usually treated, in modern accounts, as "Pythagorean arithmetic," before Hippocrates of Chios, and no attempt is made to separate more ancient and more recent material. It is inadmissible, however, to avoid the question of the form of this "Pythagorean arithmetic," for this is the crucial question in a consideration of its scientific character. The compilations from later antiquity offer a goodly number of arithmetical facts, but without any attempt at general proof of any of them. The rule is illustrated by a few examples, and this passes as verification. Of course, such incomplete inductive procedure leads to faults. The books of Euclid which are devoted to arithmetical matters are quite different; general and rigorous proof, more geometrico, is the rule, the numbers being represented by line segments. Is the inductive, proofless form ancient and primitive?

There did exist a continuous arithmetical tradition which bypassed Euclid. Aristotle knows of the ψῆδος numbers, which reappear in Nicomachus and Theo, whereas Euclid represents the numbers as lines; Aristotle speaks of triangular numbers, Euclid only of "plane" (that is, rectangular) and "solid" numbers. Certain intermediate stages of this extra-Euclidean tradition can be discerned, beginning with Plato's immediate pupils: Speusippus wrote Περὶ Πλευρακίων ἀριθμῶν—but he also included non-Pythagorean material: Xenocrates, too, wrote books entitled Τῶν περί τὰ μαθήματα βιβλία τέσσερα; Peri arithmōn, 'Aριθμοὺς θεωρίας (D. L. 4.13), and Philip of Opus wrote 'Aριθμητικά, Περὶ πολυγώνων ἀριθμῶν, Κύκλωμα, Μεσότητας. Later on, Hypsicles too wrote on polygonal numbers.

Certain advances beyond the pre-Aristotelian stage are clearly discernible: Aristotle speaks only of triangular and square numbers, and the expansion to polygonal numbers probably does not antedate Philip of Opus. It is more important that there is no trace, before Aristotle, of "perfect" numbers in the sense of Euclid. For the Pythagoreans, as Aristotle knew them, the number 10 is τέλειον, and neither 6 nor 8 plays any role. Nor does 28 occupy any significant position in the later tradition. Euclid's definition of "perfect number" is too abstract to catch the imagination of the lovers of symbolism. It is foreshadowed in Plato's Laws, where the number of citizens is to be 5,040 because this is a number with a particularly large number of
divisors, and in the context of the "generation" of numbers in Plato's system of derivation, it was inevitable that the question of the divisibility of larger numbers come to the fore. But the Euclidean "perfect number" is not a feature of early Pythagoreanism.

There are variations in the terminology, too. For Plato and Aristotle 2 is an even number, for Aristotle and Euclid a prime number, but in the later "Pythagorean" sources it will not fit the definition of an even number or that of a prime—in fact it is not regarded as a number at all, any more than 1, but constitutes, along with the latter, the first principle of number. This is sometimes regarded as ancient, because it is illogical, but it might better be regarded as a development from the Platonic system of derivation, in which the numbers have their origin in 2 and 8. On the other hand, the later sources show a more developed logic in the subdivision of even numbers; while in Plato and Euclid, όρια περιστάσεως (δύο χρόνοι) and περιττά όριας (τρία δίς) are defined side by side, the later writers distinguish όρια περιστάσεως from περιστάσεως in such a way that the two concepts are mutually exclusive. The conception of the όρια περιστάσεως is quite different in the Pythagoreans; here it is one that has a part in both terms. As early as Aristotle a change is evident to the more developed mathematical terminology; in a passage where he is "Pythagorizing" on his own, he calls the number 6 όρια περιστάσεως.

As far as concerns the method employing deduction and proof, the later tradition shows, in one respect, a decline, giving Euclid's rule for "perfect numbers" without Euclid's proof, but, on the other hand, Speusippus obviously proceeded in no less inductive a manner than his successors. Hypsicles was a professional mathematician, but some of his propositions were not furnished with proofs before Diophantus. The inductive, proofless kind of arithmetic, which lasted till long after Plato, may be regarded, along with a few basic rules, as belonging to the early Pythagoreans.

This is confirmed by the amazingly close agreement of this inductive arithmetic with Babylonian techniques of calculation. There too, by use of rules which are unproven, series are developed and made available for practical use in tabular form: series of squares, of cubes, and even of "Pythagorean" numbers. In fact, when so unusual a series as that of the form $n^2 (n + 1)$ makes its appearance, with the name παραμετρίσεως, in the Pythagorean tradition, we cannot but think of direct influence. It is possible that this Babylonian import was introduced by Pythagoras himself; but it was altered in the process, at least insofar as, in Aristo- xenus' words, the numbers were withdrawn from the use of merchants and "honored" for themselves. The graphic procedure with ψήφοι makes it possible to formulate impressively generalizations about numbers; but it also "reveals" each fact without deducing one from the other in an abstract chain of reasoning. It is the element of the unforeseeable which gives number games their appearance of something profound and secret. The "occult" charm of mathematics comes from the fact that the human mind forgets its own way of proceeding and loses sight of its own preconceptions; for alert mathematical analysis, that which fills the naive mind with amazement is seen as tautologous and therefore self-evident. What we find among the Pythagoreans is amazement and "reverence" for certain numbers and their properties and interrelations. "Even" and "odd" are united in "marriage"; and to them this means that cosmic forces are at work. A scheme of proof could hardly be anything but annoying because it would show the result as the logical consequence of the preconceptions, and reduce it to banality.

Even a game may be regarded legitimately as a kind of mathematics;
the axiomatic-deductive form is not the only one possible. But the distinctive achievement of Greek geometry is that, for the first time, it created such an axiomatic and deductive system. Seen in the context of this Greek geometry, Pythagorean arithmetic is an intrusive, quasi-primitive element.

By analysis of Euclid, Oskar Becker has reconstructed a set of theorems which has been widely heralded as proof of the existence of a deductive Pythagorean arithmetic,\textsuperscript{46} namely the “doctrine of odd and even,” as developed in Euclid 9.21-34. These propositions stand isolated in Euclid, a trivial appendage to the sophisticated number theory of books 7–9, which culminates in the proof that there are infinitely many prime numbers (9.20). The propositions about the even and the odd are only once applied by Euclid, in the proposition about perfect numbers (9.36), and in a proof of irrationality, given as an appendix, which Aristotle already knew.\textsuperscript{47} On the other hand, the basic importance of the even-odd antithesis for Pythagorean cosmology is well known; and since a fragment of Epicharmus uses the antithesis of odd and even, the conclusion has been drawn that the Pythagorean odd-even doctrine was already so well developed and so widely known about 500 B.C. that it would be recognized in a comic allusion.\textsuperscript{48} The fact that most of the propositions can be proved with simple pebble figures gives an important piece of confirmatory evidence; so the “doctrine of odd and even” is regarded as the best-attested element of very early Pythagorean science, in the form of deductive mathematics.

Yet this structure, apparently so firm, has fissures that must lead inevitably to its collapse. It is true that the Pythagoreans were concerned with even and odd numbers; but this is far from proving that they set forth the propositions found in Euclid, in sober mathematical formulation, and provided them with proofs, or that the propositions about irrationality and perfect numbers belonged with them from the beginning.

Becker himself assumes that Euclid reworked the Pythagorean doctrine, and reconstructs simpler proofs with pebble figures. But this transformation has deep consequences. In Euclid, the theory unrolls in systematic fashion, and one proposition presupposes the preceding, in the strictly deductive manner, whereas a proof with \( \psi \) is essentially inductive and pictorial. According to the principle of perfect induction, it can be regarded as probatory, but it does not presuppose other propositions; every set of facts is evident in itself. There is no need for a systematic structure, which is of the essence of deductive mathematics. Of course the Pythagoreans knew that odd plus odd makes even, and that odd plus even gives odd—they demonstrated this with their pebble diagrams\textsuperscript{49}—but they did not deduce one proposition from the other. They saw, directly, that the “male,” odd number showed itself dominant in association with the “female” even number.

The propositions about perfect numbers and about the irrational are on a quite different level. To be sure, Becker can derive from a pebble figure an important lemma for the proposition about the Euclidean perfect numbers,\textsuperscript{50} but to go further than this requires an abundant use of modern algebraic notation. Here is presupposed a whole chain of logical conclusions based on very precise concepts of divisibility, prime numbers, and factors, which is totally inconceivable without a written system of \( \sigma \) and a quasi-algebraic method of representing numbers by general symbols.\textsuperscript{51} In addition, it is probable on other grounds that the Euclidean idea of a “perfect number” was first developed in the Academy (above, nn. 27–28). It is not part of ancient Pythagorean speculation about the odd and even.

\footnote{Becker imagines that they represented even numbers by equal numbers of white and black pebbles; but an arrangement in two rows is even more striking (cf. Pl. Euthyphro 12d, where even number is defined as \( \text{πακεκοτος} \) and odd as \( \text{ακακεκοτος} \).}

\textbf{Euclid 9.21:} even + even = even

\textbf{Euclid 9.22:} odd + odd = even

\textbf{even + odd = odd}

(But see also above, ch. I 2, n. 31. The “power” of the odd number depends on the “one” that it has in its “middle.”)—In Euclid, 9.22 presupposes 9.21.

\footnote{Namely, the summation formula \( 1 + 1 + 2 + 4 + 8 \ldots + 2^n = 2^{n+1} - 1 \) (QSt 3, 53sh).}

\footnote{Archytas, like Euclid, represents numbers by line segments (A19).}
Only the propositions about the perfect numbers and the proof of irrationality give mathematical substance to the “doctrine of odd and even”; without them all we have is a remnant of very elementary and disconnected theorems. Becker himself believes that Euclid recast the proofs; but how simple it would have been for a pre-Euclidean mathematician to put the propositions about the odd and even into a systematic paradigm of deduction! The proof of irrationality and the proposition on perfect numbers presuppose this reformulation; they do not belong to the oldest stratum of Pythagorean arithmetic. The “doctrine of odd and even” only later became part of deductive mathematics, influenced by the mos geometricus.

There remains the division of numbers into odd and even, generally, in which one might, in spite of all, see the beginning of number theory; it is confidently attributed to Pythagoras himself. But this very point can be refuted by philological means. In all Pythagorean speculation the odd number is more highly valued; it is what “sets the limit,” is the male element, and stands in the “column of the good.” In all this is preserved, as shown both by anthropological parallels from folklore and linguistic observation, an ancient and widespread piece of number lore. But in the Greek language the even number is “well-structured,” ἄριστος, and the odd is “excessive,” περίττος; in normal usage ἄριστος is unequivocally the one with favorable connotation, and περίττος is negative, representing a transgression of the norm. Thus Greek terminology for even and odd is in its tendency diametrically opposite to the Pythagoreans’ numerical theory. This terminology is easily comprehensible from everyday use: dividing in half has from time immemorial had an important role in practical life, and it is no surprise that the words for “half” are built on a special root, which is not true of either “third” or “fourth.” In Greek lands it was often necessary to match two semi-choruses in a ritual dance, or to match pairs in an agon—in Plato’s view this is the sort of example upon which

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82 MD 51; cf. QSI 3, 544f.
83 Above, ch. I 2, n. 69.
84 Reidelmeister (52) says of the irrational: “It cannot be illustrated, only thought and inferred.” Becker’s illustrations, consisting of empty squares, are quite different from other pebble diagrams (cf. MD 52 with 40f).
85 Above, ch. I 2, n. 39; VI 2, n. 17.
86 Von Fritz, too (AnnMath 1945, 255), thinks that the irrational was first discovered in geometry; see below, ch. VI 3. For Plato, the expression ἀλογος ἀπέρ γραμματικός is already proverbial (Rep. 534δ); and ἀφαίρετος is also a term coming from geometry. If it is ἄριστος (Rep. 546c), “not expressible in numbers,” this presupposes the existence of the corresponding object. True to its ontological basis, Greek mathematics, unlike the modern, only develops concepts for “existing” magnitudes.

87 Heath, Math. I 70: “The distinction between odd (περίττος) and even (ἄριστος) doubtless goes back to Pythagoras.” Rostagni, Vero 27: this distinction, “nel suo valore scientifico, tutt’altro essere d’origine pitagorica.” (though one might wonder if it has “scientific value” in the Pythagorean tradition). See also Sarton, Hist. 204.
88 Above, ch. I 2; below, ch. VI 4.
89 On ἄριστος see Solon 3.39 Diehl, Theognis 154, 946; on περίττος, Empedocles fr. 13. Line 18 of Cleanthus’ hymn to Zeus is significant: αὐλο καὶ τὰ περίττα ἐπιστοίς ἄρτια ἑρεμεῖαι.
90 ἦμαν is Indo-European in root and meaning.
VI. PYTHAGOREAN NUMBER THEORY

children should practice calculation. Either the procedure is successful, because the number is “symmetrically constructed,” or one person is left over—περιττόν. It did not need a Pythagoras to produce the insight that the problem “comes out right” with 2, 4, 6, 8, or 10 persons, but not with 3, 5, 7, or 9. Just as in Latin, English, and German,68 the Greek distinction between even and odd grew out of elementary, everyday calculations; but then, among the Pythagoreans, it took on a different and unpredictable significance.69

The term λόγος, in its mathematical sense of “relation, ratio, proportion,” has been attributed by von Fritz to the Pythagoreans, and, conjecturally, to Pythagoras himself.66 Its origin lies, he thinks, in Pythagorean musical theory: the λόγος, a “Mittteil von etwas Wesentlichem an einem Gegenstand,” “the communication of something essential about a thing” (81), is, in relation to music, the numerical ratio present. If one knows this, he knows the nature (Wesen) of the interval, and can reproduce it.68 And similar reasoning holds in the realm of geometry; the ratio 3:4:5 determines the shape of a right triangle, and with its help one can reproduce the right angle. Thus the λόγος would be “the group or bundle of numbers that lie hidden in a thing, by use of which it can be not only described but reproduced” (83).

Now a distinctive feature of the Greeks’ calculation of proportions, and in particular of Pythagorean musical theory, is the occurrence of certain terms like ἐπίπεδον λόγος, ἐπίστευσαι λόγος, and the more general ἐπίστευσαι λόγος, which from a German or an English point of view seem odd. Their sense is quickly evident; ἐπίπεδον, for example, “a third in addition,” means 1 + ¼ or ¾. But the question remains, what made these ratios so important that they alone, in the Greek language, have special names, while a fraction like ¾, unlike ¼ (ἐπίπεδον), can only be expressed in a cumbersome circumlocution. The answer is simple: they were terms used in the calculation of interest. Whoever lends money expects to get his principal back and a specified fraction of it in addition. This could be ἐπίπεδον (¼, or 33⅓%), ἐπίστευσαι (½, or 20%), ἐπίστευσαι (¾, or 16⅔%), sometimes ἐπίστευσαι (½, or 12 1/2%), at the lowest ἐπίστευσαι (1/10; this was what the gods received).71 It is certain that the practice of loaning money at interest went back before the time of Solon, and though there is no direct evidence, it can hardly be doubted that the expressions mentioned were in use that early—long before the day of Pythagoras.72 Thus when terms like this turn up in a musicological context—ἐπίπεδον and ἐπίστευσαι in Philolaus (fr. 6) and the general ἐπίστευσαι λόγος in Archytas (A19)—they are borrowed from everyday speech.

