Lore and Science in Ancient Pythagoreanism

Walter Burkert

Translated by Edwin L. Minar, Jr.

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If Pythagoras does not present himself to our minds as a sharply outlined figure, standing in the bright light of history, this is not merely the result of accidents in the course of historical transmission. From the very beginning, his influence was mainly felt in an atmosphere of miracle, secrecy, and revelation. In that twilight period between old and new, when Greeks, in a historically unique achievement, were discovering the rational interpretation of the world and quantitative natural science, Pythagoras represents not the origin of the new, but the survival or revival of ancient, pre-scientific lore, based on superhuman authority and expressed in ritual obligation. The lore of number is multifarious and changeable. That which was later regarded as the philosophy of Pythagoras had its roots in the school of Plato. Outlines of an earlier reformulation of Pythagorean doctrine in the manner of the φυσιολογία of the fifth century can be detected in the fragments of Philolaus. As the old and the new interpenetrated and influenced each other, the picture of Pythagoras became distorted until, with the victory of rational science, he came to seem its true founder.

To investigate these interrelationships is still a somewhat risky undertaking; but an attempt has been made to take more account than has previously been done of the variety in the kinds of evidence available and, above all, to clarify the ramifications and the divagations of the tradition.

I am indebted for much advice and encouragement to my teachers Otto Seel, Reinhold Merkelbach, and Helmut Berve, as well as to Dr. Ludwig Koenen and to Dr. Burkhart Cardauns, who also helped me with the proofs. To all of these I offer hearty thanks.

Erlangen
April 1962

Walter Burkert

Preface to the English Edition

In revising this book for translation, it has been impossible to add references to all the literature on the subject which has appeared since 1962. I have tried to concentrate on the ancient evidence, to cut down polemics, and to incorporate whatever I have learned in these years, notably from some reviews of the German edition, and from continuous discussions with B. L. van der Waerden. In the question of the "discovery of the irrational," I have taken a stand which is less critical of the tradition; and more thorough acquaintance with ancient religion has pushed the concept of "shamanism" further into the background. But though a good number of passages have been revised, and though there are some small rearrangements in the order of treatment, still the book has remained, in all the main lines, the same.

New and comprehensive accounts of Pythagoreanism have been given by Kurt von Fritz, H. Dörrie, and B. L. van der Waerden in Pauly-Wissowa (XXIV 171–300; Supp. X 843–864), and by W. K. C. Guthrie in his *History of Greek Philosophy*, vol. I; and the same year (1966) saw the appearance of two books with the title *Pythagoras and Early Pythagoreanism*, by C. J. de Vogel and J. A. Philip. Discussion is certain to continue. I cannot claim to have provided a definitive history of Pythagoreanism, or a complete account of Pythagoras, the man and the genius. Still I trust that this book gives a full and perspicuous presentation of the evidence and thus will be useful even to those who are not inclined to draw the same conclusions from it.

My special thanks are due to Edwin L. Minar, Jr., who not only completed the laborious task of translation in a spirit of most pleasant collaboration, but to whom is due the original initiative which brought about the English edition. The responsibility for the content, and for all that may be wrong in it, remains mine.

Schwerzenbach-Zürich
August 1970

Walter Burkert

Contents

The "Pythagorean question" has sometimes been compared with the Homeric question. Not that the details of the problem would especially suggest this; what does remind one of that most famous of philological controversies is the difficulty of the argument and the lack of agreement on methodology, as well as the multiplicity and contradictory character of the solutions advanced. Another similarity, and not the least striking, lies in the tremendous importance of the questions about the life, activity, and influence of Pythagoras of Samos. Over the origins of Greek philosophy and science, as over the beginning of Greek literature, lies the shadow of a great traditional name. The attempts of scholarship to grasp the underlying historical reality keep getting entangled in contradictions; where some think they discern the figure of a world-historical genius, others find little more than empty nothingness.

Pythagoras' influence was a lasting one. The ancient tradition of the history of philosophy made him the ancestor of the "Italian School" and therefore, after Thales, the second, and more important, originator of *philosophia*—in fact, the inventor of the word. The doctrine transmitted under his name, that numbers are the principles of what exists, that the "One" is its primal ground, became part of the amalgam of Neoplatonism. In the trend set by Iamblichus, Pythagoras was the high priest, par excellence, of the divine wisdom. He then became, in the trivializing school tradition of the Middle Ages, the master of the quadrivium, and in particular the inventor of arithmetic. The early modern period discovered Pythagoras as the creator of natural science, which was just then being reborn; what Copernicus and Galileo taught was regarded by their contemporaries as a revival of Pythagorean science.[1]

Subsequently, as research based on historical perspective increasingly

[1] E.g., Campanella in a letter to Galileo of Jan. 13, 1611 (T. Campanella, *Lettere*, ed. V. Spampanato [Bari, 1927] 165). Further literature in Capparelli, I 29ff; see below, ch. IV 3, nn. 1–2.

replaced the uncritical appropriation of ancient culture, the traditional picture of Pythagoras, imposing though also vague in outline, inevitably gave way more and more before criticism. In the scholarly controversy that followed scarcely a single fact remained undisputed, save that in Plato's day and then later, in the first century B.C., there were *Pythagoreioi*. The "wisdom of Pythagoras," however, has also had passionate defenders, who opposed to criticism a countercritique: one-sided, self-sufficient methodology, they protested, had substituted hypothesis for tradition.

When we set out to survey the most important attitudes and trends in modern Pythagorean scholarship, the point of departure must be the work of Eduard Zeller.[2] In it the material is not only collected, with a completeness scarcely to be surpassed, but sifted with uncommon methodological rigor. The criterion for the value of a tradition is its age, and Zeller arrives at the verdict, often cited since in agreement and disagreement: "The tradition about Pythagoreanism and its founder thus has more and more to tell, the further it lies, chronologically, from the events . . ." (I 364). This "expansion of the tradition" (ibid.) arose basically from "dogmatic preconceptions, partisan interests, dubious legends, and spurious writings" (365). The most important source, nearly the only one which is left, is in the reports of Aristotle, in his surviving treatises. A second primary source is found in the fragments of Philolaus, in which August Boeckh, in his day, claimed to have found a firm foothold amidst the bog of Pythagorean pseudepigrapha.[3] But Aristotle speaks of "Pythagoreans," not of "Pythagoras"; so the figure of the master fades off in a mist of nonhistorical legend. Pythagoras is still recognized as the "founder of a religious society" and teacher of transmigration (411), but all philosophical significance is denied his ethical and religious doctrines (557ff). Alongside this stands, without connection, the number philosophy of the Pythagoreans. This can be reconstructed from Aristotle and is confirmed by Philolaus, though aside from him the Pythagoreans remain anonymous and scarcely datable. Sharply separated from all this ancient material is the neo-Pythagorean school, which (Zeller thinks) arose, compounded of Platonic and Aristotelian elements, not before the first century B.C. (III 2.92ff).

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Zeller's work had a decisive influence, especially in Germany; it dominates Diels's arrangement of testimonia in the *Fragmente der Vorsokratiker*. In the chapter on Pythagoras (DK 14), the biographical reports are assembled, along with the reports as to whether or not Pythagoras wrote anything.[4] In the separate chapter on "The Pythagorean School" (DK 58), the most important section is the collection of "ancient Peripatetic" material (58B),[5] following Iamblichus' catalogue of Pythagoreans (58A), and itself followed by the *acusmata* (58C), the *Pythagorikai apophaseis* of Aristoxenus (58D), and the allusions to "Pythagorists" in the Middle Comedy (58E). Between these two chapters are found, in chronological order as far as possible, the testimonies on individual Pythagoreans, notably Hippasus (DK 18), Philolaus (DK 44), and Archytas (DK 47). Hermann Diels follows Zeller also in the question of the genuineness of the Philolaus fragments.

Zeller's solution, however, left a number of problems unsolved, and later research entered in with supplement, modification, and criticism. Above all, a gap had opened between Pythagoras the religious founder and the number philosophy of anonymous Pythagoreans; to connect these disparate elements and to show their original unity was bound to be an extremely enticing challenge. For at this time the tide of system building in philosophy, which had borne the work of Zeller, was ebbing, and this very change made possible a deeper understanding of cultural history. As the boundaries of philosophy became fluid, the connections with pre-scientific, religious-mythical thought became clearer. So the task appeared to be to comprehend how religion and philosophy could be united in Pythagoras: mysticism and science (or at least the germ from which science sprang). Then came the high tide of the evolutionary idea, and it began to seem possible, with its help, to explain the contradictions of the tradition and to give everything its place in an extended and detailed history of Pythagoreanism.

The direction was set by August Döring (1892); his thesis became most influential because he was followed by John Burnet in the later editions of his *Early Greek Philosophy*.[6] The unity of science and religion is found in the ideal of catharsis; scientific activity is the highest form

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of purification and conduces toward the salvation of the soul.[7] Further, we have the principle that "The more primitive any Pythagorean doctrine appears, the more likely it is to be that of Pythagoras himself" (Burnet, *EGP* 99); in this way it is inferred that Pythagoras had not only a number theory but an astronomical system. A more advanced stage is discernible in the "Pythagoreans" of Aristotle, but his reports too are broken down into different categories and are supplemented from Plato as well as later sources.

The picture of Pythagoreanism built up in this manner is far more colorful than Zeller's skeptical account, but it depended so largely on inferences and hypotheses that opposition was inevitable. On the basis of the same leading idea, of development from religion to rational science, completely different and mutually irreconcilable reconstructions were offered. The most significant contribution, after Burnet, was F. M. Cornford's article "Mysticism and Science in the Pythagorean Tradition";[8] but his conclusions were importantly modified by his own pupil J. E. Raven.[9] Here should be mentioned also the independent accounts of Léon Robin, Abel Rey, and Pierre-Maxime Schuhl. The fact that each scholar had to erect a whole new superstructure shows the weakness of the foundation.

Nevertheless, the attempts to improve on Zeller are based on additional source material, which, though not newly discovered, was for the first time revealed in its true purport. For this was a further weakness in Zeller's position; he had been too hasty in rejecting the later tradition. More exact study showed, here as elsewhere, the fallacy in the principle *recentiores, ergo deteriores*. Erwin Rohde initiated the careful analysis of the sources of the lives of Pythagoras by Porphyry and Iamblichus, and this led back, for substantial portions, to authors of the fourth century B.C.—Aristoxenus, Dicaearchus, Heraclides Ponticus, then Timaeus. With growing optimism others followed along the path Rohde had pioneered.[10] It was not only in the realm of political history that many new insights could thus be attained; the problem of the philosophy and science of Pythagoras also began to appear in a

[7] Döring, *AGP* 1892, 505; Burnet, *EGP* 97f; below, ch. II 6.

[8] *CQ* 1922–1923; cf. *PlParm* 1–27 and *CAH* IV (1926) 544–552.

[9] Raven *PyEl*, repeated in more cautious tones in KR 216ff, 307ff.

[10] See below, ch. II 1. Special gratitude is due Armand Delatte, who completely studied through the later tradition, including even its most abstruse areas. All the parallel material is collected and set out very perspicuously by Delatte in his edition of Diogenes Laertius' life of Pythagoras, and by Ludwig Deubner, building upon Delatte's work, in his edition of Iamblichus' life of Pythagoras.

new light. The reconstruction of Aristotle's *Protrepticus* from passages in Iamblichus seemed to yield statements by Aristotle himself about the philosopher Pythagoras.[11] But most important of all, thirty-five years after the first appearance of Zeller's *Philosophie der Griechen*, Diels had shown in his *Doxographi graeci* (1879) that Theophrastus was the ultimate source of the wide-branching doxographical tradition. After that it was difficult to attribute the reports of the *Placita* to the neo-Pythagoreans, as Zeller had done (I 467ff). Diels classified them with the "ancient Peripatetic tradition" (DK 58B15), and they play their important role in all the above-mentioned reconstructions of a developing Pythagoreanism.

There was also an attempt to discover indirect sources; one sought to find Pythagorean material reflected in other pre-Socratic thinkers, whether by way of influence or of polemic. The most important step in this direction was the thesis of Paul Tannery, that Parmenides took the cosmology of the *doxa* section of his poem from the Pythagoreans and that Zeno's polemic was directed against their number theory. This brought some very ancient evidence into the field, which could help to classify, to supplement, and even to test the reports of Aristotle. This hypothesis was developed in many ways, and only very seldom placed seriously in question.[12] Similar endeavors were of course directed toward Plato, whose connection with Archytas is firmly fixed in the biographical tradition. To be sure, it is especially difficult in this case to achieve any certainty. The native capacity of Plato's mind to remold and reshape is too great, and in each specific instance it is a controversial question to what extent particular statements of the dialogues may be interpreted as historical facts. The theory of Burnet and Taylor, which takes every portrayal in the dialogues as historical fact, not only transforms the entire *Timaeus* to a Pythagorean document of the fifth century B.C., but presents us with a Socrates who is an advocate of the theory of ideas and an adept of Pythagorean wisdom.[13] Though this radical solution has attracted no following, the Pythagorean origin of the theory of ideas, and especially of the

[11] Iam. *Protr.* 51.7ff; Arist. *Protr.* fr. 11 Walzer. On this, cf. Burkert, *Hermes* 1960, 166ff, below, ch. I 3, n. 77. I. Düring, *Aristotle's Protrepticus* (Göteborg, 1961) p. 189, holds to the Aristotelian origin of this passage in Iamblichus (B18–20 Düring); this forces him to smooth over the difficult transition at Iam. *Protr.* p. 51.6f by conjecture and to ignore the characteristic *τὸ ἐστιν;* at Iam. *Protr.* 4.11; as a result, the hints toward source analysis that the text of Iamblichus provides are arbitrarily removed.

[12] Cf. below, ch. III 3 and VI 3, n. 46.

[13] Cf. Burnet, *EGP* 277f, 308f; *ThPl* 151ff; Taylor, *Tim.* and *VarSocr* (Oxford, 1911) 1/8ff.

doctrine of recollection,[14] is still being discussed; and it is generally taken as proved that Plato owed his scientific knowledge to the Pythagoreans, especially in the realm of astronomy.[15]

Zeller had scarcely more than touched on the natural sciences, never doubting the fundamental importance of the Pythagoreans in the development of Greek mathematics, astronomy, and music (I 405f). Here was a further incentive to look for more precise results. After the fundamental investigations of Paul Tannery, the comprehensive expositions of Sir Thomas Heath, and the competent studies of Abel Rey, detailed reconstructions of Pythagorean mathematics were given by Oskar Becker, B. L. van der Waerden, and Kurt von Fritz. Even though every attribution to Pythagoras himself remains unprovable, still the reconstruction seems to lead back almost as far as his era.[16]

Thus analyses and reconstructions have in many ways given occasion to think less skeptically than Zeller about the sources for Pythagorean history. More and more the tendency has been to put a higher valuation on the later tradition. Even apocryphal works were, correctly, taken seriously as evidence pertinent to intellectual history, and some were credited with an early date, in particular the *Memoirs* (*Hypomnemata*) that had been excerpted by Alexander Polyhistor.[17] Research in the religious history of late antiquity led in a similar direction. Franz Cumont and Jérôme Carcopino made extensive use of Pythagorean tradition in the interpretation of funerary symbolism of the imperial period. From this point of view there was no difference discernible between early and late Pythagoreanism; it was rather as though a powerful and continuous stream flowed from an ancient source. The numerous studies of Pierre Boyancé also follow this tendency; their aim is to grasp the "origine pythagoricienne" behind late material.

Understandably, the "Italian philosophy" was interpreted (and still is) in an even more trustful spirit in Italy. Augusto Rostagni's book *Il verbo di Pitagora* may be regarded as the most ingenious, though at the same time the most daring, attempt to comprehend science and mysticism as a unity and to trace them back to Pythagoras himself.[18] The concern of Rodolfo Mondolfo has been cautious but resolute defense

[14] Cf. below, ch. I 2, n. 82; II 6, nn. 22–24.

[15] On Plato's astronomy, below, ch. IV 1–2; cf. also H. Cherniss, *Lustrum* 4 (1959) 50ff.

[16] Cf. below, ch. VI.

[17] Cf. n. 5. Delatte, *Pol.* 121ff, maintains that Archytas' *On Law and Justice* is genuine (but see below, ch. I 3, n. 156). See also Mondolfo in *ZM* 382ff.

[18] On this, cf. below, ch. III 1, n. 7; see also ch. II 1, n. 37; II 2, n. 38.

of the tradition against the attacks of critics.[19] Even the translations and explications of the Pythagorean source material collected in the *Fragmente der Vorsokratiker*, by Antonio Maddalena (1954) and Maria Timpanaro Cardini (1958–1964), have the goal of confirming the age and philosophical significance of Pythagoreanism; and chauvinistic enthusiasm for Pythagoras runs riot in the bulky works of Vincenzo Capparelli.[20]

Over against all these attempts to achieve a more positive view than Zeller are energetic movements of scholarly criticism which have even called into question testimony accepted by that scholar. The genuineness of the Philolaus fragments was attacked by Carl Schaarschmidt in 1864, and in 1868 by Ingram Bywater. While Zeller's authority held up for a while in Germany, Burnet followed Bywater and therewith ensured the predominance of the negative verdict on the Philolaus fragments which still holds in the English-speaking world.[21]

Rejection of these Philolaus fragments is an essential element in the thesis of Erich Frank, whose book *Plato und die sogenannten Pythagoreer* (1923) towers over everything else that has appeared since Zeller on the history of Pythagoreanism, in the qualities of critical vigor, penetration, and firmness of judgment. To be sure, its merits are counterbalanced by one-sidedness and obvious perversities. Frank's methodological contribution was that he consistently held to the history of the natural sciences—mathematics, music, and astronomy—as basis for the reconstruction of Pythagoreanism. The first result was to date the development much later; all Pythagorean science, he thought, had come into existence in the circle of Archytas, about 400 B.C., influenced by the fully developed atomism of Democritus. The philosophy of the "so-called Pythagoreans," however, the number theory, was dependent on the late Plato, and was basically a creation of Speusippus, who had also himself forged the book attributed to Philolaus.[22] The figure of Pythagoras fades into the mist,

[19] Mondolfo added to his translation of Zeller an ample "Nota sulle fonte della conoscenza e ricostruzione storica del Pitagorismo" (313–385), and published two parts of it separately: "Sui frammenti di Filolao," *RivFil* 15 (1937) 225–245 (=ZM 367–381), and "Platone e la storia del Pitagorismo," *A&R* 39 (1937) 235–251 (=ZM 335–345).

[20] *La sapienza di Pitagora* (Padua, 1941–1944), 2 vol. with 1,536 pages; *Il contributo pitagorico alla scienza* (Padua, 1955); and *Il tenore di vita pitagorico ed il problema della omoiosis* (Padua, 1958).

[21] Cf. Zeller I 369.3 (vs. Schaarschmidt); below, ch. III 1, n. 14.

[22] This thesis was advanced by Ernst Howald, too, in *Essays on the History of Medicine Presented to Karl Sudhoff* (Zürich, 1924, pp. 63–72; see his very favorable discussion of Frank's book, *JAW* 197 [1923] 166ff), and worked out, though in a somewhat superficial manner, in the dissertation of Jenny Bollinger (Zürich, 1925).

and all the much-discussed Pythagoreanism of the fifth century becomes a mirage.

Frank's book, teeming with arbitrary theories and ex parte judgments, has been severely criticized;[23] and curiously enough, Frank himself expressed quite different views in later writings.[24] Nevertheless, the book still has importance, above all because of the extreme way in which the problem is put: "Plato and the Pythagoreans"—their mutual relationship is in fact the central problem of any historical investigation of Pythagoreanism, and Frank was right in perceiving that the influence did not go entirely in one direction. There is Platonic material which at a later date was wrongly labeled Pythagorean, and the generation of Plato's immediate disciples—Speusippus, Xenocrates, and Heraclides—played the decisive role in this development. Just as correct, and important, is the idea of regarding "Pythagorean" mathematics and science in the context of non-Pythagorean science, which stems from the Ionian and Eleatic philosophy of nature, and inquiring to what extent the Pythagoreans may have been on the receiving side.

In fact, notwithstanding the reconstructions of "Pythagorean mathematics," the date and importance of Pythagorean influence on Greek mathematics had been called into question as long ago as 1907 by Gustav Junge, then by Heinrich Vogt, and above all by the profound and influential work of Eva Sachs, *Die fünf platonischen Körper* (1917). The reports relating to Pythagoras himself are eliminated, what is firmly attested is dated later, and the non-Pythagorean science of the Greeks is revealed. Finally, William Arthur Heidel tried to push further on in this direction.[25] If we add that the reports of the "religious leader" Pythagoras were subjected by Walther Rathmann to so devastating

[23] As late as 1951 a direct attack was published: G. de Santillana and W. Pitts, "Philolaus in Limbo, or: What Happened to the Pythagoreans?" *Isis* 42 (1951) 112–120.

[24] Though one read in 1923 that "all those discoveries attributed by later writers to Pythagoras himself were in fact achievements of certain Southern Italian mathematicians of the time of Plato" (vi), he wrote later (*Knowledge, Will and Belief: Wissen, Wollen, Glauben: Collected Essays*, ed. L. Edelstein [Zürich, 1955] p. 82) "it can hardly be doubted that Pythagoras was the originator of this entire scientific development. He was a rational thinker rather than an inspired mystic." Though he had in 1923 held it to be impossible that Plato had taken over Pythagorean material (Philolaus) in *Philebus* 16ff—"if one reads the *Philebus* without prejudice, one has the impression, throughout, of being in the presence of the mature result of a long philosophical career" (p. 304)— still, he declares quite plainly in 1940 that Plato was dependent, in this passage, on Pythagorean ontology (*AJP* 61 [1940] 49 *Knowledge, Will and Belief* 100). On the other hand, his radical hypercriticism comes to the fore again in a review of von Fritz's *Pol.*, *AJP* 64 (1943) 220–225.

[25] *AJP* 1940; cf. below, ch. VI.

an analysis that scarcely anything remained, and that the keen analysis of Harold Cherniss shook in a dangerous way the very foundation of Zeller's structure, that is, the reliability of Aristotle's data, it seems that the last vestige of a possible consensus has disappeared, and it is no wonder if resignation spreads.

The material seems to fall into the pattern each inquirer is looking for. The historian of science rediscovers Pythagoras the scientist; the religiously minded show us Pythagoras the mystic; he who believes in a synthesis above rational analysis tries to show that in Pythagoras the *coincidentia oppositorum* is comprehended in a Basic Idea; the anthropologist finds "shamanism"; and the philological scholar may play off against one another the contradictions of the tradition, so that critical virtuosity may sparkle over a bog of uncertainty. Pythagoreanism is thus reduced to an impalpable will-o-the-wisp, which existed everywhere and nowhere.

The very life of philology is the struggle between the tendencies toward faith in the tradition and skepticism of it. In the case of Pythagoreanism, to be sure, the skeptical tendency has from the outset powerful arguments on its side. The apparent primary sources, writings of Pythagoras and his pupils, some of which are preserved while others are known to us through secondary reference, are with very slight exception unquestionably apocryphal. There is no longer even any discussion of their authenticity, except for the Philolaus fragments and some of the Archytas material.[26] Of course, "forgery" has its own importance in intellectual history; but in the discussion of early Greek thought there is no place for that which—like the book of Ocellus or of Timaeus of Locri—is obvious imitation of Platonic and Aristotelian material.

In addition, the secondary sources, comprising the tradition about the life and doctrines of Pythagoras and the history of his school, contain a good many sheer impossibilities[27] and, still more often, undeniable contradictions on very important questions—whether Pythagoras was present at the revolt in Croton, whether the numbers

[26] See now Thesleff, *Texts* (245 pages!); as against 94 lines of text from Archytas, which Diels regarded as genuine, there are 46 pages of ps.-Archytas material.

[27] E.g., Pythagoras was taken prisoner in Egypt by Cambyses (525 B.C.), stayed 12 years in Babylon, and then returned to Samos in time to leave the island, in 532 B.C., because of the tyranny of Polycrates (cf. below, ch. II 2, n. 16). He met Phalaris about 570, and was the teacher of Empedocles, who was born about 490. The musical experiments which are attributed to Pythagoras are physically impossible (below, ch. V 1).

should be thought of as corporeal or incorporeal, whether Pythagoreans believed that the earth moves or not.[28] Each side of each question is advocated by ancient authorities, sometimes with explicit polemic. Even the most extreme credulity comes to a limit here; it is simply impossible to accept "the Pythagoras of tradition," because there is no single tradition.

Finally—less striking but scarcely less disquieting—in many cases late tradition gives the name of Pythagoras, where older tradition, dealing with the same topic, does not do so.[29] This circumstance is even more suspicious when a number of apparently ancient testimonies crumble at the touch of analysis—the name of Pythagoras is interpolated in Aristotle's *Metaphysics*,[30] added by Iamblichus in a passage from Aristotle in his *Protrepticus*,[31] supplied by Proclus, after Iamblichus, in a fragment of Eudemus,[32] inserted by Porphyry in a passage stemming from Theophrastus.[33] When observations of this kind multiply, Zeller's suspicion about the "expansion of the tradition" seems justified.

On the other hand, criticism is always subject to counter criticism. Though many sources may be late and not very reliable, more must lie behind them all than a simple zero. "Pythagoreanism without Pythagoras," without chronological position or a place in the history of thought, is not only unsatisfying to the scholar, but impossible in itself. A minimalism that eliminates every aspect of tradition which seems in any respect questionable cannot help giving a false picture.

These very difficulties in evaluating the Pythagorean tradition reveal certain characteristic differences between Pythagoras and the other pre-Socratics, which inevitably set the direction for a new study of the problem. Just as a city which was continuously inhabited over a period of time, by changing populations, presents to the archaeological investigator far more complicated problems than a site destroyed by a single catastrophe and then abandoned, the special difficulty in the study of Pythagoreanism comes from the fact that it was never so dead as, for example, the system of Anaxagoras or even that of Parmenides. When their systems had been superseded and lost all but their philological and historical interest, there still seemed to be in the spell of Pythagoras' name an invitation to further adaptation, reinterpretation, and extension. And at the source of this continuously changing stream lay not a book, an authoritative text which might be reconstructed and interpreted, nor authenticated acts of a historical person which might be put down as historical facts. There is less, and there is more: a "name," which somehow responds to the persistent human longing for something which will serve to combine the hypnotic spell of the religious with the certainty of exact knowledge—an ideal which appeals, in ever changing forms, to each successive generation.

Scholarship cannot succumb to this spell. Its first task must be, since the original phenomenon cannot be grasped directly, to interpret interpretations, to single out and identify the different strata of the tradition and to look for the causes that brought transformation to the picture of Pythagoras.[34] Zeller, in his day, had seen the task and solved it in his own way, blaming neo-Pythagoreanism for all the "late" distortions. Since the results of source analysis are hardly compatible with this thesis, inquiry must take a new start.[35] Perhaps the chances of success are better than before; our knowledge of the development of fourth-century philosophy has grown, thanks to studies of Academic and Peripatetic tradition, notably by Werner Jaeger[36] and Fritz Wehrli,[37] and to the elucidation of the Platonic system of first principles carried on, after Julius Stenzel, by Hans Joachim Krämer and Konrad Gaiser. The flood of works on the pre-Socratics has provided us with means of distinguishing fifth-century thinking, in the wake of Parmenides, from later philosophy. And in relation to this, we should be able to identify an even earlier stratum.

At the same time, the intention in what follows is to give its full value to each of the various aspects of the Pythagoras tradition. One

[28] Cf. below, ch. II 2, nn. 46–47; I 3, n. 52; III 1, nn. 67ff.

[29] Cf. below, ch. I 2, n. 76; II 6; IV 1, nn. 19, 32, 38; VI 1, nn. 34–37.

[30] Cf. below, ch. I 2, n. 6.

[31] Cf. above, n. 11.

[32] Cf. below, ch. VI 1.

[33] Por. *Abst.* 2.28, p. 158.5 14 Nauck; cf. J. Bernays, *Theophrastos' Schrift über die Frömmigkeit* (Berlin, 1866) 119f.

[34] The Pythagorean movement still lingers on today. The book of Jean Mallinger, *Pythagore et les mystères* (Paris–Brussels, 1944) is dedicated to the "revered head of the Pythagoreans of Belgium," and his conviction is that "Pythagoras answers, in an amazing way, all the questions and all the needs which today's anxiety brings forth" (p. 7). The same author has written a *Note sur les secrets ésoteriques des Pythagoriciens* (Paris, 1946). In July and August, 1955, there was held in Brussels, Athens, and Samos a "Pythagorean World Congress" (cf. Schottlaender 333). Since then, Tigani, the site of ancient Samos, has been renamed Pithagorion.

[35] The task of disentangling the various branches of the tradition has been emphasized especially by Olof Gigon; see *Entretiens sur l'antiquité classique* I (Vandoeuvres–Geneva, 1952) p. 141.

[36] Jaeger showed how the Old Academy and Peripatos projected their own ideal, at each period, upon Pythagoras (*Arist.* 99f; *SBBln* 1928, 395f, 415ff).

[37] Cf. his commentary on the relevant fragments of Aristoxenus, Dicaearchus, Heraclides, and Clearchus.

can take science seriously as science and at the same time comprehend the meaning and function of that which was present before science, and which continues to influence it. One can acknowledge the structural unity of philosophy and still recognize that the history of thought is not exhausted in the reconstruction of doctrinal formulae. It is inherent in the forms of life, it cannot be separated from the unique individual, and it cannot be repeated.

Most studies of Pythagoreanism have dealt with only one restricted aspect; even Zeller confined himself to the development of philosophical concepts, left mathematics aside, and bracketed out religious and ethical questions; and later works have been even more specialized, whether in the philosophical area, in that of mathematical, astronomical, and musical problems, or that of religion. The very thing that might seem rash and hasty, in view of the fundamental differences of interpretation, is what the nature of the situation demands: as many-sided a treatment of the problem as is possible. For many of the contradictory conclusions have come from investigating and tracing the course of single paths of development, with no thought of the way in which these may converge with other, equally important lines. Any attempt to date Pythagorean philosophy and science back as far as possible, even to Pythagoras himself, must take account of its connection with the religious-cultic and primitive sides attested for the movement; and any investigation of Pythagoras the "shaman" must endeavor to make sense of the later development of Pythagorean science. The division of labor which seems so reasonable brings along with it the danger of a vicious circle. It can happen that the historian of science builds his reconstruction on a philologically inadequate foundation; the philologist takes over the seemingly exact result of the historian of science; the philosopher, on the basis of this criterion, rejects contradictory evidence—and so on. The many-sided treatment which can prevent misunderstandings of this kind must be the objective of the classical scholar; collection, interpretation, and critical analysis of the ancient evidence is the necessary common foundation. Mathematical and philosophical thought, as well as parallels from comparative religion, can never yield more than possibilities; as to the historical facts, the sources are decisive.

At the beginning of the following studies stands, not the analysis of the oldest evidence, but discussion of the tradition of Pythagorean philosophy; the task turns out to be not to separate older and more

reliable from later and more dubious material, but to judge between traditions which are contradictory but of equal antiquity. In this matter a remarkable stroke of luck has provided us with an important new piece of source material, a fragment of Speusippus on Pythagorean philosophy first published in 1953. This makes certain, what a careful analysis of the sources would in any case make likely, that a Platonizing interpretation of Pythagoreanism, which had a decisive influence on the later tradition, goes back to Plato's immediate disciples and differs sharply from the reports of Aristotle. The latter's evidence thus becomes more important than ever; for he alone warns us to separate Pythagorean and pre-Platonic from Platonic material. Only from the point of view gained by the revelation of this contrast is it possible to evaluate Plato's own testimony, and in particular the allusions in the *Philebus*. In this way we find incontrovertible evidence—as against Frank and Howald—of a pre-Platonic and pre-Socratic philosophy of the Pythagoreans.

Study of the oldest, pre-Platonic tradition can thus be supplemented by those pieces of evidence which stand outside the Platonic influence, and were not affected by the reinterpretations mentioned above. Once more the reports of Aristotle become especially important, the fragments of his lost monograph on the Pythagoreans. The Pythagoras story, which used to be, for the most part, written off as the unfortunate product of the obfuscation of historical facts, may be understood as the expression, precisely, of a definite historical reality. Pioneers of this line of interpretation were Karl Meuli and E. R. Dodds. To the legend belong the *acusmata*, in their essence doubtless extremely old. The result that emerges is a rather distinctive picture of a shamanistic "Wise Man" and a Life, or Way of Life (*bios*), dominated by ritual—a Pythagoreanism foreign to all exact science.

The study of the Philolaus fragments harks back to the results of the first chapter. A point of view is achieved from the distinction of Platonic and Aristotelian traditions about Pythagoras, from which one can see clearly the genuineness of at least part of these fragments. The attempt is made, then, to make these understandable in the framework of fifth-century thought, as the attempt of a Pythagorean to come to terms with the natural science (*physiologia*) of the Ionians and the Eleatics.

The history of the natural sciences can confirm this result, for on one hand the Philolaus testimonia take their position in the history of fifth-century thought, while on the other the origin of the exact

sciences is seen to lie outside the Pythagorean realm. The renown of Pythagoras as the inventor of mathematics and mathematical natural science is explicable as a distortion of perspective; a pre-rational interpretation of the cosmos, along with arithmological speculation, is seen in hindsight as rational science.

My aim has been to take full account of the ancient evidence;[38] and the modern literature has been consulted as fully as was practicable. As far as possible the originator of each particular thesis or argument has been named, but it was out of the question to try to provide an encyclopedic doxography of all pertinent views on each problem. I hope nothing decisive has been overlooked.

[38] It is often not necessary to cite all parallel passages. Citation of a passage in Iamblichus or in Diogenes Laertius is always to be taken as also a reference to the collection of parallels in the editions of Deubner and Delatte, respectively.

I. *Platonic and Pythagorean Number Theory*

I. THE PLATONIC THEORY OF IDEAL NUMBERS

Πλάτων πυθαγορίζει—from the time of Aristotle, this finding has often been repeated,[1] but there is little clarity as to the extent and the manner in which Plato borrowed Pythagorean doctrine, or as to what Pythagoreanism was like before Plato. This applies especially to those attempts to derive the Ideas from numbers, to equate them with numbers, or even to replace Ideas with numbers, which Aristotle and others attribute to Plato and his pupils, in particular to Speusippus and Xenocrates. These theories looked somewhat like a capitulation to Pythagoreanism on the part of the aging Plato, until Julius Stenzel set the task "to understand the concepts of Plato's late philosophy in their own context and to incorporate into Plato's philosophical development, inseparably, those 'Pythagorean mystical' elements that have up to now been traced back to external, foreign influence."[2] Since then, much scholarly work has been devoted to the Platonic "doctrine of principles,"[3] and to its interpretation in Platonic terms, starting from Platonic presuppositions. Still the question of pre-Platonic sources, of pre-Platonic Pythagorean doctrine, persists. The crucial point is to grasp firmly what the difference is between this and the later, Platonic philosophy. An investigation must start from a sketch of the Platonic theory of principles, since we know more about its context.

The foundation for the study is the evidence of Aristotle; he alone sets up Platonic and Pythagorean doctrines side by side, specifying

[1] Esp. *Met.* 987a29 ff.; cf. e.g. Aët. 2.6.6; Cic. *Rep.* 1.16, *Fin.* 5.87, Apul. *Flor.* 15, p. 60, Apul. *Plat.* 1.3, ch. I 4, n. 47.

[2] *ZG* 108.

[3] Handy bibliography in the German reprint of Stenzel *ZG* and in Gaiser, who also gives a convenient collection of the sources. The most comprehensive work remains L. Robin, *La théorie platonicienne des idées et des nombres d'après Aristote* (Paris 1908, repr. 1963). New evidence was adduced by Merlan (1934) and Wilpert (1940, 1941). The paper of H. Gomperz (1930) deserves special mention, because it has been generally overlooked. A new, energetic attempt at reconstruction and interpretation has been made in the books of Krämer (1959) and Gaiser (1963); among more recent contributions, see Theiler *Isonomia*; Dönt; K. H. Ilting, *Phronesis* 13 (1968) 1–31.

their points of agreement and—what is almost more important—their differences.[4]

This might appear an extremely unfavorable point of departure. Precisely as the result of the amount of recent research in this field, it remains one of the most controversial in Platonic studies, without any prospect of early agreement. There is controversy as to how the connection of ideas and numbers is to be understood, how the whole matter is related to the well-known philosophical theories expressed in Plato's dialogues, and what is to be attributed to Plato himself and what to his various pupils. And the most learned studies in the field, the books of Harold Cherniss,[5] have shaken the very foundation of the discussion with the thesis that most of the supposed evidence is wrong inference, or even falsification, by Aristotle, and that the true Plato is to be found only in the dialogues and has nothing to do with these speculations. At the other extreme, Krämer and Gaiser hold that the backbone of Plato's philosophy is a "doctrine of principles" (*Prinzipienlehre*) which can be found even in allusions in the dialogues. The lively, sometimes acid, discussion which has been going on for years will not easily be brought to a solution.[6]

Fortunately for the reconstruction of Pythagorean doctrine, not much depends on the solution of these problems. What the relation is between Plato's dialogues and the "doctrine of principles," whether it represents an early or only a late stage in Plato's philosophy, whether a particular doctrine is to be traced to the historical Plato, to Speusippus, Xenocrates, or even to Aristotle's interpretation, becomes irrelevant when the issue is to establish the difference between Platonism in general and earlier Pythagoreanism. In what follows, the adjective "Platonic" may be understood as applying to "Platonists" and "Platonism," regardless of the question of their relation to the philosophy of Plato himself.

For, though Cherniss has rightly emphasized the independence of

[4] Astonishingly little attention has been paid to Aristotle's distinction of Platonism from Pythagoreanism, though Zeller (I 465ff) collected the evidence. Thus down to Raven (KR nos. 320, 405, 406) and Guthrie (I 256–262; cf. below, ch. I 2–3). Platonic doctrines are included among Pythagorean documents. Frank denied himself a "more exact source-analysis of the Aristotelian and post-Aristotelian reports," because of the inadequacy of "the space available" (n. 388); thus he missed what would have refuted his thesis (cf. below, ch. I 2) that all "Pythagorean" philosophy is post-Platonic.

[5] Cherniss, *Plato*, vol. I—the second volume is not expected to appear—and *Riddle*, esp. pp. 20ff.

[6] For a polemical survey, cf. Krämer, *MH* 21 (1964) 137–167; *Philologus* 110, (1966) 35–70; earlier criticism of Cherniss' theses: De Vogel, *Mnemosyne* 1949; Ross, *PTI* 142ff.

Plato's pupils and the differences between their doctrines,[7] still it is not to be denied that there is a common tendency in the area of "first philosophy"—to use Aristotle's term—in their essays at basic ontological doctrine. The aim is to trace ideal Being to its basic principles (ἀρχαί) and to understand this Being in terms of these principles, just as the Platonist learned to understand the world in which we live from the point of view of the ideas. Heinrich Gomperz proposed the convenient term "system of derivation," though this characterizes the movement of thought in a one-sided manner. It is rather the ascent from the empirical to the Higher, the vision of the Idea through the world of experience that surrounds us, and the realization of the *agathon*, the One, in the complexity of the ideas that Platonists strive for, than conceptual "deduction" or "derivation" (Arist. *EN* 1095a32).

The problem of unity and multiplicity of ideas, of their interpenetration and their distinction by the method of diaeresis was, as the dialogues show, of increasing concern to Plato.[8] The borderline between dialectical exercise and metaphysical seriousness seems to be deliberately blurred. There occurs a curious reticence when the conversation touches upon the most essential questions, even in early dialogues.[9] In the *Timaeus*, Plato gives more definite indication that it is the question of the πάντων ἀρχή (the first principle of all things) that is being bracketed out (48c): Plato has reduced the multiplicity of the world to the four elements, the elements to regular solids, and these to triangular surfaces; "but the principles that are still prior to these god knows, and he among men who is dear to him" (53d). The *Republic* introduced the Good as the Highest, the Sun in the realm of ideas, "beyond being" (509b). The opposite to the good, in all the later dialogues, is described as an indefinite oscillation in two directions, toward "great and small," "more and less," "stronger and weaker," and it is the Good that constitutes measure and definiteness in this continuum.[10]

It is from this point of view that we can approach the reports about Plato's oral teachings,[11] in particular the lecture *On the Good*. Our

[7] *Riddle* 60off.

[8] *Parm.* 129aff (and passim), *Phlb.* 15a, *Soph.* 251aff. Cf. Krämer 429ff.

[9] *Prot.* 357b. Cf. Krämer 389ff.

[10] This has been worked out in a convincing way by Krämer.

[11] In one passage Aristotle cites explicitly the "unwritten doctrines" of Plato (*Phys.* 209b15; cf. Cherniss, *Plato* 113ff, 166f, Krämer 416ff), and he clearly refers to them in other passages (Ross, *PTI* 143ff); once (992a22) he speaks of Plato's "repeated" pronouncements.

principal source for these is the comments of Aristotle.[12] Besides, written records of this lecture by Speusippus, Heraclides, Xenocrates, and Hestiaeus are attested,[13] and it is possible that a detailed report in Sextus Empiricus was derived, by a roundabout route, from one of these.[14] Simplicius has preserved an important fragment from a report of Plato's pupil Hermodorus;[15] besides, Aristoxenus reports, on the basis of an oral account by Aristotle, on the external circumstances and the content of the lecture,[16] and there are some relevant statements in Theophrastus' *Metaphysics*. Reconstruction may proceed here on a comparatively broad basis.[17] It is hard to get rid of the tradition which traces the theory of ideal numbers to this lecture;[18] proceeding from it, Plato's students developed the theory in their own way. In reporting on it, Aristotle's attitude varies according to the subject he is dealing with. The ideal numbers are not treated in *Περὶ ἰδεῶν*,[19] whereas in

[12] Frr. 27–31, on which see esp. Wilpert, *Hermes* 1940. Even Cherniss concedes that Alexander of Aphrodisias used the book directly, though obviously no one did so after him (*Plato* 119n. 77; *Riddle* 27ff).

[13] Simpl. *Phys.* 151.6ff, 453.25ff; for Heraclides, D.L. 5.87; for Xenocrates, D.L. 4.13.

[14] Sext. Emp. *Math.* 10.248–284. Cf. ch. I 3.

[15] *Phys.* 247.30ff; see esp. Wilpert, *Hermes* 1941, 227ff, and *Zw Fr* 183ff; De Vogel, *Mnemosyne* 1949, 205ff; Cherniss, *Plato* 285ff; Krämer 282ff.

[16] *Harm.* 2, p. 30 M.: the *logoi* had to do "with mathematics, numbers, geometry, astronomy, and, finally, that the good is single" (translating Macran's conjecture ‹τ›ἀγαθόν and taking τὸ πέρας as adverbial. Cf. Cherniss, *Riddle* 87 n. 2, Ross, *PTI* 148 n. 1, 244, Krämer 423; differently interpreted in De Vogel, *GP* I 274 n. 1).

[17] Cherniss tries to undermine this foundation with the observation that in the reports about the lecture *On the Good* the point corresponds to the One (Alex. *Met.* 55.20ff; Simpl. *Phys.* 454.19ff; also Sext. Emp. *Math.* 10.259ff), while according to Aristotle Plato eliminated the concept of the point (992a20ff; Cherniss, *Plato* 167ff, *Riddle* 28f). De Vogel's attempted compromise, *Mnemosyne* 1949, 306ff, is not completely convincing. In the passages of Aristotle, however, which refer to Plato, there is no mention of the point as a "monad having position"; the first number is 2 (cf. 1081a21ff), and the first geometrical entity is the line, whereas Speusippus adopts the point corresponding to One. Thus there is some inexactitude in the later reports (cf. Krämer 418 n. 76: the commentators are using mathematical terminology). This tends to confirm the reliability of Aristotle.

[18] While Cherniss seeks to isolate the lecture *On the Good* as much as possible (*Riddle* 12), Ross speaks of a "course of lectures" (*PTI* 148), and Krämer states: "The discourses on the Good provide . . . the standard expression for the teaching activity of Plato in general" (409); he could refer to πολλάκις Pl. *Ep.* 7.342a and Arist. *Met.* 992a22, but his argument from the "imperfect of repetition" (407) in the passage from Aristoxenus (*Harm.* 2 p. 30 M.) is not conclusive; he would understand "every time when the lecture turned out to be on mathematics . . . this would seem, I think, paradoxical to the audience." But ὅτε δὲ φανείησαν may as well be optative in indirect discourse (W. Theiler, *AGP* 50 [1968] 29 n. 1). Aristotle used to tell the anecdote as a warning how not to proceed in lecturing, how to avoid disappointment by correct advertising. Evidently not the audience alone had been disappointed. Thus this experience has—against Krämer—nothing to do with the "test" described at *Ep.* 7.340b, the deliberate discouragement of merely curious auditors by demonstrating the difficulty of philosophy.

[19] Theiler, *Isonomia* 92, draws chronological conclusions from this. That the theory of ideas was first set up without recourse to ideal numbers is stated by Aristotle at *Met.* 1078b11.

his discussions of "first philosophy," he has a tendency to put a disproportionate emphasis on number theory: "If the Ideas are not numbers, it is impossible for them to exist at all; for from what kind of principles will the Ideas come?" (1081a12f).

The most important hints at these unwritten doctrines in the corpus Platonicum are to be found in famous passages of the seventh *Letter*; no wonder therefore that the discussion about the "esoteric" Plato is intimately bound up with the question of the genuineness of that letter.[20] Again, however, the answer is of limited importance for the investigation undertaken here: it is generally agreed that the letter is either by Plato, or by one of his immediate pupils, and, even in the latter case, the doctrines contained, especially the "philosophic digression" have about the same degree of authenticity as the reports on the lecture *On the Good*.[21] Dionysius, the letter says, has learned from Archytas (338c) and Dion something of the doctrines which Plato himself had not yet wanted to communicate to him, *parakousmata* (338d), chance bits of information about "the greatest matters" (341b), "something of that which is highest and first in the philosophy of nature" (344d; cf. 341d). It concerns "truth about virtue and vice" (349a-b), and at the same time "truth and falsity in the whole of being" (344b), one singular object of learning (*mathema*), which is not easy to communicate (341c). The "greatest teaching" is the Good (*Rep.* 505a), thus the close relation between the comments in the seventh *Letter* and the later memoranda of his pupils *On the Good* is obvious. As the audience was startled that "the Good" should concern mathematics and astronomy, the letter binds up "philosophy of nature" with "virtue and vice." Plato states however, "There is not any writing of mine on these matters, nor will there ever be, for this is a thing which cannot be put into words like other doctrines" (341c)—a sentence as famous as it is controversial in its interpretation. While Cherniss believes that anyone who takes these words as genuine and

[20] The authenticity of the seventh *Letter* had been a cornerstone of Platonic scholarship, especially in Germany, since Wilamowitz' *Platon* (1918), but Cherniss pronounced a negative verdict (*Riddle* 13), and proofs of inauthenticity were undertaken by G. Müller, *Archiv Philos.* 3 (1949) 251–276 (contra, H. Patzer, ibid. 5 [1954] 19–36, and B. Stenzel, *AJP* 74 [1953] 383–397)—and above all by L. Edelstein, *Plato's Seventh Letter* (Leiden, 1966) (contra, K. von Fritz, *Platon in Sizilien*, [Berlin, 1968]). The aid of the computer has been invoked: M. Levison, A. Q. Morton, A. D. Winspear, *Mind* 77 (1968) 309–325; but to attribute not only the seventh *Letter* but the introduction to the *Timaeus* and the *Critias* to Speusippus can hardly be the final solution.

[21] The philosophical digression is introduced as a λόγος . . . πολλάκις . . . ῥηθείς (342a). For interpretation, cf. H. G. Gadamer, "Dialektik und Sophistik im siebenten platonischen Brief," *SBHeid* 1964, 2; K. von Fritz, *Phronesis* 11 (1966) 117–153. On ὄνομα, λόγος, εἴδωλον, Pl. *Pol.* 285c–286a; οὐσία, λόγος, ὄνομα, *Leg.* 895d-e.

relates them to the *On the Good* must in consistency give up trying ever to understand Plato, Krämer emphasizes that these "ultimate matters" are not "inexpressible" in an absolute sense, but only for the great mass of mankind.[22] Yet Plato himself shows how serious he is about this "inexpressibility" by tracing its cause, in an excursus, to the relation between Being and the means of knowing. There are four means or steps of "knowing" an object: by name, by definition, by image, or by knowledge to which mind and right opinion are added (*ἐπιστήμη καὶ νοῦς ἀληθής τε δόξα*, 342c). But the fifth, Being itself, stands apart as "that which is the object of knowledge and truly exists" (342b). Mind (*nous*) comes closest to this (342d) but even mind does not grasp it completely and unambiguously. Each of the four kinds of knowing comprehends a qualitative aspect (*poion ti*) as much as the Being which the soul is seeking (342e):[23] "There are two things, the Being and the qualitative aspect, and it is not the 'what kind' but the 'what' that the soul seeks to know. But each of these four proffers to the soul the thing that is not being sought, and thus fills every man's mind with puzzlement and unclarity" (343b-c). Because of this inadequacy—even on the part of mind and knowledge—it is easy to contradict and refute where the "fifth" is concerned: "When we are under the necessity of separating out and revealing the fifth element, anyone who likes to do so has the means of confuting us" (343d). The ring of disappointment at the "contempt" (341e, 345b) to which the most sublime is exposed may echo the contempt (*ὑποκατεφρόνουν* Aristox.; above, n. 16) shown by hearers of the lecture *On the Good*. Therefore the only course possible, the letter says, is to take those who are already on the right path and are "related to" truth (344a), and lead them on, in patient practice, to the point where Being reveals itself to them in a sudden illumination: "Knowledge of each thing, and Mind, blaze forth in his view as he strains to the limit of human capacity" (344c).[24]

If this is taken seriously as Plato's view, the dialogues are devaluated as lacking seriousness, being only preliminary guideposts to what ought

[22] Cherniss, *Riddle* 13; Krämer 25ff, 401, 457ff; Gaiser 4f. According to Pl. *Leg.* 968e, the most important *mathemata* are not secret, (*ἀπόρρητα*), but cannot be told in advance (*ἀπρόρρητα*).

[23] The distinction *τί-ποῖον* appears as early as *Gorg.* 448e, *Meno* 71b. To modern logic, the *τί* question seems to be almost void of sense (R. Robinson, *Plato's Earlier Dialectic* [Oxford, 1953²] 49–53) which shows that it is not modern logic for which "the soul is longing." According to *Tim.* 49d (cf. Arist. *Met.* 1033b21), the object of the senses, as contrasted to the idea, is only a *ποῖόν τι*, never a *τί*, but this statement is not identical (against Wilamowitz, *Platon* II 294; Krämer 304, 459 n. 155) with the doctrine of the seventh *Letter*, where even *ἐπιστήμη* cannot grasp the *τί*.

[24] *συντείνων* MSS, em. E. Sachs (Wilamowitz, *Platon* II 295 n. 2): *συντείνοντι*.

to happen in a philosophical life. But also, the oral teaching can never be a final "system," made up out of non-contradictory rules and terms. For not even knowledge firmly grasps its object. Plato is not satisfied with elegant "principia mathematica," he asks for the Good—not in the sense of an "ought" imposed by some authority, but as the goal of all striving, the prop and the meaning of existence. This can only be perceived in an individual experience similar to religious revelation. But the only way to it is the common quest of indefatigable dialogue, which necessarily leads to the most stringent rules of dialectic, to logic and mathematics. There will be no final result, which can be taken down on paper or papyrus, so that any written account must be misleading. When Plato's students wrote about "the Good," they deviated from this attitude;[25] but it was doubtless a necessary task, once the discussion had grown into a variety of conflicting interpretations, to put down what Plato himself had taught. Independent philosophizing passed over very quickly, in the Academy, into history of philosophy —interpretation of the authoritative thought of the dead Master.[26]

For the historian, it is exact doxography that matters; so what follows is an outline of *doxai* on principles attributed to Plato. The highest principle of Platonic ontology is the One; alongside it[27] stands the Indefinite Dyad, a principle that is also described as great-and-small, many-and-few, exceeding-and-exceeded, and unequal.[28] It is responsible for every kind of multiplicity, contrast and change in the realm

[25] Krämer (412) thinks that the seventh *Letter* would allow *ὑπομνήματα* in the sense of the *Phaedrus* (276d; 278a), but 344d expressly rejects *ὑπομνήματα*. Ross, *PTI* 158, on the other hand, thinks that Plato forbids prose writings, and that the semipoetic dialogues are left untouched; but 344c5 seems to allude to the main works of Plato.

[26] Aristotle, of course, is the exception; he combined criticism with his notes. Fr. 27: "one ought to remember that we are but men [i.e. subject to failure] not only in the pursuit of happiness, but also in carrying out a demonstration."

[27] There is no word about any derivation of the "Indefinite Dyad" from the One; the later, so-called Pythagorean tradition presents monistic as well as decidedly dualistic interpretations, cf. below, ch. I 3.

[28] Arist. fr. 28 = Alex. *Met.* 56.8ff, Simpl. *Phys.* 453.33ff; *ἡ τοῦ ἀνίσου δυάς*, *Met.* 1087b7; *μὴ ὄν*, *Phys.* 192a7. There was some doubt whether the term *ἀόριστος δυάς* could be traced to Plato himself (Ross, *PTI* 184; *Met.* I 169), until the new fragment of Speusippus (*Plato Latinus* III 38; below, ch. I 3) was published. Since Speusippus himself called the second principle *πλῆθος*, it is from older tradition that he took the term "interminabilem dualitatem," i.e. from Plato. Aristotle expressly refers to the "unwritten doctrines" for the concept of *μεταληπτικόν* (*μέγα καὶ μικρόν*), *Phys.* 209b16. *Epin.* 990d–991b, a passage intentionally obscure, alludes to the role of the Dyad (on this passage, A. R. Lacey, *Phronesis* 1 [1955–1956] 81–104). Ross (*PTI*) tries to render the concept by "bare plurality" (which, strictly speaking, is rather Speusippus' *πλῆθος*); Becker (*ZwU* 18) by "logical extension," though Plato wants us to think of this "extension" as a duality, a deviation in two directions from the center, the measure, the One. Krämer has shown that this pattern of thought is discernible in ethical discussions of the late dialogues (140ff, 244ff).

of Being, as against the unity, identity, constancy brought about by the One. The One is identical with the Good;[29] the Indefinite Dyad is the ground of all evil. It is also called Not Being (Arist. *Phys.* 192a7). In Aristotle's terminology, the two principles are also related as form and matter.

Both principles "beget"[30] numbers from themselves. First 2 comes to be, as the "indefinite dyad" is limited by the One and transformed to the definite number 2 which consists of two equal units.[31] According to Aristotle, the other numbers arise in the natural succession 3, 4, 5, etc.;[32] but Aristotle emphasizes the difficulty—nay, impossibility—of deriving in this way other numbers than those of the type 2^n.[33] The numbers that arise in this way are independent entities, which cannot be combined in arithmetical calculations; their units are *οὐ συμβλητοί*, as Aristotle puts it.[34]

The relation of these numbers to the mathematical numbers which are used in calculation is hard to establish. Speusippus and Xenocrates put forth different solutions for this question: the former eliminated ideas and only recognized mathematical numbers as the ground of reality; Xenocrates equated ideal and mathematical numbers.[35] The main concern of these philosophers, however, is not to lay the foundations of mathematics, but to explain the world by means of its principles. The ideal numbers are not only the ideas of particular numbers—"twoness," "threeness," etc., but somehow govern the structure of reality: they are ideas themselves. It is not clear how this connection

[29] This is why the lecture was entitled *On the Good*; cf. Aristoxenus (above, n. 16), Arist. *Met.* 988a14, who, according to his own philosophy, says Plato "assigned" the Good to the One, as if this were a second step, whereas for Plato there was basic identity.

[30] *γεννᾶν* is used as a technical term, notwithstanding its mythological connotations. The ontological priority is established according to the principle *ἀναιρουμένου τινὸς μὴ συναναιρεῖσθαι*: of two concepts, the one which can be thought without presupposing the other is "prior"; thus number antecedes geometrical magnitudes, the line antecedes the plane, etc. (1019a1; Alex. *Met.* 55.22).

[31] 1081a23, 1083b23, 1091a24, fr. 28 = Alex. *Met.* 56.8–35.

[32] 1080a24, 1081a21, 1081b30, 1080a33, Ross, *PTI* 191.

[33] 1091a10. The Indefinite Dyad "produces duality" 1082a15; on the generation of the number Four, 1081b21, 1082a13. At 987b33 Aristotle says numbers can be "easily" generated out of Indefinite Dyad, *ἕξω τῶν πρώτων*, a much debated expression (Wilpert, *ZwFr* 207; Ross, *PTI* 188; Becker, *ZwU* 8), since the meaning "prime number" does not fit 1091a10. The One is said to be the cause of the Odd (1084a36), "falling into" the even number (1084a4), constituting the middle of even numbers (1083b28). Thus it would seem that the series of integers is produced by doubling and by adding one (cf. Alex. *Met.* 57.24), and still Aristotle states there can be no addition in the sphere of ideal numbers (1081b12, 35, 1080a23; Ross, *PTI* 192ff; cf. Wilpert, *ZwFr* 214ff).

[34] 1080a23; 1081b35.

[35] The evidence is collected by Ross, *PTI* 151.

of ideas and numbers is to be understood, in detail. While Aristotle says simply that the ideas are numbers, Theophrastus speaks of an "attaching" (*anaptein*) of the ideas to certain numbers, and thus allows us to imagine a looser relationship.[36] The attempt of Stenzel and Becker to understand the ideas as "numbers" on the basis of the method of division (*diaeresis*) prominent in Plato's late dialogues is not convincing.[37]

The most familiar example, the first step in the derivation of the world from numbers, often cited by Aristotle, is the sequence of geometrical dimensions: the line corresponds to the number 2, the plane surface to 3 (the triangle being the simplest conceivable plane), the solid (tetrahedron) to 4. It is recognized that Speusippus[38] and Xenocrates[39] worked out this derivation. Speusippus has the point at the beginning as corresponding to the One.[40] Aristotle also quite clearly attributes to Plato the derivation of line, plane, and solid "after the numbers";[41] here the "unlimited dyad," the "great-and-small," appears as "short and long," "broad and narrow," "deep and shallow."[42] One text derived from the lecture *On the Good*[43] tells us that the numbers 2, 3, 4 correspond to this derivation, but the testimony is unreliable insofar as, like Speusippus, it has the point corresponding to the One.[44] That Aristotle attributed to Plato himself the

[36] Ross, *PTI* 216 n. 1, has collected the passages of Aristotle; Theophr. *Met.* 6b11, who is followed by Ross 218: Plato "assigned numbers to ideas," without identification; other attempts at interpretation: De Vogel, *Mnemosyne* 1949, 311; Wilpert, *ZwFr* 170.

[37] After Stenzel's vague suggestions, Becker tried to give a precise solution in *QSt* 1 (1931) 464–501, defended *ZwU*; criticism in Cherniss, *Riddle* 54ff; Ross *PTI* 195ff. The main argument against Becker is that the "ideal number" would change with every new definition of an idea, and the same number would be attributed to quite different ideas. It is true that Alex. *Met.* 57.6 says that *διαίρεσις* is *γένεσις ἀριθμῶν*; but the number 2 is not generated by diaeresis (above, n. 33), nor is the sequence line–plane–solid a diaeresis.

[38] Fr. 4 = *Th. ar.* 84.10 *πρώτη μὲν γὰρ ἀρχὴ εἰς μέγεθος στιγμή, δευτέρα γραμμή, τρίτη ἐπιφάνεια, τέταρτον στερεόν.*

[39] Fr. 39; cf. fr. 34 = 1028b24.

[40] 1085a32: *ἕτεροι δὲ ἐκ τῆς στιγμῆς (γεννῶσιν τὰ μεγέθη)· ἡ δὲ στιγμὴ αὐτοῖς δοκεῖ εἶναι οὐχ ἓν ἀλλ' οἷον τὸ ἕν*, cf. *Top.* 108b7, b26; Cherniss, *Plato* 131 n. 82.

[41] This is clear from the passages where questions are raised about the relation of the ideal magnitudes so produced to the "intermediate" realm of mathematical magnitudes (992b13ff, 1080b23ff, 1085a7ff). Since neither Speusippus nor Xenocrates accepted this intermediate realm, the reference must be to Plato (Ross, *PTI* 206ff).

[42] 992a10ff, 1085a7ff, 1088b4ff, 1089b11ff, *Περὶ φιλ.* p. 78.20ff Ross.

[43] Sext. Emp. *Math.* 10.278ff; Alex. *Met.* 55.18ff and Simpl. *Phys.* 454.19ff do not mention this relation to the numbers.

[44] See above, n. 17. At 1085a7ff, Aristotle distinguishes between those who begin with the "great-and-small" and "others" (Speusippus: above, n. 38) who begin with the point.

derivation of the line from 2, the plane from 3, and the solid from 4, seems probable on the basis of two passages, though both are very controversial.[45] The well-known reduction of the physical world in the *Timaeus*, adequately evaluated by modern natural scientists,[46] seems to find here its natural sequel. If the elements are traced back to polyhedra, and polyhedra, the simplest of which is the tetrahedron, to triangular planes, and the question of their "origins," the "more ultimate principles" (ἀρχαὶ ἄνωθεν) is explicitly excluded (53d), the reduction in the present passage leads further, to the line and finally to the ultimate principles, the One and the Indefinite Dyad.

Aristotle further explicates the doctrine of ideal numbers in connection with psychological theory: Plato is said to have formed the soul out of "elements," in the *Timaeus*, in pursuance of the thought that like is known by like;[47]

[45] (a) At 1090b20ff, after his criticism of Speusippus (b14ff), Aristotle turns his attention to those "who posit ideas" (*οἱ τὰς ἰδέας τιθέμενοι*): *ποιοῦσι γὰρ τὰ μεγέθη ἐκ τῆς ὕλης καὶ ἀριθμοῦ, ἐκ μὲν τῆς δυάδος τὰ μήκη, ἐκ τριάδος δ' ἴσως τὰ ἐπίπεδα, ἐκ δὲ τῆς τετράδος τὰ στερεά . . . ἀλλὰ ταῦτά γε πότερον ἰδέαι ἔσονται . . . καὶ τί συμβάλλονται τοῖς οὖσιν; οὐθὲν γάρ, ὥσπερ οὐδὲ τὰ μαθηματικά* (cf. b13) *οὐδὲ ταῦτα συμβάλλεται. ἀλλὰ μὴν οὐδ' ὑπάρχει κατ' αὐτῶν οὐδὲν θεώρημα, ἐὰν μή τις βούληται κινεῖν τὰ μαθηματικὰ καὶ ποιεῖν ἰδίας τινὰς δόξας . . . οὗτοι μὲν οὖν ταύτῃ προσγλιχόμενοι ταῖς ἰδέαις τὰ μαθηματικὰ διαμαρτάνουσιν· οἱ δὲ πρῶτοι . . .* (and here follows criticism of the connection of ideal number and mathematical number in Plato). This passage has most often been interpreted as applying to Xenocrates (fr. 38 Heinze; Ross, *Met.* 2.481; Cherniss, *Plato* 568, *Gnomon* 1959, 45ff; De Vogel, *Mnemosyne* 1949, 303). On the other hand, Ross (*PTI* 208f) and Saffrey (25ff) hold that Xenocrates is referred to individually only from *ἐὰν μή τις* on, and that in the earlier part both Xenocrates and Plato were meant. Ross's argument that in b26 the "mathematicals" are the "intermediate realm" of Plato and not of Xenocrates, will not hold up against Cherniss (*Gnomon* 1959, 47), who has recognized here an allusion to the previously discussed theory of Speusippus. Saffrey argues that the question whether line, surface, and solid are ideas or a separate class must be asked with reference to Plato, because Xenocrates equated ideas and mathematicals. Aristotle poses the same question at 992b13ff (cf. above, n. 41), with an unambiguous reference to Plato. The clause *οὐδ' ὑπάρχει γε κατ' αὐτῶν οὐθὲν θεώρημα* cannot be directed against Xenocrates, who was precisely the one who wanted to force ideas and mathematicals together (*προσγλίχεσθαι*). Thus after all the interpretation of Ross and Saffrey is preferable.

(b) In 1036b12ff, the question is discussed: What belongs to the Form (*εἶδος*)? Do flesh and bones belong to the Form of Man? Does the line, or the continuous, belong to the Form of triangle or circle? Some deny this, *καὶ ἀνάγουσι πάντα εἰς τοὺς ἀριθμούς, καὶ γραμμῆς τὸν λόγον τὸν τῶν δύο εἶναί φασιν· καὶ τῶν τὰς ἰδέας λεγόντων οἱ μὲν αὐτογράμμην τὴν δυάδα, οἱ δὲ τὸ εἶδος τῆς γραμμῆς, ἔνια μὲν γὰρ εἶναι ταὐτὰ τὸ εἶδος καὶ οὗ τὸ εἶδος (οἷον δυάδα καὶ εἶδος δυάδος), ἐπὶ γραμμῆς δὲ οὐκέτι . . .* Once more he deals first with Speusippus, who did not accept any Ideas, and then distinguishes two directions in the theory of Ideas, and the probability is that they are those of Plato and Xenocrates. Cherniss (*Plato* 567, *Gnomon* 1959, 44) takes *αὐτογράμμην*, because it lacks an article, as predicate noun with both *δυάδα* and *τὸ εἶδος τῆς γραμμῆς*, appealing to "the elementary rules of Greek grammar." These do not apply, however, in technical philosophical language (cf., just previously, b13: *γραμμῆς τὸν λόγον τὸν τῶν δύο εἶναι*). The one named first, he thinks, Xenocrates designated the dyad, while Plato designated the "idea of line" as "line in itself"; thus the doctrine of ideal numbers and the ordinary theory of ideas stand

> and in the same way it was laid down in the work entitled *On Philosophy* that the Animal-itself is composed of the Idea itself of the One, along with the primary length and breadth and thickness, and the rest in a similar manner. Again, putting it differently, mind was said to be the One, knowledge two (because it goes in a straight line to the One), whereas the number of the plane figure is opinion and that of the solid, sensation . . . And, since the soul seemed to be productive of both motion and knowledge, some have compounded it of both . . . (*De anima* 404b18ff).

Is it a doctrine of Plato[48] or of Xenocrates[49] that is reported here? In the expression "in the work entitled *On Philosophy*" (*ἐν τοῖς Περὶ φιλοσοφίας λεγομένοις*) Aristotle is referring to his own dialogue,[50] which therefore developed further the hints of the *Timaeus* (53d) about the "first principles of things" (404b17), and included mention of the series of dimensions.[51] But the expression "again, putting it differently"

side by side. In this case, however, the explanation beginning *μὲν γάρ* must be understood as applying to the first instead of the second solution (*Gnomon* 1959, 44 n. 3), and the phrase beginning *οἱ δέ* is not only a tautology but destroys the continuity. In 1043a33ff, where the same problem is treated, there is a consistent connection of dyad and line. Therefore the translation must be, "Of those who accept the ideas, some call the dyad the line itself, others the form of line" (this is consistent with Ross, *Met.* 2.203 and van der Wielen 144ff). For, to paraphrase the justification, with the dyad, for example, the form is nothing else than the dyad itself, whereas with the line—the ideal line—one must distinguish between its form—the dyad—and a second constituent (the short-and-long). The equation of line and dyad belongs to Xenocrates, and we may attribute the other version of the theory of ideal numbers to Plato. To be sure, the argumentation of the *μὲν γάρ* sentence seems to be directed against Xenocrates, and would thus have to go back to a third Platonist, who may have defended Plato's proposal. There were too many partners to the discussion, too many varieties of the doctrine, for our reconstruction to attain absolute certainty.

[46] Cf. Becker in *Fs. Gadamer*; W. Heisenberg, *Naturwissenschaften* 45 (1958) 227–234.

[47] *Tim.* 35aff; cf. ch. V 1, below.

[48] So Ross, *PTI* 210, De Vogel, *Mnemosyne* 1949, 304f, Saffrey passim. No reliance may be placed, admittedly, in the fact that Simpl. *De an.* 28.7ff and Philop. *De an.* 75.34f refer to *On the Good*. Cf. Cherniss, *Plato* 119 n. 77.

[49] So Cherniss, *Plato* 565ff, *Riddle* 14f; Theiler, *Arist.* 94.—Kucharski sought to prove that Pythagorean doctrine was in the background here, relying on the late evidence of Aëtius and Theo Smyrnaeus (below, ch. I 3, n. 109) and on the alleged "caractère artificiel et naif" of the doctrine (*Arch.* 43)—though it does not seem to have been too primitive for Xenocrates. First he disposes of the theory of ideas (*Tétr.* 36f, 47ff), though it is unmistakably presupposed (*οἱ μὲν γὰρ ἀριθμοὶ τὰ εἴδη αὐτὰ . . . ἐλέγοντο*), then later (*Arch.* 33f, 39f) seems willing to consider whether this "point neuralgique" may after all show Platonic influence. But according to Aristotle's explicit testimony the theory of ideas is Platonic and not Pythagorean. Cf. also Saffrey 8ff.

[50] Cherniss, *Plato* 568f; Ross, *PTI* 210; De Vogel, *Mnemosyne* 1949, 304f; Saffrey 7ff; Theiler, *Arist.* 93. The same form of citation is found at *Phys.* 194a36.

[51] In 992a10ff, Aristotle presents the derivation of line, plane, and solid in the first person. (On such expressions in the early books of the *Metaphysics* see Jaeger, *Arist.* 171, 188, Eng. ed.) In addition, Alexander refers for this (*Met.* 117.24) to the *Περὶ φιλοσοφίας*.

(ἔτι δὲ καὶ ἄλλως) adds still another version of the doctrine, without citation of the source. The identification of the One with mind (νοῦς) is attested for Xenocrates, as well as the series knowledge-opinion-sensation,[52] and he knows the derivation-series line-plane-solid (above, n. 39). But then Xenocrates' definition of soul[53] is introduced as something new, a conclusion of "some"; therefore what precedes can not refer to Xenocrates,[54] and it is a tempting conclusion that what follows the citation of the *Timaeus* and its interpretation in the work *On Philosophy* is a reference to "unwritten doctrines."[55]

The close connection of all this with Plato is shown by a remarkable passage in the *Laws* (894a):[56]

> The origin of each thing takes place . . . when a first principle, taking on increment [the line], passes into its second transformation [the plane] and from this to its neighbor [the solid], and having made three transformations makes perception possible to those who perceive it.

Plato is dealing with the relation of the soul to the physical world, the priority of soul over matter. Thus there are present in Plato both the series line-plane-solid and the equation of the last stage with perception,[57] though to be sure there is nothing about the application of numbers.

Finally, we must take account of Theophrastus' statement that "most" of those who posit the One and the Indefinite Dyad as first principles only carry the derivation to a certain point:

[52] Fragments 15 and 5. Cf. Heinze 2ff, Cherniss, *Plato* 570f. The series νοῦς–ἐπιστήμη–δόξα–αἴσθησις is familiar to Aristotle himself (*De an.* 428a4, *Met.* 1074b35).

[53] Frr. 60–65 Heinze.

[54] Set forth by De Vogel, *Mnemosyne* 1949, 304. Cherniss, *Plato* 573f, emphasizes the οὕτως, which he refers to γνωριστικόν, as a connection with what precedes, and he does not cite the "some" in the middle. But the word κινητικόν introduces a new topic; and, besides, οὕτως is to be referred to the main clause (Theiler, *Arist.* 94). Theiler (ibid.) acknowledges that "Aristotle would have had to express himself differently, if he had already cited Xenocrates"; but then he separates off the last clause as an addendum, and still, because of 1090b20ff, attributes what precedes to Xenocrates. Yet he recognizes that there is a difference from the definition of soul, which was "perhaps only formulated later." So we have both the early and the late Xenocrates—too complicated a solution.

[55] Cf. *Phys.* 209b11ff.

[56] The connection was already noted in England's commentary. Cf. Cornford, *PlParm* 198; Kucharski *Tétr.* 71ff. Also Pl. *Leg.* 819c7: ἀριθμοί . . . μῆκη, πλάτη, βάθη.

[57] The series ἐπιστήμη–δόξα–αἴσθησις is also found at *Parm.* 155d. Cf. 142a, 164b, *Tim.* 28a, 37bc, 52a.

> for having generated numbers and planes and solids, they practically omit the rest, except for a brief mention, just enough to make clear that some are from the Indefinite Dyad . . . and some from numbers and the One, like soul . . .

Speusippus treats the matter similarly, he says, but Xenocrates differently (*Met.* 6a25ff).[58] Again we have an ontology, separated from Speusippus and Xenocrates, in which the series number-line-plane-solid is present alongside the connection of numbers and soul. Thus for the pioneering exposition of the theory of ideal numbers, with its implications for the physical world and the soul, we find ourselves led back again and again to Plato, and apparently to the lecture *On the Good*.

The commentators on Aristotle, from Alexander of Aphrodisias on, are unanimous that the doctrines developed in *On the Good* were Pythagorean,[59] and Aristotle also says that in his theory of the first principles Plato "mostly" followed the Pythagoreans, though he did have "something of his own" to add.[60] The seventh *Letter*, in the passage on these doctrines (338c), makes reference to Archytas: from his circle, it is suggested, Dionysius might have had knowledge of doctrine which Plato, himself, had not imparted to him.[61] But, even if this relationship may be regarded as certain, still the question becomes all the more urgent what it may have been that suggested this course to Plato. That he did not take over someone else's system unchanged may be assumed from the start, and not only because of its psychological improbability. When Plato went to Magna Graecia for the first time, at the age of nearly forty, his intellectual attitudes must have been fairly well established. The main background for Plato's ontology is clearly Eleatic dialectics: it is not by chance that Parmenides and the "Eleatic stranger" play a leading part in the late dialogues; the One and the Good had been equated already by Euclides of Megara, a doctrine

[58] There follow remarks on Hestiaeus and Plato. L. Tarán (*AJP* 87 [1966] 471) draws the conclusion that Plato cannot therefore have been referred to earlier. But μέχρι τῶν εἰρημένων (b15) looks back to a23ff.

[59] Alex. *Met.* 55.20, Simpl. *Phys.* 151.13ff, *De an.* 28.7ff, Philop. *De an.* 75.34ff. Whether Aristotle himself spoke of Pythagoreans in his Περὶ τἀγαθοῦ is, however, doubtful; where Alexander quotes exactly (56.35) there is mention only of Plato.

[60] 987a30ff, Cf. ch. I 2.

[61] Plato only mentions Archytas here to explain how Dionysius might have arrived at these παρακούσματα. Frank assumes (243.1) that Plato himself spoke of Pythagoreanism. Cf. ch. I 4 below.

Plato is echoing in the *Protagoras*.[62] Above all, Aristotle clearly distinguishes between Platonism and Pythagoreanism; starting from his statements, it is possible to judge the originality of Plato, as well as to find out what Pythagorean philosophy was like before Plato.

It is true that Aristotle's reports on the history of philosophy harbor the numerous and serious sources of error which Cherniss, in particular, indefatigably exposed. But in default of other sources, there is nothing else to do than—with due caution—to follow Aristotle's hints. Perhaps we may hope that those distortions, and the faulty perspective, which is present to an equal degree in the reports on Platonists and on Pythagoreans, will prove negligible in assessing the differences between the two.

2. THE PHILOSOPHY OF THE PYTHAGOREANS ACCORDING TO ARISTOTLE

Aristotle does not present his reports on the Pythagoreans as an impartial historian of philosophy, but always in the context of his own exposition, which has purposes of its own. The thing that lends the confrontation with the Pythagoreans its special immediacy for Aristotle is their connection with the Academic teaching on first principles, which he criticizes vigorously while at the same time making it his own point of departure.[1] And, in the organization of his inquiries about first principles (*ἀρχαί*) in the first two books of the *Metaphysics*, the Pythagoreans are not only treated separately, among the "pre-Socratics" (985b23ff), but are even more often treated in comparison with Plato.[2] Their doctrines are set forth in detail in the discussion of the difficulties in the Academic number theory (in books M and N).[3] The latter is the real target of Aristotle's polemic; Pythagorean material may be cited for its own sake, in order to achieve completeness, or it may be used to win a point from the Platonists.[4] There are further detailed reports in the *Physics*, in the discussion of the concept *ἄπειρον* (202b30ff). Isolated reports on items of scientific theory are

[62] D.L. 2.106; Cic. *Acad.* 2.129. Pl. *Prot.* 329cd (O. Gigon in *Phyllobolia: Festschrift P. von der Mühll* [Basle, 1947] 100). Cf. Krämer 505ff.

[1] Thus Aristotle finds the pattern of *ἕν* and *ὑπεροχὴ καὶ ἔλλειψις* in nearly "all" of the "early" thinkers (*Phys.* 189b8ff).

[2] 987a29ff, 989b29ff; cf. 990a27ff, 996a4ff.

[3] Elsewhere in the *Metaphysics*, there is only one allusion each in *Z* (1036b18), *I* (1053b12), and *Λ* (1072b31).

[4] Cf. 1083b8f: *ὁ δὲ τῶν Πυθαγορείων τρόπος τῇ μὲν ἐλάττονας ἔχει δυσχερείας. . .*; 1090a30: *οἱ μὲν οὖν Πυθαγόρειοι κατὰ μὲν τὸ τοιοῦτον οὐθενὶ ἔνοχοί εἰσιν. . .*

also found in other works; the famous cosmic system, with its moving earth and harmony of the spheres, is discussed in the book *On the Heavens*.

In addition, Aristotle devoted two special books to the exposition and criticism of Pythagorean doctrines, and also wrote on Pythagoras. He himself alludes once (*Met.* 986a12) to his "more exact" discussions. Plutarch, Alexander of Aphrodisias, Aelian, and especially Iamblichus have preserved important material from these books, which supplements the reports of the didactic treatises.[5]

It has been emphasized repeatedly that Aristotle, in his extant works, consistently speaks of *Πυθαγόρειοι*, not of Pythagoras,[6] and this seems

[5] The list in D.L. 5.25 (on which see P. Moraux, *Les listes anciennes des ouvrages d'Aristote* [Louvain, 1951] 243ff, I. Düring, *Aristotle in the Ancient Biographical Tradition* [Göteborg, 1957] 67ff) names one book each *Πρὸς τοὺς Πυθαγορείους* and *Περὶ τῶν Πυθαγορείων*; the catalogue of Ptolemaeus (Moraux 289ff, Düring 208ff) gives two books *On the Doctrines of Pythagoras*. The first book is cited by Aëtius 1.18.6 (Arist. fr. 201 Rose), the second by Alex. *Met.* 75.16f, Simpl. *Cael.* 392.16ff (Arist. frr. 202, 205 Rose); obviously the two books had been combined into one (Moraux 301). The Hesychius catalogue has only one book, *Περὶ τῶν Πυθαγορείων* (Moraux 198). The various titles under which the work is cited are given by Wilpert (*Hermes* 1940, 373 n. 3), who has also considerably augmented the material to be drawn from Alexander, beyond Rose's collection. For corrections relating to Rose's collection, which was taken over in essentials by Ross, see below, ch. II 4, n. 4. Further material is to be found in Iamblichus; see below, n. 112, ch. II 4, n. 5, ch. II 5, nn. 14–17. Plutarch uses the work (Gell. 4.11.11ff = Arist. fr. 194 Rose), sometimes without naming it (*De E* 388b–c; *Quaest. Rom.* 288c, with fr. 199 and 203 Rose; *De Is. et Os.* 364a, with fr. 196 Rose; *Numa* 8, with fr. 191 Rose).—The arithmological fragment published by Delatte (*Litt.* 167ff) from a codex of the 16th century is derived in essentials from Alex. *Met.* 38.8ff, with added details. It shares with the Alexander MSS (at 38.21) the corruption *καρπῶν* (instead of *καιρῶν*, found in Ascl. *Met.* 36.6, who is dependent on Alexander), with the principal MSS (at 38.19) *ἐτῶν* (the parallel tradition for Alex., MSS L and F, and Ascl. has *μηνῶν*) and (at 38.16) the nonsensical *στερεός* (the correct *τετράγωνος* in L and F). If the fragment agrees with corruptions which came after the split in the transmission of the text of Alexander, it is dependent on a late form of that text and not an independent witness to a source antedating Alex., as Delatte thinks.—Rose (*De Aristotelis librorum ordine et auctoritate* [Berlin, 1854] 81ff, *Aristoteles pseudepigraphus* [Leipzig, 1863] 193f) denied the authenticity of the book on the Pythagoreans, as indeed of all the lost works of Aristotle. (Thus Rohde, Q 103 n. 1 and passim, also speaks of "pseudo-Aristotle.") This rejection still finds champions today, especially because of the collection of miracle tales the book contained (Wehrli, *Herakleides* 71, on the other side, Zeller I 396, n. 1; Corssen, *RhM* 1912, 36ff). There can be no question of a neo-Pythagorean forgery, as the citations in Bolus–Apollonius *Hist. mir.* 6 give a terminus ad quem. The question whether Aristotle himself or one of his pupils collected the material is irrelevant for its reliability (Diels, *AGP* 1890, 467 n. 39, Hölk 22, Delatte, *Litt.* 273f).

[6] *οἱ καλούμενοι Πυθαγόρειοι*, 985b23, 989b29, *Mete.* 345a14, *Cael.* 284b7; *τῶν Ἰταλικῶν τινες καὶ καλουμένων Πυθαγορείων*, *Mete.* 342b30; *οἱ περὶ τὴν Ἰταλίαν, καλούμενοι δὲ Πυθαγόρειοι*, *Cael.* 293a20; *Ἰταλικοί*, *Met.* 987a31. At 986a29f the chronological statement about Pythagoras is an interpolation (Zeller I 597, n. 2; Ross, *Met.* I 152; Jaeger, *Paideia* I 163, n. 72 [Eng. tr.] and his note on the passage in the OCT *Met.*). It may stem from Iam. *VP* 104; it is not found in Alexander's paraphrase or in the principal MS, but Asclepius read it (*Met.* 39.25) On *MM* 1182a11 and fr. 207 Rose, see below, ch. I 3, n. 164).

also to have been the case in the lost works, as far as philosophical doctrines are concerned.[7] Often he adds the word καλούμενοι, and the extent to which this expression, "the so-called Pythagoreans," shows some kind of reservation has been vigorously debated.[8] There lies in the Greek expression about the same kind of nuance that is expressed nowadays by quotation marks. But the studious avoidance of Pythagoras' name is intentional.

The doctrine of the Pythagoreans usually appears as an undifferentiated unity; or in any case there is no sure foothold for the modern attempts to discern various stages in its development.[9] To be sure, "others of this same group" (ἕτεροι τῶν αὐτῶν τούτων) are named as originators of the "table of opposites,"[10] differences on specific points are mentioned twice,[11] and several times there is a restrictive "some" (τινές).[12] Yet there is never a distinction between "early" and "late" Pythagoreans.

For the comparison of Plato and the Pythagoreans we may begin with the first book of the *Metaphysics*.[13] Plato's doctrine is at first connected closely with the Pythagoreans: "in most respects following them, but possessing also some features of its own which set it apart from the philosophy of the Italians" (987a30f). To be sure—and we must be mindful of this qualification—this relates exclusively to the question of what principles (ἀρχαί) the early thinkers proposed, and how they are related to the Aristotelian classification. Plato and the Pythagoreans both accepted numbers as the principles (987b24)—number not as the number of other assumed objects, but as an independent entity, οὐσία. In this sense Pythagoreans and Plato are regularly mentioned in conjunction, and are set over against all other thinkers earlier than Aristotle,[14] and in this conjunction Aristotle regularly attacks them. At the same time, though, Aristotle emphasizes the difference that, while Plato separates the numbers, as ideas, from the sensible world and even sets between them the mathematical realm as a realm of its own, for the Pythagoreans things "are" numbers, they "consist of" numbers.[15] Aristotle is puzzled at this: the Pythagoreans introduced principles, he says, that would have been quite suited to lead beyond the merely perceptible into the higher realms of Being, but then they never speak of anything but what is perceptible, and "squander" (καταναλίσκουσιν) their principles on this world of ours, as though there did not exist anything but what is perceptible, what the sky encloses (989b29ff). What differentiates Plato from the Pythagoreans is "separation" (χωρισμός), the "introduction of the Forms" (987b31), which Aristotle traces back to the dialectic developed by Socrates.[16]

If the numbers are identical with things, they are space- and time-bound. Aristotle actually speaks of their origin, in a cosmogonic sense,[17] and says that they are extended, that their units possess magnitude.[18] It is naturally against this thesis that the principal logical and physical objections are directed,[19] and Zeller wanted to exclude the report about "extended units" completely, as Aristotelian

[7] Citation of the title as Περὶ τῆς Πυθαγόρου φιλοσοφίας, Aët. 1.18.6, proves nothing (in spite of Guthrie I 200): the later tradition often has "Pythagoras" instead of "Pythagoreans"; cf. D.L. 3.8, with Arist. *Met.* 987a32ff; Olympiod. *In Meteor.* 50.9ff, on 342b30, and n. 5 above.

[8] Frank uses the adjective *sogenannt* in the title of his book, as does Bollinger (and see her pp. 15f). Cf. the expression τὰ καλούμενα Ὀρφικά, Arist. *Gen. an.* 734a19, *De an.* 410b27, with Philoponus' comment ἐπειδὴ μὴ δοκεῖ Ὀρφέως εἶναι τὰ ἔπη. Contra, Cherniss (*Pres.* 384f) calls attention to οἱ καλούμενοι γεωργοί (*Pol.* 1290b40), and speaks of "designations in the currently recognized sense" (*Gnomon* 1959, 37f); cf. ZM 354. Von Fritz (*AnnMath* 1945, 249.38) suggests that there may be a certain reserve intended, because it was unusual for a philosophical school to be named in this way after its founder; but cf, Ἀναξαγόρειοι, Pl. *Crat.* 409b, *Dialex.* 6.8 (DK II 414.13), Ἡρακλείτειοι, Pl. *Tht.* 179e, D.L. 9.6. Zeller's observation is important, that there are names of political factions in -ειοι (I 446 n. 1; Minar 21). E.g., Διώνειοι, Pl. *Ep.* 7.334c; Διαγόρειοι, *Hell. Oxy.* 10; Κυλώνειοι, Iam. *VP* 248. Cf. F. Poland, *Geschichte des griechischen Vereinswesens* (1909) 73ff. Since the formations in -ειοι are regular only for s-stems (βίη Ἡρακληείη, but ἀγορά–ἀγοραῖος), the form Πυθαγόρειοι must depend on earlier examples, but probably in the realm of the ἑταιρίαι.

[9] Cornford himself admits that it is difficult to divide Aristotle's reports between the two "radically opposed" schools he reconstructs.

[10] 986a22. Cf. the end of this chapter.

[11] *Mete.* 345a14ff (Milky Way), *De an.* 404a16ff (motes in the air and souls). The theory of comets in *Mete.* 342b30ff and the system of the 10 heavenly bodies are mutually exclusive (below ch. IV 1).

[12] τινές is even used in relation to the theory of numbers in *Cael.* 300a17 (glossed over by Zeller I 450f, 451 n. 2), and in relation to particular questions *Cael.* 293b21, *Mete.* 342b30, *Sens.* 445a16. In Aristotle, τινές can also refer to a single author, as *Pol.* 1290b4 (Hdt. 3.20), *Gen. an.* 756a5 (Hdt. 2.93).

[13] On the problems of the analysis of 987a29ff, in connection with 1078b12ff, see Cherniss, *Plato* 177 n. 100, and 191f.

[14] τὸ μέντοι γε ἓν οὐσίαν εἶναι, καὶ μὴ ἕτερόν τι ὂν λέγεσθαι ἕν. . . 987b22 (cf.996a6f), 1001a9ff, 1053b11ff, *Phys.* 203a4 (also on the ἄπειρον).

[15] 987b28 (cf. τὸν ἀριθμὸν τὰ ὄντα, 1083b17); 1090a22; ἀριθμοὺς τὸν ὅλον οὐρανόν, 986a21 (cf. 986a3); τὰ σώματα ἐξ ἀριθμῶν, 1083b11ff (cf. *Cael.* 300a14ff; 986b6; 990a21; 1080b2, 16, 18), ἐξ ἀριθμῶν τὰ ὄντα, 1090a23, 32. But we also find ἐξ ἐκείνων (sc. σωμάτων) ὄντων τῶν ἀριθμῶν, 1083b18; τοὺς ἀριθμοὺς ἐν τοῖς αἰσθητοῖς, 1080b1. The lack of χωρισμός, 1080b16, 1083b10, 1090a23, *Phys.* 203a6ff; on this point the Pythagoreans are to be praised (above, n. 4)—Aristotle himself argues against χωρισμός!