But the calculation of interest is in fact called λογίζομαι: λογίζομαι

61 Pl. Leg. 810b.
62 Cf. Hes. fr. 278 M.—W. (from the Melampodia): μέρους εἰς άρθρον, άτρα μέτρον γε μέτρον, κ.τ.λ. (περισσεύει, i.e. the bushel measure holds 9,999 figs).
63 Neither par and impar nor gerade and ungerade are borrowed from Greek. English odd comes from the Old Norse, and originally designates the third or other “odd” man in a council, whose vote decides in case of a tie (NED VII 88ff).
64 Epicharmus plays with the ideas of odd and even (DK 238a fr. 170,7 ff Kibbel = Alcimus FGrHist 565F6 = D.L. 3.11; cf. above, ch. II 3, n. 58; the authenticity of the fragment was denied by Wilamowitz, among others, Platon II 28.2): καὶ τὸν ἄρθρον τοῦ περισσεύει, καὶ τὸν ἄρθρον περισσεύει λόγον, λόγον τῷ περισσεύει λόγῳ, διακειται καὶ τῷ κτίσκου αὕτης εἴμειν; . . . Luckily, in this case we know the comic context in which the words occurred. A debtor is trying to free himself of his debts by using the claim that he is no longer the same person; we know from Chrysippos (ap. Plut. Comm. not. 1083a; cf. Pl. Tht. 132d–e) that this περὶ αὐτών τῶν λόγων did occur in Epicharmus. When something is added or subtracted, what is left is no longer the same thing; all of us are constantly gaining weight . . . and so on. The proposition is proved from two examples, involving number and size respectively. This restriction is not specifically Pythagorean, but essential to the argument, which is only valid with relation to quantity. The further subdivision of numbers into odd and even seems less essential (Rostagni, Verbo 281), and therefore an indication of external, presumably Pythagorean, influence; but, for the idea to be clear enough for the comic stage, it was necessary that the general concepts be replaced by something more specific. This is why the idea of measure is expressed from the start as περισσεύει and the idea of number as even/odd. By the addition of a single pebble a number changes from odd to even or the reverse; it is precisely the notion of odd and even which makes it clear that the very smallest alteration signifies a fundamental change. Even if one were to assume that there is a relation between this passage and the Pythagoreans, the “number theory” presupposed would be the inductive and graphic, rather than the deductive. See also von Fritz, RE XXIV 203–205.
65 ABC 1955, 81ff (cf. RE XXIV 199).
68 Xen. Vect. 3.9.
69 Demosth. 34.23.
70 Demosth. 50.17.
71 IG 7.12 (443/432 B.C.); cf. IG VII 2263 = SIG 544 (Oropus, 3rd century B.C.).
72 ἐπίστευσαι τὸ ἐπίστευσαι: IG 14.25 (446/441 B.C.), 39.35 (446 B.C.). Xen. Hell. 1.7.10 et al. ἐπίστευσαι means here simply “a tenth,” as ἐπίστευσαι means “a fifth” (Ar. fr. 201 = Harpocr. s.v. ἐπίστευσαι, with further ref.; also IG VII 1607.1 = SIG 972.1); but it is precisely the tenth or fifth that must be added, “extra,” because the other party lays claim to it. In the Sargasso battle, the Locrians outbid the Crotoliates by vowing a ninth—ἐνακτεῖνα instead of δεκακτεῖνα (Justin 20.3.3).
73 That proportion was used at this time is also shown by the expression ἐπίστευσαι (on which see D. Lotze, “Hektemoroi und vorholischer Schulrecht,” Philologus 102 [1958] 1–12).
VI. PYTHAGOREAN NUMBER THEORY

The connection of proportion and music, resulting in the equation of *διάστημα* with λόγος, remains to the credit of the Pythagoreans; and in one aspect of the theory of proportion, the doctrine of “means,” Pythagorean influence is a possibility. The three μεσότητες, the arithmetic, geometric, and harmonic means, are generally regarded in the tradition as a discovery of Pythagoras. 80 The fact that all three means have a role in the *Timaeus* 81 could rouse suspicions about the tradition involving Pythagoras himself. But Theaetetus already knew the system of the three means, and used it, in a rather forced manner, as the point of departure for his classification of irrational lines; 82 thus the

means are presumably older, and they are closely related to Pythagorean music theory. The name of the “harmonic mean” is to be explained directly from the latter; the Mese is the harmonic mean of the octave Nete-Hypate. 83 In fact, it is attested that Archytas and Hippasus introduced the term *άρμονική μέσοτητα* in place of the older *μεσοτήτα μέσοτητα* 84 and the series 6, 8, 9, 12 is presupposed in the experiment which Aristoxenus ascribes to Hippasus. 85 If this third and most complicated of the means was known, then the two others surely were. 86 It would be conceivable that the harmonic mean was discovered in the context of Pythagorean music theory; 87 but it also has another use: along with the arithmetic mean it provides increasingly closer approximations of the square root; 88 and, if, as Lamblichus says, the “most perfect proportion,” 12:9 = 8:6, was introduced by Pythagoras from the *apotome* to harmony, as is stated by Eudemus, the Peripatetic. Also, 2.17 p. 138: “those who have written concerning these things declare that the Athenian Theaetetus assumed two lines commensurable in square and proved that if he took between them a line in ratio according to geometric proportion, then the line named the medial was produced, but that if he took (the line) according to harmonic proportion, then the apotome was produced.” On the concepts medial, binomial, and apotome, see Euclid 10; on the connection with the three means, Junge-Thomson 17. This fragment of Eudemus is important because its content is independent of the *Theaetetus* of Plato and is thus reliable evidence for the existence and importance of the mathematician Theaetetus. It was cited by Sachs (135, 177), Frank (272; cf. above ch. V 2, n. 41), and von Fritz (*RE* V A 1354f).

84 I.e., if one assigns the larger number to the Nete (above, ch. V 1 n. 47): 6 is Hypate, 12 is Nete, 8 (the harmonic mean) is Mese, and 9 (the arithmetic mean) is Parmase.

85 *Iam.* In *Nic.* 100.22 = *DK* 18.15: ἥ σεκε μὲν ὑπεναντία λεγομένη... ὑπὸ δὲ τῶν πέρι Ἀρχιτάν ἀνθίσθη καὶ ἦπιπασον ἀρμονική μεταλθέσθη. Also *Archytas* fr. 2, *DK* I 435.201: τρία θ’ ὑπεναντία (μέση), ἵνα καθολικό ἀρμονικόν; I 436.81: ἓ 8’ ὑπεναντία, ἵνα καθολικό ἀρμονικόν. In two other passages Hippasus and Archytas are named together, in reference to the doctrine of means (*Iam.* In *Nic.* 113.16ff, 116.1ff). Tannery concluded from this (*MS* II 190, HS: H 394) that Archytas must have cited Hippasus. Actually, Lamblichus states in the two later passages, in contradiction to his own earlier account, that Hippasus and Archytas had already discovered the fourth, fifth, and sixth means (cf. n. 92); but it is possible that he himself misunderstood his source.—Philolaus, too, knew the harmonic mean (A24).

86 *Iam.* In *Nic.* 101.15ff. *Iam.* In *Nic.* 100.15ff.

87 The geometric mean as διομος, 31c-d; the arithmetic and harmonic means, 36a, cf. *Plut.* 991a; Sachs 120ff.

88 The geometric mean as διομος, 31c-d; the arithmetic and harmonic means, 36a, cf. *Plut.* 991a; Sachs 120ff.

89 This is attested by Eudemus in a fragment preserved only in an Arabic version (overlooked by Wehrli): Pappus, *Comm. on Euclid* X, ed. G. Junge and W. Thomson (Cambridge, 1930; earlier ed. by Woepcke, Paris, 1855) 1.1 p. 63: “it was nevertheless Theaetetus who distinguished the powers (i.e. the squares) which are commensurable in length, from those which are incommensurable (i.e. in length) [this is from Pl. *Thet.* 147d et seq.] and who divided the more generally known irrational lines according to the different means, assigning the medial line to geometry, the binomial to arithmetic, and
VI. PYTHAGOREAN NUMBER THEORY

Babylon, we may be on the track of something genuine, namely that the arithmetic and harmonic mean were used in Babylonian calculating technique to find the square root. If so, a rule used in Babylonian calculation has been transposed into Pythagorean number speculation, just like the rule involved in the "Pythagorean theorem." The original name itself, ἐνυστία μεσότης ("subcontrary"), is comprehensible from its use as a tool in calculation; one forms first the "numerical mean," then its appropriate "reversal." Here too, then, Pythagorean musical theory is an outgrowth of practical methods of calculation. The further development of the doctrine of means only begins with Eudoxus; the accomplishment of the Pythagoreans around and after Hippasus is rather in the application and interpretation of known methods than in the foundation of a theory of numbers.

An unquestionable piece of scientific Pythagorean arithmetic is Archytas' proof that a superparticular proportion cannot be divided into equal parts by a mean proportional. In contrast with the inductive

98 Lam. In Nic. 118.23ff: εὖρομεν' ἐκ τούτου οὖν ἐκ τῶν Βαβυλωνίων καὶ τοῦ Πυθαγόρου πρώτου εἶναι "Εὐδομήνα ὕθελον. The proportion in question is, greater number: arithmetic mean = harmonic mean = smaller number, or, greater number: harmonic mean = arithmetic mean: smaller number (also, therefore, greater number × smaller number = arithmetic mean × harmonic mean). The Babylonians did not know the concept of proportion (Becker, Arch. f. Mus.-W. 1957, 150); but the "mean" may very well have been employed in calculation.

99 Becker, MD 65.

100 Above, n. 9.

101 He discovered the fourth, fifth and sixth means: Eudemus fr. 133 = Procl. In Eucl. 67.5ff = D22 Lasserre; lam. In Nic. 101.1ff (above, n. 84); Nicom. Ar. 2.28 and lam. In Nic. 113.16ff only call the arithmetic, geometric, and harmonic means "ancient." Philip of Opus also wrote Μεσότητας (Suda s.v. φιλοσόφος). The seventh to tenth means were discovered, according to lam. In Nic. 116.4ff, by the Πυθαγόρειος Μυωνίδης and Euphranor, not only later than Eudoxus but also later than Eratosthenes; not all Pythagorean-athenean is pre-Platonic. It does not seem possible to discover any further detail about Myonides and Euphranor (though a Euphranor wrote Περί ἀθλητῶν, Ath. 4.184e; cf. also Nicom. Ar. 2.28).—Thymaridas, whose ένυστία is mentioned at lam. In Nic. 62.11ff, has been placed "in the time of Plato or somewhat earlier" (Tannery, MS 1106ff, II 112ff; HSfH 356; also Becker, QS 4, 165ff; MD 44s), because the proposition in question can be proved by a (complicated) pebble figure; but for the chronology this does not mean any more than a mere possibility. Heath, Math. I 94, lays down that the terminology used about the ένυστία by lamblichus agrees with that of Diophantius (probably 3rd century a.d.); and Diels (DK I 447 n.) emphasized that the definition of the μοῖρας as περάνωσα συνότης, which is ascribed to the same Thymaridas by lam. In Nic. 111.2, cannot be dated earlier than Plato. Thymaridas is named as a pupil of Pythagoras at lam. VP 104, and edifying anecdotes about him are given at lam. VP 145 (after the "tripartitum" 13 L. 8.9 f) and 239; in the catalogue of Pythagoreans (p. 145.5) he is listed as a Parian. How tradition and legend got connected with a specific mathematical problem remains a mystery.

99 Archytas A19 — Boeth. Mus. 3.11. See also ch. V.

pebble games, a general proposition is here proved deductively, and the numbers, represented by letters, are obviously thought of as line segments, as in Euclid. A number of arithmetical concepts, like proportion, divisibility, the smallest numbers in a given ratio, relative prime, and mean proportional, are confidently handled; the structure follows conventional order: statement of what is "given," statement of theorem, proof; and the method of proof is the reducto ad absurdum. Above all, Archytas presupposes a whole series of arithmetical propositions and expressly cites an auxiliary theorem. Tannery, who called attention to the proof of Archytas transmitted by Boethius, concluded from this that Archytas must have had a kind of Elements of Arithmetic, and van der Waerden undertook to reconstruct, systematically, this number theory presupposed by Archytas, coming to the conclusion that in all essentials the material of Euclid's seventh and eighth books must already have been in existence; book 7, he thought, "existed in written form before 400 B.C." and "had been taken over by Euclid without significant alteration." Book 8 was the work of Archytas himself. If this were correct, we should have an imposing edifice of Pythagorean arithmetic of an entirely different kind from what Speusippus, Aristotle, Theo, and Nicomachus lead us to expect.

It is impossible to discuss in detail here the analysis of Euclid and the problem of evaluating his work as a mathematician, but we may

94 Unlike Euclid, the author designates with letters not the ends of line segments ("line AB"), but the segments themselves; DE means the number D plus the number E. This is not, however, an ancient feature (as maintained by Tannery, and van der Waerden, MtAms 1947-1949, 134), but the method followed by Boethius (also Mus. 4.2, in the translation of Sect. can. 3). Every exposition in the history of mathematics "modernizes."

95 To be sure, the structure can be the work of the intermediate source.

96 Archytas A19: "... qui enim sunt minimi in eadem proportione, quibuslibet alis numeris, hi priimi ad se invicem sunt," corresponding to Eucl. 7.22.

97 Tannery, MS 1106ff, cf. Heath, Math. I 90; Eucl. II 295; Becker, MD 44s.

98 MtAms 1947-1949, 146.


100 Van der Waerden, S 197: "Euclid is by no means a great mathematician," though he is "the greatest schoolmaster known in the history of mathematics" (196). This judgment calls for contradiction; the Elements does constitute a scientific achievement, rather than merely a schoolbook. And it is based in part on a mistranslation. In the scholiou to Euclid, p. 644.8 Heiberg, one reads, after the account of the discovery of the regular solids by the Pythagoreans and Theaetetus, Εὐκλείδης δὲ ἐπηγράφεται καὶ τὸν τὸ βιβλίον διὰ τὸ στοιχεῖον τάξιν ἐπιστευκόμενον, "This book too [sc. 13] bears Euclid's name, because he gave it the order of stoicheia" (see Burket, Philologus 1959, 189ff). Cf. Schol. Eucl. p. 73.4 (after Proclus 69.4ff), συνήγειας εἰς στοιχεῖον, τάξιν αὐτοίς καὶ ἀποδίδεις ἀρχικὴν ἐπίθεσις... Van der Waerden, however, translates (S 173), "This book also carries Euclid's name, because he embodied it in the Elements," as though he had taken it over from an earlier author without alteration or addition.
VI. PYTHAGOREAN NUMBER THEORY

refer once more to the large number of mathematicians who were active in the brief period between Plato and Euclid.\(^{102}\) Several of these were concerned, precisely, with writing or rewriting στοιχεία.\(^{102}\) The miracle of Pythagorean arithmetic would indeed be amazing, if in spite of this, Euclid had taken over “without significant alteration” expositions that were over a hundred years old.

Here we are confronted again with a basic problem of mathematical history: to be sure, it can be shown with perfect exactitude what propositions Archytas must have been presupposing as proven—but only on the presupposition, on our part, that Archytas built up a complete theory of number, in accordance with the demands of Euclidean precision. Even the suggestion that Archytas was concerned to prove a proposition in number theory is a dubious formulation. Boethius cites the proof in the context of music theory; and among the works of Euclid, it is found in the *Sectio canonis*, not in the number theory of the *Elements*. There is no question that for Archytas the musical application of the theorem—the indivisibility of the intervals of the octave, the fifth, the fourth, and the whole tone—was of primary importance. To what extent he was able to refer to a previously existing arithmetic, and to what extent he understood and fulfilled the demands of a complete, deductive system, remains to be established.

Scholars have laid emphasis on the fact that Archytas’ proof was identical with the one found in the *Sectio canonis* (3).\(^{103}\) Boethius, however, characterized Archytas’ proof as “nihilum fluxa” and replaced it with another, consisting of a literal translation of the *Sectio canonis*.\(^{104}\) Thus the tradition which Boethius was following emphasized not so much the agreement as the difference between Archytas’ proof and that of the *Sectio canonis* and found Archytas’ inadequate. In order to understand Boethius’ criticism, we must compare the two proofs in detail.

The first step they have in common: for the given superparticular proportion \(a:b\) is substituted that of the two smallest numbers in the same ratio; \(m:n = a:b\). The proof continues in the *Sectio canonis*, as follows: (1) \(m\) and \(n\) are by definition, as the smallest numbers in the given ratio, prime to each other.\(^{105}\) (2) According to the definition of the superparticular proportion, the difference between \(m\) and \(n\) is a common divisor of \(m\) and \(n\). (3) Thus \(m - n = 1\). (4) Accordingly there is no mean proportional between \(m\) and \(n\), \((\epsilon\ and by extension there is none between any numbers that stand in the same ratio, \(a:b\). Archytas, for his part, first establishes step (2), mistakenly introducing, in the statement of the reason, the concept “minimi.”\(^{107}\) Then he sets up proposition (3), as an assertion and proves it in detail, by reductio ad absurdum of the opposite. This is a basically superfluous repetition,\(^{108}\) made necessary by his view that 1 is not a number.\(^{109}\) The only crucial statement is (1), which is cited in full. After another repetition,\(^{110}\) steps (4) and (5) follow without detailed rationale; the necessary, and not quite evident, auxiliary theorem is not cited.\(^{111}\)

The repetitions here could be regarded as no more than a stylistic defect, but the fact that necessary and nonnecessary presuppositions are not distinguished strictly is more disturbing; and for this reason we can hardly suppose that it is only some copyist’s fault that the necessary auxiliary theorem is not cited. A particularly arresting formulation is found in the citation, in Euclid, of another lemma: “qui enim sunt minimi in eadem proportione quibuslibet aliis numeris, hi primi ad se invicem sunt, et solam differentiam retinent unitatem.”\(^{112}\) Tannery excised the last five words as “absurd,”\(^{113}\) and the more recent paraphrases leave them out;\(^{114}\) but the criticism of Boethius, that is, that

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\(^{102}\) This depends on Eucl. 7.22.

\(^{103}\) Citing Eucl. 8.8.

\(^{104}\) DK I 429.35: “quia omnis igitur sunt minimi in eadem proportione C, DE et sunt superparticulares . . .” Only their superparticularity is necessary for the conclusion.

\(^{105}\) DK I 429.36: “DE numerus C numerum partem una sua eiusque transcendent; sit haece D”; then 429.38: “si enim est numerus D et pars est eius qui est DE, metietur D numerus DE numerum, quocirca et E numerum metietur; quo fr, ut C quoque metietur; utrumque igitur, C et DE numeros metietur . . .”

\(^{106}\) DK I 429.37f: “dico quia omnis [57a] D [i.e., \(m - n\)] non erit numerus sed unitas.”

The proof of irrationality in Eucl. 10 App. 37 proceeds in a quite similar manner.

\(^{107}\) DK I 430.5: “igitur DE numerus C numerorum unitate transcendent.”

\(^{108}\) I.e., Eucl. 8.8; cf. n. 106. In the list of propositions presupposed by Archytas which van der Waerden sets up (SA 111), the auxiliary proposition Eucl. 8.8 is one of the most important links.

\(^{109}\) DK I 430.3; cf. Eucl. 7.22.\(^{110}\) MS III 248. (He thinks this is an interpolation by Boethius; but shall we suppose he made interpolations only to argue against them?)

\(^{111}\) E.g. Heath, Math. I 215; van der Waerden, SA 111.
VI. PYTHAGOREAN NUMBER THEORY

4. Pythagorean Geometry and Mathematical Secrets

in a sense, Pythagorean philosophy in general—achieved its final form in Platonism. Before Archytas there were number games accompanied by the "interpretation" of, and "reverence" for, number.

3. PYTHAGOREAN GEOMETRY AND MATHEMATICAL SICHEIRTS

A passage of Iamblichus, to whose Aristotelian provenance we have already drawn attention, reads as follows:1


The Pythagoreans, having devoted themselves to mathematics, and admiring the accuracy of its reasonings, because it alone among human activities knows of proofs, and seeing [equally] the facts about harmony, that they happen on account of numbers, generally admitted [and (seeing) the mathematics of optics depending on (diagrams), they deemed these (facts of mathematics) and their

Rep. 530, i.e. the circle of Archytas. Κ αἴτιος πραγματεία in relation to λογίαι also mentioned by the Xenophontic Socrates (Mem. 4.7.8).—The assertion so often repeated, that Plato differentiated λογική, as the practical art of calculation, from ἀριθμητική, as the theory of number, and that the latter was an achievement of the Pythagoreans (Tannery, HStC 381; Heath, Math. I 13f; Dodds, Gorg. 199; K. Vogel, "Beiträge zur griechischen Logistik," SbMii, math.-ph. Kl., 1936, 35f; Becker, MD 45).

is an error long ago corrected (J. Klein, "The griechische Logistik und die Entstehung der Algebra," QSt 3 [1936] 23f; A. Wedberg, Plato’s Philosophy of Mathematics [Stockholm, 1935] 22f). Plato demands mathematics as pure theory for λογική as well as for ἀριθμητική, and there is a practical "arithmetikē" as well as a theoretical "arithmetikē" or "logistikē." (Phlb. 56c). The difference between "counting" and "calculating." (Chrm. 166a, Gorg. 451b; cf. Rep. 522c, Epin. 978e; the two together, Rep. 522c-525a, Phlb. 273c, Hr. mi. 367a; logiakọ́n, Euthyd. 290c, Rep. 525a, Pol. 259c, Thl. 145c; cf. also Rep. 510c: οἱ περὶ λογικῆς... ὁ προσέμενθεν τοῖς περιτόμοις καὶ τῷ ἄρτῳ... with Thl. 198c: ἀριθμητική as the ἐπιστήμη ἄρτοι καὶ περιτόμοι. The differentiation of theoretical arithmetic and practical "logistics" was set up by Geminus (by contrast to the "Pythagorean" quadrivium: Procl. In Eucl. 38.11f; Geminus-Antiôs ap. Hero Def. 135.5 p. 98 Heiberg). Olympiodorus foisted it onto Plato (Olymp. In Gorg. 450b p. 31.47f Norvin; cf. Schol. Chrm. 165c).

1 Lam. Comm. math. α. p. 78.8-18; cf. above, ch. 12, n. 112.

118 Boeth. Lute. 3.11 p. 286.7ff (no longer included in DK): "Et secundum Archytas quidem rationem idicus in superparticulāri nullus medicus terminus cadit, qui aequālītur dividit proportionem, quoniam minimi in eadem proportione sola different unitate, quasi vero non etiam in multipli proportione minimi eadem unitatis differentiam sortium terrum, cum plures videat esse multiplices praeter eos, qui in radiicus [πυθαγόρειος = minimi, Archytas A17] collocati sunt, inter quos medius terminus scindens aequālītur eadem proportionem possit aptari. Sed haec, qui arithmeticos numeros diligenter inspectaret, facilis intelligēt. Addendum vero est, id ita evenire, ut Archytas putat, in sola superparticulāri proportione; non autem universaliter est dicendum. Perhaps Boethius did not himself understand his "reprehensio" (285.8). The concluding sentence is correct; what Archytas (apparently at least) expressed in general terms, that the "difference" of the "minimi" is one, is valid only for the superparticular proportion.

119 It would also be possible that an intermediate source transposing Archytas' proof into the later form created the confusion; or that archaic terminology was misunderstood, and Archytas meant, with a phrase like ὅπως εἴχαν τὴν ἀριθμόν διαφορά, the "common divisor" (the common divisor is found by alternate subtraction).

117 Above, ch. 2. But the title ἀριθμοῦ is also found in Democritus (B11 o).

116 The programmatic title of a work of Xenocrates (DI. L. 4.13).

115 Pl. Rep. 525c; on the subject in general, aside from Rep. 522c et seq., Philb. 56d, Thl. 195d, Epin. 990c. When Plato, at Rep. 525c, cites αἱ περὶ τῶν ἀριθμῶν, the "professional," for their axiom that the monad is indivisible and fractions are not allowed in the calculation of proportions, he may well mean the same Pythagoreans he speaks of at
principles to be, generally, causative of existing things, so that whoever wishes to comprehend the true nature of existing things should turn his attention to these, that is to numbers [and the geometrical entities among existing things] and proportions, because it is by them that everything is made clear.

In the two places where geometry is mentioned the course of the argument is deranged. To begin with the second, λόγος ("proportions") is without an article, and, as regards the sense, is intrusive after the clearly established dichotomy of arithmetic and geometry; and amidst the Aristotelian formulations the phrase τὰ γεωμετρούμενα ἐν τῶν ὄρθων sounds like late Platonism. To delete the offending words gives τῶν ἁρμονίων καὶ λόγων, which belong together and suit the context. In the earlier passage, too, there are several difficulties. The word ἔνωσις is peculiar and unexampled, and δὲν destroys the continuity. τὰ περὶ τῶν δὲν μαθήματα is not only unclear, but factually wrong; it is not the fact that mathematics is mathematical or geometrical that moved the Pythagoreans so deeply, but that an everyday concern like music, impinging on us directly through the senses, turns out to conform to mathematical rules. Thus τὰ περὶ τῶν δὲν μαθήματα is not only analogus to τὰ περὶ τῶν ἁρμονίων. Again, to delete the reference to geometry not only makes ἔνωσις superfluous, but gives a smoother and more meaningful sentence. Aristotle, in the Metaphysics (983b31), names the connection of music and number as one of the roots of Pythagorean speculation, but never brings this into relation with geometry. Aristotle, too, in the one relevant passage remaining to us (fr. 23), speaks only of "concern with numbers," which Pythagoras especially "honored" and advanced. It seems likely, then, that Iamblichus, in a passage drawn from Aristotle, added the references to geometry, so as to emphasize the many-sidedness of Pythagorean μαθήματα, whereas in the original testimony geometry plays no role, in contrast to that of arithmetic.

The most important achievement that we know of that was made by a Pythagorean in the field of geometry is the famous solution, by Archytas, of the problem of doubling the cube. Successful accomplishment of this task, talked of at least a generation but never yet performed, stimulated further action. The improved solutions, from Eudoxus and Menelaus to Eratosthenes, do not diminish but confirm the credit of the πρῶτος εὐπρήξις. But the mathematician who was Archytas' immediate predecessor in this regard was Hippocrates of Chios. It was he who had reduced the problem of doubling the cube—or finding \( \sqrt{2} \)—to that of the "two mean proportionals," and thereby given it a planimetric sense which first made the solution possible. Thus Archytas, in his decisive geometrical achievement, is not the representative of a special, Pythagorean tradition, but of the main tradition of Greek geometry in general, represented in his time by Hippocrates of Chios, and, if he was successful, it was probably mathematica quaedam facultate, non pythagorea.

The real problem of Pythagorean geometry, therefore, in the history of thought and of mathematics is this: are we to assign the well-attested geometrical accomplishments of Pythagoreans to the time before or after Hippocrates of Chios? Only in the former case could we speak of the Pythagoreans as founders of Greek geometry; in the latter, it is only a matter of individual contributions to the development and perfection of something already there, quite respectable achievements but on an equal basis with others, and themselves dependent on older and more basic work. Only Heidel put the problem in this fashion; "Pythagorean geometry" is usually treated, as though these perfectly natural, before Hippocrates. It is assumed that he had before him at least the content of the first four books of Euclid, and that these are of Pythagorean origin. This is in spite of the fact that the tradition names Hippocrates as the first author of στοιχεία.

Citing Eudemus, Proclus refers to two passages in the first book of Euclid to the Pythagoreans—though not to Pythagoras. These are the

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8 Archytas A14 = Eudemus fr. 141 = Eutocius In Archim. IIP 84 Heiberg. On the problem of duplication of the cube, see Heath, Math. I 244ff; Becker, MD 73ff. To be sure, Archytas' construction cannot be executed with ruler and compass.

9 Hippocrates, DK 22.4 = ps-Eratosth. Epist. ad Ptol. ap. Eutocius In Archim. IIP 88.17 Heiberg; also Procl. In Eucl. 213.7ff (not in DK).

10 Heidel, AJP 1940.

Tannery, Géom. 106ff (cf. above, ch. VI 1, n. 44); Rey 240 (despite his skepticism in principle, 230ff); van der Waerden, SA 135, with reserve, Heath, Math. I 166ff.
for history of mathematics dependent on him, did not have anything more to say about Pythagoreanism in the first book of Euclid. This in turn gives us certain clues to the evaluation of Pythagorean contributions in the Euclidean corpus: there are two Pythagorean theorems in the first book, and other propositions, proofs, and constructions are ascribed to Thales or Oenopides. The fourth book is Pythagorean, as the fifth is from Eudoxus. What we are concerned with, then, is certain individual, restricted portions, to be ranked along with what other mathematicians “discovered”; there is no reason to posit a unique contribution by the Pythagoreans or a substratum of Pythagorean Elements. More than once, before Euclid, Στοχεία had been written, and each time their structure and methods of proof had been thought through again.

The chronology of these Pythagoreans is not guaranteed by any external testimony, whether we go up to the early fifth century or down to the fourth, everything depends on combinations. Since Theaetetus, who was still very young in 399 B.C. and fell in battle in 369 B.C., brought to completion the treatment of the regular polyhedra, the “Pythagoreans” in question must be placed earlier. Eudemus names Theaetetus, Archytas, and Eudoxus, mentioning their individual accomplishments, so that the anonymous Pythagoreans probably belong to the time before Archytas. The decisive question of their

11 Procl. In Eucl. 379.2 = Eudemus fr. 136 = DK 38B23 (Eucl. 1.32).
12 Procl. In Eucl. 4:19.15 = Eudemus fr. 137 = DK 38B20 (Eucl. 1.44); cf. Iam. Comm. math. sc. p. 75.21, Plutarch, above, ch. VI 2, n. 7.
13 It can be shown that the scholia to the first book are mostly from Proclus (cf. e.g., above, ch. VI 2, n. 6), and those to the tenth book from Pappus (cf. e.g., below, n. 57). Both cite Eudemus.
14 Schol. Eucl. 273.3: εἰρήμα δὲ τοῦ βαλλόν τῶν Πυθαγορείων, 273.13: τὰ δὲ ἐκείνη τοῦ προέκυκλον βαλλόν τις (read ίς) δυσβαλλόν τῶν εὐρήματα. This testimonia are missing in DK, and not mentioned by Heath (Math. I), van der Waerden (SA), or Becker.
17 Procl. In Eucl. 45.5ff; cf. above, ch. VI 1, n. 119.
VI. Pythagorean Number Theory

1. Pythagorean Geometry and Mathematical Secrets

relation to Hippocrates of Chios depends on inner criteria, on the relation of concepts, propositions, constructions which seem to be presupposed by the Pythagoreans or by Hippocrates. In a closed system like geometry, so many combinations of the same propositions are possible, and so many proofs for the same conclusions, that it is never possible to establish chronological priority with mathematical certainty.

The application of areas was known to Plato, but Hippocrates of Chios, for a problem soluble by this method, used the method of "inclination" or "verging" (vēdou); it looks as though the application of areas was at least not fully developed in Hippocrates. A special case of the application of areas is the construction of the golden section. The star-pentagon or "pentagram" and the dedekadron play a role in Pythagoreanism, but both figures had been known, in a purely empirical way, for centuries, without any mathematical construction. Euclid gives the construction of the golden section, and therefore of the regular pentagon, in book 4, which is ascribed "as a whole" to the Pythagoreans; and it depends on constructions in book 2. Hippocrates

n. 12. In spite of this, Frank's formulation is too sweeping: "What is ordinarily called Pythagorean mathematics is in essentials the work of Archytas, Thales, and Euclides." (232). Thales is never called a Pythagorean, and Eudoxus is treated in the tradition as an original thinker.

24 On the idea and the method of the application of areas, see Heath, Math. I 150ff; Becker, MD 60ff; van der Waarden, SA 118ff. Euclid 2 has a generalized and therefore more complicated form of the application of areas. — παράπλευρα, Pl. Rep. 527a; a specific problem, Pl. Meno 86c (cf. Heath, Math. I 298ff; Becker, Gnomon 28 [1956] 225). διαρκείας, to which Plato alludes here, is a method developed by Leon, one of his contemporaries (Eudoxus fr. 133 = Procl. In Eucl. 66.22). Just as Plato learned from Eudoxus in the field of astronomy (above, ch. IV 2), so in mathematics he learned from his contemporaries, not from "ancient" Pythagoreans.

25 Heller, AbhBl 1958, 9, comes to the conclusion that Hippocrates still did not know the method of application of areas. See also Becker, MD 59ff.

26 Above, ch. II 4, n. 81; below, n. 65. That the magical efficacy of the pentagram does not depend on mathematically exact construction is shown by the fact that they are often drawn quite irregular. See, e.g., the Gallic coins in W. Deonna, Bull. de l'Ass. Pro Aventicum 16 (1954) 47; he also mentions (p. 24 n. 4) an empirical method of casting metallic dodecahedra, with no recourse to Euclidean geometry. — Heath conjectures that the "golden section" was a discovery of the Pythagoreans (Math. I 160ff; Eud. 2.97ff); cf. Becker, MD 63. Heller, AbhBl 1958, 9, tries to reconstruct an earlier νεών construction of the regular pentagon. In Eudoxus fr. 133 = Procl. In Eucl. 57.6 we read, (Εὐθανατο ό τοῖς τόμοις ἄφθινα λαβόντα παρὰ Πλάτωνας εἰς πλῆθος προσήχοντο καὶ ταῖς ἀναλογίαις ἐν ἄσυμοις ἐποίησαν. Sachs 97, 1281), Sarton (Hist. 442ff), and Heller (AbhBl 1958, 12f) interpret this as referring to the golden section, which would imply that the report of the scholiast on Eucl. 4 (above, n. 14) is false. But the formulation in Proclus is so general that this conclusion does not hold up. The topic may be the new, systematic development based on Euclid's new theory of proportion (as Heller thinks), or it could be the systematic development of Eucl. 2. In the first 11 of the 14 propositions of the second book the problem is the division (θέματα) of a line.