[16] 1078b30ff; O. Gigon, "Die Sokratesdoxographie bei Aristoteles," *MH* 16 (1959) 174 212.

[17] Below, n. 35.

[18] τὰς μονάδας ἔχειν μέγεθος, 1080b19; cf. b33, 1083b15, 990a12.

[19] Logical objections: mathematical number has no magnitude, 1080b12ff. *Cael.* 300a18f; physical: the force of gravity (990a14f, 1090a33f, *Cael.* 300a19) and the phenomenon of movement (990a8) both remain unexplained.

interpretation.[20] But at least there can have been nothing to refute such an Aristotelian interpretation. The Pythagoreans did not differentiate between number and corporeality, between corporeal and incorporeal being.[21] Like all the pre-Socratics, these Pythagoreans take everything that exists in the same way, as something material. For every Platonist this exposition of Aristotle's is an excruciating annoyance, which even the superficiality of the commentators of late antiquity could not cause them to overlook. Syrianus felt this most keenly, and devised a truly modern remedy:

> Aristotle has no reliable or adequate objection to the *ἀρχαί* of the Pythagoreans. For the most part, if I am to speak the truth frankly, he does not even hit them, but launches his objections against hypotheses he has invented himself.[22]

Proclus too, and others, are certain that only a misunderstanding or a malicious distortion could be to blame.[23] But what is an embarrassment for the Platonist is treasure-trove for the historian: here we have a piece of Pythagorean doctrine that was not subsumed into Platonism.

The differences extend further. "Elements" of the numbers are, according to Pythagorean doctrine, the "even" and the "odd"; the "odd" is at the same time "limit," the "even" is "unlimited." In the pair limit-unlimited we have a primeval cosmic opposition lying behind the number which is the world.[24] To Greek linguistic feeling, "limit" is the positive principle;[25] it is conceived at the same time as

[20] Zeller I 486. On "number atomism" and Ecphantus, see below, nn. 66, 74.

[21] So also Raven in KR 247 n. 1. This makes it improbable that the word *ἀσώματος* was coined by the Pythagoreans, as H. Gomperz tried to show (*Hermes* 67 [1932] 155–167). (On Philolaus B22, see below, ch. III 2, n. 45.) We may speak of the "immanence of number," but it is impossible to equate Pythagorean doctrine with the conception of Aristotle, whose theory of abstraction was possible only after the development of the theory of ideas.

[22] Syrian. *Met.* 80.20ff; cf. 83.12ff.

[23] Procl. *In Tim.* I 16.29; cf. Ascl. *Met.* 34.15ff. Simplicius' style of interpretation is fairer, though likewise permeated by reverence for Pythagoreanism: *πῇ καλῶς ἐκείνων λεγόντων ὁ Ἀριστοτέλης πρὸς τὸ φαινόμενον ἀντεῖπε τοῦ λόγου* (*Cael.* 386.8, on Arist. *Cael.* 284b6). Cf. Simpl. *Phys.* 453.10ff, 652.6.

[24] *τοῦ δὲ ἀριθμοῦ τὰ στοιχεῖα τό τε ἄρτιον καὶ τὸ περιττόν,* 986a18. At 1004b31 the pairs odd-even and limit-unlimited are separated, but this may be merely a slip. The point is enumeration of oppositions of any kind. (Cherniss, *Pres.* 47, 186–188, assumes that for the second pair Platonists are included, but there is no evidence for this.) Limit-unlimited comes first at 990a8 and 986a23 (the table of opposites); only limit-unlimited at 1091a17 (cf. *EN* 1106b28, fr. 47); only odd-even at *Phys.* 203a10ff; the explicit equation of even and unlimited at *Phys.* 203a10ff. On 987a15 cf. below, n. 38.

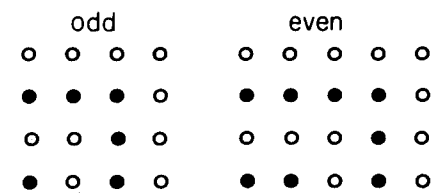
[25] The basic importance of this opposition of limit and unlimited was set forth by Heidel (*AGP* 1901), in opposition to Zeller. He leaves the question open whether the opposition of odd and even was originally equal or subordinate to it (390f); but if one considers that it must have been from a Platonic perspective that Aristotle brought the

masculine, the "unlimited" as feminine, and correspondingly the odd number is also masculine, and the even feminine.[26] Aristotle gives a complicated explanation of the correspondence of odd and limit, even and unlimited, at the basis of which lies the representation of numbers by arrangements of pebbles.[27] Since Burnet the significance of this

number aspect of Pythagorean doctrine into the foreground, one will be still more convinced of the importance of the cosmological limit-unlimited. Cf. D. N. Levin, *Ethical Implications of the πέρας-ἄπειρον Dichotomy as Seen Particularly in the Works of Aeschylus*, Diss. Harvard, 1957 (summary in *HSCP* 63 [1958] 519–522).

[26] This is why the number 5 (=2+3) is "marriage" (*γάμος*). Aristotle alludes to this at *Met.* 1078b23 and fr. 203=Alex. *Met.* 39.8ff. Also Plut. *De E* 388a–c, *Quaest. Rom.* 288c. Cf. below, n. 31.

[27] *Phys.* 203a13: *περιτιθεμένων γὰρ τῶν γνωμόνων περὶ τὸ ἓν καὶ χωρὶς ὁτὲ μὲν ἄλλο ἀεὶ γίγνεσθαι τὸ εἶδος, ὁτὲ δὲ ἕν.* On this see Stob. I prooem. 10(=DK 58B28), Alexander ap. Simpl. *Phys.* 457.12ff, Philop. *Phys.* 394.1ff, Themist. *Phys.* 80.13ff; with more detail Theo 31.15ff, Nicomachus *Th. ar.* 9.16ff (with the textual emendations of Becker *ZwU* 24f), Nicom. *Ar.* 1.9.4, 2.17, Iam. *In Nic.* 73.15ff. The gnomon spoken of here is an instrument for measuring right angles, like a carpenter's square. The result of its application as described is shown in the adjacent figure. Cf. Burnet, *EGP* 103f; Ross,



Met. I 148f; Ross, *Phys.* 542ff; Raven, *PyEl* 130f, 188ff; KR 243ff; Becker, *Grdl.* 34ff; Guthrie I, 242ff. Aristotle's expression is not clear: *καὶ χωρίς* is an old *crux interpretum*; but the inclusion of the opposition square-rectangle in the table of opposites favors the usual interpretation; with the odd numbers squares result, with the even rectangles. For other, more complicated explanations, see Taylor, *CR* 40 (1926) 149ff, and M. Timpanaro Cardini, *Physis* 3 (1961) 105ff.—Simpl. *Phys.* 455.20ff (=DK 58B28; cf. Philop. *Phys.* 389.11, 391.25, Themist. *Phys.* 80.9f) attributes a different interpretation to certain *ζητηταί*: the even number can be halved, but halving continues to infinity *κατὰ τὴν διχοτομίαν*. Precisely because of the apparent irrationality of this, some have sought to find here an ancient doctrine, like W. A. Heidel, *AGP* 1901, 395f (followed by Burnet, *EGP* 288f; Ross, *Met.* I 149), and with variations Raven in *PyEl* 193 and KR 244f. What the exegetes mean, however, can be deduced from Porphyry ap. Simpl. *Phys.* 453.25ff: the number 2 is the principle of division which proceeds to infinity, and only to this extent is there a connection between even numbers and the principle of infinity. These considerations derive from the thought of the Indefinite Dyad, and are therefore not early Pythagorean.—Raven, *PyEl* 130ff, and Kucharski ("Les principes des Pythagoriciens et la dyade de Platon," *Archives de philos.* 22 [1959] 175–191, 385–431) have stated, not incorrectly, that in the gnomon procedure One and Dyad are given equal rank as principles of the odd and even numbers respectively: but the further thesis, that the Dyad in this function could have been called Indefinite, and that therefore all the later tradition was right in designating the concept of the Indefinite Dyad as Pythagorean, cannot rest on the formulations cited from Nicomachus, Theo, and Iamblichus. All the detailed accounts ascribe to Pythagoras the unequivocally Platonic concept of the Indefinite Dyad, *καθ' ὑπερβολὴν καὶ ἔλλειψιν* (see below, ch. I 3), but Aristotle speaks in this passage of *ἄπειρον*, not of *ἀόριστον*, and nowhere else makes any mention of an important role played by "twoness" in Pythagoreanism. When Philip says (105), "the illustration is Aristotle's," he is forgetting that it is reported in indirect discourse.

allusion to the graphic basis of Pythagorean number speculation has been recognized;[28] it is not the cipher or numeral (like "5") which serves as pictorial representation of a number, but the shape of an area —the sort of thing we are familiar with from dice or dominoes. Léon Brunschvicg introduced the apt allusion to the constellations, where, too, in each case a limited number of points of light defines an object: Orion, the Bear, the Lion, etc.[29] One need not believe, however, that any one pebble arrangement was the sole point of departure for the basic identification of limit and odd. Even numbers can of course be halved, until one comes to odd numbers, and then the halving process is "at its limit." More general considerations also enter in: like the positive evaluation of limit, the habit of prizing odd numbers has for long been deeply rooted in popular feeling.[30] Plutarch, what is more, in a passage where at least for part of the way he is following Aristotle, gives a perspicuous explanation, also from the viewpoint of the pebble figure, of the masculine character of the odd number and the feminine of the even. The even number, he says, has at its middle an empty space, capable of reception, whereas the odd number has a middle member with procreative power.[31] This direct symbolism must be regarded as old, and not only because of its attestation; it has a connection, at least subliminally, with the general Greek association of masculinity with the word *περαίνειν*.[32]

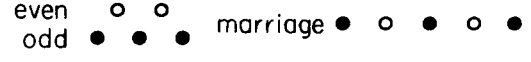
From the two primal principles rises the One; from it, number.[33] The similarity of this to the Platonic system, to the derivation of

[28] *EGP* 101ff. Zeller (I 483ff) had denied the spatial character of numbers in Pythagoreanism. Cf. above, n. 20.

[29] *Les étapes de la philosophie mathématique*, 3rd ed. (Paris, 1929) 33: "si non l'origine, du moins l'illustration saisissante de la doctrine pythagoricienne."

[30] On this, see below, ch. VI 4.

[31] Plut. *De E* 388a–c. The first part, the explanation of the *ἓν ἀρτιοπέριττον*, corresponds to Arist. frr. 199 and 203 (Alex. *Met.* 40.18ff), *Quaest. Rom.* 288c; Stob. I proem 10; cf. Aristox. fr. 23. The One also has a mid-position in odd numbers according to the Platonic doctrine (Arist. *Met.* 1083b28).



[32] I.e., in the sexual sense of *περαίνειν*.—Cherniss (*Pres.* 17 n. 68, 38) conjectures that Aristotle first thought of the equality of even and unlimited under the influence of the Platonic conception of the Unlimited Dyad. But not only the connections shown here, but also the pebble procedure, which Aristotle surely did not invent, would count against this. —*Μονάς* masculine and *Δυάς* feminine are also found in Xenocrates (fr. 15), in a mythical form of expression. Cf. also Philolaus fr. 20a.

[33] 986a17: *τοῦ δὲ ἀριθμοῦ στοιχεῖα τό τε ἄρτιον καὶ τὸ περιττόν, τούτων δὲ τὸ μὲν πεπερασμένον τὸ δὲ ἄπειρον, τὸ δὲ ἓν ἐξ ἀμφοτέρων εἶναι τούτων (καὶ γὰρ ἄρτιον εἶναι καὶ περιττόν), τὸν δ' ἀριθμὸν ἐκ τοῦ ἑνός,*

number from the One and the Indefinite Dyad, is obvious. Aristotle, too, sets Unlimited alongside Indefinite Dyad, but at the same time emphasizes what is "peculiar" (*ἴδιον*) in Plato's formulation: "the fact that in place of the Unlimited, treated as singular, he posits a dyad, and derives the Unlimited from the great-and-small—this is peculiar to him" (987b25).[34] The term Indefinite Dyad is thus by no means Pythagorean in origin; but the difference goes still deeper. The Unlimited, unitary and undifferentiated, was according to Pythagorean conceptions "outside the heaven" and penetrated the world by being "breathed in" by the heaven to separate natural things (*φύσεις*) from one another, being "enclosed and partitioned off" (*ἐναπολαμβανόμενον*) in the limited.[35] Obviously the concept of empty space, as well as that of air, is present here.[36] In any case the Unlimited of the Pythagoreans is principle and constituent of the cosmos at the same time, whereas the Unlimited Dyad of the Platonists is conceived not cosmically but ontologically, even logically. It is not in any place, but is a transcendental principle, in which everything in the realm of experience takes part, and even, in a different way, in the realm of Ideas;[37] it belongs to the theory of ideas, just as the cosmic Unlimited of the Pythagoreans fits consistently into a world without a conception of incorporeal being.

Insofar as *ἄπειρον* and *ἀόριστος δυάς* are comparable, the One falls naturally into the company of *πέρας*, and Aristotle occasionally speaks

[34] Similarly, *Phys.* 203a15.

[35] *Phys.* 203a6: *... εἶναι τὸ ἔξω τοῦ οὐρανοῦ τὸ ἄπειρον ... τὸ ἄπειρον εἶναι τὸ ἄρτιον· τοῦτο γὰρ ἐναπολαμβανόμενον καὶ ὑπὸ τοῦ περιττοῦ περαινόμενον παρέχειν τοῖς οὖσι τὴν ἀπειρίαν ...* 213b22: *εἶναι δ' ἔφασαν καὶ οἱ Πυθαγόρειοι κενόν, καὶ ἐπεισιέναι αὐτὸ τῷ οὐρανῷ ἐκ τοῦ ἀπείρου † πνεύματος ὡς ἀναπνέοντι καὶ τὸ κενόν, ὃ διορίζει τὰς φύσεις, ὡς ὄντος τοῦ κενοῦ χωρισμοῦ τινὸς τῶν ἐφεξῆς καὶ [τῆς] διορίσεως· καὶ τοῦτ' εἶναι πρῶτον ἐν τοῖς ἀριθμοῖς· τὸ γὰρ κενὸν διορίζειν τὴν φύσιν αὐτῶν ...* For the obviously corrupt wording Raven proposes (KR no. 315), with Diels, *πνεῦμά τε*; von Fritz (*RE*) suggests *ὡς πνεῦμά τι ἀναπνέοντι* [*καὶ*] *τὸ κενόν*. Cf. fr. 201 Rose: *τὸν μὲν οὐρανὸν εἶναι ἕνα, ἐπεισάγεσθαι δ' ἐκ τοῦ ἀπείρου χρόνον τε καὶ πνοὴν καὶ τὸ κενὸν ὃ διορίζει ἑκάστων τὰς χώρας ἀεί.* See also *Cael.* 279a11ff, and Cherniss, *Pres.* 214f. On the relation of this to Xenophanes, see below, ch. III 3 n. 17 (with context).

[36] The difference between space and air was established by Anaxagoras (A69) and Empedocles (B100). To draw chronological conclusions from this fact, leading to an early dating of these Pythagorean doctrines (Raven, *PyEl* 28ff, 44ff) is, however, dangerous; the "breath" of mythical cosmogonies may have survived merely in the comparison of void and breath; cf. above, n. 35. The logical difficulty, that the Unlimited becomes a limiting agent, only becomes acute because of Aristotle's terminology.

[37] *ἀόριστος* is quite seldom attested before Plato, but *ἄπειρον* as *περιέχον* is familiar to the pre-Socratics from the time of Anaximander (A10, 14; cf. Anaxagoras B 1, 2). Later Platonists find it important to note that the term "unlimited" is to be understood not in the sense of spatial extension or corporeal multiplicity, but ontologically (Plut. *Quaest. conv.* 719c–d), as "indefinite," not "infinite."

as though the two were to be identified.[38] Yet precisely because this is so obvious—even absolutely necessary—from the point of view of Platonism, one must believe the definite testimony of Aristotle that in the Pythagorean view the One had its origin from Limit and Unlimited together: *τὸ δ' ἓν ἐξ ἀμφοτέρων εἶναι τούτων* (986a19). For the Platonist this is no less a vexation than the lack of incorporeality; *μᾶλλον δ' ἀμφότερα ἐκ τοῦ ἑνός* are the words with which Asclepius impatiently interrupts his paraphrase of this passage of the *Metaphysics* (38.25). The One has a share in each of the opposite forces; it is "even" and "odd" at the same time, *ἀρτιοπέριττον*;[39] it is perfectly in keeping that it should be, as late sources say, bisexual, *ἀρσενόθηλυ*.[40] This One comes into being and develops further—it *is* nothing else than the world before its further evolution. The "first principles," Limit and Unlimited are, then, what was there before the world came into being. To the Pythagoreans, number philosophy is cosmogony: *κοσμοποιοῦσιν* (1091a18). The further development of number is also cosmogonic:

> They say clearly that when the One had been constructed—whether of planes or surface or seed or something they cannot express, then immediately the nearest part of the Unlimited began "to be drawn and limited by the Limit."[41]

The One becomes a Two as the Unlimited penetrates it. Here is one

[38] Aristotle characterizes the One (*ἕν*) and the Unlimited (*ἄπειρον*) as *οὐσίαι* (above, n. 14), and to this extent sets them side by side (987a17); he calls the One "element and principle" at 1080b31, and only names the Unlimited as "principle" at 988a26 (differently, 986b5ff). At 987a16, if one strikes out *καὶ τὸ ἕν*, with MS E, the identification of *ἕν* and *πέρας* is perfect, for then the pairs *τὸ πεπερασμένον καὶ τὸ ἄπειρον* at a15 and *αὐτὸ τὸ ἄπειρον καὶ αὐτὸ τὸ ἕν* at a17 correspond; but, since Alex. *Met.* 47.11 read *καὶ τὸ ἕν* (whereas the equivalence of One and *πέρας* would seem natural to any later, Platonic-minded reader), we must keep the full text. Aristotle is enumerating unsystematically: both the primary pair of opposites, and the One, and number in general, are *οὐσίαι* for Pythagoreans; in his repetition, Aristotle leaves out the *πέρας* that goes with the *ἕν*, following, consciously or not, Platonic ways of thinking (cf. Cherniss, *Pres.* 45 n. 175, 224ff). In Aristotle's own philosophy, *πέρας* is *οὐσία* and *περαινόμενον* is *ὕλη* (*Cael.* 293b13); and he can, under the influence of Platonic terminology, even ascribe to Empedocles the doctrine that *τὸ ἕν* is a *στοιχεῖον* (*Gen. corr.* 315a23).—One cannot derive from the table of opposites the equation one=limit=good (as KR 241). This would imply that a rectangle has crooked sides, since "rectangular" is in the same column as "crooked."

[39] Frr. 199 and 203 Rose (the latter, Alex. *Met.* 40.18ff); Plut. *De E* 388a–c; "for it is both even and odd," Arist. *Met.* 986a20. Raven (KR 317f) concludes from the table of opposites that "one" was originally regarded as odd; against this, cf. above, n. 38.

[40] Nicom. *Th. ar.* 4.1, 4.17ff, Macrob. *Somn. Sc.* 1.6.7.

[41] *φανερῶς γὰρ λέγουσιν ὡς τοῦ ἑνὸς συσταθέντος, εἴτ' ἐξ ἐπιπέδων εἴτ' ἐκ χροιᾶς εἴτ' ἐκ σπέρματος εἴτ' ἐξ ὧν ἀποροῦσιν εἰπεῖν, εὐθὺς τὰ ἐγγιστα τοῦ ἀπείρου ὅτι εἴλκετο καὶ ἐπεραίνετο ὑπὸ τοῦ πέρατος* (1091a13ff). The *ὅτι* cannot be deleted; it shows a citation is to follow, and probably separates paraphrase from quotation. Cf. below ch. III 1, n. 92; *Met.* 1080b20f, *Phys.* 213b22ff, with fr. 201, and above, n. 35; also 989b34: *γεννῶσί τε γὰρ τὸν οὐρανόν.*

of the most widespread cosmogonic themes, "the separation of Heaven and Earth":

> *ὡς οὐρανός τε γαῖά τ' ἦν μορφὴ μία.*
> *ἐπεὶ δ' ἐχωρίσθησαν ἀλλήλων δίχα . . .*[42]

The process was modified by the Pythagoreans, with their ideas of the earth in motion and the central fire. But in these very ideas is apparent a complete equivalence of the things separated: the "Hearth" of the universe and its fiery envelope, "Zeus's castle"[43] and Olympus.[44] This separation has happened in the past: when Aristotle quotes exactly, verbs in past tenses suddenly appear.

The growth of the cosmos is described, in the manner of the pre-Socratics, as the growth of a living being, and embryological concepts form part of the background.[45] The One begins to breathe,[46] and, as the breath flows in, it assumes a more complicated structure. Similarly, the Hippocratic book *De natura pueri*[47] teaches that the embryo develops as the seed, in the womb, takes in air, and is divided and articulated by it: "Each of these [bodily structures] is articulated by the breath; for as they are aerated by it they separate according to their natural affinities." In Aristotle's account the origin of the One from seed is at least mentioned as a possibility.[48] Thus it becomes obvious that the ancient idea of macrocosm and microcosm is at work in Pythagorean teaching.[49] It is not a matter of an ontology featuring the

[42] Eur. fr. 484; on the idea, W. Staudacher, *Die Trennung von Himmel und Erde*, Diss. Tübingen, 1942.

[43] *Ζανὸς πύργος, Διὸς φυλακή, Διὸς θρόνος* Arist. fr. 204=Simpl. *Cael.* 512.12ff; *Ζανὸς φυλακή* Procl. *In Eucl.* 90.17.

[44] Philolaus A16; cf. Parmenides fr. 11.2, Emp. fr. 44, *Hebd.* 2, below, ch. III 2. In a similar way, Anaximander already has fire envelop the earth "like bark," and then be "broken away" (A10; Kahn 57f).

[45] Cf. H. C. Baldry, "Embryological Analogies in Pre-Socratic Cosmogony," *CQ* 26 (1932) 27–34.

[46] *ἕλκειν* is a technical term for breathing: Hippoc. *Nat. puer.* 12, *Nat. hom.* 5, *Flat.* 3, *Carn.* 6, Diog. Apollon. A31, Emp. A74.

[47] VII 498 L. The connection with Pythagorean doctrine was shown by Olerud, 53ff. Cf. VII 488 L.: *ἡ γονὴ ὑμενοῦται φυσωμένη. . .* Philolaus also has a similar doctrine, that a living creature after its birth "immediately" draws in breath and cools itself (A22; cf. Guthrie I 278f).

[48] As emphasized by Cornford, *CQ* 1923, 10 n. 1; *PlParm* 19; cf. KR 251, Guthrie I 276ff.

[49] On this, Kranz, *NGG* 1938 and *ABG* 1955; Olerud passim. To the context of these "anthropomorphic" conceptions of the cosmos belong the speculations about the "right" and "left" side of the world (Arist. *Cael.* 284b6ff, 285b25ff, fr. 200, fr. 205). We need not here go into problems of detail, such as the contradiction that Alexander found in Aristotle's statements (fr. 205); see Zeller I 547 n. 1, Cherniss, *Pres.* 186 n. 178. J. Cuillandre's study is unhelpful (*La droite et la gauche dans les poèmes homériques en concordance avec la doctrine pythagoricienne et avec la tradition celtique* [Paris, 1944]). On "right" and "left" in general, G. E. R. Lloyd, *JHS* 82 (1962) 56–66; in Oriental cosmology, K. Sethe *NGG*, ph.-h. Kl., 1922, 197–242.

derivation of number-in-itself; their number theory is cosmogony.[50]

Here Pythagorean doctrine is quite in line with pre-Socratic traditions. The cosmogony of Leucippus is closely related: he too has two kinds of entity at the "beginning"; the many atoms and the empty space between that makes possible their separation. The first step toward organization in the cosmic whirl is that a "membrane" or "caul" (ὑμήν—here too a term used in embryology) detaches itself from the nucleus, and the cosmos grows as matter flows in from the exterior.[51] But beyond the pre-Socratic horizon a mythical one also becomes visible: the separation of Heaven and Earth, and above all the double nature of the primally existent. For what can only be explained artificially, in arithmetical terms, is familiar to the mythologist—the primal being that is bisexual.[52]

Perhaps a quite specific mythical cosmogony forms the background of the Pythagorean number theory. There are striking similarities of detail in the Orphic cosmogony which in the romance of Pseudo-Clement is given by Apion as an example of pagan theology. The problems of transmission are exceedingly complicated,[53] but the basis is unquestionably a hexameter poem ascribed to Orpheus. Allegorical interpretation of Orphic poems, from a philosophical point of view, goes back at least to the fourth century B.C., as the papyrus from Dherveni has proven;[54] so it is quite possible that in the tradition of philosophical exegesis ancient material has been preserved. In specific

[50] Cherniss (*Pres.* 39ff, 44ff, 224f, 387f) tries to show that the Pythagoreans had no doctrine of the origin of number, and that Aristotle only produces this impression by his projection of Platonic ways of thinking. To establish this he must, at *Met.* 1091a13ff, make a radical separation of the cosmic One ("the universe itself") and the "numerical unit": "Aristotle is confusing . . . the cosmogony with the number-theory" (p. 39). But Aristotle says unequivocally that the Pythagoreans knew only one kind of number, the cosmic (990a21), that is, that they thought of number theory as cosmogony, of cosmogony as the development of arithmetic.

[51] Leucippus A1=D.L. 9.32: *τούτου δ' οἷον ὑμένα ἀφίστασθαι . . . αὐτόν τε πάλιν τὸν περιέχοντα οἷον ὑμένα αὔξεσθαι κατὰ τὴν ἐπέγκρισιν (ἐπέκρυσιν MSS) τῶν ἔξωθεν σωμάτων.* (On the text and interpretation see J. Kerschensteiner, *Hermes* 87 [1959] 441–448, esp. p. 446.) The agreement is still greater when the question is not of an abstract *πέρας* but of a plurality of *περαίνοντα* (below, ch. III 2).

[52] Cf. Olerud, pp. 130–136; H. Baumann, *Das doppelte Geschlecht*, (Berlin, 1955); M. Delcourt, *Hermaphrodite* (Paris, 1958).

[53] More detailed treatment of the following, *A&A* 14 (1968) 107ff. See Clem. *Hom.* 6.3ff, *Rec.* 10.17ff; 30ff; Kern, *Orph. frag.* 55–56. On the Clement narrative in general, see the introductions to the new editions by B. Rehm (*Homilien* [Berlin, 1953]) and B. Rehm and F. Paschke (*Rekognitionen* [Berlin, 1965]); also Rehm's article in *RAC* 3 [1957] 197ff). Eusebius knew the Apion dialogues (*Hist. eccl.* 3.38.5).

[54] *Arch. delt.* 19 (1964) 17ff. Interpretation of Orphic material in Cleanthes and Chrysippus, *SVF* II 906, 907, 1078.

details, this "Orphic" text gives an impression of antiquity,[55] and it parallels to a surprising extent the first stages of the Pythagorean number theory. In the beginning was an abyss, a "boundless sea," a limitless chaos. In it there came to be, by and by, and for no particular reason, a "bubble," which began to grow and become firmer. It sucked in the surrounding *πνεῦμα*, its "skin" became hard, and soon there floated on the sea of boundlessness a glittering sphere: the world egg. In this there developed a living creature, like the sphere in shape, winged, bisexual. It broke the egg and "appeared" in radiant brilliance: Phanes! Then the two halves of the broken shell fitted themselves together "harmoniously," while Phanes took position at the utmost boundaries of the heavens, a secret, spiritual light; and from the "procreative" content of the egg arose the realms of the world.[56]

Orphism and Pythagoreanism were almost inextricably intertwined in the fifth century (cf. ch. II 2), so that it is understandable that, within the pre-Socratic domain, Pythagorean doctrine developed as a transposed version of Orphic cosmogony. We have only a few crumbs of knowledge about further details of their cosmogony: ten heavenly bodies came into being, which circle about the central fire. There were 10 because 10 is the "perfect" number;[57] and it seems also to be significant that, counting from the periphery, the sun is in seventh

[55] Apion's source is not to be identified with the cosmogony of "Hieronymus and Hellanicus" (*Orph. frag.* 54, 57ff), where Kern placed the Apion testimonies. The "Rhapsodic Cosmogony" is closer, if *πνεῦμα* is a synonym for *αἰθήρ* (*Orph. frag.* 70). There, however, Chaos and Aether are "begotten" by Chronos (*Orph. frag.* 66), whereas, according to Apion, Chaos had always been there (*Hom.* 6.3.1, 6.4.1, *Rec.* 10.30.3), and Chronos plays no active role. But Chronos is also missing in the Orphic cosmogony known to Aristotle (*Met.* 1071b27) and Eudemus (fr. 150), and in the parody of Aristophanes (*Av.* 693ff). The bisexual Phanes seems to be reflected in the myth of Plato's *Symposium* (189d; cf. K. Ziegler, *NJb* 1913, 529ff; *RE* s.v. Orphische Dichtung 1361f). The breathing in of a *πνεῦμα* seems already to have a part in the Egyptian story of the world egg (S. Morenz, in *Aus Antike und Orient: Festschr. W. Schubart*, 1950, pp. 71ff; cf. also *Quellen des alten Orients: Die Schöpfungsmythen*, Zürich 1964, p. 90) and in Mochus, *FGrHist* 784F4; Gen. 1.2.

[56] *Hom.* 6.4.1ff (*Orph. frag.* 55): *ἀπείρου τινὸς βυθοῦ ἀεὶ ῥέοντος . . . 2: συνέβη ποτὲ . . . εὐτάκτως ῥυῆναι . . . ὥσπερ ἴλιγγα καὶ μεῖξαι τὰς οὐσίας . . . ὥσπερ . . . πομφόλυξ . . . καὶ τὸ περικείμενον πνεῦμα ἐπισπάσασθαι . . . 3: ὑπὸ τοῦ περιειληφότος θειώδους πνεύματος ἀναφερόμενον προέκυψεν εἰς φῶς μέγιστόν τι τοῦτο ἀποκύημα . . . καὶ τῇ περιφερείᾳ τῷ ᾠῷ προσεοικὸς καὶ τῷ τάχει τῆς πτήσεως . . .* 6.5.4 (*Orph. frag.* 56): *ζῷόν τι ἀρρενόθηλυ . . .* 6.6.2 (*Orph. frag.* 56): *τὸ μὲν κύτος τὴν ἁρμονίαν λαμβάνει καὶ τὴν διακόσμησιν ἴσχει, αὐτὸς δὲ ὥσπερ ἐπ' ἀκρωρείας οὐρανοῦ προκαθέζεται. Rec.* 10.32.1: "[Chaos] primo omnium tempore multo concretam genuit quandam quasi bullam, quae . . . aliquanto tempore circumacta per superficiem materiae, ex qua quasi ex vulva processerat, rigore frigoris obdurata et glacialibus augmentis semper increscens . . ." 10.30.4: "peperisse ac protulisse ex se duplicem quandam speciem, quam illi masculeminam vocant . . . et hoc esse principium omnium."

[57] Arist. *Met.* 986a8ff; cf. below, ch. IV 3.

place.[58] If we see number here in a quite different function—not as a spatially extended figure but as an ordinal—yet the statement that the whole universe is ἁρμονία καὶ ἀριθμός leads off in a still different direction:[59] number is "harmony" not only in a general way, as that which transcends the opposition of Limit and Unlimited, but even more so on the basis of the recognition that all musical intervals are determined by numerical ratios.[60] This music then moves out into the cosmic realm, in the idea of the "harmony of the spheres"; a contributing factor in this development is to be found in scientific insights, in a modest knowledge of the periodicity of the movements of the planets, and in hypotheses about their distances and speeds.[61] All the same, the special position of these items of "knowledge," as compared with "speculations," does not seem to have been grasped. Rather, we find even more bizarre statements: 1 is mind (νοῦς), 2 opinion, 3 the number of the Whole, 4 (or 9) "justice," 5 "marriage," 7 "right time" (καιρός), and 10 is "perfect."[62] Aristotle tries to understand all this as attempts at scientific definition, though he also complains of "superficiality."[63] It is even more amazing, though also consistent, that these abstract concepts (or perhaps more correctly: these powers) each takes its specific position in the cosmos—for the cosmos itself does consist of numbers.[64]

From our perspective, we may distinguish at least four quite different functions of number on which the Pythagorean doctrines are built: number as the symbol of certain concepts or powers of ordering; number as the designator of order, position, or rank; number as determiner of spatial extent (the pebble figures); and finally number as ratio and mathematical formula, as natural law. Aristotle already was complaining, in his day, that very different things were being equated here;[65] no wonder that historians of philosophy, with so many different points of departure, have arrived at quite disparate results. Some interpret the Pythagorean number theory as a radical materialization of number and find it a kind of atomism, while others see in it a philosophy of mathematical form, an idealism closely akin, if not identical to, the Platonic theory of ideas.

The "number atomism" interpretation goes back to Cornford.[66] In his account of Pythagorean doctrine, Aristotle speaks of a plurality of extended monads,[67] and he often alludes to the definition of the point as a "monad having position."[68] If we interpret this as a comprehensive key idea, to be taken along with the pebble figures, the "star pictures" (constellations), and the procedure of Eurytus, who would determine the "number" of man or horse by making an outline picture with pebbles[69]—the result is the thesis that the Pythagoreans understood the materialized point as a kind of atom. They thought of all bodies as consisting of such point-atoms, and therefore things "are" numbers in the most literal sense; that is, they are the number of atom-point-units which they at any given moment contain. Does not Aristotle himself say that the Atomists "in a way" claim that things "are numbers or composed of numbers" (*Cael.* 303a8)?

Still, though Aristotle's refutation sometimes presupposes an atomistic view,[70] Cornford's theory cannot claim to give the final answer. Every atomic theory operates with invisibly small "units," so that every visible body consists of an incalculably large number of them; and in fact Cornford speaks throughout of "indefinite plurality."[71] But is an incalculably large number still a "number"? Would

[58] Fr. 203=Alex. *Met.* 38.20ff.

[59] 986a2f, 21.

[60] 985b31: τῶν ἁρμονιῶν ἐν ἀριθμοῖς ὁρῶντες τὰ πάθη καὶ τοὺς λόγους. Cf. below, ch. V.

[61] Cf. below, ch. IV 4.

[62] Below, ch. VI 4.

[63] 987a20, 1078b21ff, *EN* 1132b21ff, *MM* 1182a11.

[64] 990a18ff; on this, Alex. *Met.* 75.15, with the all-too-brief remark that in his second book *On the Pythagoreans* Aristotle dealt with the arrangement of the numbers in the heaven. Alexander says here that the One had the mid-position, and thus must have been counting from the middle out; in 38.21ff=Arist. fr. 203, however, where we learn that the sun is 7, the enumeration goes in the other direction. One would not expect purely arbitrary improvisations in Alexander, so that the irrational ambiguity must be Pythagorean. For surprising combinations see Aristotle's collection, presented with an air of mockery, in 1093a13ff. Here there is Platonic material intermingled, but the expression οὐλομέλεια τοῦ οὐρανοῦ is most probably Pythagorean. (οὐλομέλεια is an Ionic word, attested most often in the Hippocratic writings: DK I 458 n.; attributed to "Orpheus" at *Th. ar.* 48.6.)

[65] 990a22ff: one should distinguish, he says, between the number "out of which the world is composed," the "magnitudes being composed" (26), on one hand, and numbers as causes of properties like opinion or opportunity. (Cf. 986a16, 987a22ff, 1036b17, and 1092b8ff; and 1093a1ff, where the opponents in mind are mainly the Platonists.)

[66] Cornford, *CQ*; *PlParm* 56ff. Cherniss, especially, is in agreement (*Pres.* 39f, 387); he thinks this interpretation gives him a basis from which to criticize Aristotle's reports. Raven expresses doubts, *PyEl* 67ff, KR 247 (though on pp. 248 and 249 objects are said to be "aggregations of unit-point-atoms.")

[67] 1080b19, 1083b15; cf. above, nn. 15, 18. 1084b26ff is aimed at the Platonists.

[68] Below, ch. I 3, nn. 86–87.

[69] Arist. *Met.* 1092b10ff; Theophr. *Met.* 6a19ff, after Archytas. Further reports by ps.-Alex. *Met.* 827.9 (DK 45) are of dubious authenticity.

[70] At *Met.* 1083b8ff, Aristotle asserts, as refutation of Pythagorean views, that there are no "indivisible magnitudes." But, on his own principles, Cherniss ought not to have assumed from this (*Pres.* 39f) that the doctrine attacked had precisely the content which the refutation presupposes.

[71] *CQ* 1922, 137, *PlParm* 60.

it not be a better conclusion—on the sorites principle—that a unit amounts just about to nothing? If this doctrine is supposed to be early Pythagorean, we must reject the majority of the testimonies of Aristotle. According to all the sources, the fundamental thing for the Pythagoreans was the numbers from 1 to 10, not myriads; the important thing is form, not statistics, and this includes the method of Eurytus, who looked for the significant points, the specific criteria of the shape.[72] Nor can Pythagorean atomism be deduced from the arguments of Zeno;[73] and how could anyone put more trust in conjectures about putative opponents of Zeno, whom he does not himself name, than in what Aristotle ascribes explicitly to the Pythagoreans? If Ecphantus was indeed "the first" to attempt an atomistic interpretation of the number theory, this is an attempt to modernize the theory, rather than a revelation of its original significance.[74]

Another line of thought mentioned by Aristotle is almost always taken as Pythagorean: the "limit" (*πέρας*) is the essential nature (*οὐσία*) of a thing, so that a plane surface is more *οὐσία* than the physical object, a line more than a plane, a point—and thus a unit—still more than a line. Since this argument is contrasted by Aristotle, in one passage, to that of Plato, Speusippus, and Xenocrates (though Xenocrates is not named), it has been thought that by a kind of process of elimination we may decide for the Pythagoreans.[75] In addition, a doxographical report names *μαθηματικοί* as adherents to the limit doctrine, and people have assumed without hesitation that "mathematicians" must mean Pythagoreans.[76]

[72] So Raven, *PyEl* 104f, KR 313f.

[73] Below, ch. III 3.

[74] DK 51.2; G. Vlastos, *Gnomon* 25 (1953) 32f; below, ch. IV 3, n. 17.

[75] The passages are 1002a4ff, 1028b16ff, 1060b12ff, 1090b5ff, and perhaps *Cael.* 298b33. Diels included this material (DK 58B23–24), as did Kirk and Raven (320, as "pre-Parmenidean," and 405, in the chapter on Philolaus and Eurytus; the phrase "by a process of elimination," p. 316). See also Ross, *Met.* II 162, Kucharski, *Tétr.* 27f, Guthrie I 259. Zeller saw the right answer (I 484 n. 1: "Plato"; to be sure, the problem of the "points" arises again; cf. above, ch. I 1, n. 20), as did Cherniss (*Pres.* 40ff; *Plato* 132ff) and Saffrey 27f.

[76] Ps.-Galen *Phil. hist.* 18 (*Dox.* 611.2, 23; 613.1), Sext. Emp. *PH* 3.32, *Math.* 9.364, 367ff. For the assumption that references to *μαθηματικοί* are to Pythagoreans, cf. Frank, n. 174, and *Logos* 9 (1920–1921) 246 n. 1; Rougier 23, 64, etc. Examination of the data reveals that *μαθηματικοί* are advocates of astronomical theories in Aët. 2.15.5, 2.16.2, 2.16.7, 2.30.7, 2.31.2, 2.29.6; astrological, 5.18.6. The statement in Aët. 2.31.2 on the distance of the moon from the earth is decisive: it is the view of Aristarchus of Samos (*Dox.* 63). At Aët. 4.14.3 Pythagoreans and *μαθηματικοί* are named as sponsors of the theory of mirror images, and with this belongs the theory of vision in 4.13.9 (the same comparison with the stretching out of the hand): The astronomer Hipparchus is named here: *ἔνιοι δὲ καὶ Πυθαγόραν τῇ δόξῃ ταύτῃ συνεπιγράφουσιν ἅτε δὴ βεβαιωτὴν τῶν μαθημάτων* . . . That is, even in antiquity, we have a conjecture based on the postulate that mathematics is Pythagorean. Ps.-Plutarch (Aët. 4.14.3) left out the *μαθηματικοί* and speaks only of Pythagoreans; this is how the tradition burgeons. What we must hold to is that the *μαθηματικοί* of the doxographical tradition are professional mathematicians and astronomers of the Hellenistic age. (This is also true for Por. ap. Stob. 1.49.61, in spite of Delatte, *Litt.* 126. Cf. also Cic. *Div.* 2.91 Sext. Emp. *Math.* 10.174 Plut. *Prim. frig.* 953a.)

To be sure, the underlying consideration, that there can be a surface without a body, but no body without surface, line, and point[77]—this "conceptual experiment" is fundamental for the system of derivation found in Aristotle's *On the Good.*[78] But for the Pythagoreans, who know of nothing else than what is sensually perceptible, there can scarcely be a "limit" without a body (*ἄνευ σώματος*, 1002a6). In fact the limit doctrine appears once where the point of view is that of *χωρισμός*,[79] and this shows it cannot be Pythagorean. There is a passage in which the thought seems to be that the limit is immanent: To a mention of the limit doctrine, Aristotle adds the question, whether things of this kind exist inside or (as Plato, Speusippus, and Xenocrates have it) outside the realm of the sensible.[80] Yet some Platonists maintained the immanence of mathematicals.[81] For the Pythagoreans even the primary One is three-dimensional, or corporeal. The reduction of the physical world in the schema of body-surface-line-(point) belongs to the Platonists, not to the Pythagoreans, who knew only the one world of the sensible.

Now it is true that a much-cited passage of Aristotle does seem to confirm an idealistic interpretation of the Pythagorean number doctrine, the famous sentence in which the difference between Plato and the Pythagoreans seems to shrink down to nothing but an alteration of terminology: *τὴν δὲ μέθεξιν τοὔνομα μετέβαλεν [Πλάτων]· οἱ μὲν γὰρ Πυθαγόρειοι μιμήσει τὰ ὄντα φασὶν εἶναι τῶν ἀριθμῶν, Πλάτων δὲ μεθέξει, τοὔνομα μεταβαλών* (987b10ff). If it was only that "he changed the word," then, it is suggested here, the relation of number and things in Pythagorean doctrine was of just the same kind as that of

[77] 1002a6: *καὶ τὰ μὲν* (sc. *ἐπιφάνεια, γραμμή, στιγμή*) *ἄνευ σώματος ἐνδέχεσθαι δοκεῖ εἶναι, τὸ δὲ σῶμα ἄνευ τούτων ἀδύνατον.*

[78] Arist. fr. 28 = Alex. *Met.* 55.22ff (cf. above, ch. I 1): *τῶν δὲ σωμάτων πρῶτα τὰ ἐπίπεδα εἶναι –τὰ γὰρ ἁπλούστερά τε καὶ μὴ συναναιρούμενα πρῶτα τῇ φύσει—, ἐπιπέδων δὲ γραμμαί . . . γραμμῶν δὲ στιγμαί . . .*

[79] 1090b5ff; *διὰ τί οὖν χωριστὰ ἔσται* at b13.

[80] 1028b18: *ἔτι παρὰ τὰ αἰσθητὰ οἱ μὲν οὐκ οἴονται εἶναι οὐδὲν τοιοῦτον, οἱ δὲ πλείω καὶ μᾶλλον ὄντα ἀΐδια, ὥσπερ Πλάτων . . . Σπεύσιππος . . .*

[81] 998a7ff (immanence of mathematicals, separation of forms); cf. 1076a38ff. Other Platonists, not precisely identifiable, are included along with Plato, Speusippus, and Xenocrates at 1085a13, 1087b13ff, 1088b28f.

idea and thing in Plato; and all attempts to prove a Pythagorean origin for the theory of ideas make use of this argument.[82]

The difference between this and the reports elsewhere that for the Pythagoreans things "are" numbers, seems insuperable. The concept of development might seem to offer a solution of the dilemma, but its sponsors offer opposite formulae. If the identification of thing and number seems to Burnet primitive and therefore old,[83] to Cornford the idea of "imitation" seems "mystical," and therefore original.[84] Others have decided to eliminate one of the two versions as a faulty tradition, but, here again, sometimes one and sometimes the other is rejected.[85]

Amazingly enough, Aristotle himself does not seem to have noticed the discrepancy; the phrase *ἀριθμοὺς εἶναι . . . αὐτὰ τὰ πράγματα* stands just a little further on (b28). It is quite unlikely that Aristotle was following different sources; could the contradiction be only apparent?[86] Evidently Aristotle knew the Pythagoreans had used the word *μίμησις*. He surely must have intended to belittle Plato's originality in the doctrines which Aristotle himself disliked. Thus the thought that this Pythagorean *μίμησις* was the same as *μέθεξις*, and therefore implied a theory of ideas, will be seen to be interpretation. Aristotle does give some clues as to how imitation can be understood without a theory of ideas. The word "immanence" is no help (above, n. 21), but he does say that the Pythagoreans supposed they saw *ὁμοιώματα* between things and numbers,[87] and it is only natural to express such similarity or correspondence by the word *μίμησις*. Similarly, in the Hippocratic writings the relation of microcosm and macrocosm becomes a matter

[82] Cornford, *PrSap* 46: "The Platonic theory of ideas is described by Aristotle as a variety of Pythagoreanism . . ." For a Pythagorean theory of ideas, cf. Burnet, *EGP* 308f, *ThPl* 151ff, *Plato's Phaedo* (Oxford, 1911), introd. pp. xliii–xlvi; Taylor, *VarSocr* 178ff, Delatte, *Pol.* 108f.

[83] *EGP* 99ff, 307ff; now add Philip 73f.

[84] *CQ* 1922, 143f; 1923, 5.

[85] Since the notion of imitation seemed more fruitful philosophically, it is accepted by Gilbert, *AGP* 1939, 39f; by Jaeger, *Paideia* I 163 (Eng. tr.), for whom, in the other reports, "he is no doubt making the mistake of translating into material terms their theoretical identification of numberness and existence"; by Ross, *PTI* 217 (only "a primitive savage" could simply identify thing and number); and by Maddalena, 265 n. 24. Taking the opposite position, Cherniss, *Pres.* 386ff, says the imitation-statement came from a source who wished to contest Plato's originality, perhaps Aristoxenus (392: cf. Aristox. fr. 23). Similarly, Frank thinks of Speusippus (256).

[86] See the attempts at compromise by Zeller (I 451f) and Rey (356ff), who ingeniously suggests that things "are" numbers when one considers their basic nature, but "imitate" numbers when one considers their properties. Also see Raven *PyEl* 62ff, Guthrie I 230f.

[87] 985b27; *ἀφωμοιῶσθαι*, b33, *ἀφομοιῶν* (Eurytus), 1092b13. These references are adduced by Zeller (I 451f) and also discussed by Mondolfo (*ZM* 354f).

of "imitation," but—and here is the surprising fact—this imitation may be turned either way. One may just as well say that the human body "imitates" the cosmos as that the parts of the cosmos "imitate" human organs.[88] In the same way, either the arts imitate nature or nature imitates the arts.[89] Imitation is a two-sided correspondence, which makes it possible to interpret separate things following the same pattern, but without implying differences of rank or a relationship of ontological priority. Indeed, it scarcely makes a difference whether one says that stone "imitates" bones, or that earth "is" flesh.[90] When we place the Pythagorean theory in this pre-Socratic context, Aristotle's statement about "imitation" falls into place with the rest. Nothing more is meant than the correspondence of cosmos and number, in the sense that one explains and illuminates the other.[91] In post-Platonic thought one can scarcely speak of imitation without assuming that it implies a gradation of kinds of Being, especially since Plato often characterizes the relation of sensible object and Idea as *μίμησις*.[92] To this extent the surprising report of Aristotle is comprehensible. For the historical placement of the Pythagorean number doctrine, the result is more a confirmation than a correction, to say nothing of an earth-shaking contradiction.[93]

Again and again it becomes clear that the Pythagorean doctrine cannot be expressed in Aristotle's terminology. Their numbers are "mathematical" and yet, in view of their spatial, concrete nature, they are not.[94] They "seem" to be conceived as matter (*ὕλη*) and yet they are something like form (*εἶδος*).[95] They are, in themselves, being

[88] For the first-named expression see Hippoc. *Vict.* 1.10, for the second *Hebd.* 6.1, lines 13, 23 (*imitatio*). For Plato, to be sure, there is a basic difference: *οὐ γὰρ γῇ γυναῖκα μεμίμηται . . . ἀλλὰ γυνὴ γῆν* (*Menex.* 238a).

[89] Hippoc. *Vict.* 1.11ff.

[90] *Hebd.* 6.1. (If we may trust the Latin translations, the predicate was missing in the Greek, in this case.) "Imitate" and "be" are related in the same way as simile and metaphor: the latter is morphologically earlier, but the two can stand side by side.

[91] Cf. *Phys.* 213b26: *καὶ τοῦτ' εἶναι πρῶτον ἐν τοῖς ἀριθμοῖς*—"first" because most clearly and therefore most essential: 987a22: *ᾧ πρώτῳ ὑπάρξειεν ὁ λεχθεὶς ὅρος, τοῦτ' εἶναι τὴν οὐσίαν τοῦ πράγματος*; *Phys.* 203a12f: *σημεῖον δ' εἶναι τούτου τὸ συμβαῖνον ἐπὶ τῶν ἀριθμῶν.* 1020b4f is not specifically Pythagorean, but contemporary mathematics (Eucl. 7, defs. 17–18).

[92] *Tim.* 38a, 39e, 48e–f, etc.

[93] Cherniss (*Pres.* 386ff) finds a third version of the relation of thing and number in the expression, "the elements of numbers are the elements of all things" (986a1; cf. 985b25, and for the formulation 987b18; contra, Guthrie I 229f).

[94] The Pythagorean number is "mathematical" (1080b16) but "not unitary" (*μοναδικός*, 1080b19, b32); still one can say, "an arithmetical number is unitary" (1083b16); so that the corporeal number of the Pythagoreans is not mathematical (1083b12).

[95] 986b6: *ἐοίκασι δ' ὡς ἐν ὕλης εἴδει τὰ στοιχεῖα τάττειν*; 986a16: *καὶ ὡς ὕλην τοῖς οὖσι καὶ ὡς πάθη τε καὶ ἕξεις*; cf. also above, n. 65.

(οὐσία), and yet are not *quite* so.[96] They cannot be expressed in the Aristotelian framework of the four principles, or in the categories of form and matter; great as the temptation has been, both for Aristotle and for modern scholars, to understand the opposition of Limit and Unlimited as identical to that of form and matter, the explicit statements that the One partakes of both Limit and Unlimited, and that number is a kind of material (ὕλη), stand in the way. Missing are the impact of the theory of ideas and the dialectic, the classification of Being into stages of differing reality (οὐσία), the reduction of the sensible world to immaterial principles. Neither the system of Aristotle nor the conceptual framework developed by the Academy forms any part of the background of these Pythagorean doctrines; rather, they obstruct our access to them and impede our understanding of them. When one puts these observations alongside the traces of cosmogonic myths which dominate the apparently abstract pattern of the genesis of the "numbers," there can remain no doubt. What Aristotle presents as the philosophy of the Pythagoreans is truly pre-Socratic, unaffected by the achievements of Socratic-Platonic dialectic, and not to be measured by their standards. *Οἱ γὰρ πρότεροι διαλεκτικῆς οὐ μετεῖχον* (Arist. *Met.* 987b32).

Thus the thesis of Frank and Howald, that the whole number philosophy of the "so-called Pythagoreans" was developed only within the precincts of the Old Academy, is refuted from within the theory itself.[97] Indeed, Aristotle says, repeatedly and unambiguously, that the Pythagoreans are to be dated earlier than Plato.[98] Elsewhere, to be sure, Aristotle's chronological indications are imprecise enough to make one think that he was not himself quite clear about the order of events.[99] The surprising anonymity of the doctrines is of a piece with this chronological indefiniteness. The only personal name that certainly belongs with this philosophy is "Pythagoras"; but Aristotle purposely avoids it. The "Pythagoreans" belong among the "later" pre-Socratics.[100]

[96] *ἄπειρον, πέρας, ἕν, ἀριθμός* as *οὐσία*, above, n. 14; nevertheless, *σαφῶς μὲν οὐ διήρθρωται παρ' ἐκείνων*, 986b5f; cf. *Phys.* 204a33f.

[97] Frank 258: "Thus there is nothing in the whole of Aristotle's account of the Pythagorean philosophy which could not be derived from Speusippus or similar Platonists." Cf. his next pages; similarly Howald, *Ps. Sudhoff* 70ff; with amplification, Bollinger 40ff.

[98] 987a29ff, b32, 1053b12, 1078b21.

[99] Their relation to the Atomists is expressed very vaguely at 985b23, in the words *ἐν δὲ τούτοις καὶ πρὸ τούτων* (cf. Alex. *Met.* 37.6ff). At 1078b21 the Pythagoreans are dated earlier than Democritus.

[100] At 1002a11ff Platonists and Pythagoreans are mentioned as "those who came later and seemed wiser" (*οἱ δ' ὕστεροι καὶ σοφώτεροι . . . δόξαντες*). This counts against Philip's thesis that the whole doctrine goes back to Pythagoras himself (34f).

As to the sources to which Aristotle owes his knowledge, there are two different clues. The story about Eurytus is expressly referred by Theophrastus to an account given by Archytas,[101] and here we obviously have oral tradition transmitted through Academic connections in Magna Graecia; Speusippus and Xenocrates had been in Sicily with Plato.[102] In the second place, Aristotle occasionally plays off the Pythagorean doctrines against the Academy in such a way as to make the conclusion unavoidable that he is using written sources without Academic coloring.[103] Therefore he must have had at least one original Pythagorean document.

The scanty indications about Archytas given by Aristotle and his pupil Eudemus reveal an advanced stage of Pythagorean doctrine, not far from Platonism. Archytas proposes definitions whose purpose is differentiation between matter and form,[104] while Pythagorean definitions had been "superficial,"[105] and their number theory made no distinction between form and matter. Archytas designated the "irregular," or the "indefinite," as cause of motion,[106] whereas in the Pythagorean number theory the cause of motion remained unclear (990a8). If we add that the procedure of Eurytus, too, seems to be an attempt at a more comprehensive systematization of the number theory,[107] and that Eurytus is to be dated earlier than Archytas, the terminus ante quem recedes still further back into the fifth century. But Aristoxenus and later authors mention one name in conjunction with that of Eurytus—namely, Philolaus (below, ch. III).

When one attempts to understand the Pythagorean nnmber theory in the context of fifth-century pre-Socratic thought, it is impossible not to become aware, along with the mythical-cosmogonical

[101] *Met.* 6a19: *ὅ περ 'Αρχύτας ποτ' ἔφη . . .*

[102] Plut. *Dion* 22 (the third Sicilian journey). Cf. Merlan, *Philologus* 103 (1959) 203; Timaeus *FGrHist* 566F158.

[103] Esp. 1091a13ff.

[104] 1043a19ff=DK 47A22. To be sure, Aristotle is "systematizing," as his remarks show; but in the interest of a genuine definition Archytas had already gone beyond the Pythagoreans' symbolic numbers.

[105] Above, n. 63.

[106] Eudemus fr. 60 Wehrli: *Πλάτων δὲ τὸ μέγα καὶ τὸ μικρὸν καὶ τὸ μὴ ὂν καὶ τὸ ἀνώμαλον . . . τὴν κίνησιν λέγει . . . βέλτιον γὰρ αἴτιον λέγειν ταῦτα ὥσπερ 'Αρχύτας . . . τὸ δὲ ἀόριστον, φησί, καλῶς ἐπὶ τὴν κίνησιν οἱ Πυθαγόρειοι καὶ ὁ Πλάτων ἐπιφέρουσιν (οὐ γὰρ δὴ ἄλλος γε οὐδεὶς περὶ αὐτῆς εἴρηκεν)* . . . This second sentence, introduced by Simplicius with the words "and a little further on," obviously refers to the preceding, and the general expression "Pythagoreans" stands for Archytas (DK 58B32 removes the context.) Aristotle gives Plato's doctrine, without naming him, and adding his own criticism, at *Phys.* 201b20ff *Met.* 1066a10ff. In subject matter, Eudemus closely follows his teacher.

[107] Eurytus gets ironical praise from Theophrastus because he goes further than anyone else into absurdity!

background and their proximity to Leucippus, of a certain kinship with the Eleatics.[108] In terms of Aristotle's criticism, the Eleatics and the Pythagoreans find themselves near neighbours. Both schools, and they alone among the pre-Socratics, pressed forward to the brink of immaterial being, but in neither case were they able to make any real use of this advance. For the Eleatics, too, though "transcending sense perception" (*Gen. corr.* 325a13), "still conceived of 'existing things' as being only sensible things" (*Met.* 1010a1ff; cf. *Cael.* 298b21), just as the Pythagoreans squander their principles on the sensible world (989b29ff), while at the same time mistreating it in the process: "They seem to be speaking of another heaven and other bodies than the perceptible" (1090a34), and the theses of the Eleatics about our world actually seem to "border on madness" (*Gen. corr.* 325a19).

A striking fact about the examples that Aristotle gives for the Pythagorean equation of numbers and things is that it is never a question of the relation of individual thing and individual number—aside from the isolated fooleries of Eurytus—but of the correspondence of a plurality of things to the system of numbers, and in particular the correspondence of alterations in things to alterations in the number series. As Aristotle puts it, *πάθη* of things correspond to *πάθη* of numbers.[109] To the alternation of even and odd in the number series corresponds an alternation of Unlimited and Limit in the world. The *πάθη καὶ μέρη* of the heaven are reflected in numbers (986a5), as is the structure of the musical scale (*τῶν ἁρμονιῶν τὰ πάθη*, 985b32). It is not, after all, transcendent Being that the Pythagoreans are thinking about, but that-which-is, thought of as something coming to be, and many-shaped: *ὄντα καὶ γιγνόμενα* (985b28, 990a20).

Plurality and becoming: these were the problems that Parmenides had set for philosophy. He called both of them unthinkable and unsayable, because they were incompatible with his basic principle, *ἔστι*. The logical postulate of Truth, according to which any meaningful thought or speech presupposes a Truth, seemed to require the existence of an absolute Being, and "Truth" could only be expressed as "Being," *ὄν*. To the philosophers after Parmenides this seemed

[108] Cornford and Raven tried by different means to show a relation with the Eleatics. Cf. also below, ch. III 3. The interpretation which follows is indebted to C. H. Kahn, "The Greek Verb 'To Be' and the concept of Being," *Foundations of Language* 2 (1966) 245–265.

[109] Cf. 985b29, 32, 986a5, 990a19, 1088a17, 1090a21.

irrefutable; but still, few followed him to the logical conclusion, of denying becoming and plurality. But how was Parmenides to be overcome? Faced by this problem, the Pythagoreans were able to enlist the help of the technique of calculation. It must have seemed difficult even to a Zeno to deny that the thing "is valid" or "is" which we use as the paradigm of elementary correctness—namely, that 1 and 1 is 2, and that 2 and 2 is 4. And yet in this procedure something apparently new is gained out of the presuppositions, multiplicity and alteration emerge. Out of a few fundamental figures, the numbers from 1 to 10, there develops an inextricably complicated system. The "generative" character of mathematics broke through the inflexibility of the Eleatic system: plurality and becoming are not unthinkable and unsayable after all, but they can be thought and expressed in the form of numerical operations. These thus become, of course, the only legitimate form of expression about that-which-is; and the cosmogonic myth becomes the dry calculation of a process of division or addition. By contrast with Parmenides or Melissus, however, this is a relaxation of the stiffness of the denial of reality, and a new attempt to get back to everyday reality.

One could even see the foundation of mathematical science as latent in the principle that only mathematical expression is valid expression—only that its content, in empirical knowledge, is scanty; mathematics is still in its earliest stages, ringed about by all sorts of connotations and arbitrary conventions. It was only the development of mathematics from elementary arithmetic to deductive geometry, at a later time, that made a mathematical science possible—a development that leads, by way of Archytas, to Eudoxus (cf. below, chs. IV, VI).

"The Pythagoreans were the first to take up mathematics . . . and, having been brought up in it, came to believe that its principles are the principles of existing things."[110] This is Aristotle's psychological explanation of the origin of the Pythagorean doctrine. Modern scholars, too, have inclined toward psychological explanations, except that the driving force is thought to be not habituation, but the overwhelming impression of a discovery made: that of the numerical proportions in musical intervals.[111] On the other hand, a passage of Iamblichus, obviously derived from Aristotle's book on the

[110] 985b23ff; cf. ch. VI 1.

[111] Guthrie I 238.

Pythagoreans,[112] speaks of the factual justification that the Pythagoreans had to offer: "Whoever wishes to comprehend the true nature of actual things, should turn his attention to these things, the numbers and proportions, because it is by them that everything is made clear." Number is *οὐσία*, that about things which can, with a claim to truth, be expressed (Arist. *Met.* 987a19); nothing is known without number.[113]

A perfectly certain interpretation of a philosophy is impossible when it is known to us only indirectly and mostly in the context of polemic. Its place in the history of thought, however, can be satisfactorily assessed when we are clear about the traditional background (the cosmogony) and the philosophical problems current at the time (Eleaticism).

[112] Iam. *Comm. math. sc.* 25 p. 78.8–21. (On the deletion of the intrusive reference to geometry, see below, ch. VI 3, nn. 2–5, with context.) Compare the passages from Aristotle in the right-hand column.

οἱ δὲ Πυθαγόρειοι διατρίψαντες ἐν τοῖς μαθήμασι καὶ τό τε ἀκριβὲς τῶν λόγων	Cf. *Met.* 985b24: *οἱ Πυθαγόρειοι τῶν μαθημάτων ἁψάμενοι . . .* *EN* 1094b13: *τὸ ἀκριβὲς . . . ἐν τοῖς λόγοις*
ἀγαπήσαντες, ὅτι μόνον εἶχεν ἀποδείξεις ὧν μετεχειρίζοντο ἄνθρωποι, καὶ ὁμολογούμενα ὁρῶντες [ἔνισον] τὰ περὶ τὴν ἁρμονίαν, ὅτι δ' ἀριθμῶν [καὶ τὰ περὶ τὴν ὄψιν μαθήματα διὰ ‹δια›γραμμάτων], ὅλως αἴτια τῶν ὄντων ταῦτα ᾠήθησαν εἶναι καὶ τὰς τούτων ἀρχάς· ὥστε τῷ βουλομένῳ θεωρεῖν τὰ ὄντα πῶς ἔχει, εἰς ταῦτα βλεπτέον εἶναι, τοὺς ἀριθμοὺς καὶ [τὰ γεωμετρούμενα εἴδη τῶν ὄντων καὶ] λόγους, διὰ τὸ δηλοῦσθαι πάντα διὰ τούτων. ὡς οὖν οὔτ' ἐγκαιροτέρων ἂν οὔτε τιμιωτέρων ἀνάψαντες ἑκάστων τὰς δυνάμεις ἢ εἰς τὰ πάντων αἴτια καὶ πρῶτα σχεδὸν ὁμοτρόπως καὶ τὰ ἄλλα τούτοις διώριζον.	*Met.* 985b31: *τῶν ἁρμονιῶν ἐν ἀριθμοῖς ὁρῶντες τὰ πάθη . . .* 986a3: *ὅσα εἶχον ὁμολογούμενα . . .* 985b25: *τὰς τούτων ἀρχὰς τῶν ὄντων ἀρχὰς ᾠήθησαν εἶναι πάντων.* 1023b23: *λόγος ὁ δηλῶν ἕκαστον (ὁ διορισμὸς) ἔγκαιρος.* Pl. *Pol.* 282e. Theophr. *Met.* 6b13 . . .*εἰς τὰς ἰδέας ἀνάπτων, ταύτας δ' εἰς τοὺς ἀριθμούς . . .* 1078b22: *ὧν τοὺς λογους εἰς τοὺς ἀριθμοὺς ἀνῆπτον.* *De an.*404b21: *τὰ δ' ἄλλα ὁμοιοτρόπως.*

This passage cannot have been formulated by Iamblichus himself (following Arist. *Met.* 985b24ff), for there is not the slightest hint of the immateriality of the numbers, which was so important to any Platonist (cf. *Comm. math. sc.* p. 74.9ff). Since in general the later tradition does not usually follow Aristotle, an intermediary source (a re-working of 985b24ff, but free of Platonizing touches) would also be unlikely. Rather, since both the preceding material in Iamblichus (below, ch. II 5) and that which follows (Arist. frr. 52–53, guaranteed by Procl. *In Eucl.* 28.13ff and Cic. *Tusc.* 3.69) come from Aristotle, the answer must be that here too we have an independent fragment of Aristotle.

[113] Alex. *Met.* 40.12 = Arist. fr. 203; cf. below, ch. III 2, and above, n. 91.

Finally we come to the "table of ten opposites," which Aristotle sets apart from the rest of the Pythagorean number theory which he treats: "Other members of this same school say there are ten principles, which they arrange in two columns of cognates" (986a22, Ross tr.):

(1)	limit (*πέρας*)	:	unlimited (*ἄπειρον*)
(2)	odd (*περιττόν*)	:	even (*ἄρτιον*)
(3)	one (*ἕν*)	:	plurality (*πλῆθος*)
(4)	right (*δεξιόν*)	:	left (*ἀριστερόν*)
(5)	male (*ἄρρεν*)	:	female (*θῆλυ*)
(6)	resting (*ἠρεμοῦν*)	:	moving (*κινούμενον*)
(7)	straight (*εὐθύ*)	:	crooked (*καμπύλον*)
(8)	light (*φῶς*)	:	darkness (*σκότος*)
(9)	good (*ἀγαθόν*)	:	bad (*κακόν*)
(10)	square (*τετράγωνον*)	:	oblong (*ἑτερόμηκες*)

Though the good has the second-to-last position, the arrangement is clearly made from a normative point of view. In the *Nicomachean Ethics* Aristotle speaks explicitly of a "column of goods";[114] there, too, he attributes the system to the Pythagoreans, but adds, "and Speusippus, too, seems to have followed them" (1096b6). The inclusion of movement in the same column as the unlimited or indefinite (*ἄπειρον*, *ἀόριστον*) is alluded to by Aristotle in the general remark that "the principles in the second column, because they are negative (*στερητικαί*) are indefinite (*ἀόριστοι*)."[115] This is in a context which, according to the definite statement of Eudemus, applies especially to Plato, or better, to the interpretation and systematizing of Plato in the Academy. At the end of the *Metaphysics*, Aristotle alludes again to the "column of the beautiful (*καλόν*)," sets forth his opinion of the right point of view, and in his final sentence shows again what the real object of his criticism is : the "separation" of the Academics.[116]

Thus the "table of opposites" is quite closely connected with Academic doctrines; we have here a continuous transition between Pythagorean and Platonic. There is one small suggestion of Archytas (n. 115). Hermodorus' report of the *On the Good* reveals a similar train of thought: an initial threefold division of Being is traced back to a

[114] 1096b5; 1106b28f. Here the Platonic Unlimited Dyad is brought into close connection with Pythagoreanism (Krämer 347).

[115] *Phys.* 201b25 = *Met.* 1066a14. Cf. above, n. 106; on *Cael.* 284b6ff, above, n. 49.

[116] 1093b11ff; b27: *μὴ χωριστὰ εἶναι τὰ μαθηματικά.* Further, cf. 1072a31, *Gen. corr.* 319a15, 1054a29ff, with a reference to *Περὶ ἐναντίων.*

twofold division, in which one side has "equal, abiding, harmonized," (*ἴσον, μένον, ἡρμοσμένον*) and the other "unequal, moving, unharmonized" (*ἄνισον, κινούμενον, ἀνάρμοστον*).[117] The connection with Speusippus is particularly close.[118] Therefore it is not surprising if later Platonists, and also pseudo-Pythagorean works, keep introducing similar "tables of opposites."[119]

On the other hand, Aristotle considers it possible that Alcmaeon already knew the "table of opposites." This does not mean, however, that Aristotle dates it to the beginning of the fifth century,[120] but that he cannot, or will not, make a positive statement on the chronology; the table may be older or later than Alcmaeon. It may be that he knew it only from the oral tradition, passed on, later, by Speusippus. To think in terms of polarities, of antithetical pairs of concepts, is an old human habit.[121] The fact that there are ten pairs, however, and the inclusion of square and rectangle, do not give the impression of being ancient; and Alcmaeon is close to Ionian philosophers like Heraclitus. There is even more uncertainty latent in the possibilities of interpretation than in the chronology. Is such a rigid schema the result of true philosophical reflection, or of a primitive way of thinking? Or is it the expression of a strictly regimented way of life? We can see what the Platonists made of it, but it is not a helpful foundation for a reconstruction of Pythagorean philosophy.

[117] Simpl. *Phys.* 247.30ff.

[118] The idea that "good" does not belong at the beginning, but only appears as a later development, comes from Speusippus (1072b30ff, applied to the Pythagoreans and Speusippus in common; cf. 1075a36ff, 1091a29ff, a34: *τῶν νῦν τινες*), Cherniss, *Pres.* 241 n. 111. In addition, the pair one-plurality, which occupies third place in the "table of opposites" is fundamentally important to Speusippus.

[119] The following (a and b) are probably based on Aristotle: (a) Plut. *De Is. et Os.* 48.370e (the order, compared with Aristotle's, 9 3 1 6 7 2 10 4 8; the "good" is at the beginning; instead of "plurality" he has "dyad"). (b) Por. *VP* 38 (from Diogenes Antonius?): monad-dyad, light-darkness, right-left, equal-unequal, abiding-moving, straight-circular. (c) Eudorus ap. Simpl. *Phys.* 181.22ff: ordered-disorderly, definite-indefinite, known-unknown, male-female, odd-even, right-left, light-darkness. (d) "Pythagoras" in Varro *Ling.* 5.11: finitum-infinitum, bonum-malum, vitam-mortem, diem-noctem, status-motus. (e) Ps.-Archytas p. 19, 5–13 Thesleff: ordered-disorderly, limited-unlimited, speakable-ineffable, rational-irrational, binding-bondspoiling, etc. (f) "Eurysus," Stob. 1.6.19: speakable-ineffable, ordered-disorderly, rational-irrational. (g) Philo *Qu. in Exod.* 2.33: odd and god-even and mortal, equality-inequality, similarity-dissimilarity, same-different, unification-dissolution, better-worse. Cf. also Tim. Locr. 1.

[120] So Raven, *PyEl* 10f, KR 293.1, De Vogel, *Rev. philos.* 1959, 34f, Guthrie I 233, 245ff; on the other hand, the table is dated in the second half of the fifth century by Zeller 460 and Philip 37.

[121] G. E. R. Lloyd, *Polarity and Analogy* (Cambridge, 1966), esp. 31–51. There is a tempting similarity to Iranian dualism, noted, as it seems, by Aristoxenus (fr. 13 Hippol. 1.2.12).

3. THE LATER NON-ARISTOTELIAN TRADITION AND ITS SOURCES, SPEUSIPPUS, XENOCRATES, AND HERACLIDES PONTICUS

The most important of the later sources for Pythagorean philosophy are the *Pythagorean Memoirs* (*Πυθαγορικὰ ὑπομνήματα*) excerpted by Alexander Polyhistor, the *Life of Pythagoras* (*Πυθαγόρου βίος*) excerpted by Photius, the reports of Aëtius, and (most extensive of all) those of Sextus Empiricus.

It will become increasingly clear as we proceed that the so-called *Memoirs* (or *Hypomnemata*) can surely not be an original Pythagorean writing of the fourth century B.C., as Wellman and Delatte tried to prove. After the careful study of Festugière, we may date them with some confidence to the end of the third century B.C.[1]

The life of Pythagoras which Photius read has now been attributed by Theiler, on good grounds, to Eudorus.[2]

Sextus Empiricus gives four rather comprehensive accounts of Pythagorean doctrines, of which two pairs are more closely connected with each other than with the rest. The discussion "on number" (*PH* 3.151–167) is repeated in much greater detail at *Math.* 10.248–309.[3] There is a detailed exposition of the doctrine of ideal numbers, followed by a refutation of the concept of participation and "separation"; and because of a remarkable agreement with Hermodorus' account of Plato's doctrine of principles, the text has rightly been employed in the reconstruction of these Platonic theories; it must derive from one account of Plato's lecture *On the Good*.[4] It must be remembered,

[1] D.L. 8.24–33 (on the extent of the passage, see below, ch. II 4, n. 4) = *FGrHist* 273F93 = DK 58B1a. Its authenticity was maintained by Wellmann, *Hermes* 1919; this led to its inclusion in later editions of DK. Delatte (*Vie* 198–237) reached the same conclusion independently of Wellmann. Wiersma (*Mnemosyne* 1942, 97ff) introduced qualifications. Zeller (III 2.103–108) dated it in the first century B.C., and was followed by Jacoby, *FGrHist* 273F93. Festugière gave conclusive arguments against authenticity in *REG* 1945. In *Philologus* 1961, 26, I tried, by combination with the "letter of Lysis," to establish a third-century B.C. date for the book.

[2] In *Parusia* (1965) pp. 209ff. Immisch (*SBHeid* 1919, 7) had sought to show that the author was Agatharchides. Against this, see Wilamowitz, *Platon* II² 84.1, Ueberweg–Praechter 157*, and K. Reinhardt, *RE* XXII 763–768 (on the influence of Posidonius). Ps.-Justin *Coh. ad gent.* 19 cites the book (cf. 438b33, 439a19): Von Fritz (*SBMü* 1960, 6) considers the word *σεβαστικοί* (438b19) to be of imperial date, but as early as Epicurus we find *σέβασις* and *σεβασμός* (fr. 141. *Sent. Vat.* 32).

[3] Raven, *PyEl* 105ff, rashly attributes all four reports to the same source. In the following argument we need not cite more than *Math.* 10; *PH* does not add anything. At *PH* 3.152 the irrelevant *οἱ ἀτμοί* should be emended (comparing *Math.* 10.252ff) to *αἱ ἄτομοι* (Mutschmann should have taken this conjecture of Pappenheim into the text).

[4] Heinze, 37ff; Merlan, *Philologus* 1934, 37ff; Wilpert, *Hermes* 1941, *ZwFr* 121ff; De Vogel, *Mnemosyne* 1949, 209ff; Krämer 282ff; above, ch. I 1.

however, that Sextus ascribes the whole, unambiguously, to the Pythagoreans, and even to Pythagoras himself.[5] It is scarcely possible to determine the immediate source Sextus is following;[6] it is later than Epicurus, who is cited in it, and even later than Asclepiades of Bithynia.[7]

A differently organized exposition, under the heading "On the Criterion," deals with the relationship between mathematical proportion and nature (*Math.* 7); and the same subject is treated much more briefly, in a different order, and with some supplementary material, in the book "Against the Arithmeticians" (*Math.* 4).[8] In the former is found the famous citation of "Posidonius in his exposition of Plato's *Timaeus*" (*Math.* 7.93). The bitterly debated question, whether this means that Posidonius wrote a commentary on the *Timaeus*, we may here leave unsolved;[9] Reinhardt and Schmekel agree that there is much more from Posidonius than a single sentence.[10] Thus we have, in this passage, an exposition of Pythagoreanism as it was seen by Posidonius. It is an important question, nevertheless, whether Reinhardt was right in singling out the explanation of the Pythagorean Tetractys (94–100) as "a piece of book-learning that Sextus could have gotten from anywhere."[11]

This is in fact the most popular exposition of Pythagorean doctrine, which has very close parallels in Philo,[12] Theo,[13] and Anatolius.[14] In Sextus the passage merely serves the purpose of clarification, not of development of the thought; but since Posidonius comes first to mind if we think of a possible common source of Sextus and Philo, it is natural to ask whether Posidonius himself may not have been the one to take over and work in a piece of "book learning."

The elimination of the "interpolation" does make difficulties. Reinhardt himself had trouble deciding where it begins, and in any case this must be in the middle of a sentence.[15] At its end lies an incontestably Posidonian train of thought, set off by the phrase καὶ ἄλλως (101),[16] but section 99 is introduced in just the same way, so that the transitional flourish merely demonstrates the unity of the passage's structure. The controversial section is referred to twice in the sequel. The concluding sentence (109)[17] repeats a verse cited there,[18] though one cannot simply regard it as separable from the Stoicizing commentary. Further, in the Posidonian passage (101ff), there are mentioned as incorporeal point, line, and plane, "which we also discussed

[5] οἱ περὶ Πυθαγόραν *Math.* 10.248, 250; cf. 255, 263, 282, 284; Πυθαγόρας 261. Similarly *PH* 3.152, 157, 163. Plato's theory of ideas is mentioned, polemically, in *Math.* 10.258. It is hard to understand why Ross should say (*PTI* 186) that Sextus was "vague as to the authorship of this scheme."

[6] Using the difference in the derivation of line, plane, and solid mentioned in *Math.* 10.281f, Schmekel (*Mittl. Stoa* 403ff) constructed two opposed neo-Pythagorean systems, a monistic one going back to Posidonius and a dualistic one transmitted by Antiochus. This remains far from certain. It is true of both the "point" and the derivation of the Indefinite Dyad from the One that they are not taken directly from Plato (cf. Wilpert, *ZwFr* 174f, above ch. I 1, n. 17).

[7] The citation of Epicurus at *Math.* 10.257 (fr. 276 Usener) comes from Sextus' source, since *PH* 3.152 also alludes to it. The ὄγκοι of *Math.* 10.252, 254 are probably those of Asclepiades (cf. 7.201f; Theiler, *Isonomia* 90f). Theiler, *Parusia* 208 suspects that Eudorus is the source of the whole exposition; but Eudorus modifies the "Pythagorean" system by setting the One above the pair of opposites (below, n. 45).

[8] 7.94, cf. 4.2–3; 7.99–100, cf. 4.4–5; 7.95–98, cf. 4.6–9. The account in *Math.* 4 is more "by way of brief illustration" (4.4, 10), but it alone has the statement on the number 10 (4.3; lacking 7.94 but alluded to Philo *Op.* 47), as well as the connection of tetractys and soul (cf. Anat. 32).

[9] Cf. Reinhardt, *Poseidonios* 414ff; *RE* XXII 569, with references.

[10] Schmekel, *Mittl. Stoa* 405ff, Reinhardt, *Poseidonios* 415f, 418f. The threefold division of things into συναπτόμενα, ἡνωμένα, διεστῶτα (Sext. Emp. *Math.* 7.102; Reinhardt, *Kosmos* 34ff) is unmistakably Posidonian; and there is a clear piece of evidence in the mention of the Colossus of Rhodes (Sextus 7.107; Reinhardt, *Poseidonios* 419).

[11] Reinhardt, *Poseidonios* 416, *RE* XXII 725. In agreement, among others, is M. Pohlenz, *Hermes* 76 (1941) 2.

[12] *Op.* 47, cf. Sext. Emp. *Math.* 7.94; Philo 48, cf. Sext. Emp. *Math.* 7.95–98; Philo 49, cf. Sext. Emp. *Math.* 7.99–100. The agreement in order and formulation is striking, even though in some parts Sextus is fuller, and in some parts Philo. Philo's mention (50) of the game of putting a nut on top of three others to make a pyramid (καρυατίζειν, cf. Anat. 32.3ff), provides an explanation for the word ἐπαιωρήσωμεν in Sext. Emp. *Math.* 7.100. (Consequently, at *Math.* 4.5, instead of the inappropriate ἐπιθεωρήσῃ, whose correct use is illustrated by *PH* 3.154, we must read ἐπαιωρήσῃ.)—A short summary of the same train of thought is given by Philo *V. Mos.* 2.115.

[13] Sext. Emp. *Math.* 7.95–98, cf. Theo Sm. 93.21ff. In particular the "first tetractys" of Sextus 7.95 is only comprehensible through Theo 94.10ff. For "the nature of the whole," Sext. Emp. *Math.* 7.93, cf. Theo Sm. 94.4; for the oath by the tetractys (Sext. Emp. *Math.* 7.94), cf. Theo Sm. 94.6f; in addition the explanation τὸν μὲν παραδόντα λέγοντες Πυθαγόραν in Sextus, and in Theo τὸν παραδόντα Πυθαγόραν λέγουσιν. Theo's source here is obviously Thrasyllus (93.18 refers to the Thrasyllus quotation at 87.4; cf. Schmekel, *Mittl. Stoa* 409 n. 3). Theo's passage is also relevant in the *Timaeus* discussion: the "second tetractys" is the sequence of numbers in the *Timaeus*.

[14] Pages 31f (abbreviated in *Th. ar.* 29.10ff); esp. p. 32 = *Th. ar.* 30.4–14 corresponds almost word for word with Sext. Emp. *Math.* 4.6–9. Also cf. *Th. ar.* 29.10: πρώτη ἡ τετρὰς ἔδειξε τὴν τοῦ στερεοῦ φύσιν, with Philo *Op.* 49: πρώτη γὰρ αὕτη (ἡ τετρὰς) τὴν τοῦ στερεοῦ φύσιν ἔδειξε.

[15] Reinhardt, *Poseidonios* 416.3, places the end of the citation of Posidonius in the middle of section 93, but in his paraphrase (p. 418) he finds it necessary to add the concluding sentence of 93 about number and reason. In *RE* XXII 725, he adds the beginning of 94 to the Posidonius citation.

[16] Above, n. 10.

[17] κοινῷ τε λόγῳ πᾶσα τέχνη ἐστὶ σύστημα ἐκ καταλήψεων (*SVF* I 73), τὸ δὲ σύστημα ἀριθμός· τοίνυν ὑγιὲς τὸ ἀριθμῷ δέ τε παντ' ἐπέοικεν.

[18] 7.94. In the same way, the end of section 98, and 100, allude to the tetractys oath in 94. Reinhardt (*Poseidonios* 416 n. 1) interprets this as characteristic of the "interpolated" passage.

a little earlier" (104). Now since in a later passage, which is certainly Posidonian, there are mentioned precisely the "incorporeal ideas . . . consisting in the borders of bodies,"[19] the present passage, too, will have to be attributed to Posidonius. The continuity of the whole passage is irreproachable: the criterion of truth for the Pythagoreans is the "reason that is achieved from mathematics" (92), since like is known from like; the next logical step is to show to what extent nature is mathematically constructed, and thus akin to mathematical reason.[20] The solution is in the thesis that "everything is like number," which is then explained in various instances. If Posidonius, à propos of the *Timaeus*, wished to speak about the recognition of like through like, he could not ignore the fact that in the *Timaeus* the soul, which recognizes, is created as a number pattern, as the physical world is made up of mathematically determined triangles. Obviously, Posidonius is in part dependent on Aristotle, who, following a similar line of thought, brought the *Timaeus* and the theory of ideal numbers into connection, and also treated of this matter in the dialogue *On Philosophy*.[21]

Thus it remains probable that the whole section in Sextus, that is to say the most prevalent type of exposition of the Pythagorean theory of numbers, was transmitted by way of Posidonius.[22] The additional material in the parallel passage[23] is probably to be explained by the fact that Posidonius was abridged twice, in Sextus' work, in different ways. It is important to see that not only in the attention he pays to the *Timaeus*, but in details of his interpretation, Posidonius goes back to the Old Academy.[24]

[19] *Math.* 7.119 repeats almost word for word what "Posidonius in his exposition of Plato's *Timaeus*" had said in 93. Point, line, and plane are incorporeal in 99–100; and in 104 they as well as time can be given as examples of incorporeality.

[20] Reinhardt is wrong in missing, in the "interpolation," the relation to the problem of knowledge (*Poseidonios* 416).

[21] *De an.* 404b9ff has the same quotation from Empedocles as Sextus 7.92 (above, ch. I 1, nn. 47–55); cf. Chalcid. 50.

[22] Cf. F. E. Robbins, "Posidonius and the Sources of Pythagorean Arithmology, *CP* 15 (1920) 309–322; "The Tradition of Greek Arithmology," *CP* 16 (1921) 97–123; V. DeFalco, "Sui trattati aritmologici di Nicomaco ed Anatolio." *Riv. indo-greco-italica* 6 (1922) fasc. 3–4, pp. 51–60. Robbins assumes a pre-Posidonian source; DeFalco points out the fundamental importance of Posidonius' exposition. These studies may be consulted for the individual problems of filiation in the very complicated tradition; cf. also W. Theiler, *Gnomon* 28 (1956) 284f, and P. Boyancé, *REG* 76 (1963) 82ff.

[23] Above, n. 8.

[24] Exegesis of the *Timaeus* by Posidonius is also attested by Plut. *De an. procr.* 1023b (a passage that would also fit into a discussion of the criterion of truth; cf. Merlan, *Philologus* 1934, 211f); and Theo names Posidonius, at 103.18, in the course of an account of number speculation taking off from the *Timaeus*. Cf. Merlan, *Philologus* 1934, 197ff, *PlNeopl* 34ff; below, n. 75.

Sextus Empiricus occasionally uses the name *Πυθαγόρας* in addition to *Πυθαγόρειοι*; but the doxographer Aëtius even uses *Πυθαγόρας* more often than *Πυθαγόρειοι*. The two coalesce into the expression *οἱ ἀπὸ Πυθαγόρου*, in the later tradition,[25] but some trace of the difference has remained: one branch never has *Πυθαγόρειοι* where the other has *Πυθαγόρας*.

Now, we immediately notice a fact of great importance: the majority of the reports about *Πυθαγόρειοι* can be confidently referred to Aristotle as source,[26] but this is not so of even one of the reports about *Πυθαγόρας*. Pythagoras is frequently named in the same breath as Plato, Pythagoreans never.[27] Thus from the external form of the tradition itself, it is clear that alongside the Aristotelian tradition about the Pythagorean philosophy there was another, which dared to name Pythagoras himself and connects him closely with Platonism. And to be sure, in discussing Sextus' reports we constantly kept finding it necessary to refer to the Academy.

"The first principle of all things is the Monad; from the Monad comes the Indefinite Dyad to serve as matter for the Monad which is cause, and from the Monad and the Indefinite Dyad come the numbers." So runs the beginning of the *Memoirs* excerpted by Alexander (D.L. 8.25). Aëtius' phrasing is similar: "Among their principles are the Monad and the Indefinite Dyad. For him, one of the principles is directed toward the active or formal cause . . . and the other toward

[25] Aët. 3.2.1: *Πυθαγορείων*, Stob. *τῶν ἀπὸ Πυθαγόρου*, Plut.; 1.15.7: *οἱ ἀπὸ Πυθαγόρου*, Plut., but Stobaeus *Πυθαγόρειοι* continuing from 1.15.2; see Diels's note; 1.9.2: *οἱ ἀπὸ Πυθαγόρου*, Stob., Plut.; but Theodoret has, in more detail, *Θαλῆς μὲν καὶ Πυθαγόρας*... (cf. Diels, *Dox.* 46); 4.14.3 belongs with 4.13.10: *Πυθαγόρας*, as does 1.16.1 to 1.24.3, but 2.9.1 to 1.18.6: *Πυθαγόρειοι*. 1.14.2 remains problematic. (Plutarch has the correct text, as Diels says in his note. There is a relationship to Cleanthes: Aët. 1.14.5, 2.14.2, cf. 2.2.1; Por. ap. Euseb. *Praep. evang.* 3.7.4)

[26] Mostly particular questions in natural philosophy: Aët. 1.15.2, cf. Arist. *Sens.* 439a30. The material appended in 1.15.7 is simply attribution of the doctrine of Empedocles (1.15.3), who was considered a Pythagorean, to Pythagoreans generally.—At 1.18.6 Aristotle is explicitly cited (*Phys.* 213b23, fr. 201); 2.9.1 belongs in the same context. At 2.29.4 Aristotle is cited along with Philip of Opus (not in Rose; Ross p. 142). Cf. 3.1.2 with *Mete.* 345a14ff, b10ff, and 3.2.1 with *Mete.* 342b30ff; the doctrine of Hippocrates of Chios (342b36) and his pupils is attributed to the "Pythagoreans," as well as the anonymous doctrine of 345b10ff, which belongs also to Hippocrates. There remain 2.13.15 ("Heraclides [fr. 113a Wehrli] and the Pythagoreans"), 2.30.1 ("some of the Pythagoreans, including Philolaus" [DK 44A20]), and 2.22.5, which probably goes with 2.13.15. Of 10 lemmata, 6 are surely taken from Aristotle; others are from Heraclides, Empedocles, and Philolaus.