27 Namely Eucl. 2.12-13 (the extension of the "Pythagorean theorem" to scalene triangles) and 2.14 (the transformation of rectangle into square, construction of mean proportionals, and the geometrical construction of square roots)—the culminating propositions of the second book.


29 Heath, Aristotle 23ff, on Arist. Met. 10.53224, ἀνάγκει. The necessary auxiliary line is drawn from the base of the triangle upward in Eucl. 1.32, in Eudoxus fr. 136 through the apex of the triangle and parallel to the base. — Γενέμιος (Eutocius on Apollonius, II 170.4 Heiberg) states that the proposition about the sum of the angles of a triangle was first proved separately for equilateral, isosceles, and scalene triangles, πρῶτον ἐν τῷ ἰσοστρέφον καὶ πάλιν ἐν τῷ ἰσοστρέφον καὶ ἰσόστρεφον τό ἰσοδίδων καὶ τό ἰσοδίδων τό ἰσοδίδων. ... (An. post. 732a25). See Heath, Math. I 136, Eucl. I 315f, following Heiberg. But Becker has shown that the individual proofs can be grasped easily and clearly from considerations of symmetry (Grnl. 27, MD 39).


31 Tannery, Gnom. 86; DK 41.12-14 (Eudoxus fr. 138); above, ch. VI 1, nn. 128-130.

32 I.e. 24° (correct value, 23° 27' 3').
Hippocrates of Chios. They belong in the period, approximately, between 430 and 400 B.C.; it remains obscure what individuals are lurking beneath the general appellation of “Pythagorean” (above, n. 23).

The most important “discovery” was that of the application of areas. Scholars now agree that the point of these exercises is primarily algebraic; they provide an equivalent for quadratic equations. In Babylonian mathematics they had been solved algebraically, and the individual examples of the application of areas correspond exactly to the methods developed there. Thus the “geometrical algebra” of the Greeks is revealed as the transposition of Babylonian techniques of calculation into geometrical form. The occasion for the development of this seemingly complicated “geometrical algebra” was the discovery of the irrational, which made impossible, according to Greek views, the solution of quadratic problems by use of number. In fact, the discovery of the irrational is also ascribed to Pythagoreans, or even to Pythagoras himself; but the actual situation is extremely hard to grasp because of the confusion of ancient legend and allegory, and the modern conjectures they have inspired. The ancients speak of this situation in terms of “secrecy” and “treason,” the moderns of the “Grundlagenkrise der griechischen Mathematik.”

Pythagorean lore was at least in part secret, as is attested by Aristotle and Aristothenes, and as is natural in an archaic social order. On the other hand, Pythagorean “secrecy” was undoubtedly misused in later times, as a carte blanche to permit the publication of forgeries as newly discovered books, and brand the discoveries of later thinkers as plagiarism of Pythagoras. Names that occur in such context are Empedocles, Philolaus, and Oenopides, a certain musical theorist called Simon, and Plato. The oldest authorities are Timaeus of Tauromenium and Duris of Samos, who both seem to show a local patriotic partiality to Pythagoras. It is significant that, in the case of Oenopides, Eudemus, the earlier witness, has nothing to say of plagiarism. A more famous and more important story is that of the mathematical treason of Hippasus, bringing with it the impressive anecdote that he was drowned at sea, as punishment; and this same story is bound up with the discovery of the irrational.

The discovery of the problem of the irrational in geometry, and the development of the ability to cope with it, is a fundamental accomplishment of Greek mathematics which holds a lasting fascination for modern historians of science. The tradition of secrecy, betrayal, and divine punishment provided the occasion for the reconstruction of a veritable melodrama in intellectual history. The realization that certain geometrical magnitudes are not expressible in terms of whole numbers is thought of as “une véritable scandale logique,” bound to shake the very foundations of the Pythagorean doctrine, which maintained that “everything is number”; for to the Greeks, number and irrationality are mutually exclusive. Thus one comes to speak of a Grundlagenkrise—a crossroads or dilemma as to the very foundations of Greek and Pythagorean mathematics in the fifth century—and to see in the tradition about the death of the “traitor” a reflection of the shock and despair that this discovery must have brought: “O that the irrational

40 Duris FGrHist 76F3 = Por. VP 3 = DK 56.2: Pythagoras’ son Arinnaeus is said to have set up in the sanctuary of Hera (on Samos) a dedicatory monument with the epigram

Παραγωγή ϕιλος υπὸ 'Αριμνηστός μ' άνθήκη,
πολλὰς ἐξερχόμενα εἰς λόγος σοφίας.

τότε δὲ ίμαντα Σίμων τὸν ἀρμονίκο καὶ τὸν κανόνα συντεταγμένον ἐξενεγήκει τὸς ἱστον, εἶναι μὲν οὖν ἐπά τὸς ἀνάγεγραμμενος σοφίας, διὰ δὲ τὴν μίαν, ἢς Σίμων ῥήματος, συναφασθήσας καὶ τὸς αὐτὸς εἰς τὸν αὐτήματι γεγραμμένας. Diels (DK 1 445 n.) thinks the σοφία referred to are the μεθοδεύς, but the seventh of them was not discovered till after Eratosthenes (above, ch. VI 2, n. 93). The word σοφία suggests rather the calculation of a scale (as Wilamowitz, Platon II 94). There were therefore, aside from the Pythagorean musicologists, other ἀρμονικοὶ in competition with them.

41 Above, ch. III 1, n. 28.

42 Tanner, MSc I 268, Celem. 98, HSCH 259; Arist. Met. 1021a: ὁ γὰρ ἀρμόδιος σύμμετρος. In Diophantus the expression occurs that the number sought γίνεται ὁδὸς ἢ ρήσος (e.g. 4.10); see esp. Hase-Schoelz 65f; below, n. 71.

43 Hase-Schoelz; cf. Heath, Math. I, 135; Brunschvig, Etapes 45ff, Le rôle du pythagorisme dans l'évolution des idées (Paris, 1937) 21ff (heading: “Une découverte scandaleuse”); von Fritz, AnnMath 1945, 244f; Sarton, Hist. 283ff; Becker, MD 13f, 71ff; Heller, AbhBH 1958, 11, Junge, CCRM 1938, 53ff (though at 67f, Junge rejects the idea of “Geheimhaltung aus schlechtem Gewissen,” and maintains, correctly, that the Pythagoreans were apparently not upset by the matter).
had never been discovered!44 But had the painful fact of its existence really been held as a carefully guarded arcanum imperii?45

An important prop for this theory of a Grundlagenkrisis, and also a useful chronological point of reference, was the interpretation of the polemics of Zeno of Elea as relevant to the history of mathematics. This was inaugurated by Helmut Hasse and Heinrich Scholz, who argued that his critique was directed specifically against some “unclean” (unsaubere) mathematics of infinitesimals, by means of which the Pythagoreans supposedly attempted to escape the consequences of irrationality.46 If this is correct, the discovery of the irrational must have taken place before 460 B.C., which would fit in well with the conjectural dating of Hippasus.47

More recent interpretation of Eleatic philosophy does not confirm the attempt to place Zeno in the history of mathematics.48 His arguments are sometimes given a specific mathematical purport which is in no way suggested by his words, and which antiquity did not find in them, assigning him, as it did, to the ranks of the φύσικοι rather than the μαθηματικοί.49 Even when one distills out the purely mathematical content of his arguments, and does find the concept of the infinitely small and of the infinite series, still these series converge in a rational value, not in irrational proportions.50 Certain as it is that Zeno’s arguments are relevant to mathematics and even contributed in a certain way to the development of Greek mathematics,51 still, from the historical point of view ontology is prior to mathematics.52

Furthermore, it is not attested in any ancient source that Hippasus discovered the irrational, or divulged this knowledge; side by side stand diverse accounts, mutually exclusive, of the secrecy and the publication of Pythagorean mathematics. According to the version of the Pythagorean mathematici, obviously recorded by Aristotle,53 Hippasus was the first to “publish and construct” the “sphere of the twelve pentagons,” that is, the dodecahedron; he was drowned at sea as a punishment for this offense, but got the reputation of being the discoverer. According to the contrary version of the acusmatici, Hippasus was clearly the man who, by his discovery, inaugurated a new trend in Pythagoreanism, different from the original. There is no direct mention, here, of the irrational, the “betrayal” of which appears in a somewhat different account. Plutarch, who is our oldest witness for this, speaks of the secrecy, and the prohibition of putting doctrines down in writing, in the Pythagorean group: “And when their treatment of the abstruse and mysterious processes of geometry had been divulged to a certain unworthy person, they said the gods threatened to punish such lawlessness and impiety with some signal and widespread calamity.”54 We cannot equate this episode with that of Hippasus; the latter was a Pythagorean, so that his initiation into the “difficult and secret procedures” was therefore not any kind of “divulgence,” and his death was not a κοινόν κακόν.

The next witness is Pappus.55

This science (or knowledge) had its origin in the sect (or school) of Pythagoras, but underwent an important development at the hands of the Athenian, Theaetetus.... Indeed the sect (or school) of Pythagoras was so affected by its reverence for these things that a saying became current in it, namely, that he who first disclosed the knowledge of surds or irrationals and spread it abroad among the common herd perished by drowning; which is most probably a parable by which they sought to express their conviction that firstly, it is better to conceal (or veil) every surd, or irrational, or inconceivable in the universe, and, secondly, that the soul which by error or heedlessness discovers or reveals anything of this nature which is

45 “Im schlimmsten Sinn unsaubere Methoden,” Haase–Scholz 10.
46 Hasse-Scholz 8ff, and J. Zafirioupolo, L’Etoile éléa (Paris, 1950) 178ff. On the other hand, Mondolfo (Inf. 238ff) and Junge (CEM 1958, 54ff) think that the fact of irrationality was not concealed but openly admitted, and that this stimulated the development of Zeno’s arguments.—Philolaus As6 aims to avoid the irrational, but this is not “unclean” mathematics; it is not mathematics at all.
47 Von Fritz, AnnMath 1945, 245ff; cf. above, ch. II 5, nn. 69–76.
49 εἶ πολλὰ ἔστιν (fr. 1, fr. 3) is supposed to mean “if it is legitimate to regard a line as an aggregate of infinitely many, small ‘atomic’ lines” (Hasse–Scholz 10); see above, ch. III 3.
50 Stressed by Heidel, AJP 1940, 25 n. 54; Hasse and Scholz maintain (13) that Zeno belongs in the history of mathematics, rather than in that of Sophism.
51 Emphasized by Becker, Gnemon 27 (1955) 267.
52 Zeno, to some extent, obstructed the development of the calculus of infinitesimals among the Greeks; and Archimedes uses his ἐπίδοσις, which is equivalent to integration, only as a heuristic method, which must be followed by a rigorous proof (see, e.g., van der Waerden, SA 224ff).
53 Cf. above, ch. VI 1, n. 138; III 3, n. 54.
56 1, p. 65; 1, p. 64 Junge–Thomson (cf. above, ch. VI 2, n. 82). Pappus is dated, approximately, 300 A.D.
in it or in this world, wanders (thereafter) lither and thither on the sea of non-identity (i.e. lacking all similarity of quality or accident), immersed in the stream of the coming-to-be and passing-away, where there is no standard of measurement. This was the consideration which Pythagoreans and the Athenian Stranger held to be an incentive to particular care and concern for these things…

The scholia to Euclid have preserved an abbreviated version of the Greek text. and lamblichus, too, knows the tradition Pappus is following. But he also has another, according to which the traitor was only symbolically killed—a tomb was erected with his name. Lamblichus sets the three versions side by side—the symbolic “death” of the betrayer of irrationality, the drowning of the man who revealed the dodecahedron, and the drowning of the one who divulged the fact of irrationality. In addition, lamblichus has a different story about how Pythagorean geometry became known, without any quarrel or catastrophe: permission was given an impoverished Pythagorean to earn a living by giving lessons in geometry. It is conjectured that this version was originally related to Hippocrates of Chios.

Can we reconcile these various versions, in a spirit of compromise, with the thesis that Hippasus discovered or made known the dodecahedron “as well as” (“und ebenso”) the irrational? or, as Kurt von Fritz ingeniously suggested, that Hippasus discovered the irrational in the case of the dodecahedron, that is to say, of the regular pentagon? The fact of irrationality, of infinite reciprocal subtraction, is easily seen in the case of the “Golden Section”; but there is no hint, in the tradition, that this was the point of departure for the discovery of the irrational. There are two branches of the story of the traitor: on one hand Hippasus, the dodecahedron, and drowning at sea, and on the other the discovery of irrationality but no name mentioned, and various reports about the nature of the penalty. The explanation of the offender’s death as symbolical is found in the report of Nicomachus, who referred to the letter of Lysis, and the motif of drowning, of which Plutarch makes no mention, may also be an accretion to the story of the betrayal of irrationality.

All the accounts involving some kind of mathematical secret and its

(Comm. math. sc. 77.34) with the key word ἐνδόξω: ἐνδόξω δὲ τὰ μαθήματα, ἐπεὶ ἐξαναφέρεται, διὸ οὐκ ἐνδόξω μάλατα, θεοδορός τε ἐν Κυκλοπίας καὶ ἐπικρίνεται ἐν Χῖως. The language has peculiarities (plural verb with neuter plural subject; διόν = δόν, an ironic and poetic usage, in any case not Aristotelian; and an impossible participle in the dual). The content corresponds to Procl. In Eucl. 66.64 = Eudemus fr. 133, from whose source lamblichus probably derived the interpolated passage. (This was noticed by Tannery, MS VII 112, and von Fritz, AnnMath 1945, 245, though the latter wrongly attributes to Eudemus the preceding report about Hippasus, which is not closely connected with the sentence in question. See also Heller, AbhBl 1958, 7.) — Aristotle reports that Hippocrates of Chios lost his property (DK 42.2); and this seems to be the source of this version (Sachs 120.1). — Tannery, MS VII 113ff, Gymn. 84ff, HScH 124, takes the report seriously, and suggests that after the catastrophe of the Pythagorean rule, under the pressure of material need, the Pythagoreans published the supposed Tradition suivant Pythagore. (See above, ch. VI 1, n. 44.)

Becker, MD 77.


To my knowledge, the connection of the regular pentagon with the irrational is never emphasized in the tradition. The paradigmatic example for the irrational is always the diagonal of the square (below, nn. 75, 81); and von Fritz, in an earlier article (REV A 1813), assumed that the diagonal was “twiceless” the point of departure for the discovery.

Iam. VP 74ff (from Nicomachus according to Rohde, Q 138; at Iam. VP 253 = Por. VP 38, Nicomachus cites the letter of Lysis) and Clem. Al. Strom. 5.57 (from Nicomachus?) citing the letter of Lysis, which was addressed to a certain Hipparchus (Hipparchus D.L. 8.42; but Clement, lamblichus, and the MSS of the collection of letters all have Hipparchus). The latter had the temerity to διακόπτει παράδοσή, and is given the threat, if he does not mend his ways, τιθάσθαι μου. The complete text is in Hercher, Epistologr. gr. p. 601-601; Thesleff, Texts 111-114. See Burkert, Philologus 1951, 1ff, where it is conjectured that the letter was forged in the 1st half of the 3rd century B.C., as an introduction to the Hypomnemata.
betrayal share the same inherent improbability, which was perceived by Tannery.\footnote{44} Fame and profit can only result from an invention if it is welcomed by an expectant public, so that a mathematical discovery is only of interest in mathematical circles—\textit{mathematica mathematicis scribuntur}. Nobody who is not already schooled in mathematical logic is going to be deeply impressed by the fact of irrationality. But if mathematical logic was already in existence, the soil was prepared for further development; and in mathematics it is especially easy for different students to come to identical results. If mathematics had been an exclusive possession of the Pythagoreans, its betrayal would have been meaningless; but if the problems and methods of deductive mathematics were already present outside the Pythagorean circle—a fact not to be doubted—then this \textit{ekefen} could at most hasten a process that was already going on.