[27] Pythagoras and Plato: 1.23.1, 4.4.1, 4.7.1, 4.7.5, 4.9.10, 5.20.4. Pythagoras, Plato, and Aristotle: 2.10.1, 4.20.1, 5.4.2. Pythagoras and Plato in contrast: 1.10.2–3, 4.2.3–5, 2.4.1–2.

the passive or material."[28] Further on the Monad is also called mind, god, and good, and the Dyad is called divinity (δαίμων) and evil.[29] The long passage in Sextus builds on the same foundation: "Pythagoras said that the first principle of existing things is the Monad . . . and being added to itself . . . it produces the so-called Indefinite Dyad . . ." (*Math.* 10.261; cf. 276) ". . . and the rest of the numbers were produced from these . . . The Monad had the position of the active cause, and the Dyad that of the passive matter" (277). In the Photius passage the continuity is broken, but the same general outline is discernible: alongside a "Monad, i.e. the one that is comprehended by intellect," which is distinguished from the number 1 (438b14), and which is "first principle of all things" (439a19), stands the Indefinite Dyad, which is perceived "according to excess and defect";[30] and "from the Monad and the Dyad" come the numbers (439a2).

The doctrine attested in these four passages is as homogeneous in content and terminology[31] as it is flagrantly contradictory to everything that Aristotle ascribes to the Pythagoreans. There is not a word about Limit and Unlimited, about odd and even, about the One as a bisexual entity, or of cosmogonic development. On the other hand, every aspect corresponds to the system of derivation which Aristotle ascribes

[28] 1.3.8. This report is linked by the expression πάλιν δέ to another one: "The first principles, he said, are the numbers and the proportions contained in them, which he also calls harmonies, and the compounds of these two are the elements, which are called geometrical figures (γεωμετρικά)" (cf. Cic. *Acad.* 2.118). Similarly in 1.10.2, "Pythagoras posited the so-called forms . . . in the numbers and in their harmonies and in the so-called geometrical figures, as inseparable from bodies." This is confronted with Plato's "separation" (1.10.3), so that here the numbers of Pythagoras are being interpreted as immanent. The "geometrical" construction of the elements out of numerical proportions presupposes *Tim.* 53b. Now the whole section begins with the φιλοσοφία legend of Heraclides (frr. 87–88 Wehrli; Burkert, *Hermes* 1960, 159ff), which also had something in it about "numbers" in the world (at least if Iam. *VP* 59 is even partly based on Heraclides [Burkert, ibid. 161ff]). Thus one may conjecture that this passage of Aëtius is based ultimately on material from Heraclides—who was influenced by Platonic ways of thinking.

[29] Aët. 1.7.18 with ps.-Galen *Phil. hist.* 35 (*Dox.* 618.12).

[30] 438b34: μονὰς μὲν γὰρ . . . ἐνομίζετο ἡ ἐν τοῖς νοητοῖς οὖσα, ἓν δὲ τὸ ἐν τοῖς ἀριθμοῖς· ὁμοίως δὲ καὶ δύο τὸ ἐν τοῖς ἀριθμητοῖς. Immisch omitted this second sentence from text and commentary, obviously by oversight. The discrepancy between ἀριθμοῖς and ἀριθμητοῖς cannot be removed by a simple correction. There ought to be a three-part gradation: (1) the principles Monad and Dyad, (2) the numbers 1, 2, 3, etc. (for this differentiation between Monad and Dyad on one hand and the one and the two "in the numbers [ἐν τοῖς ἀριθμοῖς]," Sext. Emp. *Math.* 10.276, Lydus *Mens.* 2.6, Philo *Qu. in Gen.* 4.110), and (3) countable objects, ἀριθμητά. (Detailed differentiation of numbers and countables in Sext. Emp. *PH* 3.157ff, *Math.* 10.284ff, 4.11ff; Theo Sm. 19.15ff.)

[31] To be sure, the Indefinite Dyad is derived from the Monad at Sext. Emp. *Math.* 10.261, and less clearly at *Hypomn.* 25, though there is no suggestion of this in Photius and Aëtius, any more than in the reports about the *On the Good*. Cf. above, n. 6.

to Plato and his pupils, in distinction from the Pythagoreans. The pair of Monad and Indefinite Dyad is Platonic,[32] the concept of the Indefinite Dyad is purely Platonic, the idea of excess and defect (great-and-small)[33] is explicitly characterized by Aristotle as peculiar to Plato,[34] the matching of form and matter is Platonic-Aristotelian, like that of active and passive,[35] and Aristotle labored in vain to apply these categories of his to the "so-called Pythagoreans."[36]

The one question remaining in abeyance is that of "separation" (χωρισμός)—hardly surprising, since we are dealing with Hellenistic and largely Stoicizing tradition. Once Aëtius expressly denies separation in the Pythagoreans,[37] but in another paragraph he speaks of "participation" (μετοχή),[38] and in Sextus too the concept of participation plays a central role in both exposition and criticism.[39] In any case, all agree that the causes are, according to Pythagorean teaching, incorporeal[40]—a formulation that Posidonius, too, could accept.[41] There is thus unanimity that the sensible world is derived from higher principles,

[32] In the reports on Plato the word is ἕν, though Xenocrates fr. 15 has *Monás*.

[33] Anon. Phot. 438b35ff, Sext. Emp. *Math.* 10.263ff, esp. 275. In Aëtius and in the *Hypomnemata* the Indefinite Dyad is not closely described.

[34] *Met.* 987b26f.

[35] For the concept of ὕλη Pythagoras is grouped with the Stoics at Aët. 1.9.2: "Thales, Pythagoras, Anaxagoras, Heraclitus, and the whole flock of Stoics (ὁρμαθός, Theodoret) say that matter can be turned and altered and transferred, and that all of it is always flowing and intermingling." Cf. *Hypomn.* 25, on the elements; Aët. 1.24.3: "matter is passive"; cf. 1.16.1; Sext. Emp. *Math.* 10.277, 9.366, etc. This too suggests the *Timaeus*; cf. Arist. fr. 207 (below, n. 164), Xenocrates fr. 28 (below, n. 124). At Aët. 1.23.1 the definition of movement as "a difference or otherness in matter" is ascribed to Pythagoras and Plato; Eudemus fr. 60 attributes this definition to Plato, and distinguishes him in this from Archytas (above, ch. I 2, n. 106).

[36] Above, ch. I 2, n. 95.

[37] 1.10.2; cf. above, n. 28. In the anonymous *Prolegomena in Platonem* 5.36–38 (p. 201 Hermann) it is stated that Plato put his ideas ἐν τῷ παραδειγματικῷ, Pythagoras and Aristotle ἐν τῷ ποιητικῷ.

[38] 1.11.3.

[39] Sext. Emp. *Math.* 10.261: "(the monad) by participation in which each thing is called one (ἓν λέγεται)." From a similar source, Theo 21.2 has "(the monad) . . . for by participation in it each thing is called one." (In the sequel this is contaminated with different matter. The whole may come from Moderatus, to whom Stob. 1 proem. 8 attributes the beginning, and Theo the passage 18.3ff.) Its model is to be found in Eucl. 7 def. 1: "monad: that according to which each existing thing is called one." The "Pythagorean" sources introduce the idea of participation, and then Sextus' criticism (*Math.* 10.284ff) concentrates on this point.

[40] "The first causes are incorporeal, "Aët. 1.11.3; more fully Sext. Emp. *Math.* 10.250ff, 7.99f, 104; "the monad . . . that is among mentally apprehended things," Anon. Phot. 438b34; "all bodies come from the monad," 439a23f. In *Hypomn.* 25, the "perceptible bodies" only come into being after point, line, plane, and (geometrical) body. Philop. (*De an.* 82.19) significantly names Pythagoreans and Xenocrates (fr. 29) together as proponents of "incorporeal principles" (ἀσώματοι ἀρχαί).

[41] Sext. Emp. *Math.* 7.119; cf. above, n. 19.

and there is presupposed an ontological gradation like that of Plato's system of derivation. This too contradicts the statements of Aristotle, according to which the "so-called Pythagoreans" did not go beyond the sensible.[42] What the Hellenistic tradition presents as the philosophy of Pythagoras is, according to Aristotle's positive pronouncement, Platonism and not Pythagoreanism.

This tradition about Pythagoras, inconsistent with Aristotle, apparently came to dominate the field completely.[43] A few indications will show this well enough.[44] The Platonist Eudorus of Alexandria (first century B.C.) says that, "according to the highest *logos*," the first principle of the Pythagoreans was the One, and "according to the second *logos*," an "opposite nature" along with the One. The opposition is developed with a "table of opposites," and summarized in the formula "One (or Monad) and Indefinite Dyad."[45]

Moderatus transposes the Pythagorean number doctrine into logical-conceptual language,[46] but starts from the presupposition that One and Dyad are the principles. He elicits from the *Parmenides* a doctrine of the stages of the One which Plato is supposed to have taken from the Pythagoreans.[47] According to Hippolytus, Pythagoras posited the "unbegotten (*ἀγέννητος*) Monad" as first principle, from which the Dyad and the other numbers had their origin.[48] On the other hand Numenius ascribes a radical dualism to Pythagoras: the

[42] Above, ch. I 2, n. 15.

[43] Theo says, 20.5ff, that "the later ones" had introduced Monad and Dyad as principles, but that "the disciples of Pythagoras" (*οἱ ἀπὸ Πυθαγόρου*) had posited "all the terms set out in series, by virtue of which even and odd are conceived," i.e. the system of number concepts. The sharp distinction between *τὰ ἐν αἰσθητοῖς τρία* and *τριάς*, between *ἀριθμητόν* and *ἀριθμός* (above, n. 30), shows that this is not in agreement with Aristotle. Rather, these are Moderatus' own views (cf. Por. *VP* 49–51). The historical form of his statement is obviously based on the accidental collocation in the doxography (Aët. 1.3.8; above, n. 28).—Limit and Unlimited became important again especially after Porphyry's commentary on the *Philebus*; cf. Iam. *Comm. math. sc.* pp. 12ff, and passim, Procl. *In Eucl.* 5.14ff and passim; Nicomachus (*Ar.* 2.18.4) brings in Philolaus (below, ch. III 1).

[44] Collected by Zeller I 465ff; cf. De Vogel, *Pythagoras*, 207ff. See esp. Plut. *V. Hom.* 145 (on the source question, Diels, *Dox.* 88ff), Nicom. *Th. ar.* 11.16, 12.9ff (the Indefinite Dyad as *ἔλλειψις καὶ πλεονασμός* and as *ὕλη*), Por. *VP* 38, Iam. *In Nic.* 61.21. Cf. also Festugière, *Rév.* IV 18ff.

[45] Apud Simpl. *Phys.* 181.10ff. (Theiler, *Parusia* pp. 205–208; H. Dörrie, "Der Platoniker Eudoros von Alexandreia," *Hermes* 79 [1944] 25–39). The distinction of a highest One above the pair of opposites comes from Pl. *Parm.* 137c–141d; cf. Dodds, *CQ* 1928, 135ff. Eudorus also looks back to Xenocrates and Crantor (Plut. *De an. Procr.* 1013b).

[46] Por. *VP* 48–51; cf. above, n. 43.

[47] Por. ap. Simpl. *Phys.* 230.34ff.

[48] *Ref.* 1.2.2, 1.2.6, 4.51.4, 6.23.1: "the principle of all things is unbegotten . . . the Monad, but the Dyad was begotten." Here the difference between the Indefinite Dyad and the number 2 is obliterated—clearly a Gnostic interpretation.

Indefinite Dyad (*duitas indeterminata*) was coeval with the divinity, the Monad, and the relation of the two is that of form and matter (*species* and *silva*).[49] It is clear that even the more detailed expositions of the neo-Pythagoreans rely on the same tradition; the unmistakable Indefinite Dyad shows that the whole construct is, if Aristotle was right, *ἴδιον Πλάτωνος*.

In fact, people did realize the inconsistency with the reports of Aristotle. This appears not only from the polemics of the neo-Platonists,[50] but from a curious document, attributed to Theano, who was usually known as the wife of Pythagoras;[51]

> I have learned that many of the Greeks suppose Pythagoras said that everything came to be from number. This statement, however, involves a difficulty—how something that does not even exist is even thought to beget things. But he did not say that things came to be from number, but according to number. For in number is the primary ordering, by virtue of whose presence, in the realm of things that can be counted, too, something takes its place as first, something as second, and the rest follow in order.[52]

Here we have the neat separation of *ἀριθμός* and *ἀριθμητόν*, the concept of *μετουσία* ("presence" above), in almost verbal agreement with formulations in Sextus and Theo,[53] and in addition direct polemic against a tradition according to which things come to be "from numbers." "Many of the Greeks" have learned this false representation of the Pythagorean doctrines; here there is unmistakable polemic against Aristotle.[54] The method is subtly indirect: an "original" document is witness against his interpretation. And who is qualified to offer authentic exegesis, if not Pythagoras' wife and student herself? To be sure, Syrianus was in a still more favorable situation; he read the *Ἱερὸς λόγος* of Pythagoras himself, which named *Πρατεύς* and *Δυάς*

[49] Chalcid. 293ff = fr. 14ff Thedinga; cf. J. C. M. Van Winden, *Calcidius on Matter* (Leiden, 1959) 103–121.

[50] Above, ch. I 2, nn. 22–23.

[51] Cf. below, ch. II 2, n. 28.

[52] Stob. 1.10.13 = p. 195.12 Thesleff. It is a remarkable idea, that number "does not even exist," in blatant contradiction of all neo-Pythagoreanism and Neoplatonism. The unhesitating equation of being and corporeal being shows that the passage is Hellenistic in date.

[53] Above, n. 39.

[54] Probably against the lost writings on the Pythagoreans rather than the didactic works, which can scarcely have had much influence outside the school. The word "beget" is a barbed reference to Aristotle's accounts (*Met.* 1091a13ff).

as first principles and made number, the "ruler of shapes and forms," the basis of the "origin of all things."[55] Aristotle, accordingly, must have been mistaken . . . This is how the gap between the different traditions created the demand for apocrypha.

It is possible that the Pythagoras items of the *Vetusta placita* were "reconstructed" in the circle of Posidonius or Antiochus, with the help of apocryphal writings.[56] But these are only intermediate members in a chain of tradition that goes back further, to weighty authorities. Theophrastus remarks in his *Metaphysics*, on the problem of the good:

> Plato and the Pythagoreans make the distance (*ἀπόστασιν*) a great one, but they think everything wants to imitate it. And yet they set up a kind of antithesis, of the Indefinite Dyad and the One. In the former resides the Unlimited, the disorderly, and, so to speak, all kinds of formlessness, yet the nature of the whole would not be possible without it.[57]

Theophrastus' starting point is that Plato and the Pythagoreans, both, posited a "good" at a "great distance" from nature as a whole. Everything strives toward it, "and yet" they cannot get along without an opposite principle. The opposition is explained as that between the One and the Indefinite Dyad, in which all that is unbounded and unordered, by its nature, is pent up. Both the term Indefinite Dyad, the whole idea of a principle "in itself," and the opposition of "formlessness" and the form-giving One are according to Aristotle Platonic and not Pythagorean; still Theophrastus sees the two as a unity.[58] So the non-Aristotelian tradition, which is orthodoxy for later generations, is already present in Theophrastus, even though he speaks of Pythagoreans and not of Pythagoras. His contradiction of Aristotle may not be explained away as concision of language or hasty composition.[59]

Since Theophrastus was surely only a transmitter of this conception of Pythagorean doctrine, he must be dependent on predecessors who went still further than Aristotle[60] in connecting Plato with Pythagoreanism—to the point of identification. The only candidates would be Plato's immediate pupils, the members of the Old Academy. As a conjecture, this suggestion was made many years ago;[61] the proof came to light with a fragment of Speusippus which was first published in 1953. It had made its way into Proclus' commentary on the *Parmenides*, whose final portion is only preserved in the Latin translation of William of Moerbeke.

For the good neo-Platonic thesis that the One is still higher than Being, Proclus refers to what Speusippus had reported as the opinion of "the ancients" ("tamquam placentia antiquis," i.e. *ὡς ἀρέοκοντα τοῖς παλαιοῖς*):

> le unum enim melius ente putantes et a quo le ens, et ab ea quae secundum principium habitudine (i.e. *καὶ τῆς κατ' ἀρχὴν ἕξεως*) ipsum liberaverunt. existimantes autem quod, si quis le unum ipsum seorsum et solum meditatum sine aliis secundum se ipsum ponat, nullum alterum elementum ipsi apponens, nihil utique fiet aliorum, interminabilem dualitatem entium principium induxerunt (i.e. *τὴν ἀόριστον δυάδα τῶν ὄντων ἀρχὴν εἰσήγαγον*).[62]

It is clear that as far as Proclus could tell, Speusippus was speaking simply of "the ancients (*οἱ παλαιοί*)."[63] But there is only one possible interpretation of this. Speusippus is not presenting his own system, in

[55] Syrian. *Met.* 9.37ff, 123.1ff. On the reconstruction of this *Sacred Discourse*, see Delatte, *Litt.* 191ff; p. 164.21ff Thesleff.

[56] Diels (*Dox.* 181), following Zeller (I 467ff), derives Aëtius' testimonies, so far as they are not from Aristotle, from the pseudepigrapha. (On Posidonius, cf. below, n. 133).

[57] *Met.* 11a27ff. The Ross–Fobes translation takes *ἀπόστασιν* as "the distance between the real and the things of nature," but the point of the whole thing is the discussion of *ἀγαθόν* and *κακόν*. Speusippus has it that *ἀγαθόν* is *σπάνιον* and the *κακόν* is greatly preponderant (11a18ff); Plato and the Pythagoreans make the distance between the two great, though the second principle is indispensable in the world, *ἰσομοιρεῖν ἢ καὶ ὑπερέχειν*, 11b6. Theophrastus sets his own conviction against both of these, that *τὰ μὲν οὖν ὄντα καλῶς ἔτυχεν ὄντα* (11a25f).—Eudemus, too, after first distinguishing the two, comprehends both Plato and the Pythagoreans under the concept of the indefinite (above, ch. I 2, n. 106).

[58] Theiler, *Parusia* p. 205, reads *καίτοι ‹τοῖς› καθάπερ ἀντίθεσίν τινα ποιοῦσιν*, and opines that this formulation does not include the Pythagoreans. However, it also does not exclude them, and in content the *καίτοι* sentence belongs very closely with the preceding. The *ἀγαθόν* is the model of everything real, "and yet" (read *ἅμως* 11b5 instead of *ὅλως*) it is not sufficient by itself; the opposite principle is equally necessary. (Cf. below, n. 66.)

[59] This is Zeller's proposal. Speaking of Theophrastus, he says (I 472 n. 4), ". . . in the whole report Pythagorean and Platonic material is combined in such a way that it appears impossible to determine, from it alone . . . what belonged to each." Aristotle does say, however, that the Indefinite Dyad is not at all to be attributed to the Pythagoreans.

[60] *Met.* 987a30ff (above, ch. I 2).

[61] Cf. Frank 260.1, though he overlooks the difference from Aristotle.

[62] *Plato Latinus III: Parmenides: Procli commentarium in Parmenidem interpr. G. de Moerbeka*, ed. R. Klibansky and C. Labowsky (London, 1953) 38ff. Klibansky reconstructs the Greek text, p. 86. He suspects that the source is Speusippus' *On Pythagorean Numbers* (fr. 4 Lang), with Nicomachus as intermediary; but this cannot be proven. Merlan has shown (*PlNeopl* 96–140) that Iamblichus, in *Comm. math. sc.* 15.6ff, was able to use Speusippus in detail, perhaps through some intermediary.

[63] Proclus repeats, at the end of the quotation: "testatur et iste hanc esse antiquorum opinionem."

which the Indefinite Dyad was replaced by the concept of "plurality (*πλῆθος*)," along with other modifications.[64] But he cannot be using the expression "the ancients" of Plato either, even disregarding the plural form; he only outlived Plato eight years. Therefore Speusippus was referring to the Pythagoreans, and possibly thinking of Pythagoras himself. Plato himself speaks of Pythagorean views as an "ancient" revelation.[65]

Speusippus then, quite contrary to the statements of Aristotle, attributes to the "ancient" Pythagoreans the opposition of "One" and "Twoness," the concept of the Indefinite Dyad, and in general a principle *αὐτὸ καθ' αὑτὸ καὶ χωρὶς νοούμενον* ("ipsum seorsum et solum meditatium"). The line of thought is so close to that of Theophrastus, who was a generation younger,[66] that we can no longer harbour any doubt as to the source from which the latter acquired his non-Aristotelian conception of Pythagorean doctrine. Plato's nephew and successor claimed that the basic thought of the Platonic doctrine of ideal numbers was Pythagorean.

Speusippus was not alone in this trend. Xenocrates, too, wrote *Πυθαγόρεια*,[67] and we have one statement about Pythagoras from his pen: "Pythagoras, as Xenocrates says, discovered that the musical intervals did not originate without number, either."[68] Xenocrates, then, attributed to Pythagoras himself a specific scientific discovery about numbers in music, but even more than this: the word *καί* ("either," above) indicates that he traced to him some other number theory as well.[69] Now Xenocrates interpreted the origin of the world soul in the *Timaeus* as derivation of number from One and Indefinite Dyad,[70] and the connection of number and music, which according to Xenocrates Pythagoras also discovered, is definitely presupposed in this

[64] Speusippus did, however, come out for placing the One above Being (fr. 34e = Arist. *Met.*, 1092a14ff, and the testimony discovered by Merlan, *PlNeopl* 96ff, in Iamblichus). A point of departure for this line of thought can be seen in Pl. *Rep.* 509b, and better in Plato's *Parmenides*; cf. Dodds, *CQ* 1928, 129f.

[65] *Phlb.* 16c; below, ch. I 4.

[66] The elevation of the One in Speusippus corresponds to the "great distance" of Theophrastus. Both emphasize the necessity of the second principle, without which "nihil utique fiet aliorum," *οὐχ οἷόν τε ἄνευ ταύτης τὴν τῶν ὅλων φύσιν . . .*, as in Arist. *Met.* 1088b35ff.

[67] D.L. 4.13.

[68] *Πυθαγόρας, ὥς φησι Ξενοκράτης, εὗρισκε καὶ τὰ ἐν μουσικῇ διαστήματα οὐ χωρὶς ἀριθμοῦ τὴν γένεσιν ἔχοντα*, fr. 9 Heinze — Por. *In Ptol.* 30.2ff. It is unlikely, in spite of Heinze 5ff, that more than this sentence goes back to Xenocrates; cf. below, ch. V 1.

[69] Cf. Sext. Emp. *Math.* 10.260, who gives it as Pythagorean teaching that *ἡ ἁπλῆ γραμμὴ οὐ χωρὶς ἀριθμοῦ νενόηται . . .*

[70] Fr. 68 H. — Plut. *De an. procr.* 1012d-e; following this, Tim. Locr. 95c.

section of the *Timaeus*.[71] Furthermore, Xenocrates developed from the *Timaeus* his definition of soul as "number moving itself,"[72] and precisely this definition of soul is ascribed by the doxographers to Pythagoras,[73] as well as the doctrine of the One and the Indefinite Dyad. The later tradition about Pythagoras is largely based on the exegesis of the *Timaeus* by Xenocrates, who understood the ideas contained in Plato's dialogue as the teaching of Pythagoras.

Speusippus, too, interpreted the *Timaeus* and developed a definition of soul out of it,[74] which was taken over by Posidonius,[75] namely that the soul is *ἰδέα τοῦ πάντῃ διαστατοῦ*, and Posidonius adds *κατ' ἀριθμὸν συνεστῶσα ἁρμονίαν περιέχοντα*. Posidonius interprets the *Timaeus* as evidence for Pythagorean doctrine,[76] and in this too he is following Speusippus, for the new fragment is proof that the latter saw Platonism and Pythagoreanism as a unity.

A third writer deserves mention in this context, Heraclides Ponticus. He attributed to Pythagoras the invention and definition of the word *φιλοσοφία*, and this ascription made its way, via the doxographers, into all the ancient handbooks. Yet it is disproved by the semantic history of the word. It had meant close acquaintance and familiarity with *σοφία*; and Plato was first to define it as in insatiable striving, and set it in opposition to the possession of wisdom. This was after the Sophists and their claims had roused popular animosity.[77] Heraclides probably combined with this a number theory that took its direction from the *Timaeus*; at least he ascribed to Pythagoras the sentence, "The knowledge of the perfection of the numbers of the soul is happiness."[78]

Interpretation of the *Timaeus* and the orally transmitted theory

[71] Cf. below, ch. V 1.

[72] Plut. *De an. procr.* 1012d-e, and Xenocrates frr. 60–65.

[73] Aëtius 4.2.3, from which is derived Theodoret 5.17. Also Nemesius 102 M., Meletius *An. Ox.* III 146.30; cf. Cic. *Tusc.* 1.20 Aëtius adds that Xenocrates is following Pythagoras (4.2.4, from Stob.).

[74] Fr. 40 Lang = Iam. ap. Stob. 1.49.32; cf. the discussion of Merlan, *Philologus* 1934, 197ff; Cherniss, *Plato* 509ff; Merlan, *PlNeopl* 36ff. Cherniss sets out the relationship to the *Timaeus*.

[75] Plut. *De an. procr.* 22.1023b, on which see Macrob. *Somn. Sc.* 1.14.19, and the "Platonic" definition of soul at D.L. 3.67. On the addition made by Posidonius, cf. Sext. Emp. *Math.* 4.6–9 and Anat. 32 = *Th. ar.* 30.4ff (above, n. 8, 14, 24; Merlan, *Philologus* 1934, 197ff).

[76] Reinhardt has to concede this, since the citation of Posidonius at Sext. Emp. *Math.* 7.93 is made in the section on the Pythagoreans.

[77] Burkert, *Hermes* 1960, 159ff; cf. M. Landfester, *Das griechische Nomen φίλος und seine Ableitungen* (Hildesheim, 1966).

[78] Heraclides fr. 44 Wehrli — Clem. Al. *Strom.* 2.130.3; cf. above, n. 28.

of ideal numbers combine, for Plato's pupils (including Aristotle),[79] into a system which for the Platonists is regarded as the doctrine of Pythagoras, which they themselves follow, with more or less radical modifications; Aristotle, on the other hand, characterizes the same system as Platonic and contrasts it with other, obviously older, less advanced pronouncements which he calls the doctrine of the "Pythagoreans." The later tradition, beginning with Theophrastus, follows the Platonists, and to a great extent forgets the reports of Aristotle. Not completely, to be sure: that which Aristotle ascribes to the "Pythagoreans" in general, appears in the doxographical reports under the name of Philolaus.

This relationship of the different branches of tradition can also be shown in the further development of the doctrines. The derivation of the world from number takes place, according to "Pythagoras," by the development of the point ("a monad having position") out of unity,[80] and then of the perceptible world from the point in the hierarchical succession line, plane, solid.[81] As a rule the number 2 is assigned to the line, 3 to the plane, and 4 to the solid.[82] Sextus emphasizes the difference from another derivation, according to which the line comes to be through a continuous movement (*ῥύσις*) of the point, the plane through movement of the line, and the solid through movement of the plane. Here there is no need for any other "first principle" than the One.[83] Yet the two ideas get mixed together.[84]

We have shown that this derivation of the physical world is firmly anchored in the Platonic number theory;[85] it remains to show that none of all this is ascribed to Pythagoreans by Aristotle, and that it even contains contradictions of what is—according to Aristotle—Pythagorean.

[79] Cf. connection of the *Timaeus* with the *ἄγραφα δόγματα* at *Phys*, 209b11ff, and the sequence *Timaeus*–ideal numbers, *De an.* 404b18ff, which is found again in Posidonius (above, n. 21).

[80] Procl. *In Eucl.* 95.21ff, Schol. Arist. *Met.* 401a4 (as Pythagorean), and the passages cited in n. 82.

[81] *Hypomn.* 25, Anon. Phot. 439a19ff. (Skipped by Aët. 1.3.8.)

[82] Sext. Emp. *Math.* 10.278ff, 7.100 (cf. Philo *Op.* 49: Posidonius). Further, inter alia Philo *Op.* 98, *V. Mos.* 2.115, Theo Sm. 97.17, 100.20, 101.11 (with variations 96.9ff), Nicom. *Th. ar.* 20.9f, 23.11ff, Procl. *In Eucl.* 97.17ff, 114.25f; below, n. 120; ch. III 2, n. 41.

[83] Sext. Emp. *Math.* 10.281f, cf. 7.99 (Posidonius); 3.19f; 9.430; Eratosthenes ap. Sext. Emp. *Math.* 3.28, cf. Theo Sm. 83.21; Simon Magus ap. Hippol. *Ref.* 4.51.3, cf. 5.9.5; Procl. *In Tim.* III 137.2. On Schmekel's views, above, n. 6.

[84] Thus in Posidonius, already, the "nut game" and the "flowing" obviously belong together. Cf. Sext. Emp. *Math.* 7.99f, Philo *Op.* 49f, *Decal.* 25, Plut. *De I:* 390c-d.

[85] Above, ch. I 1, nn. 41–45.

In his philosophical glossary Aristotle takes over as valid the definition of the point as "a monad having position"[86] and mentions it often, from the point of view of "separation," as a formula used by the Platonists.[87] He never connects it with Pythagoreans; and he cannot do so consistently with his allegation that the "units" of the Pythagoreans possess "magnitude,"[88] for "point" and "magnitude" are mutually exclusive terms. For this very reason the derivation of the physical world through the series surface, line, point cannot belong to these Pythagoreans; the world of perceptible bodies is for them the only reality, beyond which they do not go.[89] Thus Aristotle never connects the line-plane-solid formula with Pythagoreanism, but always with Platonism.[90] Indeed there is one passage in which he clearly points out the difference. In his *De caelo* he explicitly disagrees with all those who "make up all bodies from planes" (299a1ff), and an express reference (300a1) shows that he has the *Timaeus* especially in mind. He asserts that to be consistent one must carry such an analysis further, to the point, not only as far as the *ἄτομος γραμμή* (299a5ff), and this is directed against Plato and Xenocrates.[91] The refutation revolves about the problem of weight, which remains unexplained in these geometrical constructions; and he appends the casual remark, "and the same thing happens to those who construct the world out of numbers: for there are some who see nature as composed of numbers, like some of the Pythagoreans"—here too the problem of weight is untouched.[92] The Pythagoreans who "compose the world out of numbers" are thus contrasted with "those who compose bodies of

[86] *Met.* 1016b24ff; cf. *De an.* 409a6, *An. post.* 87a31ff, 88a33, *Met.* 1084b25.

[87] *Phys.* 227a27: "if, as they say, the point and the monad have a separate existence of their own," *Met.* 1069a12, 1077b30, 1084b23 ("if number is separable." b2). *Top.* 108b7ff applies to Speusippus (cf. below, n. 105); against attribution to the Pythagoreans, Heidel, *AJP* 1940, 27, n. 56, Wilpert *ZwFr* 218 n. 46.

[88] Above, ch. I 2, n. 18.

[89] Above, ch. I 2, n. 15.

[90] The evidence for Speusippus and Xenocrates, above, ch. I 1, nn. 38–39. That the *πέρατα* doctrine of *Met.* 1028b16ff is not applied to the Pythagoreans was shown above, ch. I 2, nn. 77–81. The passage *Met.* 1036b12ff is almost always thought to refer to the Pythagoreans: *καὶ ἀνάγουσι πάντα εἰς τοὺς ἀριθμούς, καὶ γραμμῆς τὸν λόγον τὸν τῶν δύο εἶναί φασιν.* (Ps.-Alex. *Met.* 512.23ff—"two" as *τὸ πρῶτον διαστατόν*, line 37.—Ross, *Met.* II 202 sees the source as Aristotle's lost book. KR accept it [as pre-Parmenidean!], no. 317 [cf. 316]. Also called Pythagorean by Zeller I 511 n. 1; von Fritz, *ABG* 1955, 83; Cherniss, *Pres.* 225; Guthrie I 257.) One reason is that this is set up against *οἱ τὰς ἰδέας λέγοντες*; but it is Speusippus who does not believe in ideas, but only in mathematical numbers. In the context, the point is the separation of form and matter (*χωρίζειν*, a34, b7; *εἶδος*, b2), so that the Pythagoreans cannot be meant.

[91] Above, ch. I 1, n. 17.

[92] *Cael.* 300a14ff. Cf. above ch. I 2, n. 19.

planes" (299a3); they are united only in a common omission, their failure to face the problem of weight.[93]

The related thought, that the point in movement makes the line, the line in movement the plane, and the plane in movement the solid,[94] appears in Aristotle's polemic against Xenocrates. He seeks to reduce the latter's definition of soul to absurdity by pointing out that according to these geometrical doctrines—introduced with the words *ἐπεί φασι* (409a4)—even the "self-moving number" would have to consist of lines.[95] In order to demonstrate a contradiction, Aristotle seizes upon a statement of the opposite party—the Platonists.

Archytas had evidently gone deeply into stereometric problems; Aristotle's statement that "the Pythagoreans" called a surface *χροιά*[96] (Plato is said to have introduced the term *ἐπίπεδον*[97]) may belong in this context. According to Diogenes Laertius, Archytas introduced the idea of movement into geometry,[98] and in fact Archytas' solution of the "Delian problem,"[99] reported by Eudemus, makes good use of such concepts: a right-angled triangle, revolved about one leg, produces a cone, the end of a straight line, when turned about, describes an arc, etc. Archytas, according to Eudemus' evidence, also concerned himself with the explanation of movement in general.[100] Thus the conjecture seems justified that this comprehensive treatment of geometrical magnitudes by the use of the idea of movement comes precisely from

[93] Bywater (31) already perceived this difference, and used it against the authenticity of Philolaus fr. 12.

[94] Cf. above, n. 83.

[95] *De an.* 409a1ff; cf. Cherniss, *Pres.* 389, *Plato* 396 n. 322, Theiler, *Arist.* 101, Guthrie I 262ff. Whether Aristotle is playing off the conception of Speusippus against Xenocrates (Cherniss), or whether Xenocrates could have spoken in this way himself (Theiler refers to fr. 39, though it is "not quite uncontaminated")—this is a question we need not answer.

[96] *Sens.* 439a30, whence Aët. 1.15.2; cf. Arist. *Met.* 1091a16 and Nicom. *Th. ar.* 25.15: *ταὐτὸν ἐν μονάδι, ἕτερον ἐν δυάδι, χροιὰ ἐν τριάδι, σῶμα ἐν τετράδι.* The latter, however, is obviously tinged with archaism, and not genuine tradition; it takes its origin from the doxographical report. The opposition of *ταὐτόν-ἕτερον* originates in the Platonic dialectic.—Further, cf. Hero Alex. *Deff.* p. 20f Heiberg, Aristid. Quint. p. 110. 25 Winnington-Ingram.

[97] D.L. 3.24. The concept of *ἐπίπεδον* is presupposed by Plato, *Meno* 76a and *Pol.* 299e; in a different sense *Tht.* 173e = Pi. fr. 292, as well as Democr. fr. 11d *Αἰτίαι ἐπίπεδοι* ("terrestrial," as distinguished from 11c "aerial"; B155 is not a direct quotation). The idea of *χροιά* is alluded to at Pl. *Meno* 75b-c. C. Mugler, "Sur l'histoire de quelques définitions de la géometrie grecque, II: La surface," *AC* 27 (1958) 76-91, presents an ingenious but ill-founded reconstruction; he puts the *χροιά* idea earlier than Parmenides.

[98] 8.83 (referring to the problem of doubling the cube): *πρῶτος τὰ μηχανικὰ ταῖς μαθηματικαῖς προσχρησάμενος ἀρχαῖς μεθώδευσε καὶ πρῶτος κίνησιν ὀργανικὴν διαγράμματι γεωμετρικῷ προσήγαγε.*

[99] Eudemus fr. 141 Wehrli = DK 47A14.

[100] Above, ch. I 2, n. 106.

Archytas.[101] Here, too, Archytas went further than the older Pythagoreans.[102] It is true that geometric figures had also been represented with pebbles: 3 is an "odd line"—or a triangle, or a "gnomon," 9 is an "odd square";[103] but here the strict derivation of spatial shapes (*γεννᾶν*) was not yet felt as a problem. What ontological status Archytas gave to the series point, line, plane, solid, can hardly be determined.[104]

Aristotle, in any case, attributed the derivation series point, line, plane, solid, to Platonism, not to Pythagoreanism, and in this differs from the later tradition, which saw in it the teaching of the Pythagoreans, and indeed of Pythagoras himself. Once more we suspect the pupils of Plato as the source of the non-Aristotelian tradition, and especially Speusippus, since the "point" always plays a role in these "Pythagorean" accounts,[105] and once again we find the direct proof: in his book *On Pythagorean Numbers*, from which Nicomachus has preserved us a considerable excerpt, Speusippus ascribed to them this very concatenation: "For 1 is the point, 2 the line, 3 the triangle, and 4 the pyramid."[106] Eva Sachs already had guessed that this book was one of the most important sources of the later tradition about Pythagoras.[107] Whether or not the newly discovered fragment of Speusippus about the Pythagorean "principles" came from the same book, in any case it is with Speusippus that the tradition begins which contradicts the reports of Aristotle.[108]

It is now no longer surprising to find that the series of types of

[101] Frank 370f, accepted by Wilpert *ZwFr* 174.6. Frank goes on, in an imaginative vein, to construct a "dynamic system of Archytas" (124ff), but far exceeds the limits of what can be known of the matter. Cf. Cherniss, *Pres.* 388f.

[102] Cf. above, ch. I 2, nn. 104–106.

[103] Cf. Philolaus A26.

[104] "Pure" mathematics, divorced from ontological problems, is hardly to be assumed in Archytas. Arnobius 2.9 (*Dox.* 172) names Pythagoras and Archytas: "causam in numeris ponit." But Plato's philosophy, through dialectics and theory of ideas, was bound to change whatever he took over. The connection of the numbers 2, 3, and 4 with line, plane, and solid, which seems more primitive than the motion idea of Archytas, might be an attempt to derive these concepts by purely logical means, without introducing motion.

[105] For Speusippus the point is *οἷον τὸ ἕν* (Arist. *Met.* 1085a33); *ὅτι στιγμὴ ἐν γραμμῇ καὶ μονὰς ἐν ἀριθμῷ· ἑκάτερον γὰρ ἀρχή* (*Top.* 108b26; cf. above, ch. I 1, n. 40). Quite similarly, Sext. Emp. *Math.* 10.278 has *τὸ σημεῖον κατὰ τὸν τῆς μονάδος λόγον τετάχθαι.*

[106] Fr. 4 Lang = *Th. ar.* 82.10ff (= DK 44A13, KR 319); the sentence cited is at *Th. ar.* 84.10f. KR, p. 255, have "little doubt" that Speusippus is reproducing genuine and early Pythagorean doctrine, and Guthrie agrees (I 260f). Saffrey would prefer to brand the fragment a neo-Pythagorean forgery, but there are no arguments for this—not even the absence of the title from the list of Speusippus' writings in D.L. 4.4, for this is expressly labeled as incomplete.

[107] 65f.

[108] Cf. above, n. 62.

cognition, which in Platonism is set up as parallel to the hierarchy of geometrical solids, and also with the numbers from 1 to 4 (1 *νοῦς*, 2 *ἐπιστήμη*, 3 *δόξα*, 4 *αἴσθησις*),[109] likewise makes its appearance as the teaching of Pythagoras;[110] the Platonic system, in all its aspects, is being passed off as Pythagorean. Here too, along with partial agreement, we find a contradiction to the statement of Aristotle that the Pythagoreans equated *δόξα* with the number 2.[111] The relationship between Xenocrates and the series of types of cognition, and a certain kinship between them and the *Timaeus*, have already been mentioned.[112]

But we must go over the same ground once more. "Pythagoras" taught, according to Aëtius,[113] that earth came to be from the cube, fire from the pyramid (or tetrahedron), air from the octahedron, water from the icosahedron, and finally, from the dodecahedron, the "sphere of the All." The agreement with the *Timaeus*, extending to actual verbal coincidence, is obvious.[114] Less striking, but no less clear, is the contradiction of what Aristotle says about the Pythagoreans: "They have not said anything at all about fire or earth or the other material things of this sort, because, I suppose, they did not have anything distinctive to say about perceptible things."[115] Here we may add an item

[109] Above, ch. I 1, nn. 48–54,

[110] Aët. 1.3.8; cf. 4.4.5 (Theodoret 5.20), Theo Sm. 97.24ff (the eighth "tetractys"), ps.-Archytas p. 38.12 Thesleff = Iam. *Comm. math. sc.* 35.27ff. From the fact that the explanations in Aristotle (*De an.* 404b21ff) and in Aëtius are different, Kucharski wishes to conclude that they have a common source. In fact, Aëtius is hardly likely to be dependent on *De anima*, but the common source is not therefore a Pythagorean writing, but the exposition of Platonists.

[111] Alex. *Met.* 39.13ff = Arist. fr. 203: *νοῦν δὲ καὶ οὐσίαν ἔλεγον τὸ ἓν . . . δόξαν δὲ τὰ δύο διὰ τὸ ἐπ' ἄμφω μεταβλητὴν εἶναι.*

[112] Above, ch. I 1, nn. 52, 57.

[113] Aët. 2.6.5, and Ach. *Is.* 6 p. 37.29ff Maass, probably from Posidonius. Cf. the further material collected by Sachs 9ff.—Sext. Emp. *Math.* 10.283, *Hypomn.* 25, and Anon. Phot. 439b17 only enumerate the elements, without mentioning the regular solids—doubtless the result of abbreviation in the excerpting process. On Procl. *In Eucl.* 65.15, which is usually attributed to Eudemus (fr. 133 W.), see below, ch. VI 1.

[114] On the dodecahedron, *Tim.* 55c: *ἐπὶ τὸ πᾶν ὁ θεὸς αὐτῇ κατεχρήσατο.* Aët. 2.6.5: *ἐκ δὲ τοῦ δωδεκαέδρου τὴν τοῦ παντὸς σφαῖραν.* On the problem of the fifth element cf. Guthrie I 270ff, P. Moraux, *RE* XXIV 1171–1263.

[115] *Met.* 990a16. Because the Pythagoreans do not distinguish between the perceptible and the mathematical, in Aristotle's view they are incapable of explaining the peculiar characteristics of the physical world. The ironic explanation, in the sentence quoted, is Aristotle's, and this in itself shows that the Pythagoreans did not have a doctrine about the elements. The *Timaeus* attempts to explain the properties of the elements (the sharp, pointed pyramid is fire, the stable cube is earth, the nearly spherical icosahedron is water, and so on). Raven (*PyEl* 152ff) tries to neutralize Aristotle's evidence with *Cael.* 293a30ff, where fire is characterized as *τιμιώτατον*, which he thinks is an indication that there did after all exist a Pythagorean doctrine of the elements. Aside from the fact, however, that such a value judgment is not a "theory" in the sense required by the search for *ἀρχαί*, Aristotle separates this passage from the preceding Pythagorean material with the words *πολλοῖς δ' ἂν καὶ ἑτέρως συνδόξειεν.* He means the Platonists (cf. below, ch. IV 2, n. 16).

from the scholia to Euclid, possibly from Eudemus, whose significance was seen by Eva Sachs.[116] The Pythagoreans, it says, treated of only three regular solids—pyramid, cube, and dodecahedron, and Theaetetus was the first to add the octahedron and icosahedron. But again Speusippus, contra Aristotle, treated the derivation of the elements from the regular solids as Pythagorean; he wrote in his book *On Pythagorean Numbers*, ". . . and about the five shapes, which are given to the cosmic elements" (*Th. ar.* 82.17 DeFalco).

The keystone of the argument is the declaration of Aëtius, "Pythagoras says that the cosmos is a created thing (*γενητόν*) in the realm of thought but not in that of time (*κατ' ἐπίνοιαν . . . οὐ κατὰ χρόνον*)."[117] "Pythagoras" is here taking a position in the controversy which developed over interpretation of the *Timaeus*, whether the creation of the world by the Demiurge is to be understood literally. Speusippus and Xenocrates asserted that it was merely a mode of exposition *διδασκαλίας χάριν*, an essay in mental construction (*ἐπινοίᾳ*).[118] It is this that gives point to Aristotle's polemic, "There ought to be no difference of opinion as to whether or not the Pythagoreans posit generation (*γένεσις*) or not; for they say clearly . . ."[119] Plainly, Speusippus and Xenocrates gave out their interpretation of the *Timaeus* as Pythagorean doctrine, and Aristotle is refuting them with the actual words of a Pythagorean document. Aristotle already presupposes and rejects the conception of Pythagoreanism that makes its appearance in the later doxographical tradition. It is that of Speusippus, Xenocrates, and Heraclides, which amalgamates interpretation of the *Timaeus* with the Platonic "system of derivation."

There remain to be discussed a number of reports in the doxography, most of them concerning less vital questions, that cannot be placed in direct connection with the evidence of Aristotle; the latter did not have occasion, of course, to mention every detail of Pythagorean teaching. For these additional testimonies too, however, it can be shown that they go back to Plato's disciples, or, insofar as they preserve older material, were transmitted by them.

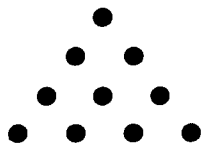
[116] Schol. Eucl. 654.3ff; cf. below, ch. VI 3 (not in DK); the *Suda* s.v. Theaetetus; Sachs 76ff. Timpanaro Cardini's allegation (31) that Sachs brought forward against the Pythagorean tradition "nient' altro che affermazioni arbitrarie" is astonishing.—Diels included Aët. 2.6.5 among the Philolaus materials, as 44A15, comparing B12; contra, Sachs 43ff; cf. below, ch. III 2, n. 183.

[117] Aët. 2.4.1; cf. Sext. Emp. *Math.* 10.255: *κατ' ἐπίνοιαν . . .*

[118] Arist. *Cael.* 279b32, with the ancient comments and scholia, assembled by Lang as Speusippus fr. 54; Xenocrates frr. 33, 54 H.

[119] *Met.* 1091a13ff; above, ch. I 2, n. 41.

The kernel of Pythagorean wisdom is the "tetractys," or "four-group," made up of the numbers, 1, 2, 3, 4, which add up to 10. They are represented in a pebble figure, in the form of the "perfect triangle;"



and the available sources, from Posidonius on, show how these four numbers contain not only the basic intervals—fourth, fifth, octave, and double octave—but also, according to the Platonic pattern, point, line, plane, and solid.[120] The harmonic ratios, the "perfection" of 10, and the role of the pebble figures are all part of what Aristotle attributes to the Pythagoreans; but the derivation of spatial shapes is in implicit contradiction to what he says.[121] Speusippus' book *On Pythagorean Numbers*, on the other hand, presented this series; and half of his presentation was devoted to the number 10, with emphasis on the asseveration which recurs again and again in the literature on the tetractys, that all people, Greeks and barbarians alike, count to 10 and then return to 1.[122] But there is a relation to Xenocrates as well. It is frequently mentioned that the Pythagoreans, in their oath by the tetractys called it the "fount of ever flowing nature (*παγὰν ἀενάου φύσεως*)."[123] But Xenocrates called matter *ἀέναος*,[124] and, though this may also hold an allusion to an expression of Plato's, it is tempting to believe that the verse on the tetractys was known to him.[125] For another verse, too, is commonly cited in the literature along with this one: *ἀριθμῷ δέ τε πάντ' ἐπέοικεν*[126] and Themistius cites this very line in a passage where he refers to Xenocrates' book *On Nature*.[127] Though a good many bits of Themistius' own wording may have worked their way in, alongside the paraphrase of Aristotle, still he was well informed about Xenocrates.[128]

Thus from an element of Pythagorean tradition which at first seems to contain an ancient, pre-Platonic kernel, a clear trail leads us back to Speusippus, and a somewhat less clear one to Xenocrates. It is the latter to whom the connection seems closer in the matter of the *daimones*. It is quite probable that Pythagoras himself spoke of *δαίμονες*, but the process of transition from religious experience and teaching to systematic philosophy is obscure. In any case, the expressions in the later tradition, about *daimones* as souls or as some kind of intermediate being, were the work of Platonists. Aëtius (1.8.2) names Thales, Pythagoras, Plato, and the Stoics as advocates of doctrines about *daimones*. Plutarch is more cautious: in two parallel passages he names, first, "Plato, Pythagoras, Xenocrates, and Chrysippus" (*De Is. et Os.* 25.360d), and then again only "Plato, Xenocrates, and Chrysippus" (*De def. or.* 17.419a). That Pythagoras is not named first, in despite of chronology, or even may be left out, suggests the conjecture that what is meant is really "Pythagoras as cited by Xenocrates."[129] If, as Aristotle says (fr. 193), the Pythagoreans considered it the most natural thing in the world for a person to "see" a *daimon*, and if they also saw the myriads of souls dancing as motes in a sunbeam,[130] this is a long way from the Platonic concept of immateriality. There is also the graded series god-*daimon*-hero-man, which was attributed to the

[120] Sext. Emp. *Math.* 7.94–100, with the parallels (above, nn. 8, 12–14), Aët. 1.3.8, Theo Sm. 87.5ff, 93.17ff, etc. The most detailed treatment is that of Delatte, *Litt.* 249–268. Sometimes it is also called *τέταρτος ἀριθμός* (cf. Theiler, *Gnomon* 28 [1956] 284f.) Lucian (*V. auct.* 4) represents Pythagoras as saying, "What you suppose is four is really ten, and a perfect triangle, and our Oath." Cf. also Burnet, *EGP* 102f; below, ch. II 4.

[121] Above, n. 92.

[122] *Th. ar.* 83.7: *ὀρθῶς τε καὶ κατὰ φύσιν εἰς τοῦτον καταντῶμεν παντοίως ἀριθμοῦντες Ἕλληνές τε κεἰ πάντες ἄνθρωποι*, (Arist.) *Pr.* 15.3.910b23ff: *διὰ τί πάντες ἄνθρωποι, καὶ βάρβαροι καὶ Ἕλληνες, εἰς τὰ δέκα καταριθμοῦσι . . . εἶτα πάλιν ἀναδιπλοῦσιν*; Sext. Emp. *Math.* 4.3: *ἐπ' αὐτὸν φθάσαντες πάλιν ἀναλύομεν ἐπὶ τὴν μονάδα* (at 7.94 the corresponding sentence has dropped out); Philo *Op.* 47: *περὶ ὃν ὡς καμπτῆρα εἰλοῦνται καὶ ἀνακάμπτουσι*. Quite similar is Anat. p. 39 = *Th. ar.* 86.3; cf. Theo Sm. 93.19, 99.17, Anon. Phot. 439a5, Hippol. *Ref.* 6.23.3. There is a remarkably close resemblance between the passage from the *Problemata* and the following of Aët. 1.3.8: *μέχρι γὰρ τῶν δέκα πάντες Ἕλληνες, πάντες βάρβαροι ἀριθμοῦσιν, ἐφ' ἃ ἐλθόντες πάλιν ἀναποδοῦσιν ἐπὶ τὴν μονάδα*. The common source is obviously Speusippus.

[123] Below, ch. II 4.

[124] Fr. 28 H. Aët 1.3.23.

[125] For this combination, see Zeller II 1.1014, n. 3; Cherniss, *Plato* 484f, *Gnomon* 1959, 41; against it, Heinze 14. Delatte is dubious, *Litt.* 253. Pl. *Leg.* 966e. Cf. also the doxographical reports on Pythagoras' teaching about *ὕλη ῥευστή*; above, n. 35.

[126] Sext. Emp. *Math.* 7.94, 4.2, Theo Sm. 99.16, Plut. *De an. procr.* 1029f, Iam. *VP* 162, etc. Delatte, *Litt.* 14f, would like to show that the line is from an ancient hexameter *Ἱερὸς λόγος*, but the pareomiac (with a catachrestic dependence on *Il.* 22.71 and Tyrtaeus 7.27) may be independent.

[127] Xenocrates fr. 39 H. = Themist. *an.* 11.20ff.

[128] Themistius knew this book of Xenocrates, as is shown by fr. 61 Heinze (Cherniss, *Plato* 399 n. 325, vs. Heinze p. 65). Saffrey (38ff) would like to posit a "neo-Pythagorean revision" of the physics of Xenocrates, for to him "Pythagorean" means "non-Xenocratic." But the connections shown here leave no room for doubt; Xenocrates himself "Pythagorizes."

[129] Heinze 78ff. M. Detienne seeks to recover original Pythagorean material: *REA* 60 (1958) 271–279; *RHR* 155 (1959) 17–32; *Homère, Hésiode et Pythagore* (Brussels, 1962); *La notion de Daimon dans le pythagorisme ancien* (Paris, 1963); on the last, Burkert, *Gnomon* 36 (1964) 563–567; cf. below, ch. II 2.

[130] *De an.* 404a17ff; "a piece of Pythagorean superstition," says Zeller I 561, and Cherniss agrees (*Pres.* 291 n. 6). But cf. Democr. ap. Arist. *De an.* 404a3ff (DK 67A28). The *Hypommemata* have here preserved ancient material (32): *εἶναί τε πάντα τὸν ἀέρα ψυχῶν ἁμπλεων*.

Pythagoreans from the time of Aristoxenus.[131] There is no direct way of checking this; nevertheless, this hierarchical series is common in Plato,[132] while Aristotle ascribes to the Pythagoreans a three-term series, with Pythagoras himself as middle member between god and man (fr. 192).

In addition to such beliefs about souls and *daimones*, the doxographical tradition assigns to Pythagoras or the Pythagoreans the threefold division of the soul into *νοῦς*, *θυμός*, *ἐπιθυμία*,[133] and the authority of Posidonius is cited for this.[134] But it is apparent by now that this is no isolated phenomenon; in other respects as well Posidonius appears as a link in the Pythagoras tradition.[135] Now, this tripartition of the soul is closely related to the theory of the "three ways of life," which Heraclides Ponticus cites as coming from Pythagoras.[136] It is possible that the soul doctrine insinuated its way into the tradition by the same route. Anyway, here too Platonic and Pythagorean material is seen as a unity, and Aristotle's reports lead in a different direction.[137]

The *Hypomnemata* present a somewhat different division of the psyche (30) "into *νοῦς* and *φρένες* and *θυμός*. *νοῦς* and *θυμός* are also present in the other animals, but *φρένες* only in human beings." Von Fritz has tried to show, on the basis of a thorough semantic investigation, that in this respect the author is reproducing a completely un-Platonic and therefore pre-Platonic doctrine.[138] From Empedocles on, *φρονεῖν* and *νοεῖν* were generally equated, and in all post-Platonic philosophy *νοῦς* is a specific criterion of humanity; therefore a theory which

[131] D.L. 8.23, Iam. *VP* 100, 144, 175, Por. *VP* 38; cf. Aristox. fr. 34, Iam. *VP* 37, Zaleucus ap. Stob. 4.2.19, *Carm. aur.* 1–4.

[132] Pl. *Crat.* 397c-e, *Rep.* 392a, 427b, *Leg.* 717b, 738d, 801e, 818c, 910a, *Epin.* 984d.—Inscriptions from Dodona (Collitz II 1582, 1566, 1585b), apparently from Epirus, Corcyra, and Aetolia, point toward a ritual background.

[133] Posidonius in Galen *Plac. Hipp. et Plat.* 4.7, 5.6 (V 425, 478 K.), with the cautious formulation *αὐτοῦ μὲν τοῦ Πυθαγόρου συγγράμματος οὐδενὸς εἰς ἡμᾶς διασῳζομένου, τεκμαιρόμενος δὲ ἐξ ὧν ἔνιοι τῶν μαθητῶν αὐτοῦ γεγράφασιν.* Aët. 4.4.1, cf. 4.7.5 ("Pythagoras and Plato"). It is common in the pseudo-Pythagorean literature: Archytas fr. 64 Nolle = Iam. ap. Stob. 1.49.34; Aesara (or Aresas) ap. Stob. 1.49.27 = Thesleff, *Texts* pp. 48ff; Metopus ap. Stob. 3.1.115 = Thesleff, *Texts* p. 116; Theages ap. Stob. 3.1.117 = Thesleff, *Texts* p. 190. Cf. also Pollux 2.226.

[134] For this reason the tradition is often defended: Burnet, *EGP* 296 n. 2, Taylor, *Tim.* 263f, 496ff, Cornford, *CQ* 1922, 147. Joly hesitates, pp. 78f, with refs. Philolaus fr. 13 has nothing to do with this (below ch. III 2, nn. 149–157).

[135] Above, nn. 8–22, 74–75.

[136] Frr. 87–88 Wehrli; cf. Burkert, *Hermes* 1960.

[137] Not only the motes in the sunlight, but also the allusion to metempsychosis at *De an.* 407b22 (below, ch. II 3). On *ψυχή* and *ἁρμονία*, see below, ch. III 2, nn. 165–168.

[138] "*Noûs*, *noeîn*, and Their Derivatives in Pre-Socratic Philosophy," *CP* 41 (1946) 33f; similarly Wellmann, *Hermes* 1919, 235ff; Delatte, *Vie* 222.

assigns *νοῦς* to animals as well must go back to the first half of the fifth century B.C. Also, *φρένες*, he says, was not used from the fourth century on.

Still, *φρένες* does occasionally appear in later writings,[139] and it remains arguable that at least a certain kind of *νοῦς* belongs to animals.[140] More important, however: the evidence of the *Hypomnemata* is not isolated. Aëtius tells us (5.20.4), "Pythagoras and Plato say that the souls of the so-called irrational animals have reason too, but they do not act in accordance with reason because of the faulty composition [mixture] of their bodies, and because they do not have the faculty of speech (*τὸ φραστικόν*). We can see this in the case of apes and dogs; for they think (*νοοῦσι*) but do not speak (*φράζουσι*)." What Aëtius says about thinking (*νοεῖν*) and speaking (*φράζειν*) now makes clear the distinction between *νοῦς* and *φρένες* in the *Hypomnemata*. There are other ways, too, in which the two sources resemble one another in their treatment of "Pythagorean" psychology.[141] In addition, Aëtius speaks of Pythagoras and Plato in a single breath, in this specific context, so that we cannot single out particular un-Platonic or pre-Platonic elements. Here, too, we have an interpretation of Plato that, like Aristotle, restricts immortality to the highest psychic activities, and therefore divides the intellectual segment of the psyche in the same way as the Stoic differentiation between *λόγος ἐνδιάθετος* and *λόγος προφορικός*.[142] This is not a trace of pre-Platonic Pythagoreanism.

There remain two ancient-sounding reports of Aëtius on doctrines of Pythagoras. The first says that necessity hedges the world about: *ἀνάγκην περικεῖσθαι τῷ κόσμῳ*,[143] and the second is a definition of time

[139] Cf. Festugière *REG* 1945, 43ff; e.g., Chrysippus, *SVF* III p. 27.8; Anon. Lond. 4.14ff, Phld. *Po.* 5.19, *Rhet.* 1.240 S. When the "diaphragm" is misplaced in the head (*Hypomn.* 30), we scarcely get the impression of having the ancient or original version ("correctly" *Hebd.* 6: moon—*φρήν*—mid-point—seat of intelligence).

[140] According to Aët. 4.5.12, Democritus followed Parmenides and Empedocles in equating *νοῦς* and *ψυχή* (cf. Arist. *De an.* 404a27ff). Strato (fr. 107ff Wehrli) equates *νοῦς* and *αἴσθησις*; in such circumstances one could scarcely deny the lower animals any share in *νοῦς*.

[141] Aët. 4.5.10: *Πυθαγόρας τὸ μὲν ζωτικὸν περὶ τὴν καρδίαν, τὸ δὲ λογικὸν καὶ νοερὸν περὶ τὴν κεφαλήν.* Cf. *Hypomn.* 30: *νοῦς* and *φρένες* in the brain, *θυμός* in the heart.—Aët. 4.7.5: *Πυθαγόρας Πλάτων τὸ μὲν λογικὸν ἄφθαρτον, τὸ δὲ ἄλογον φθαρτόν.* Cf. *Hypomn.* 30: *τὸ μὲν φρόνιμον ἀθάνατον τὰ δὲ λοιπὰ θνητά.*

[142] Aëtius cannot be drawing from the *Hypomnemata*, since he would not have found the references to Plato there. The *Hypomnemata* are probably older than the *Vetusta placita* (cf. the Stoic *δυσκρασία*, Aët. 5.20.4).

[143] Aët. 1.25.2; Zeller (I 542 n. 2) is inclined to equate Necessity with the fire that surrounds the world.

as "the sphere of the encompassing (*σφαῖρα τοῦ περιέχοντος*)." This seems to be known to Aristotle, who refers to it in connection with the definition of time abstracted from the *Timaeus*[144] and dismisses its reasoning, "that everything is in time and likewise in the sphere of the all" as "somewhat simple-minded." Indeed the association of the celestial sphere and all-embracing time was widespread in the fifth century[145] and evidently reflects a pre-philosophic way of thinking. It makes its way into Pythagorean doctrine as attested by Aristotle: the cosmos breathes in time, along with the Unlimited;[146] it is still discernible in Aristotle's allusion to the divine *aion* encompassing the world,[147] though transformed by Plato's distinction between *aion* and *chronos* (*Tim.* 37d). Later tradition points to Orpheus for this imaginative concept: Chronos hatching the world egg.[148] And Ananke, too, occurs in Orphic mythology, expressly associated with Chronos surrounding the world.[149] It is possible that Orphic cosmogonies were exercising some influence as early as the fifth century and that they were in closest connection with Pythagoreanism. The way by which

[144] Aët. 1.21.1, Arist. *Phys.* 218a33 (*τὸν χρόνον*) *οἱ μὲν γὰρ τὴν τοῦ ὅλου κίνησιν εἶναί φασιν, οἱ δὲ τὴν σφαῖραν αὐτήν.* For the attribution of the first definition to Plato (*Tim.* 37c-d, 39b-c), cf. 223b21, Theophr. *Phys. op.* fr. 15 (*Dox.* 492), Eudemus fr. 82 W., Aët. 1.22.1, Plot. 3.7.8. Simplicius (*Phys.* 700.16ff, 785.14) thinks the definition of "Pythagoras" is to be found in the *Categories* of the ps.-Archytas (24,15 Thesleff) (*χρόνος*) *κινάσιός τινος ἀριθμὸς ἢ καὶ καθόλω διάστημα τᾶς τῶ παντὸς φύσιος.* Speusippus (fr. 53 L.) and Xenocrates (fr. 40 H.) have different definitions of time.—According to Plutarch (*Quaest. Plat.* 1007b) Pythagoras defined time as *ὅλου ψυχή.* (Cf. the ps.-Archytas definition of time with Speusippus' definition of soul, above, n. 74.)

[145] Hermippus the comic poet, fr. 4 (= Stob. 1.8.36; ca. 430–420 B.C.):

ἐκεῖνός ἐστι στρογγύλος τὴν ὄψιν, ὦ πονηρέ,
ἐντὸς δ' ἔχων περιέρχεται κύκλῳ τὰ πάντ' ἐν αὑτῷ . . .
ὀνομάζεται δ' ἐνιαυτός, ὢν δὲ περιφερὴς τελευτὴν
οὐδεμίαν οὐδ' ἀρχὴν ἔχει . . .

Cf. *Hebd.* 16: "annus autem, in quo omnia circumeunt, habet in se ipso omnia."—The heaven is *χρόνου ποίκιλμα* in Critias B18 (cf. B25, 33f). In Indian mythology too, Kala or Time is described as being the starry heaven (Olerud 135).

[146] Arist. fr. 201 (cf. *Cael.* 279a11). This is not identical with the Pythagoras testimony Aet. 1.21.1: according to these Pythagoreans, the sphere of the universe came to be, but nobody spoke of coming-to-be of time before Plato (Arist. *Phys.* 251b17); and the role of the celestial sphere for measuring time was to change radically through the introduction of a moving earth (ch. IV 3).

[147] *Cael.* 279a18-b3.

[148] Kern, *Orph. frag.* 54, 57, 70.

[149] Kern, *Orph. frag.* 54, 126, 162; Nicomachus *Th. ar.* 81.19: *τὴν Ἀνάγκην οἱ θεολόγοι τῇ τοῦ παντὸς οὐρανοῦ ἐξωτάτῃ ἅντυγι ἐπηχοῦσι.* The Ananke of Parmenides (B 8.30, 10.6) is assigned to the mid-point of the universe (A37). On Orphism and Pythagoreans, above, ch. I 2, nn. 53–56; below, ch. II 3.

these two isolated statements came to be handed down in the doxography as lore of "Pythagoras" is not clear; but there is one obvious guess: the Plato interpretation of the Old Academy played a decisive role. For as the definition of time (*chronos*) is treated by Aristotle in the context of exegesis of the *Timaeus,* so *ananke* belongs to the *Republic,* where the universe is whirling on the knees of Ananke (616c).[150]

Finally, there is the basically important doxographical statement that Pythagoras coined the term "cosmos,"[151] *ἐκ τῆς ἐν αὐτῷ τάξεως* (Aëtius). This makes him the creator of one of the most influential concepts of Greek natural philosophy. The history of the word in pre-Socratic philosophy, as far as we can trace it, speaks, however, decisively against the thesis of such a programmatic invention of the concept at the very beginning of Greek philosophy;[152] we rather see the word gradually developing, from the prosaic "arrangement" of things in parts of the universe to the "ordering" of the whole world. There is, nevertheless, again an obvious guess how the alleged invention came to be attributed to Pythagoras: it fits very well into the anecdote of Heraclides Ponticus which had Pythagoras explaining the word *philosophia*—an invention of Pythagoras which also made its way into all the doxographical handbooks.[153]

Nevertheless, it is *communis opinio* that the Pythagoreans at least, if not Pythagoras himself, played a decisive role in the development of the Greek idea of "cosmos."[154] One always cites Plato, who introduces, against Callicles, the doctrine of "wise men" that friendship and equality hold sway in the world, and that this is why it is called

[150] Clearchus (frr. 3–4 Wehrli) wrote a commentary on the mathematical passages of the *Republic.*

[151] Aët. 2.1.1 = DK 14.21 (*τὴν τῶν ὅλων περιοχήν*), D.L. 8.48 (*τὸν οὐρανόν*); the additional notice about Parmenides relates only to the spherical shape of the earth, as Diels saw, *Dox.* 492); Anon. Phot. 440a27f (*τὸν οὐρανόν*); Achilles *Is.* 5 (*τὸ πᾶν*); Schol. A *Il.* 3.1 (*ἡ τῶν ὅλων τάξις*); Iam. *VP* 162. On the cosmology attributed to "Pythagoras" in the doxography, below, ch. IV 1.

[152] Cf. J. Kerschensteiner, *Kosmos: Quellenkritische Untersuchungen zur vorsokratischen Philosophie* (Munich, 1962); H. Diller, "*Der vorsokratische* Gebrauch von *κόσμος* und *κοσμεῖν,*" in *Festschrift Snell* (Munich, 1956) pp. 47–60; Kirk, *Heraclitus* 311ff (only the sense "arrangement of things" in Heraclitus; contra, Dodds, *Gorgias* 338f, Kerschensteiner 97ff).—Kranz, *ABG* 1955, 13f, traces the word *κόσμος* back to Anaximander, as do Diels, Jaeger, Gigon (*Ursprung* 91), Hölscher (*Hermes* 1953, 264), and Kahn (219ff). Kerschensteiner (29ff) qualifies this.

[153] So Kirk, *Heraclitus* 313.1, Kerschensteiner 229; cf. above, n. 77, and below, ch. IV 1.

[154] M. Pohlenz, *Aus Platos Werdezeit* (Berlin, 1913) 152ff; Kranz, *Philologus* 1938, 439, *ABG* 1955, 35, DK *Nachträge* I 502.17; Dodds, *Gorgias* 337ff; Krämer 142, 233; Guthrie I 206ff.

"cosmos."[155] Here Plato adduces the evidence of geometry, in a manner that cannot be well explained from the dialogue itself, and there is much to be said for the conjecture that Plato is alluding to the close affinity between geometry and the study of first principles which he had learned from Archytas.[156] It is also possible, however, that he was already foreshadowing ideas of his own, later formulated in the *Timaeus*; and the close relationship of this passage of the *Gorgias* to other thinkers of the fifth century, especially Empedocles and Euripides, ought not to be overlooked because of concentration on the Pythagorean allusions.[157] In spite of the distinctly Pythagorean flavor of the

[155] *Gorgias* 507e–508a: "Wise men say, Callicles, that heaven and earth, gods and men, are held together by the principles of sharing, by friendship and order, by self-control and justice; that, my friend, is the reason they call the universe 'cosmos,' and not disorder or licentiousness. Clever though you are, you seem not to have paid enough attention to these matters; it has, in fact, escaped you what a mighty power is exercised, both among men and gods, by geometrical equality. And it is your neglect of geometry which brings about your opinion that one should strive for a share larger than that which other men possess" (tr. W. C. Helmbold).

[156] Cf. above, nn. 98f. Kerschensteiner 222f, Dodds, *Gorgias* 339f, and others understand *ἡ ἰσότης ἡ γεωμετρική* in the sense of the "geometrical proportion" that was considered the principle of aristocratic justice, in contrast with arithmetic equality, the principle of democracy: Pl. *Pol.* 257b, Arist. *EN* 1131b13ff, *Pol.* 1301b29ff; cf. Pl. *Rep.* 558c, *Leg.* 757bc, Isoc. 7.21. Ps.-Archytas 34.3–14 Thesleff has a different scheme, in which the harmonic mean is aristocratic, the geometric democratic, the arithmetic oligarchic. (Cf. Delatte, *Pol.* 95ff, with further references; the spuriousness of the Archytas material is proven even by the use of such a word as *ἀπάθεια*, 33.18 Thesleff). But an "equality" that distributes "more to the greater and less to the lesser" (Pl. *Leg.* 757c) would be scarcely appropriate to refute the *pleonexia* of Callicles; and "geometrical proportion" usually is *γεωμετρικὴ ἀναλογία*, or *γεωμετρικὴ μεσότης*, not simply *γεωμετρικὴ ἰσότης*. Thus "geometrical equality" should be understood in a more general sense, as *ἡ τοῦ ἴσου ἀναλογία* in Archytas A23a—the power of mathematics that governs the world.—Plato, in comparing justice to musical harmony, and therefore to the "harmonic" mean (*Rep.* 443d; Morrison, *CQ* 1958, 213ff), shows that he is not committed to any specific pattern.

[157] Olympiodorus, on the passage, and the scholiast, name "the Pythagoreans and in particular Empedocles." And, to be sure, *Φιλότης* is a cosmic power in Empedocles, "equal in length and breadth" (B17.20); mathematical proportion plays a role (B96, cf. 135). Most similar to the *Gorgias* passage is the exhortation of Jocasta in Eur. *Phoen.* 535ff about the "equality" that binds man to man, as it governs the course of days and years. The same kind of protreptic is presupposed by Soph. *Aj.* 670ff. But cf. also (after Diog. Apollon.), Hippoc. *Flat.* 5 (DK 64C2) and Pl. *Crat.* 412d, on the *διεξιόν*.—A reference to Pythagoreans is seen in the word *σοφοί* (Dodds, *Gorgias* 297), in comparison with *Meno* 81a, *Rep.* 583b (Adam II 379, ad loc.), and *Gorg.* 493a (below, ch. III 2, n. 48). But the word is much more widely used. Plato seems to apply the word *σοφός* to Empedocles (*Lys.* 214b), to Heracliteans (*Lys.* 215d), and to Sophistic rhetors (*Lys.* 222e, *Meno* 75c, *Phdr.* 266c. *Symp.* 185c). Cf. also *Crat.* 402a (Heraclitus), *Phdr.* 235c (Anacreon), *Rep.* 365c (Simonides), *Tht.* 157b (Heracliteans?). R. S. Bluck, too, doubts that *σοφοί* is used here of the Pythagoreans (*CR* 11 [1961] 30). The ancient tradition connects with Anaxagoras the well-known verses of Euripides which say that the contemplation of the "unchanging cosmos" restrains from evil deeds (Eur. fr. 910 = Anaxagoras A30); and even as early as Anaximander (fr. 1) there is a justice involved in cosmic process.

passage, the mention of geometry certainly does not point back to the oldest stratum of Pythagorean mathematics, in which arithmetic was central.[158] Thus in no instance does the doxography on Pythagoras the philosopher bring us back to solid ground; it all seems to derive from the Platonic exegesis of the Old Academy.

Zeller's solution of the problem of the Pythagorean tradition must, then, be modified. It is not true that we can make a chronological distinction between the oldest and therefore most reliable accounts, those of Aristotle, and a later accretion which cannot be checked and is therefore suspect; nor is it true that neo-Pythagoreanism, no matter how early one makes it begin, is to be held accountable for the changing course of the tradition. In the earliest evidence available two conceptions of Pythagorean philosophy are in sharp contrast with each other. The non-Aristotelian tradition is even the older of the two, insofar as it can be traced to Speusippus. This tradition from the Old Academy equates the Pythagorean philosophy with the doctrines of the *Timaeus* and with the Platonic number philosophy. The highest principles, immaterial, are the One and the Indefinite Dyad; from them come the pure, incorporeal numbers; the numbers produce the pure geometrical shapes, line, plane, and solid, as well as the perceptual functions of the "Living Creature itself"; from the mathematical regularity of the regular polyhedra come the elements and therewith the multifariousness of the empirical world; and this whole process takes place, in this order, only in thought, which traces back changeless Being to its ultimate principles and understands it by means of them.

In each of these points the exposition of Aristotle offers contradiction. The Pythagoreans do not think of separating the numbers from the physical world, but stay in the realm of the perceptible; the numbers are spatially extended shapes, and things "are" numbers; order, correspondence, harmony within the empirical world are comprehensible in terms of number, which is the key to understanding and shows forth its power in divers ways. The world, which is single, has come to be, as harmony, out of Limit and the Unlimited, and its articulation expresses the perfection of number. There are hints of mythical ways of thought, and obviously there is little logical or systematic consistency.

That in fact the interpretation of the Old Academy does form part of the background for Aristotle is shown by his clearly polemical form

[158] Below, ch. VI 2.

of expression: there ought to be no difference of opinion as to whether or not they attributed generation to the cosmic numbers.[159] Aristotle at times, to be sure, shows that he has roots in the other tradition. In the *De caelo* he joins in the Pythagorean number juggling,[160] in his *On Philosophy* he follows right along in the Pythagorean tracks in explaining the perfection of musical harmony, taking his departure from ἄπειρος and περαίνουσα φύσις;[161] in the first book of the *Metaphysics* he treats Plato and Pythagoreans as quite closely connected, whereas in books *M* and *N* he scarcely mentions anything but their disagreements.[162]

In consideration of these matters, the question of a possible Platonic "early period" in Aristotle's thought is bound to rise. The *De caelo* and the first book of the *Metaphysics* are thought to be early works;[163] it is possible that in his understanding of the Pythagorean philosophy, as well as in other matters, Aristotle only freed himself gradually from the picture of it drawn in the Old Academy. But, given the complicated and controversial character of the problem of Aristotle's philosophical development, this cannot be anything but a conjecture. It may help us, however to understand the surprising fragment 207, without taking refuge in athetesis: "In his writings on Archytas, Aristotle says that Pythagoras too called matter 'other,' because it is in flux (ῥευστήν) and always becoming 'other' than it was."[164] This obviously has connections with the "matter in flux (ὕλη ῥευστή)" which according

[159] *Met.* 1091a13ff.

[160] *Cael.* 268a10.

[161] Fr. 47 = Plut. *Mus.* 24.1139f; attribution to *On Philosophy* (fr. 25 Walzer) by E. Bignone, *Ann. d. Scuola Norm. Sup. d. Pisa* Ser. II 2 (1933) 287, *L'Aristotele perduto* II (Florence, 1936) 361f. Mondolfo (ZM 373) thought it might be attributed to the book on the Pythagoreans, but he can scarcely be right, since that book contained criticism as well as exposition (fr. 205). Ross (*Aristotelis fragmenta selecta*, p. 93) accepted the false conjecture ἀρτίου καὶ περισσοῦ for ἀπείρου καὶ περαινούσης . . . φύσεως; cf. below, ch. III 2.

[162] We cannot here go into the question of the attempt of Cherniss to reconstruct a common original for *Metaphysics A* and *M* (*Plato* 189ff).

[163] Jaeger, *Arist.* 175ff, 316ff. For a brief survey of the later research, cf. De Vogel *GP* II 10–19.I. Düring, *Aristoteles* (1966) 254–270, assigns *A* and *MN* to about the same period.

[164] Fr. 207 = Damasc. *Princ.* 2.172.16ff. Because of its inconsistency with Aristotle's other statements, the authenticity of this fragment was denied by Zeller I 470 n. 3, Cherniss, *Pres.* 17 n. 68, De Vogel, *Pythagoras* 214ff. Rostagni, whose reconstructions of Pythagoreanism follow Platonic paths, defended its genuineness (*Verbo* 43.1), citing *Met.* 1087b26, whose contrast of "other" and "one" is attributed by ps.-Alexander 798.23 to Pythagoreans (cf. Ross, *Met.* II 471). But, since the question is that of the characterizations of matter (ὕλη), Aristotle cannot be speaking of the Pythagoreans (cf. above, ch. I 2, n. 95). What we have is exegesis of Plato (cf. esp. *Parm.* 157b).

to the doxographers was part of the doctrine of Pythagoras and Plato.[165] If the doxography goes back, in general, to the Old Academy, it is possible that Aristotle too, either in an early work, or in an exoteric discussion, perhaps in a looser, dialogue form, adopted the Pythagoras of his colleagues.[166] Maybe he was attempting a philosophical interpretation of a traditionary pronouncement of Pythagoras, in which the word "other" (ἄλλο) occurred.

But if the tradition rejected by Zeller proves to be that of the Old Academy,[167] it is by no means thereby rehabilitated. It would be rash to assume that if we merely accepted tradition the problem would be solved. For what we have is not a unitary tradition, but irreconcilable contradiction, and one cannot endorse one side without rejecting the other. Scholarship must decide between the Platonic and the Aristotelian line, for only one of them can be historically correct.

It is natural to try to settle the matter by use of the idea of development. The accounts of Aristotle have a decidedly antique air; they seem "pre-Socratic,"[168] while the doxography stemming from the Platonists presupposes a sophisticated use of dialectic. But it is out of the question to suppose that Aristotle is referring to Pythagoreans of the fifth century, the Platonists to contemporaries. Aristotle says explicitly that not Pythagoreans but Plato introduced the notion of the Indefinite Dyad, and Speusippus speaks of the "ancients," Xenocrates and Heraclides of Pythagoras himself. For the same reason we must reject the hypothesis, occasionally considered as a last resort, that Pythagoreans of the fourth century could have taken over the theory of ideas and the dialectic developed in the Academy;[169] Plato's disciples are not talking about younger contemporaries, but about Pythagoras. Either Aristotle arbitrarily reinterpreted Pythagoreanism as primitive

[165] Above, n. 35.

[166] Zeller explained *MM* 1182a11 (cf. 1194a29) as a cruder version of 1078b21 with Πυθαγόρας instead of Πυθαγόρειοι. Dirlmeier defends the authenticity of *MM* (*Aristoteles Werke* 8, Berlin 1958), and it might well transmit early Academic tradition.

[167] Zeller saw clearly the contradiction of the traditions (I 465ff), but did not come to this conclusion, for several reasons: He did not originally perceive the dependency of the doxography on Theophrastus, and this led him to explain away the direct testimony of Theophrastus (above, n. 59). Also, he relied solely on the dialogues for his interpretation of Plato, rejected the reports of Aristotle, and therefore did not see the inner connection of the "system of derivation." Finally, it is only the new fragment of Speusippus that proves the traditions are in contradiction even in the matter of the ἀρχαί, and not only in relation to line–plane–solid and the doctrine of the elements.

[168] Above, ch. I 2.

[169] E.g., Ross, *Met.* II 471 (cf. above, n. 164), Kucharski (above, ch. I 1, n. 49).

speculation, or the tradition of the Old Academy is, from a historian's point of view, fiction.[170]

The decision can scarcely be in doubt: Speusippus, Xenocrates, and Heraclides equate the doctrine of their master Plato, and therewith also their own philosophical positions, with the wisdom of Pythagoras, whereas Aristotle sees both Academic and Pythagorean philosophy from a certain distance, and even from the stance proper to an active polemic.[171] Plato's pupils ignore the intellectual developments of a century and a half, a period of tremendous intellectual turmoil; their picture of Pythagoras is, in the history of thought, an impossibility.

The consequences of this decision must be faced, without further attempts at compromise. The "derivation system" is an achievement of Plato and the Academy, a genuine *transposition platonicienne*[172] of an older, Pythagorean number philosophy, making use of some improvements introduced in the circle of Archytas. The tradition that attributes the "derivation system" to Pythagoreans, or even to Pythagoras himself, is to be taken for what it is: evidence for the Old Academy, for Speusippus, Xenocrates, and Heraclides, but not for historical Pythagoreanism.[173] This involves, as mentioned above, nearly the whole of the post-Aristotelian tradition on Pythagoreanism. At least in the philosophical realm,[174] the only usable evidence for

[170] The contradiction of the two traditions has mostly been overlooked, except by Zeller; it was only the interest devoted to the Platonic number philosophy that brought it clearly into the foreground. To be sure, people have often perceived the connection between the later tradition and Platonism. (On Sext. Emp., above, n. 4; but Raven, *CQ* 1951, 147ff, saw in Sext. Emp. *Math.* 10.248ff at least partially Pythagorean material, perhaps going back to Archytas. On Aëtius, cf. Frank 260 n. 1.) Less clearly seen was the contradiction with Aristotle. Delatte (*Vie* 234) asserts that the *Hypomnemata* show *aucune influence* of Platonic doctrine, in spite of the Indefinite Dyad in section 25; on the other hand Wellmann (*Hermes* 1919, 227) thought he could prove by Aët. 1.3.8 that the Indefinite Dyad was genuine Pythagoreanism. The two testimonies stand or fall together. (Rightly, on *Hypomn.* 25: Wiersma, *Mnemosyne* 1942, 97; Festugière *REG* 1945, 10ff, with refs.) Often, too, the difference is simply ignored, as by Cornford, *PlParm* 3ff.

[171] Similarly, the Platonists always present Parmenides as simply Platonic, while Aristotle emphasizes the Eleatics' "primitive" inability to get beyond sense perception (αἴσθησις) (DK 28A24–25).

[172] A phrase coined by A. Diès, *Autour de Platon* (Paris, 1927) 400ff.

[173] Throughout antiquity, and beyond, the interpretation of Pythagorean doctrines offered by Plato and his disciples held the field; but, though the stream that was later accepted as Pythagoreanism came from Plato, one ought not to designate Plato and his pupils as Pythagoreans. They are thinkers sui generis, and later "Pythagoreanism" is very largely a *dimidiatus Plato*. This book comprehends as Pythagoreanism that which was not "transposed" by Plato, that which existed before Plato and contemporary with him as the intellectual position and the way of life of persons who called themselves Pythagoreans. (On the origins of "neo-Pythagoreanism" see below, ch. I 4.)

[174] We shall see in chapters IV and VI that the character of the tradition is no different in the fields of mathematics and astronomy.

what is Pythagorean, aside from pre-Platonic testimonies and Aristotle, is in statements that agree with them and are not contaminated with Platonic material.[175]

One might still hesitate to accept a conclusion that leads to disagreement with the overwhelming majority of the sources. But it is supported by Plato himself; the significance of this evidence becomes clear only when it is seen as foreshadowing the conflict of interpretation which arose among Plato's successors.

4. PYTHAGOREANISM IN PLATO AND THE ORIGIN IN PLATONISM OF THE PYTHAGOREAN TRADITION

Modern scholarship on Plato has taught us to see, in the totality of his work and in the individual dialogues, the inner consistency and the necessary unity. The tendency to look for concrete historical relationships, for the origin of particular inspirations, or the target of specific allusions, has receded into the background.[1] Not the least important cause of this is a certain resignation, for it is quite impossible to determine and delimit, from the study of Plato alone, all of his "sources." In Plato, every thesis or argument derives its importance from its truth value, not from its origin in one source or another. Foreign material is no longer foreign, but an integral part of the Platonic structure. This is why the question of the nature of historical Pythagoreanism is perhaps hardest of all to answer, from Plato alone.

Plato's dialogues do not suggest strongly that Pythagoreanism was the determinative influence upon him. Aside from Socrates, three thinkers in particular stand out: Heraclitus, Anaxagoras, and Parmenides.[2] The importance of Parmenides seems to increase in the later dialogues: alongside the one *Timaeus* stand *Parmenides*, *Sophist*, and *Politicus*. All the same, the few references to Pythagoreanism are of special relevance in their Platonic context.

Pythagoras is named in a single passage in the *Republic*, in the final reckoning with Homer. Has Homer, Socrates asks, earned the laurels of a lawgiver like Lycurgus, Charondas, or Solon, or even

[175] This result is decisive in the case of the Philolaus fragments (ch. III, below). To be sure, bits of genuine Pythagoreanism make their appearance in the Platonists' tradition, as with the tetractys (above, n. 120). But in each case authenticity must be proven.

[1] Cf. Diès, Ed. of *Philèbe* xxii; Wilpert, *Phileb.* 576.

[2] Also Empedocles and the Sophists. It is true that the "stranger from Elea" was in later times called a Pythagorean (Clem. Al. *Strom.* 1.48.2).

speculation, or the tradition of the Old Academy is, from a historian's point of view, fiction.[170]

The decision can scarcely be in doubt: Speusippus, Xenocrates, and Heraclides equate the doctrine of their master Plato, and therewith also their own philosophical positions, with the wisdom of Pythagoras, whereas Aristotle sees both Academic and Pythagorean philosophy from a certain distance, and even from the stance proper to an active polemic.[171] Plato's pupils ignore the intellectual developments of a century and a half, a period of tremendous intellectual turmoil; their picture of Pythagoras is, in the history of thought, an impossibility.

The consequences of this decision must be faced, without further attempts at compromise. The "derivation system" is an achievement of Plato and the Academy, a genuine *transposition platonicienne*[172] of an older, Pythagorean number philosophy, making use of some improvements introduced in the circle of Archytas. The tradition that attributes the "derivation system" to Pythagoreans, or even to Pythagoras himself, is to be taken for what it is: evidence for the Old Academy, for Speusippus, Xenocrates, and Heraclides, but not for historical Pythagoreanism.[173] This involves, as mentioned above, nearly the whole of the post-Aristotelian tradition on Pythagoreanism. At least in the philosophical realm,[174] the only usable evidence for

[170] The contradiction of the two traditions has mostly been overlooked, except by Zeller; it was only the interest devoted to the Platonic number philosophy that brought it clearly into the foreground. To be sure, people have often perceived the connection between the later tradition and Platonism. (On Sext. Emp., above, n. 4; but Raven, *CQ* 1951, 147ff, saw in Sext. Emp. *Math.* 10.248ff at least partially Pythagorean material, perhaps going back to Archytas. On Aëtius, cf. Frank 260 n. 1.) Less clearly seen was the contradiction with Aristotle. Delatte (*Vie* 234) asserts that the *Hypomnemata* show *aucune influence* of Platonic doctrine, in spite of the Indefinite Dyad in section 25; on the other hand Wellmann (*Hermes* 1919, 227) thought he could prove by Aët. 1.3.8 that the Indefinite Dyad was genuine Pythagoreanism. The two testimonies stand or fall together. (Rightly, on *Hypomn.* 25: Wiersma, *Mnemosyne* 1942, 97; Festugière *REG* 1945, 10ff, with refs.) Often, too, the difference is simply ignored, as by Cornford, *PlParm* 3ff.

[171] Similarly, the Platonists always present Parmenides as simply Platonic, while Aristotle emphasizes the Eleatics' "primitive" inability to get beyond sense perception (αἴσθησις) (DK 28A24–25).

[172] A phrase coined by A. Diès, *Autour de Platon* (Paris, 1927) 400ff.

[173] Throughout antiquity, and beyond, the interpretation of Pythagorean doctrines offered by Plato and his disciples held the field; but, though the stream that was later accepted as Pythagoreanism came from Plato, one ought not to designate Plato and his pupils as Pythagoreans. They are thinkers sui generis, and later "Pythagoreanism" is very largely a *dimidiatus Plato*. This book comprehends as Pythagoreanism that which was not "transposed" by Plato, that which existed before Plato and contemporary with him as the intellectual position and the way of life of persons who called themselves Pythagoreans. (On the origins of "neo-Pythagoreanism" see below, ch. I 4.)

[174] We shall see in chapters IV and VI that the character of the tradition is no different in the fields of mathematics and astronomy.

what is Pythagorean, aside from pre-Platonic testimonies and Aristotle, is in statements that agree with them and are not contaminated with Platonic material.[175]

One might still hesitate to accept a conclusion that leads to disagreement with the overwhelming majority of the sources. But it is supported by Plato himself; the significance of this evidence becomes clear only when it is seen as foreshadowing the conflict of interpretation which arose among Plato's successors.

4. PYTHAGOREANISM IN PLATO AND THE ORIGIN IN PLATONISM OF THE PYTHAGOREAN TRADITION

Modern scholarship on Plato has taught us to see, in the totality of his work and in the individual dialogues, the inner consistency and the necessary unity. The tendency to look for concrete historical relationships, for the origin of particular inspirations, or the target of specific allusions, has receded into the background.[1] Not the least important cause of this is a certain resignation, for it is quite impossible to determine and delimit, from the study of Plato alone, all of his "sources." In Plato, every thesis or argument derives its importance from its truth value, not from its origin in one source or another. Foreign material is no longer foreign, but an integral part of the Platonic structure. This is why the question of the nature of historical Pythagoreanism is perhaps hardest of all to answer, from Plato alone.

Plato's dialogues do not suggest strongly that Pythagoreanism was the determinative influence upon him. Aside from Socrates, three thinkers in particular stand out: Heraclitus, Anaxagoras, and Parmenides.[2] The importance of Parmenides seems to increase in the later dialogues: alongside the one *Timaeus* stand *Parmenides*, *Sophist*, and *Politicus*. All the same, the few references to Pythagoreanism are of special relevance in their Platonic context.

Pythagoras is named in a single passage in the *Republic*, in the final reckoning with Homer. Has Homer, Socrates asks, earned the laurels of a lawgiver like Lycurgus, Charondas, or Solon, or even

[175] This result is decisive in the case of the Philolaus fragments (ch. III, below). To be sure, bits of genuine Pythagoreanism make their appearance in the Platonists' tradition, as with the tetractys (above, n. 120). But in each case authenticity must be proven.

[1] Cf. Diès, Ed. of *Philèbe* xxii; Wilpert, *Phileb.* 576.

[2] Also Empedocles and the Sophists. It is true that the "stranger from Elea" was in later times called a Pythagorean (Clem. Al. *Strom.* 1.48.2).

those of a practical adviser like Thales or Anacharsis? Or (600 a-b),

> if not in public life, is Homer said in private life to have been, during his lifetime, influential in the education of any persons who cherished him and his association with them and passed on to their successors some kind of "Homeric way of life," as did Pythagoras? He was greatly loved, in this way, and his followers even to the present day speak of a "Pythagorean way of life," and seem in some way to stand out from the rest of mankind.

What gives this passage its importance is its connection with Plato's own lot. For him, too, the most desirable career would have been great influence in the polis; after this was denied him, he decided to be, at least "in private life . . . influential in the education" of individuals: he founded the Academy.[3]

In contrast to this "private circle" of Plato, the character Timaeus in Plato's dialogue is shown as one who combines political activity and experience with profound philosophical learning, so that he exemplifies the synthesis of philosopher and statesman which was Plato's highest ideal. To be sure, he is accompanied by Critias and Hermocrates,[4] and it is possible for the hypercritical to point out that he is not characterized *expressis verbis* as a Pythagorean.[5] But for a man from Italy who has been deeply engaged in studying "the nature of the universe" (*περὶ φύσεως τοῦ παντός*, 27a), it is hard to think of any other identification than with the *Ἰταλικοί, καλούμενοι Πυθαγόρειοι*.[6] It can also be inferred from his praise for the lot of the *συνήκοοι* of a great man in the *Laws* (711e) that Plato was inclined to see his ideal of the philosopher-king as realized in the circle of the Pythagoreans; for this *ἅπαξ λεγόμενον* is obviously the Attic version of the Pythagorean *ὁμακόοι* (cf. below, ch. II 4). There is no reason to regard Timaeus as a historical person. Frank speculated that he served as a mask for Archytas, but the fact that his home city is given as Locri Epizephyrii calls Philistion to mind.[7] It has already been mentioned that even Speusippus

[3] The relationship between the Academy and the Pythagorean society has often been emphasized. Boyancé brought out the significance of their common cult of the Muses (*Muses* 249ff). Cf. also Morrison, *CQ* 1958, 211f.

[4] 20c–d.

[5] A. Rivaud, *Timée* (Budé ed., Paris, 1925) 18.

[6] Above, ch. I 2, n. 6; cf. Pl. *Ep.* 7.338c; above, ch. I 1.

[7] Frank 128f, with notes 375, 375a, 376. There is nothing among the testimonia collected in the Timaeus chapter of DK (49) which might not have been derived from Plato. The argument sometimes heard, that the characters of Plato's dialogues, and especially the title characters, are regularly historical, is in all probability not valid for the *Philebus*, and therefore not generally. (Cf. also Wilamowitz, *Platon* II 84, Cornford, *Tim.* 2f.)

and Xenocrates thought of the doctrine of the *Timaeus* as Pythagorean wisdom.[8] All the same, for historical purposes the doctrine of the *Timaeus* can serve as a source for Pythagorean doctrine no more than—or rather, just as little as—the *Parmenides* can for the historical Parmenides;[9] and the theory of the elements can be shown to be non-Pythagorean.[10] The fact, though, that Plato uses an Italiote as the spokesman for his own cosmological scheme, is an indication that in Magna Graecia Plato had at least found an impulsion or an inspiration toward a view of the cosmos that seemed to him significantly different from the system of Anaxagoras.

As with "Pythagoras," Plato names "Pythagoreans" in just one passage. They call music and astronomy "sister sciences," he says, and try to find the numbers that represent musical intervals.[11] What Aristotle says about the "harmony of the spheres" is here given additional depth; rather than criticizing absurdities, Plato sets forth the leading idea. We are also taken beyond Aristotle by the indication that the theory of the numerical nature of the intervals was peculiar to the Pythagoreans and distinguishes them from other, non-Pythagorean musical theorists.

Having come so far, we can confidently point out other Platonic allusions to Pythagoreans, like a passage in the *Cratylus* whose importance has been emphasized by Boyancé.[12] More important, however, is a section of the *Philebus*.

Socrates-Plato attacks the problem of *ἡδονή* by relating it to the more general problem of the "one and many," unity, plurality, and their mutual interpenetration, not only in the objects of experience but in the realm of ideas itself (14d et seq.). An ancient tradition, he says, shows us the way toward solution of the problem:

> There is a gift of the gods—so at least it seems evident to me—which they let fall from their abode; and it was through Prometheus, or

[8] Ch. I 3.

[9] An expression like that of Pl. *Parm.* 162b, *τῷ τε ὄντι τοῦ μὴ εἶναι καὶ τῷ μὴ ὄντι τοῦ εἶναι μέτεστι*, is diametrically opposed to the doctrine of the historical Parmenides.

[10] Above, ch. I 3, nn. 115–116. At *Tim.* 48b (cf. 53c), the derivation of the elements is introduced as something completely new, which has never previously been discussed.

[11] *Rep.* 530d and 531b–c; below, ch. V.

[12] *Crat.* 405c; Boyancé, *Songe* 97 n. 4, *Muses* 101, *REG* 1941, 147ff. (Though the reconstruction, from the *Cratylus*, of a Pythagorean "doctrine d'Euthyphron" goes too far. Here again the question rises whether the "Pythagoreanism" so disclosed, which is supposed to underlie nearly all the etymologies of the *Cratylus*, is not rather Platonism. Plato is playing, in a "Pythagorizing" vein, with various preexisting forms.)

> one like him, that it reached mankind, together with a fire exceeding bright. The men of old, who were better than ourselves and dwelt nearer the gods, passed on this gift in the form of a saying: all things (so it ran) that are ever said to be consist of a one and a many, and have in their nature a conjunction of Limit and Unlimitedness . . . (16c; tr. Hackforth).

Our task is, he says, not to proceed immediately from the One to the Many but to comprehend, stage by stage, the numerical structure that lies between the One and the Unlimited—and herein lies the difference between dialectics and eristic—just as the grammarian knows the number and nature of the sounds which, in the unitary realm of language, determine the multiplicity of linguistic expression, or as the musician becomes master of the infinite realm of tones by his knowledge of the limited number of the intervals. After the special problem of the *Philebus* is then once more formulated, the ontology based on the opposition of πέρας and ἀπειρία is used profitably in its solution.[13]

In antiquity Syrianus, Proclus, and Damascius saw a connection between the Limit-Unlimited pair of this passage and Pythagoreanism, and specifically with the fragments attributed to Philolaus.[14] In general, modern scholars have assumed an almost self-evident Pythagorean origin for this material,[15] although Frank[16] defended the view that this was not a borrowing at all, but a purely Platonic argument, provided, in Plato's way, with a poetic-mythical garb. Porphyry, in fact, explained the *Philebus* by reference to Aristotle's *On the Good*.[17]

Now, it can be shown from the reports of Aristotle himself that the doctrines of the Pythagoreans on Limit and Unlimited cannot have been developed out of Platonism, as Frank thought.[18] But aside from that, we can find a much stronger argument in the *Philebus* itself. In the exposition of the example from music, "the men of old," οἱ πρόσθεν, are mentioned again. They recognized the number and kind of the

[13] Cf. Wilpert, *Phileb.*; R. Hackforth, *Plato's Examination of Pleasure: A Translation of the Philebus, with Introd. and Comm.* (Cambridge, 1958); Taylor, *Phileb.* There is a close relation between *Philebus* 14d et seq. and *Parm.* 158b et seq.; cf. Cornford, *PlParm* 213.

[14] Syrian. *Met.* 9.37ff, Procl. *In Tim.* I 84.4, 176.29, II 168.29, *Theol. Pl.* 1.5, 3.7, Damasc. *Princ.* I 101.3 R.

[15] Zeller I 457 n. 1, 480.1; Ueberweg–Praechter 306; Taylor, *Phileb.* 52ff; Diès, *Philèbe* xxii (with hesitation); Raven, *PyEl* 180ff; Hackforth, *Phileb.* 20f.

[16] 304. Similarly, Bollinger 74. Mondolfo's citation of individual occurrences of the words πέρας and ἄπειρον in pre-Socratic philosophy (Zeller–Mondolfo 378f) does not constitute an adequate refutation of this. It would be conceivable that Plato himself followed hints from a number of different predecessors.

[17] Simpl. *Phys.* 453.30ff (Arist. fr. 28).

[18] Above, ch. I 2.

intervals which "we have learnt, conformably to the teaching of the men of old days who discerned them, to call 'scales' (ἁρμονίας)" (17d). They discovered in the bodily movements of the performers a similar organization, "that must, we are told, be numerically determined and be called 'figures' and 'measures,'" and advanced the view that "this is always the right way to deal with the one-and-many problem." Thus these musical theorists reach out beyond the boundaries of their specialty,[19] adopting a postulate that takes in everything, and the latter is identical with that of the "men of old," previously formulated (16c-d). Now, in the *Republic*,[20] Plato distinguishes two tendencies in musical theory, and decides that only one of them, that of the Pythagoreans, is worth serious attention, because they measure musical intervals by numerical proportion. In the *Philebus*, Plato is at first less precise: τὰ διαστήματα ὁπόσα ἐστὶ τὸν ἀριθμὸν τῆς φωνῆς . . . καὶ ὁποῖα (17c-d). This looks like mere classification, such as one also finds in non-Pythagorean musicology; but the following expression, καὶ τοὺς ὅρους τῶν διαστημάτων is a technical term in the theory of proportion.[21] Rhythms and meters are measured "by numbers" (17d); and coming back to this theme later (25d-e), Plato states unmistakably that musical harmony depends on numerical proportions. Thus it is natural to suppose that from the beginning Plato was thinking of the same musical theorists;[22] from what Plato himself says it emerges that the ontology of the *Philebus* has its roots in Pythagoreanism.

So we come into possession of a piece of pre-Aristotelian evidence for a Pythagorean philosophy of some scope, musically oriented. A prime necessity, however, is to distinguish precisely between what Plato inherited and what it became in the alembic of his own mind. But a complete answer to this problem cannot be derived from Plato's words alone. The best we can achieve is a lower limit, so to speak: what is derivative is that which does not follow from the context and the purpose of the dialogue. Insofar as what we learn in this way

[19] There is no indication of any such thing in the treatment of "grammar" (18b–d).

[20] 530d et seq.; above, n. 11.

[21] In the system of Aristoxenus, too, one finds ὅροι; they are the notes, with their names (Aristox. *Harm.* 49.20 M., Pl. *Rep.* 443d). But these are only "comprehensible" in the σύστημα; the same interval may be exemplified by quite different notes, while in Plato they are "comprehended" (ἐπειδὰν λάβῃς, 17c) before the σύστημα. In the ratio theory the ὅροι are determined by the interval peculiar to each case (e.g. 9 : 8 is a whole tone).

[22] Richter (89ff) believes that at *Phlb.* 17c Plato is rehabilitating the non-Pythagorean theory of music, and only at 23c goes into the ratio theory, from a new point of view.

coincides with the testimony of Aristotle, the reconstruction may be regarded as confirmed.

The fundamental problem, that all being is at the same time one and many, had been formulated long ago (14c). It is also discussed in other dialogues and brought into connection with the problems raised by the Eleatics.[23] Empirical things (γιγνόμενα) had already been designated, in passing, as unlimited (ἄπειρα, 15b), in contrast to the "unities" represented by the ideas. What is new in the pronouncements credited to "the ancients" (16c) is, first of all, the antithesis of πέρας and ἀπειρία. This feature alone is also found at the beginning of the later passage resuming the argument: τὸν θεὸν ἐλέγομέν που τὸ μὲν ἄπειρον δεῖξαι τῶν ὄντων, τὸ δὲ πέρας (23c). In the conclusion drawn from this (16d), the opposition of one and many again comes to the fore: the method put forward as ancient tradition is that of Platonic diaeresis, which Plato was already calling for in the *Phaedrus*. Still, the special emphasis on "number" as intermediary between the "One" and the "Infinite" is new. In the example from music theory this "number" suddenly appears with a different function: the measures and tones are not only "diaeretically" divided, but "measured" in numerical proportions. When the argument is resumed, the "mixture" of Limit and Unlimited consists in the fact that "number" is introduced into things: numerical relations like equal, double, etc. (25d). It is not only musical harmony that depends on this process, but health of body and the proper rhythm of the seasons—thoughts already brought forward by Eryximachus in the *Symposium*.[24] Still different is the function of "number" in another context of thought: everything dependent on τέχνη was discovered by its means (16c). Here it is a question of counting, measuring, weighing; one is reminded of an allusion in the *Politicus* to the thesis of πολλοὶ τῶν κομψῶν . . . ὡς ἄρα μετρητικὴ περὶ πάντ' ἐστὶ τὰ γιγνόμενα.[25] So right from the beginning there

[23] Esp. *Parm.* passim, *Soph.* 251a

[24] 186a (medicine), 187a (music), 188a (seasons). There it is Eros that is named, not number; the speech is not simply Pythagorean, but Platonic artistry in exposition. But Pythagorean themes do seem to be used.

[25] *Pol.* 284c. Plato dismisses these people lightly and ironically, because they are not used to looking at things κατ' εἶδη. He takes over their leading ideas, but only in a significantly refined form. What Plato develops out of Pythagorean thought in the *Philebus* is what he misses in the *Politicus*. All the same, it may be that Pythagoreans are in his mind here too (as scholars have mostly supposed, since Campbell; cf. Raven, *PrEl* 186f). In one case the fruitful development of the basic idea takes his attention, in the other a more easygoing, popularizing application of it is rejected (e.g., in medicine; see below, ch. III 3).

is interpenetration of various factors. What is Pythagorean, because it is, in the Platonic milieu, novel and a bit awkward, is the pair πέρας-ἀπειρία as well as a certain emphasis on number and proportion. More we cannot say, from the analysis of Plato's text; but if we add the testimony of Aristotle, his words and Plato's complement one another. The conception of the world as a harmony of Limit and Unlimited, permeated by number, that great creator of order, was for Plato a point of departure and a guidepost on his quest for intellectual mastery of the riddle of the plurality and unity of being.[26]

When the argument is resumed (23c), "a god" is again named as revealer of the pair of opposites ἄπειρον-πέρας. Plato, independently (τιθώμεθα), posits a "mixed" class and also introduces, as a novelty, a fourth class (the αἰτία). Here, then, we are on Platonic ground. This is where the Unlimited first receives its more precise specification, which could not have been foreseen. Whereas it was previously matched with plurality (πλῆθος, 16d, 17e), it is now explained with the help of pairs of opposites like warmer and colder, strongly and mildly, more and less, rather and less rather (24a). There is emphasis on both the importance and the difficulty of this way of thinking: "if things are said again and yet again, there is some prospect of the two parties to a discussion being brought to a tolerable agreement" (24e). In every "more and less rather" lies "the nature of the Unlimited" (24e). The place of a simple conception of a spatial or numerical "Unlimited" is taken by the continuous "indefiniteness" on both sides of the limiting measure. The place of the Unlimited is taken by the Indefinite Dyad, as we may express it in the light of the reports about *On the Good*.[27] The analysis of Being in the *Philebus* is a foreshadowing, an early beginning which makes use of the thought of others, moving toward the ontology developed in *On the Good*.[28] As Hermodorus expresses it, τὴν ὕλην ὁ Πλάτων κατὰ τὸ ἄπειρον καὶ ἀόριστον ὑποτιθέμενος ἀπ' ἐκείνων

[26] Kucharski, "Le *Philèbe* et les *Eléments harmoniques* d'Aristoxène," *Rév. philos.* 84 (1959) 41–72, and H. Koller, "Die dihäretische Methode," *Glotta* 39 (1960) 6–24, believe that Plato is developing the method of diaeresis according to the model of music theory, but a contrary argument would be that there is nothing in the *Phaedrus*, *Sophist*, or *Politicus* to suggest any such connection (though in *Phdr.* 270d he speaks of "numbering" the forms), whereas the feature which is peculiar to the *Philebus*, πέρας-ἀπειρία is attested by Aristotle as Pythagorean.

[27] Hermodorus' evidence is at n. 29 below. Merlan has drawn attention, rightly, to the role of the problem of τὸ μᾶλλον καὶ ἧττον δέχεσθαι in Aristotle's doctrine of the categories (*Philologus* 1934, 35ff). Even modern writers are unanimous that there is a relationship to the *On the Good*; cf. Taylor, *Phileb.* 50ff; Ross, *Met.* I 171; Wilpert, *Phileb.* passim.

[28] On questions remaining open in the *Philebus*, cf. Wilpert, *Phileb.* 58of.

αὐτὴν ἐδήλου τῶν τὸ μᾶλλον καὶ τὸ ἧττον ἐπιδεχομένων, ὧν καὶ τὸ μέγα καὶ τὸ μικρόν ἐστιν.[29]

What Plato presupposes as "source" is no more, and nothing else, than what Aristotle treats as the doctrine of the Pythagoreans. There is nothing of the One and the Indefinite Dyad, but Limit and Unlimited; and if we are right in combining with the *Philebus* the passage in the *Politicus*, no theory of ideas. At the same time, we see Plato's philosophy, emerging from these Pythagorean stimuli, moving in the direction of the ontology of the lecture *On the Good*. Our decision, regarding the tradition about Pythagoreanism, between that of the Old Academy and that of Aristotle, is thus irrevocably confirmed: it was not Speusippus, Xenocrates, and Heraclides, but Aristotle, who gave authentic information on the Pythagoreanism that was there before Plato. And what is more, we can understand, from Plato, how the former tradition arose, with its distortion of the historical picture.

For Plato's affirmation of the divine origin of the doctrine of Limit and Unlimited is more than a glittering sequin on the fabric of the exposition. It signifies that its truth is beyond doubt; and Plato feels that this imposes on him the obligation to grasp the truth of this idea and its all-encompassing significance. Such a divine revelation is not something finished and complete, but a task to fulfill—like that of "coming to the aid of the god" (βοηθεῖν τῷ θεῷ) in the *Apology*.[30] It is precisely because of the authority of such a doctrine that interpretation must set in immediately; and in this process Plato's own thoughts and those of others become almost inextricably intertwined.

Here is a type of interpretation untroubled by minutiae of historical accuracy and only interested in the sense intended. Actually, the Greek question τί λέγει; does not mean, "What words are used?" but "What does this mean?" This method is characteristic of Plato, but surely not of him alone.[31] When, in the dialogue named after him, Protagoras speaks of Homer and Hesiod, Orpheus and Musaeus as Sophists, Socrates overtrumps him with the claim that the most σοφισταί, students of φιλοσοφία, are to be found in Crete and Sparta.[32] A similar passage in the *Laws* is more serious. At first the Spartan and Cretan constitution is represented as aimed toward warfare as its only goal (625d). This object is rejected as comprehending only one portion of ἀρετή. The result of this, however, is not rejection of the Cretan and Spartan νόμοι—they are of divine origin, handed down by Zeus and Apollo. Therefore, ὥσπερ τό τε ἀληθὲς οἶμαι καὶ τὸ δίκαιον ὑπέρ γε θείας (πολιτείας) διαλεγομένους λέγειν (630d-e), the interpretation must be revised; and to the bewilderment of the Cretan and the Spartan, the object of their constitutions appears all of a sudden as much more comprehensive, and by no means restricted to the military aim.[33]

This is the spirit in which, on the basis of the *Philebus*, we must judge the relationship with Pythagoreanism; here too the tradition is set before the philosopher as a task which has to be thought through. And as the ancient claim, that Sparta's constitution came from the god of Delphi, must be taken seriously, the "divine" origin of Pythagorean teachings, too, is more than a *façon de parler*. Aristoxenus tells us that Pythagoras got his doctrines from the Delphic oracle,[34] but Aristotle says that Pythagoras himself was believed to be the "Hyperborean Apollo."[35] Taken together with the *Philebus*, this can only mean that the Pythagoreans, even as early as those whom Plato knew, understood their own philosophical activities as developments of the basically identical doctrines of their master, the divine Pythagoras, "but that it all comes from that great man (εἶναι δὲ πάντα ἐκείνου τοῦ ἀνδρός)."[36] We must suppose, then, that this way of thinking was already established in Plato's day. It explains the surprising uncertainty of Aristotle as to the chronological relationships of the Pythagoreans.[37] He refused to accept the assertion that all this was Pythagoras' teaching, but he had no basis for more precise chronological determination (and perhaps no interest in it).

But where Aristotle looks upon it with critical aloofness, Plato's disciples join him in taking their place within the Pythagorean tradition. In this respect too, the *Philebus* confirms the conclusion we were able to reach from the contradiction of Aristotelian and Academic accounts of Pythagoreanism. Plato's school sees in its own philosophical treatment of the problem of ultimate principles a continuation of Pythagoreanism, so that ancient material is reinterpreted accordingly. This Platonic interpretation of Pythagorean philosophy became

[29] Simpl. *Phys.* 247.30ff.

[30] *Ap.* 21b.

[31] Cf. the sharply ironical, but by no means entirely inaccurate, formulation of Cherniss, referring to Aristotle (*Riddle* 30): "Aristotle is one of those who cannot be refuted by an author's words because he is sure that the author was unable to say what he really thought."

[32] *Prot.* 316d, 342ab.

[33] Cf. also W. Keber, *Platos Stellung zur Sparta-Ideologie* (Diss. Münster, 1957).

[34] Aristox. fr. 15 D.L. 8.8., 21.

[35] Below, ch. II 3.

[36] Iam. *VP* 88 *Comm. math. sc.* 77.22; cf. below, ch. II 5. Iam. *VP* 198: καλὸν δὲ τὰ πάντα Πυθαγόρᾳ ἀνατιθέναι τε καὶ ἀπονέμειν καὶ μηδεμίαν περιποιεῖσθαι δόξαν ἰδίαν . . . (similarly ibid. 158). The proverbial αὐτὸς ἔφα D.L. 8.46 (with the parallels given Delatte, *Vie* ad loc.).

[37] Above, ch. I 2, nn. 99, 120.

dominant in the ancient tradition. Aristotle was the only one to contradict it, and shows us thereby what had been there before Plato; and in fact what Plato presupposes is what Aristotle criticizes.

It is not necessary at this point to pursue in detail the further allusions to Pythagoreans in Plato, whether certain or conjectural.[38] What has been said is enough to serve at least as a sketch of the general trends. The true problem of the Pythagorean tradition lies in Platonism, for Platonizing interpretation took the place of historical reality. One can only guess at the reasons why Plato and his pupils saw themselves as continuators of Pythagoreanism. Personal contacts were certainly important; they were already present in the Socratic circle. Simmias and Cebes of Thebes, "hearers" of Philolaus, appear even in Xenophon as pupils of Socrates,[39] and we may believe that, as the *Phaedo* has it, the Pythagorean Echecrates of Phlius had some connection with Socrates.[40] Plato's friendship with Archytas is attested by the seventh *Letter*,[41] and the later biographies of Plato agree that a principal motive of his first journey to Magna Graecia was to establish contact with the Pythagoreans there.[42] One must proceed cautiously in trying to determine what extent of doctrinal agreement there may have been between Plato and the Pythagoreans. There is a considerable danger of projecting onto the Pythagoreans, unjustifiably, the unique style of thought and presentation that were Plato's. We must remember the possibility of individual influences that could be at work in the impressiveness and magnetism of such a personality as Archytas. It is worth remembering that in the seventh *Letter* immortality figures as a belief not subject to skeptical doubt (335ab; cf. *Rep.* 498d). In addition to external influences

[38] On Philolaus, below, ch. III 2; on the *ψυχή-ἁρμονία* doctrine, below, ch. III 2; on the harmony of the spheres, below, ch. IV 4; on the "nuptial number," below, ch. VI 4; on the origin of the soul, ch. V; on *Gorg.* 493a, below, ch. III 2; on *Gorg.* 507e, above, ch. I 3, nn. 155–157.

[39] Pl. *Phd.* 61d–e, Xen. *Mem.* 1.2.48, 3.11.17.

[40] Echecrates as a Pythagorean: Aristox. fr. 18–19 (cf. DK 53); ps.-Plato, *Ep.* 9, where Echecrates appears as a friend of Archytas, is doubtless a rhetorical exercise with no historical authority.—The Echecrates from whom Timaeus claims to have received information about Locrians (*FGrHist* 566F12=Polyb. 12.10.7) is, in spite of the chronological problems, likely to be the same as the Pythagorean from Phlius (cf. Oldfather, *RE* Supp. III 417f; Jacoby IIIb, *Komm.* 552, *Noten* 326, 195; Echecrates of Locri as a teacher of Plato: Cic. *Fin.* 5.87, Val. Max. 8.7.3). Cf. also below, ch. II 5.

[41] 338c, 339b. . . *Ἀρχέδημον, ὃν ἡγεῖτό με τῶν ἐν Σικελίᾳ περὶ πλείστου ποιεῖσθαι, τῶν Ἀρχύτῃ συγγεγονότων ἕνα.* According to this Archytas had "pupils." Cf. also 350a.

[42] D.L. 3.6, Cic. *Fin.* 5.87 (from Antiochus), *Rep.* 1.16, Apul. *Plat.* 1.3, etc. (with inconsistencies in details). Cf. also Wilamowitz, *Platon* II 82f. In Anon. Phot. 438b18 Plato is called *Ἀρχύτου τοῦ πρεσβυτέρου μαθητής*; this is polemic against the other view, that Archytas had been a pupil of Plato (Immisch 46). The meaningless *senior Archytas* (Apul. *Plat.* 1.3) and *Ἀρχύτας ὁ πρεσβύτερος* (Iam. *VP* 104) is a misunderstanding of the same source.

there is Plato's peculiar hesitation ever to speak dogmatically, in his own name. When Socrates receded into the background, in comparison with Plato's deepening interest in mathematical science, he was glad to present, as the spokesman of his "likely account" in the *Timaeus*, a man from Magna Graecia, even though the cosmic design presented there went far beyond the words of the predecessors who had helped stimulate him. Finally, it seems to have struck Plato sometime that a way toward the solution of the growing problem, in the theory of ideas, of the *κοινωνία τῶν εἰδῶν*, opened itself up in the form of number concepts. And it was in Pythagorean number speculation that he found this solution foreshadowed.

It is scarcely surprising that Plato's disciples subsequently saw Pythagoras only through the eyes of Plato.[43] But it is also comprehensible that the doxography took its information about Pythagoras from the writings of Plato's pupils,[44] when we realize that there was nowhere else a tangible account of the philosophy of Pythagoras, clearly outlined and provided with a fixed technical terminology—as was done for the doctrine of Parmenides in his poem. There was no "Word of Pythagoras," and in the absence of authentic sources whoever wanted to mention Pythagoras could do nothing but rely on those expositions that spoke confidently about a "philosophy of Pythagoras." Aristotle spoke only of "Pythagoreans," aside from the fact that the material was scattered through Aristotle's works and presented in polemical fashion, while the positive accounts of the Platonists provided conveniently organized reports. In addition, the Platonists' expositions, as the only ones to suit later philosophical taste, were for that reason more likely to give an impression of authenticity.[45]

Even in the *Life of Pythagoras* which Photius excerpted,[46] Pythagoreanism, Academy, and Peripatos are seen as making an unbroken unity: the ninth successor of Pythagoras is Plato, the tenth Aristotle. For the most part, though, Plato and Pythagoreans are separate in the later tradition. The Pythagorean tradition freed itself, cut loose from the Academy, and as a result of this separation from its origin it was

[43] Speusippus and Xenocrates also had personal contact with Pythagoreans; above, ch. I 2, n. 102.

[44] On the problem of the sources of the *Vetusta placita*, see ch. I 2, n. 76, ch. I 3, nn. 56, 111, 142.

[45] For the playing off of "more genuine" Pythagoreans against Aristotle, see below, ch. III 1. With reference to the reports of Aristotle, Philoponus clearly formulates his principle for interpretation of Pythagoreanism (*De an.* 70.2): *εἰ μὲν γὰρ τὸ φαινόμενον ἐλαμβάνομεθα, καταγέλαστον ἔσται καὶ οὐ σοφῶν, ἀλλὰ γραῶν λῆρος· εἰ δὲ ἐκεῖνοι σοφοί, δεῖ ζητεῖν τι τῶν φαινομένων σεμνότερον.*

[46] 418b17ff (cf. above, ch. I 3, n. 2).

able to develop a life of its own. One cause for this lies in the fact that tradition about Plato's oral teaching lost in importance, in comparison with the steady influence of the Platonic dialogues. The doctrines formulated in the latter were well known, while the theory of ideal numbers sank into desuetude.

The most important factor, however, was the effect of the direction of Academic development on the tradition about Pythagoras. From Aristotle's time it had been customary to see Plato as a synthesis of Socrates and Pythagoras.[47] In the generation of Plato's pupils the "Pythagoreanism," metaphysical speculation, carried the day. But, when a reaction set in and the Academy became predominantly "Socratic," Pythagoras necessarily lost his place. This is just what happened, from the time of Arcesilaus and the "Middle Academy"; the school of Plato went over to skepticism, and there had to be a parting of the ways with Pythagoreanism.

There were necessarily two aspects to this breach. Insofar as the Academics following the Socratic-skeptical trend still felt themselves to be Plato's successors, they had to push diligently aside everything "dogmatic," and especially the mathematical-scientific and metaphysical teaching of the school, as not genuinely Platonic. They found another origin for it: Pythagoras. On the other hand, what this process discarded retained, even for the rationalistic Hellenistic world, a certain fascination. Those who were attracted by it could no longer attribute it to Plato, against the authority of the Middle Academy, but found it necessary to reach back for the authority of "Pythagoras." And when he took the limelight, Plato and his pupils were stigmatized as plagiarists.

To Cicero, "Academic" and "skeptical" mean the same thing, and in fact Plato was often invoked by the skeptics.[48] This tradition may have affected the report of Sextus Empiricus on the theory of ideal numbers;[49] use was made of an exact transcript of the lecture *On the Good*, but the whole is presented as refutation of the Pythagoreans. Plato's name first occurs only quite incidentally (10.258), but in the skeptical refutation he is cited at length (10.302ff). Criticism of the theory of ideal numbers takes the form of a struggle, in alliance with Plato, against Pythagoras.

[47] Arist. *Met.* 987a29ff; cf. Dicaearchus fr. 41, Cic. *Rep.* 1.16, *De or.* 1.42, *Fin.* 5.87, *Tusc.* 5.10, Numenius ap. Euseb. *Praep. evang.* 14.5.9, Aug. *De civ. D.* 8.4.

[48] Cic. *Acad.* 2.74, 1.46, Sext. Emp. *PH* 1.221, *Proll. in Pl.* 10. 3–8 (p. 205 Hermann). Cf. Burkert, "Cicero als Platoniker und Skeptiker," *Gymnasium* 72 (1965) 175–200. The first ps.-Xenophontic letter (Hercher p. 788) is also a polemic of the "Socratics" against Plato's "Pythagorizing." The reproach is made against the Platonists: *Αἰγύπτου ἠμάθησαν καὶ τῆς Πυθαγόρου τερατώδους σοφίας.*

[49] Above, ch. I 3, nn. 4ff.

The contrary position is expressed in Numenius' work *On the Difference (διαστάσεως) between the Academics and Plato.*[50] For Numenius, Plato and "the great Pythagoras" are about the same thing, and whatever Socrates had to offer also came from Pythagoras (5.7). Plato's direct disciples had followed their master, and "it was primarily because of them that Pythagoras came to be highly honored" (5.2); Arcesilaus and his pupils had wandered from this path, and with scorn and anger Numenius condemns this apostasy.

But long before Numenius this attitude had produced remarkable results. From the third century B.C. on, the apocryphal Pythagorean writings appear, each trying to outdo the other,[51] presenting Platonic-Peripatetic doctrines as original pronouncements of Pythagoras and his pupils. The interpretation of Pythagoreanism that Speusippus, Xenocrates, and Heraclides had given is hypostasized in this revival, and the purported originals make the intermediaries superfluous. Thus the criticism of Pythagoreans themselves is turned against the Platonists:

> . . . that Plato and Aristotle, Speusippus and Aristoxenus and Xenocrates, as the Pythagoreans say, appropriated what was fruitful, with slight modification, but collected some superficial or inconsequential things, whatever is brought forward by those later malicious slanderers in an effort to refute and mock the school, and put these down as the special doctrines of the sect . . .[52]

Thus even these Pythagoreans admit, implicitly, that what was essential, or "fruitful," in their doctrines agrees with Plato and Aristotle, and that they could make no good use of what historical tradition had to offer as the ἴδια of Pythagoreanism. Later Pythagoreanism is stamped

[50] Numenius fr. 1ff=Euseb. *Praep. evang.* 14.5–9.

[51] On the apocrypha, see Burkert, *Philologus* 1961.—Antiochus of Ascalon led the Academy back to dogmatic ways, and left skepticism, in turn, homeless. Subsequently, in just as unhistorical a way, it sought out a new founder to whom it might trace itself, Pyrrho. (Cf. A. Weische, *Cicero und die Neue Akademie* [Münster, 1961] 105ff.)

[52] *τὸν Πλάτωνα καὶ Ἀριστοτέλη Σπεύσιππόν τε καὶ Ἀριστόξενον καὶ Ξενοκράτη, ὥς φασιν οἱ Πυθαγόρειοι, τὰ μὲν κάρπιμα σφετερίσασθαι διὰ βραχείας ἐπισκευῆς, τὰ δ' ἐπιπόλαια καὶ ἐλαφρὰ καὶ ὅσα πρὸς† διασκευὴν† καὶ χλευασμὸν τοῦ διδασκαλείου ὑπὸ τῶν βασκάνων ἕτερον συκοφαντούντων προβάλλεται, συναγαγεῖν καὶ ὡς ἴδια τῆς αἱρέσεως καταχωρίσαι . . .* (Por. *VP* 53). The source of this passage (transition from Moderatus to Nicomachus) is impossible to determine. *ἀνασκευήν* is my suggestion for M's *διασκευήν*; cf. Sext. Emp. *Math.* 6.4, 8.196, 9.55, etc. (P. Shorey, *CP* 27 [1932] 175f, proposed *διαπυρμόν* or *διαστροφήν*, each less likely on paleographical grounds.) It is not true, without qualification, that Plato, Speusippus, and Xenocrates only left the Pythagoreans *ἐπιπόλαια*. Plato makes almost no explicit pronouncements about Pythagoreanism, while Speusippus and Xenocrates translate and modernize it. This is not enough, however, to satisfy the "imperialistic" attitude of the later Pythagoreans. On Plato's "appropriating" Pythagorean doctrines, see also Iam *VP* 131.

so deeply with Platonism that it has no longer any conception of its real origin.

One might therefore define later Pythagoreanism as Platonism with the Socratic and dialectic element amputated. In fact, Plato remained the principal source for all later Pythagoreans—Plato's myths, and in particular the *Timaeus*. The apocrypha, presenting the supposed originals, could make no headway against this overwhelming influence. Apollonius of Tyana taught "doctrines about the physical world similar to the opinions of Plato's *Timaeus*," and though Proclus considered the "Timaeus Locrus" document genuine and put it at the head of his commentary on the *Timaeus*, it was not this sorry scribble but Plato's *Timaeus* itself that he classified, along with the *Chaldaean Oracles*, as far excelling all other literature.[53] Scholars have shown in different ways that Neoplatonism is quite closely dependent on the Old Academy,[54] and "Pythagoreanism" too belongs in this category. It is also basically Platonism, existing at a time when Plato (as interpreted in Pythagorean fashion) had lost his position in the Academic school. Later, neo-Pythagoreanism converges, in the philosophical realm, with Neoplatonism.[55]

And yet Pythagoreanism is not sufficiently characterized by that which is lacking, as compared with Plato. The inherited material that was lumped together under Pythagoras' name, undigested or mutilated though it may have been, was in this process raised to a new dignity and endowed with unexampled authority. The whole body of apocryphal literature lies within the realm of religion. The ontology of the Old Academy was oriented toward the divine; the Pythagorean pseudepigrapha meet a subconscious religious need of the Hellenistic period. And when, from the first century B.C. on, people once more come forward to declare themselves Pythagoreans, their most noticeable characteristic is that they are seeking (or even, as for example in the case of Apollonius of Tyana, claiming to possess) a superhuman, divine wisdom. And it may be that in this very point—not in details of doctrine but in the claim to possess divine knowledge—we are most likely to find an element of its real origin, in the influence of Pythagoras of Samos.

[53] Philostr. *VA* 6.22, Marinus *V. Procl.* 38.

[54] Dodds, *CQ* 1928, Merlan, *PlNeopl.*

[55] Alongside Pythagoreanism as doctrine stands Pythagoreanism as a way of life. While the Hellenistic apocrypha promoted Pythagoreanism in a literary way, the real point of "neo-Pythagoreanism" from the times of Nigidius Figulus and Anaxilaus of Larisa was to take it seriously in everyday life. Once more there were people who devoted themselves to a *Bίος Πυθαγόρειος* (Burkert, *Philologus* 1961).

II. *Pythagoras in the Earliest Tradition*

1. SOURCE PROBLEMS

Platonizing interpretation brought basic change to the Pythagoras tradition, so that a historical reconstruction of what it was like before Plato's time must be based on the pre-Platonic evidence. This can be supplemented by reports that were not drawn into the process of Platonic transmogrification; but, to judge these with any approach to confidence, we must first survey the nature of the tradition as a whole. It is no longer admissible simply to reject what is late attested, as Zeller did; any such item might have its origin in an ancient and authentic source. After Rohde, the most important advances in the analysis of the sources were made by Delatte and Lévy. Significant results have been achieved; but the essential thing is to distinguish between what is certain and what is merely conjecture.[1]

Most of our material on the life and activities of Pythagoras is collected in the eighth book of Diogenes Laertius, in Porphyry's *Life of Pythagoras*, and especially in Iamblichus. We may add the tenth book of Diodorus, of which only fragments are preserved, and the very short sketch in Justin. Photius' excerpt from an anonymous life of Pythagoras has very little that pertains to history or biography. Iamblichus is most detailed; his plan was to present the *κατὰ Πυθαγόραν φιλοσοφία* in a work designed to run to ten books, because ten is the perfect number. The *Pythagorean Life* (for his subject was the appropriate way of living, rather than the biography of Pythagoras as such)[2] was followed by the *Protrepticus*, the book *On General Mathematical Knowledge* (*De communi mathematica scientia*), and, as a treatment of the special sciences, a commentary on Nicomachus' *Introduction to*

[1] An example of analysis carried too far is Bertermann's dissertation. In his concluding table (75ff), the whole text of Iamblichus' *Vita Pythagorica* is traced back, sentence by sentence, through two or three intermediaries for each, to sources of the fourth century B.C. But his evidence is mainly vague associations and scarcely ever attains the status of proof. For an example of the way in which hasty source analysis can lead to error, see my discussion of the "letter of Lysis," *Philologus* 1961, 17–24.

[2] For convenience, the commonly accepted title *Life of Pythagoras* has been retained here.

Arithmetic. Later in the series came the book on numerical theology (*Theologumena arithmeticae*). Further parts are not preserved, and perhaps not all of them were written. It was Iamblichus who set the direction for the later Neoplatonists, toward a definite equation of Platonism and Pythagoreanism.[3]

Our first question must be as to the direct sources of Iamblichus. Here analysis is easiest in the case of the *Theologumena arithmeticae*; one source, Anatolius *On the First Ten Numbers*, has been recovered, and the second, Nicomachus '*Αριθμητικῶν θεολογουμένων βιβλία β'*, we have in an excerpt by Photius (*Bibl.* 187). The manuscripts of the *Theologumena arithmeticae* often name Anatolius and Nicomachus, and the book proves to be, essentially, a cento made up from the two older ones. What is not Anatolius is mostly quotation of Nicomachus, including his own quotations.[4] The contribution of the author, or rather compiler—and in spite of doubts this was probably Iamblichus himself—is merely arrangement and introduction.

Certainty was reached by Erwin Rohde on another point, the relation between Porphyry and Iamblichus. The latter did not use Porphyry directly; instead of the general *Philosophic History* he used special works on Pythagoras.[5] Thus we have access to important sections of Nicomachus' biography of Pythagoras, and in their original wording.[6]

[3] It is to be ascribed to the influence of Iamblichus that the chapter on Pythagoras in Porphyry's *Philosophic History* became separated from the rest and is therefore the only surviving section (cf. Harder xv).

[4] V. de Falco, "Sui *Theologumena arithmeticae*," *Riv. indo-greco-italica* 6 (1922) fasc. 1/2, 49–61; H. Oppermann, *Gnomon* 5 (1929) 548–558. If Iamblichus had inserted the source citations himself they would have been evenly distributed; but, as it is, they are found almost exclusively in sections based on Nicomachus.

[5] That Iamblichus does use Porphyry is maintained, after Zeller I 365 n. 1, by E. Norden (*Agnostos theos* [Berlin, 1913] 344 n. 2) and more recently by J. A. Philip (*TAPA* 90 [1959] 185–194. Rohde's proof to the contrary (Q 125ff) was based on the fact that the Nicomachus sections of Porphyry occur, without exception, as verbal quotations in Iamblichus, while apart from them there are only occasional points of contact. In addition, Iam. *VP* 233, compared with Por. *VP* 59, proves that Iamblichus is copying the wording of Nicomachus, not that of Porphyry. Iam. *VP* 253 has the Doric forms proper to the Lysis letter (Hercher, *Epistologr. gr.* p. 603), which are not preserved by Porphyry (58). Iam. *VP* 170 has Metapontum, correctly, where Por. *VP* 4 names Croton (Nauck liv, 36; below, ch. II 2, n. 18). Even Iamblichus could not have spoken of "general agreement" (*VP* 248) if he had had before him the contradictory versions of Por. *VP* 56–57.

[6] Por. 20–31 = Iam. 30, 33, (241), 34, 60, 61, 62, 36, 63, 134–135, (142), 136, 64–67; Por. 59–61 = Iam. 233–237. The material promised in the sentence with which the extant text of Porphyry breaks off is in Iam. *VP* 189–194. In these cases Porphyry names Nicomachus; on the other hand, Iamblichus names Nicomachus at *VP* 251. Further, Iam. *VP* 252 p. 135.10–17 Deubner = Por. *VP* 55 p. 47.18–48.1 Nauck; Iam. p. 135.18–136.13 = Por. 57f pp. 49.16–50.11. Thus we can be certain that these passages are all from Nicomachus (contra Jäger, 59ff, who supposes the source is the "handbook," as well as Delatte, *Pol.* 219, and Minar 68 n. 64, who take Neanthes as the direct source; correctly Rohde, *Q* 115f, Corssen, *Philologus* 1912, 332ff, Lévy, *Sources* 92 n. 1; the fact that Iamblichus puts passive for active at p. 135.17, and thus has the persecutors being killed, is his own misunderstanding, (Lévy, *Sources* 97). Since, further, Iam. *VP* 248 p. 133.12–14=Por. *VP* 54f pp. 46.24–47.2 show verbal agreement, and in addition Iam. *VP* p. 133.8f introduces the version of the Pherecydes episodes which is later (p. 135.11) attributed to Nicomachus, the excerpt from Nicomachus must begin as early as Iam. *VP* 248, and Por. *VP* 54, and the Aristoxenus citation (fr. 18) must come from Nicomachus (as fr. 31=Iam. *VP* 241 = Por. *VP* 59). This had already been deduced by Rohde, from Iam. *VP* 251: *Νικόμαχος δὲ τὰ μὲν ἄλλα συνομολογεῖ τούτοις* (Q115f; Lévy, *Sources* 116, attributes the Aristoxenus citation to the third source of Iamblichus, the "handbook"; Diels, DK 14.16 n., names Apollonius as the source of the Aristoxenus citation, doubtless a mere slip, passed on by Timpanaro Cardini, p. 55). Nicomachus cited Aristoxenus, but preferred to follow another version. Most authorities suspect this is Neanthes, who is cited by name at Por. *VP* 55 p. 48.1 (von Fritz, *Pol.* 5, after Corssen), but Lévy thinks it was Satyrus (*Sources* 61ff; cf. D.L. 8.40), and that Neanthes is the basis rather for D.L. 8.39=Hesychius. It is not likely that the citation of Dicaearchus, Por. *VP* 56f (fr. 34 W.) was in the text of Nicomachus (Corssen, *Philologus* 1912, 341f, according to von Fritz, *Pol.* 7, already in Neanthes), since Iamblichus does not seem to know it (above, n. 5). Nor does Dicaearchus fr. 33= Por. *VP* 18 appear in Iamblichus; Porphyry has inserted into the Nicomachus text a passage from the "handbook" source (the same citation at D.L. 8.40=fr. 35b W.).

It also becomes obvious that Porphyry copies more mechanically than Iamblichus. Where Porphyry gives two connected excerpts from Nicomachus, Iamblichus may distribute the same material in different chapters, according to an artificial outline of his own.[7] Without the parallels in Porphyry it would be a hopeless undertaking to attempt to identify in the mosaic those tiles that make up the contribution of Nicomachus.

Further, there is in Porphyry a connected passage from Moderatus on number theory,[8] and twice he cites Antonius Diogenes' romance on *The Wonders beyond Thule*, though it is not quite clear where the quotations end.[9] His basis for the remainder must be a handbook containing many citations of its sources, and for this part the points of contact with Diogenes Laertius are especially numerous.

[7] Iamblichus' skill in combining disparate quotations can best be seen from his use of Plato; cf. Merlan, *PlNeopl* 148ff.—Small variations between Porphyry's and Iamblichus' wording are frequent. Generally, Iamblichus is fuller; and it may be that in the verbosity of his exposition he introduces expansion as often as Porphyry abbreviates.

[8] 48–53; cf. above, ch. I 3.

[9] Por. *VP* 10ff and 32ff; Lydus *Mens.* 4.42 p. 99 W. shows that Porphyry's source in *VP* 44 is still Diogenes Antonius. Rohde first (Q 126) assigned 10–17 and 32–45 to Antonius, then later (*Rom.* 272 n. 2), on the ground that in his novel Antonius could not have made explicit citation of sources, only 10–14, 32–36, and 44. (In 15 Dionysophanes is cited, in 41 Aristotle). On the other hand Jäger (36 ff, 43–47) and K. Reyhl (*Antonios Diogenes*, Diss. Tübingen, 1969, 20–31) hark back to Rohde's original thesis, excluding only the direct citations. To be sure, the mention of *ἄδυτα* connects Por. *VP* 17 and 34. Reyhl adds Por. *VP* 46–47, which is quite improbable, because Iam. *VP* 228 has copied the same text, but Iamblichus did not use Antonius Diogenes—and Por. *VP* 54–55, rightly comparing *φυσιογνωμονήσας* §54 and §13, though this is squeezed into a context from Nicomachus (above, n. 6).

Another direct source, Apollonius of Tyana,[10] is once explicitly cited in Iamblichus (254–264), and once shown to be a source by parallel passages in Porphyry.[11] Rohde showed convincingly that two other continuous sections, the speeches of Pythagoras in Croton[12] and his meeting with Phalaris,[13] have the same origin.

Rohde went on to propose a mechanical two-source theory, according to which Iamblichus drew material exclusively from Nicomachus and Apollonius; he tried to assign each chapter to one of the two, though allowing that Iamblichus may have worked out some individual passages on his own.[14] This two-source theory was vigorously attacked by Méautis, and definitely refuted by Lévy.[15] At least

[10] The identity of this Apollonius with the wonder-worker of Tyana was contested, after Wyttenbach, by Méautis (91), but has probability on its side. Apollonius was a conscious and enthusiastic Pythagorean.

[11] Iam. *VP* 3–8, 11; cf. Por. *VP* 2. Rohde (Q 128f) and Lévy (*Sources* 105ff) give the whole section 3–25 to Apollonius; but it is more probable that Iamblichus' compilatory method begins at the very beginning, though perhaps in an especially careful vein. (Cf. the connection of Iam. *VP* 19 with Nicom. *Th. ar.* 53.1ff.)

[12] Iam. *VP* 37–57 is bracketed with the Apollonius passage 254ff by 4 separate concordances: 49 and 262; the temple of the Muses 50 and 264; the Pythaeum (or Pythium) 50 and 261; *θεῖος*, 53 and 255.

[13] Iam. *VP* 215–222; cf., for Apollonius before Domitian, Philostr. *VA* 8.1ff (Rohde, Q 164ff; Lévy, *Sources* 109, *Lég.* 50ff). Boyancé (*REA* 1934) tried to trace the section to the *Abaris* of Heraclides Ponticus. In refutation, Miss De Vogel points out the Stoic and late Platonic terminology (*Mnemosyne* 18 [1965] 388–396, *Pythagoras*, 304–306). Festugière (*REG* 50 [1937] 474–476) adduces comparisons with hermetic-gnostic material. The rapid summary at 219 shows that Iamblichus was not composer but compiler.

[14] According to Rohde Iamblichus' own hand can be seen in 103–105, 157–162, 167–186, 198–199, 214, 223–228, 240–241, 244–247. Where doublets occur, Rohde always gives the one version to Nicomachus, and the other to Apollonius; but we should bear in mind the possibility of free self-citation by Iamblichus.—Iam. *VP* 159f=*In Nic.* 5.27ff is attributed by Rohde to Nicomachus' biography of Pythagoras (Q 156f); but are we to suppose that Iamblichus would really cite Nicomachus to comment on Nicomachus, and in addition that he would have been able to find such a detailed parallel to what Nicomachus says at *Ar.* 1.1? The passage is tolerably well integrated into Iam. *VP* 159f, while it seems a disconnected, interpolated block at *In Nic.* 5.27ff; Iamblichus is repeating his own paraphrase.

[15] Méautis 87ff, Lévy, *Sources* 111ff. E.g., Nicomachus (Iam. *VP* 30=Por. *VP* 20) names 2,000 auditors of Pythagoras, Apollonius (Iam. *VP* 254, 260) gives 300, but Iam. *VP* 29, like D.L. 8.15, 600. (Iam *VP* 29 is marked as an interpolation by Deubner, but, given the compilatory habits of Iamblichus, incongruities are always to be expected.) Iam. *VP* 35 had already been assigned to a "handbook" by Rohde (Q 131). Further: Iamblichus presents the miracles of Pythagoras in two versions, one according to Nicomachus and one according to Aristotle. If it can be shown that the versions taken from Nicomachus are almost always altered in a distinctive manner, so as to blunt the paradoxes, it is quite improbable that Nicomachus himself recorded both versions (as Rohde says, Q 152f); and why should Porphyry always have chosen the non-Aristotelian formulation? (Cf. below, ch. II 3.)—The sentence about the 3 books of Pythagoras (Iam. *VP* 199) cannot come from Nicomachus, who thought Pythagoras left no writings (*VP* 57; cf. Iam. *VP* 146, where doubtless Nicomachus is to be understood as one of the *ἀλλόγιμοι καὶ ἀξιόπιστοι* Pythagoreans who attributed the *Ἱερὸς λόγος* not to Pythagoras but to Telauges; cf. Euseb. *Hist. eccl.* 6.19.8). This sentence, however, shows verbal coincidence with D.L. 8.15 (below, ch. III 1).

a third source is to be recognized, of the "handbook" type, whose presence can be detected in coincidences between Diogenes Laertius and the "handbook" segments of Porphyry. Actually, there is no need to restrict Iamblichus to three books; it was more convenient, if he wanted to write on music (ch. 26), to find something in Nicomachus' book on music than to seek out an appropriate passage in his life of Pythagoras.[16] And Iamblichus could have read personally, as Stobaeus did, the *Πυθαγορικαὶ ἀποφάσεις* of Aristoxenus.[17]

Thus our analysis leads, in the first instance, mainly to sources of the first or second century: Moderatus lived under Nero,[18] Apollonius under Domitian, and Nicomachus is dated by the fact that Apuleius translated his *Introduction to Arithmetic* into Latin.[19] Yet these neo-Pythagoreans are themselves only intermediaries. Nicomachus cited his authorities meticulously, Apollonius did not. So Rohde judges Nicomachus very favorably: he is an intelligent compiler, and no forger. Toward Apollonius he is very negative: "one is best advised not to believe anything he says."[20] But this judgement can only be reckoned as true a parte potiori. Apollonius too used good sources, and it is precisely his material that has provided most opportunities for those who, from time to time, have tried to discover really ancient lore.[21] Nicomachus, on the other hand, who calls himself a Pythagorean, is so intimately concerned in his narrative that, at least in selection, arrangement, and interpretation, his personal contribution must not be underestimated.

For the earlier stages of the tradition Diogenes Laertius is particularly important, though his work is hard to analyze. He has woven together material from various handbooks, and his "card-file" method makes it almost impossible to discern connections of any larger elements. Still, there is an example of parallel tradition in the Pythagoras

[16] Iam. *VP* 115ff=Nicom. *Ench.* 6f. Rohde (Q 126f, 146f) assumes that Nicomachus had an identical passage in his life of Pythagoras.

[17] The excerpts from the *Πυθαγορικαὶ ἀποφάσεις* are quite distinctive in style (Iam. *VP* 101–102, 174–176, 180–183, 200–213, 230–233). In addition, at the beginning of the first excerpt the title is expressly cited (101). The Stobaeus excerpts are frr. 34–37, 39–41 Wehrli. The collection in DK 58D is somewhat helter-skelter.

[18] Cf. W. Capelle, *RE* XV 2318–2320. Plutarch mentions Moderatus at *Quaest. conv.* 8 7 1.

[19] Cf. F. E. Robbins, in M. L. D'Ooge (tr.), *Nicomachus of Gerasa, Introduction to Arithmetic*, with Studies in Greek Arithmetic by F. E. Robbins, L. C. Karpinski (New York, 1926) pp. 71f. The peculiar *RE* article by F. Kliem (XVII 463f) deals only with Nicomachus' *Arithmetic* and (briefly) the *Theologumena*, completely omitting consideration of the extant *Harmonicum Enchiridium* and the largely reconstructable life of Pythagoras.—Proclus thought he was a reincarnation of Nicomachus (Marinus *V. Procl.* 28). This may place Nicomachus' death in 196 A.D. (J. M. Dillon, *CR* 19 [1969] 274).

[20] *Q* 172.

[21] Below, n. 37.

article of the *Suda* and a Platonic scholium (*Rep.* 600b) which is almost identical with it; both are referred to Hesychius of Miletus. The same exposition is discernible in Diogenes Laertius, though broken up by numerous insertions. So one of the main sources can be distinguished, even if its name is unknown.[22]

The work of Neanthes of Cyzicus must have been an important intermediary source.[23] He brought together several versions of the origin of Pythagoras, including that of Aristoxenus;[24] like Hesychius he names Pherecydes and Hermodamas as Pythagoras' teachers, and enumerates the brothers of Pythagoras in the same terms as the source common to Diogenes Laertius and Hesychius.[25] Thus in Neanthes we find the earliest example of the handbook provided with source citations and variants. Perhaps it was the one that set the trend;[26] in any case it furnished an exposition that was widely used—by the source of Diogenes Laertius, by Nicomachus-Iamblichus, by Clement and Porphyry. It is striking that Neanthes is often named along with Hippobotus.[27] Obviously one had cited the other; probably Hippobotus, who wrote *On Sects* (D.L. 1.19), made use of the work of Neanthes, whose special concern was with the "mythical." Thus the sequence Neanthes–Hippobotus may be one link in the handbook tradition.

Callimachus' student Hermippus wrote several influential books on Pythagoras; Josephus calls him the "most distinguished" of the biographers of Pythagoras.[28] The fragments we have contain the most

[22] Cf. Delatte, *Vie* 9–63, where the older literature is discussed. "Handbooks" are responsible for the coincidences between Diogenes Laertius and Clement, Hippolytus, Porphyry, and Iamblichus. (On this, cf. Jäger passim.)

[23] *FGrHist* 84F26–33. The book in question is the fifth book of his *Μυθικά* (F29). This Neanthes is probably to be dated about 200 B.C., and distinguished from a historian of the same name of about 100 years earlier (Jacoby, *FGrHist* IIc, 144; for the early date, von Fritz, *Pol.* 6).

[24] F29=Por. *VP* 1–2; the emendation of *Κλεάνθης* to *Νεάνθης* is guaranteed by Clem. Al. *Strom.* 1.62.2; Aristox. fr. 11a–b; Neanthes himself believed that Pythagoras came from Syria—the most unusual view; he must, however, have mentioned the canonical version, that he came from Samos, and in that case he must have had three versions side by side, as do Por. *VP* 1 and Clement, loc. cit.

[25] *Εὔνοστος* in Neanthes (Por. *VP* 2) and Diogenes Antonius (Por. *VP* 10) and *Εὔνομος* in D.L. 8.2 and Hesychius can only be secondary variants.

[26] Lévy, *Sources* (cf. above, n. 6) also attributes D.L. 8.39–Hesychius to Neanthes, comparing Neanthes F30 (= Por. *VP* 55). Here Neanthes also is harking back to Aristoxenus (fr. 18 Iam. *VP* 249).

[27] Nicom. (Por. *VP* 61 Iam. *VP* 189 Neanthes F31), *Th. ar.* 52.8ff (= Neanthes F33 Aristox. fr. 12). Hippobotus is named along with Neanthes for differing versions in Clem. Al. *Strom.* 1.62.2; that of Hippobotus (Pythagoras a Samian, attributed by D.L. 8.1 to Hermippus) may have been also in Neanthes (above, n. 24). At D.L. 8.72, also, Neanthes (F28) follows Hippobotus.

[28] *Ap.* 1.163. This is a somewhat tendentious passage, for Josephus found in Hermippus indications of the dependence of Pythagoras on the Jews. The fragments are collected in *FHG* III 41f.

eccentric material in the whole Pythagorean tradition; Rohde considered the book "a malicious satire on Pythagoras."[29] But, although it can be shown occasionally that Hermippus distorts older source material in a rationalizing and ironical spirit,[30] still it cannot all be arbitrary invention. A student of Callimachus will, with one degree or another of seriousness, be engaged in collecting precisely that which is antique and odd, so that Hermippus too may have some accurately preserved material.[31]

Finally, the most important sources to which analysis can lead us, because they are the oldest, are Heraclides Ponticus, Aristoxenus, and Dicaearchus, along with Aristotle. In their case, significant chapters can be discerned, documented by direct quotation; attempts to enlarge the evidence by building up more complicated reconstructions hardly get beyond the stage of hypothesis.[32]

The greatest unknown is the historian Timaeus. He is cited by name

[29] *Q* 111. Especially bizarre is the report that, in a war between Acragas and Syracuse, Pythagoras lost his life in the course of a retreat, because he refused to run through a field of beans. (The same motif in Neanthes *FGrHist* 84F31=Iam. *VP* 189ff, in the story of Myllias and Timycha.)

[30] Hermippus (D.L. 8.69) gives a rationalistic interpretation of Heraclides' *Περὶ τῆς ἄπνου* (frr. 76ff Wehrli), namely that Empedocles did not revive a dead woman, but "cured a woman of Acragas named Panthea whom the doctors had given up."

[31] The dating of Epimenides to the time of Solon (contra Pl. *Leg.* 642d) had been regarded as an unfounded invention of Hermippus, until it was corroborated by Arist. *Ath. Pol.* 1 (cf. Diels, *SBBln* 1891, 387ff).

[32] Arist. frr. 190ff; cf. above ch. I 2, and below, ch. II 2, 4, 6. Heraclides names Pythagoras in frr. 40–41, 44, 87–89 Wehrli.—Aristox. frr. 2, 11–41, 43, 47–48, 50; Dicaearchus fr. 33–36, 41. Attempts at reconstruction have been directed mainly toward the *Abaris* of Heraclides (cf. Wehrli, pp. 84ff). It has been supposed that two important scenes can be regained. (1) The meeting of Abaris and Pythagoras, Iam. *VP* 91–93 (Corssen, *RhM* 1912, 38f; A. Rehm, ibid. 423f; Lévy, *Sources* 35, *Lég.* 48; Boyancé, *REA* 1934, added Iam. *VP* 215ff; cf. above, n. 13). The connection of the two wonder-workers may, however, be earlier; it is difficult to equate the divine Pythagoras of the encounter with Abaris with the Pythagoras who in fr. 87–88 espouses a definitely human brand of *φιλοσοφία*. (Rehm's assumption, 423f, that in different works Heraclides depicted different Pythagoras types is a counsel of desperation; the depiction of the death of Empedocles, fr. 83, seems to show that Heraclides liked to clothe the divine in a mist of mystery, so that he can scarcely have imagined Pythagoras as simply the Hyperborean Apollo.)—(2) A *katabasis* or descent to the underworld by Pythagoras, reflected in Hieronymus fr. 42 (D.L. 8.21) and Hermippus (D.L. 8.41) (Diels, *AGP* 1890, 469; Corssen, *RhM* 1912, 22, 281; esp. Lévy, *Sources* 26f, 34ff, *Lég.* 79ff; for an older source, Norden, Vergil VI, 35 n. 1; Delatte, *Litt.* 109). Heraclides fr. 75 is corrupt; and Wehrli's conjecture of *τὸ δεύτερον* for *τὸ δένδρον* would destroy the foundation of the attempt to establish a connection with Hieronymus (Wehrli p. 86). Since Heraclides locates the next world in the realm of the stars, he cannot be behind the literally understood word *κατάβασις* in Hermippus (cf. below, ch. IV 4). The mention of Abaris as flying, in fr. 51c, is explicitly assigned to the book *On Justice* (and there is no justification for identifying this work, on this account, with the *Abaris*; cf. Wehrli, 84, contra Boyancé, *REA* 1934, 322ff). Also, no one could have guessed, without the explicit attribution, that the Pythagoras anecdote in frr. 87–88 was from the *Περὶ τῆς ἄπνου*.

for a series of very specific reports about Pythagoras,[33] so that, a fortiori we must assume that he gave a general exposition of Pythagoreanism. He apparently showed a decided sympathy with Pythagoreanism, rooted in western Greek local patriotism.[34] Suspected of holding Timaeus material are Justin,[35] Diodorus,[36] and especially Apollonius.[37] In principle, the task is to define a kind of Vulgate tradition, not derived from either Aristoxenus or Dicaearchus. Delatte and Rostagni produced, independently, bold reconstructions, not without contradicting each other, particularly in the decisive matter of chronology. These constructs have, rightly, been subjected to sharp criticism. It cannot be proved that everything in Iamblichus and the other late accounts goes back to an authority of the fourth century B.C., nor that Timaeus would be the only eligible candidate for this position. There are an

[33] *FGrHist* 566F13, 14, 16, 17, 131, 132.

[34] One might conjecture that Timaeus was first to trace the origin of the name Magna Graecia (*Μεγάλη Ἑλλάς*) to the influence of Pythagoras (Nicom. in Iam. *VP* 30=Por. *VP* 20, Iam. 166, Cic. *Tusc.* 5.10, *Lael.* 13).

[35] 20.4. For the attribution to Timaeus, cf. A. Enmann, *Untersuchungen über die Quellen des Pompeius Trogus für die griechische und sicilische Geschichte* (Dorpat, 1880). Justin 18 is equivalent to Timaeus F131, and Justin 11 corresponds to Iam. *VP* 56, which directly follows a fragment of Timaeus (F17). The different reports about the luxurious living of the people of Croton (Tim. F44–45. Justin 1–2) can be reconciled (von Fritz *Pol.* 46f.) Still, this does not prove that Timaeus is Justin's only source. Is the impression of the unity of Justin's exposition perhaps only the result of its severe compression, and therefore to be credited to the epitomator rather than to the unitary source? Pompeius Trogus did use a number of different sources (O. Seel, *Die Praefatio des Pompeius Trogus* [Erlangen 1955] 18ff).

[36] 10.3–12. Diodorus is following a highly rhetorical, moralizing source, which treats the Seven Sages in the same manner (9.1–15; Schwartz, *RE* V 678f). It was later than Callimachus (10.6.4). Among its sources was Aristoxenus (Schwartz 679; von Fritz, *Pol.* 22ff), but there is no distinctive trace of Timaeus. The relation of the Damon–Phintias story in Aristoxenus (fr. 31) to the version in Diodorus (10.4.3) is controversial; in Aristoxenus, Dionysius wishes to test the steadfastness of a Pythagorean in danger of death, and therefore condemns the innocent Phintias, whereas in Diodorus the latter had really planned an attempt on the tyrant. Wehrli (*Aristoxenus* p. 57) finds that Diodorus' version, since it is simpler, is the earlier; but Cobet (*Collectanea critica* [Leiden, 1878] 433) and Nauck (LVII 40) stressed that the famous dénouement, Dionysius' wish to be admitted as a third member of their friendship, only makes sense if there was no deadly enmity but only a test engineered by the tyrant. Thus Aristoxenus' account is after all the original, and Diodorus is not using pre-Aristoxenian material.

[37] In the speeches of Pythagoras, Iam. *VP* 37–56, 56 corresponds to Timeaus F17; there is a relationship between 42 and Timaeus F146; 56 agrees with Justin 20.4.11, and 37, 40, and 47 are related to D.L. 8.22f. Local tradition of south Italy is evident in 40, 44, 46, 50, and 52. Furthermore, Timaeus F13 corresponds to Iam. *VP* 71–72 (von Fritz, *Pol.* 39). It is inferred from the connection with Timaeus that Iam. *VP* 71–72 comes from Apollonius, Rohde, *Q* 137; Delatte, *Litt.* 85f. (The latter adds sections 74–78 and therefore misinterprets the Lysis letter; cf. Burkert, *Philologus* 1961.) Timaeus F147 is to be compared with Iam. *VP* 35. On the basis of these correspondences, the conclusions are drawn that (1) the speeches of Pythagoras in Iam. *VP* 37–57 are taken entire from Timaeus ("selon toute vraisemblance textuellement Timaios," Boyancé, *RevPH* 29 [1955] 183), and that (2) all

uncomfortable number of intermediate sources, passing the material on in a continuous process of variation, combination, and compilation; and other historians, like Theopompus[38] and Duris,[39] also made mention of Pythagoras. Also, it would be strange if Antiochus of Syracuse, Philistus, and Ephorus had nothing to say about Pythagoreans in connection with the history of Magna Graecia; but not the slightest trace of such is discernible.

We cannot divide the whole of the later tradition neatly among the fourth-century sources.[40] On the whole, the "later" tradition seems to be not so much the result of unscrupulous falsification as of simple-minded, naive compilation and transmission of whatever could be found, contradictions and all. Still, even if the source analysis takes us back as far as we could hope, to the very beginning of biographical study in the early Peripatos, Zeller's skepticism cannot be considered

other Apollonius sections have this origin, viz. Iam, *VP* 215ff (Delatte, *Litt.* 25 n. 3; disagreeing, Boyancé; cf. above n. 13), and especially Iam. *VP* 254ff (without hesitation Bertermann 37ff; Delatte, "La chronologie pythagoricienne de Timée," *Musée Belge* 1920, 5–13: "Apollonius se révèle tributaire de Timée pour son histoire du Pythagorisme presque toute entière" [13]; cf. *Litt.* 86, *Vie* 169, Rostagni, "Pitagora e Pitagorici in Timeo," *ScrMin* II 1, 3–50). On the other hand von Fritz, *Pol.* 55ff. shows that in Iam. *VP* 254ff Apollonius' method is that of "what might almost be called large-scale historical fresco painting" (61), that he uses much old material but cannot be relied on for details. Cf. Jacoby, *FGrHist* IIIb, *Komm.* 550ff, *Noten* p. 325 n. 191c. The ambivalence of the arguments is shown, for example, by the repeated connection of Pythagoras with Tauromenium (Iam. *VP* 33=Por. *VP* 21, from Nicomachus; Iam. 112, 136)—but Tauromenium was not founded till 403 B.C. Shall we say that the local patriotism of the historian from Tauromenium is in evidence here, or that Timaeus, of all people, is excluded by such a monstrous anachronism?

[38] *FGrHist* 115F72–73.

[39] Duris *FGrHist* 76F22–23; here Samian patriotism plays a part.

[40] The catalogue of Pythagoreans Iam. *VP* 267=DK 58A is a quite isolated piece of erudition: 145 of its 235 names occur only here (cf. the lists of Brown, 1vff). And it is not Iamblichus' own composition, since he elsewhere names 18 other Pythagoreans. It is independent of the ps.-Pythagorean literature, lacking 18 of the "Pythagorean" writers catalogued by Thesleff. Most surprising is the omission of the Locrian Timaeus; Diels's proposal to include the Parian Timaeus among the Locrians (DK I 447.3) is arbitrary. There is without doubt a close relation to Aristoxenus (Rohde, *Q* 171): compare the Phliasians p. 146.6 with Aristox. fr. 18–19; Damon and Phintias, p. 146.1, with Aristox. fr. 11; Hippo as a Samian, p. 146.3, with Aristox. fr. 21; Philolaus and Eurytus as Tarentines, p. 144.11, with Aristox. fr. 19; Charondas and Zaleucus, pp. 146.12, 145.8 with Aristox. fr. 43.—The form of the list of names, classified according to geography, corresponds to the documentary tradition which we see in inscriptions (e.g., *IG* II² 1697ff, 1740ff), not to the literary tradition (e.g. Iam. *VP* 266, D.L. 10.22ff, Por. *VP* 7–9, Regenbogen, *RE* s.v. Pinax, XX 1451f). There seems to be genuinely Italian material among the names; cf. A. von Blumenthal, *Glotta* 17 (1929) 104f, 152ff. They take us down to the first half of the fourth century (cf. also Oldfather, *RE* XI 74). The only possible candidate for authorship seems to be Aristoxenus himself, working in the documentary method of the earliest Peripatos (cf. *FGrHist* 124F23). Philochorus (*FGrHist* 328T1) wrote α *Συναγωγὴ ἡρωΐδων ἤτοι Πυθαγορείων γυναικῶν*. Is there perhaps a relation between this and the second part of the catalogue?

refuted.[41] It is precisely with the authors in the circle of Plato and Aristotle that the real problems begin.

Most obvious is the contradiction between Aristoxenus and Dicaearchus, regarding the catastrophe that overwhelmed the Pythagorean society. One of the two reports must be basically wrong: either Pythagoras withdrew to Metapontum before the outbreak of the unrest and died there (as Aristoxenus says) or he and his followers were hounded from city to city (as Dicaearchus has it).[42] Like his doctrines, the life of Pythagoras also becomes a mirror image of real controversies in the schools.[43] On the one hand there is the controversy over the primacy of the theoretical or the practical life (*βίος θεωρητικός*, *βίος πρακτικός*). In this respect Heraclides thinks of Pythagoras as the apostle of pure "theory"; to Aristoxenus and Dicaearchus he is an active politician.[44] Then, in the matter of his doctrines on the soul, Heraclides, following Plato, advocates the immortality of the soul and seeks proofs for its independence of the body. This explains his interest in Abaris, in the metempsychoses of Pythagoras, and the "apparently dead woman" of Empedocles. Like Xenocrates, he expands Plato's myths into a system that comprehends both science and religion, and joins together astronomy and immortality.[45] We need not be bound by Timaeus' characterization of him as a *παραδοξολόγος*;[46] but in his treatment of the anecdote about *φιλοσοφία* we can show that, contrary to history, he projected Plato's conception onto Pythagoras.[47]

Dicaearchus said that "soul" is a mere word (frr. 7ff), and there is an unmistakable irony in his account of Pythagoras' doctrine of metempsychosis. Pythagoras was once, he says, a beautiful courtesan.[48] Dicaearchus has Pythagoras, at the end, stumbling from one catastrophe to another; and the Locrians, who were famous for their *εὐνομία*, denied him admittance to their city.[49]

Aristoxenus dealt most fully with Pythagoras and his pupils. He was

[41] Cf. the introduction, above. Not only Capparelli (I 351 and passim) but also Morrison (CQ 1956, 135) maintain that Zeller's skepticism can no longer be upheld in view of the results of source analysis.

[42] Below, ch. II 2.

[43] Cf. the discussion of Wehrli in his commentaries on the fragments of the Peripatetics.

[44] Jaeger, *SBBln* 1928, 396 n. 1, 415ff, after Rohde, *KlSchr* II 110.

[45] Below, ch. IV 4.

[46] *FGrHist* 566F6 Heraclides fr. 84.

[47] Burkert, *Hermes* 1960, 159ff.

[48] Fr. 36. The irony was noticed by Rohde, *Psyche* II App. 10 (Eng. tr.), and Wehrli, p. 53.

[49] Fr. 34–35. Here there is polemic against Aristoxenus (fr. 43), who made Zaleucus a pupil of Pythagoras.

a native of Tarentum, and cited in evidence the acquaintance of his father with Archytas,[50] as well as his own acquaintance with the "last" of the Pythagoreans.[51] Clearly, he put himself forward as an expert in Pythagorean matters, just as he was, at the same time, an authority in musical theory. He considered false, however, the numerical theory of the tone intervals, which is specifically attested by Plato and Aristotle as Pythagorean,[52] and did not believe in the immortality of the soul, which he called a *ἁρμονία*, with all the consequences drawn from that idea in Plato's *Phaedo*.[53] Thus Aristoxenus has to steer a complicated course.[54]

In his *Harmonics*, Aristoxenus disposes in a single sentence of the music theory advocated by the Pythagoreans down to Archytas, without even naming the Pythagoreans;[55] what does not suit him is not mentioned. Aristoxenus characterizes the Pythagoreans with whom he was acquainted, as the "last," suggesting that with them the school ended—but there was a Pythagorean named Lycon who wrote against Aristotle, and there are also other traces that lead into the later fourth century.[56] Aparently Aristoxenus does not recognize these men as Pythagoreans. Finally, the assertion that Pythagoras was fonder of beans than anything else can only be veiled polemic against the taboo on beans attested by Aristotle and Heraclides.[57]

The *Πυθαγορικαὶ ἀποφάσεις* show the lineaments of a rational, clearly articulated ethic, oriented toward practical needs. Its political precepts are surprisingly similar to those of Plato's *Republic*, though the predecessors of Plato named by Aristotle in this connection are Phaleas

[50] Fr. 30; cf. frr. 47ff; Plut. *De gen.* 592f: on fr. 2, von Fritz, *Gnomon* 32 (1960) 495.

[51] Frr. 18–19; cf. fr. 1; below, ch. II 6.

[52] Below, ch. V 1.

[53] Frr. 118ff.

[54] Aristoxenus is the fullest of the ancient sources for Pythagoreanism, and therefore the question of his credibility is especially important and much discussed. The very rationality which characterises Aristoxenus' Pythagoreans, far from mysticism and magic, seems a favorable sign to some scholars, suspicious to others. On the positive side, following Krische, are, for example, Delatte (*Litt. ip. Pol.* 213), Rostagni (*ScrMin* I 137ff), Howald (*JAW* 197 [1923] 163), with qualifications von Fritz (*Pol.* 27ff), Wuilleumier hesitatingly (602ff). On the negative side: Lévy (*Sources* 44ff), Frank (260, *AJP* 1943, 221 [where he wrongly depends on the formulation of Euseb. *Praep. evang.* 15.2=Aristox. fr. 64, followed in this by van der Waerden, *SA* 107]).

[55] Page 32 M., emphasized by Frank, *AJP* 1943, 221f. It should be kept in mind, in this connection, that, according to Aristocles in Eusebius, Aristoxenus makes malicious reference to Aristotle without calling him by name (fr. 64; cf. Wehrli, p. 68).

[56] On this, below, ch. II 6.

[57] Below, ch. II 4 n. 124. The report on Pythagoras as an eater of meat in fr. 25 is biased in the same direction; cf. below, ch. II 4, n. 111.

and Hippodamus, and not the Pythagoreans.[58] When Aristotle says that none of the "ancients" had dealt in detail with the nature of *Τύχη*, whereas Aristoxenus presents as "Pythagorean sayings" precisely what the *Eudemian Ethics* expounds as *τύχη*,[59] then it is obvious that fourth-century ethics, and not ancient Pythagorean tradition, is being set forth. Finally, Zaleucus and Charondas, contrary to any chronological scheme, are made into pupils of Pythagoras,[60] and we learn that Pythagoras was the first to introduce weights and measures into Greece.[61]

Not that Aristoxenus invented it all; he had sources of information, but like the Platonists he interpreted Pythagoreanism in accordance with his own preconceptions.[62] And this is the basic fact that finally emerges from analysis of the sources: though the late tradition may be traced back to the writers of the fourth century B.C., this does not lead us onto firm ground but into the precarious territory of that day's controversies. Instead of reliable facts one finds the shifting claims to a Pythagorean tradition apparently already in a state of flux, for the benefit of the peculiar concerns of the pupils of Plato and Aristotle. One may attempt to discover historical truth from amidst the polemic—not, naturally, isolating the individual testimonies, but considering each along with the others that contradict it. But the foundation of historical research must be the pre-Platonic evidence because this belongs to a time before Pythagoreanism sank into the quicksand of

[58] Cf. A. Rivaud, "Platon et la 'politique pythagoricienne,'" *Mél. Glotz* (Paris, 1932) 779–792: "Aristoxène a reconstruit, avec le secours des textes de la République, une politique pythagoricienne qu'aucun auteur pythagoricien n'avait jamais formulée expressément" (784). M. Pohlenz (*NGG*, ph.-h. Kl,. 1924, 19–32) inferred as the source of the ps.-Demosthenic first oration *Against Aristogeiton* a Pythagorean writing *Περὶ νόμων*. But aside from the problems of detail involved in such a reconstruction, it would not lead us to pre-Platonic terrain. The trial of Aristogeiton took place about 324 B.C., so that there might be influences from Socratics, Academics, or Peripatetics, not even excluding Aristoxenus.

[59] Aristox. fr. 41; *EE* 1246b37ff, 1248b2, 1214a15; the testimony of Aristotle about the "ancients," *Phys.* 195b35ff. But compare 196b5; H. Täger, *De Aristoxeni libro Pythagorico* (Diss. Göttingen, 1922). This unpublished dissertation has unfortunately remained almost unknown; even Wehrli does not use it. Yet Wehrli too says (59): "The hallmark of the *'Αποφάσεις* is to lay claim to Academic-Peripatetic material on behalf of the Pythagoreans."

[60] Fr. 43; followed by Posidonius (Sen. *Ep.* 90.6).

[61] Fr. 24; cf. below, ch. VI 1.

[62] The *Πυθαγορικαὶ ἀποφάσεις* seem to be a modern substitute for the *acusmata*. Wehrli (61f) refers, on fr. 37, to Arist. *Met.* 985b30. Concise formulations of popular "wisdom" —*καιρός* 7, *ὡρίζοντο γὰρ ἐπιπολαίως* (987a22) are analyzed and more fully developed by Aristoxenus.

the school controversies.[63] We may reasonably expect information about the early school from those later authors who are critical of Pythagoreanism rather than from those who identify themselves with it and are therefore forced into radical reinterpretations of it. From this point of view the testimony of Aristotle again emerges as especially important, for he clearly, in his lost book on the Pythagoreans, was concerned to collect material, rather than to interpret it in such a way as to fit into his own system.

It is customary to complain about the sparseness of early testimonies. Still, whereas we have no explicit reference to Anaximander or Parmenides by any fifth-century author, there is a quite imposing array of references to Pythagoras.[64] Interestingly, most of these had already been assembled in ancient times; we have them as quotations in the lives of Pythagoras.[65] The history of Pythagoreanism was already, at that time, the laborious reconstruction of something lost and gone.

2. HISTORICAL BACKGROUND

It is only in post-Aristotelian sources that biographical and historical details regarding Pythagoras and the Pythagoreans are to be found. The most important accounts are those of Aristoxenus and Dicaearchus, but they differ diametrically on one vital point—the role of Pythagoras in the revolution in Croton. Considering these authorities and also the later tradition, of which some indeterminable portion comes from Timaeus, one is tempted to say that there is not a single detail in the life of Pythagoras that stands uncontradicted. It is possible, from a more or

[63] In a still more radical spirit, Gigon (*Ursprung* 123f) bases his account exclusively on fifth-century testimonies, and Fränkel (*DPh* 354ff) relies entirely on Xenophanes, Heraclitus, and Empedocles. The most ancient evidence is collected in Zeller *SBBln* 1889 985, 996 *KlSchr* I 458–472, Frank 356f, Timpanaro Cardini 12ff.

[64] In summary: Xenophanes DK 21B7=D.L. 8.36; Heraclitus B40=D.L. 9.1, B81=Phld. *Rhet.* I c. 57; 62+Schol. Eur. *Hec.* 131, B129=D.L. 8.6; Empedocles B129=Timaeus *FGrHist* 566F14=D.L. 8.54=Nicom. (Por. *VP* 30=Iam. *VP* 67); Hdt. 2.81, 4.95, cf. Hellanicus *FGrHist* 4F73; Ion of Chios DK 36B2=D.L. 8.8, Clem. Al. *Strom.* 1.131, 134 D.L. 1.120; Glaucus of Rhegium D.L. 9.38; Democritus A33=D.L. 9.38; DK 68B60a; *Dissoi logoi* 6.8. Contemporary with Plato: Isoc. *Bus.* 28; Alcidamas D.L. 8.56; Alcidamas, Arist. *Rhet.* 1398b9; Anaximand. Hist. *FGrHist* 9T1=*Suda* s.v.; Andron of Ephesus D.L. 1.119, Euseb. *Praep. evang.* 10.3.6 (DK 7A6); Theopompus *FGrHist* 115F72 73. On Antisthenes, see below, ch. II 2, n. 38.

[65] Most discussed in antiquity were the verses of Empedocles, B129 (cf. the previous note). D.L. 8 adds Xenophanes, Heraclitus B129, Ion B2. Clement dates Pythagoras by Heraclitus B40 (*Strom.* 1.129.4). Herodotus' story of Zalmoxis influenced the later tradition in many ways (cf. D.L. 8.2, Por. *VP* 14f, Iam. *VP* 173).

less critical selection of the data, to construct a plausible account;[1] but it is bound to rest on shaky foundations, for no documentary evidence has appeared.[2]

A mainstay in discussion of the chronology[3] is the report of Aristoxenus, according to whom Pythagoras left Samos for Italy in 532/531 B.C. because of the oppressive tyranny of Polycrates.[4] On the other hand, Eratosthenes identified the philosopher Pythagoras with the Pythagoras of Samos who was an Olympic victor in 588 B.C.[5] The tradition provides examples of other early dates, too,[6] and in the other direction Alcidamas and Timaeus make Empedocles the direct pupil of Pythagoras, which would push his date far down into the fifth century.[7] Xenophanes and Heraclitus refer to Pythagoras;[8] for Herodotus he does not belong to a very distant past.[9] A statement in Iamblichus, which probably goes back to Aristotle, places him after the Seven Sages,[10] and an old tradition makes him a pupil of Pherecydes of Syros.[11] Thus various strands of evidence lead to the second half of the sixth century and to this extent support Aristoxenus, even though there is in his reference to Polycrates a certain tendentious quality.

Pythagoras came from Samos,[12] and his father's name was Mnesarchus.[13] We cannot determine the age of certain local traditions of Samos

[1] Cf. Zeller I 380ff. The most important of the older studies is Krische. See further Delatte, *Pol.*; von Fritz, *Pol.* and *RE*; Minar; Ciaceri II 86ff, 337ff; Dunbabin 359ff, 366ff, 369ff.

[2] A striking piece of original evidence for the extraordinary fame that surrounded Pythagoras as early as the 5th century is perhaps to be seen in the "Pythagores" coins of Abdera. See Seltman, *Greek Coins* (London, 1933[1], 1955[2]) 142ff, pl. xxviii 11 (cf. *Numismatic Chronicle* 6.9 [1949] 21); J. M. F. May, *The Coinage of Abdera* (London, 1966) pp. 157, 167, with pl. 13 no. 218 (P183); also, addendum p. 176 with text-cut and description of no. 218/1 (P183/1). The date is given as 430/420 B.C. On the obverse the coins of Abdera show the picture of a griffin, after that of the mother-city Teos. The reverse has, after about 450 B.C., the name of the mintmaster in charge (who may have been the priest of Apollo, Abdera's patron deity), and in the *quadratum incusum* a symbol which changes from one mintmaster to another and, at least in some cases, bears an obvious relationship to his name—as when Python uses a tripod, Euagon a prize amphora, Nicostratus a soldier, or Molpagores a girl dancing (nos. 345–348, 293–295, 313–315 May). The coins of the mintmaster ΠΥΘΑΓΟΡΗΣ display an idealized, bearded head, in two different types. The design must be related to the person of the mintmaster, and may be related to his name. Seltman's bold conclusion, that the person represented is the famous Pythagoras himself, seems likely. G. M. A. Richter's suggestion that the mintmaster portrayed himself (*Greek Portraits* IV [Brussels, 1962] 17–19) has been disproved by the appearance of a second, different type. The representation must be that of an ideal person, not a living individual; see W. Schwabacher in *Stockholm Studies in Class. Archaeology* 5 (1968) 59–63. In any case, we know Pythagoras was known in Abdera at this epoch (Democritus, D.L. 9.38; Herodotus cites the Greeks living on the Black Sea and Hellespont for the relation between Pythagoras and Zalmoxis; cf. below, ch. II 3). To be sure, the interpretation cannot be proven, and there are other coins of Abdera with heads that cannot be identified.—Certain ancient portraits have been thought to depict Pythagoras: (1) a bust from Herculaneum, the copy of a non-Attic work of the 4th century B.C. (K. Schefold, *Die Bildnisse der antiken Dichter, Redner und Denker* [Basel, 1943] 100f); (2) a neoclassical statue in Rome (Schefold 160.1); (3) a bust in Ostia (G. Becatti, "Ritratto di un vate antico," *Bolletino d'Arte* 34 [1939] 97–110). In addition, Pythagoras appears on coins of Samos from Trajan to Decius (Head 606; Schefold 173.17–18; *British Museum Coins, Ionia*, nos. 237, 257, 287, 351, 365), and on a contorniate from the 4th century A.D. (Schefold 173.19).

[3] Cf. Zeller I 381 n. 1; Rohde, *Q* 118ff; Jacoby, *Apollodor* 215ff; Lévy, *Sources* 2ff; von Fritz, *Pol.* 68ff and *RE* 179–185.

[4] Aristox. fr. 16 W., followed by Apollod. *FGrHist* 244F338–339; cf. IIIb *Noten* p. 326 n. 198. In spite of this, Aristoxenus made Zaleucus and Charondas pupils of Pythagoras (fr. 43).

[5] *FGrHist* 241F11: D.L. 8.47; ibid. 48, an epigram on this Pythagoras by Theaetetus of Cyrene (time of Callimachus), and (49) an anonymous epigram which gives his father's name as Crates. Eratosthenes' list of Olympic victors must have been reliable for the period in question; i.e. the Olympic victor of 588 is a historical figure (Rohde, *Q* 118). Lévy (*Sources* 20ff) sees the whole as part of the *légende d'enfance* that grew up about Pythagoras: even as a boy he won an Olympic victory.

[6] Pliny the Elder (*HN* 2.37) dates an astronomical discovery of Pythagoras to 612/609; and at 36.71 he places him in the reign of Psemetnepserphres (=Psammetichus I, 664–610 B.C., or Psammetichus II, 594–589; naturally the two could easily be confused).—Eusebius (*Arm. Chron.* p. 14.30 Karst) places Pythagoras under the successor of Sanherib (i.e. after 681 B.C.; cf. Jacoby, *FGrHist* IIIa 296, on 273F79: not Alexander Polyhistor, but an addition by Eusebius). The Numa story (Jacoby, *Apollodor* 225f; cf. Burkert, *Philologus* 1961) and the encounter with Phalaris (Iam. *VP* 215ff) were invented without consideration of chronological matters.