This dilemma, however, does not apply in the case of the dodecahedron. As in the case of musical discoveries (ch. V i), the tradition about Hippasus, though surrounded by legend, makes sense. In the background of the mathematical problem of the dodecahedron there stands the dodecahedron as a cult object. Numerous dodecahedra made of bronze have been found in Gaul and thereabouts; and one made of stone has been found in northern Italy, dating back to prehistoric times.\footnote{45} Their significance and use is unclear; the best conjecture seems to be that they were a kind of dice, used for oracular or mantic purposes. In Plato’s \textit{Timeaus} the dodecahedron appears unexpectedly as the image of the whole (55c); it is widely supposed that the Pythagorean tradition was in his mind, the one that presupposed the Hippasus story and was not without relation to the Italo-Gallic region.\footnote{46} The dodecahedron may well have been important as a \textit{symbolon} in the Pythagorean school, like the pentagram; Hippasus’ offense was in analyzing the sacred object, publicly, by mathematical means. Whether he actually made a mathematical construction of it is uncertain,\footnote{47} and it is all the more doubtful whether, in this process, he stumbled onto the problem of the irrational.

It is striking that in the accounts of the “betrayal of the secret of the irrational” the oldest, that of Plutarch, is the least precise, that Pappus speaks only of a legend which arose as a result of the discovery, and that all the reports stress the deep significance rather than the facts of the matter.\footnote{48} In Plutarch it is clear that the word \textit{árrygos}, set in quotation marks, as it were, by \textit{λεγόμενον}, is to be understood in a double sense. The “ineffable because irrational” is at the same time the “unspeakable because secret.” Carefully guarded secret doctrines (\textit{árryga})—which are dangerous to the uninitiate—played an important part, in Plutarch’s day, in all the mysteries and similar organizations.\footnote{49} Thus Pythagorean philosophy was, as Nicomachus puts it, \textit{árrygos} \textit{en tois στήθεσι διαφυλάχθησα}.\footnote{50} Not that it was arbitrarily “forbidden”—\textit{ápárrygos—but it was “unsayable.” The fascination of the \textit{árrygon} lies in the pretense to indicate the fundamental limitations of human expression, which are at the same time transcended by the initiate. It is no wonder that this fascination was felt in the realm of mathematics as well; Athenagoras reports of the Pythagorean Lysis the definition, \textit{άρμον άρρητων δύναται τόν θεόν}.\footnote{51} This exciting double sense of the word \textit{árrygos} is what makes the story of the discovery and betrayal of the irrational an \textit{exemplum} for Plutarch, and even more for Pappus, who is probably following some Platonic source. When we see that a name is not mentioned, and that the details vary from one version to another, it is tempting to think that it was precisely the

\footnote{44} Tannery, \textit{Géom.} 82f. In spite of this, he accepted the tradition of secracy, because of the supposed “primitiveness” of the mathematics of Oenopides (n. 31).

\footnote{45} F. Lindemann, “Zur Geschichte der Polyeder und der Zahlzeichen,” SMMü, math.-ph. Kl., 26 (1897) 625–678, described 28 dodecahedra, including one of statite from Monte Loffa, northern Italy, dated 9th to 6th century B.C. (cited by Sachs 83f). On the (more than 50) Celtic dodecahedra, see J. de Saint-Venant, \textit{Dodécédres perles en bronze creux ajouté de l’époque gallo-romaine} (Nevers, 1907); L. Saint-Michel, Bull. de l’Ass. G. Budé 3rd ser. 4 (1951) 92–116; and W. Deonna, Bull. de l’Ass. Pro Aventico 16 (1934) 19–89, which makes it seem probable that they were used in a “jeu divinatoire.” Saint-Michel and Deonna ascribe the Celtic dodecahedra to Pythagorean influences, but the prehistoric example would rather indicate that here, again, Pythagoras was dependent on older material.

\footnote{46} Cf., e.g., Taylor, \textit{Tim.} 377–378.

\footnote{47} Sachs 82: “That the mathematical construction of the dodecahedron took place before that of the octahedron—when all the solids were known—is unthinkable.” She translates \textit{γράφωμα “sich zeichnen” (8)}, and supposes that the Pythagoreans knew the dodecahedron only in an empirical way. The expression \textit{διδασκαλουστηρον αφαιρεσις} in Pl. Phd. 110b (cf. Plut. Quast. Plat. 5:1003d), shows that (as today) balls for children were made in the shape of dodecahedra, of 12 pieces of leather; this makes it seem unlikely that the dodecahedron was first made known, outside of Pythagorean circles, by Hippasus.

\footnote{48} Pappus expressly cites the “Athenian stranger” from Pl. Leg. 819d, and in general his exposition is strongly influenced by Platonism.

\footnote{49} \textit{árrygos} \textit{iêròs} as early as Herodotus 5.83, 6.135, Aristoph. Nuh. 302, Eur. I.41, Bacch. 472, etc. On the \textit{árrygos} in Plato, see Friedländer, \textit{Plato I} 64ff = I 59ff Eng. tr. From late antiquity, see for example Plut. De Is. et Os. 25: δια της μωσαϊκης ιερως περικυκλώμενα και τολεδον \textit{árrygos} διασώζεται; De au. Procr. 4.1013c (the argument that the world was created, and therefore not eternal): δια της διάνοιας και \textit{árrygos} οὐδενον δειν περικυκλώντας και \textit{διειβαίνει}, Hippol. Ref. 1 praef. 1 (on the Gnostics) \textit{άποκρύπτεις τα \textit{árryga} εκεντε μυστηρια . . .}

\footnote{50} Nicom. ap. Por. VP 37 = Iamb. VP 252.

\footnote{51} Athenag. Suppl. 6: DK 46.4 – p. 114,15 Thesleff. Of course this is as spurious as the other Pythagorean quotations in Athenagoras, but still apparently the earliest extant reference for the expression “irrational number” (see above, n. 43).
ambiguity of the word ἀρρητός which provided the germ of the tradition, or at least that this was an influential factor in its development. One had the story of Hippasus as told by Aristotle, the Lysis letter with its threat of symbolic death, and, in general, the careful secrecy of the Pythagorean group. Pythagorean lore was ἀρρητός like that of the mysteries,²⁷ and the penalty of betrayal was death. Pythagorean lore was mathematical in nature, and in geometry ἀρρητόστα play a role which is mysterious to a layman; how easy a step to equate the two senses of ἀρρητός and create the story of the secret of the irrational and its betrayal! The authority of Eudemus fails us twice, in Pappus and in Proclus;²⁸ but it may underlie the scholia on Euclid.²⁹ Thus it is possible that he mentioned the fact of the Pythagoreans’ discovery, giving the cue for the development of the legend. But the involvement of Hippasus is scarcely better attested than that of Zeno, so that another apparent chronological foothold is lost.

The deep significance of the discovery, so dramatically expressed in the catchword Grundlagenkrise, is not attested in the sources. “Nowhere in the many passages about the irrational in Plato and Aristotle can we detect any reference to a scandal, though it would surely still have been known in their day.”³⁰ In addition, the inherent connection of the problem of the irrational with Pythagorean speculation and philosophy, which some have supposed they saw, is doubtful. The so-called Pythagorean theorem, when applied to a square, would necessarily lead to the irrational \(\sqrt{2}\); but Babylonians, Indians, and Chinese knew the theorem without knowing about the irrational. Side numbers and diagonal numbers belong in the same context but do they represent a way of understanding or of avoiding the irrational?³¹ The arithmetical proof of the irrationality of \(\sqrt{2}\) is regarded as early, but it does not emerge naturally as a consequence of the Pythagorean pebble games.³² Music theory advances as far as the problems of the irrational, but stops there and declares them nonexistent.³³ The irrational belongs to the domain, not of arithmetic, but of geometry.

For the Pythagoreans who were concerned with the number theory described by Aristotle, and for the cosmology summarized in the phrase “everything is number,” the irrational has obviously no importance. Eurytus, who cannot be dated earlier than about a generation before Archytas,³⁴ tried to determine “the number” of specific objects, oblivious that the discovery of the irrational had long before cut the ground from beneath the Pythagorean theory of numbers. What is more, Aristotle, though he knew about it, did not use the fact of irrationality as an argument against the Pythagorean doctrines he criticizes, while the book On Indivisible Lines does use irrationality against the “automatic lines” of Xenocrates.³⁵ Clearly Pythagorean number theory and deductive mathematics lie on two different planes; “all things are number” never means “all magnitudes are commensurable.” In Pythagorean number theory the relations of existing things are interpreted, and the “nonexistent” is left out of account.

In attempting to date the discovery of the irrational, we have the known fact that Theodorus of Cyrene proved the irrationality of square roots in the cases between \(\sqrt{3}\) and \(\sqrt{17}\), so that the proof for \(\sqrt{2}\) was known before his time.³⁶ Democritus seems to have been

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²⁷ The comparison with the Eleusinian mysteries is drawn in the letter of Lysis, p. 112 ff. Thesleff = Hercher, Epit. Gr. p. 602 ff. = Iam. VP 75.
²⁸ Pappus cites Eudemus for the achievement of Theaetetus (above, ch. VI 2, n. 82), but not in connection with the Pythagoreans (above, n. 50). After the introductory sentence on Pythagoras follows a citation of Plato (Thit. 1430); then Apollonius of Perga is mentioned, and Eudemus makes his appearance only after a second Platonic citation (Thit. 1476), so that the entire passage cannot be ascribed to him. See above, n. 68.
²⁹ Schol. Eucl. pp. 415. 7-416.13, characteristically speaking of Ἀριστείτης; cf. the introductory sentence of Pappus (above, n. 50): “This science had its origin in the sect of Pythagoras, but underwent an important development at the hands of the Athenian Theaetetus.” An expedient of practical geometry was to speak of two different “measures,” e.g. of the side and the diagonal of the square, Schol. Eucl. p. 416. 8-10, Arist. Met. 1053a14ff (misunderstood by Ross, Met. II 283; not explained by Heath, Aristotle 2187), Pl. Parm. 140b-c.
³¹ Above, ch. VI 2, n. 14-18.
³² Above, ch. VI 2, n. 54.
³³ Above, ch. VI 1, n. 4.
³⁴ DK 43. How far these Pythagoreans were from being true mathematicians is shown by the fact that their “units,” according to Aristotle, “had magnitude,” while even Protagoras presupposes a definition of the point, among geometers, as without extension.
³⁵ Ps. Arist. L. In. ins. 65,633ff. Junge pointed to Eurytus, in this connection, as early as 1907 (Symb. Joach. 230ff); cf. CEM 1958, 57ff.
³⁶ Pl. Thit. 147d. As to the method of proof Theodorus may have used, there has been a long discussion, not yet concluded, among modern scholars. Whereas Vogt (Bibl. Math. 1909-1910, 101ff) and Junge (CEM 1958, 412ff) reconstructed a geometrical proof making use of reciprocal subtraction, and van der Waarden (SA 142ff) advocated an algebraic-mathematical method; von Fritz (RE V A 1813ff), Chernous (Rev. of Metaph. 4 [1957] 41ff), and A. Waserer ("Theaetetus and the History of the Theory of Numbers," CQ 52 [1958] 165-179) favored an apagogical method analogous to the numerical proof of the irrationality of \(\sqrt{2}\) (above, ch. VI 2, n. 47). The irrationality of \(\sqrt{2}\) can very easily be proved for the square by the method of reciprocal subtraction (van der Waarden, Math. Ann. 1947-49, 699f; SA 177; Becker, MD 733). The resulting διάγραμμα is identical to that of the "side" and "diagonal" numbers, above, ch. VI 2, n. 16.
concerned with the problem; but how far back into the fifth century we should date the discovery is an open question. If we try to determine the position of Hippocrates of Chios, the results are as ambivalent as those relating to the problem of the application of areas (above, n. 25). He obviously has no trouble constructing a line with a length equal to \(\sqrt{2}\) and \(\sqrt{3}\) is also a geometrical problem for him. One finds the square root by constructing the mean proportional; “geometrical algebra” is on the way. But, on the other hand, Hippocrates uses a definition of the equality of ratios which is not applicable to irrational magnitudes. The problem in the theory of proportion which this raises was solved only by Eudoxus, and a passage in the Academicorum index seems to allude to a forward step between Hippocrates and Eudoxus: in Plato’s time, it says, (τέθειν εἰς κορυφήν) . . . τὰ περὶ τῶν ἀριθμῶν προβλήματα, τῶν περὶ Ἐθνον μετάστησάτων τῶν διαφοράς ἤπειρον. 

88 He has a title (B1) ἐν τῷ ὑπάρχου ἱπποτικὸν περὶ ἱπποτικοῦ καὶ πολλῶν a’’ (πολλῶν = πολλῶν, δὲν). Vogt (Bibl. Math. 1909–144ff) contests the reference of this to the problem of the irrational, since he wishes to show that the word ἱπποτικὸν in the sense “irrational” is post-Platonic, overlooking Pl. Rep. 534d. One can only guess how Democritus solved the problem: for a spatial atomism see S. Luria (QS) 2 (1932) 106–18) and Heath, Aristotle 976f, for the distinction of geometrical space from the atoms (following Simpl. Phys. 82f, Schol. Arist. 466b14) are R. Philpap, Hermes 64 (1929) 177ff, and Michel 675 n. 1.

89 διαφοράς ἔλεγεν θείος, Eudemus fr. 140.

90 Hippocrates takes his departure from the proposition that similar segments are related as the squares of their bases. The proof is that circles are related as the squares of their diameters: ὅσοι γὰρ οἱ κύκλοι πρὸς ἄλλους έχουσι, οὕτως καὶ τὰ ἱμιμετα- ἐμα ἑκά τοῦ μέρους ἐνα οὐκ ἢ κύκλον (Simpl. Phys. 61.11ff; cf. Eucl. 7 def. 21; the exact proof was first offered by Eudoxus; for a conjecture about Hippocrates’ proof, see Becker, Gnomon 27 [1955] 267 n. 4). This sentence is regarded by Tannery (MS 1 399ff), Diels (ed. of Simpl. in CAG), and Becker (QS) 2, 411–419 as an addition of Simplicius, and Wehrli omits it. The ascription to Eudemus was defended by Rudio (Bibl. Math. 3 [1902] 1–62—where he weakens his argument with the claim that τῆς μία first means sector, and then in the following sentence; refuted by Tannery, MS III 119–130 = Bibl. Math. 3 [1902] 342–349). See also the discussion of Heath, Math. 187ff. The citation from Euclid inserted by Simplicius ends before ὅσοι γὰρ . . . The word ὅσοι (p. 61.8) is followed by the indication of the course of proof, which is followed, as a consequence (ὅσοι καὶ, p. 61.14); Wehrli and Diels have the citation of Eudemus take up with καὶ, without ὅσοι), by the sentence about the angles in similar segments. Therefore, the proposition using the definition of proportionality, which is valid only for numbers, not for geometrical magnitudes (Euclid 7 instead of the general theory of proportion in book 5) may be regarded as evidence from Eudemus about Hippocrates. After Zeuthen, van der Waerden called attention to the passage, MiAnn 1940–1941, 137f, cf. from Fritz, ABB 1959, 60ff, 69ff.

85 See Becker, MD 102ff; van der Waerden, SA 187ff; Heath, Math. 1.32ff. A “pre-Euclidean” theory of proportion, which was applicable to irrational magnitudes, and which operates with reciprocal subtraction, was reconstructed by Becker (QS) 2 [1932] 311–333; cf. Ged. 7ff, MD 103f; von Fritz, ABB 1955, 92; van der Waerden, MiAnn 1947–1948, 688f; SA 177ff; Heath is skeptical, Aristotle 80ff.

86 Page 16 Mekler. A perfectly certain restoration is hardly possible.

The conquest of the irrational in Greek mathematics, and along with it the development of “geometrical algebra,” clearly was a gradual process. There were beginnings before Hippocrates of Chios, but the decisive breakthrough was later. The Pythagoreans to whom Eudemus attributed the application of areas and perhaps also the “discovery” of the irrational may well belong to this later period. Whether it was the same mathematician who first grasped the idea of the διανυσματικὸν and “demonstrated” it by alternate subtraction of side and diagonal of a square, who exploited the superiority of geometry to algebra in the development of the application of areas, and who perceived the difficulty that the ἱπποτικὸν γραμμαὶ do not fit into the calculation (ratio, λόγος)—or whether several men worked, one after the other, on these problems—we can no more answer these questions than we can give their names. But in any case, the beginnings of the development that is brought to a climax by Eudoxus are not far from Theodorus of Cyrene. This strictly mathematical development had little relation to the cosmological number theory of men like Philolaus and Eurytos. Pythagoreans made significant contributions to the development of Greek geometry, but the thesis of the Pythagorean foundation of Greek geometry cannot stand, any more than the legend of a great mathematics held secret.

4. NUMBER AND COSMOS

Aristotle represents the Pythagoreans’ number theory as arising from their study of exact mathematics, and this relationship may well have seemed obvious, or even necessary. To be sure, there is no evidence, and no way of showing by philological means, that the Pythagoreans

87 Frank concludes from a passage in Plato’s Laws (819d) that in his time the problem was quite new, so that before 400 it was “completely unknown among those most keenly interested in scientific matters” (228): it was brought to Athens by Theodorus in 399 B.C. But since at least half a century elapsed between 400 and the composition of the Laws, the word ἰδίος in 819d cannot be pressed (cf. von Fritz, AnnMath 1945, 243 n. 6). Vogt, Bibl. Math. 1909–1910, dated the decisive discovery about 410, van der Waerden (MiAnn 1947–1949, 133) before 420, and from Fritz, thinking of Hippasus, about 450 (ABB 1955, 84 n. 159). A simple point of departure for the discovery was provided by the traditional (partly Babylonian) approximative values for the diagonal of a square. When the Greeks endeavored to find an exact result to replace the approximations, they discovered the fact of incommensurability, in the same way that they recognized the squaring of the circle as a problem.

1 Above, ch. VI 1, n. 68.
were the originators of mathematics as a deductive science; and it is impossible to discern any relationship between the doctrine of transmigration, which is reliably attested for them, and the science of mathematics. Number does, however, dominate the Pythagoreans’ general view of the world. In the relations among numbers they found the essence of musical harmony, and they knew or discovered a number of propositions that are mathematically significant. Is not this sedulous preoccupation with number precisely what we mean by mathematics? Do we not see here, in epitome, the origin and basis of natural science? To this question we must answer no.

Number and mathematical science are by no means equivalent. Numbers go back in origin to the mists of prehistoric times, but mathematical science, properly speaking, did not emerge earlier than sixth- and fifth-century Greece. People knew numbers before mathematics in the strict sense; and it was in the pre-scientific era that the “number mysticism” arose, or “number symbolism” or “numeology,” which continues even now to exert a certain influence. No one could overlook the fact that this kind of thing was present in Pythagoreanism; Aristotle names first of all, among the ἰδιωματα which the Pythagoreans thought subsisted between numbers and things, the equation of certain numbers with διάκλησις, ψυχή καὶ νοῦς, and καιρός (Met. 985b27ff), and only with a “furthermore” goes on to add the mathematical theory of music. It is not necessary to attempt here a complete presentation of Pythagorean number speculation; in particular, the very luxuriant tradition from later antiquity can be left aside, since the demonstrably ancient material, attested by Aristotle, is sufficient to establish the basic facts.

One is νοῦς and αἰθιότος; two is δύο; three is the number of the whole—beginning, middle, and end; four is justice—equal times equal—but it is also, in the form of the tetractys, the “whole nature of numbers”; five is marriage, as the first combination of odd and even, male and female; seven is opportunity (καιρός) and also Athena, as the “virginal” prime number; ten is the perfect number, which

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2 In English the hybrid “numeology” has become standard, though the term “arithmology,” introduced by Delatte, would be preferable on philological grounds. There is no question of “mysticism” in the proper sense, involving union of the inner and outer world; and the “symbolism” referred to is not an arbitrary assignment of “signs,” but the discovery of apparently natural correspondences and interrelations (ψυχη καὶ νοτια, Plut. fr. 9).—There is no comprehensive exposition of ancient number symbolism, like that provided by Dornseiff for letter mysticism (Das Alphabet in Mystik und Magie, 2nd ed., Leipzig, 1923). On particular topics, see E. Wölfflin, “Zur Zahlensymbolik,” ALL 9 (1896) 333-351; H. Usener, “Dreizeit,” RHM 58 (1902) 1-47, 161-208, 321-362 (cf. RAC 5.vv. Drei, Dreieck). An exhaustive collection of material is W. H. Roscher, Die enneadischen und hebdodischen Frieten und Wochen der altesten Griechen (AbhLpz 21.4, 1903); Die Sieben- und Neunzahl im Kultus und Mythos der Griechen (AbhLpz 24.1, 1904); Die hebdodischen Lehren der griechischen Philosophen und Arzte (AbhLpz 24.6, 1906); Enneadische Studien (AbhLpz 26.1, 1907); Die Tesserakonten und Tesseractokontenlehren der Griechen und anderer Völker (BerLpz 16 [1909] 177); Die Zahl 5 in Mythos, Kultus, Epos und Taktik der Hellenen und anderer Völker (AbhLpz 33.5, 1919); O. Weinreich, Trisakoonkistik Studien (Giessen, 1916) 93ff; Germain passim. V. F. Hopper, Medieval Number Symbolism (New York, 1938, with a good summary of the ancient phenomena).

3 For this tradition see, along with Zeller I 495-507, the works of Robbins and de Falco (above, ch. I 3, n. 22). The Theologomena arithmeticae gives the most detailed summary, and de Falco’s apparatus provides a convenient key to the parallel accounts. Posidonius was involved in the transmission of Pythagorean number doctrine (Theo Sm. 103.16ff; Sext. Emp. Math. 7.9iff; above, ch. I 3, nn. 8ff), but the crucial attestations are earlier. Speusippus’ book Περὶ Πυθαγορικῶν ἀριθμῶν (fr. 4) was doubtless very important; but there seems to have been an indeterminable number of developments and revisions, many in pseudepigrapha (Porphyrius Περὶ τῆς ἐβδομαδός, Th. 57.15, Megillae Περὶ ἀριθμῶν, Th. 34.21; cf. Jerome Epist. 49.19). The origin of the Pythagorean mathematics in number mysticism is emphasized by Joel, Geschichte der antiken Philosophie I (Tübingen, 1921) 358ff; Rey 395; Schult, Essay 258ff; Junge, DeMath 342ff; van der Waerden, SA 93ff; Zeller (495ff), but cf. 453 n. 3; he treats the symbolism as “development and application” of the number theory, and Burnet refuses to see it as the starting point, regarding the musical discovery as primary (ECG 107).

4 The data given in Aristotle’s treatises are supplemented by Alexander from Aristotle’s lost book on the Pythagoreans. At Met. 985b30 he speaks of ψυχη καὶ νοτια, upon which Alexander comments (39.13), νοτια δε καὶ αἰθιότοι ἔχουν τὸ ἐν τῷ γὰρ ψυχην ὦ τοῦ νοτια ἐνα (sc. Aristotle). That is to say, Alexander corrects Aristotle; by ψυχη καὶ νοτια, he means precisely νοτια (cf. Pl. Crat. 400a: νοτια καὶ ψυχη, referring to Anaxagoras’ Nudex). This word was all he found attested; at the same time he supplemented Aristotle’s testimony with the word αἰθιότος; he is not paraphrasing, but commenting by the citation of further source material—obviously Aristotle’s lost book on the Pythagoreans (οἰδιακον sc. Aristotle), Alex. Met. 38.8).

5 Arist. Met. 990a23, Alex. Met. 39.16; ὄνομα, Alex. Met. 74.12 (on Arist. Met. 990a22; cf. Plut. De Is. et Os. 75, Th. ar. 9.6, Plot. 511.4, Procl. in Alc. 1.104). But here, Alexander’s word ψαλασσω apparently does not refer to Aristotle (75.27). For ὀνυχα see Arist. Met. 990a24, and Th. 39.11, 35.1.

6 Arist. Cael. 268a11. The Orphic verse, according to which Zeus is beginning, middle, and end (Orph. frag. 21), is alluded to in the Derveni papyrus, and then in Plato (Lec. 713e). The proclamation of the god is prior to the general Pythagorean formulation (cf. also Pl. Parm. 145a), showing that in this respect Pythagoreanism is dependent on purely religious, or “Orphic,” sources. Tannery (MSc II 184ff, HSc 190f) and Roscher (AbhLpz 1906, 188) supposed that there was an Orphic number symbolism earlier than the Pythagorean; but the evidence cited is late (see Delatte, Litt. 208ff, on the supposed Ἱμμνος to number). Whether the use of number as an effective force went further in Orphism than in other ritual, is quite uncertain.

7 Arist. Met. 985b30, 1078b23, EN 1132b2, MM 1182a11, Alex. Met. 38.8ff; see above, ch. I 3, n. 120, ch. II 4, nn. 154ff.

8 Arist. Met. 1078b23, Alex. Met. 39.8; cf. above, ch. I 2, n. 31. The interpretation of 6 as marriage (= 2 x 3) is later (Th. ar. 43.16ff, with the parallels cited).

9 Arist. Met. 985b30, 990a23, 1078b22, Philolaus fr. 20 (above, ch. III 2, n. 52). According to Alex. Met. 39.11, the number 7 is Athena, because among the first 10 numbers “it neither begets another nor is begotten by any.” On the connection of gods and numbers, see Xenocrates fr. 15, Plut. De Is. et Os. 10, 75, Stob. 1 proem. 10, Th. ar. passim.
comprehends the whole nature of number and determines the structure of the cosmos, and with it the symbolic interpretation of numbers.¹⁰

Numbers are alternately even and odd, and this antithesis is the same as that in the cosmos between “limiting” and “unlimited”; odd numbers correspond to the more highly valued principle, that of “limit,” and are also male. One is an exceptional case, being simultaneously even and odd, female and male. Geometrical figures, too, have specific functions and their “character.” Philolaus assigns masculine divinities to the triangle, and feminine to the square. It is probably correct that the pentagram was a symbol of recognition among the Pythagoreans, and the dodecahedron seems also to have played some such role.¹¹

Of course there was nothing like a complete system; there were overlappings and contradictions of all kinds—ἀνάγκη πολλά συμβαίνει τά ανάρ, as Aristotle says.¹² One might be tempted to ignore all this, as a side growth on the main stem of Pythagorean wisdom; but more careful examination reveals that this kind of lore is to be recognized not as a branch but as a root, and one which goes very deep.

In primitive cultures, numbers are not abstract concepts with a content which is mathematical and quantitative, but mysterious beings: “each number has its own individual physisognomy, a kind of mystic atmosphere, a ‘field of action’ peculiar to itself.”¹³ The comparative material offered by anthropology need not be considered here except insofar as we find phenomena similar to the Pythagorean. Not that we start by assuming connections and influences, but that the peculiarities of certain psychological phenomena, more likely to be obscured by concepts of Greek science and mathematics, may be clarified in this manner.

In certain Negro peoples the king of the gods is invoked thrice; the queen four times; the amulet worn by a man has 3 knots, that of a woman 4. The birth of a boy is celebrated after 3 days, that of a girl after 4; a dead son is mourned for 33 days, a daughter 44. If we add that the number 5 plays a part in the marriage ritual, we have 3 as the male number, 4 as the female, and 5 as the number of marriage—among the Negroes of the Sudan.¹⁴ This sounds so Pythagorean that one is almost tempted to think of some kind of direct connection. In fact, this possibility cannot be excluded entirely,¹⁵ but the same situation—3 as the man’s number, 4 as the woman’s—is found in the ritual of Alaskan Eskimos.¹⁶ At other points on the globe’s surface, 4 belongs to the man and 3 to the woman; and the numbers 9 and 7 are found for man and woman, as well as 3 and 5, 4 and 5, and 6 and 5. The range of the evidence extends from the South Seas, over Africa, to the American Indians. There are traces of a similar use of 3, 4, and 5 among the Germans;¹⁷ and in Rome a girl was named 8 days, a boy 9 days after birth,¹⁸ while 5 torches were used in the wedding ceremony.¹⁹ Above all, we find that with very few exceptions festivals of the gods were set on odd days of the month.²⁰


¹¹ Fettweis, Zeitschr. f. philos. Forsch. 1950–1951, 179; Archeion 1932, 210ff, referring to Plut. De Is. et Os. 56 (3 is Osiris, 4 Isis, 5 Horus). Plutarch combines Egyptian and Greek material; there was certainly some Hellenistic Greek influence in the Sudan.


¹⁴ Plut. Quaest. Rom. 102, Macrobr. Sat. 1.16.36, Festus p. 120 M. Plutarch naturally interprets this as Pythagorean; in the 1st century B.C., Castor of Rhodes connected Roman and Pythagorean (FGrHist 250F6).

¹⁵ Plut. Quaest. Rom. 2. The use of the word cera (κεραίω) shows that Greek influence was present.

¹⁶ Exceptions: the Regifigium, the Equinox, and the procession to the Argei (see G. Wissowa, Religion und Kultus der Römer [Munich, 1912] 436f, Censusorius 20.4, Festus p. 109 M. T. Mommsen sees Pythagorean influence (Die römische Chronologie [Berlin, 1894] 15.12), as does F. Bümker, Ovid, Fasten I (Heidelberg, 1957) 15. W. Aly wishes to use this to date the establishment of the calendar of festivals (ARW 33 [1936] 59); and
When Babylonian sages name the 13th, 15th, and 17th days as favorable for setting a ridgepole,\textsuperscript{21} they are differentiating the odd numbers, as favorable, from the even. In East Prussia odd days of the month used to be regarded as lucky, in Pomerania the even.\textsuperscript{22} In ancient folklore, a remedy for headache was made by crushing an odd number of berries;\textsuperscript{23} it was recommended that a hen be set to hatch an odd number of eggs,\textsuperscript{24} and that a herd of animals consist of an odd number.\textsuperscript{25} Here Pythagorean influence cannot be ruled out,\textsuperscript{26} but the basic idea is older, and the practices are independent of Pythagoras.

The number 7 has a special significance in ancient Egyptian medicine,\textsuperscript{27} as in that of certain Indian tribes.\textsuperscript{28} Solon uses it in dividing human life into periods,\textsuperscript{29} as do the Etruscans.\textsuperscript{30} Among the Babylonians, each divinity has a specific number which belongs to it—for example, Ishtar has 15, perhaps reminiscent of the 5 which is γάμος.\textsuperscript{31}

ancient tradition, relying on just this kind of observations, made Pythagoras the teacher of Numa (on this, see Burkert, Philologus 1961). See also above, ch. II 4, nn. 128, 136. On the other hand, Macrobius (Sat. i.13.5) speaks of "secretum hoc est ante Pythagoram partim natura." K. Latte, Römische Religionsgeschichte (Munich, 1969) 199, called attention to the fact that the reverse number of days, used by the Romans, these festivals come on even-numbered days in the latter half of the month; but it may be that the reverse numeration was the oldest (A. K. Michels, The Calendar of the Roman Republic (Princeton, 1967) 139).—The role of 3, 9, and 27 in the Roman cult of the dead is contrary to Pythagorean doctrine (H. Diels, Sylphische Blätter (Berlin, 1890) 40ff; cf. below, n. 56. Thus in any case the roots of Roman number symbolism are earlier than Pythagoras.

\textsuperscript{21} Meisner II 278.
\textsuperscript{22} Handwörterbuch des deutschen Abertlugs VIII 164ff.
\textsuperscript{23} Plin. HN 2.156; cf. 24.82, 28.36, 28.33 ("Pythagoras").
\textsuperscript{24} Varro Rust. 3.9.12, Columella 8.5.8, Plin. HN 10.151, 161, 18.231, Pallad. 1.27.1, Geop. 14.7.1,3. See Roscher, AbhLpz 1904, 65 n. 153. This is common in Römische Religionsgeschichte (Munich, 1969) 199.
\textsuperscript{25} Geop. 18.2.8; also attested for Germany, Relig. i. Gesch. u. Gegenwart V 2068.
\textsuperscript{26} On Helenistic pseudopegrapha dealing with geom wic et et related topics, see M. Wellmann, "Die Georgika des Demokritos," AbhLpz 1921 n. 4; "Die Φωνή των Βολων Ευσκύρων und der Magier Anaxilaos aus Larissa," AbhLpz 1928 n. 7. Their influence is seen as early as Cato Agr., Wellmann 1921, 34ff.
\textsuperscript{27} Papyrus Ebers, ca. 1500 B.C.; Roscher, AbhLpz 1906, 108.
\textsuperscript{28} L. Lévy-Brühl, How Natives Think (above, n. 13) 212.
\textsuperscript{29} Fr. 19 Diehl; often cited in ancient numerology, where the treatment of 7 was generally the fullest.
\textsuperscript{30} Varro ap. Cens. 14.6. Varro, who likes to "Pythagorize," is of course not entirely reliable on such a matter. He had himself buried "Pythagorico modo" (Plin. HN 34.160).
\textsuperscript{31} Meisner 131; cf. also A. Jermi, Handbuch der alles Orientalischen Geisteswissenschaft (Berlin 1929) 265f. F. X. Kugler, "Der Ursprung der Zahl Inbolute,... in pythagorischer Beleuchtung,..., Klio 11 (1911) 484-496, wished to use this as an indication that the whole of Pythagoras' number symbolism was derived from Babylon; but the development is a complex one, and number symbolism can be shown to have existed in Greek culture before Pythagoras (Germain 61ff). The highest god of the Hittites, the weather god, is represented ideographically by the sign for 10 (Wörterbuch der Mythologie ed. H. W. Hausig [Stuttgart, 1961] I 209).