[7] Alcidamas ap. D.L. 8.56, Timaeus *FGrHist* 566F14=D.L. 8.54. There is no reason to doubt the text, as Lévy does at *Sources* 54 n. 3; neither the dating of Pythagoras nor that of Empedocles by Timaeus is known. (For contradictory reconstructions, see Delatte, *Musée Belge* 1920; Rostagni, *ScrMin* II 1 pp. 3–50; cf. von Fritz, *Pol.* 47ff, *RE* 180–184; Jacoby *FGrHist* IIIb *Komm.* 552.) At Iam. *VP* 44 Pythagoras is represented as speaking of 7 Crotonian Olympic victors; and this could only apply to the years between 508 and 496 (cf. A. Moretti, *Olympionikai* [Rome, 1957]). Can this come from Timaeus (cf. ch. II 1, n. 37)?

[8] Zeller felt (I 640 n. 1; cf. Rathmann 38) that in fr. 7 Xenophanes speaks of Pythagoras as though he were already dead; and Heraclitus (fr. 40) speaks of Pythagoras in the same breath as Hesiod—though scarcely on chronological grounds. Cf. below, ch. II 6.

[9] 4.96: cf. below, ch. II 3.

[10] Iam. *VP* 83; cf. below, ch. II 4.

[11] See below, ch. II 3. The synchronism of Pythagoras and Alcmaeon at Arist. *Met.* 986a29 is an interpolation; cf. above, ch. I 2, n. 6.

[12] Hdt. 4.95; Hellanicus *FGrHist* 4F73; Isoc. *Bus.* 28; Hermippus ap. D.L. 8.1; etc. He was, however, a "Tyrrhenian" according to Theopompus *FGrHist* 115F72, Aristox. fr. 11a–c W., and Aristotle (MSS Aristarchus) fr. 190, from one of the "Tyrrhenian islands of the northern Aegean." (The statement in Plut. *Quaest. conv.* 727b–c that he came from Etruria, is Plutarch's improvisation.) He was a Syrian from Tyre according to Neanthes *FGrHist* 84F29 (cf. Apollonius ap. Iam. *VP* 5f, 7, 13). Lycus (?) ap. Por. *VP* 5 says λέγουσι γὰρ αὐτὸν οἱ μὲν εἶναι Σάμιον οἱ δὲ Φλιάσιον, οἱ δὲ Μεταποντῖνον (cf. below, ch. II 3, n. 11). Phlius appears in the genealogies, varying in detail, of D.L. 8.1 and Paus. 2.13.2. (For Pythagoreans from Phlius, cf. Aristox. fr. 19 and Echecrates in Pl. *Phd.*; for Pythagoras and Phlius, Heraclides fr. 87 W. A Φλιάσιος σωμασκητής is distinguished as a homonym by D. L. 8.46; cf. below, ch. II 4.) Zeller's conciliatory suggestion (after K. O. Müller) that Pythagoras came from a "Tyrrhenian-Pelasgian family that had migrated from Phlius to Samos" (I 380.2) will scarcely find support any more.

[13] Heraclitus fr. 129, Hdt. 4.95. In Iamblichus the name is consistently miswritten as "Mnemarchus." It is given as Marmacus in the genealogy of D.L. 8.1 (cf. von Fritz, *RE* XXIV 172; Mamercus as son of Pythagoras, Plut. *Aem. Paul.* 1); *Demarato* (or *de Marato*?) *natus*, Justin, 20.4.3.

in which Pythagoras figured.[14] As early as Herodotus he is connected with Egypt.[15] In the tales of his journeys to visit the Phoenicians, the Chaldaeans, and the Magi there is a good deal of imaginative conjecture, though he unquestionably had some sort of contact with the Orient.[16] The principal site of his activity was southern Italy, and the traditions center about the cities of Croton and Metapontum.[17] It is well attested that his death took place in the latter city.[18]

Not surprisingly, Pythagoras' teaching is permeated with the kind of religion characteristic of Magna Graecia. Typical of this is the prominence of the chthonian divinities—Demeter, Persephone, Dionysus—and of eschatological beliefs, especially the type that produced the numerous representations of the journey of the deified dead into the Beyond.[19] This religious situation is probably older than Pythagoras. The prominence of Aphrodite beside Persephone in the votive tablets from Locri[20] surely cannot be derived from Pythagoreanism, but there is nothing distinctively Pythagorean in the famous gold tablets, either.[21] Pythagoras entered a religious world of a peculiar character, in which Mediterranean, Italic, and pre-Doric, Achaean elements were amalgamated.[22]

It is remarkable that only Croton and Metapontum, among the south Italian cities, are noted for the worship of Apollo. There seem to have been quite ancient Apollo cults in Metapontum,[23] as well as in Macalla, which belonged to Croton.[24] The facts that Croton used the tripod of Apollo on its oldest coins (about 550 B.C.),[25] that Caulonia,

[14] Antiphon (Por. *VP* 9 = Iam. *VP* 26f) speaks of a *Πυθαγόρου καλούμενον ἔτι καὶ νῦν ἡμικύκλιον, ἐν ᾧ Σάμιοι περὶ τῶν κοινῶν βουλεύονται*, and an *ἄντρον* before the city. Local patriotism may play a part in the report of Duris (*FGrHist* 76F23) about an epigram and dedicatory offering of Pythagoras' son Arimnestus in the Heraeum at Samos. A "Samian poet" is cited by Apollonius (Por. *VP* 2 = Iam. *VP* 5). On Samian coins, see n. 2 above.

[15] 2.81 (below, ch. II 3); Isoc. *Bus.* 28; Hecataeus of Abdera *FGrHist* 264F25 = Diod. 1.69.4, 96.2, 98.2. The letter of introduction from Polycrates to Amasis (Antiphon ap. Por. *VP* 7 = D.L. 8.3) is an invention based on Agesilaus' letter to Nectanebis in behalf of Eudoxus (D.L. 8.87). See further Zeller I 387.1.

[16] Aristoxenus alleged that he paid a visit to "Zaratas" (fr. 13 = Hippol. *Ref.* 1.2.12; on the extent of the Aristoxenus material see Wehrli 50f and W. Spoerri, *REA* 57 [1955] 267–290). Alexander Polyhistor followed him in this (*FGrHist* 273F94; cf. Jacoby IIIa 294ff). It was a chronologically impossible idea to imagine that Pythagoras got from Egypt to Babylon as a prisoner of Cambyses (*IG* XIV 1297, II 20 = *FGrHist* 252B7; Apul. *Flor.* 15, p. 56; *Th. ar.* 53.1ff; Iam. *VP* 19; the episode is modeled after the Democedes story). Further references in Zeller I 384ff, as also for his supposed contacts with the Thracians (Zalmoxis, below, ch. II 3, n. 202), with Arabs, Jews, Indians, and the Druids of Gaul. The god Men and the taboo on white roosters are certainly from Asia Minor (ch. II 4, n. 47), and the wearing of trousers is Persian or Scythian (Ael. *VH* 12.32).

[17] Alcidamas in Arist. *Rhet.* 1398b10f: *Ἰταλιῶται Πυθαγόραν (τετιμήκασιν)*. The legends are mainly localized in Croton (of which Caulonia was a colony) and Metapontum (ch. II 3). Pythagoras is Metapontine to Lycus (?) ap. Por. *VP* 5; Brotinus (DK 17.1), Theano (Iam. *VP* 267, pp. 146.22 and 132), and Hippasus (DK 18.1–2) are sometimes from Croton, sometimes from Metapontum.

[18] Arist. fr. 191, Aristox. fr. 18, Dicaearchus fr. 34–35 (cf., for further material, Zeller I 417ff; Delatte, *Pol.* 203ff, *Vie* 241ff, Lévy, *Lég.* 63ff). In Metapontum, Cicero saw "Pythagorae ipsum illum locum, ubi vitam ediderat, sedemque" (*Fin.* 5.4; this may of course be a rediscovery for the benefit of tourists). According to Porphyry (*VP* 4 = *FGrHist* 566F131), Timaeus spoke of the house of Pythagoras in Croton, and the same report is given, but with Metapontum as the city, by Favorinus (D.L. 8.15), Justin (20.4.18), and Iamblichus (*VP* 170). Val. Max. 8.15.1 is corrupt. It is more likely that Porphyry made an error (or his source; cf. Rohde, *Q* 133.1; Delatte, *Vie* 183; Lévy, *Sources* 54.1, *Lég.* 65) than that Timaeus purposely substituted Croton for Metapontum (as Jacoby, *FGrHist* IIIb *Komm.* 551; Jäger 20ff).

[19] Cf. Giannelli, Wuilleumier 469 560. Wilamowitz (*Platon* II 84) had already called attention to the religion of the western Greeks and its connection with Pythagoreanism.

[20] Quagliati, *Ausonia* 3 (1908) 136–234; W. A. Oldfather, *Philologus* 69 (1910) 114–125, *RE* XIII 1349ff, Delatte, *Pol.* 7; see now H. Prückner, *Die Lokrischen Tonreliefs* (Mainz 1968), who points out their connection with the ritual prostitution in the temple of Aphrodite in Locri (Justin 21.3, Clearchus fr. 43a W.). The ancient authorities contradict each other on the connection between Locri and Pythagoreanism; Aristoxenus (fr. 43) calls Zaleucus a Pythagorean, but Dicaearchus (fr. 34) tells how the Locrians refused Pythagoras admission to the city. On "Timaeus of Locri" see above, ch. I 4, n. 7.

[21] DK 1B17–21. A new tablet, of the fourth century, has been discovered at Pharsalus (N. M. Verdelis, *Arch. eph.* 1950–1951, 98ff). Six of the total of twelve were found in southern Italy (Petelia, Thurii). It is usual to call them "Orphic-Pythagorean" (cf. Nilsson, II 223ff; Ziegler, *RE* XVIII 1386ff). On the other hand, some have claimed that they are Pythagorean but not Orphic (H. M. R. Leopold, *Mélanges d'Archéologie et d'histoire* 39 [1921–1922] 170f; Wilamowitz, *GldH* II 202f; Thomas 130ff; cf. Cumont, *Symb.* 371, n. 1, 377, n. 6, 423, n. 5; Bidez, *Eos* 9.2). This seems less likely, however, since the oldest of all has been discovered in Thessaly. The central figures are Persephone and Dionysus (who was worshiped in Metapontum as Dionysus Eriphius: Thomas 140ff). Eubuleus, also present, makes one think of Eleusis (Thomas 138f; cf. 146ff). Nothing points unequivocally to Orpheus, but specifically Pythagorean elements cannot be demonstrated, either. There is no direct mention of metempsychosis on the tablet from Pharsalus (or that from Petelia). Vague associations, like the occurrence of the cypress (cf. D.L. 8.10) and the choice between right and left (cf. above, ch. I 2, n. 49; on the "Y," the *littera Pythagorae*, see Burkert, *Philologus* 1961, 230) cannot suffice to show that Pythagoreanism is the source of the whole. We must also consider the very old silver tablet from Posidonia (*IG* XIV 665; cf. Giannelli 147f), with the inscription *ΤΑΣ ΘΕΟ Τ‹Α›Σ ΠΑΙΔΟΣ ΕΜΙ* (like DK 1B18.8: *δεσποίνας δ' ὑπὸ κόλπον ἔδυν χθονίας βασιλείας*). Behind this lies an essentially pre-Pythagorean mystery rite, in which Pythagoreans could of course participate.

[22] On relations between Metapontum and Messenia, see ch. II 3, n. 242. On relations with the Etruscans, Furtwängler, *Gemmen* III 202f, 254ff; Ferrero 120ff; van Essen (skeptical). J. Gagé tries to show that Dodona was a center of religious influence for Italy (*RHR* 145 [1954] 137–167; 146 [1954] 18–50, 129–139; 147 [1955] 1–31).

[23] Apollo Lyceus and Apollo Carneus; cf. Giannelli 61f.

[24] Ps.-Arist. *Mir. ausc.* 107, 840a15; cf. Giannelli 162–167.

[25] The coins of Croton have often been brought into connection with Pythagoreanism. Cf. Head, 99 (negative); Giannelli 152–153 (doubtful); Boyancé, *Muses* 238; Mary White, *JHS* 74 (1954) 36–43; A. M. de Guadan, *Numisma* 34 (1958) 9–24. The most enthusiastic advocate of the Pythagorean interpretation is C. Seltman, who even supposed that Pythagoras, who was the son of a gem engraver (D.L. 8.2), designed these coins himself (*Numismatic Chronicle* 6.9 [1949] 1–21, *Greek Coins* [London, 1955²] 77f). This is refuted by the fact that the oldest coins of Magna Graecia were issued by Sybaris, not Croton; cf. De Vogel, *Pythagoras* 52–54.

a colony of Croton, showed on its coins Apollo *καθαρτής* with a stag,[26] that in about 470 Metapontum issued Apollo coins[27]—these facts doubtless have implications about the soil in which Pythagoras' doctrine took root. His unique success had a quite individual kind of background, which combined the piety of chthonic mysteries with worship of Apollo the "purifier."

The traditions about the persons surrounding Pythagoras scarcely take us out of the realm of legend. The name of Theano is famous; she is usually called Pythagoras' wife, but sometimes his daughter, or only his pupil;[28] but Theano is also the name of the wife of the mythical king Metapontus![29] Bro(n)tinus, named in the book of Alcmaeon, is sometimes called father and sometimes husband of Theano,[30] and both are placed sometimes in Croton and sometimes in Metapontum.[31] Pythagoras' son and successor is most often Telauges,[32] but he also has other sons, daughters, and successors.[33] Of the other "Ancient Pythagoreans" collected in the *Fragmente der Vorsokratiker*, Cercops is probably a figment of ancient philology,[34] Petron perhaps an invention,[35] Paron a misinterpretation of Aristotle,[36] and Parmiscus is only attested in the cult legend of Delos.[37]

[26] Oldfather, *RE* XI 81ff; K. Lange, *Götter Griechenlands* (Berlin, 1940) pl. I and n. 117; E. Simon, *Charites: Studien zur Altertumswissenschaft* (Bonn, 1957) 41f; S. P. Noe. *The Coinage of Caulonia* (New York, 1958); L. Lacroix, *Rev. Belge de numism.* 105 (1959) 5–24.

[27] Below, ch. II 3, n. 150.

[28] Cf. von Fritz, *RE* V A 1379–1381; *διεβοήθη τοὔνομα*, Dicaearchus ap. Por. *VP* 19. Wife of Pythagoras: Hermesianax fr. 2, 85 D.; D.L. 8.42 = Schol. Pl. *Rep.* 600b; *Suda* s.v. Pythagoras; Iam. *VP* 146, 265. Daughter of Pythagoras: Anon. Phot. 438b31. Pupil, wife of Brotinus: D.L. 8.42; *Suda* s.v. Theano; Iam. *VP* 267 p. 146.22. Those who ascribed complete celibacy to Pythagoras naturally could not make Theano either his wife or his daughter.

[29] *RE* V A 1379. On Metapontus, cf. Kroll, *RE* XV 1326.

[30] DK 17 (Brontinus); Alcmaeon fr. 1; father of Theano, D.L. 8.42; husband of Theano, ibid. *Suda* s.v. Theano, Iam. *VP* 267.

[31] Above, n. 17.

[32] Von Fritz, *RE* V A 194–196; Dittmar, *Aischines* 214ff; successor of Pythagoras: Anon. Phot. 438b30; son of Theano: Schol. Pl. *Rep.* 600b = *Suda* s.v. Pythagoras (Hesychius); D.L. 8.43; cf. Iam. *VP* 146, Euseb. *Praep. evang.* 10.14.14, Por. *VP* 4, etc.

[33] His son Mnesarchus: Anon. Phot. 438b29, Iam. *VP* 265, Schol. Pl *Rep.* 600b = *Suda* s.v. Pythagoras, Euseb. *Praep. evang.* 10.14.14; his son Arimnestus: Duris *FGrHist* 76F23 (Por. *VP* 3); his son Mamercus: Plut. *Aem. Paul.* 1; "Damon" Schol. Pl. *Rep.* 600b (a corruption of "Damo," his daughter [D.L. 8.42 = letter of Lysis, Hercher, *Epistologr. gr.* p. 603, Iam. *VP* 146]); other daughters: Aesara, Arignote, Myia: cf. Wellmann, *RE* s.v. Aisara, Arignote, Damo; von Fritz, *RE* s.v. Myia 2.—Iam. *VP* 265: *διάδοχος . . . πρὸς πάντων ὁμολογεῖται . . . γεγονέναι Ἀριστᾶιος.*

[34] Ch. II 3, n. 60.

[35] DK 16; cf. Jacoby on *FGrHist* 554F5 (Hippys of Rhegium; contra, Schmid I 2, 701ff); W. Vollgraff, *Mnemosyne* 4.2 [1949] 91ff; Burkert, *Philologus* 1959, 185f).

[36] Ch. II 4, n. 24.

[37] Ch. II 3, n. 191.

The tradition of the political activity of Pythagoras is consistently related to Croton, rather than to Metapontum. When it is reported that Pythagoras made a series of speeches before the boys, the young men, the women, and the magistrates,[38] this may well reflect an archaic, club-like organization of society.[39] But the reconstruction of the political events is difficult.

Two events were basically important in determining the history of Croton during this period, its defeat by Locri at the Sagras River[40] and the victory over Sybaris which is dated 510 B.C. From that time on, according to the evidence of coins,[41] Croton exercised some kind of hegemony in southern Italy. It clearly ended, suddenly, about 450 B.C., shortly before the revival of Sybaris-Thurii. The Pythagoras tradition contains reports, with various small modifications, of a great catastrophe: the house of Milo, which was the meeting place of the Pythagoreans in Croton, was burnt down by their opponents, and only a few of those present escaped.[42] Since one of the survivors, Lysis,

[38] Dicaearchus, fr. 33 (Por. *VP* 18), names magistrates, youths, boys, and women; the speeches are given in a different order (youths, magistrates, children, women) in Iamblichus (*VP* 37–57, from Apollonius, after Timaeus: above, ch. II 1, nn. 12, 37). In an explanation by the Socratic philosopher Antisthenes of the word *πολύτροπος* in *Od.* 1.1 (scholium on the line from the *Homeric Questions* of Porphyry), is found the sentence *οὕτω καὶ Πυθαγόρας λέγεται πρὸς παῖδας ἀξιωθεὶς ποιήσασθαι λόγους διαθεῖναι πρὸς αὐτοὺς λόγους παιδικούς, καὶ πρὸς γυναῖκας γυναιξὶν ἁρμοδίους, καὶ πρὸς ἄρχοντας ἀρχοντικούς, καὶ πρὸς ἐφήβους ἐφηβικούς.* There is no cogent reason to strike this sentence of the Antisthenes material, as being an addition by Porphyry (with H. Schrader; L. Radermacher, *Artium scriptores, SBWien* 227 [1951] 121f). Cf. also F. D. Caizzi, *Antisthenis fragmenta* (Milan, 1966) p. 107. Interest of some Socratic in Pythagoras may be reflected in Plut. *De curios.* 516c. But the content of Pythagoras' teaching was unknown, according to Dicaearchus ap. Por. *VP* 19. The speeches given in extenso by Iamblichus (*VP* 37–57) are a later production (*pace* Rostagni, *ScrMin* I 3ff, De Vogel, *Pythagoras* 70–147; cf. *Gymnasium* 74 [1967] 459).

[39] Cf. also Morrison, *CQ* 1956, 145ff. According to Timaeus *FGrHist* 566F131, Pythagoras' daughter was as virgin the leader of virgins, as matron the leader of the matrons of Croton. Cf. Commentary on Alcman 10 Page, fr. 1 iii.

[40] This event is usually dated ca. 530 B.C., but Kiechle (50f) would put it ca. 550, von Fritz (*RE* 183) ca. 520.

[41] Kahrstedt, *Hermes* 1918; von Fritz, *Pol.* 68ff; C. M. Kraay, "The Coinage of Sybaris after 510 B.C." *Num. Chron.* 18 (1958) 13–37; De Vogel, *Pythagoras* 55–57.

[42] The principal source is Aristoxenus, fr. 18. According to Dicaearchus, who refers to oral tradition current in his own day, such uprisings took place "everywhere" (fr. 34), and Polybius (2.39) speaks of the burning of Pythagorean *συνέδρια*, using the plural. Presumably Aristoxenus wanted to gloss over the extent of the anti-Pythagorean activities (von Fritz, *Pol.* 30). Cylon is named as the leader of the opposition; it is noteworthy that Aristotle names him as opponent of Pythagoras in his list of literary feuds (fr. 75 = D.L. 2.46, 8.49; in the former passage the reading *Κύλων καὶ Ὀνάτας* is defended by Diels, DK 14.15, against Menage's conjecture *Κύλων Κροτωνιάτης*; he refers to Iam. *VP* p. 143.23 Deubner).

later became the teacher of Epaminondas in Thebes,[43] this termination of Pythagorean dominance cannot have taken place before 450 B.C., and therefore certainly not during the lifetime of Pythagoras. It is tempting to interpret these events along with the numismatic evidence, to the effect that from about 510 to 450 B.C., under the leadership of a Pythagorean oligarchy, Croton experienced a period of prosperity and power, which then was broken up by internal strife. After this catastrophe, Tarentum became the center of Pythagoreanism.

It is a question, however, what Pythagoras himself had to do with these events. After the defeat at the Sagras, according to Justin's account, he succeeded in reforming the Crotoniates' character to such a degree, by "daily praises of virtue," that the victory over Sybaris became possible. Then, according to Timaeus, directly after this victory, they fell into luxurious living (*τρυφή*).[44] According to a detailed account in Diodorus, Pythagoras, functioning as the embodied conscience of Croton, had a part in bringing about the war against Sybaris. A number of factors conspire, however, to make this account seem very suspicious: the quite different account of Herodotus, the inherent improbabilities, and, most of all, the structure of Diodorus' exposition, which is very like the plot of a tragedy.[45] The destruction of Sybaris was the worst atrocity wrought by Greeks against a Greek city in that era; the attempt to make the unheard-of comprehensible

[43] Aristox. fr. 18 (Iam. *VP* 249f), D.L. 8.39, etc. (DK 46). Alcidamas, as reported in Arist. *Rhet.* 1398b18f, must refer to this (*Θήβησιν ἅμα οἱ προστάται φιλόσοφοι ἐγένοντο καὶ εὐδαιμόνησεν ἡ πόλις*; cf. Diod. 15.52). Frank, who spoke of "literary fiction," has neglected to take account of the earliness of the evidence (294.1). Wuilleumier (564f) expresses doubts of the story's historicity. On Philolaus' residence in Thebes, cf. Pl. *Phd.* 61e, on Simmias and Cebes, above, ch. I 4, n. 39.

[44] Justin 20.4.5ff, Timaeus *FGrHist* 566F44–45. According to Apollonius (Iam. *VP* 255) the revolt against the Pythagoreans resulted from conflict over conquered Sybarite territory. Dunbabin (360) points out that no Olympic victors from Croton are listed for the years 548–532. This can be combined with the report of Justin and so used for dating the battle at the Sagras (above, n. 40). But is it certain that Croton's recovery was due to Pythagoras?

[45] Diodorus 12.9.2ff. The source is neither Ephorus (Diels, hesitantly, DK 14.14) nor Timaeus (Lévy, *Sources* 59f), but some later author (E. Schwartz, *RE* V 685f). Herodotus' version (5.44f) is different. An illogical feature of Diodorus' account is that the tyrant Telys of Sybaris (Herodotus calls him king) first banishes his opponents and then, after they have sought refuge at Croton, demands their return. What follows is modeled, down to details, on tragedies like Aeschylus' *Suppliants* and Euripides' *Heraclidae* and *Suppliants*. The exiles have taken refuge at the altars in the marketplace of Croton, emissaries of Telys demand their surrender and threaten war, the popular assembly is in doubt what to do until Pythagoras addresses them and reminds them of the obligation to protect suppliants, and the war and the famous victory follow. According to Andron (ch. II 3 below), Pythagoras foretold to a friend the conquest of Sybaris, which makes him a seer, without active participation in the events, as in Iam. *VP* 133, 177 (Apollonius?)

was bound to give rise to legends, and the contrast of "Sybaritic" luxury with Pythagorean sobriety was a strong stimulus to the creation of moralistic and edifying fiction.

According to Aristotle and Aristoxenus, Pythagoras withdrew to Metapontum before the outbreak of trouble and died there;[46] but according to Dicaearchus, Pythagoras himself was in Croton at the time of the revolt,[47] and in several accounts he is even said to have perished in the fire.[48] Since the escape of Lysis cannot have occurred before 450 or so, the reports of Dicaearchus and the later writers cannot be accepted. Probably two anti-Pythagorean movements have been combined, one in Pythagoras' lifetime and another about 450.[49] It is clear, however, that there was no established tradition, no author seems to use documentary evidence;[50] everything depends on oral tradition, in which "Pythagoras" quickly comes to mean "the Pythagoreans," in much the same way as the Ionians speak of Ion and the Dorians of Heracles.

When Plato contrasts Pythagoras with the lawgivers as one who became, *ἰδίᾳ τισὶν*, a guide to the good life, he is not thinking of him as a political figure.[51] In some respects, Pythagoras is connected with

[46] Arist. fr. 191, Aristox. fr. 18; similarly Apollonius (Iam. *VP* 255), Philod. *Rhet.* II 180 Sudhaus, Justin 20.4. Others accounted for Pythagoras' absence from Croton by saying he had gone to Delos to care for, and bury, his old teacher Pherecydes (Satyrus ap. D.L. 8.40, Nicom. ap. Por. *VP* 55, Iam. *VP* 252). This was contradicted as early as Dicaearchus (fr. 34 = Por. *VP* 56; to judge by the form of the citation, in indirect speech, the sentence in question belongs with the Dicaearchus quotation, and not, as Lévy thought, *Sources* 51, n. 4, to Porphyry).

[47] Frr. 34–35. Others expanded the story: his disciples made a bridge over the fire with their own bodies, so that the master might escape (Por. *VP* 57; Tzetzes *Chil.* 11.80ff). Hermippus' version is unique: Pythagoras died in the war between Acragas and Syracuse, because he refused to escape by crossing a field of beans. (D.L. 8.40, Schol. Pl. *Rep.* 600b, *Suda* s.v. Pythagoras; at D.L. 8.93 this is combined with his flight from the fire. It is hardly possible to determine whether the similar story of Myllias and Timycha at the beanfield, recounted by Neanthes *FGrHist* 84F31 = Iam. *VP* 189ff, is primary, as Lévy thought, *Lég.* 68, or derivative.)

[48] D.L. 8.39, Schol. Pl. *Rep.* 600b, *Suda* s.v. Pythagoras (Neanthes? cf. above, ch. II 1, n. 6), Hippol. *Ref.* 1.2.16, Plut. *Stoic. rep.* 1051c; cf. Arnob. 1.40, Mara bar Serapion (W. Cureton, *Spicilegium Syriacum* [London, 1855] p. 73f).

[49] Minar 53ff; with modifications Morrison, *CQ* 1956, 149.

[50] Dicaearchus (fr. 34) refers expressly to oral tradition. The so-called *ὑπομνήματα* of the Crotoniates cited by Apollonius (Iam. *VP* 262) are suspect. Reference to such a source may point to Timaeus (cf. Morrison, *CQ* 1956, 149); but the whole thing may be a fabrication, for the "memoirs" are supposed to prove that the arbitration judgment by the representatives of Tarentum, Metapontum, and Caulonia, which was unfavorable to the Crotoniates, was purchased by bribery; cf. Delatte, *Pol.* 218; von Fritz (*Pol.* 65f) thinks the "memoirs" "may have been contained in a local chronicle which might have been embellished by legends." Cf. Lévy, *Sources* 115.9.

[51] *Rep.* 600a. This is emphasized by Delatte, *Pol.* 17f; Frank, *AJP* 1943, 222, n. 7; R. Joly, *Mém. Ac. R. de Belgique* 51.3 (1958) 28.

Metapontum more closely than with Croton,[52] while the political activity and the burning-episode belong to Croton.[53] Some modern scholars have gone so far in a skeptical direction as to assert that Pythagorean political activity in Croton is an invention—Aristoxenus and Dicaearchus, they suppose, projected upon the Pythagoreans their own ideal of the *βίος πρακτικός*, and thus invented Pythagorean politics together with an appropriate historical background.[54]

There is, however, an often forgotten testimony of Theopompus, in the midst of a fragment of Posidonius about the tyrant Athenion of Athens.[55] Athenion had been a member of the Peripatetic school, but at the first opportunity he cast aside the mask of philosophy and became a tyrant, thus illustrating "the Pythagorean doctrine regarding treachery, and the meaning of that philosophic system which the noble Pythagoras introduced, as recorded by Theopompus in the eighth book of his *History of Philip*, and by Hermippus the disciple of Callimachus."[56] Theopompus must have said that the secret, but genuine, goal of the philosophy introduced by "the excellent Pythagoras" (*ὁ καλὸς Πυθαγόρας*) was tyranny.

There are other traces of an anti-Pythagorean tradition, representing Pythagoras and his pupils to be tyrants. Appian writes, in a context very similar to that of Posidonius, "also in Italy, some of the Pythagoreans, and in other parts of the Grecian world some of those known as the Seven Wise Men, who undertook to manage public affairs, governed more cruelly, and made themselves greater tyrants than ordinary despots."[57] Diogenes Laertius names a *Πυθαγόρας Κροτωνιάτης, τυραννικὸς ἄνθρωπος*, who is said to have been a contemporary of the philosopher—a desperate attempt to get rid of the tradition of Pythagoras as tyrant;[58] and elsewhere the revolt against the Pythagoreans is represented as a blow for freedom from tyranny.[59]

The existence of this tradition as early as the fourth century helps us understand why it was so important to Aristoxenus to show Pythagoras as an opponent of Polycrates and an émigré in search of freedom, and to emphasize the readiness of the Greeks of southern Italy to accept Pythagorean dominance or leadership.[60]

Theopompus' interpretation is as tendentious as that of Aristoxenus.[61] But, if the Pythagorean dominance in Croton can be seen in two such different lights in the fourth century, as a model of free government under aristocratic guidance or as a detestable tyranny, the underlying reality cannot be an invention of Aristoxenus and Dicaearchus, nor one by Theopompus, but only a tradition that could be interpreted in more than one way. We must believe Dicaearchus' testimony that the memory of Pythagorean rule and of revolt against it was still alive in Magna Graecia in his day. Plato, too, alludes to the Pythagoreans' connection of politics and philosophy (above, ch. I 4).

There is no inconsistency between this and the religious and ritual side of Pythagoreanism. In fact, cult society and political club are in origin virtually identical. Every organized group expresses itself in terms of a common worship, and every cult society is active politically as a *ἑταιρία*.[62] Pythagoreanism fits into this picture and can be seen to have firm rootage in the social and political conditions of the time.

[52] Pythagoras is once called a Metapontine (Lycus [?] ap. Por. *VP* 5; cf. below, ch. II 5, n. 66), but never a Crotoniate. He died in Metapontum (above, n. 18). Theano, too, belongs to Metapontum (above, nn. 28–29).

[53] Considering the novelistic quality of his whole account, there is no significance in Plutarch's localization of the burning-episode in Metapontum (*De gen.* 583a).

[54] Cf. ch. II 1, nn. 43, 44, 58. The skepticism was expressed most extremely by Frank, *AJP* 1943, 222f.

[55] Theopompus *FGrHist* 115F73, cited in Posidonius *FGrHist* 87F36 = Ath. 5.213f, given here in Gulick's Loeb tr. Cf. Delatte, *Vie* 251. Hermippus, who is intermediary source between Theopompus and Posidonius, wrote *βίοι τῶν ἀπὸ φιλοσοφίας εἰς τυραννίδας καὶ δυναστείας μεθεστηκότων* (*Acad. Ind.* col. 11, p. 29 Mekler).

[56] On the history, see U. von Wilamowitz-Moellendorff, "Athenion und Aristion," *SbBln* 1923, 39–50.

[57] *Mithr.* 28, in White's Loeb tr. He is speaking of the tyranny of the Epicurean Aristion, who came to power shortly after Athenion. The source may well, again, be Hermippus.

[58] D.L. 8.46; cf. Delatte, *Vie* 251. For similar concern about Pythagoras' "image," and the assertion that he authorized athletes to eat meat, cf. below, ch. II 4, n. 111.

[59] D.L. 8.39, Tert. *Apol.* 46.13, Arnob. 1.40; cf. Delatte, *Vie* 251, *Pol.* 16. Apollonius, too, hints at this theme (Iam. *VP* 257ff).

[60] Free choice as the basis of Pythagorean rule: Iam. *VP* 249 = Aristox. fr. 18. Von Fritz (*Pol.* 18ff), discussing Aristoxenus' bias, suggests that the whole passage from Nicomachus (Por. *VP* 21f = Iam. *VP* 33f) in which Pythagoras is represented as converting various tyrants and freeing many cities was drawn from Aristoxenus; and in this he is followed by Wehrli (ad Aristox. fr. 17). But considering the manner of citation, this seems uncertain: the sentence for which Aristoxenus is specifically cited, about Lucanians, Peucetians, Messapians, and Romans as pupils of Pythagoras, is missing in the parallel passage, Iam. *VP* 33f but stands by itself at D.L. 8.14 and Iam. *VP* 241. It is, then, an isolated citation, of interest to later generations because of the mention of the Romans; note also that it breaks the continuity at Por. *VP* 21f. It is hard to attribute to Aristoxenus the anachronism of including Tauromenium (above, ch. II 1, n. 37; Frank makes much of this, *AJP* 1943, 220).

[61] His thrust at Pythagoras may be intended as a covert blow at the Academy, against which Theopompus wrote a pamphlet (*κατὰ τῆς Πλάτωνος διατριβῆς*, Ath. 11.508c–d), and against which the reproach was so often directed, that Plato's pupils became tyrants (cf. Ath. loc. cit., from Herodicus).

[62] Thus in Athens the desecration of the Herms and the profanation of the mysteries constituted a political act, as the "Bacchanalia" were regarded in Rome as a *coniuratio* (Livy 39.8.1, 14.3). Plato's Academy was a cult organization (on this, Boyancé, *Muses* 249ff), and Dion's partisans were united, among other things, by initiation into the mysteries (Pl. *Ep.* 7.333e, Plut. *Dion* 56).

In some reports the name of Pythagoras has an almost mythic function: a belief, a political act, or a mishap experienced by Pythagoreans is described by an assertion that Pythagoras taught, did, or suffered this or that. But we must resist the temptation to interpret Pythagoras as a mythical figure—the "speaker from Pytho" as an incarnation of the Hyperborean Apollo,[63] his fiery death as deification,[64] and the like. Even what is legendary has its relation to specific historical events. There is no doubt of the historical reality of the Pythagorean society and its political activity in Croton; but the Master himself can be discerned, primarily, not by the clear light of history but in the misty twilight between religious veneration and the distorting light of hostile polemic. Pythagoras and the Pythagoras legend cannot be separated.

3. METEMPSYCHOSIS AND "SHAMANISM"

That Pythagoras taught the doctrine of metempsychosis is generally regarded, and rightly, as the one most certain fact in the history of early Pythagoreanism. For it is alluded to in the most ancient piece of evidence about Pythagoras, the well-known lines of Xenophanes,[1]

καί ποτέ μιν στυφελιζομένου σκύλακος παριόντα
φασὶν ἐποικτῖραι καὶ τόδε φάσθαι ἔπος·
παῦσαι μηδὲ ῥάπιζ', ἐπεὶ ἦ φίλου ἀνέρος ἐστὶν
ψυχή, τὴν ἔγνων φθεγξαμένης ἀίων.

[63] This interpretation appears as early as Aristippus the younger (D.L. 8.21 = Giannantoni A162): *ὅτι τὴν ἀλήθειαν ἠγόρευεν οὐχ ἧττον τοῦ Πυθίου.* Apollonius has it somewhat differently, in the context of the birth legends (Iam. *VP* 7: *ὅτι . . . ὑπὸ τοῦ Πυθίου προηγορεύθη*). Pythagoras "sounds like a *nom de guerre*," says Nilsson (*A History of Greek Religion* [Oxford, 1925, 1950²] 202). But there are many personal names of similar form (Athenagoras, Diagoras, Hermagoras), and the name Pythagoras itself is not rare (cf. the *RE* articles).

[64] Cf. Heracles on Oeta, whom Peregrinus Proteus imitated in A.D. 167, as Timanthes of Cleonae had done (Paus. 6.8.4). Lévy, *Lég.* 71.1, refers to Croesus, who was transported from the pyre, according to the legend, to the land of the Hyperboreans, R. Eisler (*Orpheus the Fisher* [London, 1921] 11f) rashly equates Pythagoras with Apollo, and Apollo in turn with the sun-god, who makes his way to the west and is there burnt in his house . . .

[1] DK 21B7 = D.L. 8.36 (*A. P.* 7.120; *Suda* s.vv. Xenophanes, *στυφελίξαι*). Whether Pythagoras is the person referred to was questioned by O. Kern, *AGP* 1 (1888) 499, by Rathmann, 37f, and by Maddalena, 335ff. Maddalena adds that, even if Xenophanes did mean Pythagoras, the doctrine in question was not metempsychosis but only the equality of the human and the animal soul; but this overlooks the explicit formulation *φίλου ἀνέρος ψυχή ἐστιν*. The passage is regarded as the best piece of evidence about Pythagoras by Zeller (I 557), Wilamowitz (*GldH* II 190), Kranz (*Hermes* 1934, 226f; DK I 490.38ff), Mondolfo (in *ZM* 314ff), Thomas (71f), Long (17), Dodds (*Irr.* 143 n. 55), and others.—

Diogenes Laertius says these lines refer to Pythagoras. In the fragment itself there is no name mentioned, but Diogenes explicitly cites the beginning of the poem, so that at least his source, where Xenophanes is cited for evidence on Pythagoras, was based on the complete text. The first words ("And once . . .") show that other, similar anecdotes had preceded this one; thus the subject was a well-known person, and not some anonymous "Orphic." It is worth noting that Xenophanes also attacked Epimenides;[2] not only the Homeric religion was subject to his criticism, but non-Homeric religious manifestations such as doctrines about the soul and about ritual purification.

An allusion of Aristotle is equally explicit. He complains of his predecessors that in their theories about the soul they paid far too little attention to the necessary presuppositions about the body: "They try to say what kind of thing the soul is, but do not go on to specify about the body which is to receive the soul, as though it were possible, as in the tales of the Pythagoreans, for just any soul to clothe itself in just any body."[3] In his critique of various philosophers, he introduces this ironical comparison with the Pythagoreans' "myths"[4] as though they were something well known. What he cares about here is not distinctions among individuals, but among species. The soul "clothes itself" (a common expression in the doctrine of metempsychosis)[5] in any kind of body, human or animal; and this failure to distinguish between human and animal gives rise to the same scandalized tone that we hear in Xenophanes. Theophrastus, on the other hand, in arguing against sacrificing animals, tries to bring animals and human beings closer together. The former, too, have souls, quite like those of human

Apollonius of Tyana imitated his Master, recognizing in a lion a later incarnation of King Amasis (Philostr. *VA* 5.42; see Lévy, *Sources* 2 n. 7, 134).

[2] D.L. 9.18 (DK 21A1), 1.111 (DK 21B20).

[3] Arist. *De an.* 407b20 (DK 58B39) and 414a22. Doubts that the passage refers to metempsychosis were expressed by Rathmann 18f, and Maddalena 338ff. The latter claims that metempsychosis implies a judgment of the dead, so that it is not the entrance of "just any" soul into "just any" body; but what Aristotle is talking about is the character of souls in general, and not of those of individuals. Maddalena suggests that he is thinking of the dust particles in the air (*ξύσματα*), alluded to at *De an.* 404a18, though this doctrine is rather compatible than otherwise with metempsychosis. Cherniss (*Pres.* 325 n. 130) leaves the matter undecided. See also Nilsson, *Op.* II 663, and Theiler, *Arist.* 98, and the Myllias anecdote, Arist. fr. 191.

[4] *μῦθος* is used by Aristotle in a sense very similar to the modern one. Cf. 1074a38.

[5] Kranz, DK I 504.7; *εἰσδύεσθαι* Hdt. 2.123; similarly D.L. 8.77, Por. *Abst.* 1.1, Max. Tyr. 10.2, Diod. 5.28.6. Democritus can also speak of atoms as *ἐνδυόμενα εἰς τοὺς πόρους* (A165); cf. Hippoc. *Vict.* 1.25; but the ancient commentators have no hesitation about the reference of the present passage to metempsychosis. Philop. *De an.* 140.3ff cites Empedocles fr. 117 in explanation of it; cf. Olympiod. *In Phd.* A X 1, p. 56.17ff Norvin.

beings, or perhaps even identical with them, "as Pythagoras taught."[6]

Porphyry gives a description, taken from Dicaearchus, of the arrival of Pythagoras in Croton: Pythagoras had great success, and was invited to give lectures before the civic leaders, the young men, the boys, and the women.

> As a result of these events, a great reputation grew up about him, and he won many disciples from the city itself; not only men but women too . . . [including Theano], as well as many from the non-Greek territory nearby, kings and nobles. Now the content of his teaching to his associates no one can describe realiably, for the secrecy [σιωπή] they maintained was quite exceptional. But the doctrines that became best known to the public were, first, that the soul is immortal, then that it migrates into other species of animals, in addition that at certain intervals what has once happened happens again, so that nothing is really new, and finally that we ought to regard all living things as akin. Pythagoras is said to have been the first to introduce these opinions into Greece.

Whether all this is from Dicaearchus is controversial and cannot be definitely decided by philological means.[7] It is in Porphyry's manner, however, to quote long passages; and his second citation from Dicaearchus is an extensive section, as can luckily be proven from the parallels.[8] The skeptical tone of the expression is noteworthy; no one, he says, knows for sure. This cannot come from Porphyry, who cites, from different sources, details about the mathematical knowledge and procedures of Pythagoras;[9] nor from a source like the "handbook" used

[6] Por. *Abst.* 3.26: *εἰ φαίνοιτο κατὰ Πυθαγόραν καὶ τὴν ψυχὴν τὴν αὐτὴν εἰληχότα (τὰ ζῷα)*, from the concluding sentence of the argument attributed at 3.25 to Theophrastus. The sentence is hardly an addition by Porphyry, who expressly denied the extension of metempsychosis to animals (*De regressu an.* fr. 11 Bidez).

[7] Por. *VP* 19; section 18 is given as fr. 33 by Wehrli. The transition from indirect to direct discourse, which may seem suspicious, has already taken place in 18. Section 19 was attributed to Dicaearchus by Rohde, *Q* 126, Jäger 40, Burnet, *EGP* 92, Rostagni, *ScrMin* I 42 n.1, Lévy, *Sources* 50, Mondolfo in ZM 363f. The attribution was contested by Rathmann 3f, on the ground that the report is incompatible with Dicaearchus' partisanship for the *vita activa* (though we know at least, from fr. 36, that Dicaearchus did treat of Pythagoras' metempsychosis doctrine). Maddalena (76 n. 21) suggests that the uncertainty expressed does not comport with the public instruction reported in section 18 (but this is intended as ironical), and also that Aristotle had some exact knowledge of Pythagorean doctrines (but not, we may reply, doctrines of Pythagoras himself). Wehrli, following Rathmann, omits section 19.

[8] Por. *VP* 56 57 fr. 34 W.; the conclusion D.L. 8.40 fr. 35 W.

[9] Por. *VP* 6f, citing the *Memoirs*. In the next sentence Eudoxus is cited (fr. 36 Gisinger, 325 Lasserre); Gisinger is wrong in assigning the whole passage, including the citation of the *Memoirs*, to Eudoxus.

by Diogenes Laertius, which knows of writings by Pythagoras.[10] In fact, such an expression is scarcely conceivable after the image of Pythagoras that originated with Speusippus, Xenocrates, and Heraclides had become canonical. This points, then, to the antiquity of the source, and specifically to Aristotle's pupil Dicaearchus, who was so skeptical about the Platonic-Pythagorean doctrine of the soul.[11] One of the doctrines designated here as "best known to the public," that of the cyclic recurrence of all things, is attested as Pythagorean by Dicaearchus' colleague, Eudemus.[12]

Ion of Chios had already named Pythagoras as the one who knew all about the soul, in an epigram which, according to Diogenes Laertius, refers to Pherecydes of Syros:[13]

> *ὣς ὁ μὲν ἠνορέῃ τε κεκασμένος ἠδὲ καὶ αἰδοῖ*
> *καὶ φθίμενος ψυχῇ τερπνὸν ἔχει βίοτον,*
> *εἴπερ Πυθαγόρης ἐτύμως σοφός, ὃς περὶ πάντων*
> *ἀνθρώπων γνώμας εἶδε καὶ ἐξέμαθεν . . .*

Likewise, the tradition of the Greeks that lived on the Black Sea and the Hellespont, as reported by Herodotus,[14] connects the *Γέται ἀθανατίζοντες* with Pythagoras. In neither of these cases is anything said about metempsychosis, but Pythagoras is obviously the best-known name in the realm of immortality and the afterlife.

It is no less important to recognize, however, that the later tradition frequently attempts to ignore or interpret away the doctrine. There is not a word about metempsychosis in Aëtius; and, what is more, he projects the Platonic doctrine of the soul onto Pythagoras. The

[10] D.L. 8.6f = Schol. Pl. *Rep.* 600b = *Suda* s.v. Pythagoras.

[11] Cf. ch. II 1, n. 48. A similar agnostic expression in Por. *VP* 5 is probably also old: "If you happen not to know the country or the city whose citizen this man was, do not be concerned; for some say he was from Samos, some from Phlius, some from Metapontum." The name of the author of this is, however, uncertain: MSS *Λεῦκος* (*Λύκος*? *Λύκων*?). See below, ch. II 5, n. 66.

[12] Fr. 88 W (DK 58B34 = Simpl. *Phys.* 732.26); cf. fr. 89 W. (D.L. 1.9).

[13] DK 36B4 fr. 30 Blumenthal = fr. 5 Diehl (D.L. 1.120). Rathmann disputes the reference to Pherecydes (44f), but this cannot be checked. He says the last verse is an interpolation, because the *μέν* of line 1 has no corresponding *δέ*; but for a fragment this kind of argument is inconclusive. Maddalena (348f) emphasizes that metempsychosis is not explicitly mentioned.—In line 3 the manuscript reading is *εἴπερ Πυθαγόρης ἐτύμως ἦ σοφὸς περὶ πάντων*; the obviously correct emendation is that of Sandbach, *Proc. of the Camb. Philol. Soc.* 185 n.s. 5 (1958–1959) 36. The wording of line 4 is awkward. For the subject matter Kranz (*Hermes* 1934, 227f) compares Heraclitus fr. 129 (followed by Mondolfo in ZM, 317f, and Sandbach, loc. cit.). A similar expression about Pythagoras is found at Ov. *Trist.* 3.3.62.

[14] 4.95 96; below, n. 203.

Memoirs have no place for palingenesis; souls that are pure rise to the "highest," and the impure are given over to the Erinyes.[15] A Stoic source in Sextus[16] gives as the reason for Pythagorean vegetarianism the basic relationship of all living beings, from the gods down to animals without reason, by virtue of the πνεῦμα τὸ διὰ παντὸς τοῦ κόσμου διῆκον. Xenocrates had adduced the relationship of man and beast as a reason not to eat meat, and Theophrastus expressed a similar idea.[17] As against Empedocles' eloquent advocacy of vegetarianism, with the gruesome picture of a father killing his own son (B136–137), these later philosophers have sublimated metempsychosis into a generalized, philosophical doctrine of soul.[18] Xenocrates, to be sure, developed a theory of *daimones*, but did not assume metempsychosis;[19] and, whereas the main line of the tradition makes Pythagoras a reincarnation of Euphorbus, Xenocrates makes him the son of Apollo.[20] Once more the same conclusion: insofar as the later tradition contradicts the most ancient evidence, it agrees with the thought of the Platonists, and especially Xenocrates; he reinterpreted the Pythagorean tradition to make it conform to his way of thinking. The fact that in this case the later tradition is not unanimous in discarding metempsychosis, along with Xenocrates, is not due to a continuous Pythagorean tradition, but to the tremendous influence of the Platonic myths and the dissenting views of other members of the Academy and Peripatos, especially Heraclides Ponticus.[21]

[15] 31; see above, ch. I 3. In the *Memoirs*, as in Aëtius, only the highest part of the soul is immortal. The neo-Pythagorean Alexander of Abonuteichus rejected metempsychosis in somewhat similar terms: the "soul" develops and then perishes, but the important thing is the "spirit" (φρήν) which emanates from the mind of "Zeus." (Lucian *Alex.* 40; in the background is Pl. *Tim.* 30b; cf. Plut. *De fac.* 944e.)

[16] Sext. Emp. *Math.* 9.127f; on this *pneuma* theory which was attributed to Pythagoras see Cic. *Nat. d.* 1.27.

[17] Fr. 98 H. = Por. *Abst.* 4.22. Xenocrates also speaks of the danger of "assimilation" to the souls of irrational creatures through the eating of meat (fr. 100 H. = Clem. Al. *Strom.* 7.32.9; here Polemo is named as well as Xenocrates). For Theophrastus, see above, n. 6.

[18] Sext. Emp. *Math.* 9.127, 129 cites these same lines of Empedocles, but does not mention metempsychosis.

[19] On the theory of *daimones*, see Heinze 78ff; for the rejection of metempsychosis, 147.

[20] Fr. 22 H. = Iam. *VP* 7 (cf. Por. *VP* 2). Eudoxus and Epimenides are also cited for this view; cf. the statement of Speusippus about the birth of Plato, fr. 27 L. = D.L. 3.2. Cf. below, n. 143.

[21] Clearchus, like Heraclides, championed the immortality of the soul and its independence of the body (Wehrli p. 47). In fr. 38(= Ath. 4.157c, DK 44B14) he represents the Pythagorean Euxitheus (doubtless a fictitious person) as speaking about the imprisonment of the soul in the body and the prohibition of suicide (cf. Pl. *Phd.* 61e).

The doctrine of metempsychosis is set forth, with some detail, in works of Pindar,[22] Empedocles,[23] Herodotus (2.123), and Plato.[24] The question how much of this can be accepted as testimony on Pythagoreanism depends on one's judgment of the complicated and much discussed phenomenon of "Orphism." Scholars' conceptions of Pythagoreanism and of Orphism are inevitably as interdependent as the pans of a balance. A "minimalist" attitude to the Orphic tradition rapidly raises the importance of Pythagoreanism,[25] while hypercriticism toward Pythagoreanism peoples Greece with *Orpheotelestae*.[26] There is no such thing as a *communis opinio* on Orphism, especially since the sensational discovery of the papyrus of Derveni[27] has shaken many established views.[28]

A few details, however, are likely to survive examination. There were Ὀρφικά, purported poems of Orpheus, perhaps circulating in differing versions, and including at least a theogony and cosmogony, of the sort for which the papyrus of Derveni provides a philosophical commentary.[29] There were Ὀρφεοτελεσταί,[30] who with reference to these writings gave private initiations to mysteries, in which the punishments in store for the uninitiated in the next world were vividly depicted. For the initiates there was the βίος Ὀρφικός, an ascetic life featuring specific abstinences, and especially vegetarianism.[31]

[22] *Ol.* 2.56ff, fr. 133 Schr. (= 127 Bowra). Cf. Long 29ff, Nilsson I 692f, E. Thummer, *Die Religiosität Pindars* (Innsbruck, 1957) 121–130.

[23] *Katharmoi*, passim. Cf. Long 45ff.

[24] *Phd.* 81b, *Menex.* 81a, *Rep.* 614ff, *Phdr.* 248d, *Gorg.* 525c (with Dodds's note). Cf. Thomas passim, Long 63ff, von Fritz, *Phronesis* 1957; also R. S. Bluck, "The *Phaedrus* and Reincarnation," *AJP* 79 (1958) 156–164, "Plato, Pindar, and Metempsychosis," ibid. 405–414.

[25] Wilamowitz (*GldH* II 182ff) is followed in this trend especially by Thomas, Linforth, and Long. As a result, not only metempsychosis, but even the Gold Tablets (ch. II 2, n. 31) turn Pythagorean.

[26] See esp. Rathmann.

[27] A part of it is published by S. G. Kapsomenos, *Arch. delt.* 19 (1964) 17–25. Cf. R. Merkelbach, *Zeitschr. f. Papyrologie u. Epigraphik* 1 (1967) 17–32; W. Burkert, *ibid.* 2 (1968) 93–114.

[28] The most critical study of the Orphic tradition is Linforth's, followed by L. Moulinier, *Orphée et l'Orphisme à l'époque classique* (Paris, 1955). The most favorable treatment of the Orphic tradition, since Kern, is that of Ziegler, *RE* XVIII 1321ff. A middle position is sought by Guthrie, *Orpheus*; Nilsson, *Op.* II 628ff; Dodds, *Irr.* 147ff. For bibliography, see Nilsson, *Op.* II 628, *GrR* I 678ff.

[29] Orphic writings are mentioned in Eur. *Alc.* 967, *Hipp.* 954, Pl. *Rep.* 364e (βίβλων δέ ὁμαδὸν παρέχονται Μουσαίου καὶ Ὀρφέως), Arist. *De an.* 410b28 (ἐν τοῖς Ὀρφικοῖς ἔπεσι καλουμένοις); cf. *Gen. an.* 734a19, and fr. 7. See Nilsson, *Op.* II 630ff.

[30] The word is found in Theophr. *Char.* 16.11, Plut. *Lac. apophth.* 224e. For the facts, see Pl. *Rep.* 365a (μὴ θύσαντας δὲ δεινὰ περιμένει).

[31] Ὀρφικοὶ βίοι, Pl. *Leg.* 782c; ἄψυχος βορά Eur. *Hipp.* 952.

Metempsychosis is not attested directly for Orphism in any ancient source[32]—only the preexistence of the soul. It is undergoing punishment in its confinement to the body, which is both a prison and a protection.[33] Souls are borne by the wind into the body;[34] it is an almost unavoidable supplement to suppose that other living creatures, at their death, have "breathed out" these same souls. According to Plato, priests of the mysteries teach reincarnation:[35] "priests and priestesses who have paid careful attention to being able to explain their ministry" (*Meno* 81a).

Herodotus ascribed to the Egyptians a fully developed theory of metempsychosis; and, since the Egyptians never had such a doctrine,[36] it is clear that Greek ideas are being projected onto the foreigners. His manner of introducing the doctrine suggests south Italy.[37] Herodotus makes the ominous addition: "There have been some Greeks, both earlier and later, who have subscribed to this doctrine, giving it out as their own. Though I know their names, I am not putting them down." All possible combinations have been tried in the attempt to decide what Herodotus is concealing, but with no certain result.[38]

[32] Wilamowitz, *GldH* II 194: "eine orphische Seelenlehre soll erst einer nachweisen." That Orphism taught metempsychosis is contested by Krüger, *Orph.* 37f, Thomas 55f, Long 89ff. On the other side of the argument are Rohde, *Psyche* II 121ff (Eng. ed. 341ff), Nilsson, *Op.* II 663ff, *GrR* I 691ff, Dodds, *Irr.* 149 with n. 94.

[33] Plato *Crat.* 400 b–c; also *Phd.* 62b, Xenocrates fr. 20. Neither the Attic *φρουρά* nor *περίβολος* can have occurred in a hexameter poem.

[34] Arist. *De an.* 410b27; cf. Kern, *Orph. frag.* 223.

[35] *Leg.* 870d–e; cf. Wilamowitz, *Platon* I 249. Perhaps we may add the verse of Rhadamanthys, Arist. *EN* 1132b27.

[36] Cf. H. Bonnet, in *Reallexikon der ägyptischen Religionsgeschichte* (Berlin, 1952) 76f. This is regarded as a closed question; Kees, for example, does not even mention it.

[37] 2.123: "The Egyptians say that Demeter and Dionysus are the rulers of the underworld. *And also*, they were the first to proclaim the following doctrine . . ." Herodotus adds the doctrine of metempsychosis, because the preceding context reminded him of Greek ideas. Demeter and Dionysus in close connection point to southern Italy; it is from there that the triad of Ceres, Liber, and Libera was introduced to Rome in 493 B.C.

[38] Available for choice are the Orphics, Pythagoras, and Empedocles. (Pherecydes is hardly eligible; cf. Long 13f.) The possible combinations are (1) Orphics and Pythagoras (Zeller, *SBBln* 1889, 993; Nilsson, I 701; Morrison, *CQ* 1956, 137; and others), (2) the Orphics and Empedocles (Rathmann 48ff; though he does not exclude the first possibility), (3) Pythagoras and Empedocles (Long 22, Timpanaro Cardini 21f). Maddalena (346f) concludes that Pythagoras is not intended, from the fact that Herodotus does name him at 4.95. It has been thought that it was Empedocles whom Herodotus did not wish to name, for the reason that he was still alive (Stein in his note on the passage; Burnet, *EGP* 88 n. 5). Herodotus is reticent in the realm of cultic *ἀπόρρητα* (e.g. 2.61, 2.47, 2.170f), but also as to what he considers inferior (4.43, 1.51; thus Long, 23 n. 60, thinks Herodotus does not wish to mention a plagiarist; Morrison, *CQ* 1956, 137, suspects he is thinking of Eleusis). If one adopts the longer version at 2.81, Orphism "is" Pythagorean for Herodotus, so that Pythagoras and Empedocles must be intended; and the echo of a theory of the elements (a journey through creatures of land, water, and air) does remind one of Empedocles fr. 115, even though there are details that do not match exactly (no mention of fire; 3,000 instead of 30,000 years = 300 years Kern, *Orph. frag.* 231).

In another passage Herodotus mentions Pythagorean and Orphic teaching in the same breath, and compares it with Egyptian,[39] but the decisive sentence is transmitted in two versions, and it is matter for vigorous controversy whether one branch shows interpolation or the other abbreviation. The Egyptians may not enter any sanctuary in woolen clothing, nor bury their dead in woolen shrouds, he says. Then the Florentine branch continues, *ὁμολογέουσι δὲ ταῦτα τοῖσι Ὀρφικοῖσι καλεομένοισι καὶ Πυθαγορείοισι*, but the Roman branch has *ὁμολογέει δὲ ταῦτα τοῖσι Ὀρφικοῖσι καλεομένοισι καὶ Βακχικοῖσι, ἐοῦσι δὲ Αἰγυπτίοισι καὶ Πυθαγορείοισι*. One cannot, on principle, prefer one of the manuscript families.[40] It should be clear, though, that a contamination of the two versions, as found in the textus receptus, is unacceptable.[41]

The short version is, naturally, smoother,[42] but the more complicated thought of the longer version also makes sense: "these [customs] agree with those [Greek customs] that are called 'Orphic' and 'Bacchic,' but are in reality Egyptian and Pythagorean." In this version *ταῦτα* is the subject, and the datives are therefore to be taken as neuter; whereas in the other case masculine datives correspond to the personal subject (the Egyptians).[43] This comparison favors the longer version, for the ancient testimonia speak of *Ὀρφικά*, not *Ὀρφικοί*.[44] The following words, too, *τούτων τῶν ὀργίων*, would come more naturally after a preceding neuter;[45] in addition, the use of *ὁμολογέειν* seems more

[39] 2.81. Wilamowitz believes in interpolation (*GldH* II 189 n. 1 = 187² n. 1), as do Rathmann 52ff and *PhW* 54 (1934) 1178ff, Linforth 38ff, Long 24f, Timpanaro Cardini 81. The longer text is defended by Krüger, *Orph.* 13f, Boyancé, *Muses* 94.1, Dodds, *Irr.* 169 n. 80. Guthrie (*Orpheus* 16, 272 n. 4) accepts the contaminated textus receptus without question.

[40] Cf. Schmid I 2.672 and Jacoby, *RE* Supp. II 515ff. According to the evidence of the papyri, the twofold tradition does not go back to ancient times (A. H. Paap, *De Herodoti historiis in papyris et membranis Aegyptiis servatis* [Leiden, 1948] 95ff), and in Byzantine times there was no longer any reason for interpolation. There is no way to tell whether, in his brief reference to the passage (*Apol.* 56), Apuleius was using the shorter text (as Linforth 47) or was himself abridging.

[41] That is, the longer version but with the verb *ὁμολογέουσι*. After Zeller *SBBln* 1889, 994, and Burnet, *EGP* 88 n. 4, had separated the words *καὶ Πυθαγορείοισι* by a comma, Legrand modified to *καὶ ‹τοῖσι› Πυθαγορείοισι* (Coll. Budé, 1936). Maddalena, 236f, calmly omits as interpolated everything after *Ὀρφικοῖσι*.

[42] "More reasonable and intelligible," says Linforth (45); but this is merely the result of its brevity.

[43] Linforth would like to deny this (43); but see 2.80: *συμφέρονται . . . Αἰγύπτιοι . . . Λακεδαιμονίοισι, Ἑλλήνων οὐδαμοῖσι*. The expression *καλεόμενοι . . . ἐόντες δέ* is not elsewhere directly attested, but *ἐόντες . . . καλεόμενοι δέ* is not infrequent (2.178, 3.99).

[44] Wilamowitz *GldH* II 199.

[45] Dodds, *Irr.* 169 n. 80; Rathmann, *PhW* 54 (1934) 1183, artificially separates *τούτων τῶν ὀργίων*: "the rituals of these."

natural in the longer text.[46] On the other hand, it is hard to reach a definite conclusion as to the extent to which the variants *ὁμολογέει* and *ὁμολογέουσι* fit in better with the assumption of interpolation or omission.[47]

In any case Herodotus states that there is a connection between Orphism and Pythagoreanism in the realm of ritual. In addition, the longer text contributes not only an indication that Orphism is connected with Dionysus, but a theory about its origin, namely that it comes from Pythagoras, who got his teachings from Egypt. There is of course a relation between this and the presentation of the doctrine of metempsychosis, and probably also other features such as the statement that the Egyptian priests abhor beans, which has not been confirmed by Egyptian evidence.[48] Thus Greeks in Egypt connected beliefs about the afterlife and religious customs with the name of Pythagoras in the same way as Greeks living in the area about the Hellespont and the Black Sea did with the beliefs of the Getae about immortality (Hdt. 4.95). The difference is that in the latter case the Greeks' feeling of superiority to the barbarians led them to make Zalmoxis the pupil of Pythagoras, while in Egypt their awe of the ancient foreign culture produced an opposite result, and Pythagoras became the pupil of the Egyptians.

In general, modern scholars have seen Orphism as the older and more comprehensive movement and considered Pythagoras to have been influenced by it,[49] but the reverse theory is not only in the longer

[46] Cf. Plut. *De Is. et Os.* 364f: *ὁμολογεῖ δὲ καὶ τὰ Τιτανικὰ καὶ Νυκτέλια τοῖς λεγομένοις* . . . and Plut. *Caes.* 9 (on the Bona Dea festival) *αἱ γυναῖκες πολλὰ τοῖς 'Ορφικοῖς ὁμολογοῦντα δρᾶν λέγονται.* Rathmann, *PhW* 54 (1934) 1182f, emphasizes that *ὁμολογέειν* is elsewhere used by Herodotus only of persons; but in the sense "agree" = "resemble," the accusative of specification *ταῦτα* would be odd, too. The verb is construed with *οὐδέν* and *κατά* at 1.142, 2.18, 6.54.

[47] Linforth argues acutely (46) that an interpolator would also have had to alter *ὁμολογέουσι* to *ὁμολογέει* (a point missed by all those who accept the textus receptus), whereas an abridgment would not have needed to alter *ὁμολογέει* to *ὁμολογέουσι*, so that the discrepancy has been caused by interpolation. Yet we read, in the passage beginning in section 80, *συμφέρονται δὲ καὶ τόδε . . . τόδε . . . συμφέρονται*, so that it was easy to alter a following *ὁμολογέει δὲ ταῦτα*, by assimilation, to *ὁμολογέουσι*, and when the neuters became masculines the abbreviation was necessary.

[48] Hdt. 2.37, with B. A. van Groningen's notes (Leiden, 1946).

[49] A *Ἱερὸς λόγος* which Iamblichus read told of Pythagoras being initiated by the Orphic Aglaophamus (Iam. *VP* 146). It was regarded as certain that Pythagoras got his doctrine of metempsychosis from Orphism by Rohde (*Psyche* II 107f = 336f, Eng. ed.), Zeller I 68ff, 563, Rathmann, Guthrie (*Orpheus* 216ff), Nilsson I 701f, Kern (*Die Religion der Griechen* II [Berlin, 1935] 162). See also Jaeger *Theol.* 105 = 145f Eng. ed.; *HTR* 52 (1959) 135–147; R. S. Bluck, *Plato's Meno* (Cambridge, 1961) 61ff, 274ff. On the other

version of Herodotus; Ion of Chios also made use of it. In his *Τριαγμοί*, whose authenticity is guaranteed by Isocrates,[50] he said that Pythagoras "had written some poems and attributed them to Orpheus" (*ἔνια ποιήσαντα ἀνενεγκεῖν εἰς 'Ορφέα*).[51] This report was unearthed by ancient scholars interested in the controversy over whether Pythagoras left any writings; but it is a misunderstanding to conclude from it that Ion knew works of Pythagoras, who referred in them to Orpheus.[52] *Τὰ εἰς 'Ορφέα* (or *Μουσαῖον*) *ἀναφερόμενα* is a common designation for the literature which circulated in the name of Orpheus or Musaeus.[53] What Ion meant was that the real author of certain poems circulating under the name of Orpheus was Pythagoras—that the *'Ορφικά* were *Πυθαγόρεια*. It was not works of Pythagoras that Ion knew, but Orphic poems whose origin he was trying to determine. Herodotus says emphatically that the poets who were reputed to have lived before Homer were actually later (2.53); and this is aimed specifically at Orpheus. Clearly, to an educated person of the age of the Sophists it already seemed incredible that works had been preserved that were

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side, Lobeck maintained the thesis that Orphism was the borrower from Pythagoras (*Aglaoph.* 388; also, 247ff, 330f, 358f, 698). Also see R. Turcan, RHR 150 (1956) 137.

[50] Isoc. 15.268 (DK 36A6). Harpocration (s.v. Ion, DK36A1) writes of the work *Τριγμοί*: *ὅπερ Καλλίμαχος ἀντιλέγεσθαί φησιν ὡς 'Επιγένους.* Epigenes wrote *περὶ τῆς εἰς 'Ορφέα ἀναφερομένης ποιήσεως*, giving an allegorical interpretation (Clem. Al. *Strom.* I 131.5, 5.49), and also wrote commentary for Ion's tragedies (Ath. 11.468c). Linforth 116ff identifies him, conjecturally, with Epigenes the pupil of Socrates (Pl. *Ap.* 33e, *Phd.* 59b, Xen. *Mem.* 3.12.1. Dodds's objection, *Irr.* 171, is refuted by the Derveni papyrus). If he is right, the text of Harpocration may be kept, though the usual thing is to emend it (*ὑπὸ 'Επιγένους* Bergk, *ὡς καὶ 'Επιγένης* Diels, *καὶ 'Επιγένης* Jacoby; Pfeiffer, in his note on Callim. fr. 449, sides with Diels and Jacoby). It is worth noting that Clement, too (*Strom.* 1.131.5), brings together Ion (DK 36B2) and Epigenes (DK I 105.31). The sentence in the *Suda* *ἔγραψε* (sc. Orpheus) *Τριαγμούς, λέγονται δὲ εἶναι Ἴωνος τοῦ τραγικοῦ* (s.v. Orpheus [DK 1A1]) can hardly be anything but a misunderstanding of the item in Harpocration (Lobeck 388f, Kern, *Orph. frag.* p. 318).

[51] DK 36B2 = fr. 24 Blumenthal = D.L. 8.8; almost the same wording in Clem. Al. *Strom.* 1.131.4 = *FGrHist* 392F25 = Kern, *Orph. frag.* test, 248 (*Ἴων . . . Πυθαγόραν εἰς 'Ορφέα ἀνενεγκεῖν τινα ἱστορεῖ*). Cf. Linforth 109ff.

[52] Tannery (*MSc* IX 226ff) combined this with the story of Aglaophamus (Iam. *VP* 146) and ascribed both to the same forger. Gigon says (*Ursprung* 125), "Ion is acquaintd with writings of Pythagoras, and claims to know from them that Pythagoras attributed the doctrines they expound to the ancient bard Orpheus"; and Kranz expresses himself in similar terms (*Hermes* 1934, 227). The correct interpretation is given by Lobeck (389), Linforth (188), and Nilsson (I 701).

[53] *τὰ εἰς 'Ορφέα καὶ Μουσαῖον ἀναφερόμενα* Phld. *De piet.* p. 80 G. = *Orph. frag.* 30; Epigenes' title, above, n. 50, Tatian 41 = *Orph. frag.* test. 183; cf. Phld. *De piet.* p. 66 G. = DK 14.17, DK 11B1, Clem. Al. *Strom.* 1.131, Schol. BT *Il.* 18.570. *Suda* s.v. Orpheus = DK 1A1: *ἀναφέρονται εἰς 'Ονομάκριτον*; here, too, *ἀναφέρειν εἴς τινα* means "attribute authorship to someone."

written by a member of the Argonautic expedition.[54] On the other hand, they had to account for the existence of the "Orphic" literature,[55] and there were various ways to do this. One could construct a number of characters all named Orpheus, and this was done as early as the fifth century by Herodorus of Heraclia.[56] Or, one could put the responsibility upon other writers—ancient epic poets,[57] Onomacritus,[58] or Pythagoras and various Pythagoreans. These methods also crossed over one another, and this is probably the explanation of the reports about an "Orpheus of Croton,"[59] or "Cercops the Pythagorean."[60] Epigenes, especially, was prone to explain everything by Pythagoreanism;[61] and Plato, too, shows that he knows Magna Graecia is a suitable place for such material to come from, when he attributes a myth about the underworld to a *κομψὸς ἀνήρ, ἴσως Σικελός τις ἢ Ἰταλικός*.[62]

Perhaps Heraclitus, who in one fragment (40) reproaches Pythagoras along with Hesiod for useless *πολυμαθίη*, had already connected Pythagoras with Orphism: *Πυθαγόρης Μνησάρχου ἱστορίην ἤσκησεν μάλιστα πάντων καὶ ἐκλεξάμενος ταύτας τὰς συγγραφὰς ἐποιήσατο ἑαυτοῦ σοφίην, πολυμαθίην, κακοτεχνίην*. The genuineness of the fragment has been suspected, because its purpose in Diogenes Laertius is to prove

[54] See also Arist. fr. 7.

[55] Unlike the Pythagoreans, the Orphics committed their teachings to writing from the beginning, as is shown, aside from the evidence cited in n. 29 above, by the vase paintings showing a scribe standing before Orpheus' singing head, Linforth 122ff. (Incidentally, their use of writing gives us a historical terminus post quem.)

[56] *FGrHist* 31F42.

[57] Prodicus, the composer of a *Minyas*, in which a trip to Hades was described (Kinkel, *EGF* p. 215ff), becomes the composer of the Orphic *katabasis* (*Suda* s.v. Orpheus = DK 1A1; Wilamowitz, *GldH* II 197 n. 3). On Cercops see n. 60 below.

[58] On the role of Onomacritus see Nilsson, *Op.* II 645ff. It is certain that he collected and edited; but the idea that he composed whole poems (as Philop. *De an.* 186.26, *Suda* s.v. Orpheus) or "invented" the story of Zagreus (as Paus. 8.37.5), is philological conjecture.

[59] Asclepiades ap. *Suda* s.v. Orpheus, where we also find an Orpheus from Camarina. Nilsson assumes (*Op.* II 644) that the poems in question actually came from Croton and Camarina.

[60] The testimonia are in DK 15. A tradition transmitted by Aristotle names a Cercops as a rival of Hesiod (Arist. fr. 75 = D.L. 2.46), and the epic poem *Aegimius* was sometimes ascribed to Hesiod (frr. 294–301 M.–W.) and sometimes to Cercops of Miletus (Hes. fr. 301 M.–W. = Ath. 11.503d; cf. also Schmid I 1.254). It was natural to identify the rival of Hesiod with the composer of certain poems that stand in rivalry to Hesiod's *Theogony*—the Orphic theogonies; and, if then the *Ὀρφικά* were classified as Pythagorean, Cercops became straightway a Pythagorean. There is no need to distinguish two men named Cercops (*pace* DK I 106.6ff, Timpanaro Cardini 69 n.).

[61] Epigenes ap. Clem. Al. *Strom.* 1.131.5 attributes the *κατάβασις* and the *Ἱερὸς Λόγος* to Cercops, the *Πέπλος* and the *Φυσικά* to Brotinus (DK 17).

[62] *Gorg.* 493a; cf. below, ch. III 2, n. 48.

that Pythagoras left writings.[63] But since it does not prove this at all, because all it claims is that Pythagoras plundered the writings of others, and not that he wrote anything himself, its authenticity has been generally recognized since the discussions of Wilamowitz and Reinhardt.[64] There remains what seemed to Wilamowitz the "excruciating difficulty" ("peinliche Schwierigkeit") of guessing what "writings" Heraclitus could have been thinking of. Surely not Homer and Hesiod, who were known to all Greeks. But aside from them, and Anaximander, there was scarcely anything in the purview of Greek culture that Pythagoras could have used for such a purpose, except *Ὀρφικά* and the like. Perhaps the word Heraclitus chose, *κακοτεχνίη*, points in the direction of the *ἀγύρται καὶ μάντεις*.[65]

If this is right, Heraclitus saw the relation of Pythagoras and Orphism in the opposite way from Herodotus and Ion.[66] Where they attribute Orphic doctrine to Pythagoras, he makes Pythagoras the borrower. The contradiction is not particularly surprising, for even at that early date no one could be precise about chronology, in matters of this kind. Each arranged the facts in the way that best suited his purpose: whoever wanted to find a tangible personality in the chaotic mass of *Ὀρφικά* hit upon Pythagoras, and those who wanted to cast doubt on his originality used Orphism for this purpose.

Thus the oldest sources show Pythagoras, unlike Orpheus, as a tangible personality of the historical period, but their doctrines as connected or even identical. There is no support in these sources for the modern attempts to discern a difference in doctrine between

[63] B129 D.L. 8.6; it was branded spurious by Diels, *AGP* 1890, 451f, and included among the "dubious or forged" in DK. Zeller (*SBBln* 1889, 988; I 393 n. 5) wished to strike out at least the mention of writings: *ἐκλεξάμενος ταῦτα[ς τὰς συγγραφάς]*.

[64] For its genuineness, after Bywater: Burnet, *EGP* 134 n. 2; Reinhardt, *Parm.* 235.1; Wilamowitz, *GldH* II 188 n. 1; Rathmann 39; Kranz, *Hermes* 1934, 227f; Morrison, *CQ* 1956, 136. Reinhardt called the reference to *ἱστορίην* in *ταύτας τὰς συγγραφάς* "barbarische Härte" (cf. fr. 5). There could have been something in the preceding context that would have made the word *συγγραφαί* easier to understand. (H. Gomperz, *Hermes* 68 [1933] 41, tries to show that fragments 40, 41, 129, and 81 followed each other, without a break, in that order; but this cannot be proven.)

[65] See below, n. 226. Onomacritus worked *ἐκλεγόμενος*, according to Hdt. 7.6 (a reference supplied me by M. West). The reference is to Orpheus, according to Rathmann, 91, per contra, Kranz, *Hermes* 1934, 116, and Mondolfo in ZM 317 point out that (according to later tradition) Pythagoras collected Egyptian, Babylonian, and Persian traditions. Are we to suppose, however, that Heraclitus thought Pythagoras capable of doing research in foreign-language books? (To be sure, a certain Antiphon states that Pythagoras learned Egyptian, D.L. 8.3.)

[66] Rathmann 43f.

Orphism and early Pythagoreanism.[67] It is only too easy for modern notions to intrude. If one believes, with Nietzsche, in a primal opposition of "Apollonian" and "Dionysian," then Pythagoras and Orphism must stand in the same polar relationship; and if, under the influence of later evidence, one regards the philosophy of number and the foundation of exact science as the essential ingredient of Pythagoreanism, the antithesis of Apollonian rationality and Dionysian mysticism fits in very nicely. We must bear in mind, however, that as the Greeks thought of them, Apollo and Dionysus were brothers; the supposed clear differentiation of Pythagoreanism from Orphism is simply not attested in the oldest sources.

In the historical and social realm, much more than in that of doctrine, we do seem to detect a certain difference between the two groups. Much of the evidence about the Orphics, as in general about the unofficial mystery cults, clearly reflects the activities of mendicant priests.[68] Pythagoreans, however, both in Croton and probably elsewhere in southern Italy, held position of dominance in their cities, for a time. This means that something related to Orphism had emerged from the anonymity of back-alley ritual and become respectable; known persons, not apocryphal writings, are active in this movement—*Πυθαγόρειοι*, not *'Ορφικά*. Legend knew of royal seers like Melampus; the abnormal is sometimes rated higher and sometimes lower than the normal. While the cults based on the supposed writings of the mythical singer of antiquity remained suspect in the eyes of many, in the person of Pythagoras the ancient figure of the wandering purveyor of salvation had taken on a new radiance which cast its gleam as far as Plato's philosopher-king.[69]

A general observation may advance our argument somewhat. Mystery cults offering the promise of salvation, even along with cosmogonic myths, are conceivable without a doctrine of metempsychosis; and the fact that Orpheus, but not transmigration, is often connected with Eleusis[70] might be an indication that there was at least one branch of "Orphism" without metempsychosis. In that case, this doctrine must be an innovation upon a general Orphic background; for its introduction, Pythagoras is the only obvious candidate. There is a possibility, deserving serious consideration, that it came from India.[71]

It is the Pythagorean variant of Orphism that is manifest in Pindar, Empedocles,[72] Herodotus, and Plato, connected always with south Italian or Sicilian tradition. What details are to be attributed to Pythagoras himself remains an open question.[73] Does every living creature have an immortal soul that migrates from one incarnation to another? Do plants have such souls?[74] Do they only enter certain species of animals?[75] Do only certain special individuals, even among men (*δαίμονες*, Empedocles calls them) undergo this wondrous experience?[76] To what extent does the doctrine imply clearly formulated

[67] "Apollinizzare l'orfismo" is Ciaceri's characterization of the achievement of Pythagoras ("Orfismo e Pitagorismo nei loro rapporti politico-sociali," *Atti R. Acc. di Arch., Lett. e Belle Arti* n.s. 12 [Naples, 1931–1932] 209–223). At the same time, Ciaceri interprets Orphism as "democratic" and Pythagoreanism as "aristocratic." Linforth (43) considers the conjunction of "Bacchic" and "Pythagorean" in the longer version of Hdt. 2.81 to be an argument for athetesis. Maddalena (363 n. 98, 327 n. 25) would use athetesis in this same passage to such effect as to remove completely any connection of Orphic and Pythagorean, and Delatte also (*Litt.* 4f) is for carefully separating them. Cameron (5ff) assumes an independent development from a common source; and both Rohde (Q 104; somewhat differently in *Psyche* II 107 = 336f. Eng. ed.) and Dodds (*Irr.* 143, 149 n. 95) also interpret the two movements as parallel developments. Frank (357 n. 168) and Boyancé (*Muses* 93ff; cf. *REG* 1941, 161 n. 2) stated that they were inextricably intertwined. Nock emphasizes that in contrast to Orpheus, Pythagoras is "a tangible figure" ("Herodotus 2.81," *Studies Presented to F. L. Griffith* [London, 1932] 248). Note the confident assertion in Herodotus 4.96 that Pythagoras lived many years later than Zalmoxis.

[68] Cf. the anecdote of King Leotychidas and the Orpheotelestes Philippus, Plut. *Lac. apophth.* 224c. It need not be true historically, but is good evidence for the general atmosphere. For the typical figure of the *ἀγύρτης*, which is in itself quite old, cf. Aesch. *Ag.* 1273f, Ar. *Pax* 1045ff, *Av.* 959ff, Demosth. 18.260, 19.249, 19.281 (on Aeschines' mother), Antiphanes fr. 159, Clearchus fr. 47 W.

[69] Cf. ch. I 4. On the aristocratic character of Pythagoreanism see Minar, Ciaceri (above, n. 67), and Kerényi 35. G. Thomson, in arbitrary manner, calls Pythagoras "the first great exponent of democratic thought" (*Aeschylus and Athens*, 2nd ed. [London, 1946] 210).

[70] Ar. *Ran.* 1032; Dem. 25.11; *Marmor Parium*, *FGrHist* 239A14–15; Hecataeus of Abdera, Diod. 1.96, cf. 4.25; Plut. fr. 212 Sandbach.

[71] F. Cornelius, cited by K. von Fritz, *Gnomon* 40 (1968) 8 n. 1; Kranz, *Emped.* 29f. Dodds (*Irr.* 172 n. 97) suggests that Greek and Indian doctrines of metempsychosis have a common origin in shamanism.

[72] Empedocles is regularly connected in the ancient tradition with Pythagoras, from Alcidamas on (below, ch. II 2, n. 7) but only indirectly with Orpheus: Apollonius has Orpheus sing about *Neikos* (1.498), and fr. 141 is also cited as a verse of Orpheus (*Geop.* 4.15.8). The modern tendency to speak of Orphism in relation to Empedocles (see O. Kern, "Empedokles und die Orphiker," *AGP* 1 [1888] 498–508) comes from the fact that the image of Pythagoreanism is determined by ideas about number philosophy and exact science.

[73] See C. Hopf, *Antike Seelenwanderungsvorstellungen*, Diss. Leipzig, 1934; W. Stettner, *Die Seelenwanderung bei Griechen und Römern* (Stuttgart, 1934); Long, passim; von Fritz, *Phronesis* 1957; R. S. Bluck, *AJP* 79 (1958) 405–414.

[74] According to ordinary usage, the word *ζῷα* (Hdt. 2.123, Dicaearchus ap. Por. *VP* 19), as well as *ἔμψυχα*, would exclude plants (Rohde, *Psyche* II 180 n. 4 = 404 n. 82 Eng. ed.), but Empedocles (frr. 117, 127) and Heraclides (fr. 89 W.) include them (Long 23), and perhaps this is implied in the taboo on beans (below, ch. II 4).

[75] Iam. *VP* 85 (there are no human souls in sacrificial animals).

[76] Kerényi (18) showed that the "Eastern-Western" concept of the similarity of all souls is by no means self-evident; Rathmann (10) had emphasized that the story of Euphorbus does not imply a comprehensive doctrine of metempsychosis. On Arist. fr. 191, see below, n. 129.

beliefs, and how important is the word *ψυχή*?[77] Was there present at its beginning the significant semantic innovation whereby the "soul," as distinguished from the body and independently of it, is regarded as the "complete coalescence of life-soul and consciousness"[78] —a world away from the Homeric conception—or is "soul" primarily a mysterious, meta-empirical Self, independent of consciousness, as some important witnesses seem to indicate?[79] Is the soul newly incarnated immediately after the death of the old body,[80] in which case Hades becomes unnecessary, or is there an intermediate phase, which would leave Hades there, as a way station?[81] Is the process of palingenesis the work of blind natural forces—a creature at the moment of birth sucks in the soul[82]—or is it the execution of a penalty assessed in a judgment of the dead?[83] Is there an endless, cyclical movement,[84] or is there a fall at the beginning and a salvation at the end which is permanent—or perhaps has as its alternative an eternal damnation, in which case the concepts of Elysium and Tartarus again become

[77] Empedocles speaks of *δαίμονες*. In Xenophanes fr. 7 it is not stated that the dog "has" a soul; it "is" the soul of the friend, and "cries out" (Fränkel *DPh*² 311).

[78] Jaeger *Theol.* 99ff = Eng. ed. 83. See Long 2ff. It is beyond doubt that metempsychosis, or at any rate related religious concepts, played a significant role in the development of the notion of *ψυχή* (see Dodds, *Irr.* 140ff; on *ψυχή* in Heraclitus, B. Snell, *Entdeckung des Geistes* [Hamburg, 1955³] 36ff = 17ff Eng. ed.), but they do not presuppose it.

[79] Only exceptional persons like Pythagoras remember their previous incarnations. Pindar even sees a polar relationship between the soul and "consciousness": The *αἰῶνος εἴδωλον*, which comes from the gods, sleeps when one is awake, and is active while he sleeps (fr. 131; Aesch. *Eum.* 104; Hippoc. *Insomn.* VI 640 L.; Arist. fr. 10). In relation to Indian ideas of metempsychosis, too, one finds it necessary to speak of a meta-empirical "self" (Long 10).

[80] This is suggested in Hdt. 2.123 and Schol. BT *Il.* 16.857. Empedocles has no underworld; for him earthly existence is Hades (as Rohde, *Psyche* II 178f = 381 Eng. ed.; Dodds, *Irr.* 174 n. 114; wrongly rejected by Long 59ff).

[81] Cf. Serv. *Aen.* 3.68. Some of the *acusmata* speak of Tartarus (Arist. *An. post.* 94b33), and of dead persons under the earth (Ael. *VH* 4.17); Pindar and Plato insert a stay in Hades between reincarnations. The *katabasis* stories involving an "underground dwelling" are rooted in the traditional conception of Hades. The bean taboo may also be relevant here (ch. II 4).

[82] Hdt. 2.123: *ἐς ἄλλο ζῷον ἀεὶ γινόμενον ἐσδύεται.* Schol. T *Il.* 16.857: *Πυθαγόρας φησίν, ὡς ἀναχωροῦσα ἡ ψυχὴ ἐν ἐκείνῳ γίνεται [ἐν] ᾧ ἂν γεννωμένῳ σώματι ἢ φυτῷ καταντήσῃ* (similarly Schol. B); the soul of Patroclus complains of its fate, because it fears an unworthy rebirth. Cf. also Arist. *De an.* 407b22 (above, n. 3). Stettner regards this amoral metempsychosis as the primary type (7ff, 29ff). Kern, *Orph. frag.* 223 tries to compromise.

[83] So Pindar, Plato, and the *acusma* about breaking bread (Arist. fr. 195; Iam. *VP* 86). Arist. *EN* 1132b21ff connects Pythagoreanism with a saying of Rhadamanthys (above, n. 35).

[84] This is suggested in Hdt. 2.123, Eudemus fr. 88, Dicaearchus ap. Por. *VP* 19. Cf. the astrologers in Varro (Aug. *De civ. D.* 22.28, Serv. *Aen.* 3.68).

relevant?[85] Each of these questions is answered in more than one way in the ancient tradition, and there would be small prospect of success in an undertaking to crystallize out a sharply defined "doctrine of Pythagoras." There is a veritable maze of conflicting tendencies—mythical tradition persisting along with newly developing concepts of the world, the tradition of the mystery cults, ethical demands, and a growing recognition of natural law. It is only too easy for the modern scholar, from the vantage point of his own rationalistic and systematic activities, to suppose that at the beginning there was a unified, carefully worked out, and firmly defined theory.

On such a topic as men's conceptions of a future life, a topic bearing on matters so completely outside normal experience, a consistent and unified doctrine can only exist in the form of scripturally established dogma maintained by ecclesiastical authority. In oral tradition, or in the belief of a group no matter how closely knit, though agreement in essentials may persist, varying interpretations are bound to emerge. Conceptions of the afterlife are and have always been syncretistic.[86] It is only theology, coming along rather late in the tradition, that is interested in smoothing out the differences. Thus it is not to be presupposed that Pythagoras would commit himself to answer all possible questions; the main point was not a well-rounded system, but the working power of the doctrine. If there was no established "Holy Word," new interpretations were bound to develop among the disciples, on the ipse dixit, *αὐτὸς ἔφα* principle. Only dead dogma is preserved without change; doctrine taken seriously is always being revised in the continuous process of reinterpretation.

The metempsychosis doctrine does present a new start, after all, even in the hybrid character that its various aspects reveal. It does not offer mythical narrative, a picturesque story which gives the interpretation of a ritual, but a general doctrine which claims to be immediately true.[87] While even in Hesiod, no matter how carefully

[85] The version of Pindar and Plato (though *Rep.* 611a and *Tim.* 41d introduce the idea that the number of souls in the world must remain constant). The idea of a fall and a salvation is most prominent in Empedocles and in Plato's *Phaedrus*. Reward and punishment in the next world are very important in Orphism (above nn. 30 and 68).

[86] In Egypt, even the oldest pyramid texts show a good many mutually inconsistent ideas about the afterlife (see Kees).

[87] This is true even if at first the metempsychosis doctrine did not apply to all living beings, or even to all men. The story of Euphorbus was intended not as an exceptional case but as an example. On the other hand, there do seem to be special cases in the accounts of Epimenides (below, n. 166) and Aesopus (n. 176). Plato is thinking of metempsychosis in this way when he evaluates it as an effort *λόγον διδόναι* (*Meno* 81a).

each detail of his exposition is planned, the relationships and the laws of the universe are only indirectly stated, in genealogies and mythical episodes, the doctrine of metempsychosis directly states a general law. It does not merely explain particular situations; past and present, preexistence and life after death are comprehended in a single thought. And, insofar as animals and plants are included in it, the unity and homogeneity of the universe are grasped.[88] In the thought of the preexistence and immortality of the soul lies an attempt at consistent thinking, a groping for something like an eternal, imperishable Being —a foreshadowing of the ontology of Parmenides.[89] To this extent, even leaving aside the ideas of science and philosophy, Pythagoras, as teacher of metempsychosis, is not simply the prophet of old-fashioned piety, but at the same time, in his own way, a thinker leading, through ancient forms, to a new level of consciousness.

A doctrine like that of metempsychosis, which transcends normal human ways of knowing, can find a guarantee only in supernatural experience, in the world of the divine or quasi-divine. If Pythagoras knew the facts about the fate of the soul in this life and the next, he must have had superhuman powers and faculties; the prophet must be able to refer to his own example.[90] It is natural, then, that elements of the Pythagoras legend are, from the beginning, connected with the doctrine of metempsychosis.

It is often seen as matter for regret that miraculous tales have attached themselves to the figure of Pythagoras and make it difficult for the scholar to disentangle the thread of historicity from the web of legend and fiction.[91] In the circumstances it is very tempting to use expressions like "neo-Pythagorean" or "late antiquity" to classify these tales.[92] One feels confident, in any case, that they represent a

[88] In the saying ascribed to Thales, too, *πάντα πλήρη θεῶν*, the word *πάντα* points in this direction; there are not sacred and profane realms, but a unified world.

[89] Cf. Kahn's judgment of Greek doctrines of metempsychosis (*AGP* 1960, 34): "by its rigor and its generality, such a doctrine is no longer primitive." Nilsson, *Op.* III 51 sees the origin of the doctrine of metempsychosis in "pure logic"—an exaggeration, but it gets at an important aspect of the matter.

[90] Cf. Empedocles' expression (fr. 117) *ἤδη γάρ ποτ' ἐγώ*, and also Epimenides, DK 3B2. Orpheus and Musaeus are also, of course, sons of gods (Pl. *Rep.* 363c, *Tim.* 40d).

[91] Zeller, *Vortr.* 30; Rathmann 17f: "fabulis tamquam adopertum."

[92] Zeller (I 365) says that a great part of the miraculous tales is "neo-Pythagorean" (cf. *Vortr.* 42f, and Schottlaender 342). Kerényi (15) speaks of "spätantike Pythagoraslegenden." Rathmann (25ff) tries to show that all the stories of Pythagoras were derived from the legends of Epimenides, Abaris, Orpheus, and Zalmoxis; but this is refuted by the "golden thigh" theme. Maddalena (358) sees that everything essential is attested in 4th-century sources, but will not admit that it may go back further.

secondary growth, layers that must be stripped off until what the scientific historian recognizes as "facts" can be seen. Only in a few cases has it been recognized that these miraculous stories do not conceal but reveal reality, that they give us a clue to the impression made on contemporaries by an actual person,[93] and that they may even contain facts of a special character. The Pythagoras legend is the oldest available layer of the tradition on Pythagoras; it is attested earlier than any of the "historical" details of his life in Aristoxenus and Dicaearchus, and is presupposed by the Platonizing reinterpretation of Pythagoras in the Old Academy.

As early as Empedocles, Pythagoras appears as a superhuman figure:

> *ἦν δέ τις ἐν κείνοισιν ἀνὴρ περιώσια εἰδώς,*
> *ὃς δὴ μήκιστον πραπίδων ἐκτήσατο πλοῦτον,*
> *παντοίων τε μάλιστα σοφῶν τ' ἐπιήρανος ἔργων·*
> *ὁππότε γὰρ πάσῃσιν ὀρέξαιτο πραπίδεσσιν,*
> *ῥεῖ' ὅ γε τῶν ὄντων πάντων λεύσσεσκεν ἕκαστον*
> *καί τε δέκ' ἀνθρώπων καί τ' εἴκοσιν αἰώνεσσιν.*

Timaeus understood the words as applying to Pythagoras; others thought of Parmenides.[94] This clearly shows that Empedocles did not give any name,[95] but also that the anonymous figure appeared as the teacher and master of Empedocles. The praise of Epicurus by Lucretius, and the praise of Lucretius by Vergil, are in the same tradition; neither gives a name, any more than Ovid does when he introduces Pythagoras.[96] Therefore, neither some anonymous Orpheotelestes[97] nor a

[93] Rohde, in 1871 (Q 105), recognized the "ganz echten Mythencharacter." See also Lévy passim, Gigon, *Ursprung* 131, Dodds, *Irr.* 144f, with nn. 63–64. Lévy thought the Pythagoras saga had, indirectly, even influenced the gospels (a view disputed, rightly, by Fr. M.-J. Lagrange, *Rev. bibl.* 45 [1936] 481–511; 46 [1937] 5–28, and M. Goguel, *RevPh* 14 [1938] 241–270). He tried to show that the historical tradition about Pythagoras, in Aristoxenus and others, was a rationalization of the legend. Rostagni, too (*Verbo* 121 n. 1) speaks of the "gospel" of Pythagoras.

[94] Fr. 129 (D.L. 8.54 = Timaeus *FGrHist* 566F14, and Nicomachus ap. Por. *VP* p. 56 Lam. *VP* 67).

[95] Rathmann 42f, Maddalena 343ff. For Pythagoras as teacher of Empedocles, see below, ch. III 3, n. 59; for Parmenides as his teacher (obvious from the fragments), Theophrastus ap. D.L. 8.55 = *Dox.* 477.18.

[96] Luc. 1.62ff, Verg. *G.* 2.490ff, Ov. *Met.* 15.60ff (Rostagni, *Verbo* 266f, Long 18 n. 16). Rostagni mentions the report that Pythagoreans avoided pronouncing Pythagoras' name (*Verbo* 231; Iam. *VP* 88, probably from Aristotle; also Iam. *VP* 150, 255).

[97] Rathmann 138.

man of the Golden Age[98] can be intended. The mental power of the great man extends over "ten and twenty human lifetimes";[99] his mind is superior to the change of generations—is it his immortal soul that remembers "every detail"?

Heraclides has Pythagoras tell this tale:[100] he had once been Aethalides, the son of Hermes, and received from the latter the gift of remembering everything, both in life and in death. Thus he knew that, as Euphorbus, he had been slain by Menelaus in the Trojan War, and that he had subsequently been Hermotimus, then Pyrrhus, a fisherman of Delos, and finally Pythagoras. Here we are on the shaky ground of Academic and Peripatetic controversy. Dicaearchus and Clearchus give a very different list of Pythagoras' previous incarnations: Euphorbus, Pyrandrus, Aethalides, a beautiful prostitute named Alco, and Pythagoras.[101] It looks as though each one treated the Pythagoras tradition as his whim or fantasy dictated.

Actually, it can be shown that there was an independent tradition involving Aethalides,[102] Hermotimus,[103] and the Delian fisherman,[104]

[98] Zeller, *SBBln* 1889, 990; I 396 n. 1, 584 n. 5. Against this interpretation Rostagni (*Verbo* 229ff) cites fr. 132, which shows that in Empedocles' view there are at all times "divine" men. The attribution to Pythagoras has been defended also by Kranz, *Hermes* 1935, 112 n. 2; Mondolfo in ZM 329: Long 17ff: Timpanaro Cardini 17f. Maddalena, too, concedes this (64.3), but contests the connection with metempsychosis, since Nicomachus interprets the words as referring to Pythagoras' hearing the harmony of the spheres; but it may be that Nicomachus, or one of his predecessors, was trying to get rid of the metempsychosis theory.

[99] *ἀνθρώπων αἰών* = generation ; cf. Aesch. *Sept.* 744 (Long 21, 51); wrongly "in seinen zehn und zwanzig Menschenleben," DK. Homer's Calchas knows "what is, what will be, what was" (*Il.* 1.70), the sibyl, too, defies time (Heraclitus fr. 98); cf. also Parmenides fr. 4.1.

[100] Fr. 89 W. = D.L. 8.4; repeated Hippol. *Ref.* 1.2.11, Por. *VP* 45 (Euphorbus at the beginning), Tert. *An.* 28f (Pyrrhus before Hermotimus), Schol. Soph. *El.* 62 = *Suda* s.v. *ἤδη* (*Πύθιος* instead of *Πύρρος*), Hieron. *In Ruf.* 3.40 (from Por. *VP* 45; Callicles instead of Aethalides), Schol. Ap. Rh. 1.646 (a double version, with Hermotimus once corrupted to *Ἡλεῖός τις* and placed after Pyrrhus, and the second time left out). See Rohde, *Psyche* II 417ff = 598ff Eng. ed., Rathmann 9 n. 67. Which book of Heraclides is the source is uncertain (the *Abaris* according to Corssen, *RhM* 1912, 28; A. Rehm ibid. 423; Lévy, *Sources* 40; Wehrli, hesitantly, suggests the *ἄπνους*).

[101] Dicaearchus fr. 36, Clearchus fr. 10 = Gell. 4.11.14.

[102] Pherecydes of Athens, *FGrHist* 3F109 = Schol. Ap. Rh. 1.645. (See Jacoby's note; the attribution to Pherecydes of Syros, DK 7B8, can scarcely be right.) Aethalides is a son of Hermes. He has the privilege that his soul may dwell part of the time on earth and part of the time in Hades; this is reminiscent of the Dioscuri, not of metempsychosis. Aethalides belongs to Lemnos; this may have something to do with the supposed Tyrrhenian origin of Pythagoras (ch. II 2, n. 12).

[103] Below, nn. 177–178.

[104] Pyrandrus is probably the same as Pyrrhus. The skill of a certain "Delian diver" was proverbial (D.L. 2.22, 9.12, Herondas 3.51). There is a swimmer near the ship of Theseus as it lands in Delos, on the François vase. Perhaps some Delian ritual lies in the background.

but not Pythagoras. Obviously Heraclides first put the two traditions together and the "beautiful prostitute" may be a sarcastic addition of Dicaearchus.[105] Present throughout, however, is the assumption that Pythagoras had "proven" his theory of metempsychosis by recounting his previous incarnations; and also present throughout is the puzzling figure of "Euphorbus."

In Antiphanes' comedy *Νεοττίς*,[106] first presented not long after 342 B.C., appears a miser who gives his slaves nothing to eat:

> *ἄνθρωπος ἀνυπέρβλητος εἰς πονηρίαν,*
> *τοιοῦτος οἷος μηδὲν εἰς τὴν οἰκίαν,*
> *μηδ' ὧν ὁ Πυθαγόρας ἐκεῖνος ἤσθιεν*
> *ὁ τρισμακαρίτης, εἰσφέρειν ἔξω θύμου.*

He is alluding to Pythagoras' ascetic precepts, but the decisive word for the interpretation is *τρισμακαρίτης*. *Τρὶς μάκαρες Δαναοὶ . . .* (*Od.* 5.306) would be a familiar phrase to every Greek, but *μακαρίτης* is only used of one who has "gone to his reward," that is, who is dead. The poet is punning: the "thrice blessed" Pythagoras is at the same time the "thrice dead." One should not lay undue stress on the number 3, which has an intensive force, and is also explained by the conventional formula *τρὶς μάκαρες*. But this much is certain: that it was well enough known in Athens, about 342 B.C., that Pythagoras had lived through several lives, that a comic poet could count on an allusion to the idea being understood.[107]

Late testimonia often name no one but Euphorbus as an earlier incarnation of Pythagoras,[108] and since this name is mentioned by both

[105] If Pythagoras experienced everything, he must once have been a woman. It is hard to tell whether Clearchus was taking the matter seriously, whether he was following Dicaearchus, or whether it was only a process of abridgment, in the course of the tradition, that left him with the same series as Dicaearchus.

[106] Antiphanes fr. 168 Kock = Ath. 4.108e. The date of the *Neottis* can be deduced from fr. 169 = Ath. 6.223e, where allusion is made to Demosthenes' position in the negotiations with Philip about Halonnesus in 342 B.C. (Ath. in his comment on the fragment, hypothesis to ps.-Demosth. 7).

[107] The passage is missing in the index of Kaibel's edition of Athenaeus, s.v. Pythagoras, and (doubtless for this reason) is omitted in DK and Weiher. Zeller cites it (III 2.93), but mistranslates *τρισμακαρίτης* as "preiswürdig." (LSJ has it right.) It is possible that Antiphanes is dependent on Heraclides; he seems to allude to him in fr. 113 Kock (Heraclides fr. 10 W.).

[108] Callim. fr. 191.59 Pf., Diod. 10.6.1, Hor. *Carm.* 1.28.11 (where the word *iterum* excludes further incarnations), Nicom. (Por. *VP* 26 = Iam. *VP* 63), Hippol. *Ref.* 1.3.3. At *Th. a.* 52 8ff De Falco, Androcydes, Eubulides, Aristoxenus, Hippobotus, and Neanthes are named together, but it is impossible to make out how much may go back to Aristoxenus (fr. 12, with Wehrli's comment p. 50). That Empedocles spoke of "Euphorbus" (Rostagni, *Verbo* 240ff) is unlikely; ancient scholarship on Pythagoreanism would hardly have failed to mention such a fact.

Heraclides and Dicaearchus, the conclusion has been drawn, rightly, that this detail of the tradition is older than Heraclides.[109] Before the beginning of scientific historiography, the historical consciousness of the Greeks relates mainly to two periods, the immediate past and the Homeric period known from the *Iliad* and the *Odyssey*; so that when Pythagoras spoke of his earlier lives, he had to establish his presence in the earlier period.[110] But why did he choose precisely this not very distinguished participant in the Trojan War? Some suggested answers are based on the etymology of the name,[111] but the most persuasive interpretation is that of Karl Kerényi, who found a clue to the riddle in the words of Homer. As Patroclus is dying he says to Hector,[112]

[109] Rohde, *Psyche* II 418 = 599 Eng. ed.; Corssen, *RhM* 1912, 45ff. (Contra, Lévy, *Sources* 34 n. 1: the naming of Euphorbus alone is a result of secondary abbreviation; but, if that is the case, why do Aethalides, Hermotimus, and Pyrrhus never get named by themselves as earlier incarnations of Pythagoras?)—Pythagoras recognized, in Argos, the shield of which Menelaus had despoiled the body of Euphorbus (Diod. 10.6.2, Ov. *Met.* 15.163f, Schol. T *Il.* 17.28). This happens in the Heraeum near Mycenae according to Nicom. ap. Por. *VP* 27 = Iam. *VP* 63; cf. Paus. 2.17.3. No place is given in Hor. *Carm.* 1.28.9ff, Hippol. *Ref.* 1.3.3. Maximus Tyrius (10.2) speaks, erroneously, of a temple of Athena. Heraclides (fr. 89) has Hermotimus find the shield, in the sanctuary of Apollo at Didyma, corrupted in Tert. *An.* 28 to "Delphi." This has the appearance of being secondary, and if so the shield story is at least older than Heraclides. Nicomachus finds the story silly: *τὰ γὰρ ἱστορούμενα περὶ τῆς ἀσπίδος παρίεμεν ὡς πάνυ δημώδη.*

[110] This consideration would seem to indicate, again, that originally only one earlier incarnation, as Euphorbus, was mentioned. It may have been only later that people calculated a certain number of years. At *Th. ar.* 52.8ff (Androcydes, Eubulides, Aristoxenus, Hippobotus, and Neanthes; cf. n. 108) the period is 216 years. "Pythagoras himself," D.L. 8.14 mentions 207 years, though this is surely to be corrected to 216, which is in itself a significant number (the cube of 6). This "writing" of Pythagoras, cited D.L. 8.14, is to be identified with the "tripartitum" of D.L. 8.6f, because of its Ionic dialect (Diels, *AGP* 1890, 469). See further Rohde, *Psyche* II 419 = 599f Eng. ed., Corssen, *RhM* 1912, 243, Delatte, *Vie* 181, Lévy, *Sources* 76. O. Skutsch, *CP* 54 (1959) 115, conjectures that Ennius' peacock originally belonged among the incarnations of Pythagoras; he would insert it between Euphorbus and Pythagoras. Scholars have mostly supposed that the source of the whole tradition was an old *katabasis* poem (Rohde, Q 106.1; dubiously *Psyche* II 419 = 600 Eng. ed.; Norden is very positive: it was "an ancient Orphic-Pythagorean poem of the sixth century" (Vergil VI, 5 and 21) which "was regarded as almost canonical"; cf. Dieterich, *Nekyia* 129, Ganschinietz, *RE* X 2410. There is no reason to believe, however, that there ever was an ancient written source, or that, if the tradition was oral, it was transmitted in the form of a poem. What we have to reckon with is oral narrative and maxim (*acusmata*), not literary genres.

[111] Delatte, *Vie* 157, interprets Euphorbus as the "good shepherd," and assumes the existence of a Phrygian cult of a "good shepherd" and an "Orphic" apocalypse deriving from it; but this is all pure hypothesis. Skutsch, *CP* 54 (1959) 114, understands Euphorbus as meaning "he who eats the right food," and connects him with Pythagorean dietary regulations. It may be that both these motifs, "shepherd mysteries" and "correct food," played a role; and generally it is well to keep in mind the possibility of multiple interpretation.

[112] *Il.* 16.849f; Kerényi 19. Rohde mentions (*Psyche* II 418 — 599 Eng. ed.) that Euphorbus' father was a priest of Apollo, but does not think this explains the matter adequately. Schottlaender (345f) cites the fact that his mother's name was *Φροντίς* (*Il.* 17.40). Corssen (*RhM* 1912, 22) calls the choice of Euphorbus incomprehensible; and Tert. *An.* all jeered at the irrationality of it.

ἀλλά με μοῖρ' ὀλοὴ καὶ Λητοῦς ἔκτανεν υἱός,
ἀνδρῶν δ' Εὔφορβος· σὺ δέ με τρίτος ἐξεναρίζεις.

If we consider the arithmetic here, it seems as though Moira, Apollo, and Euphorbus only make up two, so that two of the three must be identical. The solution that Moira is not personified here, and thus not counted as one of the group, is by no means self-evident to the ancient scholars who busied themselves with the problem.[113] "If someone wanted to say, 'I am perhaps Apollo,' he could, in Homeric terms, call himself Euphorbus," says Kerényi. The advantage of this interpretation is that it is entirely derived from the Homeric text. The name Euphorbus refers unmistakably to Homer, and the whole intellectual world of the archaic period takes its character from Homer. Innovation presents itself in the guise of Homeric interpretation.

We are told in fact that in Croton Pythagoras was thought to be "the Hyperborean Apollo." For these aspects of the Pythagoras legend, Aristotle's book on the Pythagoreans is the important source.[114] We have the miraculous stories in four versions, in the *Historia mirabilium* of Apollonius,[115] in Diogenes Laertius, Aelian, and Iamblichus.[116] Aristotle records the following items:

(a) Pythagoras was called "Hyperborean Apollo" by the Crotoniates.[117]

(b) At the same hour on the same day he was seen both in Croton and in Metapontum.[118]

[114] See the scholium on the passage.

[115] See above, ch. I 2. The material is collected by Rose as fr. 191, discussed by Corssen, *RhM* 1912, 29ff, and Lévy, *Sources* 10ff. It is scarcely to be supposed that Aristotle failed to mention the Euphorbus story, but since it was so well known he is not cited for it.

[116] The immediate source of Apollonius is obviously Bolus, as is shown by the lemma *Βώλου* at the beginning of the text (Diels, *SBBln* 1891, 393f; Wellmann, *AbhBln* 1921, 4 u; Lévy, *Sources* 11.4). Theopompus (*FGrHist* 115F70–71) is the source of ch. 5 (on Pherecydes) but not for ch. 6, as Diels wrongly supposes. All of it clearly comes from Aristotle, and Bolus is, at least here, compiler rather than forger. On Apollonius see W. Kroll, *RE* Supp. IV 45ff: none of his sources is later than the second century B.C.

[117] Also Plut. *Numa* 8, Amm. Marc. 22.16.21, and some echoes in Lucian. The wording of the principal sources is often so closely similar that there may well have been a common intermediate source (Bolus?). Nicomachus, though taking departure from the same source, introduces variants that are mostly characteristic of him. He calls his authorities *ἀκόλουθοι καὶ ἀξιόλογοι* (Por. *VP* 23 — Iam. *VP* 60).

[118] (In the following notes, a passage that names Aristotle is cited in bold face type.) **Ael. 2.26**, D.L. 8.11, Iam. *VP* 140 (on the text see Deubner, *SbBln* 1935, 677ff; other proposals in Hölk 23, Corssen, *RhM* 1912, 37 n. 1; Lévy, *Sources* 14 n. 3); cf. Iam. *VP* 30; Lucian *Dial. mort.* 20.3—Nicomachus combines this with the Abaris episode, Por. *VP* 28 — Iam. *VP* 135f.

[119] **Ael. 2.26**, 4.17, Ap. *Hist.* 6. It is "Metapontum and Tauromenium" in Nicomachus Por. *VP* 27 — Iam. *VP* 134, cf. 136), "Thurii and Metapontum" in Philostr. *VA* 4.10. Apollonius of Tyana imitated the feat: he was in Smyrna and Ephesus on the same day (Philostr. loc. cit.).

(c) When Pythagoras stood up among the spectators at Olympia, people saw that one of his thighs was of gold.[119]

(d) He reminded Myllias of Croton that he had been King Midas.[120]

(e) He stroked a white eagle in Croton.[121]

(f) As Pythagoras was crossing the Casas River, the river hailed him in an audible voice, "Greetings, Pythagoras!"[122]

(g) As a ship was entering the harbor of Metapontum, he predicted that a dead man would be found in it.[123]

(h) In Caulonia, he correctly predicted the appearance of a white bear.[124]

[119] **Ap. *H.m.* 6,** Ael. 2.26, 4.17, D.L. 8.11, Iam. *VP* 140, Plut. *Numa* 8. Cf. Lucian *V. auct.* 6, *Gall.* 18, *Dial. mort.* 20.3. Olympia is named in Ael. 4.17, Plut. *Numa* 8, and Amm. Marc. 22.16.21, and is probably to be supplied in Ael. 2.26; the other sources merely speak of an *ἀγών* or *θέατρον*. Lévy, *Sources* 20, assumes that Pythagoras appeared as an athlete (cf. above, ch. II 2, n. 5), but *παρέφηνε* signifies an unintentional brief disclosure, not the nakedness of an athlete (see Plut. *Praec. conj.* 142c).—Nicomachus mentions the showing of the thigh only in connection with the Abaris scene and dismisses the matter with the verb *τεθρύληται* (Por *VP* 28 = Iam. *VP* 135, cf. 91f.)—Alexander of Abonuteichus copied the golden thigh (Luc. *Alex.* 40).—On Schol. Luc. p. 124.6f, see below n. 215.

[120] Ael. 4.17, Iam. *VP* 143; alluded to by Nicomachus ap. Por. *VP* 26 = Iam. *VP* 63. Maddalena (359 n. 86) contests the attribution of this to Aristotle, but since the same item occurs in two sources in the midst of Aristotelian material, it is at least probable.—The name Myllias is used in Neanthes' horror story at *FGrHist* 84F31 = Iam. *VP* 198ff. Por. *VP* 61.

[121] Ael. 4.17, Iam. *VP* 142, Plut. *Numa* 8, Amm. Marc. 22.16.21. Nicomachus (Por. *VP* 25 = Iam. *VP* 62) shows exact verbal agreements with Plutarch (*ὑπερπτάμενον . . . καταγαγεῖν*); but he makes the whole story a proof of the validity of divination by birds, and sets the scene in Olympia, while he excludes the "golden thigh" story from Olympia (above, n. 119).

[122] **Ael. 2.26,** Ap. *H.m.* 6. The name of the river is *Κόσας* in Aelian, while Apollonius has the meaningless *κατὰ σάμον* (i.e. *Κάσας*?), and Diogenes (8.11) *Νέσσος*. In the Nicomachus tradition, Porphyry (27) has *Καύκασος* (meaningless; a mistake for *καὶ Κάσαν*? L. Bieler, *WS* 48 [1930] 201–205, tries to explain this last by assuming the loss of a line in Por.; but he overlooks the context in Nicom. and the parallel in Iam., which show that not more than one word can be wrong). Iamblichus (134) has *Νέσσος* (obviously substituted, from the "handbook source," for *Καύκασος*; cf. D.L., Lévy, *Sources* 104, 115). What we should expect is *Κάσας*, which is a river near Metapontum (Bacchyl. 11.119; Diels, *Hermes* 33 [1898] 334; Lévy, *Sources* 13). Can *Νέσσος* be a mishearing of *Νέαιθος*, the river near Croton (as Arcerius suggested)?

[123] Ap. *H.m.* 6, Iam. *VP* 142, Nicom. ap. Por. *VP* 28 (Iam. agrees exactly with Nicom.). Cf. Thrasyllus ap. Dio Cass. 55.11.3, *CCAG* VIII 4.100.1ff.

[124] **Ap. *H.m.* 6,** with a gap which is filled from Iam. *VP* 142. Nicomachus has a different version (Por. *VP* 23 — Iam. *VP* 60), according to which, in Daunia, Pythagoras converted a wild she-bear to vegetarianism, and a similar tale (Por. *VP* 24 — Iam. *VP* 61) about an ox in Tarentum which, on the urging of Pythagoras abstained for the rest of its life from eating beans.

(i) In Etruria, he bit a poisonous snake to death.[125]

(j) After predicting to the Pythagoreans the outbreak of civil strife (*στάσις*) he disappeared to Metapontum without anyone's seeing him go.[126]

Two more reports are only found in Iamblichus, in context with the Aristotelian material. It is possible that they go back to Aristotle:

(k) Pythagoras took from Abaris, the priest of Apollo from the country of the Hyperboreans, the arrow with which he traveled, and thus established himself as the Hyperborean Apollo.[127]

(l) "They say of the man who bought Pythagoras' house and tore it down, that he did not dare tell anyone what he saw, but that as a result of this crime he was convicted of sacrilege by the Crotonians and executed. For he was convicted of having stolen the golden chin which had fallen from the god's statue." Pythagoras' house is inviolate, like a sanctuary of the mysteries; the transgressor dies the death of a scapegoat.[128]

These "miracles" are portents without interpretations, revelation and occultation at once. At a certain moment there is a glimpse of the divine: the gleam of the golden thigh, the greeting of the river god, the arrival of the Hyperborean. Superhuman powers are evident in Pythagoras' prophecies, in his mastery of the animals, and in his control of space and time, as well as in the numinous dread that

[125] **Ap. *H.m.* 6.** The tone is different in Iamblichus (142): *τὸν μικρὸν ὄφιν, ὃς ἀπέκτεινε ἄλλον*. Rose corrects *ὅς* to *ὅν*, but probably Iamblichus himself changed the wording. He passes on, with the phrase *ὁμοίως δέ*, to the story of the driving out (*ἀπεπέμψατο*) of another snake from Sybaris. (It is doubtful whether the double version stems from Aristotle.) That Pythagoras himself bit the snake (like must be combated by like!) was too grotesque for Iamblichus.

[126] **Ap. *H.m.* 6.**

[127] Iam. *VP* 140f, excluded from the Aristotle fragment by Rose. Corssen, *RhM* 1912, 181, tries to show that Hermippus is its source, using the argument that Pythagoras, who simply takes his arrow away from "the unfortunate Abaris," is presented in an unfavorable light; but, if Pythagoras is the "Hyperborean Apollo," Abaris has reached his destination and does not need the arrow any more (Lévy, *Sources* 18). Lévy, (*Sources* 14f) discovered that the rare feminine *ἡ οἰστός* occurs not only in section 140 but also, precisely, in Aristotle (*Phys.* 239b7). The excursus on Abaris, sec. 141, p. 79.18–23, has a close relationship to Ap. *H.m.* 4, and probably comes from a different source from the surrounding context. In that case, Iamblichus may well be the author of the transitional passage pp. 79.23–80.1. The expression *ὁμολογοῦντα ἐποίησεν*, the "confession" of Abaris, so reminiscent of the confession of Peter after the transfiguration of Christ, can scarcely be old. Perhaps it is not coincidence that in this passage (p. 79.23f) *οἰστός* is twice masculine and once feminine, in the manuscripts. (Deubner restores the feminine throughout.) The parallel version of the Abaris story at Iam. *VP* 91ff has only the masculine.

[128] Iam. *VP* 143. Timaeus reports that Pythagoras' house became a shrine of Demeter (*FGrHist* 566F131). On scapegoat rituals see V. Gebhard, *RE* V A 1290ff.

attaches to his house. There is always something enigmatic about the meaning of these miracles, which is apparently revealed to the insider but not explained to the uninitiated. The relation to Apollo is ambiguous, too. According to Aristotle, the Pythagoreans taught, ἐν τοῖς πάνυ ἀπορρήτοις, that "among rational beings there is that which is god, that which is man, and that which is like Pythagoras."[129] Here a phrase like λογικὸν ζῷον betrays later terminology, though the idea of the intermediate would not contradict the identification with "Hyperborean Apollo": the third kind of being could be gods in human form. But explicitness is avoided, just as the Euphorbus story does not go beyond a "perhaps" as to the hero's divine status.

A second group of legends is also attested in fourth-century sources, but it was controversial even then whether they apply to Pythagoras or Pherecydes of Syros. In his *Tripus*, a book about the Seven Sages, Andron of Ephesus ascribed to Pythagoras what Theopompus ascribed to Pherecydes. Porphyry, who brings out the contradiction, brands Theopompus the "thief"[130]—which prompts modern scholars to take the opposite view.[131] These are the stories:

(a) Either Pythagoras in Metapontum or Pherecydes in Syros[132] took a drink of water from a well and predicted an impending earthquake.

(b) Pythagoras in Megara Hyblaea or Pherecydes in Samos predicted that a ship, sailing with a favorable wind, would sink, and this immediately happened.

(c) Pythagoras predicted the conquest of Sybaris or Pherecydes that of Messene, and in each case a friend was warned.

(d) Pherecydes or Pythagoras, on the authority of a dream in which Heracles appeared, told the Lacedaemonians not to honor gold or silver; and in the same night Heracles bade the kings to heed the words of Pherecydes or Pythagoras.[133]

These miracles all belong to the realm of prophecy. Prediction of an earthquake is also ascribed to Anaximander;[134] the addition of the "drink of water" suggests an origin in cult practices.[135] Comparison of details[136] yields no conclusive argument for the priority of either version, though there is a serious chronological difficulty in the juxtaposition of Pherecydes and the fall of Messenia.[137] It is scarcely possible to judge the date or the reliability of Andron; but there is no doubt that Theopompus was hostile to the Academy and also to Pythagoras.[138] In any case Andron attests the wide dissemination of Pythagoras legends in the fourth century. There is no reason to think of Aristotle as being dependent on Andron.[139]

Later tradition adds little beyond the miraculous catch of fish, which does seem to be earlier than Nicomachus.[140] What Aristotle reports is in tone, quite apart from the nature of its attestation, anything but characteristic of "late antiquity." It was more likely to provoke later ages to mockery or protest. Alexander of Abonuteichus claimed

[129] Arist. fr. 192 = Iam. *VP* 31; cf. Iam. *VP* 30, 143f, Schol. BT *Il.* 1.340. Ps.-Apollonius of Tyana, *Ep.* 50, counts Pythagoras ἐν γένει δαιμόνων. Corssen (*RhM* 1912, 39) and Kerényi (18) emphasize that this is not incompatible with the equation Pythagoras-Apollo.

[130] Porphyry, Φιλόλογος ἀκρόασις ap. Euseb. *Praep. evang.* 10.3.6 (DK 7A6). Theopompus' version is also attested at D.L. 1.116 and Ap. *H.m.* 5 (*FGrHist* 115F70-71), Andron's (earthquake and sinking ship) at Iam. *VP* 136.

[131] For priority of the Pherecydes version are Corssen, *RhM* 1912, 33ff, Bertermann 58f, Rathmann 28.

[132] In Samos, according to Max. Tyr. 13.5. No location is named in Cic. *Div.* 1.111 and Plin. *HN* 2.191.

[133] This story is only found in D.L. 1.117. Theopompus F71, with the addition ἄνια δὲ Πυθαγόρᾳ περιάπτουσι ταῦτα. But Porphyry mentions that Andron reported still more miracles of Pythagoras (καὶ ἕτερά τινα τούτοις ἐπαγαγών). Andron named Pherecydes as Pythagoras' teacher (D.L. 1.119).

[134] For Anaximander, see Cic. *Div.* 1.112, Plin. *HN* 2.191 (DK 12A5a). The mention of Anaxagoras (Amm. Marc. 22.16.22 [DK 59A10]) is doubtless the result of confusing the two men.

[135] At the oracle at Clarus, the priest prophesied after drawing water from a secret spring in a cave (Tac. *Ann.* 2.54; cf. M. Ninck, *Die Bedeutung des Wassers im Kult und Leben der Alten* [Leipzig, 1921] 83ff).—Apollonian prophecy is combined here with "physical" speculation about the subterranean forces that cause earthquakes.

[136] Corssen (*RhM* 1912, 36) stresses that Pythagoras is not ordinarily connected with Sicily (Megara Hyblaea). But why would a plagiarist not have put down "Croton" or "Metapontum," rather than betray himself? Aristotle has Pythagoras traveling to Sicania (above, n. 125).

[137] Cf. Kiechle 11 and 56. The fall of Messene is set at 600 B.C. (at the latest) in ancient tradition, but Pherecydes was a contemporary of Anaximander and perhaps dependent on him (von Fritz, *RE* XIX 2030f), though some authorities give an earlier date (Arist. fr. 74, *Suda* s.v.).

[138] F71; see above, ch. II 2, n. 61. This counterbalances the principal argument for the priority of the Pherecydes version, namely that the attribution is more likely to have been switched in the direction of the more prominent figure, Pythagoras.

[139] As Rohde (Q 135) thought. Corssen (*RhM* 1912, 32ff) equates the second anecdote given by Andron and the similar one in Aristotle (above, n. 123), and denies Aristotelian authorship. Porphyry's wording makes it clear, however, that Andron told the same story as Theopompus (D.L. 1.116), namely that the ship sank, whereas in Aristotle it has a corpse on board. Corssen finds Theopompus' version simpler, but one can also interpret it, with its sensational tone, as a cruder variant of Aristotle's.

[140] Nicom. ap. Por. *VP* 25 = Iam. *VP* 36; somewhat differently Plut. *Quaest. conv.* 8.8.1 729d, *De cap. ex inim. ut.* 9.91c, Apul. *Apol.* 31. According to Nicomachus the event takes place between Sybaris and Croton, according to Apuleius near Metapontum. Pythagoras either buys some fishermen's catch and sets the fish free (Plut., Apul.), or predicts the number of fish that will be in the net and then, when the prediction turns out correct, bids the fishermen set the fish free. Knowing a number in advance is a feat attributed to the seer from earliest times (Hes. *Melampodia* fr. 278 M.-W.); but here the motif of prophecy could have been an addition of Nicomachus himself.

to be Pythagoras and displayed his golden thigh. In Lucian's judgment he selected and even exceeded "all the vilest and most damaging slanders ever vented against Pythagoras."[141] The stories of miracles are widely regarded as attempts to discredit him and put down as slander. Nicomachus expresses candidly his distaste for the crude, "plebeian" miracles,[142] and tells the stories in such a way that the inexplicable is played down or ignored, so that what remains is, essentially, clairvoyance and the doctrine of the kinship of all living beings, and the exhortation to *ἡμερότης*. The golden thigh is only mentioned in connection with the wonder-worker Abaris, and is quickly disposed of, and completely unsophisticated details like appearing simultaneously at two places or biting a snake to death are omitted. In their place appear the somewhat sentimental tales about the conversion of wild animals. But even much earlier than Nicomachus these reports had been transformed. "Epimenides, Eudoxus, and Xenocrates" are cited as saying that Pythagoras was the son of Apollo.[143] This eliminates not only metempsychosis but identity with Apollo, and closeness to the gods is expressed in the customary form of divine ancestry.

The Pythagoras legend is attested for the fourth century B.C., and at least part of it was well known; what is more, it is distinct from the Platonizing interpretation current in the Academy, which made Pythagoras a doublet of Plato himself. It antedates Antiphanes, Andron, Heraclides. It was recorded by Aristotle, perhaps not without a polemical glance toward the Academy's modernizing interpretation. It was carried along, willy nilly, by others, though assiduously reinterpreted or, sometimes, rejected as slanderous, and all the while made the target of ridicule by its enemies. Finally, then, with figures like Apollonius of Tyana and Alexander of Abonuteichus, the wonder-worker reappears in reality; such activity is to be sure "late antique," but it revives preclassical patterns.[144]

[141] Lucian, *Alex.* 4 (cf. 40).

[142] Above, nn. 109, 119, 117, 121, 124.

[143] Iam. *VP* 7 = Xenocrates fr. 22 H., Eudoxus fr. 86 Gisinger = fr. 324 Lasserre. The authenticity of the Xenocrates fragment is doubted by Rohde (Q 128), Zeller (II 1.1023.5), and Lévy (*Sources* 9.5), that of the Eudoxus fragment by Schaarschmidt (44f)—without adequate grounds. Xenocrates had to reinterpret the Pythagoras legend (above, nn. 19–20). Apollonius (Iam. *VP* 5 Por. *VP* 2) quotes two lines from an elegy by a "Samian poet" in which Apollo is named as Pythagoras' father. Lévy (*Sources*, 105 n. 4, *Lég.* 6 n. 4) thinks the citation of Epimenides, Xenocrates, and Eudoxus is an interpolation in the Apollonius passage; if correct, this would speak for its authenticity, for in that case it must have been taken from a "handbook" source.

[144] The influence of the Pythagoras legend is traced by Lévy, *Lég.*, cf. above, n. 93; on its effect on Athanasius' life of St. Anthony, see R. Reitzenstein, *SBHeid* 1914, Lévy *Lég.* 129ff. Lévy, however, strongly overrated the influence of Heraclides on the legend.

Oral tradition clearly is involved in the Pythagoras legend; we must expect alterations and distortions to have occurred. Still, for its origin no terminus post quem is set other than the historical Pythagoras,[145] and faithful preservation of original traits cannot be excluded from the start. This is why we should try to understand the tradition before discarding it, even though, consciously or not, scholarship tends to proceed from the assumption that legend is always secondary, because the "historical kernel" must be an event or pattern of events amenable to common sense and not a "miracle." What if the "facts," as we see them, were experienced differently at that time? If the historical Pythagoras taught metempsychosis, this same historical Pythagoras must have claimed superhuman wisdom, he had to use his own life as an example and find himself in the Trojan War. And if he wanted to make this credible, he had to—perform miracles.

The *katabasis* of Pythagoras is very important in this context, but it is especially difficult to evaluate this tradition. Therefore we may first survey what there is of similar phenomena in archaic Greece, in the hope that this will bring various kinds of confirmation and illumination.

From ancient times Pythagoras, as miracle-worker, has been associated with figures like Aristeas, Abaris, Epimenides, Phormio, and Empedocles.[146]

An epic poem entitled *Arimaspeia*, by Aristeas of Proconnesus,

[145] In all cases of coincidence with the stories of Aristeas, Epimenides, or others, Rathmann assumes that the ascription to Pythagoras is secondary; and Corssen (*RhM* 1912, 44) believes that the motif of simultaneous presence in two places (above, n. 130) was appropriated for Pythagoras from a version of the Aristeas story not known to Herodotus (below, n. 148). But if everything is late ascription to Pythagoras, where did his fame come from? How could a cipher have exercised so much attraction?

[146] Ap. *Il.m.* 1–6: Epimenides, Aristeas, Hermotimus, Abaris, Pherecydes, Pythagoras; Iam. Al. *Strom.* 1.133.2 (cf. Tatian 41): Pythagoras, Abaris, Aristeas, Epimenides, Pherecydes, Empedocles, Phormio; Nicom. (Por. *VP* 29 = Iam. *VP* 135): Empedocles, Epimenides, Abaris, Pythagoras; Max. Tyr. 10.1ff: Epimenides, Pythagoras, Aristeas; Plin. *HN* 7.174: Hermotimus, Aristeas, Epimenides, Empedocles; Procl. *In Remp.* II 113: Aristeas, Hermodorus (cf. below, n. 177), Epimenides; Greg. Naz. (Migne 35.581, cf. 1.1.1.1, 38.47): Empedocles, Aristaeus (i.e. Aristeas), Empedotimus, Trophonius. As early as Democritus' book *Περὶ τῶν ἐν Ἅιδου* there was a treatment *περὶ τῶν ἀποθανεῖν λεγομένων ἔπειτα ἀναβιούντων* (DK 68B31). Wellmann (*AbhBln* 1921.4, 12f) thought this book was a forgery by Bolus; but it was included in Thrasyllus' tetralogically arranged catalogue of Democritus' writings; certainly spurious items, like Bolus' *Χειρόκμητα*, appear as an appendix to the catalogue (D.L. 9.49). The topic was current in the fifth century B.C., as is shown by Soph. *El.* 62f.

was in circulation in the early sixth century B.C.[147] Aristeas told how, possessed by Apollo (φοιβόλαμπτος γενόμενος) he had traveled to the country of the Issedones in the far north, and learned from them about the Arimaspi, the griffins, and the Hyperboreans who lived still further north. Herodotus adds a local legend from Proconnesus, to the effect that Aristeas died, and soon after was seen traveling abroad,[148] while his body was found to have vanished. After seven years he appeared in town again, bringing his *Arimaspeia*, and then disappeared again. Herodotus also was told, in Metapontum, that Aristeas had appeared there and bidden the natives to build an altar to Apollo. "For (he said) Apollo had visited them alone among the Italians, and he himself had accompanied him in the form of a raven . . . And even now there stands in the agora, near the statue of Apollo, a statue inscribed with the name of Aristeas, and there are laurel bushes round about."[149] Herodotus calculates that the appearance of Aristeas in Metapontum occurred 240 years after his disappearance in Proconnesus. It is hard to account for this dating, though the incident in Metapontum must have been relatively late; one is tempted to think that the Metapontine coins might be relevant which, beginning to appear about 470 B.C., show Apollo with a branch of laurel.[150]

The report from Proconnesus is inconsistent. First Aristeas dies, then his corpse disappears; possession and disappearance stand side by side. Is it that "Herodotus has combined two versions of the legend"?[151] Later reports are unequivocal: the soul leaves the body and hovers about in the air "in the form of a bird."[152] Perhaps the contradiction itself, the failure to smooth over difficulties, and the lack of a clear separation of body and soul, are signs of an archaic way of thinking.

Aristeas is connected with Pythagoras by more than the similarity of the legends in which they figured.[153] Pythagoras was regarded by the initiated as the Hyperborean Apollo and died in Metapontum near the beginning of the fifth century. But the main authority on the Hyperboreans was Aristeas, and the god who flew about the world, accompanied by his servant in the form of a raven, and finally alit in his special city, was the Hyperborean Apollo.[154] If, about 470, a mysterious prophet brought it about, by his message, that an altar and a statue were set up, then the new cult was obviously dedicated to the Hyperborean Apollo, and for whatever Pythagoreans there were in Metapontum at that time we must assume that the proclamation of Aristeas was understood, at least ἐν ἀπορρήτοις, as meaning that Apollo had lived in Metapontum in the form of Pythagoras.[155] The cult belongs with the legend; in both is reflected the activity of Pythagorean circles.

Abaris, too, is a priest of the Hyperborean Apollo, and even comes, himself, from the land of the Hyperboreans. He brought gifts from

[147] J. D. P. Bolton, *Aristeas of Proconnesus* (Oxford, 1962; cf. *Gnomon* 35 [1963] 235–240). The terminus ante quem is a representation of Arimaspi on a mirror from Kelermes (Bolton, pl. 1, ca. 575 B.C.), the terminus post quem the mention of the Cimmerian invasion. Pindar mentions Aristeas (fr. 271), and the *Arimaspea* was used by Hecataeus (Jacoby's notes on *FGrHist* 1F193–4) and by Aeschylus in the *Prometheus* (Bolton 45–64). The most important evidence: Hdt. 4.13–15.

[148] Between Artace and Cyzicus, according to Hdt. 4.14, "on the way to Croton" in Plut. *Rom.* 28, a contamination with the story which follows in Hdt.

[149] Hdt. 4.15. At Ath. 13.605c, too, there is a mention of the δάφνη χαλκῆ (δάφναι in Hdt.; cf. Giannelli 63 n. 1) of Aristeas in the marketplace of Metapontum. (Perhaps from Theopompus: see Jacoby, *FGrHist* 115F248 n.)

[150] Giannelli 62 (cf. Head² 76; S. P. Noe, *The Coinage of Metapontum* II [New York 1931] nos. 314ff, on the dating II 9ff. One coin shows an altar next to Apollo, no. 319.) Herodotus at least places Aristeas later than Homer (2.53; according to Strabo 14.635 "some" would reverse the relationship). Proconnesus was founded in the time of Gyges (Strabo 13.587, 590). If the connection with the coins is correct, and Herodotus thus learned in Metapontum, about 440, about an event of approximately a generation earlier the *Arimaspea* would be dated about 710; but this does not fit the history of Proconnesus (see also Bolton [above, n. 147] 127). All the same, we must reject E. Schwyzer's conjecture of συγκυρήσας for συγκυρήσαντα, which would show Herodotus dating his own visit to Metapontum rather than the appearance of Aristeas (*PhW* 42 [1922] 528; followed by Schmid I 1.303 n. 3). The point Herodotus is making is the time between Aristeas' two apparitions (Meuli 154.2).

[151] Rohde II 92.1 = 329 n. 109 Eng. ed.; Corssen, *RhM* 1912, 44.

[152] Max. Tyr. 10.2, Plin. *HN* 7.174, *Suda* s.v. Aristeas.

[153] According to a late version (Ap. *H.m.* 2) Aristeas was seen on the same day in Proconnesus and Sicily. (For Pythagoras, above, n. 118.)

[154] The earliest mention of the Hyperboreans: *Epigoni* fr. 3, Hes. fr. 150.21 M.-W.; the arrival of Apollo in Delphi from the land of the Hyperboreans: Alcaeus fr. 307 L.-P., and a shield strap from Olympia (600–575 B.C.) in E. Kunze, *Olympische Forschungen* II (1950) 74; further references to the flying Apollo: *Gnomon* 35 (1963) 239 n. 1. The Greeks associated the name Hyperborean with *Βορέας* and *Βορεάδες*, and this fits in well with the disappearance stories (Hdt. 4.36, Diod. 2.47.7 = Hecataeus of Abdera *FGrHist* 264 F 7). J. Harmatta has recently suggested a derivation from **βόρις*, "mountain" (*Acta antiqua* 3 [1955] 56ff); A. J. van Windekens favors derivation from the root of *φέρειν* ("conductor" into the next world?), *RhM* 100 (1957) 164–169.—According to Ath. 605c (above, n. 149), Aristeas comes to Metapontum "from the Hyperboreans."

[155] Cf. Bolton 174f. The connection between Aristeas and the Pythagoreans was emphasized by Rohde, *Psyche* II 99.2 = 333 n. 122 Eng. ed.; Diels, *Parm.* 21; Giannelli 63 64. "Aristeas of Metapontum" is found in the catalogue of Pythagoreans, Iam. *VP* p. 144.2 Deubner. According to Iam. *VP* 138, the Pythagoreans believe everything that is said about Aristeas and Abaris.—Stein, in his note on Hdt. 4.15, interprets the visit of Apollo with relation to the remarkable fertility of the land of Metapontum; but from the very beginning this was represented by the ear of grain on the city's coins. E. Pais, *Storia della Sicilia* I (1894) 548, conjectured, arbitrarily, that Herodotus had misunderstood, and that Aristaeus rather than Aristeas had been Apollo's *paredros* in Metapontum.

there, according to Attic legend, to the Proerosia festival at Eleusis.[156] Abaris has connections with other Greek cults,[157] and there were charms and oracles attributed to him.[158] Probably his meeting with Pythagoras had been recounted even before Aristotle's time,[159] though there were chronological difficulties.[160]

According to Herodotus, without ever eating, Abaris carried Apollo's arrow all over the world,[161] but as early as Heraclides it was said that he flew on this arrow,[162] and this version is regarded as the original one.[163] It may be, though, that just as in the case of Aristeas, the tradition was self-contradictory from the beginning. There were alternative ways to report the activities of the miracle-worker; Abaris could not perhaps "actually" fly, but he could claim the ability, and even, in ecstatic ritual, act it out, as it were, as a shaman. Whoever was ready in his heart to believe, would speak of "flying;" those who were ready to discard the old-time magic would report the matter in the style of Herodotus.

Epimenides, the famous Cretan "purifier," comes from a different environment. That he slept for decades in the cave of Zeus was known already to Xenophanes.[164] This puts Epimenides in the main line of specifically Cretan cult and myth. King Minos visited his father Zeus in that cave every eight years.[165] Epimenides called himself Aeacus—

[156] Lycurgus frr. 84–85 Blass, Hippostratus *FGrHist* 568F4; cf. the Hyperboreans in Delos, Hdt. 4.33.

[157] Sparta, Kore Soteira (Paus. 3.13.2, Ap. *H.m.* 4, Iam. *VP* 92, 141); Cnossus, (Iam. *VP* 92); Palladium (Firm. Mat. *Err. prof. rel.* 15).

[158] ἐπῳδαί, Pl. *Charm.* 158b, where Zalmoxis is also mentioned; Ap. *H.m.* 4, Schol. Ar. *Eq.* 729.

[159] Cf. n. 127. On Iam. *VP* 215, ch. II 1, n. 13. The source of the exposition in Iam. *VP* 91–93 cannot be determined (Heraclides?). Nicomachus (Iam. *VP* 135 = Por. *VP* 28) presupposes it.—Abaris is given as a Pythagorean in Iamblichus' catalogue, *VP* p. 145.17.

[160] Pindar (fr. 270) puts him into the age of Croesus, Hippostratus (*FGrHist* 568F4) at 568/565, "others" at 696/693. Cf. Jacoby on 568F4.

[161] Hdt. 4.36. Lycurgus, too, says that Abaris carried the arrow (fr. 85 Blass = Harpocration s.v. Abaris), as does Aristotle (Iam. *VP* 140; above, n. 127). For both linguistic and material reasons, we must reject the conjecture, repeatedly proposed (e.g. M. Mayer *RE* XV 1357f, Roscher, *Lex.* I 2837) to substitute ὡς τὸν (= τοῦτον) ὀϊστὸς περιέφε for ὡς τὸν ὀϊστὸν περιέφερε in the text of Herodotus.

[162] Heraclides fr. 51c. He is αἰθροβάτης in Nicomachus (Por. *VP* 29 = Iam. *VP* 13 and in Iam. *VP* 91.

[163] Corssen, *RhM* 1912, 47, Meuli 159f, Dodds, *Irr.* 161 n. 33 (contra Rohde, *Psyche* I 91 n. 1 = 327 n. 108 Eng. ed.).

[164] Xenophanes DK 21B20 (Epimenides lived to the age of 154). Theopompus explicitly mentions the Dictaean cave (*FGrHist* 115F67–69, Max. Tyr. 10.1; cf. 37.1), though one might be more disposed to think of the one on Ida, mentioned by "Epimenides" (DK 3B24; Rohde, *Psyche* I 129 n. 1 = 108 n. 24 Eng. ed.).

[165] *Od.* 19.178, [Pl.] *Minos* 319e, Pl. *Leg.* 624b. On the Cretan Zeus, Nilsson I 323ff.

thus making himself a brother to Minos—and claimed "that he had been reborn many times" (πολλάκις ἀναβεβιωκέναι).[166] The Cretans called him νέος Κούρης, thus bringing him into close relationship with Zeus.[167] A voice speaks to him from heaven;[168] and the Cretans sacrifice to him as to a god.[169]

Oracles and theogonic poems were in circulation, bearing the name of Epimenides.[170] Doubtless much was attributed to him by later forgers. Plato's report that Epimenides prophesied the Persian War ten years before it occurred is explained as meaning that at that time a new book of oracles "by Epimenides" appeared.[171] Legendary motifs were added, too. Nevertheless, the purification of Athens from the Cylonian curse in the time of Solon may be regarded as historical.[172] There were "caves of Zeus" in Crete, sites of the initiatory ceremonies of secret societies, who are reflected in myth as "Dactyls" or "Kouretes."[173] The initiation of the καθαρτής by sleeping in the cave of Zeus is comprehensible from this point of view. It also represents death and resurrection, for in the meantime Epimenides was regarded as dead. The long fast, made possible by the magic food ἄλιμον, as well as the tattooing, also are bound up with ritual.[174] But given these facts, his being "reborn many times" and his identification with Aeacus are not necessarily "purely Pythagorean fabrication."[175] There is never any mention of a metempsychosis doctrine of Epimenides, only the unique character of the initiate. The explanation is rather to be found in a

[166] D.L. 1.114; cf. Procl. *In Remp.* II 113. When the *Suda*, s.v. Epimenides, says ὡς ἐξίοι ψυχὴ ἀνδρὸς ἤθελε καιρὸν καὶ πάλιν εἰσῄει ἐν τῷ σώματι, the precise wording is no doubt secondary, perhaps taken over from Aristeas (as Dodds, *Irr.* 163.42); but it is hardly original in his case either.

[167] Myronianus ap. D.L. 1.115, Plut. *Solon* 12. Cf. the hymn to Zeus from Palaikastro, in which the god is addressed as Μέγιστε Κοῦρε Κρόνειε (Nilsson I 322f).

[168] Theopompus (*FGrHist* 115F69, D.L. 1.115): as Epimenides was outfitting a sanctuary to the Nymphs, a voice commanded, μὴ Νυμφῶν ἀλλὰ Διός. This Zeus was, then, worshiped in a grotto or cave.

[169] D.L. 1.114.

[170] Collected in DK 3B, and *FGrHist* 457. See esp. Diels, *SBBln* 1891, 387ff.

[171] Pl. *Leg.* 642d; Diels, *SBBln* 1891, 395, and DK I 32 n.

[172] Arist. *Ath. Pol.* 1, Plut. *Solon* 12, Neanthes *FGrHist* 84F16 (Ath. 13.602c). See Diels *SBBln* 1891, 396.

[173] Ant. Lib. 19, Por. *VP* 17, Eur. fr. 472. Cf. Nilsson I 261–264; P. Faure, *Fonctions des cretois antiques* (1964).

[174] ἄλιμον: Theophr. *Hist. pl.* 7.12.1, D.L. 1.114, Plut. *Conv. sept. sap.* 157d (DK 3A5). Use of this is attributed to Heracles by Herodorus (*FGrHist* 31F1), to Pythagoras by Diogenes Antonius (Por. *VP* 34). See Rohde, *Rom.* 275 n., Haussleiter 79ff, Dodds, *Irr.* 161 n. 42 (Diels regarded the "Orphic asceticism" of Epimenides as secondary, *SBBln* 1891, 395.) On tattooing, see *Suda* s.v. Epimenides (DK 3A2), Dodds, *Irr.* 163 nn. 43–44.

[175] Diels *SBBln* 1891, 396 n. 1.

parallel development from common origins; Pythagoras too, as the legend says, sought initiation in the cave on Ida.[176]

A journey of the soul, in pure form, was attributed to Hermotimus of Clazomenae: His soul left his body and wandered about, while the body lay as though dead, until one day his enemies burned the body while the soul was absent.[177] Hermotimus could predict future events; the Clazomenians built a sanctuary in his honor.[178] Here too there is ritual to go with the legend.

The stories of Phormio and Leonymus take us to Croton. Phormio is mentioned as early as Cratinus, interestingly enough in the comedy *Trophonius*. Theopompus relates, among other things, that Phormio was wounded, in a battle, by divine opponents, the Dioscuri. He was told by an oracle to travel to Sparta—obviously to the house of the Dioscuri; and when he laid hand on the door, he found it was the door of his own house in Croton. He was home, safe and sound.[179]

The tale of Leonymus is almost like a doublet of this.[180] In the battle on the Sagras he was wounded by Ajax, who was fighting in the ranks of the Locrians. To be healed he had to go, at the behest of the Delphic Oracle, to the "White Isle," where he met Achilles and Ajax,

[176] Por. *VP* 17 (Antonius Diogenes?), D.L. 8.3. The connection of Pythagoras and Epimenides is obviously secondary; Pythagoras is sometimes the teacher (Nicom., Por. *VP* 29 = Iam. *VP* 135; cf. Iam. *VP* 104, 221f) and sometimes the pupil (Apul. *Flor.* 15 p. 59, D.L. 8.3). For a purported piece of evidence from Epimenides about Pythagoras, see above, n. 143.—A unique rebirth story was told of Aesop, as early as the fifth century B.C. (Plat. Com. fr. 68, ca. 400 B.C.; Schmid I 4.145ff; Hermippus ap. Plut. *Solon* 6; Ptolemaeus son of Hephaestion ap. Phot. *Bibl.* 152b11). This is not "parody of Pythagorean teaching" (as Hausrath says, *RE* VI 1710; cf. Schmid I 1.675 n. 2), but has its roots in ritual. The uncomfortable feeling about the killing of the scapegoat was counterbalanced by the purported revival (A. Wiechers, *Äsop in Delphi* [Meisenheim, 1961] 37ff).

[177] Ap. *H.m.* 3 (perhaps from Theopompus, like sections 1 and 5; Rohde, *Psyche* II 95.1 = 331 n. 112 Eng. ed.), Plin. *HN* 7.174, Plut. *De gen.* 592c–e ("Hermodorus"), Tert. *An.* 44.

[178] Ap. *H.m.* 3, Tert. *An.* 44. According to Aristotle, Hermotimus had a doctrine about νοῦς before Anaxagoras (*Met.* 984b15; fr. 61 = Iam. *Protr.* 48.16f) cf. M. Detienne, "Les origines religieuses de la notion de l'intellect: Hermotime et Anaxagore," *Rev. Philos.* 89 (1964) 167–178.

[179] See Meineke, *FCG* II 1227ff; Diels, *Parm.* 17ff; Cratinus fr. 223 Kock; Theopompus *FGrHist* 115F392 = *Suda* s.v. Phormio (who adds an ecstatic journey to Cyrene). Clem. Al. *Strom.* 1.133.2 associates Phormio with Aristeas and the rest (above, n. 146); Paus. 3.11 gives a different story of Phormio the Spartan and the house of the Dioscuri.

[180] Paus. 3.19.11–13. Somewhat differently Conon *FGrHist* 26F1§18, who has no epiphany on the island, only a sacrifice. Tert. *An.* 46.9 and Herm. *Phdr.* p. 75 Couvr. = Schol. Pl. *Phdr.* 243a have him healed in his sleep. On the matter of the battle at the Sagras, above, ch. II 2, n. 40.

and then returned home sound. The "White Isle,"[181] later localized in the Black Sea, was originally identical with the "White Rock" in the underworld, and Leonymus was "the first" to visit this island. Thus in this case recovery is not possible without a regular journey to the nether world. Phormio obviously did his "traveling" in a trance state; and many thought that Leonymus had only been dreaming.

The legend of Stesichorus is bound up with the story of Leonymus: the latter brought from the White Isle Helen's instructions to Stesichorus, that by composing his palinode he might regain his sight.

Both the Leonymus and the Stesichorus stories have been supposed to go back to a Pythagorean origin,[182] and a certain amount of coincidence in place and time is not to be denied, though the epoch of Stesichorus is earlier than that of Pythagoras.[183] There was a belief in southern Italy that healing could be won by an ecstatic journey into the world beyond, to the gods. And as they thought of their ancestral gods as quite literally fighting in the ranks,[184] the healing legends too might be bound up with ritual activity.

In fifth-century Sicily, Empedocles could promise his pupils: "Thou shalt stay the power of the unwearied winds which sweep upon the earth . . . and, if thou wish, bring back their breath again. After dark rain thou shalt cause a seasonable drought for men, and after summer's drought bring on the streams that nourish the trees . . . Thou shalt bring back from Hades the strength of a man who has died."[185] The Empedocles legend reports the miracles as having been accomplished; as has been said, he was "the creator of his own legend."[186] But how could he have created it, how could he have called himself a god, if he was not able actually to perform, or at least to pretend to perform

[181] The translation of Achilles to the "White Isle" is already mentioned in the *Aethiopis* (Proel. *Chrestom.*). The localization of the island in the Black Sea is as early as Pi. *Nem.* 4.49. Cf. Rohde, *Psyche* II 371ff = 537ff Eng. ed., Roscher, *Lex.* s.v. Leuke.—*Od.* 24.11.

[182] Detienne, *RHR* 1957, 129ff. Diels, *Parm.* 17ff, suggested that an epic poem on the battle at the Sagras was the source of the story of Phormio and Leonymus.

[183] The new fragments of Stesichorus (D. L. Page, *Lyrica graeca selecta* [1968] 263–268), along with the archaeological evidence, confirm the dating of Stesichorus in the first half of the sixth century.

[184] In the battle against Sybaris, Milo led the Crotoniates in the garb of Heracles (Diod. 12.9.6). Croton was regarded as having been founded by Heracles (Ov. *Met.* 15.8ff, Iam. *VP* 50). According to Conon (above, n. 180), the Locrians left a place for Ajax in their battle order; on Ajax as divine ally, see P. von der Mühll, *Der grosse Aias* (Basel, 1930) 14ff.

[185] Fr. 111.3ff, tr. Guthrie. Cf. Hippoc. *Morb. sacr.* VI 258f L.

[186] Dodds, *Irr.* 145, after J. Bidez, *Biographie d'Empédocle* (Ghent, 1895). Heraclides Ponticus' Ἄνοδος (fr. 76–89 Wehrli) takes as its starting point the last line quoted here (fr. 111.9).

extraordinary and amazing feats? His saga must have its roots not in literature, but in reality; and indeed Gorgias testifies that "he himself had been present when Empedocles performed feats of magic."[187] According to Timaeus he restrained the destructive north winds at Selinus by the use of bags made of asses' hides—that is, by a secret sacrifice in the manner of the Hyperboreans.[188]

A "journey into the underworld" was actually part of the ceremony at the oracle of Trophonius in Lebadea.[189] The one who is to consult the oracle was borne by a wind into the depths, feet first, as the dead are carried. After the god has appeared to him, he returns to earth, often after a lapse of several days. Before the sanctuary were the springs of Lethe and of Mnemosyne, whom the initiate expected to see in the underworld, according to the evidence of the Gold Plates. Trophonius is a *Ζεὺς Χθόνιος*—Epimenides too met Zeus in the bowels of the earth. But in Lebadea there can be no doubt that the main thing was ritual, not legend. We do not know whether the visitor to the oracle was put into a trance state—there were long ritual preliminaries, and not everyone was admitted—or whether perhaps machines may have been used, of the kind used in the *katabasis* of the Roman Bacchanalia, according to Livy.[190] It is significant, however, that legend connected a Pythagorean, Parmiscus, with the otherworld journey of Lebadea.[191]

[187] Satyrus ap. D.L. 8.59. Diels, *SBBln* 1884, 344, tries to weaken the force of this evidence by referring it not to a writing of Gorgias, but to a dialogue of Alcidamas; cf. Burkert, *RhM* 1962, 48.

[188] *FGrHist* 566F30 = D.L. 8.60. One thinks of Aeolus' leather bag. See R. Strömberg, *Acta Univ. Gotoburg.* 1950.3, 71–84, who cites additional comparative material. On the Hyperboreans' manner of sacrificing, see Pi. *Pyth.* 10.33.

[189] The principal source is Paus. 9.39; cf. Hdt. 8.134, *IG* VII 3055, 4136, Rohde, *Psyche* I 119ff = 92ff Eng. ed., Radke, *RE* VII A 678–695. Nilsson (II 450) does not believe that the full development of the ritual came before imperial times. But Dicaearchus spoke of a *katabasis* (frr. 13ff), and said that "nescire ea melius esse quam scire" (fr. 17). Semos (*FGrHist* 396F10) speaks of one who consulted the oracle and lost his ability to laugh. Nilsson's assertion that the "spring of Lethe" was an idea invented after Plato, and that the spring of Mnemosyne belonged to the Hellenistic age (II 226f, *Op.* III 85–92) has been refuted by the gold plate of Pharsalus, which has the phrase *Μνημοσύνης Λίμνη* (ca. 350 B.C.; *Arch. eph.* 1950–1951, 98ff). U. E. Paoli calls attention to the significance of the fact that the visitor to the oracle is carried away feet first (*Die Geschichte der Neaira* [Bern, 1953] 43).

[190] Livy 39.13.13: "raptos a dis homines dici, quos machinae illigatos ex conspectu in abditos specus abripiant." On this, see Festugière, *Mél. d'arch. et d'hist.* 56 (1954) 94ff. (At p. 95, he conjectures that a mechanical contrivance may have been used in the Trophonius ritual.)

[191] Semos *FGrHist* 396F10, and the Delian temple inventory of 279 B.C. (*IG* XI 2.161B17, DK 20). The age of the legend is indeterminable; the older inventory lists are all fragmentary. It is possible that a dedicatory gift was ascribed later to Parmiscus. (The form *Παρμίσκος* is found in the inscription, elsewhere often *Παρμενίσκος*; cf. DK 20.) Apollonius of Tyana, too, was a visitor to Trophonius (Philostr. *VA* 8.19).

In many places there were subterranean installations which presented the underworld in physical form. The structures at Clarus are impressive; the *adyton* with the spring that gives out the oracle lies under the cella of the temple.[192] And even more amazing are the subterranean passageways at Baiae, near Cumae—if they really belong to the rites of the "Cimmerians" and not merely to the water supply.[193] There is also the mysterious *mundus* at Rome, visited by boys.[194] In connection with the cult of Demeter there were *megara*, subterranean rooms or caverns, into which offertory gifts were lowered.[195]

This last brings us back to Pythagoras. His house, says Timaeus, was made into a temple of Demeter; and woe to the uninitiated who entered it![196] In various segments of the tradition we have reports of subterranean chambers in which Pythagoras met with his disciples;[197] and above all, we learn of a *katabasis* of Pythagoras himself, although the clearest allusions to it are in the distortions of ridicule and parody. Hieronymus of Rhodes said[198] that Pythagoras descended to Hades and saw how the souls of Homer and Hesiod were atoning for what

[192] L. Robert, *Archaeolog. Reports* 1959–1960, 41f; "*Μυηθέντες ἐνεβάτευσαν*," *RPh* 22 (1948) 159. Cf. Tac. *Ann.* 2.54, above, n. 135.

[193] R. F. Paget, *In the Footsteps of Orpheus* (London, 1967), thinks he has found the megarons mentioned by Ephorus *FGrHist* 70F134.

[194] Cato ap. Festus p. 154 M.

[195] Schol. Lucian p. 275.23 Rabe, Menander fr. 870 Körte, Paus. 1.27.3, 9.8.1, Plut. *De Is. et Os.* 378c, Aelius Dion, s.v. *μάγαρον*, Paus. Att. s.v. *μέγαρον*, Hsch. s.v. *Μέγαρα*. M. Schede, *Die Ruinen von Priene* (1964²) 93.—An inaccessible subterranean megaron was discovered a few years ago in Posidonia, where a potsherd was found with the words *ΤΑΣ ΝΥΜΦΑΣ ΕΜΙ ΗΙΑ[ΡΟΝ* (B. Neutsch, *SBHeid* 1957.2). Cf. also Hdt. 6.134.

[196] *FGrHist* 566F131, Iam. *VP* 143 (above, n. 128).

[197] Antiphon *Περὶ τῶν ἐν ἀρετῇ πρωτευσάντων* (certainly not the Sophist Antiphon, as Neale tried to show in ZN I 1.393 n. 3), Por. *VP* 9 = Iam. *VP* 27: In Samos, Pythagoras *ἔξω τῆς πόλεως ἄντρον οἰκεῖον τῆς ἑαυτοῦ φιλοσοφίας ποιήσαντα ἐν τούτῳ τὰ πολλὰ τῆς ἡμέρας καὶ τῆς νυκτὸς διατρίβειν συνόντα ὀλίγοις τῶν ἑταίρων*. Iamblichus adds, probably from the same source, *τὸν αὐτὸν τρόπον Μίνῳ τοῦ Διὸς υἱῷ διανοηθείς*. Diogenes Antonius ap. Por. *VP* 34: *ὁπότε θεῶν ἀδύτοις ἐγκαταδύσεσθαι μέλλοι καὶ ἐνταῦθα χρόνου τινὸς ἐνδιατρίψειν, ἀλίμοις ἐχρῆτο καὶ ἀδίψοις τροφαῖς*, like Epimenides (above, n. 174). Hippol. *Ref.* 1.2.18: *ἐν ἀδύτοις καταγείοις ἠρεμεῖν ἐποίει μανθάνοντα*.—To this context belongs the subterranean basilica near the Porta Maggiore in Rome, on which see Carcopino, *Bas.*, passim. To be sure, the interpretation of this as a cult site is controversial. (F. J. Dölger, *De datum van het grote hypogaeum bij de Porta Maggiore te Rome* [Diss. Leiden, 1918], contra, C. C. van Essen, *Mnemosyne* 4.13 [1960] 277–280.)

[198] Fr. 42 W. D.L. 8.21 (on the attribution of this to Heraclides, see above, ch. II 1 n. 14) Levy (*Sources* 37.1, *Lég.* 82f) misunderstood the ironical conclusion and thought the reference was to the punishment of adulterers (cf. Iam. *VP* 50); he therefore had to athetize the last clause, *καὶ δὴ καὶ διὰ τοῦτο τιμηθῆναι ὑπὸ τῶν ἐν Κρότωνι*. Human life is only truly fulfilled by the consummation of marriage, and those who remain *ἀτελεῖς* will suffer—a thought that is also expressed, in a different form, in the myth of the Danaides. (See Carcopino, *Bas.* 121 n. 1, with references.)

they had said about the gods (this does not sound very archaic),[199] and also how those were punished "who would not lie with their wives; and this, of course, is why he was honored in Croton." (The gibe in this last clause is unmistakable.) Much more influential was the account of Hermippus:[200]

> After arriving in Italy, he built a little underground room and instructed his mother to write down events, as they happened, on a tablet, recording the time,[201] and to keep passing these notes down to him until he came back up. His mother did this, and after some time Pythagoras came up thin as a skeleton. He went into the assembly and announced that he had just returned from Hades. What is more, he read off to them an account of what had happened during his absence. Taken in by his words, they wept and moaned and were sure that he was some kind of divinity; so that they even entrusted their wives to him, thinking that they too would learn something from him. And they were called *Πυθαγορικαί*.

The mocking tone of this account of a "journey to the underworld" is of course unmistakable. The question remains, however: how did this sound when it was taken seriously; and how old is the story? The usual assumption is that Hermippus' account is a simple transference to Pythagoras of what Herodotus and, after him, Hellanicus[202] had reported of Zalmoxis, the Getic god; but it may be that the matter is more complicated than that.

[199] The background of this detail is to be found in Xenophanes and Plato. On the Homeric poems in (late) Pythagorean tradition, see Delatte, *Litt.* 109ff, and M. Detienne, *Homère, Hésiode et Pythagore* (1962).

[200] D.L. 8.41, Tert. *An.* 28, Schol. Soph. *El.* 62 = *Suda* s.v. *ἤδη*. Allusions at Lucian *Gall.* 18, Celsus (Origen *C. Cels.* 2.55), Eustathius on *Od.* 11.592, 24.264. See Delatte, *Vie* 244ff, Lévy, *Sources* 37ff. It is open to question whether there is a relationship to Heraclides at this point (see ch. II 1, n. 32). The fact that Tertullian, the Sophocles scholium and the *Suda* connect this passage with Heraclides fr. 89, does not prove that Hermippus himself had cited him (*pace* Corssen, *RhM* 1912, 23, and Lévy, *Sources* 40; Rohde, *Psych* II 419 = 599 Eng. ed., is hesitant).

[201] *τὰ γινόμενα εἰς δέλτον γράφειν σημειουμένην καὶ τὸν χρόνον*. Lévy, *Sources* 38, *Lég.* 129ff, translates "sealed letter," and sees in the reading of a sealed letter a further miracle (cf. Philostr. *VA* 3.16, Lucian *Alex.* 19ff). But this leaves the words *καὶ τὸν χρόνον* unaccounted for; and one would expect *σεσημειωμένην*. *σημειοῦσθαι* means "make notes," and the participle refers to *τῇ μητρί*. (The change of case is not uncommon; cf. Eur. *Med.* 57f.)

[202] Hellanicus, *Βαρβαρικὰ νόμιμα*, *FGrHist* 4F73. Here Zalmoxis promises the "return of the dead and is therefore teaching a kind of metempsychosis, while Herodotus only speaks of immortality. The complete dependence of this book of Hellanicus upon Herodotus (and Damastes of Sigeum) is stressed by Porphyry (Euseb. *Praep. evan.* 10.3 *FGrHist* 4F72), but it cannot be proven spurious (Jacoby 454).

The Greeks called the Getae *Γέτας τοὺς ἀθανατίζοντας*, for this seemed particularly noteworthy to them. "They believe that they do not die, but that when someone succumbs he goes to the *daimon* Zalmoxis" (4.94). Every four years they send the god a "messenger," in the form of a human sacrifice. "But as I learn from the Greeks who live on the Hellespont and the Black Sea, this Zalmoxis was a human being, a slave, in Samos, of Pythagoras the son of Mnesarchus" (4.95). Zalmoxis was set free and returned to Thrace a rich man. There, thanks to his Greek culture, he could easily impose on his fellows, especially since they were "a bit simple-minded" (*ὑπαφρονέστεροι*), "being acquainted with the Ionian way of living, and with manners more polite than those of Thrace in that he had been familiar with Greeks, and with Pythagoras, who was not the meanest sage in Greece." Zalmoxis built a banqueting hall in which he entertained the most prominent citizens, and promised them that his guests and their descendants would not die, but would live forever in enjoyment of everything good. In the meantime he had built an "underground chamber" to which he now withdrew for three years, mourned as dead. Then, in the fourth year, he reappeared; and now the Thracians believed in him, and believed in the conquest of death and in immortality. "Concerning this underground dwelling," says Herodotus, "I am neither excessively doubtful nor excessively credulous. I do think, though, that this Zalmoxis lived many years before Pythagoras" (4.96).

It is not difficult to see the connection between the Thracians' ritual and myth.[203] Just as, once upon a time, Zalmoxis had disappeared for three years and reappeared in the fourth, so the god is summoned every four years, by the "messenger"—summoned to a ritual meal, which provides a guarantee of immortality.

Pride in Greek culture and Greek cleverness are obvious in the account the colonists gave Herodotus. They are far above Thracian barbarism; a slave among Greeks can be a god among Thracians and make them believe anything he chooses.

Hermippus' story has been called a *freche Uebertragung*.[204] But the

[203] On this see F. Pfister, "Zalmoxis," *Studies Presented to D. M. Robinson* II (St. Louis, 1951) 1113–1123 (p. 1113 on the various forms of the name in Greek). There is now epigraphical attestation of "Zalmodegikos" as the name of a Thracian prince (D. M. Pippidi, *Studii clasice* 1 [1961] 53–66; *SEG* 18 [1962] no. 288).

[204] Corssen, *RhM* 1912, 43; similarly Rohde, *Q* 106 n. 1, Lévy, *Sources* 39, *Lég.* 133. Since Hermippus derived the doctrines of Pythagoras from "Jews and Thracians," (Joseph. *Ap.* 1.165), he doubtless knew Herodotus' Zalmoxis story, and probably also accepted his dating of Zalmoxis before Pythagoras.

Greeks were often inclined to attribute to foreign peoples things that were really Greek, and Herodotus in particular mentions many things in his accounts of foreign peoples which he intentionally omits when writing about Greeks. The most striking example is the Egyptians' alleged doctrine of metempsychosis, but the situation seems to be the same with abstinence from beans[205] and with the novella about Rhampsinitus' treasure house.[206] Herodotus tells about werewolves among the Neuroi of Scythia, but has nothing to say about similar phenomena in Arcadia.[207]

The striking thing about the Getae is their belief in immortality, and this must have been the reason Herodotus' informants thought of Pythagoras. Ionian manners and Ionian cunning could have given Zalmoxis ideas in other realms, but Pythagoras was the authority in questions of the afterlife and of immortality,[208] as we learn not only from Ion but from Herodotus himself. This reputation must have been familiar to the Greeks in plain and memorable terms. Herodotus assumes that one knows what he means when he says Pythagoras was "not the weakest" *σοφιστής* of the Greeks. Therefore, it will be well to consider whether specific details of the Pythagoras tradition are reflected in the Zalmoxis story.[209]

It is doubtful whether the "subterranean chamber" really belongs in the Zalmoxis tradition. According to a report from the age of Caesar, there stood beside the king of Thrace a revered priest or shaman who was a successor of Zalmoxis—at that time a certain Decaeneus, beside King Burebistas—and he dwelt, as Zalmoxis did, on the holy mountain Cogaeonus.[210] Given the strong tendency for

[205] Above, n. 48.

[206] Hdt. 2.121. The cult legend of Trophonius and Agamedes is very like it, while on the other hand no Egyptian reference to it has been found (cf. Radke, *RE* VII A 680, Lévy, *Lég.* 187). In the Rhampsinitus story as with Trophonius, a *katabasis* is involved. According to one version (Schol. Ar. *Nub.* 508), Trophonius was starved in a subterranean chamber, as Pythagoras, too, died of starvation (Satyrus ap. D.L.8.40, Dicaearchus fr. 35).

[207] Hdt. 4.105—Plin. *HN* 8.81, Varro ap. Aug. *De civ. D.* 18.17, Pl. *Rep.* 565d.

[208] After Zeller (*SBBln* 1889, 992), Rathmann reluctantly draws this conclusion from Hdt. 4.94ff. So does Maddalena (347ff), though he tries also to find that this is in contradiction to the doctrine of metempsychosis. Cf., however, the version of Hellanicus, n. 202 above. On Ion fr. 4, and Hdt. 2.81, see above, nn. 13, 39.

[209] Boyancé, *Muses* 134, finds here the most ancient reference to the special significance of cult meals for the Pythagoreans. Morrison, *CQ* 1956, 137ff, goes further, trying to find a social and political significance: in the *andron* Pythagoras presents his doctrines to the assembled Ionian bourgeois, a *σοφιστής* like Xenophanes. But it seems that the essential point is in the following trick, without which not even the Thracians would believe in immortality.

[210] Strabo 7.297f.

religious traditions to cling to holy places, we may believe that in Herodotus' time, too, the Thracians thought of Zalmoxis as being on his holy mountain, and not in an underground dwelling somewhere. But if this is so, this is a Greek motif; and it may have been that the Greeks imputed to Zalmoxis rather a slavish imitation of Pythagoras.

Hermippus' account surely cannot, in all respects, be derived from Herodotus. The notes passed down into the underground room could be explained as an elaboration of the trick; Pythagoras must know who has died in order to describe his experiences in Hades in a credible way. But how does it happen that his mother is his confidante and assistant? It is highly unlikely that Pythagoras brought his mother with him to Croton, and such an idea is never mentioned in the tradition. What we have, then, is a rationalizing version of something quite different. Pythagoras brings with him from Hades *τῆς μητρὸς παραγγέλματα* (commands of "the mother"), a message from the divine *Μήτηρ*—Demeter.[211] Thus the "little dwelling" becomes a sanctuary of Demeter, as Timaeus says Pythagoras' house was (above, n. 196). In this case, however, Hermippus has an element of the story that does not derive from Herodotus, and whose significance is no longer understood—therefore something quite ancient, and belonging originally to the Pythagoras legend. Another feature of the story that makes the impression of being genuine and ancient is the wasting away of Pythagoras. Intensive fasting always forms a part of the routine of shamans and fakirs.[212] This is not mentioned in Herodotus' account either,[213] so that Hermippus' report has independent value as evidence alongside that of Herodotus. It shows Pythagoras in the role of a hierophant in the cult of Demeter.[214]

The most remarkable detail of the Pythagoras legend, his golden thigh, points in the same direction. Antiquity understood this as a sign of divinity, but we find no explanation of just how this is so.[215]

[211] Cf. the intentionally ambiguous characterization of Callias as *ἱερεὺς τῆς μητρὸς καὶ τῆς θυγατρός* in Andoc. *De myst.* 124 (L. Koenen, *Studien zur Textgeschichte und Textkritik* [Cologne, 1960] 87.)

[212] Cf. the *ἄλιμον* of Epimenides (above, n. 174).

[213] A four-year fast is impossible, as Herodotus knew. But the four-year cycle was an essential feature of the Thracian ritual (above, n. 203); so this detail had to be dropped from the Zalmoxis story.

[214] Cf. Ael. fr. 10, on a megaron that may only be entered by hierophants.

[215] It was obviously considered a purported proof of divinity as early as Aristotle (fam. I F 140, above, n. 119). A scholium on Lucian (p. 124.6f Rabe) tells us that the thigh of Pythagoras was imprinted with an image of Apollo (*ἐντετυπωμένον*). Cf. Dodds's cautious remarks *Ir.* 163 n. 43. Lévy, *Sources* 12, *Lég.* 49 n. 1, interprets both the golden thigh and the image of Apollo as phenomena of the sun's glare; but why precisely the thigh?

Long ago Wilhelm Mannhardt made reference to myths of dismemberment and revival;[216] but here again, the legend is based on the actuality of the initiation rite. It was merely imitation of Pythagoras when Alexander of Abonuteichus, at a mystery ritual, let a glimpse of his golden thigh be seen, but it is worth noting that this festival itself was organized on the pattern of the Eleusinian.[217] More important is what Prudentius says in our most detailed description of the cult of the Mother, namely that persons dedicate themselves to the Mother and receive her "seal" (*sphragitis*), which is burned into them with red-hot needles; and at the burial of an initiate the "dedicated member" of the body was covered with a gold plate.[218] Tattooing of the Galli is attested for the Hellenistic period.[219] But, before all, the myths tell over and over of the favorite of the Great Mother being wounded in the thigh,[220] as also of the thigh wounds of those who attempt to make their way into the underworld.[221] Only he who bears the sign can descend into the pit with impunity.[222] In the same way, Pythagoras' golden thigh is the sign of the initiation which makes it possible for him to travel to Hades.

Are we to suppose, then, that Hermippus' whole story, including the golden thigh, is literally true? In any case it contains reflections of ancient ritual practices which survived for a very long time.[223] And though there is no direct testimony on the matter before Aristotle, Hieronymus, and Hermippus, we must take account of certain possible allusions in earlier literature. In the *Electra*, for example, Sophocles

[216] *Germanische Mythen* (Berlin, 1858) 74. Against Mannhardt, Meuli (160ff) cites Siberian myths of "golden heroes" and "golden gods."—On dismemberment and revival in shaman initiations, see Eliade, 45ff, 53.

[217] Lucian, *Alex.* 38, 40.

[218] Prudent. *Perist.* 10.1076ff; F. J. Dölger, *Sphragis* (1911) 41ff.

[219] *Etym. magn.* s.v. *Γάλλος*, Plut. *De adul. et am.* 56e, 3 Macc. 2.29.

[220] Adonis; for Atys (or Attis), see Hdt. 1.34–45.

[221] Heracles bitten by the serpent in the tail of Cerberus, Apollod. 2.5.12; cf., for example, a volute crater in Munich, no. 3297 (Furtwängler–Reichhold pl. 10).—On Theseus, Schol. Ar. *Eq.* 1368; on the death of Miltiades, Hdt. 6.134. Cf. *Phronesis* 14 (1969) 22–27.

[222] The Galli in Hierapolis, Strabo 13.629, Damasc. *Isid.* 131; Dionysus: Hor. *Carm.* 2.19.32.—The Ethiopians tattooed their children on the knee with an image of Apollo, according to Lydus *Mens.* 4.53 p. 110 Wuensch.—Each of the Seleucid kings, descendants of Apollo, had a birthmark on his thigh (Justin 15.4.3–5).

[223] There are surprisingly similar stories of the prophet living underground and subsequently appearing in Iranian-Arabic tradition (G. Widengren, *Iranisch-semitische Kulturbegegnung* [1960] 62–66).

has Orestes say that he will not scruple to give a false report of his death:[224]

> ἤδη γὰρ εἶδον πολλάκις καὶ τοὺς σοφοὺς
> λόγῳ μάτην θνήσκοντας· εἶθ', ὅταν δόμους
> ἔλθωσιν αὖθις, ἐκτετίμηνται πλέον.

"The wise" win special honor by being regarded as dead for a time and then returning. The stories of Aristeas and Zalmoxis, as told by Herodotus,[225] provide excellent examples of this; but would it be these persons that would most readily come to an Athenian spectator's mind, or rather Pythagoras—*Ἑλλήνων οὐχ ὁ ἀσθενέστατος σοφιστής* (Hdt. 4.95)? He was, after all, Zalmoxis' teacher; and it is to him that the ancient scholia refer.

Heraclitus calls Pythagoras the chief of swindlers,[226] and accuses him of having made, from the books of others, his own *σοφίη, πολυμαθίη, κακοτεχνίη*.[227] Commentators have emphasized the words *σοφίη* and *πολυμαθίη*, and paid much less attention to the climactic word *κακοτεχνίη*. *Τέχνη* is not a doctrine, but an activity; *κακοτεχνία* is a technical term for the subornation of perjury, and in general designates disingenuous ruses by which anyone attains an end.[228] Thus Heraclitus is accusing Pythagoras of being a charlatan who, by ignoble deception, has attained to fame. The kind of procedure that Herodotus attributes to Zalmoxis, Hermippus to Pythagoras, and that the story of Aristeas in Metapontum presupposes, is unquestionably a glaring example of *κακοτεχνίη*; and it seems quite likely that Heraclitus, as well as Herodotus and Sophocles, had heard of a ritually enacted *katabasis* of Pythagoras.

[224] Soph. *El.* 62ff. To be pronounced dead was to the primitive mind a serious matter, an event scarcely to be undone (cf. Plut. *Quaest. Rom.* 5).

[225] G. Kaibel refers to Zalmoxis (*Sophokles Elektra* [Leipzig, 1896] 79). On the relationship of Herodotus and Sophocles, see Jacoby, *RE* Supp. II 234ff. Heracles, Theseus, and Odysseus scarcely are relevant cases here, since, though they were thought for a time to be dead and gone, this was involuntary, and not the result of their *σοφία*.

[226] *κοπίδων ἀρχηγός*, fr. 81 (Timaeus *FGrHist* 566F132). Cf. Wilamowitz, *Hermes* 62 (1927) 277, Reinhardt, *Hermes* 63 (1928) 107–110.

[227] Fr. 129; above, n. 63.

[228] Burnet, *EGP* 134.2, Reinhardt, *Parm.* 235f. Ciaceri II 94. Hera's *ἀπάτη* (*Il.* 15.14) is *κακοτέχνος*, so is the corruption of the Pythian priestess (Hdt. 6.74), the introduction of lying witnesses (Dem. 47.1, 49.56, Pl. *Leg.* 936d), false artifice used by a musician (Ath. 14.614f). Rhetoric is *κακοτεχνία* to Epicurus (fr. 51 Us. = 19.3 Arrighetti).—Cf. also (Strabo 10.474), in the context of *ἀγυρτικόν* and *γοητεία* the expression *τὸ φιλότεχνον τὸ περὶ τὰς Διονυσιακὰς τέχνας καὶ τὰς Ὀρφικάς*. M. Marcovich, *Philologus* 108 (1964) 41, compares *ψευδέων τέκτονας καὶ μάρτυρας* (fr. 28); the threat uttered in this fragment puts us in mind of fr. 66 and 14, and we are in the realm of the *μύσται*.

Werner Jaeger wrote of Pythagoras, "The modern fashion of describing him as a sort of medicine-man has no claim to serious consideration." But an examination of the most ancient evidence makes it difficult to forget about the wonder-worker. The concept that first made it possible to take this aspect of the tradition seriously was that of shamanism, introduced by Meuli and Dodds.[229]

The word "shaman" comes from the language of the Tunguses of Siberia, and the phenomenon of "shamanism" was first studied in relation to certain Siberian tribes. The shaman has the ability, in an ecstatic state which is voluntarily induced by means of a definite technique, to make contact with gods and spirits, and in particular to travel to the Beyond, to heaven or to the underworld. Shamanism is the focus of these peoples' religious and intellectual life; the shaman's special task is to bring back health for the sick from another world and to conduct the souls of the dead to their new home. There are female as well as male shamans. The ability to achieve the shaman's ecstasy, and the technique necessary, are won, by those who have a special vocation, by a long ceremony of initiation, which includes the imparting of a certain mythical "knowledge," comprehending earth, heaven, and underworld.