The number 4 plays a dominant role in the cosmology of the American Indians: corresponding to the 4 directions and the 4 winds, the gods are divided into groups of four, and colors, actions, and tribal territories are divided according to the same principle.\textsuperscript{32} It is number that orders the universe.

In Chinese thought, number symbolism was developed into an amazingly delicate and complex system, which comprehends cosmos and man, nature and social order.\textsuperscript{33} The basic antithesis of Yang and Yin is manifested in the antithesis of odd and even number; and it is odd number which corresponds to the active, male principle, Yang. The significance of the numbers from 1 to 10 is of fundamental importance. They are divided into two groups of five, from 1 to 5 and from 5 to 9, and then brought into connection with the five elements, the points of the compass, the seasons, the main types of taste or flavor, the colors, and, above all, with the five notes of the pentatonic scale. The number 4 is the number of procreation. The numbers are grouped together in a figure made with counters or pebbles—which is a representation of the entire universe.\textsuperscript{34} The Chinese know the numerical laws of the musical intervals, and express the relative height of a tone in terms of a series of numbers, which is also used in determining lengths for bamboo flutes. But this is not a matter of physical theory, but of analogies with cosmic regularities; in the ratio 3:2 or 4:3 is expressed the relationship of Yang and Yin;\textsuperscript{35} the numbers that occur have a value and significance of their own. Unity, in this context, is not regarded as a number and can be added or subtracted at will,\textsuperscript{36} in order to produce a result in which the relationship in question can be expressed in "significant" numbers. Thus, for example, the interval of a fourth can be 81:60 instead of 80:60; and the arithmetical complications in the calculation of the scale, in which the Pythagoreans got themselves tangled, can readily be avoided. The objective is not, in fact, accuracy, but the revelation of connections and correspondences. The five notes of the pentatonic scale are also arranged in the form of a pentagram.\textsuperscript{37}

\textsuperscript{32} Lévy-Brühl, How Natives Think, 210; Schuh, Essai 259 n. 5.
\textsuperscript{33} For the following see Granet, esp. pp. 200ff; H. Köster, Symbolik des chinesischen Universismus (Stuttgart, 1958) 44ff; Fung Yu-Lan, A History of Chinese Philosophy, tr. D. Bodde (Princeton, 1952-1953) II 93ff. For references and advice in this field, I am much indebted to Prof. H. Steininger.
\textsuperscript{34} Arrangement Ho-t'u, Granet 177f; connected with the Ming t'ang, Granet 178ff.
\textsuperscript{35} See above, ch. V 1, n. 64, on superparticular proportions.
\textsuperscript{36} Compare the German "acht Tage" for a week, along with "14 Tage" (French "quinze jours") for 2 weeks (and English "fortnight").
\textsuperscript{37} Granet 230ff.
VI. PYTHAGOREAN NUMBER THEORY

The Chinese knew the "Pythagorean theorem," but used it only with whole numbers. Here again the unit does not count, so that 8, 9, 12 can represent the sides of a right triangle, and 5, 5, 7 those of a right isosceles triangle. The objective, again, is not mathematics, but the aim of making the proportions in the roof of a Chinese house correspond to the proportions of the cosmos.

We read in a text of the 11th period which features the study of "emblems and numbers." Heaven, earth, the yin and yang, and wood, fire, earth, metal and water, make nine; together with man, they make ten. Heaven's number is with this made complete. It is as though we had an ancient Pythagorean text before our eyes. Also, as the Platonic "system of derivation" follows the Pythagorean, in China a philosophy emerged, neo-Confucianism, which is strongly reminiscent of Platonism.

Perhaps one ought not to exclude completely the possibility of direct connections between the Pythagoreans and the Chinese; but whatever tenuous lines of connection between East and West there were before the beginning of the modern age, they are not so important as the basic, underlying idea or attitude. Speculation about numerical relationships in the cosmos is world wide, firmly established in ritual, and capable of being elaborated into a rich and ingeniously structured system without either presupposing or giving rise to mathematics in the proper sense of the word. To suppose, therefore, that number speculation is derived from mathematics and that consequently Pythagorean mathematics is primary (as others have done besides Aristotle) is no better than a petitio principii.

The nature and origin of number symbolism has also been treated

from the point of view of C. G. Jung's school of psychoanalysis. The mass of data collected is amazing; number symbolism continues to exert a surprising influence even in the subconscious mind of modern man. The fascination of the "perfect" number 3 was exploited by the Third Reich; and the pentagram lives on in the flags and emblems of the United States and the Soviet Union. Thus for the psychologist the numbers are archetypes. To be sure, the connection of mind and symbol does not yet seem to be completely clear or capable of formulation with scientific precision. The meanings of the individual numbers are sometimes almost uniform, but sometimes entirely ambiguous, giving the impression of being arbitrarily interchangeable. It is surely mistaken to derive number symbolism from certain particular natural observations, as Roscher referred the special significance of 7 and to the phases of the moon. It is also clear that certain elementary calculations play an important role. One learns to count and calculate in childhood, and from the beginning the numbers are apprehended as things, with certain characteristics; they preserve this peculiarity even, at an unconscious level, in the mind of the adult. For example, the odd numbers, which cannot be divided evenly, are felt as uncomfortable, even perverse—and therefore, if one knows how to use them, powerful and lucky. A prime number like 7 is especially hard to manage and therefore very significant; and here an additional psychological factor is at work—it is easy to picture in one's mind a group of six, or a hexagon, but much harder to imagine a heptagon. Thus many kinds of threads join to compose this complicated fabric.

Like the numbers, the simpler geometrical shapes have an "archetypal" symbolism. The square symbolizes the earth both for the Chinese and for certain Indian tribes; and Philolaus "dedicates" the angle of the square to the goddesses Rhea, Demeter, and Hestia. But here too the attributions are inconsistent; there is no direct relationship

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43 L. Paneth, Zahlensymbolik im Unbewussten (Zürich, 1952); see also C. G. Jung, Aion (Zürich, 1951) 323f.
44 Paneth, 45ff, discusses the significance of 5 as the number of vitality and sexuality, without knowing the Pythagorean equation of 5 and marriage. For the pentagram on flags, see De Vogel, Pythagoras, 297–299.
45 Roscher, Ahlfé 1904, 67ff. On the other hand, H. Quiring tries to trace the importance of 7 to the planets alone (Altertum u. 1958 208–214); correctly Junge, DsMath 356.
46 Dornseiff (above, n. 2), 171ff, uses the expression Kundheitsmyth.pec.
47 Lévy-Bruhl, How Natives Think (New York, 1966) 114 (Ishaka Indians). They also represent the earth by four dots (□). Above, ch. IV 3, n. 60.
48 Endo, Hears 114 and Aphrodite.
between the triangle as a female sexual symbol40 and the male gods to whom the Pythagoreans dedicated the angle of the triangle (above n. 48). It is noteworthy that in prehistoric ornamentation there are examples which anticipate the "tetractys."41 The φιλόσ φίλοσ figures, too, with their speculative interpretation, "make sense," in a way, as "archetypal" patterns; they "speak to" certain psychic dispositions. Those mysterious dodecahedra mentioned above62 may be rooted in similar soil.

Greece, too, has its primeval, ritually significant symbolic numbers. Even in the Homeric epics one notices the preference for certain numbers,8 especially 3 and 9, and also 5 and 7—the odd numbers. The number 8 hardly occurs at all, and also apparently played no part in the ancient Pythagorean tradition.90 Certain numbers belong to certain gods; the cult of Apollo and that of Dionysus were dominated by the numbers 7 and 9.94 The significance of 3 in purification ritual was emphasized by Aristotle; as Usener perceived, it is rooted in a primitive conception of number, in which, after 1 and 2, 3 means plurality in general.95 At the symposium one set out three cranefish of wine, and poured a libation from the first to Zeus and the Olympians, from the second to the Heroes, and from the third to Zeus Soter or Teleios.96 Two belongs to the chthonic world, whereas 3 is the number of completion or fulfillment. If one is tempted to see here the early onset of Pythagorean interpretation, still even in Homer himself, δόξα means "doubt," or "confusion"; and it is not far from here to δίκαιος. In wrestling, a match has been ended, since time immemorial, when one contestant has won three falls (τριάκτυς). The special status of the first four numbers, as a group—the tetractys—is reflected even in the fact that they alone are declined, after Indo-European tradition.

According to Aristotle,97 the Athenians were divided, in early times, into 4 phylae and 12 phratries each made up of 30 families, on the model of the division of the year into seasons, months, and days. Numerical order unites society and cosmos; and even if this were to be put down as fourth-century speculation, it may well be based on something older. The number 7 is in Solon’s view the one according to which human life is to be divided into periods; and neither its interpretation as καθάρισμα nor its important role in Greek medicine can be traced to Pythagoras as originator.98 The most popular expression of the inferior status of even numbers came in the theory that 7-month and 9-month babies can live, but 8-month babies cannot.99 It is asserted in the Hippocratic book on the subject that the first movement of the embryo in the womb takes place after 3 months in the case of boys and 4 in the case of girls.100 The formation of the embryo takes 30 days for boys and 42 for girls; and the postpartum discharge lasts a proportionate length of time.101 Thus 3 is the male and 4 the female number, which in the second example is conjoined with a period measured in sevens. There is some Pythagorean material in the Hippocratic writings;102 but the

40 E.g., the expression δελθα in Ar. Lys. 151, Paus. 2.2.6.11; A. Stuber, RAC s.v. Dreieck, with refs.: M. Elad, Die römische Zahlensymbolik, tr. S. Coovv (London: Rider, 1962) 155.
42 Above, ch. VI 3, n. 65.
43 Above, passim.
44 Cf. n. 4–10. For Homer, Germain 8f; on the other hand, 8 becomes important in late antiquity and in Christianity: F. J. Déöger, Antike und Christentum 4 (1934) 153–182.
45 Roscher, Ahbh. 1904, 5ff, 54ff. For example, the Carina last 9 days; 9 men gather in each of 9 "sunshades" (tent-like structures: Ath. 4.141e); Apollo was born on the seventh day of the month, and he is ζημόγαριος at Aesch. Sept. 800. There is a 7-day, sevenfold blessing with καθάρισμα στροφάδων at the Pharmacos ritual (Hipponax fr. 11; IX 499f Diehl-Beutler; cf. Nilson I 107ff). There are 7 bunches of grapes and 7 dolphins in the famous Dionysus cup of Ekkielos.
46 Above, n. 2; Arist. Cal. 626a14; cf. Od. 11.128. Reemphasizing 3, one gets 9 and 27, which are important in the cult of the dead (Soph. OC 479ff, Germain 38f; on Roman phenomena, above, n. 20).
47 K. Kircher, Die südtale Bedeutung des Weins im Altertum (Giesen, 1910) 17ff, 34ff; Nilson, Op. 1 428ff. Zeis τέχνη, Eur. fr. 148, Ar. fr. 526, Philochorus FGGrHist 328887 (with Pythagorean interpretation). In Pythagorean doctrine, one is enjoined to bring odd-numbered offerings to the gods of heaven, even to those of the underworld (Plut. Num

14. Por. VP 38, Iam. VP 156, Schol. A II. 23.171; Plato alludes to this, Leg. 717a; Serv. Aen. 3.305, Buc. 5.66, 8.75, ascribes it—probably following Varro—to the Romans. But this is not universally valid for either Greek ritual (above, n. 55) or Roman (above, n. 205).
88 Above, n. 29. For the material in the Hippocratic corpus about “critical days,” see Roscher, Ahbh. 1906, 55–84. Hippocrates ( Epid. 1.26) distinguishes a number of κηριάτων ιτωάτων from another ιτωάτων. See also Heracleitus A18, B126 (whose genuine-ness is very doubtful, because of the dual, quite irregular for Ionic), Alcmaeon A15, Empedocles A83, fr. 153a, Hippo 1,6, Aristox. fr. 23. Criticism of Pythagorici numeri: Cels. Med. 3.4.
89 Modern medicine does not confirm this doctrine, at least in such an extreme form. Of course, ancient physicians were very seldom able to determine precisely the time of conception.
101 Nat. puer. 18, VII 500 L.; but according to Dioscorides and Strato the embryo is formed in the case of girls by the sixth, in that of boys by the seventh hekdomad (Strato, fr. 97–98 Wehrli)—here the girls are quicker, but they still have an even number.
102 Above, ch. Ill 2, nn. 113 ff; Ill 3 nn. 61, 86.
forms ἄ τριστος and ἰ τρισκεῖστε are attested even in Mycenaean Greek, and there is no obvious reason for the variation in gender in nouns from the same root, except that gender is determined by number. The birthday of Apollo is celebrated on the seventh, that of Artemis on the sixth day of the month.88 A man, says Hesiod,89 should marry at about 30,

ἡ δὲ γυνὴ τέτων ἴβλιοι, πέμπτῳ δὲ γαμοῖτο.

A girl has four years to herself after she reaches puberty, and her marriage is to take place in the fifth. So the numbers 3, 4, and 5, as those of man, woman, and marriage, are present in early Greek folk custom.

All this goes to show that the curious numerical correspondences, known as Pythagorean as early as Aristotle, are not a late development or a trivialization of a Pythagorean philosophy or mathematics, but have their roots in primitive ideas of number. The numerical symbols for male, female, opinion, marriage, opportunity, and “the whole” are earlier in origin than the time of Pythagoras; they were present not only in oriental lore but in the language and customs of the Greeks themselves. The notion that numbers have a “metamathematical,” cosmic significance, and that they reveal the principle of the order of the world and of human life, is not any kind of scientific or philosophical insight, but a readily comprehensible characteristic of pre-mathematical thinking about number. Pythagorean number symbolism is therefore much older than any natural science, mathematics, or astronomy that Pythagoras or his pupils could be imagined to have practiced. It has nothing to do with science in our sense—which is to say, the Greek sense—of the word; it neither presupposes this nor advances it. The Pythagorean doctrine that “all is number” grows directly out of “archetypal” number symbolism, which in one degree or another is worldwide in occurrence.

Of course, number symbolism can be combined with scientific knowledge. Both in China and among the Pythagoreans, it took over mathematical music theory; and in the “harmony of the spheres” we have an adaptation of mathematical musicology to the newly discovered, scientific understanding of the cosmic order. But this cannot mean more than an application and confirmation of a basic idea which was already present and was open to any kind of enrichment. Number is not quantity and measurability, but order and correspondence, the articulation of life in rhythmic pattern, and the perspicuous depiction of the whole as the sum of its parts. To see a “consistently quantitative view of the world”86 in Pythagorean number theory is a mistake.

One cannot help recognizing how closely this number symbolism is connected with the realm of the aiciousata. In both cases the origin lies in primeval custom, found in somewhat similar form among the most varied peoples and cultures; and in both cases we find the circumscription of life and thought by forms and formulas, which are laid on the Pythagorean by authority and whose explanation and interpretation are secondary in importance. Number in fact means restraint; counting is performed in successive acts, and thus time itself, composed of successive events, is number.86 The great mass of the aicusata had to do with sacrificial ritual, and its methods and times, the καυρι,87 so that correct piety (εὐθεῖα) depends on knowing number. While this reveals the orderly arrangement of time, such a point of view may lead also to thoughts of the recurrence of the same, of periodic transmigrations, or even to the recurrence of the same condition of the world.88 To the four seasons correspond the four ages of man’s life,89 the

88 D.L. 2.44; Deubner, Attische Feste (Berlin, 1932) 179, 201, 209.
89 Op. 695, 698. Naturally, 40 cannot be the usual age for women to marry, nor, hardly, can 17 for men; it is all the more remarkable how 3, 4, and 5 are worked into the formulation. A provision in Plato’s Laws is that a man may not be punished with blows until the age of 30, a woman till 40 (912b-c, 913c). According to the Republic (560e), a woman may have children from the twentieth to the fortieth year, a man from the twenty-fifth to the fifty-fifth. Perhaps it is worth noting that though groups of 4 are uncommon in ritual, it is in women’s cults that they do appear (Germain 51; the Heraca at Olympia, Paus. 5.16; Demeter Chthonia at Hermion, Paus. 2.35.7; the four daughters of Celerus, Hymn. Hom. Cer. 109f).—One hopes that, on the third day of the month, a child born will turn out to be a boy; for a girl born on this day would turn out to be unfeminine in character (Schol. V on II. 8.39, Suda s.v. Τρισκεῖστε). Naturally, there are exceptions: Heracles and Hermes were born on the fourth.—One resists the temptation to mention the 3 Dorian and 4 Ionian phyla.