The ecstatic journeys of a Hermotimus, and the ecstatic healings of a Phormio or Leonymus are obviously analogous to this, and the apparent death and the journey of Aristeas seem easier to understand, like the "flying" of Abaris on his arrow. The connection of Aristeas and Abaris with central Asia is certain.[230] Dodds speaks of the Greeks as coming into contact with shamanistic ideas from the north at the period of colonization of the Black Sea area; and he also brings Pythagoras into this context. Here we find, all together, association with gods and spirits, mastery of animals, disappearance, and simultaneous presence in different places.[231] To be sure, the *katabasis* of Pythagoras is not of a specifically shamanistic kind, any more than the miracles credited to Empedocles, Epimenides' visits to the cave, or the visit to the underworld from Lebadea.

The significance of the idea of shamanism for the history of philosophy lies in the conjecture that the new conception of the soul, which was to become the dominant one through the influence of Plato, is to be traced to this source. The independence of the soul from the body is immediately experienced and depicted in the shaman's ecstasy; and this is the reason for the particular interest taken in these phenomena by the Platonist Heraclides and the Peripatetic Clearchus (frr. 7–8). The Thracian Orpheus is not far from Scythia,[232] and in this vicinity (geographical and religious) we are but a step away from the doctrine of metempsychosis. The belief that the spirit of a powerful revenant has entered a living body is common among shamans,[233] and precisely this is attested for Epimenides. The inspired bard feels himself at one with the whole world: "I have been an eagle, I have been a sea coracle... I have been a sword in the hand, I have been a shield in battle, I have been a string in a harp."[234] It only needs a small stimulus, though its consequences are important, to make such ideas into a full-blown theory of metempsychosis.

The fact is that the very richness of the Greek tradition in stories of this type is somewhat embarrassing. At least some of them go back earlier than the colonization of the Black Sea. Meuli attempted to derive Greek epic from shamanistic poetry, and Odysseus does have shamanistic traits.[235] Then we must add Melampus' learning the language of the animals,[236] Teiresias' changes of sex,[237] Polyidus' awakening

[229] Jaeger, *Paideia* I 162 tr. Highet (= I 221 Ger. ed.).—Meuli and Dodds first worked this out in detail, but Rohde had already referred on occasion to Eskimos and Indians (*Psyche* II 97.1), and Diels had referred to shamanism (*AGP* 1897, 233ff, *Parm.* 14f, *NJb* 1922, 239f) as Lobeck, I 13 n. h, had already done.

[230] The Arimaspi have their origin in central Asiatic mythology (A. Alföldi, *Gnomon* 9 [1933] 566ff, Meuli 155ff). It has been recognized since the work of Meuli that the Scythians and Thracians had a genuine shamanism (Eliade 376ff); cf. the Thracian καπνοβάται (Posidonius *FGrHist* 87F104 Strabo 7.296) with the αἰθροβάτης Abaris (above, n. 162).

[231] Above, nn. 117–125, Eliade 103ff. Apollonius of Tyana also learned the language of animals (Philostr. *VA* 1.20). It is a common motif that an eagle brought the power of shamanism from heaven (Eliade 78ff).

[232] On Orpheus as a shaman see Dodds, *Irr.* 147ff.

[233] Dodds, *Irr.* 144f, Eliade 91ff. Shamans also claim to be descended from the sky-god (Eliade 79). Kahn emphasizes (*AGP* 1960, 32ff) that a doctrine of the nature of the soul does not necessarily develop from such ideas.

[234] Expressions of Irish bards, cited by Cornford (*PrSap* 122). "It is difficult," he says, "... to decide whether transmigration or metamorphosis is meant ... But the difference between metamorphosis and metempsychosis is, after all, not great" (123). Cf. Empedocles fr. 117, "For I have been ere now both boy and girl, and bush, and bird, and mute fish in the waves" (tr. after Leonard). Dodds derives Indian doctrines of metempsychosis from shamanism (*Irr.* 172 n. 97, citing *Acta orientalia* 17 [1939] 164ff).

[235] Meuli 164ff. E. Schwartz (*Die Odyssee* [Munich, 1924] 185ff) called attention to "shamanistic" features of the story of Odysseus—aside from his narrative of his journeys in the first person and his journey to the land of the dead, also his assumption of the role of a beggar, and especially his activity as an ἀγύρτης in Thesprotia.

[236] Hes. fr. 261 M.-W., Apollod. *Bibl.* 1.9.11. Shamanistic parallels in Eliade 105 n. 60.

[237] Hes. fr. 275 M.-W. (the *Melampodia*). Parallels from shamanism, Eliade 248.

Glaucus from the dead,[238] the motif of the winged seer,[239] and the magic quality of music, which can be ἐπῳδή, an incantation with power over gods and spirits.[240] Then there is the Messenian legend of the magician and king Periclymenus, who could change himself into any sort of creature,[241] and of Aristomenes, who made his way back out of the chasm of Ceadas.[242] In addition, if γόης, a word that combines the magic of self-transformation with the mourning of the dead,[243] originally meant something like "shaman," this takes us very far back indeed.

It is a controversial question, however, how far we may go in assuming a general spread of the kind of shamanism found among the peoples of Siberia and among the Indians, and how this phenomenon is to be fitted in to the general history of mankind.[244] It is especially difficult to be certain about the existence of an Indo-Iranian, or a specifically Iranian, shamanism.[245] The mythical motifs concerned can be traced with certainty back to the early ages of Mesopotamian civilization—Innana's trip to the underworld, Gilgamesh's search for immortality along the path of the sun beyond the great twin-peaked mountain.[246] The pertinent ritual, the cult of the Great Goddess, probably goes beyond the Neolithic to the Paleolithic Age, and we may

[238] References, *RE* XXI 1653ff. Aeschylus knows the legend, and the well-known fragment 638 of Euripides comes from his *Polyidus*. Furtwängler, *Gemmen* III 253, identified representations of the legend on archaic Etruscan gems. See also R. F. Willetts, *Klio* 37 (1959) 21ff. The Sotades cup (London D5) shows Polyidus and Glaucus in a subterranean vaulted chamber labeled "Hades" (A. S. Murray and A. H. Smith, *White Athenian Vases in the British Museum* [London, 1896], pl. XVI; J. Beazley, *Attic Red-Figured Vase-Painters*[2] [Oxford, 1963] 763 no. 2; ca. 450 B.C.).

[239] Musaeus has the gift of flight (Paus. 1.22.7) See further P. Wolters, "Der geflügelte Seher," *SBMü* 1928, 1.

[240] Cf. Boyancé, *Muses* (though he avoids the word "shamanism").

[241] Hes. fr. 33 M.-W.

[242] Paus. 4.18.5: An eagle, or the air resistance of the shield, bears Aristomenes unharmed to the floor of the ravine, just as, at the Trophonius oracle, a wind bears the inquirer into the depths.—The Dioscuri take Aristomenes' shield from him, and he finds it again in the sanctuary of Trophonius (Paus. 4.16.5–7). A 4th-century tradition connects Aristomenes with the mysteries of Andania (Paus. 4.26.8). Interestingly enough, there are close connections between Messenia and Magna Graecia; the colonists of Metapontum came mainly from Messenia (Kiechle 34ff). Ganschinietz lists more *katabasis* stories from Greece, *RE* X 2395ff. There are also relationships with Italian material: the Ausonians related of Mares, the "first man," whose form was that of a centaur, that he lived 123 years, and τρὶς ἀποθανὼν ἀνεβίω τρίς (Ael. *VH* 9.16).

[243] Cf. Burkert, *RhM* 1962.

[244] Eliade regards shamanism in a quite general way as an ecstasy-inducing technique. L. Vajda writes against this generalization (*Ural-Altaische Jahrbücher* 31 [1959] 456–485, with full bibliography); he would like to narrow it down as closely as possible—geographically, phenomenologically, and historically.

[245] Cf. G. Widengren, *Numen* 2 (1955) 62ff.

[246] *ANET* 52–57, 47–50, 88–96; *Phronesis* 14 (1969) 16–21.

conjecture that from very early there were attached to it societies of men with their initiation ceremonies, bringing the renewal of life, possibly in wild, ecstatic orgies. The sloughing of a snake's skin, attributed by the Gilgamesh epic to its having swallowed the plant of immortality, is described in an early Greek poem in the words, "Only the ψυχή remains."[247] This complete separation of body and soul does not seem, however, to belong to the early Mesopotamian or early Mediterranean tradition, whereas in the Iranian tradition the dichotomy of "bony" (that is, corporeal) and "spiritual" life is firmly established.[248] When Aelian[249] tells us that Pythagoras wore white clothing, a golden crown, and trousers, this last detail is a sure pointer toward the Iranian-Scythian area.[250] And yet these northern nomads never developed a doctrine of metempsychosis, so that we are once more tempted to consider the possibility of a direct dependence of the Greeks on the Indians (above, n. 71).

It is not possible, at this late date, to search out and map every channel of historical influence. Whether or not the expanded conception of shamanism is recognized as legitimate is a terminological question for specialists; but it has in any case performed the useful function of taking the so-called myths and legends seriously and showing how they make sense as clues to actual cult practices.

For this is the picture of Pythagoras that emerges from the study of the most ancient testimony, not influenced by Plato. He is the hierophant of Great Mother mysteries with an Anatolian stamp, and has a new doctrine, probably influenced by Indo-Iranian sources, of immortality and of the triumph over death through successive rebirths. Epimenides and Empedocles were similar "shamans," but the special character of Pythagoras' activity is seen above all in one fact—the continued existence of a society of *Πυθαγόρειοι*.

[247] Hes. fr. 204, line 139. M.-W.

[248] *Avesta*, passim. The teaching of the magi on resurrection and immortality: Theopompus *FGrHist* 115F64 = D.L. 1.9.

[249] *VH* 12.32: Πυθαγόρας ὁ Σάμιος λευκὴν ἐσθῆτα ἤσθητο καὶ ἐφόρει στέφανον χρυσοῦν καὶ ἀναξυρίδας. In exactly the same attire—ἐν χιτῶνι λευκῷ καὶ χρυσῷ στεφάνῳ καὶ ἀναξυρίσιν—the highest God, lord of death and rebirth, appears in the "Mithras liturgy." *Pap. gr. mag.* IV 699 = A. Dieterich, *Eine Mithrasliturgie* (1923[3]) 14, 15 (a reference supplied to me by M. West). Cf. the white clothing of the Pythagoreans, attested at [illegible] 19, Iam. *VP* 100 (probably from Aristoxenus, since Aristox. fr. 27 occurs in both passages; Rostagni, *Verbo* 272.1, Boyancé, *REG* 1939). Also note the golden crown in the "Grave of Archytas" at Tarentum (*AA* 1927, 137; Glotz-Cohen, *Hist. Gr.* III [Paris, 1936] 403 n. 76; Wuilleumier 548f). The Messenians, too, buried their dead with golden crown and white clothes (Paus. 4.13.2f).

[250] Cf. Hdt. 1.71—Pythagoras is represented with a turban, as in K. Schefold, *Die Bildnisse der antiken Dichter, Redner und Denker* (Basel, 1943), 100f, 160.1.

4. ACUSMATA

The oldest form of transmission of the teachings of Pythagoras is represented by the *acusmata*, which are also called *symbola*, orally transmitted maxims and sayings.[1] Our first tangible evidence about them goes back to about 400 B.C. Anaximander of Miletus (the younger), whom Xenophon names as one of those who could find the hidden meanings in Homer, also wrote an *Explanation of Pythagorean Symbola*.[2] This shows that the tradition was pre-Platonic; and if these *symbola* already needed explanation, or allegorical interpretation like the text of Homer, they must be much older, must go back in fact to pre-classical or archaic times.

It is a matter of question how much of the detail we have goes back as far as Anaximander's book.[3] Once again we find our most important evidence in Aristotle's book on the Pythagoreans. In addition to fragments cited by name,[4] there is a long passage in Iamblichus which has preserved Aristotelian material, partly in his very words.[5] Philochorus also wrote a book *Περὶ συμβόλων*.[6] But the principal source of the later tradition is the book of Androcydes, a Pythagorean, *Περὶ Πυθαγορικῶν συμβόλων*.[7] A physician named Androcydes is cited in fourth century books;[8] but there is some literary fiction involved in this context,[9] and it is not clear just what the relation may be between this and the book on the Pythagorean *symbola*. In any case the latter was in existence in the first century B.C.[10]

In the above-mentioned passage of Iamblichus, the *acusmata* are divided into three groups and given as answers to the questions *τί ἐστι; τί μάλιστα;* and *τί πρακτέον;* In the first group there are two

[1] Hölk, a student of Rohde, first clarified the main lines of the tradition. Later, Boehm collected comparative material from folklore, which helps to interpret the various *symbola*. In the following pages his numbers are given, and these should be understood also as referring to his convenient collection of parallel references. The most extensive discussion is that of Delatte, *Litt.* 269ff; the most important texts are in DK 58C.

[2] *ἔγραψε Συμβόλων Πυθαγορείων ἐξήγησιν* (*Suda* s.v. Anaximander = *FGrHist* 9T1 = DK 58C6). Corssen, *RhM* 1912, 249f, followed by Lévy, *Sources* 67.3, maintained that the *Suda* confuses Anaximander of Miletus with Alexander (Polyhistor) of Miletus, who also wrote on the Pythagorean *symbola* (*FGrHist* 273F94 = Clem. Al. *Strom.* 1.70; Jacoby, in his note on the passage, disagrees with Corssen). Xenophon, however (*Symp.* 3.6), refers to Anaximander's method of allegorical interpretation, and we cannot hope for any older or more apposite confirmation. The *Suda* dates Anaximander in the reign of Artaxerxes Mnemon (405–359). The dramatic date of the *Symposium* (422 B.C.) proves nothing, as Xenophon even introduces himself as a character (1.1; cf. Ath. 5.216d). But the mention of Stesimbrotus together with Anaximander brings the latter closer to 400 than say, to 360.

[3] The *Suda* (s.v. Anaximander) cites three *symbola*: not to step over a yoke, not to stir the fire with a knife, and not to eat from a whole loaf. The first two are favorite examples of Pythagorean *symbola* (no. 30 and 33 in Boehm; cf. Por. *VP* 42), so that one is led to suspect that the *Suda*'s datum is not taken from Anaximander's book but, for example, from some handbook. On the other hand, the third example (no. 39 Boehm) is not found elsewhere except in Hippol. *Ref.* 6.27.5, where the other two are also found (with *σάρον* instead of *ζυγόν*). Delatte (*Litt.* 286.2) supposes that Hippolytus and the *Suda* are both dependent on Anaximander.

[4] Cf. Hölk 21–40. The group of fragments collected by Rose (194–197) and printed without change by Ross must be revised: fr. 197 does not belong; Jerome copied Porphyry carelessly, and the name of Aristotle (Por. *VP* 41 = Arist. fr. 196) got attached to a passage which Porphyry expressly distinguishes from the Aristotle citation (Rohde, *Q* 139.1, Hölk 38ff; Por. *VP* 42 = Jerome belongs in the Androcydes tradition). —Fr. 195 is arbitrarily singled out by Rohde; D.L. 8.33 belongs to the *Memoirs* section (Hölk 27, 36ff, DK 58B1a, *FGrHist* 273F93). Fr. 192, first part (Iam. *VP* 30) is not an excerpt, but Iamblichus' own composition.

[5] Iam. *VP* 82–86, according to Hölk's analysis (31ff) pp. 47.11–50.17 Deubner. But one may well wonder whether p. 47.4–11 does not also have Aristotelian material, especially where the passage is connected closely with Ael. 4.17: *πειρῶνται διαφυλάττειν ὡς θεῖα σύμβολα* (Iam.) ~ *ὡς χρησμῷ θείῳ προσείχον* (Ael.); *ἀκούσματα ἀναπόδεικτα καὶ ἄνευ λόγου* (Iam.) ~ *οὐχ οἷόν τε δὲ ἦν διαπορῆσαι ὑπέρ τινος αὐτῷ ἢ . . . προσερωτῆσαι* (Ael.). The formulation in Iam., *τούτους ἔχειν βέλτιστα πρὸς φρόνησιν οἵτινες πλεῖστα ἀκούσματα σῴζουσι*, has an Aristotelian ring, too.

[6] *FGrHist* 328T1, doubtless with reference to Pythagoreanism (Jacoby IIIb *Supp.* 228f).

[7] Cf. Hölk 40ff. The oldest citation is by the rhetorician Tryphon *RhGr* III 193f Spengel. Very closely connected with this is Plut. *De educ. puer.* 17.12d–e, as is confirmed by the citation of Androcydes in *Paroemiogr. gr.* II 437, 770. Similar is Demetrius of Byzantium ap. Ath. 10.452d–e; and other passages to be included here are D.L. 8.17f, Por. *VP* 42, Hippol. *Ref.* 6.26–27 (see Hölk's tables, 50ff). It is not known, though, whether the work of Alexander Polyhistor or Anaximander of Miletus contributed importantly to the tradition along with Androcydes. One trace of Ionic dialect (*μαχαίρη*) is found as late as Plutarch (*De Is. et Os.* 354f.)

[8] Theophr. *Hist. pl.* 4.16.6 (whence Plin. *HN* 17.239), a formula for the prevention of drunkenness; Clearchus ap. Ath. 6.258b, on the flatterer (Wehrli breaks off fr. 20 before this passage, though the sentence following certainly refers to 255d = Clearchus fr. 19 W: *πᾶν ὑπομένειν* ~ *πάντα ὑποδύεται; ταπεινὰ ἤθη* ~ *εὐχέρεια; καταφρονητικοὶ τῶν ἀεὶ κρατούντων* ~ *οὐ βαρυνόμενος οὐδενὶ τῶν αἰσχρῶν*. Kaibel has the limits right in his edition of Ath.). Theophrastus is probably dependent on the letter of "Androcydes, well known for his wisdom" to Alexander the Great, whose "lack of moderation" he wished to "restrain" (Plin. *HN* 14.58). There is a citation containing a warning against overindulgence in wine and meat at Clem. Al. *Strom.* 7.33.7, and, without the author's name but in Ionic dialect, at Plut. *De esu carn.* 995e; cf. *De tranq.* 472b.

[9] The dramatic situation of the "letter to Alexander" is surely imaginary, and the person of the author may be so as well. Corssen believed in its authenticity (*RhM* 1912, 244ff), but his main argument, supposed Pythagorean dietary regulations in Androcydes, is a misunderstanding of Plin. *HN.* 17.239. He was followed by Delatte, *Litt.* 285, *Vie* 186, and by Bertermann (passim), for whom Androcydes becomes one of the main sources of the whole Pythagoras tradition. Diels (DK I 465 n.) and Lévy (*Sources* 66ff) express doubts, and a negative judgment is given by Zeller (III 2.118 n.) and Hölk (40ff).

[10] "Androcydes" coincides a number of times with the "Three Books," a forgery of about 200 B.C., attributed to Pythagoras (cf. Burkert, *Philologus* 1961; D.L. 8.7, 9f, 14). For example, one ought to pray, not for what he wants, but for what the gods want (Iam. *VP* 86, cf. Diod. 10.9.7.—"Androcydes" in Iam. *VP* 145, in the framework of an anecdote about the Pythagorean Thymaridas), and the calculation of the length of a period of reincarnation (D.L. 8.14; Androcydes ap. *Th. ar.* 52.8ff; cf. ch. II 3 n. 110). Here the priority is not ascertainable with certainty; cf. n. 73; a sentence on the quadrivium, in Ionic dialect, occurs in Nicom. *Ar.* 1.3.3.

instances of verbal agreement with Diogenes Laertius' citation from Aristotle,[11] and a third passage agrees almost exactly with one in the *Oeconomica* attributed to Aristotle. Here a textual corruption in the latter may be removed by use of the Iamblichus text.[12] Some sentences in Aelian, quoted along with other material about Pythagoras,[13] have points of contact both with fragments of Aristotle and with Iamblichus. The conclusion seems inevitable that not only the passage of Iamblichus[14] but also that in Aelian go back to Aristotle.

The other two categories of *acusmata* in Iamblichus can also be traced to Aristotle. Porphyry cites, from Aristotle, some sentences of the same type as the *τί ἔστι* adages in Iamblichus.[15] The pronouncement that Pythagoras was "the Hyperborean Apollo" is expressly attested as an *acusma*;[16] in the passage just mentioned, Aelian has some material that is closely related to what is attested by Porphyry;[17] there is similar material in Diogenes Laertius,[18] and also a remark of Aristotle himself.[19]

[11] Iam. *VP* 84, p. 49.6 D.; D.L. 8.34 (Arist. fr. 195; Diogenes' version is more detailed, Iam. *VP* 86, p. 50.8 D.; D.L. 8.35 (*ἄρτον μὴ καταγνύειν*). The last of the explanations given by D.L. (*ἐπεὶ ἀπὸ τούτου ἄρχεται τὸ ὅλον*) must go with the sentence in Iamblichus: *οὐ δεῖ οἰωνὸν ποιεῖσθαι τοιοῦτον ἀρχόμενον καταγνύντα καὶ συντρίβοντα*, whether the cosmological interpretation proposed by Delatte (*Vie* 239) is wrong, or whether Iamblichus misunderstood.

[12] Iam. *VP* 84, p. 49.4 D.: *γυναῖκα οὐ δεῖ διώκειν τὴν αὑτοῦ*; ps.-Arist. *Oec.* 1344a16 (DK 58C5): *ἥκιστα δεῖν δοκεῖν ἀδικεῖν*. Instead of the usual reading *δεῖν* [*δοκεῖν*] *ἀδικεῖ* (as in DK) should be read *δεῖν διώκειν* [*ἀδικεῖν*] (Deubner, *SBBln* 1935, 672). Cf. also Iam. *VP* 48.

[13] Ael. 4.17, *καρδίας ἀπέχεσθαι*, cf. Arist. fr. 194; (*ἀπέχεσθαι*) *ἀλεκτρυόνος λευκοῦ*, Arist. fr. 195, Iam. *VP* 84; *μὴ χρῆσθαι βαλανείῳ μηδὲ βαδίζειν τοὺς λευφόρους*, cf. Iam. *VP* 83. Thus the intervening sentence (*ἀπέχεσθαι*) *θνησειδίων* also belongs to Aristotle (cf. *Hypomn.*, D.L. 33; missing in Boehm).

[14] Iam. *VP* 85, p. 49.9ff: *ἀγαθὸν οἱ πόνοι, αἱ δὲ ἡδοναὶ ἐκ παντὸς τρόπου κακόν· ἀνακολάσει γὰρ ἐλθόντας δεῖ κολασθῆναι* is nearly quoted by Arist. *Protr.* fr. 60 (Iam. *Protr.* 47.25): *τοῦτο γὰρ θείως οἱ ἀρχαιότεροι λέγουσι τὸ φάναι διδόναι τὴν ψυχὴν τιμωρίαν καὶ ζῆν ἡμᾶς ἐπὶ κολάσει μεγάλων τινῶν ἁμαρτημάτων*. (Cf. Cic. *Consolatio* fr. 8 M., perhaps from Crantor: "luendorum scelerum causa nasci homines.") On Philolaus fr. 14, see ch. III 2 n. 47.—Death as a *μετοίκησις* (Iam. *VP* 85, p. 50.2) is Platonic (*Ap.* 40c, *Phd.* 117e; see Deubner on the Iamblichus passage); but it is also understandable in the framework of the ancient doctrine of metempsychosis.

[15] Por. *VP* 41 = Arist. fr. 196; also Plut. *De Is. et Os.* 32, Clem. Al. *Strom.* 5.50.1.

[16] Iam. *VP* 140: *καὶ ἓν τοῦτο τῶν ἀκουσμάτων ἐστί· τίς εἶ, Πυθαγόρα; φασὶ γὰρ εἶναι Ἀπόλλωνα Ὑπερβόρειον* (cf. ch. II 3, n. 117).

[17] *VH* 4.17; printed with Arist. fr. 196.

[18] 8.35: *γῆρας καὶ πᾶν τὸ μειούμενον ὅμοιον· αὔξην καὶ νεότητα ταὐτόν, ὑγίειαν τὴν τοῦ εἴδους διαμονήν, νόσον τὴν τούτου φθοράν*. This sentence, like the one that precedes it (*καὶ τῶν σχημάτων τὸ κάλλιστον σφαῖραν εἶναι τῶν στερεῶν, τῶν δὲ ἐπιπέδων κύκλον*) is excluded by Rose from fr. 195, but Delatte (*Litt.* 277, *Vie* 239) treats it as Aristotelian. The suspicious word *εἶδος*, in true pre-Socratic fashion, means nothing more than the shape of the body.

[19] *An. post* 94b33 (DK 58C1).

The traces of Aristotelian influence are still clearer in the *τί μάλιστα* group. "This is the same sort of wisdom as that attributed to the Seven Sages; for they too were trying to find not 'what is good?' but 'what most [good]?'" (Iam. *VP* 83). This is an excellent comment, and quite correct from the point of view of the history of thought. Whatever the case may be about the authenticity of the sayings of the "Seven Sages" (we have collections of adages in the *τί μάλιστα* format),[20] in the search for superlatives (*τί φέρτατον, τί μάλιστον*) there lies a very ancient kind of "wisdom," to be found not only in the *Contest of Homer and Hesiod* and in the Aesop legend, but also in Sappho (27 D.) and at the beginning of Pindar's first *Olympian*. Now the designation of the Seven Sages in this passage as *ἑπτὰ σοφισταί*, an expression already becoming obsolete in the fourth century, is attested precisely for Aristotle.[21] What is more, in the passage we have cited twice above, Aelian gives two *τί μάλιστα* sayings, one of which agrees with Iamblichus;[22] so that the similar ones in Diogenes Laertius must also be attributed to Aristotle.[23]

[20] Plut. *Conv. sept. sap.* 9.153c, D.L. 1.35, Stob. 1.1.29a, 1.4.7a, 1.8.40a, 1.18.1e; cf. also 1.1.30ff and Ephorus ap. Diod. 9.26f. The saying *σοφώτατον χρόνος*, which occurs in its context, is ascribed by Eudemus fr. 90 W. to Simonides. Lévy (*Sources* 4 n. 1) believes that the collection of sayings of Thales in D.L. 1.35 is presupposed at Iam. *VP* 83; but this unprovable. The attempt of O. Brendel to reconstruct an original series of these sayings (*AA* 41 [1936] 26f) is also problematic. Cf. B. Snell, *Dichtung und Gesellschaft* (1965) 103.

[21] Pl. *Prt.*, assigned to the *On Philosophy* because Aristotle dealt with the Seven Sages in that work (fr. 3). Isoc. *Antid.* 235 attests that *ἑπτὰ σοφισταί* was an antiquated expression. (Cf. Ephorus ap. Diod. 9.26, Androtion ap. Aristides *Or.* 46, II 407 Dind.)—The attribution of this sentence to Aristotle yields a valuable chronological datum: *πρότεροι γὰρ οὗτοι Πυθαγόρου ἐγένοντο*. Hermippus (D.L. 1.42) names Pythagoras among the 17 who were, in various lists, included among the Seven Sages, but Dicaearchus (fr. 32 W.) does not include him.

[22] *VH* 4.17: *πάντων σοφώτατον ὁ ἀριθμός, δεύτερος δὲ ὁ τοῖς πράγμασι τὰ ὀνόματα θέμενος*. Cf. Iam. *VP* 82, p. 47.17: *τί τὸ σοφώτατον; ἀριθμός· δεύτερον δὲ τὸ τοῖς πράγμασι τὰ ὀνόματα τιθέμενον*. H. Steinthal, *Geschichte der Sprachwissenschaft bei den Griechen und Römern* I (Berlin, 1890²) 153–168, tried to show that the second part was a construct based on Pl. *Crat.* 416b–c. It is peculiar to have a "next best" given in the context of the *τί μάλιστα* formulas. It may be that this second part was an addition. It is often cited by itself in the later tradition (Cic. *Tusc.* 1.62, Varro *Ling.* 8.7, Philo *Qu. in Gen.* 1.90, 4.194, *Decal.* 23, Iam. *VP* 56, Clem. Al. *Ecl. proph.* 32.1, etc.; Boyancé, *REG* 76 [1963] 70–72). Boyancé, *REG* 1941, 172, called attention to Herodotus' statement that it was Homer and Hesiod who "gave the gods their names" (2.53). Von Fritz (*AJPh* 1960, 15) dates the *acusma* to the second half of the 5th century, but admiration of the mysterious deviser of names is already expressed in Aesch. *Ag.* 681–688.—Aelian adds the pronouncement on the mallow leaf (see n. 40 below).

[23] Above, n. 18, on sphere and circle; and cf. Archytas DK 47A23a. Of course the terminological distinction between *στερεόν* and *ἐπίπεδον* is not Pythagorean (above, ch. I 5, n. 97); but if the original formulation was *τί κάλλιστον σχῆμα; κύκλος καὶ σφαῖρα*, the more precise terms would be adopted as a matter of course by later writers, including Aristotle.

Among this group we can even distinguish a pre-Aristotelian line of tradition. Aristotle introduces a Pythagorean named Paron who is otherwise completely unknown (DK 26): *οἱ μὲν σοφώτατον ἔλεγον (τὸν χρόνον), ὁ δὲ Πυθαγόρειος Πάρων ἀμαθέστατον . . .* (*Phys.* 222b17). But in Eudemus' more detailed account the story goes differently (fr. 90 W.): *ἐν Ὀλυμπίᾳ Σιμωνίδου τὸν χρόνον ἐπαινοῦντος ὡς σοφώτατον, εἴπερ ἐν αὐτῷ αἱ μαθήσεις γίνονται καὶ αἱ ἀναμνήσεις, παρόντα τινὰ τῶν σοφῶν εἰπεῖν, τί δέ ὦ Σιμωνίδη, οὐκ ἐπιλανθανόμεθα μέντοι ἐν τῷ χρόνῳ;* Eudemus cannot be dependent on Aristotle, since he sets the scene more elaborately—Olympia, Simonides, etc.—but what is a proper name to Aristotle appears here as a participle: *Πάρων-παρών*. We cannot however, follow Simplicius' proposal to correct *Πάρων* in Aristotle to *παρών*, for without the scene as Eudemus has it, the word would be meaningless. Both are following the same source, a written source which, with the accents unmarked, could be understood in either of two ways, e.g. *ΠΑΡΩΝ ΤΙΣ ΠΥΘΑΓΟΡΕΙΟΣ ΕΛΕΓΕ . . .*[24] Thus we have a pre-Aristotelian proof of the high valuation placed on *μνήμη* by the Pythagoreans, and also of the Pythagoreans' use of proverbial wisdom in the contest of the ancient *σοφισταί*.

This shows that the entire section in Iamblichus is full of Aristotelian material. It may be that the division of maxims into three categories is likewise Aristotle's work. He was interested in logical distinctions, and in particular in the early stages of conceptual definition.[25] From the lines of parallel transmission emerges quite an extensive body of material.

What are the Isles of the Blest? Sun and moon. What is the Oracle of Delphi? *τετρακτύς· ὅπερ ἐστὶν ἡ ἁρμονία ἐν ᾗ αἱ Σειρῆνες.*[26] Pythagoras is the Hyperborean Apollo.[27] An earthquake is a mass meeting of the dead.[28] The purpose of thunder is to threaten those in Tartarus, so that they will be afraid.[29] The rainbow is the reflected splendor of the sun.[30] The sea is the tears of Cronus.[31] The Great Bear and the Little Bear are the hands of Rhea.[32] The Pleiades are the lyre of the Muses, and the planets are Persephone's dogs.[33] The ring of bronze when it is struck is the voice of a daemon entrapped in it;[34] and the ringing that people often hear in their ears is the voice of the *κρείττονες*.[35] Old age is decrease, and youth is increase; health is retention of form, disease its destruction.[36] Friendship is harmonious equality.[37]

The most just thing is to sacrifice, the wisest is number,[38] the most beautiful, harmony, the strongest, insight, the best (in the sense of the most desired), happiness, the truest, that men are wicked,[39] the holiest, a mallow leaf,[40] the most beautiful shapes, circle and sphere.[41]

One ought to beget children, for it is our duty to leave behind, for the gods, people to worship them.[42] One should put on the right shoe

[24] Diels, DK I 217 n.: "Eudem folgte also einer anderen Lesart oder Ueberlieferung." This evades the problem. The source is likely to be a Sophistic-rhetorical writing like Alcidamas' *Museum*. Eudemus probably understood his source correctly, and the whole chapter, DK 26, should be eliminated (Wehrli 106).

[25] Cf. his understanding of the Pythagorean number symbolism as attempts at definition (above, ch. I 2, n. 63). The equations he mentions like, "marriage is 5," are at home among the *acusmata*.

[26] Iam. *VP* 82; cf. below, n. 154.

[27] Above, ch. II 3, n. 117.

[28] Ael. 4.17. The idea behind this may be that of a battle or struggle (Nilsson I 702 n. 1 referring to *Il.* 20.61ff).

[29] Arist. *An. post.* 94b33.

[30] Ael. 4.17; cf. Anaximenes A7, A18, and Xenophanes fr. 32.

[31] Por. *VP* 41, Plut. *De Is. et Os.* 364a. Cf. the reference to the Adriatic as the "sea of Cronus," Ap. Rhod. *Argon.* 4.327, and the localization of the myth of the castration of Cronus on the island of Corcyra (Timaeus *FGrHist* 566F79, Lycoph. 761, *Orph. frag.* 137, 154). Cronus has a cave under Ocean, according to Por. *De antr. nymph.* 7 and Eust. *In Dion. Per.* 32.

[32] According to the tradition the Little Bear was discovered by Thales (which means that it was mentioned in the *Ναυτικὴ ἀστρολογία* which was attributed to him; Callim. fr. 191 **, with D.L. 1.23).—In this context obviously belongs Procl. *In Eucl.* 90.14: *οἱ δέ γε Πυθαγόρειοι τὸν μὲν πόλον σφραγῖδα τῆς Ῥέας ἀποκαλεῖν ἠξίουν* (referring to a seal ring worn on the hand).

[33] Below, ch. IV 1.

[34] These sentences are in Por. *VP* 41 (Arist. fr. 196), Plut. *De Is. et Os.* 32, Clem. Al. *Strom.* 5.90.1; on the last, Cook, *Zeus* II 649ff; L. Parmentier, *Mém. Acad. Belgique, Cl. d. lett.*, ser. II, 11 (1913) 31–61, with many parallel references. Cf. esp. Apollodorus *FGrHist* 224F110, scholium and Eust. on *Il.* 16.408 (*ἤνοπι χαλκῷ*); below, ch. V I.

[35] Ael. 4.17; on *κρείττονες* see Hsch. s.v.

[36] D.L. 8.35; cf. above, n. 18. Similar is the correlation of the ages of man with the seasons "Pythagoras" ap. D.L. 8.10, Diod. 10.9.5. But this is found as early as *De hebdomadibus* 4f, and in an allusive way in Heraclitus (fr. 100; cf. Reinhardt, *Hermes* 1944, 228ff; Kirk, *Heraclitus* 294ff).

[37] Timaeus *FGrHist* 566F13 (cf. Pl. *Leg.* 757a, *Hypomn.* 33, Iam. *VP* 162; Delatte, *Vie pyth.* 40). In the same passage *κοινὰ τὰ φίλων* is given as a Pythagorean saying (cf. Pl. *Lys.* 207c).

[38] Cf. above, n. 22. Iam *VP* 82 has an additional item: *τί σοφώτατον τῶν παρ' ἡμῖν; ἰατρική*

[39] Iam. *VP* 82; cf. above, n. 14.

[40] Ael. 4.17. Also, *μαλάχης εἴργεσθαι*, Iam. *VP* 109; cf. Iam. *Protr.* 125.12ff; below, n. 140. The Neoplatonist Isidorus held to this precept (Dam. *Isid.* 125).

[41] Above, n. 23; ch. IV 2.

[42] Here the important thing is not the precept but the reason given for it (Iam. *VP* 86). Plato has the same thought, philosophically expressed, *Leg.* 773e: *χρὴ τῆς ἀειγενοῦς φύσεως ἀντέχεσθαι τῷ παῖδας παίδων καταλείποντα ἀεὶ τῷ θεῷ ὑπηρέτας ἀνθ' αὑτοῦ παραδίδοναι.* It is hardly likely that the simpler formulation found in the *acusma* was spun out of the Platonic passage. Comparable is his idea that a human being is *ἓν τῶν κτημάτων τοῖς θεοῖς* (*Phd.* 62b). Men are created as servants to the gods in Babylonian myth (*ANET* 68).

first,[43] not travel by the main roads (λεωφόροι) (no. 41), not dip one's hand into holy water (no. 44), not use the public baths (no. 45), and not help a person to unload but only to load up.[44] One should not have children by a woman who wears gold jewelry (no. 21), not speak in the dark (no. 51). One should pour libations over the handle of the cup,[45] refrain from wearing rings with depictions of gods (no. 9), and not "pursue" one's own wife, since the husband, in receiving her as a suppliant at the altar, has taken her under his protection.[46] One should not sacrifice a white cock, because they are suppliants and sacred to the god Men.[47] One should never give advice except with the best intent (for advice is sacred) (no. 63),[48] nor make a detour on the way to the temple (no. 59). One should sacrifice and enter the temple barefoot (no. 3), and in battle hold one's place, so as to fall with wounds in the breast.[49] Eat only the flesh of animals that may be sacrificed; abstain from beans; do not pick up food that falls from the table, for it belongs to the Heroes.[50] Abstain from fish that are sacred (including τρίγλη, ἀκαλήφη, ἐρυθρῖνος, μελάνουρος); do not break bread;[51] put salt on the table as a symbol of righteousness.[52] It is also forbidden to eat certain parts of the sacrificed animal, and there are many specific injunctions about the time and the manner of sacrificing,[53] and about rites connected with death and burial. This is the context of the prohibition mentioned by Herodotus, of burying the dead in woolen garments.[54]

There are other rules, of just the same kind, in the tradition stemming from Androcydes. For example, one is not to stir the fire with a knife (no. 33), step over a yoke (no. 30), or sit on a bushel measure (no. 32). On rising one is to straighten the bedclothes and eliminate the traces of one's presence (no. 34),[55] as well as destroying the marks of a pot in the ashes (no. 35). One ought not to clean a chair with a torch (no. 36)[56] or to step, or make water, on nail parings (no. 48),[57] and should point a sharp knife in the other direction (no. 37), and not look at oneself in a mirror with the help of artificial light (no. 52). On a journey do not turn around at the border.[58]

Hermippus has some strange precepts: One should not pass by where an ass is lying, and should avoid "thirst-causing water."[59] A coffin should not be made of cypress, for that is the material of Zeus' scepter.[60] The same sentence appears in a remarkable list of cult rules in Iamblichus (153–156); it combines quite trivial matters ("do not kill a louse in the temple") with commandments as important as the prohibition of cremation ("in agreement with the Magi"). The prohibition of "roasting what has been boiled" is attributed in the pseudo-Aristotelian *Problemata* to "the mysteries."[61]

What we have here is a very mixed collection of sayings and maxims. The threefold division, perhaps originated by Aristotle, is artificial and is not consistently followed.[62] The rules and prohibitions regarding daily life attracted most attention in ancient times; and there were two opposite ways of interpreting them.

[43] A verse cited by a Polemon (of Ilium? Phot. *Bibl.* 553b20, no. VIII Nauck) adds that in washing one's feet one should do the left foot first. (Cf. Iam. *Protr.* 107.10, 114.7, Menander fr. 97 Koerte.)

[44] This has the ring of a purely moral exhortation to πονεῖν. Méautis (78ff) refers to stories about the underworld like Apul. *Met.* 6.18.3, in which one is forbidden to help a daemon one meets.

[45] The thought is, of course, to avoid any intimate contact with the divine (Philostr. *VA* 4.20).

[46] Above, n. 12.

[47] No. 13. The mention of the god Men certainly points toward Asia Minor as the place of origin of this. Cumont (1942) would like to find here a direct contact between Pythagoreans and the μάγοι, and points out that this prohibition is still observed today in Iran (293f). But even if that be the place of origin, the white cock is connected with Mithras and the sun, whereas the connection with Men (Shin) is Babylonian (Cumont 292, 285 n. 1). Therefore it must be a case of Anatolian syncretism rather than a direct connection with Persia. (Later the cock is sacred to the sun: Iam. *VP* 147, *Protr.* 116.12.)

[48] A widely known proverb. Cf. Hes. *Op.* 266, Epicharm. fr. 228 Kaibel, Ar. fr. 33, Xen. *An.* 5.6.4, Pl. *Ep.* 5.321c, *Theag.* 122b, Lucian *Ind.* 25, Iam. *VP* 49.

[49] This well-worn maxim of military conduct may be interpreted not only in the literary, physical sense, but in a mystical way. See Kirk, *AJP* 70 (1949) 384–393, commenting on Heraclitus frr. 24 and 136, and M. Detienne, "Des confréries de guerriers à la société pythagoricienne," *RHR* 163 (1963) 127–131.

[50] Aristotle (D. L. 8.34) gives Aristophanes (fr. 305) as authority for this. On the topic, Rohde, *Psyche* I 245 n. 1 – 202 n. 114 Eng. ed.

[51] Basically synonymous with "Do not eat from a whole loaf" (above, n. 3; no. 39 Boehm); bread must be cut with a knife, in a prescribed ritual manner.

[52] This rationale excludes other, less pleasant interpretations, like the role of ἁλῶν χόνδρος in mysteries of Aphrodite (Clem. Al. *Protr.* 2.14.2). Egyptian priests did not put salt on the table (Plut. *De Is. et Os.* 363c.)

[53] Iam. *VP* 85; cf. 152.

[54] Cf. *Inscr. Délos* 2180 = Sokolowski, *supp.* 56; Lobeck 244f; J. Quasten, *AJP* 61 (1942) 207ff.—On Hdt. 2.81 see ch. II 3, n. 39.

[55] Cf. Por. ap. Stob. 1.49.59, Ar. *Nub.* 975f, W. Deonna, *REG* 42 (1929) 171, O. Weinreich, *ARW* 28 (1930) 183–184.

[56] The torch plays a role in cult, especially in ceremonies of purification; see Diphilus fr. 126 Kock.

[57] "Not to spit on them," Iam. *Protr.* 124.1; πρὸς ἥλιον τετραμμένον μὴ ὀμείχειν, D.L. 8.17, Iam. *Protr.* 115.19, Hes. *Op.* 727.

[58] Ἐρινύες γὰρ μετέρχονται, Iam. *Protr.* 115.1, Hippol. *Ref.* 6.26.

[59] διψίων ὑδάτων ἀπέχεσθαι, Hermippus ap. Joseph. *Ap.* 1.164.

[60] Hermippus ap. D.L. 8.10.

[61] *Problemata inedita* ed. Bussemaker 3.43. Perhaps the reference is to the Dionysus ritual reflected in the Zagreus myth.

[62] Material that belongs to the τί μάλιστα category in Ael. *VH* 4.17 (ἱερώτατον τὸ τῆς μαλάχης φύλλον) appears as a prohibition in Iamblichus (μαλάχης ἐΐργεσθαι, *VP* 109).

The account in Iamblichus, which goes back to Aristotle, leaves no doubt that the precepts are intended to be taken literally (Iam. *VP* 86):

> In some cases a statement is added as to why this is to be done[63] (e.g., that one ought to have children in order to leave behind another to worship the gods in one's stead), but in other cases no explanation is given. Some of the explanations added seem to be ideally suitable, but others are far-fetched . . . The added, conjectural explanations are not Pythagorean, but originate with persons who introduced clever explanations from without,[64] in an effort to give a plausible rationale.

The most prominent topic of the *acusmata* is sacrificial ritual; and the accounts of Herodotus, Isocrates, and Eudoxus speak of the special role of ritual ἁγνεῖαι among Pythagoras and his followers.[65]

Nevertheless, the prevailing view in antiquity was that what was desired was not compliance to the letter but comprehension of the deeper meaning. As early as Anaximander allegorical interpretation was applied to this material as to Homer; and, when Aristotle used the word ἐπισοφιζόμενοι, he surely had this method in mind.[66] Our principal source for the later interpretation is Androcydes. Here the *acusmata* are regarded as αἰνίγματα, riddles, which clothe a lofty wisdom in language unintelligible to the uninitiated.[67] As Clement puts it, σφάλλεται μὲν ὁ ἄπειρος καὶ ἀμαθής, καταλαμβάνει δὲ ὁ γνωστικός.[68]

It is obvious that Androcydes must have exercised a certain selectivity, for not everything would fit together consistently. The injunction καρδίαν μὴ ἐσθίειν means that one ought not to worry;[69] but its companion, the precept not to eat the μήτρα is not mentioned. Among the forbidden fish, the μελάνουρος ("blacktail") could be interpreted to apply to men with "black" characters,[70] but τρίγλη and ἀκαλήφη do not suggest anything interesting.

In any case, this new process of exposition and reinterpretation was quite inevitable, for in an enlightened period most of this material would appear nothing but ridiculous. Thus the Pythagoreans were led to complain that Plato, Aristotle, and their pupils had appropriated what was "fruitful" in the Pythagorean doctrine, with some little revision, and left only the nonsense to them—all that was likely to make the Pythagorean teaching seem absurd.[71] Allegorical interpretation, here as elsewhere, was the necessary means of adapting ancient lore to new ways of thinking, and thus preserving its authority.

On the other hand, Aristotle's report itself points the way to a different, more historical interpretation. He points out parallels with popular belief, which we also can see reflected here and there in comedy, and also with certain usages of the "barbarians" which had preserved, as he says, "even to the present day," practices of their ancestors.[72] The Alexander Polyhistor *Memoirs*, which, surprisingly, still ignore the allegorical interpretation of the *acusmata*, add the significant comment that the abstinences they enjoin correspond to "the ones prescribed by those who perform the initiatory rites in the temples."[73]

Along with the new type of interpretation comes an alteration in the terminology. In Aristotle, and only in his report, *acusmata* is the only term used for these precepts,[74] but outside the Aristotelian tradition the prevailing word is *symbola*, though there can be no doubt that the two words refer to the same thing.[75] Along with the word ἄκουσμα

[63] ἐπιλέγεται ‹διὰ› τί δεῖ. The insertion of ‹διὰ› (a conjecture of Kiessling's merely recorded in the apparatus by Deubner) is essential. The omission would be an easy one in a majuscule manuscript: *ΕΠΙΛΕΓΕΤΑΙΔΙΑΤΙΔΕΙ*.

[64] ἐπισοφίζεσθαι is attested from the corpus Hippocraticum (LSJ); but εἰκοτολογία is only found in later texts (ps.-Archytas p. 37.1 Thesleff).

[65] Hdt. 2.81, Isoc. *Bus.* 28, Eudoxus fr. 36 Gisinger = 325 Lasserre = Por. *VP* 7.

[66] Hölk 21.

[67] For the word αἰνίγματα, see Tryphon, Demetrius, and Plutarch, n. 7 above. A certain Hippomedon asserted (Iam. *VP* 87) that Pythagoras had given λόγους καὶ ἀποδείξεις for all his sayings, but that they had been lost through the carelessness of his successors. Plutarch joins in the game of "solving" these riddles (*Quaest. conv.* 8.7, where the words οἱ παλαιοί in section 2 obviously refer to Androcydes; cf. D.L. 8.17). See also Philop. *De an.* 116.29, Olympiod. *In Phd.* p. 8.22 Norvin.

[68] Clem. Al. *Strom.* 5.57.1; similarly, an anonymous writer (Plutarch?) ap. Stob. 3.1.199: ὥστε . . . αὐτόθεν ἔχειν φῶς καὶ χαρακτῆρα τοῖς συνήθεσι τὸ φραζόμενον, τυφλὸν δὲ καὶ ἄσημον εἶναι τοῖς ἀπείροις . . .

[69] No. 15 Boehm: Demetrius, Plutarch, D.L. 8.18, etc.; cf. *Il.* 24.129.—Arist. fr. 194, Gell. 4.11.11, Ael. *VH* 4.17; cf. Iam. *VP* 109.

[70] Tryphon, Plutarch (above, n. 7), Arist. fr. 194. Androcydes may also have made additions; the proverbial σιτίον εἰς ἀμίδα μὴ ἐμβάλλειν (*Paroemiogr. gr.* II 770) only makes sense in a metaphorical interpretation.

[71] Por. *VP* 53.

[72] At D.L. 8.34, Ar. fr. 305 is quoted (above, n. 50; Rose unjustifiably omits the citation of Aristophanes from fr. 195).—καθάπερ ἔτι καὶ νῦν οἱ βάρβαροι, D.L. 8.35 (cf. Iam. *VP* 86).

[73] *Hypomn.* 33 (cf. Iam. *VP* 138). This excludes the "Androcydes" interpretation of the taboo on beans and that on blacktail (both mentioned in *Hypomn.* 33); and this also probably indicates that "Androcydes" should be dated later than the *Hypomnemata* (3rd century B.C.). The "Androcydes" interpretation of the taboo on beans—that one ought not to take part in politics—is hardly pre-Hellenistic. (A different interpretation, based on the same reason, that the bean was used to vote with, is found in Arist. fr. 194 = D.L. 8.34: ὅτι ὀλιγαρχικόν.)

[74] Iam. *VP* 82ff, 140. Elsewhere the word ἄκουσμα has the sense "musical entertainment" (Xen. *Mem.* 2.1.31, Arist. *Pol.* 1336b2, Iam. *VP* 245, etc.).

[75] Hölk 12ff.

goes the designation of the members of the Pythagorean society as ὁμάκοοι and ὁμακοεῖον for their meeting house.[76] On the other hand, the word σύμβολον carries with it the suggestion of a "symbolic" interpretation. The word is not a late addition, however,[77] but carries another implication as well, prior to any "symbolic" exegesis. In the realm of mystery religion, σύμβολα are "passwords"—specified formulas, sayings, ἐπῳδαί, which are given the initiate and which provide him assurance that by his fellows, and especially by the gods, his new, special status will be recognized.[78] The *acusmata* or *symbola* still have this function as late as Lucian's parody: τούτων γὰρ ἂν μεμνημένον ἐλπίδας ἔχειν τῆς εἰς τὴν νῆσον ἀφίξεως, says Rhadamanthys, in the *True History*, to Lucian as he leaves the Isle of the Blest.[79] Aristoxenus mentions σύμβολα as "passwords" used by the Pythagoreans.[80] The pentagram, too, was a *symbolon* in this sense; it was thought from the most ancient times to have a secret power and significance.[81] But when once the allegorical and symbolic interpretation of the Pythagorean sayings had gained a foothold, it was inevitable that the word σύμβολον be understood in this sense, whereas ἄκουσμα had no such obvious meaning and fell out of use.

The riddle as a literary form (γρῖφος) is very old, and is used in the promulgation of oracles. There can be no doubt, though, that the *acusmata* are, rather than simple, commonsense wisdom in abstruse form,[82] ancient magical-ritual commandments.[83] It is not possible here to offer a complete analysis, but we may try to establish a few points of reference.

There are obvious coincidences with rituals of Greek mystery cults, and they even share whole series of commandments. For example,

(a) The initiate at Eleusis must fast, avoid baths, abstain from domestic fowl (and the cock is especially named), as well as ἰχθύων καὶ κυάμων ῥοιᾶς τε καὶ μήλου, καὶ ἐπ' ἴσης μεμίανται τό τε λεχοῦς ἅψασθαι καὶ τὸ θνησειδίων.[84]

(b) At the Haloa, one was forbidden to eat ῥοιά, μῆλον, ὄρνιθες κατοικίδιοι, ᾠά, as well as the fishes τρίγλη, ἐρυθῖνος, μελάνουρος, κάραβος, γαλεός.[85]

(c) Preparation for the *katabasis* in the Trophonius cult included avoidance of warm baths, τρίγλη, τρυγών, μελάνουρος, and one must wear a linen garment.[86]

(d) The μάγοι, καθαρταί, ἀγύρται, ἀλαζόνες against whom the Hippocratic work *On the Sacred Disease* speaks, forbid their patients baths, τρίγλη, μελάνουρος, κεστρεύς, ἔγχελυς, certain kinds of meat, various birds including the cock, and spices. They may not wear any black garment, lie on a goatskin, lay one foot or one hand over the other; and there are other such purificatory measures.[87]

(e) In Delos, the worshipers of Zeus Cynthius approach him barefoot and clad in white. They abstain from sexual intercourse and from meat, and do not wear iron rings, keys, belts, purses, or weapons.[88]

[76] ὁμακοεῖον Nicom. (Iam. *VP* 30 = Por. *VP* 20), Iam. *VP* 74, 185. Altered to ὁμακόιον, Olympiod. *In Phd.* p. 9.10 Norvin, and copied by Schol. Pl. *Phd.* 61c, Olympiod. *In Alc.* p. 132.12, Eust. p. 856.63f; ὁμάιον, Hierocles *In CA* 27 (p. 484 Mullach), Eust. loc. cit.—ὁμάκοοι, Iam. *VP* 73 (Timaeus? above, ch. II 1, n. 37). The compound, with ὁμο- and the Doric form, indicates that an original term has been preserved. On συνήκοος (Pl. *Leg.* 711e), see ch. I 4.

[77] *Pace* Hölk 15ff.

[78] W. Müri, "Symbolon: Wort- und sachgeschichtliche Studie," *Beilage* to *Jahresber. d. St. Gymn.* (Bern, 1931) 39ff; Boyancé, *Muses* 51ff; cf. the Orphic papyrus DK 1B23, the Eleusinian σύνθημα, Clem. Al. *Protr.* 2.21 (σύμβολον = σύνθημα, "password;" Eur. *Rhes.* 572f). Cf. Plaut. *Mil.* 1016: "cedo signum, si harunc Baccharum es."

[79] *Ver. hist.* 2.28, Nock, *AJA* 1946, 153.

[80] Fr. 43 W.: (Pythagoras) εἴ τινα πύθοιτο τῶν συμβόλων αὐτοῦ κεκοινωνηκότα, εὐθύς τε προσηταιρίζετο καὶ φίλον κατεσκεύαζεν.

[81] Lucian *Laps.* 5; Schol. Lucian p. 234.21 Rabe; Schol. Ar. *Nub.* 609. The pentagram has been very widely used as a magical symbol, from the times of the early oriental civilizations. See R. Eisler, *Weltenmantel und Himmelszelt* I (Munich, 1910) 304f; J. Leite de Vasconsellos, *Signum Salomonis: Estudo de etnografia comparativa* (Lissabon, 1918); Cook, *Zeus* III 341 n. 3; W. Deonna, *Bull. de l'Ass. Pro Aventico* 16 (1954) 44ff; C. Láscaris Comneno and A. M. de Guadan, *Rev. di filos.* 15 (1956) 181–207 (with the comment of W. Burkert, *Philologus* 1961, 230 n. 4); Stapleton, *Osiris* 1958, 12f. 35ff; De Vogel 292–299.

[82] Maddalena thinks that the *acusmatici* misunderstood Pythagoras' metaphorical language (362). Von Fritz (*SBMü* 1960, 16f) regards the allegorical sense as primary in ζυγὸν μὴ ὑπερβαίνειν (but what of the variant σάρον in Plut. *Quaest. conv.* 727c, *Quaest. Rom.* 290e, Hippol. *Ref.* 6.27.4?), and also in καρδίαν μὴ ἐσθίειν. (On this, see below, n. 116; above, n. 70.)

[83] "Ritualgesetze," Rohde, Q 109; cf. Lobeck 902f, 248f; Nilsson I 703–708. "Genuine taboos," Burnet *EGP* 96. Boehm gathers comparative material. Delatte, *Litt.* 287f, favors the ethnological method, though elsewhere he hesitates (*Vie* 186f: "trop uniforme pour être toujours vrai").

[84] Por. *Abst.* 4.16, *Hymn. Hom. Cer.* 50. The avoidance of λεχώ and θνησείδια is also prescribed at *Hypomn.* 33; cf. also n. 102.

[85] Schol. Lucian p. 280.23 Rabe.

[86] Cratinus fr. 221 Kock (Ath. 7.325e), Paus. 9.39.5, 8. The proscription of meat-eating and baths is also in the Paris magical papyrus *PGM* no. IV 736.

[87] Hippoc. VI 354f L., cited by Delatte, *Vie* 232; Boyancé, *Muses* 107. A connection of these καθαρταί with Pythagoreanism is not to be excluded; M. Wellmann, *Die Fragmente der Sikelischen Ärzte* (Berlin, 1901) 29.1, conjectures that Empedocles and his school are the persons meant.

[88] *Inscr. Délos* 2529 Ziehen LS 91.

(f) The sacred law of Lycosura commands that one sacrifice barefoot, wear white clothing, and neither rings nor other gold ornaments.[89]

(g) In the Temple of Asclepius one may not pick up anything that falls to the floor (Iam. *VP* 126), and one who undergoes the sleep cure at Pergamum may not wear ring, belt, or gold ornament.[90]

It may be that in a few cases Pythagorean ritual was adopted in late times by various cults,[91] but in general the latter are independent of Pythagoreanism and older than it. Some taboos are attested from an earlier date,[92] and a good many are widely spread folk tradition.[93] Above all, the form of such authoritatively prescribed commandments and prohibitions, which are not supposed to be understood but merely obeyed, is primeval; it is entwined in the very roots of religious ritual. In what one does and does not do is manifested the identity of the group, the membership of the members and the exclusion of outsiders. The more selective the society, the more careful are the "taboos." Fasting, abstention from particular foods, and rules of sexual behavior[94] play an important role. It is of first importance that the "wise man" —the priest, the hierophant, the shaman—who claims a special position in the social organization, gain and maintain, through a special ascetic regimen, the special powers that belong to him. To this extent, the *acusmata* also have their connection with the Pythagoras legend and its cultic bases. The saying that Pythagoras was the Hyperborean Apollo is expressly attested as an *acusma* (Iam. *VP* 140).

Pythagorean silence and secrecy should also be seen in the context of cult and ritual. To be sure, the secrecy of Pythagorean doctrine later was misused by forgers as license to "discover" more and more Pythagorean writings.[95] But the testimony of Aristotle and Aristoxenus, which proves the existence of Pythagorean *ἀπόρρητα*, cannot be ignored.[96] Aristotle's testimony has to do with the intermediate position of Pythagoras between man and god. All mysteries have secrets; the ritual is interpreted in a *ἱερὸς λόγος* which may not be disclosed to the uninitiate,[97] and the initiate also learns secret passwords, *σύμβολα*, *συνθήματα*. All kinds of societies that are bound together by cult have their esoteric aspect—even political clubs, trade guilds, and those of physicians.[98] Among the Pythagoreans the practice of remaining silent for long periods—one of the most effective means of attaining inner composure—is to some extent a continuation of the practice of shamans and yogis.[99] The *ὁμάκοος* is not supposed to speak but to hear, and as early as Isocrates and the comic poet Alexis Pythagorean silence was proverbial.[100] A five-year period of silence as a test before acceptance in membership is attested by Timaeus.[101] The practice was, also, to avoid using Pythagoras' name.[102]

Aristoxenus mentions that Epaminondas called the Pythagorean Lysis, who was his teacher, "father."[103] Ludwig Edelstein brought this into connection with the precept of the Hippocratic *Oath* that the physician must regard his teacher as his father and the teacher's sons as brothers.[104] But the same usage is especially prevalent in the mysteries;[105] whoever has led the candidate to be initiated becomes his father. The Pythagoreans, too, form a "brotherhood," in accordance

[89] *SIG*³ 999 (Delatte, *Vie* 231f); similarly in Andania (*SIG*³ 736): no shoes, no gold, linen clothing; in Ialysus (*SIG*³ 338): no shoes as in the mysteries of Demeter (Callim. *Cer.* 124.)

[90] Sokolowski, 14. Cf. also L. Deubner, *De incubatione* (Leipzig, 1900) 14ff.

[91] The prohibition of beans is found in cult regulations of the imperial period: Rhodes (Sokolowski, *supp.* 108), along with the prohibition of *ἀφροδίσια* and *καρδία*; Smyrna (Sokolowski, 84) along with the prohibition of eggs. Perhaps the Titans were somehow brought into this context, but the text cannot be restored with confidence. Probably this is a case of neo-Pythagorean influence (Nock, *HSCP* 63 [1958] 415–421); an ancient ritual can of course be understood in late times as Pythagorean.

[92] Hes. *Op.* 727 (above, n. 57); *Hymn. Hom. Cer.* 50 (above, n. 84); cf. also the *Σελλοὶ ἀνιπτόποδες χαμαιεῦναι* at Dodona, *Il.* 16.234.

[93] Cf. above, n. 43.

[94] Complete celibacy is ascribed to Pythagoras at D.L. 8.19; but this is contradicted by the traditions that include his wife and children (above, ch. II 2). The *acusma* cited in n. 42 (above) implies the desirability of marriage, as do the sayings of Theano in D.L. 8.43 (cf. Hdt. 1.8; A. Raubitschek, *RhM* 100 [1957] 139), and Iam. *VP* 132, Stob. 4.23.53. Pythagoras demands strict monogamy according to Philostr. *VA* 1.13, Iam. *VP* 48, 50, *A.P.* 5.43. Special regulations as to diet and regimen, in relation to sexual intercourse, are given by Aristox. fr. 39 W., D.L. 8.9, and Diod. 10.9.3ff. "Clinias," a character in Plut. *Quaest. conv.* 654b, is negative on the matter.

[95] Burkert, *Philologus* 1961; below, ch. III 1.

[96] Arist. fr. 192 = Iam. *VP* 31, Aristox. fr. 43 W. = D.L. 8.15: *μὴ εἶναι πρὸς πάντας πάντα ῥητά*. Further references in Zeller I 409 n. 2. On the alleged secrecy of Pythagorean mathematics, see below, ch. VI 3. The whole tradition of secrecy is rejected as a late invention by (among others) Maddalena 80f, and G. Boas, "Ancient Testimony to Secret Doctrines," *Philos. Rev.* 62 (1953) 79–92. The latter completely overlooks the attestation of Aristotle and Aristoxenus. The ritual background of Pythagorean secrecy was brought out by Zeller. loc. cit., Rey 112f, and Mondolfo in ZM 414 n. 2.

[97] *ἱερός* is used here in the sense of *οὐχ ὅσιον λέγειν*, cf. Hdt. 2.61f.

[98] Cf. Hippoc. *Jusj.* and *Lex*, Ar. *Nub.* 140ff; Gigon, *Ursprung* 130f.

[99] Cf. also O. Casel, *De philosophorum graecorum silentio mystico* (Giessen, 1919), G. Mensching, *Das heilige Schweigen* (Giessen, 1926).

[100] Isoc. *Bus.* 29, Alexis fr. 197 Kock, and Dicaearchus ap. Por. *VP* 19.

[101] *FGrHist* 566F13 = D.L. 8.10, Iam. *VP* 72, etc. The technical terms *ἐχεμυθία* and *ἐχεμυθεῖν* are found in Ath. 7.308d, Plut. *Numa* 8, *Quaest. conv.* 728e, Lucian *Gall.* 2, Iam. *VP* 94, *ἐχερημοσύνη* at Iam. *VP* 246.

[102] Iam. *VP* 53, 88, 255. The hierophant at Eleusis is not called by name (P. Foucart, *Les mystères d'Eleusis* [Paris, 1914] 173ff).

[103] Aristox. fr. 18 = Iam. *VP* 250, Diod. 10.11.2, Nep. *Epam.* 2.2, Plut. *De gen.* 583c.

[104] *CMG* I 1, p. 4.5ff; L. Edelstein, *The Hippocratic Oath* (Baltimore, 1943) 34ff.

[105] A. Dieterich, *Eine Mithrasliturgie*, 1923³, 146–149; 1 Cor. 4.15. For the adoption of Heracles and the Dioscuri in Eleusis, see Plut. *Thes.* 33. On "Chaldeans," Diod. 2.29.4.

with the ancient custom of colleagues bound together in a cult. We are even told that Parmenides adopted Zeno.[106]

That the "Pythagorean life" developed from living custom, with all its complexity and paradox, rather than from clearly articulated doctrine, can be seen very clearly in the rules about abstinence from meat.[107] The self-evident corollary of the doctrine of metempsychosis would have to be complete vegetarianism. Empedocles drew this conclusion, and according to Eudoxus, Pythagoras was not only a vegetarian but avoided any association with butchers and hunters.[108] Aristoxenus, however, asserted that he only avoided eating plow oxen and rams, but was especially fond of the meat of tender young kids, sucking pigs, and cockerels.[109] Hints of Athenian ways here are intended to lessen the absurdity of Pythagoreanism, for the benefit of the reading public. There are other remarkably persistent traditions that seem to know nothing of Pythagoras as vegetarian. The famous sacrifice of an ox to celebrate a geometrical discovery is attested by a (probably) fourth-century source,[110] and the tradition is hardly later that it was precisely Pythagoras who introduced the meat diet for athletes.[111] Nicomachus smoothed out the contradictions of the evidence by use of the reports about degrees of membership within the society: the *θεωρητικοί* practiced complete abstention, whereas the *πολιτικοὶ ἢ ἀκουσματικοί* ate meat, though "seldom."[112] But the very Pythagoreans whom we must suppose to be "acusmatics," the "Pythagorists," Diodorus of Aspendus, and the Cynic-influenced Onesicritus, emphasize the radical formulation *ἐμψύχων ἀπέχεσθαι*, while Aristoxenus seems to be thinking rather of "theoretical" Pythagoreans, who still like meat.

To judge by Aristotle's testimony, the *acusmata* did not contain any simple prohibition of the eating of meat, but various specific precepts: the heart and womb of the animal, and perhaps also other similar parts,[113] may not be eaten. Here, as also in the prohibition of killing a white cock or of eating the flesh of animals that have died a natural death,[114] it is taken for granted that other kinds of meat will be eaten. Special treatment of certain parts of the slaughtered animal was part of sacrificial ritual from very early times. Karl Meuli has shown that the horror of death and the reverence for life manifest themselves in these usages and also express themselves in an attempt at recompense, that is in the rebirth of the slain creature.[115] The heart has a special role in Greek ritual.[116] This is reflected in the Orphic myth in which Athena

[106] Apollodorus *FGrHist* 244F30; *πατήρ*, Pl. *Soph.* 241d, De Vogel 240f.

[107] Cf. Zeller I 403f, and esp. Haussleiter 97–157.

[108] Fr. 36 Gisinger = 325 Lasserre = Por. *VP* 7. (The "impurity" of hunters and butchers does not exclude the eating of meat, but is actually evidence for it; see Meuli, *Opferbr.* 228; Agatharchides *GGM* I 154, on the Troglodytes.)—Vegetarianism is characteristic of the *Πυθαγορισταί* and of Diodorus of Aspendus (below, ch. II 5). See also Onesicritus *FGrHist* 134F17 (Strabo 15, p. 716), Callim. fr. 191.61f Pfeiffer. For the story that at Delos Pythagoras only worshiped at the altar of Apollo Genetor, where bloody offerings were not made, see, e.g., Iam. *VP* 25, 35, Cic. *Nat. d.* 3.88, Clem. Al. *Strom.* 7.32, Macrob. *Sat.* 3.6. Arist. fr. 489 = D.L. 8.13 and Timaeus *FGrHist* 566F147 = Cens. 2.3 mentioned the altar, but it is not certain that they brought Pythagoras into connection with it (Delatte, *Vie* 177). Among later writers, see Ov. *Met.* 15.75ff (with reminiscence of Empedocles) and Sotion ap. Sen. *Ep.* 108.17.

[109] Frr. 25, 28, 29, D.L. 8.20, Diogenes Antonius ap. Por. *VP* 36, Iam. *VP* 150 (where cockerels are explicitly mentioned). Boyancé pointed out the connection of the sacrifice of rams and pigs with Attic ritual, *REG* 1939, 40ff. On the plow ox cf. Wehrli on Aristox. fr. 29a, Haussleiter 116.3-4, Meuli, *Opferbr.* 275ff. With Aristox. fr. 27 are connected D.L. 8.19 and Iam. *VP* 98 (Boyancé, loc. cit.); the former mentions woolen clothing. At Eleusis it was said to be the command of Triptolemus *γονεῖς τιμᾶν, θεοὺς καρποῖς ἀγάλλειν, ζῷα μὴ σίνεσθαι* (Por. *Abst.* 4.22 = Xenocrates fr. 98), and Aristoxenus ascribes similar regulations to the Pythagoreans (D.L. 8.23, Por. *VP* 39, Iam. *VP* 99f).

[110] The basic testimony is an epigram by a certain Apollodorus, who should perhaps be identified with the philosopher of Cyzicus (DK 74) and dated earlier than Epicurus (below, ch. III 1, n. 51). The means chosen to avoid this testimony was to assert that Pythagoras had sacrificed an ox of dough (Diogenes Antonius ap. Por. *VP* 36; Greg. Naz. *Ep.* 198, Migne 37.324, where *σταίτινος* is misread as *πήλινος*). This is obviously transferred to Pythagoras from Empedocles (Ath. 1.3e, Favorinus ap. D.L. 8.53, *Suda* s.v. Athenaios; cf. Philostr. *VA* 1.1.2; Haussleiter 162, Delatte, *Vie* 174). For similar substitute offerings, see Hdt. 2.47.

[111] Por. *Abst.* 1.26 = Heraclides fr. 40 W. (though it is impossible to determine how much of the context goes back to Heraclides), Favorinus ap. D.L. 8.12, Por. *VP* 15. In order to explain away this evidence, some postulated a different Pythagoras: D.L. 8.13, 8.46 (here he is a Phliasian!), Plin. *HN* 23.121, Iam. *VP* 25 ("son of Eratocles"): *τούτου δὴ καὶ τὰ ἀλειπτικὰ συγγράματα φέρεται, οὐ καλῶς εἰς Πυθαγόραν τὸν Μνημάρχου τούτων ἀναφερομένων*. There was in circulation, then, a coach's handbook under the name of Pythagoras. The tradition must have been formed before Pythagorean vegetarianism was firmly established. Scholars have referred, in discussing this matter, to the famous athlete Milo of Croton, who was regarded as a Pythagorean (Haussleiter 124f; as Pythagorean, Iam. *VP* 104, Aristox, fr. 18 = Iam. *VP* 249, Strabo 6.263; as a heavy eater, Ath. 10.412ef). Iccus of Tarentum was a famous trainer (a Pythagorean according to Iam. *VP* p. 144.6; DK 25; Wuilleumier 566). The boundary between the "Pythagorean life" and rational diet fluctuates (cf. ch. III 3).

[112] Iam. *VP* 107f, 150. (The allusion in 108 to legends about Pythagoras persuading wild animals not to eat meat points to Nicomachus as the source of the passage; above, ch. II 3, Rohde, *Q* 143f.) The differentiation according to classes of membership is accepted by Haussleiter 119, Vlastos, *Philos Q.* 1952, 110 n. 62, von Fritz, *SBMü* 1960, 13. Eubulus, *Περὶ τοῦ Μίθρα ἱστορία* (ap. Por. *Abst.* 4.16) distinguishes three classes among the Magi; cf. below, ch. II 5.

[113] Arist. fr. 194 (above, n. 69). Add *ἐγκέφαλον μὴ ἐσθίειν* Iam. *VP* 109, Iam. *Protr.* 123.14, Plut. *Quaest. conv.* 635c. There is a different list in Por. *VP* 43.

[114] *Θνησείδια*: Ael. 4.17, *Hypomn.* 33; cf. above, n. 13.

[115] Meuli, *Opferbr.*, esp. pp. 185ff.

[116] It was cut out of the living animal and laid on the altar still beating (Galen *Plac. Hipp. et Plat.* V 238 K.; *Etym. magn. καρδιωσάμενοι*; *Suda*, s.v. *καρδιωσάμενος*).

saves the heart of Dionysus Zagreus when he is torn to pieces by the Titans and hides it in a sacred chest,[117] and also in the speculations of the physical philosophers, according to which the heart is the first of the organs to grow.[118]

Animal sacrifice was the focal point of the traditional religion, that is of the official cult of the polis, and to renounce it would have been more than religious reform. It would have meant a complete overturn of traditional ways. It is interesting that this is the goal of Zarathustra's gospel.[119] As far as we can judge, the Pythagoreans sought to compromise the matter; an *acusma* asks, "What is most just?" and answers, "To sacrifice." An accommodation of the doctrine of metempsychosis and the traditional way was found, because it had to be found. "The only animals into which the souls of men do not enter are those which may, according to sacred law, be sacrificed. Therefore, those who are allowed to eat meat may eat only of those animals that may be sacrificed, but of no others."[120]

In the late compromise, it seems that the ancient cult practice is still influential. Originally, and for a long time, abstinence was only a preparation for the sacred meal, so that omophagia and vegetarianism, different as they seem, are complementary.[121] In the mysteries of Demeter and Dionysus the most important sacrificial animals are sucking pigs, cocks, and kids,[122] the very animals of whose meat, according to Aristoxenus, Pythagoras was especially fond. Perhaps, then, this statement was not manufactured out of whole cloth, but was a rationalization of ritual. One report says of the Pythagoreans, "Throughout their lives they abstained from eating meat; and when, in their own stead, they made the prime-offering of an animal to the gods, they would just taste it, so that in truth they lived untouched by such things."[123]

The famous, or notorious, taboo on beans is found in similar contexts. It is attested by Aristotle, Heraclides, Callimachus, and indirectly by Empedocles;[124] only Aristoxenus denies it.[125] This was a favorite target of those who would mock Pythagoreanism. As early as Aristotle several alternative explanations of the puzzling aspects of it were available. The interpretations that are most closely related to genuine Pythagoreanism are those which connect beans with the doctrine of metempsychosis. Along with Aristotle's report that beans are like the "gates of Hades," since they alone among plants do not have joints,[126] we have a remarkable couplet in the scholia to Homer:[127]

> *ψυχῆς αἰζηῶν βάσιν ἔμμεναι ἠδ' ἀναβαθμὸν*
> *ἐξ Ἀίδαο δόμων, ὅταν αὐγὰς εἰσανίωσιν.*

It is through bean blossoms that souls return to earth for their reincarnation. Varro says that the souls of the dead are in the beans;[128] and there are several variants of the conception that a soul emerges from a plant, or a bloom, to enter into a human body.[129] This makes comprehensible the legend that Pythagoreans refused to walk through a

[117] M. Tierney, "A Pythagorean Tabu," *Mél. E. Boisacq* (Brussels, 1935) 317–321, referring to Clem. Al. *Strom.* 2.17.2, 2.22.4 and Firm. Mat. *Err. prof. rel.* 6.1ff. That Pythagoreanism is here directly dependent on Orphism is possible but not definitely provable.

[118] Empedocles A 84, Plut. *Quaest. conv.* 635e; at 636d the Orphic prohibition of eggs is interpreted in the same way; it is attested for the Pythagoreans only at *Hypomn.* 33.

[119] The fight against the sacrifice of the cow, Yasna 44.20; 92; cf. above, ch. II 2, n. 16.

[120] Iam. *VP* 85, Por. *Abst.* 1.26; cf. also Boyancé, *REG* 1939, 52 n. 2. Though this interpretation may be a counsel of desperation, it need not on that account be secondary, for the awkward situation was there from the beginning. There has, however, been an interpolation in the excerpt from Aristotle; *καθήκει* is not used impersonally by Aristotle. — Another way out was to consider sacrifice as "justified execution" (Por. ap. Stob. 1.49.59; cf. Pl. *Leg.* 870e).

[121] Cf. Eur. fr. 472.

[122] The cock and the mysteries of Demeter: Por. *Abst.* 4.16; sucking pigs sacrificed at Eleusis: Ar. *Pax* 374, with scholia; kids: the *ἔριφος εἰς γάλα* of the Gold Plates; cf. also *GRBS* 7 (1966) 99.

[123] Por. *Abst.* 2.28, which is not from Theophrastus according to J. Bernays, *Theophrastos' Schrift über Frömmigkeit* (Berlin, 1866) 119, and W. Pötscher, *Theophrastos Περὶ εὐσεβείας* (Leiden, 1964) 176. Cf. Arist. fr. 194 (D.L. 8.19): *Ἀριστοτέλης δέ φησι καὶ μήτρας καὶ τρίγλης ἐνίοτε*. Here the word *ἐνίοτε* means "at certain times." See also Iam. *VP* 85: the *acusmata* are concerned, above all, *περί τε θυσίας καθ' ἑκάστους τοὺς καιροὺς πῶς χρὴ ποιεῖσθαι*. Cf. Burnet, *EGP* 95.

[124] Arist. fr. 195 = D.L. 8.34, Heraclides fr. 41 (surely to be attributed to the book *Περὶ τῶν Πυθαγορείων*, although Pythagoras is not named), Callim. fr. 553 Pfeiffer; also Emp. fr. 141, and the frequently cited verse *ἴσόν τοι κυάμους τε φαγεῖν κεφαλάς τε τοκήων* (references in *Orph. frag.* 291 Kern, Delatte, *Faba* 36 n. 2). The subject is fully canvassed by Boehm 14ff; R. Wünsch, *Das Frühlingsfest der Insel Malta* (Leipzig, 1902) 31–46; Haussleiter 407ff; Delatte, *Faba*; M. Marcovich, *Philologus* 108 (1964) 29–39. It is well known that the bean in question is a kind of European vetch (Vicia faba); the beans used as vegetables nowadays are of American origin.

[125] Fr. 25 W.; cf. above, ch. II 1, n. 57.

[126] Arist. fr. 195 = D.L. 8.34. On the word *ἀγόνατον* see Por. *De antr. nymph.* 19 (Delatte, *Litt.* 36ff, *Faba* 36ff; R. D. Hicks is wrong in suggesting a lacuna before *ἀγόνατον*, in his Loeb edition of D.L.).

[127] Schol. T *Il.* 13.589. no. VII Nauck.

[128] Ap. Plin. *HN* 18.118. At the Roman festival of the Lemuria, one throws beans behind him, at night, "et dicunt se Lemurios domo extra ianuam eicere" (Varro ap. Non. p. 135.15 M.; somewhat differently Ov. *Fast.* 5.435ff).

[129] Attested as early as Aeschylus, in the story of Glaucus, fr. 28f; see Paus. 9.22.7, Philostr. *VA* 1.5, Lévy, *Lég.* 178ff, Ov. *Fast.* 5.231ff, H. Usener, *Kl. Schr.* IV (Leipzig, 1913) 128ff.

bean field in bloom;[130] but it scarcely brings us closer to understanding the real reason for the importance of beans.

We dare not take too lightly the rationalizing, physiological explanation, based on the difficulty of digesting beans.[131] The seer or sage is very sensitive to small physical disturbances. It may have been seen by the ancients, even before the discovery of the chemistry of proteins, that of all vegetable diets one of beans is most like one of meat. A contributing factor in this recognition may have been that a certain amino acid, present in beans, can provoke strong allergic reactions in some persons.[132] In any case the peculiarities of beans had obviously, from early times, been exploited in cult and myth. The most interesting of the explanations of the bean taboo claim amazing similarities between bean and man: the blossom of the plant, or the bean itself, are transformed through certain procedures into human form, or into the form of parts of the human body,[133] they remind of the genitalia[134] and smell like semen. At the origin of the world, bean and man emerged from the same primeval slime.[135] And when we look for the origins of these prolific fancies, we are once more led to the mysteries. Demeter, says Pausanias,[136] gave mankind all the products of the earth, with the exception of beans; and "whoever has witnessed an initiation at Eleusis or read the so-called Orphic writings, knows what I mean." We are denied this knowledge. It may be that beans were eaten in a ritual meal, and if so, Aristoxenus could be

[130] Myllias and Timycha; Neanthes *FGrHist* 84F31 = Iam. *VP* 191.

[131] The *φυσώδες* of beans is often emphasized, as in the joke used by Heniochus, fr. 4 (Ath. 9.408a), *ἔτνος κυάμινον διότι τὴν μὲν γαστέρα φυσᾷ τὸ δὲ πῦρ οὔ* (7f). Cf. Hippoc. *Vict.* 2.45; the collection of references in Delatte, *Faba* 54ff, and Boyancé, *Muses* III.2. Delatte derives the taboo on beans entirely from their effects on digestion, but with too easy a transition from *φῦσαι* to animism. The mythical account cannot be deduced from the physiological facts.—Amphiaraus avoided beans for the sake of his art of divination (*Geop.* 2.358); cf. Hdt. 4.184, Cic. *Div.* 1.62.

[132] Capparelli I 187 n. 3, II 825ff, with references.

[133] Heraclides fr. 41 (the bean in the coffin), Diogenes Antonius (Por. *VP* 44 = Lydus *Mens.* 4.42), Hippol. *Ref.* 1.2.15 (the buried bean blossom), also in a Demotic papyrus (T. Hopfner, *Offenbarungszauber* I [Leipzig, 1921] 135).—Transformation into human blood: Lucian *V. auct.* 6. See also Marcovich, cited above, n. 124.

[134] Arist. fr. 195, Gell. 4.11.10.

[135] Por. *VP* 44, Hippol. *Ref.* 1.2.14.

[136] 1.37.4; cf. 8.15.3f, Por. *Abst.* 4.16. There is some sort of connection between this and the *ἥρως Κυαμίτης* at Athens. *Hymn. Orph.* 26 prescribes, as an offering for Ge, *θυμίαμα πᾶν σπέρμα πλὴν κυάμων καὶ ἀρωμάτων*.—Cf. also the role of beans in the Greek cult of the dead: Plut. *Quaest. Rom.* 95, Lydus *Mens.* 4.42; in Rome: Plin. *HN* 18.118. The Flamen Dialis was forbidden beans (C. Koch, *Der römische Juppiter* [Frankfurt, 1937] 34ff, 84f). On the Lemuria, above, n. 128, and Delatte, *Faba* 38ff. On the Indians, L. von Schröder, *Wiener Zs. f. Kunde d. Morgenl.* 15 (1901) 187–212; on the Germans, ibid., and M. Höfler, *ARW* 2 (1899) 109. On Hdt. 2.37, see above, ch. II 3, n. 48.

acquitted, once more, of pure invention. In any case, the Pythagorean taboos are closely connected with ritual, either taken over from it or set up in opposition to it.

Side by side with the ritual material, the *acusmata* have rules and precepts that we would like to classify as rational, in the categories of ethics[137] or physics. But the remarkable thing is the juxtaposition: the rainbow is a reflection of the sun, thunder is a noise to frighten souls in Tartarus, an earthquake is a mass meeting of the dead. Most just is sacrificing, wisest is number, strongest is intelligence, holiest a mallow leaf—taboo and proverbial wisdom jumbled together. Close examination shows an amazing, inextricable tangle of religious and rational ethics. A husband must not "persecute" his wife, for he took her under his protection at the sacred altar; one should only help his fellow to load up, never to unload, for we are in this life to be punished; good counsel is sacred—a moral commandment in religious form, like the exhortation to leave children behind for the sake of the gods. The command relating to behavior in battle, too (to fall, if one must, with wounds in the chest) probably has a religious background (see n. 49).

It is striking how constantly attention is oriented toward the world of the dead, the heroes, and the *δαίμονες*. Earthquake and thunder, the clang of bronze and the ringing in one's ear, the crumbs that fall from the table—a person is always being moved and surrounded, even physically, by "stronger" powers. In the saying that the motes in the sunbeam are "souls,"[138] this feeling is expressed in an almost disconcerting manner; wherever a sunbeam falls, it is swarming with souls. Another dictum that hints at "daemonic" forces is that attributed by Aristotle to Philolaus: *εἶναί τινας λόγους κρείττους ἡμῶν*.[139] The mallow, the "holiest" thing, is a plant of the dead,[140] the *λεωφόροι* are the roads over which the dead are conveyed to their graves[141] (and this is doubtless also the reason why one may not split wood in the road),[142] and the *τρίγλη* was sacrificed to Hecate.[143] Pythagorean silence also belongs in this context; the Greeks went past *ἡρῷα* in silence, in order not to disturb the *κρείττονες*.[144] In leaving home on a trip, one must not turn to look

[137] Boehm, nos. 61–69 ("Praecepta moralia").

[138] Arist. *De an.* 404a17.

[139] *EE* 1225a30 = DK 44B16.

[140] Nilsson, *Op.* I 336f, *GrR* I 705 n. 1.

[141] Boehm on nos. 41–42.

[142] No. 42 Boehm.

[143] Apollodorus *Περὶ θεῶν*, *FGrHist* 244F109 = Ath. 7.325a-b.

[144] Epicharm. fr. 165 Kaibel: *ἀλλὰ καὶ σιγῆν ἀγαθόν, ὅκκα παρέωντι κάρρονες*; cf. Hsch. s.v. *κρείττονες*, Schol. Ar. *Av.* 1490.

back "for the Erinyes are following."[145] The prohibition of bathing may also be related to demonology.[146] No wonder that we have anecdotes pointing up how natural it seemed to the Pythagoreans to encounter spirits![147] Aristotle is our witness that the Pythagoreans "used to express great surprise if someone said he had never seen a *daimon*."[148]

The difference of the levels on which this "thought" operated does not seem to have been recognized; and here we see the continuation of a type of thinking that had already been left behind, in principle, by Anaximander and Anaximenes. Both of these sought to explain the earthquake on the basis of physical principles,[149] and thunder as well.[150] Recognition of the nature of the rainbow goes back to Anaximenes.[151] But, whereas Anaximander asks, "What is the sun? . . . How big is it?" and answers, "A circle of fire . . . the same size as the earth,"[152] the Pythagoreans ask, "What are the Isles of the Blest?" and are satisfied with the answer, "Sun and moon."

The intimations of number theory are also part of this picture. "The wisest thing is number";[153] such a sentence can be understood without any esoteric doctrine and without any "Pythagorean mathematics." More important is the tetractys, regarded as the epitome of Pythagorean wisdom. The Pythagoreans swore by Pythagoras as by "him who brought the tetractys to our generation,"

> *οὐ μὰ τὸν ἁμετέρᾳ γενεᾷ παραδόντα τετρακτύν,*
> *παγὰν ἀενάου φύσεως ῥίζωμά τ' ἔχουσαν.*[154]

The second verse of this couplet can scarcely be older than Empedocles.[155] Possibly the first line stood by itself in the beginning; with its

[145] Above, n. 58.

[146] Abstention from baths and from sexual intercourse is important not only in cult but also in magic; see A. Abt, *Die Apologie des Apuleius von Madaura und die antike Zauberei* (Giessen, 1908) 111ff, 114 nn. 5–6; C. Bonner, "Demons of the Bath," in *Studies Presented to F. L. Griffith* (London, 1932) 203–208.

[147] Pythagoras and Calliphon: Hermippus ap. Joseph. *Ap.* 1.164; Eurytus and Philolaus: Iam. *VP* 148, 139.

[148] Fr. 193 = Apul. *De deo Socr.* 20.167.

[149] Anaximander A28, Anaximenes A7§8, A21.

[150] Anaximander A23, Anaximenes A17.

[151] A7, A18; above, n. 30.

[152] Anaximander A11, 21.

[153] Cf. above, n. 22; Aesch. *Prom.* 459, Trag. adesp. 470 N., Gorg. *Pal.* 30; also below, ch. VI 1; VI 3.

[154] References above, ch. I 3, n. 120; Delatte, *Litt.* 249ff.

[155] Kranz, *Philologus* 1938, 438; *ῥιζώματα* B6.1, *πηγή* B23.10. The general idea of *φύσις* is scarcely likely to have existed before the second half of the 5th century.

negative formulation, the oath probably applied primarily to the secrecy of Pythagorean doctrine. The *τετρακτύς*, a "tetrad" made up of unequal members, is a cryptic formula, only comprehensible to the initiated. The word inevitably reminds of *τριττύς*, the "triad" of different sacrificial animals. Is the sacrificial art of the seer, involving the shedding of blood, superseded by a "higher," bloodless secret?[156] The *acusmata* provide a hint toward an explanation: "What is the oracle of Delphi?" "The tetractys; that is, the harmony in which the Sirens sing" (Iam. *VP* 85). The later tradition is more explicit: The "tetrad" of the numbers 1, 2, 3, and 4, which add up to 10 (the "perfect triangle"), contains within itself at the same time the harmonic ratios of fourth, fifth, and octave. The Sirens produce the music of the spheres, the whole universe is harmony and number, *ἀριθμῷ δέ τε πάντ' ἐπέοικεν*.[157] The tetractys has within it the secret of the world; and in this manner we can also understand the connection with Delphi, the seat of the highest and most secret wisdom.[158] Perhaps Pythagorean speculation touched upon that focal point, or embodiment, of Delphic wisdom, the bronze tripod of Apollo. Later sources speak of its mysterious ringing,[159] which must have been "daemonic" for Pythagoreans.[160]

The meaning of the shibboleth or *symbolon* "tetractys" can only be explained in a tentative fashion. In place of that which was connected with it from the beginning, in the form of belief or experience, the later sources give us more and more rationalizations. Some way or

[156] Cf. Iam. *VP* 93, 147: Pythagoras taught Abaris to use a bloodless form of divination, with numbers.