86 Frank 72: “konsequent quantitative Weltanschauung.”
87 It is not a coincidence that in the oldest references τριστός, δεκάς, τριακάς are mostly used in a temporal sense, of the 4th, 10th, and 30th day (Hes. Op. 794, 798, Hymn. Hom. Merc. 19; differently in II. 2.126). Seidenberg (above, n. 13) propounds a thesis that counting had its origin in a “creation ritual.” This is unprovable, but his collection of material on ritual counting is worth attention.
88 Iam. VP 85 (following Aristotle: above, ch. II, 4, n. 5): τὰ δὲ πλείστων έξο μήκος, περὶ τὴν θυσίαν καθ’ ἐκ τῆς τοῦ καυρίου πάς χρὴ ποιεῖται. A portion of the exposition, based on later sources, is at Iam. VP 152.
89 Eudemus fr. 88, Dicaearchus ap. Por. VP 19.
90 This is frequent in later sources: the “triparitum,” D.L. 8.10, Of. Met. 15.199-213. For parallels see Delatte, Vie 110. Perhaps Alcmaeon has this in mind in fr. 2, if it means that, while the year is continually renewed, the winter of man’s life is not followed by a new spring (above, ch. III 3, n. 97). Cf. the riddle of the Sphinx solved by Oedipus, and Hippoc. Nat. hom. 2.
of the universal regularity, to be comprehended by means of number, through which the word cosmos acquired "a deeper meaning, and one of decisive importance for the history of philosophy." Mathematical astronomy, after some stirrings in the fifth century, was only brought to full development by Eudoxus; music theory was at first more a number game than a science; and the "philosophiae naturalis principia mathematica," in the sense of Newton, were never attainable to the Greeks, even though in the Timaeus it seems that Plato dreamed of something of this kind. Modern perspectives distort our view of the ancient "wisdom" of Pythagoras, as in fact it had soon become distorted in antiquity. An impressive achievement can consist in the synthesis, systematization, and consistent development of something preexisting, to say nothing of the unaccountable workings of a personality which is not dependent on originality or the independent importance of specific thoughts or doctrines, but of itself enhances the significance of meditation and teaching. Pythagoras was no lonely, unrecognized thinker, an extremely renowned and successful "sage," the founder of a society which lasted a long time. Further, is it not a great achievement if a strong personality, by virtue of its charismatic power, gives new life to what is old, and preserves its spiritual forces through a period of turmoil and change? The schemes and projects of science always fall short; there remains an unexplained residue to cause disquiet and the yearning for completeness. To round this out into a comprehensive knowledge which could satisfy all the aspects of a man's being, this ideal was regarded as accomplished by Pythagoras. Perhaps it is here that we should see his great importance; because of him there remained alive something of the ancient lore, pushed to one side by the growth of science but never really dispensable, powers of the past which could preserve a feeling of security in an increasingly secular and material world.

It has long been known that conscious and unconscious, rational and irrational impulses, logic and mysticism, interpenetrate in a complicated and nearly inextricable fashion. As Kepler discovered his second planetary law in "Pythagorean" manipulation of regular polyhedra, so one might find it obvious that precisely the pre-philosophical lore of Pythagoras provided the stimulus for Pythagorean science. But not only does the cosmic significance of number come from pre-logical

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20 Hesiod measures cosmic distance in time (Th. 72af), Anaximander in linear terms (above, ch. IV 1, nn. 55-56).
21 Above, ch. I 2, nn. 45ff; III 2, nn. 130, 162.
22 ἑκατονταυδοκεῖον, Diod. 1.11 (Hecataeus of Abdera?), describing the "Egyptian" doctrine of microcosm and macrocosm.

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24 See Becker, F. Gadjamer 12.
number symbolism, but, even in that which Aristotle presents as the philosophy of the Pythagoreans, there emerges again and again a spirit and method directly opposite to that of exact mathematics, so that the latter cannot have arisen from the activities of the Pythagoreans. It is not an unbroken unity of science and religious-ethical teaching that we find in the Pythagorean tradition, but a groping attempt to mediate between two levels, to transpose an ancient interpretation of the world into the language of the recently founded φυσικός. In this transposition, apparently, the opposition of "limiting" and "unlimited" first attained an important part, even though the high valuation placed on "limit" is a derivative from the ancient lore. This made it possible to show a relationship between essential characteristics of being and the world of number. Other agreements were sought and happily found—significant numbers in the calculation of the scale, regularities in the pebble diagrams, orderly behavior among the stars. But there is no steady continuity or consistency; different kinds of material were present, partly retained, like the ancient numerical symbols, or the mere juxtaposition of analogous ideas, and partly added, like the medical doctrines in Philolaus (A27–28), which do not have any necessary relation to his cosmological ideas. The acausmatic and similar regulations were passed on, without further justification, as Plato attests Philolaus did (Phd. 61d–e). Thus this Pythagorean philosophy was a synthesis closely tied to the conditions of the time, and without any lasting validity. The nature of the tradition authorizes us to regard it as largely the work of Philolaus, following after some similar attempts of Hippasus.

But even Philolaus and his pupils must have been persuaded that in their thoughts they were only following and carrying out the insights of Pythagoras, and presenting in different terms what he had long ago known and meant. New interpretation of this sort is inevitable, if an authority is to maintain his position as intellectual history progresses. Pythagoras was the wisest of all men, the tradition said. For an age that still knew no science, γνώτες and τελεσταί were the "sages," and Pythagoras belongs in this context, as we can see from the oldest witnesses, whether they mock or admire him. But only a few decades later, it was impossible to accept this; so the image of Pythagoras must change. If Pythagoras was "wise," he was "wise" in the spirit of the new age, certainly not a γόνος but a scientist. The "sage" acquires his knowledge by his relation to the gods, whereas the scientist bases his on deductive proof. The "sage" works immediately, in his own person, whereas the scientist writes books. The "sage" knows the roads through Heaven and Underworld, whereas the scientist measures cosmic distances in stades. The "sage" interprets the signs of the zodiac, whereas the scientist calculates the movements of the heavenly bodies in advance. Number lore in particular must inevitably appear in a very different light; what had been a symbolic ordering and classification of the multiplicity of phenomena becomes, in hindsight, mathematics, for a nonmathematical idea of number is no longer acceptable. The delight of the pragmatic historian in a πρώτος εὑρέθη helped to transmute this interpretation into a historical datum.

Plato forced the divergent factors of rationality and morality, of physical science and speculative interpretation, into a new synthesis, though he is quite consciously continuing ancient tradition. All the more, it was Plato's pupils who placed themselves under the authority of "the ancients," and as a consequence saw Pythagoras in the light of what Plato himself had been the first to do. The pre-Socratic interpretation of Philolaus, inadequate in many respects, was supplanted by a kind of Platonism, which was regarded in succeeding centuries as the "true" Pythagoreanism. Since the content was mostly just passed along with no other justification than authority, and was accepted without further examination, it corresponded in its function to the unified conception of the world characteristic of an earlier, pre-scientific

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76 Number symbolism belongs, for Plato, in the realm of myths; and this is true, in particular, of the famous "nuptial number" of Rep. 545c. (See esp. Heath, Math. I 30ff; A. Diès, Le nombre de Platon [Paris, 1936]; more recently, A. Ahlvers, Zahl und Klang bei Platon [Bern, 1932] 11ff; M. Denkinger, "L'énigme du nombre de Platon," REG 68 [1955] 38–70; H. von Ehrenfels, AGP 44 [1962] 240–244. Clearchus had already commented on the passage, as shown by fr. 3–4 Wehrh.) The Muses present the exposition of the nuptial number παῖσανα καὶ ψαλμιδόταν, as Plato has it (Rep. 545c; this is theerriment of the superior; cf. Pind. 3.656, Phil. 536, Leg. 885c). What follows is not nonsense but the indication of meaningful mathematical connections. A relationship with Pythagoreanism is probable (above, ch. VI 2, n. 13; it is hard to decide whether Alex. Met. 75.27ff is genuine tradition or a secondary reconstruction). But this does not mean that whoever has discovered the number 12,000,000, or some other number, will have grasped the secret of the cosmos, and put himself in a position to prevent its downfall. Its decline is not to be arrested by human means (546b). The veil of riddle suggests an infinite task; it is myth, in the sense of groping for truth at the threshold of the ineffable. Even in decline and destruction, it is not chaos or blind conjecture that rules, but an eternal, intelligent order; and mathematical knowledge can enable us to grasp this in some small degree. This is the interpretation the Platonist Eratosthenes also gives to the legend of the "Delian problem," of duplication of the cube: the god's intention was, through the insoluble problem, to inspire interest in the study of geometry (Theo Sm. 2.3ff).
age. Again, since a doctrine whose appeal is to faith is normally presented in as ancient garb as possible, the name of Pythagoras sometimes obscured that of Plato. The tradition of Pythagoras as a philosopher and scientist is, from the historical point of view, a mistake. But the fascination that surrounded, and still surrounds, the name of Pythagoras does not come, basically, from specific scientific connotations, or from the rational method of mathematics, and certainly not from the success of mathematical physics. More important is the feeling that there is a kind of knowing which penetrates to the very core of the universe, which offers truth as something at once beatific and comforting, and presents the human being as cradled in a universal harmony. In the figure of Pythagoras an element of pre-scientific cosmic unity lives on into an age in which the Greeks were beginning, with their newly acquired method of rational thought, to make themselves masters of their world, to call tradition into question, and to abandon long-cherished beliefs. The price of the new knowledge and freedom was a loss in inner security; the paths of rational thought lead further and further in different directions, and into the Boundless. There the figure of the ancient Sage, who seemed still to possess the secret of unity, seemed more and more refulgent. Thus after all, there lived on, in the image of Pythagoras, the great Wizard whom even an advanced age, though it be unwilling to admit the fact, cannot entirely dismiss.
Abbreviations generally follow the standard list of the *Oxford Classical Dictionary* (that in the second edition, of 1970, is substantially identical with that of the first). The following list includes some works frequently cited, some departures from the usage of OCD, and some items omitted from it. For Greek authors, see also the list in Liddell-Scott-Jones, *Greek-English Lexicon* (Oxford, 1925–40, with Supplement, 1968), and for journals, the list in *L'année philologique*.

### I. PERIODICALS AND COLLECTIVE WORKS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AA</td>
<td><em>Archäologischer Anzeiger</em></td>
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<tr>
<td>AA</td>
<td><em>Acta antiqua Academiae Scientiarum Hungaricae</em> (only cited for works of Szabó; see bibliography)</td>
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<tr>
<td>A&amp;A</td>
<td><em>Antike und Abendland</em></td>
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<tr>
<td>AAHG</td>
<td><em>Anzeiger für die Altertumswissenschaft</em>, hrsg. von der Osterreichischen Humanistischen Gesellschaft</td>
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<tr>
<td>ABG</td>
<td><em>Archiv für Begriffsgeschichte</em></td>
</tr>
<tr>
<td>AbhBln</td>
<td><em>Abhandlungen der Preussischen Akademie der Wissenschaften</em>, Berlin</td>
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<td><em>Abhandlungen der Sächsischen Gesellschaft der Wissenschaften</em>, Leipzig</td>
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<tr>
<td>AC</td>
<td><em>L'antiquité classique</em></td>
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<td>AGP</td>
<td><em>Archiv für Geschichte der Philosophie</em></td>
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<td>AJA</td>
<td><em>American Journal of Archaeology</em></td>
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<td>AJP</td>
<td><em>American Journal of Philology</em></td>
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<td>ALL</td>
<td><em>Archiv für lateinische Lexikographie und Grammatik</em></td>
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<td><em>Athenische Mitteilungen</em></td>
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<td><em>Ancient Near Eastern Texts</em>, ed. J. B. Pritchard</td>
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<td>AnzAW</td>
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<td>Abbreviation</td>
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<tr>
<td>At-R</td>
<td>Atene e Roma</td>
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<tr>
<td>Arch. delt.</td>
<td>'Αρχαία δείκτη σι κερκυρώς</td>
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<tr>
<td>Arch. eph.</td>
<td>'Αρχαία εφημερίς</td>
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<tr>
<td>ARW</td>
<td>Archiv für Religionswissenschaft</td>
</tr>
<tr>
<td>CAG</td>
<td>Commentaria in Aristotelis graeca, Berlin (cited by page and line)</td>
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<tr>
<td>CAH</td>
<td>Cambridge Ancient History</td>
</tr>
<tr>
<td>CIG</td>
<td>Corpus inscriptionum graecarum</td>
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<tr>
<td>C&amp;M</td>
<td>Classica et mediaevalia</td>
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<td>CMG</td>
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<td>CRAI</td>
<td>Comptes rendus de l'Académie des Inscriptions et Belles-lettres</td>
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<td>DK</td>
<td>Hermann Diels, Die Fragmenta der Vorsokraker, 6th ed. by Wathler Kranz (Berlin: Weidmann, 1951-1952; repr. of the 5th ed., 1934-1937, with Nachträge; later editions are reprints of this)</td>
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<td>Dox.</td>
<td>Hermann Diels, Doxographi graeci (Berlin, 1879; repr. 1958)</td>
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<td>Felix Jacoby, Die Fragmenta der griechischen Historiker (Berlin: Weidmann, Leiden: Brill, 1923-)</td>
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<td>FHG</td>
<td>Car. and Theod. Müller, Fragmenta historiorum graecorum (Paris, 1841-1870)</td>
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<td>GGN</td>
<td>Göttingische gelehrte Nachrichten</td>
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<td>GRBS</td>
<td>Greek, Roman, and Byzantine Studies</td>
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<td>IG</td>
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<td>JAW</td>
<td>Jahresbericht über die Fortschritte der klassischen Altertumswissenschaft, founded by Conrad Burian</td>
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<tr>
<td>JIIH</td>
<td>Journal of the History of Ideas</td>
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<td>JHS</td>
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<tr>
<td>MHI</td>
<td>Museum Helveticum</td>
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<td>New English Dictionary (Oxford, 1905)</td>
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<td>Hermann Diels, Poetarum philosophorum fragmenta (Berlin, 1901)</td>
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<td>Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, B: Studien (Berlin, 1931ff)</td>
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<td>Revue des études grecques</td>
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<td>RhGr</td>
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<td>SVF</td>
<td>Stoicorum veterum fragmenta, coll. Hans von Arnim (Leipzig, 1903-1921)</td>
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<td>TAPA</td>
<td>Transactions of the American Philological Association</td>
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Ach. Is.
Achilles Tatius, Introductio in Aratum, ed. Ernst Maass (Commentariorum in Aratum reliquiae, Berlin, 1898)

Ael. NA
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Ael. VH
Aelian, Varia historia

离开
Alexander Aphrodisiensis, In Aristotelis Metaphysica commentaria, ed. Michael Hayduck (Berlin, 1891; CAG I; cited by page and line)

Anat.

Anon. Phot.
Anonymous Photii, Phot. cod. 249 (Thesleff, Texts, pp. 237-242; see also bibliography s.v. Immisch)

A.P.
Anthologia Palatina

Ap. Hm.

Apul. Plat.
Apuleius, De Platonе et eius dogmate

Arist. EE
Aristotle, Ethica Eudemia

Arist. EN
Aristotle, Ethica Nicomachea

Arist. fr.

Arist. Met.
Aristotle, Metaphysica

Arist. MM
Aristotle, Magna moralia

Aristid. Quint.
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Aristox. Harm.

Ascl. Met.
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Hdt.

Hebd.

Hipp.

Hippocr.

Hipp. Ref.

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