[157] Connected in the tradition with the oath by the tetractys; above, ch. I 3, n. 126.

[158] Boyancé, *AC* 1951, 421ff, cites the theory of the Delphians according to Plut. *Quaest. conv.* 745a (cf. *De fato* 568e), which connects the three Muses with three heavenly realms. Cf. Heinze 75ff, Dörrie *Hermes* 1954, 336ff.—Delatte, *Litt.* 260f, refers to the Delphic *Κελῃδόνες*, who were brought into connection with the Sirens (Pi. *Paean* 8.71; Snell, *Hermes* 90 [1962] 4f, Ath. 7.290e, Paus. 10.5.12).

[159] The oldest testimony seems to be that of Vergil *Aen.* 3.92. Cf. K. Schwendemann, *Jb. d. Dt. Arch. Inst.* 36 (1921) 168ff. On the role of the tripod, see also P. Amandry, *La mantique apollinienne à Delphes* (Paris, 1950) 140ff, H. W. Parke and D. E. W. Wormell, *The Delphic Oracle* (Oxford, 1956) I 24ff.

[160] Eustath. p. 1067.59: *καὶ οἱ Πυθαγορικοί φασι τὸν χαλκὸν παντὶ συνηχεῖν θειοτέρῳ πνεύματι· διὸ καὶ τῷ Ἀπόλλωνι τρίπους τοιοῦτος ἀνάκειται.* Cf. above, n. 34. For the ringing of the tripods at Dodona, see Demon *FGrHist* 327F20. Hsch. s.v. *Τρίοψ· ὁ ὑπὸ τῶν Πυθαγορικῶν ἐν Δελφοῖς τρίπους* (cf. Por. *VP* 16) shows that Pythagoreans were concerned with the Delphic tripod. The statement of Aristoxenus (fr. 15) that Pythagoras got most of his doctrines from Themistocleia the Pythian priestess, could well be a rationalizing interpretation of the connection suggested by the *acusma* about the tetractys.

For speculation about the relation of the tripod to the number 3, see Plut. *De E* 387c, Lobeck 386f.

other the secret of the world is to be found in number, but there is danger in too much conjectural reconstruction. If this is a starting point for Pythagorean science, the question remains, how far Pythagoras went in this direction. The pronouncement that number is "the wisest thing" may be made on no other basis than naive wonderment at its versatile usefulness and its indefeasible correctness, with no foundation of sophisticated number theory or mathematical philosophy. It does not take more than the amusement of an idle hour to discover and establish that $1+2+3+4=10$. Numerical relationships in the cosmic order are to be found in primitive and mythical thought, and in the same realm we can find the notion of the cosmic origin and function of music, and it is not necessary to assume special astronomical knowledge. The evidence about the tetractys, about numbers, and about music, is not necessarily on a different level from that about other *acusmata*, such as that an earthquake is a rally of the dead, that the rainbow is a reflection of the sun, or that the sun and moon are the Isles of the Blest. To what extent the seeds thus planted had already been developed in the direction of rational science and knowledge, is something which the history of the exact sciences must try to clarify.

The question of the date of this Pythagorean wisdom, thus far postponed, cannot be answered in a completely satisfactory manner. The latest terminus for the form of those that are sayings or precepts without explanation is the allegorical treatment of Anaximander the younger, about 400 B.C., and for the existence of a sizable, varied collection it is the reports of Aristotle; but in the other direction there can scarcely be any limit. In reckoning probabilities one must bear in mind that in oral tradition such a loose aggregation of adages and maxims is liable to constant change; some items are lost, but others are bound to take their place, especially when there begins to be a certain competition to know "as many as possible" of them (Iam. *VP* 82), and by the rule of *αὐτὸς ἔφα* all are attributed to the Master. Still we may be confident, in the light of the zeal to maintain the doctrine of the Master himself, that at least some original matter is faithfully transmitted. It is like a gravel pile; there is no pebble of which we can say that it must be primitive rock, but any single one may be.

We can get a little beyond this non liquet by recognizing that not only the content, in certain details, but certainly the form, as a whole, is older than Pythagoras. There are taboo-precepts in all primitive

cultures; and the aphoristic formulation is attested for the Seven Sages. Individual rules,[161] and even the number symbolism[162] and the assertion about the rainbow,[163] are attested for an earlier period. And it is unthinkable that ritual prohibitions like those of beans, heart, and baths, widespread as they are, and variously modified, can have had their origin in the doctrine of the historical Pythagoras. As befits the role of hierophant which Pythagoras played according to the legend, Pythagoreanism is attached to preexisting Greek cults.

The idea occasionally expressed,[164] that the *acusmata* were simplistic back formations from a developed, scientific doctrine of Pythagoras, is thus in principle refuted. It is true that the history of human thought shows relapses from physical science to magic,[165] but vestiges of science tend to be preserved. Insofar as Pythagoreanism agrees with more ancient material in general Greek cults, it is only by petitio principii that we could imagine a double, self-cancelling development —forward to science and back again to the starting point.

The only remaining question would be, then, whether the *acusmata* can have worked their way into the tradition from outside, at a late stage in its development. It is not impossible that this may be the case with a few individual items, but for the entire collection it is utterly improbable. There must have been a point of crystallization for any possible secondary additions. In fact, even the later "mathematical" Pythagoreans conceded that the *acusmata* came from Pythagoras.[166] Thus the modern student too may, or rather must, conclude that a nucleus of the collection goes back to Pythagoras and that the doctrine of the historical Pythagoras was presented on the level of the *acusmata* and passed on in this form.

For the pronouncements of the *acusmata* fit in with what we can learn from the Pythagoras legend: the same universe of gods, daemons, and souls interpenetrates the universe of man; we hear of Pythagoras as divine; we are reminded of metempsychosis; and all of this has its source in a "wisdom" that comprehends equally and without

[161] Above, nn. 57, 84.

[162] Below, ch. VI 3.

[163] Providing that Anaximenes (A7, 18) is earlier than Pythagoras, as in the generally accepted chronology.

[164] Krische 33ff, and more recently Maddalena 362 (above, n. 82).

[165] For example, astrology can be interpreted as a retrogressive development of Hellenistic science (see Nilsson II² 268). But it preserved, from its scientific background, the order of the planets, the spherical shape of the earth, and advanced methods of calculation.

[166] Below, ch. II 5; *ἀκάλει* (sc. Pythagoras), Arist. fr. 196.

differentiation the divine and the earthly, the rational and the religious—the lore of one who "knows more" than ordinary men.

Even if Pythagoras was adapting older material, there was naturally, in the process of choice, a certain λόγος, a kind of "reflection;" but this does not imply a rationally constructed system. Decisions may be taken in quite different strata of the psyche, as to what is appealing, or illuminating, or obvious. It is questionable to what extent one is justified in speaking of a "purification" or a "spiritualization" of religion.[167] There is, in the form and function of the *acusmata*, a forward step; but this is on a different level.

The ritual prescriptions which made their way into the *acusmata* serve for unusual occasions. Their demands apply during the initiation ceremony, or incubation, or a journey to the underworld, or for the period of convalescence; afterward, one can live as he did before. The rhythmic alternation of holiday and nonholiday, of ἱεραί and ὅσιαι ἡμέραι, is a mark of all primitive, naive religion. In such religion opposites dwell peaceably side by side—εὐφημία and αἰσχρολογία, chastity and license, earnest and jest, each in its proper time. The "sacred" animal is sacred just because one day it will be slaughtered and eaten. Vegetarianism and omophagia are not mutually exclusive. These rhythms respond to basic needs of the human psyche, and of society. But, when reason begins to reflect on man's beliefs and behavior and tries to remove discrepancies and to arrive at general, objective theses about gods and their laws, the ancient way of living must be reformed; whatever gods prescribe must be valid at any time.

If Pythagoras himself was a kind of hierophant, he found no successor;[168] the Pythagoreans were left with their *acusmata* applying no longer to festivals but to normal life, which, as a consequence, seemed to others abnormal. Prohibitions like those of beans, heart, certain fishes, and baths are now absolute and must be observed at all times; and the Pythagorean always wears white clothing. He lives every day of his life as though he were preparing for initiation at Eleusis, for incubation at Asclepius' temple, or for the journey to Trophonius. He follows not the cult rules of a certain holy site, but those of a βίος

[167] Zeller I 411: "(We shall scarcely go wrong in believing) . . . that he transformed the Dionysiac mysteries in the spirit of a more advanced moral teaching, and made them into an auxiliary of it." Perhaps the *acusma* about salt can be interpreted in this way (above, n. 52).

[168] Later tradition constructed a list of "scholarchs," but there is not even agreement among the sources on the immediate successors of Pythagoras. Cf. ch. II 2, n. 33.

which he has personally and consciously chosen.[169] To be sure, as is clear from the example of vegetarianism, an open breach with ritual piety is avoided.

As metempsychosis changed from ritual and myth to a doctrine with a claim to truth, so here, ritual bound to certain conditions changed into unconditional, permanent rules of life. In both cases Orphism, or the Ὀρφικὸς βίος, bears an embarrassing resemblance to Pythagoreanism. In Orphism, however, according to the testimony of Plato, the older practice of individual, magic rites did not die out; he makes the complaint against the Orphic ἀγύρται καὶ μάντεις that they promised individuals and whole cities expiation for their sins, at the cost of a little sacrifice and a pleasant dinner.[170] One can make use of the Ὀρφικά without being an Orphic, but he who follows Pythagoras becomes a Πυθαγόρειος.

To take the *acusmata* seriously means an almost frightening constriction of one's freedom of action in daily life. Whether a Pythagorean gets up or goes to bed, puts on his shoes or cuts his nails, stirs the fire, puts on the pot, or eats, he always has a commandment to heed. He is always on trial and always in danger of doing something wrong. No more carefree irresponsibility! Everything he does is done consciously, almost anxiously. The mythical expression of this attitude to life is a world full of souls and daemons, which affect every moment of a person's life. Everywhere are rules, regulations, and an ascetic zeal for discipline; life is πόνος, which must be endured.

In his discussion of Greek shamanism, Dodds uses the word "puritanism,"[171] and by it he means the strongly felt tension between bodily needs and those of the soul, which is to be freed from the body. If our analysis is correct, the comparison with historical Puritanism can be seen in a still broader perspective. The Puritan and Pietist movements emerged as reactions against a Christianity relying on "primitive" religious forms, in which ritual and daily life, Church and world, holiday and workday ran along parallel. Their aim was to make the whole of life a service of God; every day was to be lived like Good Friday. The dangerous area of arbitrary human choice and of

[169] Cf. Pl. *Rep.* 600b. If Iam. *VP* 96–100 is mainly from Aristoxenus, it provides a very early report of a communal society of the monastic type.

[170] *Rep.* 364b–e.

[171] *Irr.* 139(ff). A comparison has also been drawn, occasionally, between the Pythagorean life and that of Calvin's Geneva.—On "täglicher Gottesdienst" see Nilsson II² 381f. H. Gomperz (*PhSt* 50f) has good remarks on the "acusmatic" life.

carefree joy in living was narrowed as much as possible. The ecclesiastical hierarchy was replaced by the democratic conventicle.

Thus the *acusmata* go beyond ritual piety, not in their content but in the way in which they regulate man's life, and foreshadow a later development in Greek ethics, much in the way that the doctrine of metempsychosis foreshadows a later ontology. The significance of Pythagoras is to be sought not in the realm of philosophy proper but in the approaches to it, in his position as an intermediary between old and new.

5. *ACUSMATICI* AND *MATHEMATICI*

The tradition has contradictory reports about sects, hierarchical gradations, and schisms among the Pythagoreans. Very often a distinction is made between a lower and a higher degree of Pythagorean wisdom, and this goes back at least as far as Timaeus.[1] He tells of a five-year period of probation, during which the new disciple must listen in silence, and did not even see Pythagoras face to face; the voice of the sage came from behind a curtain (*σινδών*). Only after this period did the pupil become *ἐσωτερικός*. The distinction of *Πυθαγορισταί* and *Πυθαγόρειοι* is joined to that of "exoteric" and "esoteric,"[2] and there is also a distinction between *ἀκουσματικοί* and *μαθηματικοί*.[3] The *mathematici* are the "genuine," or truly "philosophizing" members, whose goal is *ἀκρίβεια*, so that they correspond to the "esoteric" Pythagoreans.[4] The position of the *πολιτικοί* is confused; sometimes they are reckoned among the *acusmatici*, sometimes among those of the highest grade.[5] There are some apparently artificial divisions into

[1] *FGrHist* 556F13 = D.L. 8.10 is very like Iam. *VP* 72(ff), so that the latter's exposition as a whole must be based ultimately on Timaeus (Rostagni, *ScrMin* II 1.21ff, von Fritz, *Pol.* 39, parallel V). See also the collection of references in Zeller I 400 n. 4, and von Fritz, *SBMü* 1960.—The king of Persia eats behind a curtain, according to Ath. 4.145b–c.

[2] Hippol. *Ref.* 1.2.4 (*τελεώτερα . . . μετριώτερα μαθήματα*); Iam. *VP* 80 (*Πυθαγόρειοι* as *γνήσιοι*, like *'Αττικοί* distinguished from *'Αττικισταί*—an interesting point chronologically). Iam. *Comm. math. sc.* 24 pp. 74.15–75.5 (*οἱ ἔξω . . . οἱ ὀνομασθέντες Πυθαγόρειοι*) See also Schol. Theocr. 14.5 on the difference between respectably living *Πυθαγόρειοι* and beggar *Πυθαγορισταί*.

[3] Por. *VP* 37 (characteristic of the *mathematici* are *ἀκρίβεια*, *περιττότερος λόγος*); Clem. Al. *Strom.* 5.59 (*γνησίως*); Iam. *VP* 29 (*φιλοσοφοῦντες* distinguished from the *πολλοί*; this digression, which breaks into the continuity of the sentence, is athetized by Deubner, *SBBln* 1935, 663f; but Iamblichus is capable of such a thing).

[4] As also in Iamblichus' superficial summary, §89.

[5] Iam. *VP* 150: *ἀκουσματικοὶ καὶ πολιτικοί* distinguished from *θεωρητικοὶ καὶ φιλόσοφοι*. Varro, however, ap. August. *Ord.* 2.20 presents the *πολιτικοί* as the highest group; cf. Iam *VP* 72, 108, 129. The demands of practical life, shown in section 88, are incompatible with Plato's idealistic demands.

three groups, which seem likely to be secondary.[6] The main point is always the existence of differences of rank or grade, but they are sometimes seen as steps in a continuous development and sometimes as names of separate groups. In this context the word *ἀκουσματικοί* remains a puzzle. If the reference is simply to the probationary period of "hearing," one would expect *ἀκουστικοί* or *ἀκροατικοί*.[7] What *acusmata* meant to the Pythagoreans we are not told in this connection.

Quite different is a notice that Iamblichus copied twice from the same source, not without introducing a serious contradiction on one point. There were, he says, two kinds of Pythagorean philosophy, as there were also two kinds of Pythagoreans, *ἀκουσματικοί* and *μαθηματικοί*. In what follows, the text of the *De communi mathematica scientia* has preserved the original version:[8]

[6] *ἀκουστικοί, μαθηματικοί, φυσικοί*: Taurus ap. Gell. 1.9; epigram of Socrates (cf. D.L. 2.47, Carcopino, *Bas.* 254.2–3: conjectural date, 1st century A.D.), *A. P.* 14.1; cf. the Stoic division of philosophy into the three parts logic, physics, and ethics.—*σεβαστικοί, πολιτικοί, μαθηματικοί*: Anon. Phot. 438b19ff, Schol. Theocr. 14.5 (*σεβαστικοί* are defined as *οἱ περὶ τὴν θεωρίαν*).–*Πυθαγορικοί–Πυθαγόρειοι–Πυθαγορισταί* as immediate pupils–pupils of pupils–*ἔξωθεν ζηλωταί*: Anon. Phot. 438b23ff, Schol. Theocr. 14.5.

[7] *ἀκουστικοί*, above, n. 6. *ἀκροατικοί* = *ἐσωτερικοί* used of the pupils of Aristotle, Gell. 20.5 (on this tradition about the Peripatos see Lucian *V. auct.* 26, Plut. *Alex.* 7, G. Boas, *Philos. Rev.* 62 [1953] 79ff).

[8] Iam. *VP* 81, 87–89. (The account of the *acusmata*, which comes mainly from Aristotle, comes between these two passages, from 82–87, i.e. pp. 47.4–51.12 Deubner. On this, see above, ch. II 4. How mechanically the insertion is made is shown by the word *τούτους*, p. 51.13, which has as antecedent the *ἀκουσματικοί* named at pp. 46.25 and 47.1. The scissors-and-paste procedure is surely Iamblichus' own.) This passage is nearly the same as *Comm. math. sc.* 25, pp. 76.16–78.8. The latter passage was written later, and has the error *ἐξαγώνων* for *πενταγώνων* at p. 77.20. It was not, however, copied from the *Vita pythagorica*, for it continues without a break, whereas the *VP* shows arbitrary alterations. To set the crucial sections parallel to each other:

Iam. *VP* 81 p. 46.26ff:

τουτωνὶ δὲ οἱ μὲν μαθηματικοὶ ὠμολογοῦντο Πυθαγόρειοι εἶναι ὑπὸ τῶν ἑτέρων, τοὺς δὲ ἀκουσματικοὺς οὔτοι οὐχ ὡμολόγουν, οὔτε τὴν πραγματείαν αὐτῶν εἶναι Πυθαγόρου, ἀλλ' 'Ιππάσου· τὸν δὲ 'Ιππασον οἱ μὲν Κροτωνιάτην φασιν, οἱ δὲ Μεταποντῖνον . . .

Iam. *VP* 87 p. 51.12ff:

οἱ δὲ περὶ τὰ μαθήματα τῶν Πυθαγορείων τούτους τε ὁμολογοῦσιν εἶναι Πυθαγορείους, καὶ αὐτοί φασιν ἔτι μᾶλλον, καὶ ἃ λέγουσιν αὐτοί, ἀληθῆ εἶναι . . .

Comm. math. sc. p. 76.19ff:

τούτων δὲ οἱ μὲν ἀκουσματικοὶ ὠμολογοῦντο Πυθαγόρειοι εἶναι ὑπὸ τῶν ἑτέρων, τοὺς δὲ μαθηματικοὺς οὗτοι οὐχ ὡμολόγουν, οὔτε τὴν πραγματείαν αὐτῶν εἶναι Πυθαγόρου, ἀλλὰ 'Ιππάσου· τὸν δ' 'Ιππασον οἱ μὲν Κροτωνιάτην φασίν, οἱ δὲ Μεταποντῖον. οἱ δὲ περὶ τὰ μαθήματα τῶν Πυθαγορείων τούτους τε ὁμολογοῦσιν εἶναι Πυθαγορείους, καὶ αὐτοί φασιν ἔτι μᾶλλον, καὶ ἃ λέγουσιν αὐτοὶ ἀληθῆ εἶναι . . .

Nothing can be changed in *VP* 87, because in the following passage the interpretation of the *mathematici* is given. Nauck and Hölk (5) proposed simply emending 81 after the *Comm. math. sc.* passage, but this is impossible. See below, n. 9 (DK I 107 n.; Deubner, *SBBln* 1935, 620): Iamblichus' source is correctly reproduced in *Comm. math. sc.* and this is the only one with historical value. The *acusmaticus* Hippasus is a conjecture of Iamblichus. (See further Delatte, *Litt.* 272f; Frank, *Logos* 9 [1920–1921] 246 n. 1; Rey 228 n. 1; von Fritz, *SBMü* 1960, 21: Timpanaro Cardini has it wrong, 80.) The text used in DK should therefore be that of the *Comm. math. sc.*

> Of these, the *acusmatici* are recognized by the others as Pythagoreans, but they do not recognize the *mathematici*, saying that their philosophic activity stems not from Pythagoras but from Hippasus . . . But those of the Pythagoreans whose concern is with the *μαθήματα* recognize that the others are Pythagoreans, and say that they themselves are even more so, and that what they say is true.

According to this, the *acusmatici* are incontestably Pythagoreans, who refuse to recognize the *mathematici*, who, they say, are really followers of the innovator Hippasus. On the other hand, the *mathematici* maintain that they are the true successors of Pythagoras, "even more" than the *acusmatici*, and that the alleged innovation of Hippasus was nothing more than a plagiarism of doctrine taught much earlier by Pythagoras; "everything is due to 'that man'"—Pythagoras.

In his *De vita Pythagorica*, Iamblichus seeks to maintain just the opposite of this: that the *mathematici* are the uncontested Pythagoreans, who refuse to recognize the *acusmatici* on the ground that they are a sect founded by Hippasus. Thus Hippasus is an *acusmaticus* in this version, and in the other a *mathematicus*. But this is not maintained consistently in the *Vita Pythagorica*; after a digression on the *acusmata* follows the other version, agreeing with *De communi mathematica scientia*. The latter, which is consistent and complete in itself, is thus shown to be primary. The cause of the discrepancy is not a slip of the pen, for in two other passages Iamblichus calls Hippasus an *acusmaticus*.[9] Actually, the account designed for the *Vita Pythagorica* is the only one conceivable to Iamblichus. For him the *μαθήματα*, the *Wissenschaft*, the incontrovertible proofs—so incontrovertible that it is not worthwhile to think them through again—[10] belong irrevocably to the doctrine of Pythagoras. It seemed to him unthinkable that anyone could contest this, to say nothing of these doubters being acknowledged by their opponents as genuine Pythagoreans. Iamblichus knows the tradition that made the *acusmatici* the lower class, the "spurious," the "many" who are not true philosophers. Here he can only believe that his eyes have deceived him, and quickly switch the two nouns.[11] We have here, then, an arbitrary alteration, whose motive is transparent; but it is not maintained consistently, and the result is confusion.

The only version usable in an attempt to reconstruct the history of early Pythagoreanism is that of the *De communi mathematica scientia*. It presents us with two rival groups of Pythagoreans: the *acusmatici* get along without proofs—and in general without the branches of learning that fall into the special category of *μαθήματα*, namely arithmetic, geometry, astronomy, and music (as we may discover from the rest of the tradition), whereas it is in these that the *mathematici* see the true, more profound wisdom of Pythagoras. What is left to the *acusmatici* is a collection of *dicta* without explanation—the *acusmata*, and this is surely the source of their name. Iamblichus is doubtless justified by the content, though it makes awkward reading, in inserting here the section on the *acusmata*. The mathematical and scientific aspect is taken from Pythagoras and attributed to Hippasus, as a later development which turned the school tradition aside from its original course. The *mathematici* have a different explanation:

> Pythagoras, they say, came from Ionia and Samos at the time of the tyranny of Polycrates, when the civilization of Italy was flourishing, and the first men in the cities became his trusted associates. The older of these he addressed in simple style, since they had little leisure, being occupied with political affairs, and he saw that it was difficult to speak to them in terms of *μαθήματα* and proofs. He thought they would be better off for knowing how to act, even without knowing the reasons, just as persons under medical care get well even though they are not told the reason for every detail of their treatment. The younger men, however, who had time to put in the effort of learning, he addressed with proofs and *μαθήματα*. They themselves, then, the *mathematici*, are the successors of the latter group, and the *acusmatici* of the former.

Hippasus, they said, only published, for his own aggrandizement, things that Pythagoras had taught long before.

When we approach the problem of the source, and therefore also the value, of this report, we are struck immediately with the impartiality with which these two contradictory versions are presented; the author does not start with a preconceived answer. In each of the other reports of lower and higher levels of Pythagorean wisdom, only one side is given; they follow the version of the *mathematici*, who distinguish themselves, as "more genuine" Pythagoreans, from others who

[9] Iam. *In Nic.* 10.20 (DK 18.11), Iam. *De anima* ap. Stob. 1.49.32. Syrianus derives his material from Iamblichus (*Met.* 123.7ff, 142.21ff, with the same apocryphal citation of Hippasus). Thus Iamblichus is the only source for the tradition of Hippasus as an *acusmaticus*.

[10] Cf. Iam. *VP* 157: *τὰ γραφέντα ὑπὸ τῶν Πυθαγορείων ὑπομνήματα, περὶ πάντων ἔχοντα τὴν ἀλήθειαν . . . καὶ πραγμάτων ἐναργῶν καὶ ἀναμφιλέκτων ὡς ὅτι μάλιστα μεστὰ μετὰ ἀποδείξεως ἐπιστημονικῆς καὶ πλήρους, τὸ λεγόμενον, συλλογισμοῦ . . .*

[11] At Iam. *VP* 87f, in the version of the *mathematici*, this exchange produces an impossible result, and Iamblichus should have dropped the whole text. But it suited his predilections so well (cf. *VP* 80, 90ff) that he could not let it go.

profess a simpler form of Pythagorean doctrine.[12] From the time of Timaeus, the account of the *mathematici* holds the field, and all that remains of the other side is the unexplained term *acusmatici*. Iamblichus' account of the schism, which gives the other side a chance to be heard along with, and even before the *mathematici*, cannot possibly be derived from the one-sided version that prevailed from Timaeus' day,[13] and therefore must be, in essentials, earlier than Timaeus.

Now the term *acusmata* is only found in the exposition of Iamblichus, which goes back to Aristotle, while the non-Aristotelian tradition uses the word *symbola*. This suggests that the report of the schism also stems from Aristotle,[14] and that Iamblichus has, in a way quite consistent with his usual compilatory method of writing, artificially rearranged two excerpts that come, ultimately, from the same source. This conjecture is strongly corroborated by the introductory sentence in *De communi mathematica scientia*: δύο δ' ἐστὶ τῆς Ἰταλικῆς φιλοσοφίας εἴδη καλουμένης δὲ Πυθαγορικῆς (p. 76.16f). This corresponds very closely, without being a direct copy, to expressions in Aristotle,[15] though elsewhere, understandably enough, Iamblichus does not use the expression καλούμενοι Πυθαγόρειοι. In addition, the language of the passage that follows in *De communi mathematica scientia* shows close kinship, without being a quotation, with expressions in the *Metaphysics*.[16] Thus we have a piece of evidence traceable to Aristotle, which in its very wording has preserved marks of its origin.[17]

Iamblichus' account of the division among the Pythagoreans, unique in content, takes on tremendous significance because of the

[12] Notice the expression ἔτι μᾶλλον and the emphasis on ἀποδείξεις at Iam. *VP* 87, taken along with the expressions collected in nn. 2–6 above.

[13] This is perceived by Hölk 5f.

[14] This solution is favored by Delatte, *Litt.* 273(ff), though he too readily takes the passage on the *acusmata* as a unity with the surrounding material about the division of the society (cf. ch. II 4). Bertermann assigned the whole to Androcydes, because in his superficial way he failed completely to notice the difference between the conception of the *acusmata* held by Aristotle from that of Androcydes.

[15] *Mete.* 342b30: τῶν Ἰταλικῶν τινες καὶ καλουμένων Πυθαγορείων; *Cael.* 293a20: οἱ περὶ τὴν Ἰταλίαν, καλούμενοι δὲ Πυθαγόρειοι. The adjective Πυθαγορικός is found at *De an.* 407b22.

[16] Above, ch. I 2, n. 112.

[17] It is hard to say whether some material from other sources may be interspersed with the Aristotelian. It is perhaps doubtful whether Iam. *VP* 86f (pp. 50.18–51.6, and 51.7–12) or *VP* 89 = *Comm. math. sc.* pp. 77.24–78.5 (discussed below in ch. VI 3) come from Aristotle. The concluding sentence, *Comm. math. sc.* p. 78.6, is Aristotelian in its phrasing (περὶ μὲν οὖν τῆς διαφορᾶς ἑκατέρας τῆς πραγματείας καὶ περὶ τῶν μαθημάτων σχεδὸν ταῦτά τε καὶ τοιαῦτά ἐστι τὰ συμβεβηκότα).

authority of its source.[18] In fact, the modern controversies over Pythagoras and Pythagoreanism are basically nothing more than the continuation of the ancient quarrel between *acusmatici* and *mathematici*. Is there nothing more in the doctrine of Pythagoras than what is indicated by the *acusmata*, with which the Pythagoras legend and the theory of metempsychosis are of course closely connected? Or was there from the beginning, behind these religious and mythical features, whose existence cannot be denied by the modern scholar any more than it could by the *mathematici*, a new, scientific approach to philosophy, mathematics, and the study of the world's nature?

Before pronouncing on this question,[19] we must discuss the report in the light of other testimony from fourth-century sources, so as to confirm its Aristotelian origin and at the same time get from reports to historical facts. Aristotle recognizes among the Pythagoreans a twofold πραγματεία: on the one hand Πυθαγορικοὶ μῦθοι, metempsychosis, the Pythagoras legend, and the *acusmata*, and on the other a philosophy of number connected with mathematics, astronomy, and music, which he never tries to trace back to Pythagoras himself and whose chronology he leaves in abeyance. In this he dissociates himself from the Platonists, who attribute to Pythagoras himself a more sophisticated version of the Pythagorean number theory.[20] Plato and his pupils thus stand in the tradition of the *mathematici*, and it is not surprising that their version carried the day. The doxographical tradition, too, was dominated by the Platonic interpretation of Pythagoreanism.

[18] The account of the division into groups is taken seriously by Rohde, Q 107ff; Hölk 5f; Delatte, *Litt.* 29ff, 271ff, *Pol.* 25ff; Jaeger, *SBBln* 1928, 416 = "On the Origin and Development of the Philosophic Ideal of Life," tr. R. Robinson, in *Aristotle*, 2nd Eng. ed. (Oxford, 1948), 455ff; Frank 69ff; Burnet, *EGP* 94; Minar 34f; Guthrie I 192f; and others. On the other hand, Zeller (I 415 n. 1) rejected the whole tradition as a late construct. It would not be unthinkable that Aristotle, having observed differences and worked out the idea of a split in a theoretical way, then came to think of this as a historical reality; but the explicit statements as to the extent to which each group recognized the other cannot be accounted for in this way.

[19] Frank (71) finds the exposition of the *mathematici* simply *unsinnig*. Speaking for the *acusmatici* as the original group are Rohde, Q 102ff, Hölk 4, (with reserve) Delatte, *Litt.* 311f; on the other side, among others, is Jaeger (*Paideia* I 221 = Eng. ed. I 162). Even von Fritz (*SBMü* 1960, 14, 18ff) is overborne by the weight of the later tradition and seeks to weaken the force of the report. The *acusmatici*, he thinks, were only concerned to reject certain developments of doctrine, whereas the *mathematici* wished to emphasize the "truth" of their findings and did not attribute these to Pythagoras. But the words of the text are unambiguous: εἶναι δὲ πάντα ἐκείνου τοῦ ἀνδρός (Iam. *VP* 88). Guthrie seems to forget the claim of the *acusmatici* (I 192).

[20] Cf. above, ch. I 2, 3, and 4.

We can identify certain Pythagoreans who may be classed among the *mathematici*: first of all Archytas with his pupils and also some of his predecessors, among whom he himself mentions the name of Eurytus; doubtless also Simmias and Cebes, the "auditors" of Philolaus; and finally Echecrates of Phlius,[21] with his friends. It is this same circle to whom Aristoxenus refers:

> For the last of the Pythagoreans, whom Aristoxenus himself saw, were Xenophilus of Chalcidice in Thrace, Phanton of Phlius, and Echecrates, Diocles, and Polymnastus, also Phliasians. They were pupils of Philolaus and Eurytus of Tarentum.[22]

Aristoxenus named Archytas as a friend of his father Spintharus.[23]

Whatever its basis may have been, Aristoxenus' assertion that Pythagoras was especially fond of beans obviously has a polemical point; and it is now clear that it is aimed at *acusmatici*. Equally clearly, his statements about Pythagoras eating meat are to be explained along the same lines;[24] and perhaps the main purpose of the *Πυθαγορικαὶ ἀποφάσεις* was to drive out of currency the enigmatic or ridiculous *acusmata*.

When we start looking for *acusmatici*, we think first of the *Πυθαγορισταί*[25] who appear in leading or secondary roles in the Middle Comedy. It is repeated again and again that they eat *οὐδὲν ἔμψυχον*,[26] not even the meat of sacrificial animals,[27] only a lot of wretched vegetables.[28] Sometimes they scarcely eat anything at all;[29] they drink plain water,[30] attract attention with their silence and their *στυγνότης*,[31] they wear a ragged *τρίβων*,[32] go about barefoot,[33] and are stiff with dirt since it is against their principles to bathe.[34] Shabbiness turns into arrogance; they are typical *ἀλαζόνες*.[35]

The comic poets mock these mendicant Pythagoreans as living so ascetically because this was the best they could do;[36] but a fragment of Aristophon reveals another motive: "He said that he had gone down to visit those below in their daily life, and he had seen all of them, and that the *Πυθαγορισταί* had far the best lot among the dead. For Pluto dined with them alone, because of their piety."[37] The interlocutor then remarks that Pluto must be a very easy-going god, to associate with such dirty ragamuffins; but in spite of the element of caricature, the goal of Pythagorean asceticism is clearly apparent: a favored life in the next world and intimate acquaintance with the gods.[38] The reference to the "thrice blessed" Pythagoras, in Antiphanes, shows that the "Pythagorists" were also concerned with metempsychosis.[39]

These Pythagoreans are placed in the south Italian scene by the very title, *Tarentini*. In a play of Alexis, who himself came from Thurii, the happy-go-lucky life of Athens is described to one of the characters; obviously the play contrasted the sobersides Pythagorean and the easy-going Athenian.[40] At the time of the play's performance—

[21] Above, ch. I 4, n. 40. A woman from Phlius belonged to the Academy (D.L. 4.2).

[22] Aristox. fr. 19 = D.L. 8.46; the same names are given in fr. 18 = Iam. *VP* 251.

[23] Above, ch. II 1, n. 50.

[24] Above, ch. II 4. Aristoxenus gives the same historical account as the *mathematici*: Pythagoras comes to Italy in the time of Polycrates (fr. 16), and acquires influence over the political leaders of the city (frr. 43, 17, 18).

[25] Cf. Zeller III 2.93f, Weiher 55–68, Méautis 10ff. The most important texts, from D.L. 8.37f and Ath. 4.160f, are printed in DK 58E. The fragments in question are the following (Kock's numbers, which are followed by Edmonds): Antiphanes *Corycus* fr. 135 = Ath. 4.161a, *Mnemata* fr. 160 = Ath. 4.161a, *Neottis* fr. 168 = Ath. 4.108e–f (above, ch. II 3, n. 106), inc. fab. fr. 226 = Ath. 2.60d; Mnesimachus *Alcmeon* fr. 1 = D.L. 8.37; Alexis *Pythagorizusa* fr. 196–197 = Ath. 4.161c–d, fr. 198 = Ath. 3.122f, fr. 199 = Gell 4.11.8, *Tarentini* fr. 219 = Ath. 11.463d–e (see Weiher 66f), frr. 220–221 = Ath. 4.161b–c, fr. 222 = Ath. 4.134a–b; Aristophon *Pythagoristes* fr. 9 = Ath. 4.161e, fr. 10 = Ath. 6.238c–d (see Méautis 12ff), frr. 12–13 = D.L. 8.38; Cratinus the younger *Pythagorizusa* fr. 6 = D.L. 8.37, *Tarentini* fr. 7 = D.L. 8.37.—There is no way of knowing whether Naevius' *Tarentilla* belongs in this context.—Delatte conjectures (*Vie* 241) that the plays of Alexis and Cratinus entitled *Tarentini* and those entitled *Pythagorizusa* were identical, in each case, and that their authorship was contested; but here again there is no basis for a firm answer.—The difference in the appellations *Πυθαγοριστής* (Aristophon: title and frr. 9, 12), *Πυθαγορικός* (Antiphanes fr. 160), *Πυθαγόρειος* (Alexis fr. 196), and *Πυθαγορίζοντες* (Alexis fr. 220) is not a matter of technical terminology (cf. Weiher 56). The dietary regulations are in any case based on the authority of Pythagoras himself (Antiphanes fr. 168).

[26] Antiphanes fr. 135, Alexis fr. 220, Mnesimachus fr. 1, and Alexis fr. 27 (Méautis 10 n. 6.)

[27] Mnesimachus fr. 1, Alexis fr. 196.

[28] *θύμον*, Aristophon fr. 10, Antiphanes fr. 168; *ἄλιμα καὶ κακὰ τοιαῦτα*, Antiphanes fr. 160.

[29] Aristophon fr. 10, Alexis frr. 221, 197, Theocr. 14.5ff.

[30] Alexis frr. 198, 220, 221; Aristophon fr. 10.

[31] Alexis fr. 197.

[32] Aristophon frr. 9, 13.

[33] Aristophon fr. 10, Theocr. 14.6.

[34] *ἀλουσία*, Alexis fr. 197, Aristophon frr. 13, 9, 12; lice: Aristophon fr. 13.

[35] Com. adesp. fr. 275 Kock fits into this context: *οὐδ' Ἰταλιώτης οὐδ' ἀλαζὼν οὐδαμῶς*.

[36] Antiphanes fr. 226, Aristophon fr. 9.

[37] Aristophon fr. 12. Méautis (16) conjectures that the person mentioned is Pythagoras himself; but can he be supposed to meet his followers in Hades?

[38] Cf. Emp. fr. 147, Pl. *Phdr.* 247a.

[39] Antiphanes fr. 168; Aristophon fr. 10 (cf. Weiher 65, Méautis 12ff) mocks palingenesis as an *ἄλλοτε ἄλλον γίγνεσθαι* (D.L. 8.36).

[40] Alexis fr. 222.

probably in the middle third of the fourth century[41]—"Pythagorists" must have been a familiar sight in Athens.

Clearly, Aristoxenus' veiled polemic has the purpose of destroying this image of Pythagoreans as a group of low-class tatterdemalions, addicted to superstitious abstinences. There is special point in his insistence that the Pythagoreans he knew personally were "the last"; "They held to their original way of life (ἤθη), and their science . . . until, not ignobly, they died out" (fr. 18 = Iam. *VP* 251). Nothing about abstinence from beans or meat—only μαθήματα; nothing about poverty, dirt, or lice—only *noblesse*, to the very end.[42] Diodorus, in a passage certainly based on Aristoxenus, places these "last" Pythagoreans about 366/365 B.C.,[43] but the comic allusions would take us a few decades later; and there was a Pythagorean Lycon who wrote after Aristotle.[44] This Aristoxenus can ignore; such persons, he implies, are not really Pythagoreans.

All the same, it is not so easy to equate the Pythagoreans of Aristoxenus with the *mathematici* and those of comedy with the *acusmatici*. According to Aristotle's report, the *mathematici* recognize the Pythagoreanism of the *acusmatici*, whereas Aristoxenus ignores conceptions deviant from his own. The *acusmata* themselves do not enjoin complete vegetarianism, but complicated rules of dietary detail; but in the comedies absolute abstinence from meat seems to be prescribed. In addition, there is an occasional mention of rhetorical, dialectical writings, or speeches, as though they were regular Sophists.[45]

Neither Aristoxenus nor Middle Comedy provides a wholly undistorted reflection of a particular type of Pythagoreanism. The former is contradicted by Eudoxus, who, as a personal pupil of Archytas and himself one of the most brilliant Greek mathematicians, must have been as close as anyone was to the tradition of the *mathematici*, and who states that Pythagoras abstained from ἔμψυχα and avoided association with hunters and butchers.[46] He recognizes, or perhaps rather exaggerates, the abstinences. Thus Aristoxenus appears to be dissociating himself from Eudoxus and the *mathematici*, insofar as he denies the abstinences and does his best to present Pythagoras' way of life as normal.

The comedies, on the other hand, make everything cruder than it was. At least one hint—not conclusive, to be sure—suggests that more precise regulations were mentioned, in the matter of abstinence from meat.[47] The main point here is, though, that comedy is not the place for hairline distinctions. Its effect comes from mixing up the incompatible. It is tempting to conjecture that there were Pythagoreans, in the first half of the fourth century, emigrating from southern Italy to Athens,[48] although there are certain difficulties in this.[49] This much is certain, though, that Plato, who had achieved extraordinary fame inside and outside of Athens, and who was also often alluded to in the comedies,[50] traveled several times to Sicily and Magna Graecia, and had an especially close relationship with the Pythagoreans of Tarentum. Also, his disciples equated their own philosophy with that of Pythagoras and in various books presented a new picture of the Samian sage. To this extent, Pythagoras was a live topic of conversation in the Athens of the fourth century, and it seems likely that the mockery of the "Pythagorists" was aimed at the Academy as well. More precisely, when the comic poets identified the loftiest philosophical attempts of the Platonists with the common, mendicant Pythagoreanism, they were ridiculing one group as well as the other.[51]

The distinction between *acusmatici* and *mathematici*, drawn in the reports of the schism, is not directly reflected in the contradictions of

[41] On the chronology: Epigraphical evidence shows that from 377 to 351, in order, Antiphanes, Mnesimachus, Alexis, and Aristophon won victories at the Lenaea (*IG* II/III² 2325), and Alexis won at the Dionysia of 347 (*IG* II/III² 2318, XI 278; see A. Pickard-Cambridge, *The Dramatic Festivals of Athens* [Oxford, 1968²] 106, 114). The years 408/405–334/331 are given as the dates of Antiphanes. Alexis was long-lived, and seems to have been active still about 270 B.C. (fr. 244). Antiphanes' *Neottis* is to be dated shortly after 342 (above, ch. II 3, n. 106), Aristophon's *Pythagoristes* perhaps in 338 (Edmonds on fr. 11); Mnesimachus' frr. 7–8 take us to the year 345. Antiphanes' *Corycus* should perhaps be dated 359, Mnesimachus' *Alcmeon* ca. 340, Alexis' *Tarentini* between 330 and 320 (see T. B. L. Webster, *Studies in Later Greek Comedy* [Manchester, 1953] 53f).

[42] Aristoxenus' Pythagoreans bathe, too (Iam. *VP* 98).

[43] Diod. 15.76.4.

[44] Below, n. 65.

[45] Alexis fr. 221: *Πυθαγορισμοὶ καὶ λόγοι λεπτοὶ διεσμιλευμένοι τε φροντίδες . . .*, Cratinus fr. 7: the Pythagorists drive ordinary people out of their minds with their rhetorical subtleties, *τοῖς ῥητόροις, τοῖς πέρασι, τοῖς παρισώμασιν, τοῖς ἀποπλάνοις, τοῖς μεγάθεσιν νουβηνικοῖς.*

[46] Fr. 36 Gisinger = 325 Lasserre = Por. *VP* 7.

[47] Alexis fr. 199 = Gell. 4.11.8 (taken along with Aristox. fr. 25): "ac de animalibus Alexis etiam poeta in comoedia quae *Πυθαγορίζουσα* inscribitur, docet." To judge by the context, the eating of certain kinds of meat must have been mentioned as being licit.

[48] Von Fritz, *Pol.* 76f, brings the appearance of the Pythagorists into connection with the fact that about 388 Dionysius I subjugated the southern Italian alliance of Croton, Sybaris, and Caulonia, and also with the report of Aristoxenus about the emigration from Italy of all Pythagoreans except Archytas (fr. 18, Iam. *VP* 250f; the text is corrupt and cannot be restored with certainty; cf. von Fritz, *Pol.* 103f and, contra, Wehrli 53; also see Aristox. fr. 2, and von Fritz, *Gnomon* 32 [1960] 495).

[49] The Pythagorists of Alexis and Cratinus come from Tarentum, but no Pythagorean had to leave the city of Archytas. Also, the comedies are some decades later than 388.

[50] See Weiher 45–55.

[51] The plays called *Tarentini*, of Alexis and Cratinus, are connected with Plato's Sicilian journey by Edmonds, and dated about 360 (a different answer in Webster, above, n. 41). The *πανήγυρις* comparison of Alexis *Tarentini* fr. 219 could be derived from Heraclides frr. 87–88 (Burkert, *Hermes* 1960, 165 n. 3).

the pictures of Pythagoreanism drawn by Aristoxenus and by the comic poets; but it does seem to be clear that alongside of the scientific Pythagoreanism with which Plato and his pupils allied themselves, and which Aristoxenus stubbornly claimed to know best, there was another quite different type of Pythagoreanism whose adherents were mendicant "teetotalers" with special hopes about the next life.

The first evidence for this tendency is the *Telauges* of Aeschines of Sphettus. Dittmar has elicited the fundamental facts about it from the exiguous fragments:[52] Telauges was depicted as a representative of the Pythagorean life, characterized by a shabby poverty proudly displayed, decked out with a *κώδιον* and beggar's scrip.

We can get a clear picture of Diodorus of Aspendus, as a historical person of the first half of the fourth century B.C.[53] A mocking verse by the musician Stratonicus claims that with a "crazy garment of skins" and "arrogant presumption (*ὕβρις*)" he gathered followers.[54] Archestratus, in his *Ἡδυπάθεια*, recommends shark meat as a delicacy, and says that anyone who will not eat the flesh of a creature that itself eats human beings should restrict himself to vegetables, should seek out Diodorus the Sage, and live the ascetic Pythagorean life with him.[55] Here Diodorus is the well-known Pythagorean, famous for his vegetarianism. Timaeus mentions the striking clothing he wore, and Sosicrates describes it more precisely; long hair, long beard, folded *τρίβων*, wallet and staff, "though the Pythagoreans before him wore shining bright clothes, bathed and anointed themselves, and had their hair cut according to the fashion."[56] Athenaeus, without naming a source, refers to him as barefoot and dirty.

This picture of Diodorus is the conventional picture of a Cynic, so that the usual thing is to dispose of him quickly as one who managed "to represent his Cynicism as Pythagorean philosophy."[57] Or we may read that "the ascetic trend caused many Pythagoreans, after the rise of Cynicism, to assume the Cynic garb and way of life."[58] Thus Diodorus would be a Pythagorizing Cynic or a Cynicizing Pythagorean—in any case some kind of hybrid. But this ignores the chronology. The testimony of Stratonicus, who died about 350 B.C., shows that we should regard Diodorus as in any case contemporary with, but more probably, earlier than, Diogenes. According to the tradition, Diodorus' rival for the honor of having invented the Cynic costume was not Diogenes but Antisthenes.[59] If, as modern scholars rightly believe,[60] Antisthenes was not yet really a cynic, then Diodorus' garb was not Cynic either; rather, the Cynicism of Diogenes is in a way a continuation of Pythagoreanism of the "acusmatic" stripe. There are unmistakable coincidences—the praise of *πόνος* and contempt for *ἡδονή*,[61] and in general the choice of a special *βίος* in contrast with the "normal" hit-or-miss life-style of the ordinary man. Just as the Pythagorean feels himself a stranger on the earth, so the Cynic tries to free himself from all ties. Threads lead from here to the Stoa as well; both Zeno and Chrysippus were interested in aspects of Pythagoreanism.[62]

But Timaeus emphasizes that Diodorus "pretended" to have associated with Pythagoreans, and Sosicrates declares that the appearance of Diodorus was a novelty and that before him the Pythagoreans had lived a more normal life. Finally, there is a statement in Iamblichus, probably derived from Timaeus,[63] that after the catastrophe of the school Diodorus of Aspendus was taken into the Pythagorean society "because of the need for members," and that, returning to Greece, he published the "Pythagorean sayings"—obviously meaning the *acusmata*.[64] The tendency of these reports is obviously to deny real Pythagoreanism to

[52] Dittmar, *Aischines* 213ff, von Fritz, *RE* V A 194ff, *Pol.* 76 n. 30.

[53] The most important testimonia, from Stratonicus, Archestratus, Timaeus, and Sosicrates, are in Ath. 4.163c–f; cf. Tannery *MSc* VII 201–210. Stratonicus died about 350 B.C. (Wilamowitz, *Ind. Schol. Gott.* 1893/1894, 16.1, Maas, *RE* IV A 326f). Timaeus cites him (*FGrHist* 566F16). Archestratus wrote about 330 B.C. Zeller (I 426 n. 3) confused the citation of Archestratus with a nearby citation of Timon, and therefore mistakenly dated Diodorus in the 3rd century. W. Crönert, *RhM* 62 (1907) 311f (dubiously Powell, *Coll. Alex.* p. 212), suggested attributing the Stratonicus verses to Cercidas; but he could scarcely have been cited by Timaeus, to say nothing of Archestratus.

[54] Timaeus *FGrHist* 566F16 = Ath. 4.163c–f: *τῷ περὶ θηροπέπλου μανίας ὕβρεώς τε περιστάσιμον στοὰν ἔχοντι Πυθαγόρου πελάτᾳ.*

[55] Fr. 23.18ff Brandt = Ath. 4.163d–e:

ὥστε πρέπει καθαρῶς ὁπόσοι τάδε μωρολογοῦσι
τοῖς λαχάνοις προσάγειν καὶ πρὸς Διόδωρον ἰόντας
τὸν σοφὸν ἐγκρατέως μετ' ἐκείνου πυθαγορίζειν.

[56] Ath. 4.163f and D.L. 6.13. This account of Sosicrates, who seems to use Aristoxenus (ch. II 3, n. 249), is in turn the basis of Schol. Theocr. 14.5 (DK I 478.37f, above, n. 6), on the differentiation between *Πυθαγόρειοι* and *Πυθαγορισταί*. Long hair is also found in the Pythagoras legend: *ὁ ἐκ Σάμου κομήτης*, Iam. *VP* 11, with the parallel passages.

[57] Zeller I 426 (where Diodorus is wrongly dated; see above, n. 53).

[58] Ueberweg-Praechter 64. Cf. Rostagni, *ScrMin* II 1.41.

[59] D.L. 6.13 (Diocles and Neanthes name Antisthenes, Sosicrates names Diodorus of Aspendus; above, n. 56).

[60] Cf. Wilamowitz, *Platon* II 163 n. 4; J. Geffcken, *Griechische Literaturgeschichte* II (Heidelberg, 1934) 30 and nn.; E. Schwartz, *Charakterköpfe aus der Antike*[2] (Leipzig, 1943) 116ff, esp. 123f. On the history of Cynicism, see K. von Fritz, *Quellenuntersuchungen zu Leben und Philosophie des Diogenes von Sinope, Philologus* Supp. 18.2 (1926); D. R. Dudley, *A History of Cynicism* (London, 1937).

[61] Above, ch. II 4, n. 14; for the comparison of *ψυχή* and *ἁρμονία* see D.L. 6.27, 65.

[62] Zeno wrote *Πυθαγορικά* (D.L. 7.4); Chrysippus cites a Pythagorean verse (Gell. 7.2.12 = *SVF* II 294 *Carm. aur.* 54).

[63] Iam. *VP* 266; cf. Delatte, *Musée Belge* 1920, Rostagni, *ScrMin* II 1.41.

[64] *διέδωκε τὰς Πυθαγορείους φωνάς*; for *φωνή*, "saying," see Pl. *Prot.* 341b: *τὴν Σιμωνίδου φωνήν . . .*, Epicurus *Ep.* 1.36.

Diodorus, though Iamblichus admits that he had been a member of the school. But the two oldest witnesses, Stratonicus and Archestratus, call him a Pythagorean, without qualification. Sosicrates, however, is dependent on Aristoxenus (above, n. 56), and Timaeus is in the same line. Thus what first was generally regarded as Pythagorean in the fourth century is later branded as "alleged" Pythagoreanism, and contrasted with that which is "genuine."

With Diodorus doubtless belongs Lycon,[65] who criticized Aristotle's extravagant way of living. His ideal must have been something in the nature of Cynic self-sufficiency. Aristocles introduces him as one "who called himself a Pythagorean," showing the same sort of reserve about his Pythagoreanism as Timaeus did about that of Diodorus. There is no reason not to identify him with the Lycon of Iasus who wrote on the Pythagorean life, emphasizing the "moderate" regimen of Pythagoras.[66] After Lycon, who must have been approximately contemporary with Aristotle, there are no more Pythagoreans of this type to be found. An echo of such activity can still be heard in Onesicritus, who named Pythagoras as one of the Greeks who, before Socrates and Diogenes, had taught doctrines like those of the Indian Gymnosophists.[67] But Onesicritus is regarded as a Cynic. The tendency in Pythagoreanism represented by Diodorus of Aspendus was absorbed by Cynicism, which took shape as the form of the self-sufficient, world-despising *βίος* which suited the demands of the age. Meanwhile the spiritual power of Pythagoreanism found, through the interpretation of the Platonists, a new vehicle adaptable to changing times.

In the fourth century, then, alongside those Pythagoreans with

[65] Aristocles ap. Euseb. *Praep. evang.* 15.2.8 (DK 57.4; I. Düring, *Aristotle in the Ancient Biographical Tradition* [Göteborg, 1957] Test. 58 i; cf. p. 391). Zeller (I 426 n. 3) connected Lycon with Diodorus.

[66] Ath. 10.418e (DK 57.3): *Λύκων ὁ Ἰασεὺς ἐν τῷ περὶ Πυθαγορείου βίου.* Capelle doubts the identification (*RE* XIII 2308f, s.v. Lycon n. 15). Iam. *VP* 267 (the catalogue of Pythagoreans) lists a Lycon as Tarentine, D.L. 5.69 knows only one *Πυθαγορικὸς Λύκων.* This Lycon is naturally to be distinguished from the historian Lycus of Rhegium (earlier than Timaeus: *FGrHist* 570, and Jacoby in IIIb *Komm.* 597ff; also Laqueur, *RE* s.v. Lykos, no. 50)—who, considering his interest in miracles, asceticism, and the like (frr. 5, 6, 7), certainly must have mentioned Pythagoras.—Por. *VP* 5 cites *Λεῦκος ἐν τῇ τετάρτῃ τῶν ἱστοριῶν.* The emendation *Λύκος* is tempting, but the second person *σοι* is not very likely in a work of history. Jacoby classes the passage among the doubtful fragments of Lycus (*FGrHist* 560F15) and is inclined to assign it to Lycon (IIIb *Komm.* 601). The *ἱστορίαι* cannot be the same as the *περὶ Πυθαγορείου βίου*, since it only takes up the origin of Pythagoras in the fourth book. There can be no certainty about the attribution in the case of the botanical data in DK 57.5 (Lycon) and 57.2 (where the MSS have Ibycus; there were also physicians named Lycus, *RE* s.v., nos. 51 and 52).

[67] *FGrHist* 134F17 — Strabo 15.716 (Dittmar saw the relevance of this to the "Pythagorist" context, *Aischines* 217f). *ἐμψύχων ἀπέχεσθαι* appears among the rules of Aśoka: *Journ. Asiat.* 246 (1958) 1ff.

whom the Academy felt akin in their philosophical and scientific endeavors, there were others of a quite different type, exemplified by Diodorus of Aspendus but presupposed also by Aeschines and the comic poets. Their characteristic mark is not *μαθήματα* but a *βίος*. They remind us of the *acusmata* (alluded to in the tradition: above, n. 64) by their avoidance of baths and of shoes, and by their vegetarianism, though the evidence, almost entirely satirical and negative, oversimplifies and distorts the picture. Allusions in comedy permit us to attribute to them a belief in metempsychosis and hopes for a better life to come. Aristoxenus acts as though this kind of Pythagorean did not exist, though his portrait of Pythagoras is specifically intended to correct the impressions they made. From Timaeus on, these are "alleged" Pythagoreans, distinguished from "genuine" ones; and from Timaeus on the tradition becomes canonical that the real teaching of Pythagoras was esoteric, as distinguished from the imperfect, exoteric preliminary stage.

Aristoxenus is corrected by Eudoxus in one point, and in a way this corroborates Iamblichus' account of the division. This report reveals details that were obscured from the time of Aristoxenus and Timaeus, for tendentious reasons. This confirms its Aristotelian origin, and it must be taken seriously as an expression of historical facts. The *mathematici*, whose successors Plato and his disciples thought of themselves as being, tried to regain the ancient wisdom of Pythagoras by scientific studies of their own, while *acusmatici* like Diodorus of Aspendus wished only to live a straitlaced life in accordance with the ancient precepts. The *mathematici*, followed in this by Eudoxus, did not attack the ritual observances taught by Pythagoras, but the *acusmatici* saw a defection from Pythagoras in the further development of scientific study. But neither tendency could endure except in altered form: "mathematical" Pythagoreanism in the reinterpretation of the Platonists, and the "acusmatic" way of life, rationalized and secularized, in Cynicism.[68] And, since the "mathematical" tradition, in its Platonic metamorphosis, became completely dominant in the literary realm, the contention of the *mathematici* also won out, that the *acusmatici* were not genuine, but only imperfect, Pythagoreans.

[68] On the question of the continuity of the Pythagorean tradition, see Burkert, *Philologus* 1961. In the Hellenistic period, or, say, in the 3rd and 2nd centuries B.C., there do not seem to have been people calling themselves *Πυθαγόρειοι*, nor is there evidence for the survival of Pythagorean cult, *ὄργια* of the *acusmatici*, which are commonly assumed to have continued. It does seem, however, that there was a good deal of interest in Pythagoreanism: and this was part of the reason for the apocryphal literature. The *Πυθαγορικτάς* from Athens in Theocritus (14.5), barefoot, pale, and hungry, may be a figure from comedy (Wilamowitz, *Platon* II 84).

Hippasus,[69] who plays a crucial role in the versions of both groups, is mentioned in certain early reports. Aristotle and Theophrastus attribute to him, as well as to Heraclitus, the doctrine that fire is the ἀρχή. Ancient scholars drew a chronological conclusion from the fact that his name comes first, but this is hardly justified.[70] Aristoxenus credits him with an acoustic experiment and brackets him with Glaucus of Rhegium.[71] Iamblichus names him three times, along with Archytas, as discoverer of the "harmonic mean," again a connection with music theory.[72] According to the report of the schism, Hippasus claimed to have discovered the dodecahedron and was drowned at sea.[73] He wrote no books.[74] The evidence seems to point toward the first half of the fifth century—including an apocryphal report that he was the teacher of Empedocles.[75] His home city was Metapontum,[76] and there is a slight trace of a connection with Phlius.[77]

Thus Hippasus is the oldest Pythagorean we know of who worked at mathematics and music theory, and also had something to say in the realm of natural philosophy—though, to be sure, not in terms of a theory of number, or of a philosophy of "limit," "unlimited," and "harmony." The independent reports of Aristotle and Aristoxenus confirm the account of the division into sects—in the version of *De communi mathematica scientia*—according to which Hippasus was one of the *mathematici*. This report shows the position of Hippasus between the fronts which were later formed. For the *acusmatici* his activity was something new, and subversive; but the *mathematici*, in order to find a firm basis for new doctrines, were forced to abandon Hippasus and brand him a plagiarist. Since he became in this way a scapegoat for both sides, it was easy for further charges to emerge, such as that he had traduced Pythagoras.[78]

It is impossible to determine whether the breach between Hippasus and the other Pythagoreans came before or after the political catastrophe about 450 B.C.; it is tempting to suppose that there was a connection between the inner and outer crises of Pythagoreanism.[79] But it is a more important question, which of the two opposed conceptions of Pythagoras should be considered correct. The name μαθηματικοί, interesting as it is, scarcely helps;[80] we must weigh the

[69] DK 18. Cf. Zeller I 603f, Wellmann, *RE* VIII 1687f (too brief), Frank 261ff (hypercritical), Timpanaro Cardini 78ff (harmonizing).

[70] Arist. *Met.* 984a7, Theophr, *Phys. op.* fr. 1 = *Dox.* 475; other doxographical data DK 18.7–9. On Hippasus as teacher of Heraclitus, *Suda* s.v. Heraclitus (DK 18.1a). On the soul as fire, DK 18.9; Arist. *De an.* 416a9.

[71] Aristox. fr. 90 = Schol. Pl. *Phd.* 108d; on the facts, see below, ch. IV 1. In the comment on the source, μέμνηται δὲ τούτων Ἀριστόξενος . . . , DK follows Hermann in printing τούτου instead of the MS reading τούτων (Greene, *Scholia Platonica*; von Fritz, *AnnMath* 1945, 245; Wehrli), which alone gives the testimony its importance: Aristoxenus mentioned not only Glaucus but the whole incident, including Hippasus. It is uncertain how Aristoxenus dealt with the contrast between Hippasus' music theory and his own (cf. Wehrli, *Aristoxenos* p. 77).

[72] DK 18.15; below, ch. VI 2.

[73] Iam. *VP* 88, with *Comm. math. sc.* 77.18ff; cf. Iam. *VP* 247; below, ch. VI 3. The death of the offender at sea reminds of the death of Protagoras (on which see von Fritz, *RE* XXIII 910f).

[74] Demetrius of Magnesia ap. D.L. 8.84 (DK 18.1); cf. below, n. 78.

[75] The "letter of Telauges to Philolaus," cited by Neanthes (*FGrHist* 84F26 = D.L. 8.55, Thesleff, *Texts* p. 189). Even Neanthes doubts the authenticity of the letter, but in any case it was in existence by about 200 B.C.

[76] Arist. *Met.* 984a7, D.L. 8.84, etc. He is from "Croton or Metapontum" according to Iam. *VP* 81 = *Comm. math. sc.* p. 76.23; from Sybaris according to the catalogue Iam. *VP* 267. (This would date him after 444).

[77] There are two versions of a genealogy of Pythagoras which connect him with Phlius: Cleonymus of Phlius-Euthyphro-Hippasus Marmacus Pythagoras, according to D.L. 8.1; Hippasus of Phlius-Euphron Mnesarchus Pythagoras according to Paus. 2.13.2. These genealogies obviously stem from the Pythagorean group in Phlius. That the uncommon name Hippasus appears in both is hardly an accident.

[78] According to Apollonius (Iam. *VP* 257ff) Hippasus was one of those who rebelled against Pythagorean dominance. Ninon produced a λόγος ἱερός, πεπλακὼς καὶ γεγραφὼς ἐξ ὧν μάλιστα αὐτοὺς ἤμελλε διαβάλλειν (Iam. *VP* 258); and there must be some connection between this and the remark of Heraclides Lembus, in his enumeration of Pythagorean writings τὸν δὲ *Μυστικὸν λόγον Ἱππάσου . . . εἶναι, γεγραμμένον ἐπὶ διαβολῇ Πυθαγόρου*. Either Heraclides is dependent on the story which later appears in Apollonius, only that Hippasus, the apostate, is substituted for Ninon; or the narrative is late and there was an actual ἱερὸς λόγος which was regarded as overly primitive and therefore discreditable, so that it was branded a forgery and a libel and put down to the account of the wicked Hippasus. In this case, however, Hippasus would not be an *acusmaticus* (as Timpanaro Cardini 81), but a *mathematicus* who wished to conceal his plagiarism and characterize Pythagoras as a primitive—as scandalous letters of Epicurus were forged not by Epicureans but by a Stoic (D.L. 10.3). Cf. Frank 70.

[79] Tannery (*Géom.* 85f), relying on the account of Apollonius in Iam. *VP* 257ff, thinks that the internal division of the school started by Hippasus finally led to civil war and to the final catastrophe. Tannery speculates (*MSc* VII 209f, on Diodorus of Aspendus) that after the collapse of their political strength the Pythagoreans restricted themselves to their religious and superstitious activities; against this is the fact that the *mathematici* whom Plato knew were all active after the political catastrophe.

[80] Whether the word μαθηματικός "concerned with the subjects of learning," came to have its narrower sense "mathematician" precisely among the Pythagoreans (von Fritz, *SBMü* 1960, 20f) or only in the Academy (below, ch. VI 1), the history of the word cannot decide whether the *mathematici* were descended from Pythagoras or Hippasus. Perhaps the terms μαθηματικοί and ἀκουσματικοί do not go back to the original schism, but were only later applied to the rival groups. Formations in -ικός become frequent from Plato on. (A. Amman, *-ικός bei Platon*, Diss. Fribourg, 1953; the earliest occurrence of μαθηματικός is Pl. *Soph.* 219c). The primary contrast is between ἀκούσματα and μαθήματα. It was in the educational movement of the 5th century that the word μανθάνειν acquired its specialized sense. Parmenides (frr. 1.31, 8.52) and Empedocles (fr. 17.14) call for μανθάνειν, and Protagoras wrote *Περὶ τῶν μαθημάτων* (D.L. 9.55; the sense is "On the Branches of Learning," and it is a mistake to apply it to mathematics, as in DK 80B7; cf. Pl. *Prot.* 318d and 313c, *Laches* 179e, 182d). In the *Clouds* the μαθητής is an established type. In Pythagoreanism, "hearing" plays a prominent part, and this too suggests that ἄκουσμα is the older idea (above, ch. II 4, n. 76). If, as time went on, certain Pythagoreans emphasized the importance of μαθήματα as contrasted with ἀκούσματα, they were integrating themselves into the intellectual development of the 5th century: they do not want "hearsay" but the comprehension of truth.

content of the two versions. The *mathematici* claimed that their studies were no more than explication of the doctrine of Pythagoras, and that "everything came from him." This principle, which seems to have been accepted in the day of Xenocrates, emanates from a completely unhistorical point of view, close to that of myth. What seems important and desirable, takes the form of a quasi-historical assertion: in the beginning, there was Pythagoras.

On the side of the *acusmatici* is the fact that the *acusmata*, the ideas of metempsychosis, and the legend of Pythagoras are early attested and primitive in character. The *mathematici*, in their effort to get free of this annoyance, resorted to the suspicious expedient of secrecy and esoterism.

If, in spite of this, efforts continue to be made to show that the *acusmatici* were not entirely in the right, that Pythagoras was "not merely" a kind of shaman, and that science too was present in Pythagoreanism from the beginning, at least in embryonic form, at the base of these efforts lies not only the force of the dominant tradition, but also the seemingly ineluctable idea of the law of development, the postulate that the later Pythagoreanism must in some way be foreshadowed in the earlier. Yet one may well ask just how much that is coherent and stable is necessary to explain the development. The question of what is scientific depends more upon form, method, and proof than upon the content or the practical function. May it not be that the conceptual and scientific impulse simply provides a new form for an ancient and pre-scientific lore or attitude? The "wisdom" of the *acusmata*, the "wisdom" of a shaman-like "divine man" can stand without the prop of science, and did so in the activity of the *acusmatici* down into the fourth century. Greek science, including Greek mathematics, may well have another and non-Pythagorean origin.

6. EARLY EVIDENCE FOR PYTHAGORAS AS A SCIENTIST?

In discussing Pythagoras scholars have always, with varying degrees of uneasiness, taken cognizance of his doctrine of metempsychosis and the accounts of his miracles.[1] But the principal endeavor of the historians of philosophy, in this area, has been to show that this aspect of the man is not the only one, and not even the most important one; they have

[1] Jaeger asks (*Paideia* I 221 = I 162 Eng. ed.), "But what connexion has all this [number theory, geometry, music theory, and astronomy] with the doctrine of transmigration...?"

looked to find at least the "germ" of the mathematics, natural science, and philosophy that were current, in later times, under the name of Pythagoras. Often a simple "not only-but also" has seemed enough; he was not only a "medicine man" but also a thinker. But may not even a "shaman" perhaps accomplish intellectual feats, without necessarily clothing them in strictly rational or conceptual form? More penetrating are the efforts to discern an inner connection between apparently heterogeneous things.

The point of departure for these must always be the little group of testimonia which, in praise or in irony, speak of the preeminent, comprehensive knowledge of Pythagoras. First comes Heraclitus: *πολυμαθίη νόον ἔχειν οὐ διδάσκει. Ἡσίοδον γὰρ ἂν ἐδίδαξε καὶ Πυθαγόρην αὖτίς τε Ξενοφάνεά τε καὶ Ἑκαταῖον* (fr. 40). *Πυθαγόρης Μνησάρχου ἱστορίην ἤσκησεν ἀνθρώπων μάλιστα πάντων καὶ ἐκλεξάμενος ταύτας τὰς συγγραφὰς ἐποιήσατο ἑαυτοῦ σοφίην, πολυμαθίην, κακοτεχνίην* (fr. 129). Then Empedocles (fr. 129):

> *. . . ἀνὴρ περιώσια εἰδώς,*
> *ὃς δὴ μήκιστον πραπίδων ἐκτήσατο πλοῦτον,*
> *παντοίων τε μάλιστα σοφῶν <τ'> ἐπιήρανος ἔργων . .*

And Ion of Chios:[2]

> *εἴπερ Πυθαγόρης ἐτύμως σοφός, ὃς περὶ πάντων*
> *ἀνθρώπων γνώμας εἶδε καὶ ἐξέμαθεν.*

Herodotus tells of Zalmoxis, who was cleverer than the Thracians, *οἷα Ἕλλησί τε ὁμιλήσαντα καὶ Ἑλλήνων οὐ τῷ ἀσθενεστάτῳ σοφιστῇ Πυθαγόρῃ* (4.95). The key terms are, then, *πολυμαθίη*, *ἱστορίη*, and *σοφιστής*.

From the accusation of polymathy, "we may infer that the later 'so-called Pythagoreans' . . . were right in naming Pythagoras as the founder of their peculiar science."[3] More must be involved than religious revelation, for the name of Pythagoras stands beside that of Xenophanes. Surely, however, the idea of polymathy must apply to all four of the men named in the fragment, and especially Hesiod, who

[2] Cf. ch. II 3, n. 13.

[3] Jaeger, *Paideia* I 221 = I 162 Eng. ed. Rohde, too (Q 105; *Psyche* II 159 = 374 Eng. ed.), finds in this word the seed of later science. Cf. Mondolfo in ZM 316f, G. Vlastos, *Philos Q* 2 (1952) 111 n. 64, K. Freeman, *The Pre-Socratic Philosophers* (Oxford, 1949²) 76 (with this word Pythagoras was classified among scientists by Heraclitus).—Against these attempts to extract "science" from *πολυμαθίη*, see Reinhardt, *Parm.* 232ff, Frank 356 n. 166, W. J. Verdenius, *Mnemosyne* 3.13 (1947) 280ff.

is the first mentioned; and in fact Hesiod's *Theogony* and the *Catalogues* are polymathy in the truest sense of the word. But Hecataeus and Xenophanes are distinctly separated from the other pair by the word *αὖτις*, suggesting that Pythagoras belongs more closely with Hesiod than with them. This can scarcely be accounted for as a chronological indication;[4] the difference in time between Hesiod and Pythagoras is greater than that between Pythagoras and Xenophanes or Hecataeus. Something in the nature of the subject matter, then, must be what connects Pythagoras with Hesiod. Hecataeus and Xenophanes have in common their emphatically modern and polemical attitude, critical of the traditional mythology; Pythagoras does not belong in their company. His "much knowledge" spans the entire world of man and, above all, the world of the supernatural, both this life and the next. This is clear from Empedocles. He stands beside Hesiod as the representative of Orpheus, so to speak—the representative of non-Hesiodic mythical teaching about the world and its gods.

Ἱστορίη is rightly regarded as a key word for the open-minded kind of inquiry based on observation, pursued by the Ionians; so some have seen Heraclitus as testifying to Pythagoras' "scientific research."[5] If it only were not for those awkward *συγγραφαί* which sound very much like "Orphic" writings, and the word *κακοτεχνίη*, which means "skulduggery"![6] When one considers the formulation by Hermesianax of Colophon (third century B.C.), *Ἡσίοδον πάσης ἤρανον ἱστορίης* (fr. 2.22 D.), one realizes the insubstantiality of all the inferences drawn from the passage of Heraclitus. *Ἱστορίη* does not necessarily imply anything more than the Hesiodic type of *πολυμαθίη*, and does not make any definite allusion to *rationale Wissenschaft*, "science."

The word *σοφιστής* is translated "scientist," too;[7] and it has even been asserted that the word proves that Herodotus "knows nothing of Pythagoras as a miracle-worker."[8] Here again the perspective is distorted. In the fifth century it is poets, more than anyone else, that

[4] This interpretation is attempted by Lévy, *Sources* 2 n. 8; H. Thesleff, *On Dating Xenophanes* (Helsinki: Soc. Scient. Fenn., Comm. Hum. Litt. 23.3, 1957) 7 (dating Xenophanes late). H. Gomperz, *Hermes* 1923, 36 n. 1, and M. Marcovich, *Philologus* 108 (1964) 40, speculate that Pythagoras, unlike Xenophanes, was already dead, and therefore classed with Hesiod.

[5] Burnet, *EGP* 85 and 97: "scientific investigation"; Rostagni, *Verbo* 131: "ricerca scientifica"; Raven in KR 228: "scientific inquiry."—On *Comm. math. sc.* p. 78.5 ff. Iam. *VP* 89, see below, ch. VI 1.

[6] Above, ch. II 3, nn. 64, 228.

[7] Burnet, *EGP* 85: "scientific man," 97: "does imply scientific ideas." Contra (rightly), Rathmann 47, Vlastos, *Philos Q* 2 (1952) 111 n. 64.

[8] Corssen, *RhM* 1912, 45.

are designated *σοφισταί*,[9] and even for Isocrates Homer is *ὁ μεγίστην ἐπὶ σοφίᾳ δόξαν εἰληφώς* (*Soph.* 2). For Herodotus, along with Pythagoras, successors of Melampus and Solon are *σοφισταί*.[10] Melampus, who according to Herodotus introduced the worship of Dionysus from Egypt, is without a doubt a *Wundermann* par excellence; and even if one supposed that Pythagoras was closer to Solon than to him, he is still far from science. The new thing that was introduced by those whom Plato called Sophists is not at all suggested by the word. "Wisdom" is an advantage admired and sought from time immemorial, but there are utterly different conceptions, from time to time, as to what it consists of, what it pertains to, and how it manifests itself. Were it certain that Pythagoras was a scientist, these words could be understood in this sense. As long, however, as only his "shamanistic" activities are early attested, our conclusion must be that no statement about the "wisdom" of Pythagoras carries us further than these activities. Quite the contrary, in fact: the Hippocratic book *On the Sacred Disease* attacks the itinerant "medicine men," who expect to heal the sick with magic rites, *μάγοι καὶ ἀγύρται καὶ καθαρταὶ καὶ ἀλαζόνες* (VI 354 L.), and adds, *προσποιέονται . . . πλέον τι εἰδέναι*. The claim to know "more," beyond the limits set for ordinary men, is the mark of the shaman; *τεκμαίρεσθαι* is that of the scientist (Alcmaeon fr. 1).

The efforts to discover a unity of scientific and religious thinking in Pythagoras, and thus a step beyond the merely ritual and mythical realm, center about the ideas of *κάθαρσις* and *ἀνάμνησις*. In both cases the question of Plato's position is difficult; that is, did he merely take over these ideas, or reinterpret them in an independent manner? Since the time of Döring it has been thought that the concept of *κάθαρσις* holds the key to understanding the connection of religion and science in early Pythagoreanism.[11] Rigorous scientific work, especially mathematics, is supposed to release the soul from its close tie to the body and in this way becomes the principal agent of "purification," upon which

[9] Pi. *Isthm.* 5.36, with schol., Soph. fr. 820 N., Hsch. s.v. *σοφιστήν*, G. B. Kerferd, *CR* 64 (1950) 8–10. This removes the necessity of interpreting the special position of the "wise man" in the metempsychosis doctrines of Pindar (fr. 133 Schr.) and Empedocles (fr. 146) as related to the "scientific" character of Pythagoreanism (Mondolfo in ZM 124ff).

[10] In Hdt. 1.29, 2.49. In Eur. (?) *Rhes.* 949 Orpheus is *σοφιστής*, and Menander seems to have dubbed Epimenides *σοφώτατος* (*Κωνειαζόμεναι* fr. 2 Koerte = fr. 308 Kock).

[11] Döring, *AGP* 1892, 505, and, following him, Burnet (*EGP* 97f), through whom, especially, the idea became current. Cf., for example, Delatte, *Pol.* 5, Rougier 101ff, Robin, *Pensée* 65, Mondolfo in ZM 646, Cornford, *PrSap* 110f.

depends the well-being of the soul in this life and the next. This is clearly stated in the *Phaedo*, which introduces Pythagoreans as interlocutors and has a good deal to say about Pythagorean matters.[12] Separation of soul and body, "dying," is the goal of all true philosophy, so that pure truth may be apprehended by the soul, in its pure state (64a)—*κάθαρσις* through *φιλοσοφία* (67cd). And in the *Republic* it is the sciences of arithmetic, geometry, astronomy, and music that lead from the world of appearances to the ideas and effect the necessary *περιαγωγή* of the soul (521c). But here we must move carefully; this line of thought is so closely bound up with the theory of ideas that without this, it becomes meaningless. Without Plato's *χωρισμός*, without the view that there are incorporeal, nonsensible objects of knowledge and science, all science must remain tied to the visible world, and this commitment becomes deeper and deeper. And Aristotle attests that the Pythagoreans were in this very situation.[13] Among these Pythagoreans there is not a trace of the idea of science as an escape from the world; it may not be a coincidence that at the point in the argument of the *Phaedo* where Socrates first speaks of dying as the goal of life, Plato has Simmias laugh (64a).

In fact, the idea of purification through science was apparently not ascribed to Pythagoras before the time of Iamblichus.[14] To be sure, an important and widely cited sentence of Aristoxenus says *οἱ Πυθαγορικοὶ . . . καθάρσει ἐχρῶντο τοῦ μὲν σώματος διὰ τῆς ἰατρικῆς, τῆς δὲ ψυχῆς διὰ τῆς μουσικῆς*, and a number of similar reports may be found in Iamblichus and others,[15] but the topic here is music in the proper sense of the word, which is understood in the light of magic and is bound about by ritual. Science is neither involved in the origin of these usages nor a necessary consequence of them—though by means of a new interpretation of music it may naturalize itself in this realm.[16] There is, however, no testimony to the occurrence of such a transposition

[12] On Simmias, Cebes, and Echecrates, above, ch. I 4, nn. 39–40; on the doctrine of *ψυχή-ἁρμονία*, below, ch. III 2.

[13] *Met.* 989b29; cf. ch. I 2, n. 15.

[14] Theo Sm. 14ff is only concerned with Plato. The Lysis letter (Hercher, *Epistologr. gr.* p. 602.12ff = Iam. *VP* p. 44.2ff, closely dependent on Pl. *Rep.* 429d) uses *katharsis* in a different sense, namely moral preparation for admission to the esoteric teaching.—In detail, Iam. *Comm. math. sc.* 22, esp. p. 69.6ff, pp. 55ff, p.84.12.

[15] Aristox. fr. 26; below, ch. V 1. The sharp distinction of soul from body reminds of Plato (*Soph.* 227c); later sources represent music as also acting on the body (Iam. *VP* 110f, etc.), which doubtless corresponds more closely to the original magical practice.

[16] Socrates in Pl. *Phd.* 61a: *ὡς φιλοσοφίας μὲν οὔσης μεγίστης μουσικῆς.*

before Plato. Festugière derived the idea of purification, in Plato, from general-Greek cult practices;[17] Boyancé replied with the significant comment that the question was not about a single, ephemeral ritual act, but about a lasting style of life, and that in this respect the Pythagoreans seem to have been Plato's predecessors.[18] But scientific activity is not necessarily comprised in the *Πυθαγόρειος τρόπος τοῦ βίου*, either.

The situation is scarcely different with recollection (*ἀνάμνησις*) and related ideas. Here, too, there is a discernible Pythagorean background: Empedocles (fr. 129) says that Pythagoras, when he put all his spiritual power to work, could survey ten and twenty generations. Surely this assumes that he recalled his earlier incarnations. The later tradition tells of a system of memory training among the Pythagoreans. They tried in the morning, or in the evening, to recall all the events of the past day, and even of the day before.[19] In the world of the dead are the springs of Lethe and of Mnemosyne, and the initiate is warned of the former and directed toward the latter.[20] So the attractive conjecture has been made that the original goal of Pythagorean exercises in concentration was to enhance the strength of the soul so that, following the example of Pythagoras, it may avoid the spring of Lethe.[21] Perhaps it was also customary practice in the mysteries to awaken in the initiate a "memory" of his divine descent.

The concept of reminiscence is basic to Plato's epistemology. All true knowledge, as knowledge of an idea, is a process of reminding oneself of something that the soul has seen before its entry into the body. To make this clear, Plato makes use of the doctrines of metempsychosis, and he demonstrates the existence of *anamnesis*, with the help of mathematics, in the famous passage in the *Meno* (80dff). Because of the tendency to regard everything mathematical as *eo ipso*

[17] A. J. Festugière, *Contemplation et vie contemplative selon Platon* (Paris, 1950²) 123ff.

[18] *REG* 1941, 164 n. 3; above, ch. II 4.

[19] Diod. 10.5.1, Iam. *VP* 165 (in the morning), Cic. *Sen.* 38. The verses cited at Por. *VP* 40 are the same as *Carm. aur.* 40ff; one also cited at D.L. 8.22 (evening). The source of this tradition cannot be determined with certainty (on Delatte's reconstructed *Ἱερὸς Λόγος* see below, ch. III 1). See also Apollonius ap. Iam. *VP* 256. According to *Th. ar.* 81.17, the number 10 is *Μνήμη* and 1 is *Μνημοσύνη*; below, ch. III 2, n. 48; above, ch. II 4, n. 24.

[20] Cf. above, ch. II 3, n. 189.

[21] Diogenes Antonius ap. Por. *VP* 45: *ὡς ἀθάνατος ἡ ψυχὴ καὶ τοῖς κεκαθαρμένοις εἰς μνήμην τοῦ παλαιοῦ βίου ἀφικνεῖται*; Procl. *In. Tim.* I 124.5; cf. *Hymn. Orph.* 77.9: *εἰς Μνημοσύνην: μύσταις μνήμην ἐπέγειρε εὐιέρου τελετῆς.* Also Emp. fr. 119; Dieterich, *Nekyia* 122; Rohde, *Psyche* II 186 n. p. 406 n. 96 Eng. ed.; Dodds, *Irr.* 173 n. 107; J. P. Vernant, "Aspects mythiques de la mémoire en Grèce," *Journal de psychologie* (1959) 1ff; B. Gladigow, *Hermes* 95 (1967) 407–420.

Pythagorean, all of this is commonly thought of as Pythagorean,[22] and if true this would show a basic correlation of metempsychosis, genuinely philosophical epistemology, and exact mathematics.

But a closer look reveals that the connection of Pythagoras with Plato, in relation to *anamnesis*, is scarcely more than an equivocation. In Plato, what is "discovered" is that group of elements of human knowledge, beyond the empirical and not discoverable in experience, which are called in modern philosophy the a priori. They can be shown most clearly in the realm of mathematics or in expressions of relationship like "equality."[23] Therefore, in Plato mathematics is a means of proof, and reminiscence as well as metempsychosis is a myth-like clarification of facts of the greatest philosophical relevance. Plato makes use of mathematics because it is necessary to his doctrine, not because of any partiality toward Pythagoreans or mystical attraction toward number. The intended goal of his argument is the theory of ideas, for it is in the ideas that Plato finds that knowledge, beyond the empirical. The object is not the extension of human experience to "ten and twenty human lifetimes," nor the projection of this life into another, very real, next life, but a vision of a quite different kind. The knowledge of the ideas is supramundane, and cannot be attained by living through even a large number of earthly lives. This is made clear in the myth of the *Phaedrus*, and the *Meno* hints at it. Any slave can "recall," because what is at issue is not just any experience, but that which is generally valid; it is explicitly stated that this slave must have attained this knowledge "when he was not a human being" (*Meno* 86a). Here we are in a realm which, according to Aristotle, was foreign to the Pythagoreans: διαλέγονται μέντοι καὶ πραγματεύονται περὶ φύσεως πάντα (*Met.* 989b33). It is impossible to credit Pythagoras' eschatological

[22] Burnet, *ThPl* 43: "I see no difficulty in referring this doctrine in its mathematical application to Pythagoras himself." He is more reserved in the edition of the *Phaedo* (72), since, as represented by Plato, Simmias and Cebes do not seem familiar with the doctrine; therefore the doctrine is attributed to Socrates: "he applied the old religious doctrine of ἀνάμνησις to science." Here the difference is seen; we only need to replace "Socrates" with "Plato."—The matter is most fully discussed by Cameron, who has the twofold thesis that ἀνάμνησις in Pythagoras comprehended "a theory of knowledge" and that "divine knowledge was Number" (20). Neither of these points is provable (cf. Cherniss, *AJP* 61 [1940] 361), and in particular there is no evidence that for the Pythagoreans "recollection" was bound up with number. (Heraclides fr. 44 is surely thinking along Platonic lines; cf. ch. II 3, n. 78.) Long (68f) emphasizes that the doctrine of transmigration as taught in the *Meno* is dependent on Pindar, but finds, in the nearby mathematical discussion "additional proof" that Pindar, too, had a "Pythagorean background"; mathematical, ergo Pythagorean. (On this see ch. VI.)

[23] So Pl. *Phd.* 72e; brief treatment of recollection *Phdr.* 249b c, differently *Rep.* 518b d, *Tht.* 148e. Cf. E. Heitsch, *Hermes* 91 (1963) 36–52. Proclus regards the doctrine πᾶσα μάθησις ἀνάμνησις as Pythagorean (*In Eucl.* p. 45.5).

doctrine with Plato's mathematics and his theory of ideas. The development of the concept of *anamnesis* into epistemology and the proof of it by mathematical means are seen to be typical "Platonic transposition."[24] What is Pythagorean is practice in concentration and memory, which have no necessary connection with mathematical science, but belong to the world of "shamanism."

Generally speaking, the argument from silence is not very persuasive. And yet one is impressed by the negative consensus of the ancient sources, down to Aristotle, in the matter of Pythagoras the philosopher and scientist. Herodotus mentions him in connection with Zalmoxis, and mentions Pythagoreanism in speaking of the rituals of Egypt; but, when he deals with the rise of geometry in Egypt (2.109), Pythagoras' name does not occur. Democritus was acquainted with Pythagoreans,[25] and wrote a book entitled *Πυθαγόρης*, but it is classified, in the catalogue of Democritus' writings, among the ethical tetralogies, along with *Περὶ τῶν ἐν Ἅιδου*.[26] The *Dissoi logoi*, too, speak of Pythagoreans, along with followers of Anaxagoras, but in relation to the teachability of σοφία καὶ ἀρετά.[27] Plato names Pythagoras just once, as ἡγεμὼν παιδείας and creator of a *Πυθαγόρειος τρόπος τοῦ βίου*.[28] It is the Society's way of life, and not any particular knowledge, that makes his fame. The doxographical sketch in the *Sophist* alludes to Pherecydes, Alcmaeon, the Eleatics, Heraclitus, and Empedocles, but the "Eleatic stranger" obviously does not think of Pythagoras when he is listing the attempts τὰ ὄντα διορίσασθαι πόσα τε καὶ ποῖά ἐστιν (242c; cf. *Tht.* 152e). Alcidamas stated that Empedocles had studied with Pythagoras and Anaxagoras, καὶ τοῦ μὲν τὴν σεμνότητα ζηλῶσαι τοῦ τε βίου καὶ τοῦ σχήματος, τοῦ δὲ τὴν φυσιολογίαν.[29] This is very doubtful as a historical statement, but it

[24] L. Robin pronounced the final verdict arguing against Burnet, many years ago: ". . . que l'ἀνάμνησις pythagoricienne peut bien être l'origine historique de l'ἀνάμνησις de Socrate et de Platon, mais que ce n'est pas un motif de transporter dans le Pythagorisme primitif la conception philosophique qui caractérise cette dernière," in "Sur la doctrine de la réminiscence," *REG* 32 (1919) 451–461, at p. 455. Cf. Dodds, *Irr.* 209f.

[25] This is attested by his contemporary Glaucus of Rhegium (D.L. 9.38). Apollodorus of Cyzicus mentioned Philolaus (ibid.). Cf. also ch. II 2, n. 2.

[26] DK 68B0a. Against its genuineness, Zeller *SBBln* 1889, 996, Wellmann *AbhBln* 1921.4, 12f, and Diels (with hesitation); but cf. above, ch. II 3, n. 146; Frank 67; Gigon, *Ursprung* 128f.—It is tempting to juxtapose *Carm. aur.* 12 with fr. 84 (cf. also fr. 244 and 264). Democritus praised Pythagoras (Thrasyllus ap. D.L. 9.38).

[27] 6.8 (DK II 414.13).

[28] *Rep.* 600ab; above, ch. I 4.

[29] D.L. 8.56, from the book φυσικός; Brzoska, *RE* I 1538. Lévy, *Sources* 2 n. 6, considers the possibility of forgery; and Burnet, *EGP* 202, would substitute *Πυθαγορείων* for *Πυθαγόρου*; but if even Timaeus (*FGrHist* 566F14) can make Empedocles a direct pupil of Pythagoras, surely a rhetor like Alcidamas could have done so.

gives an insight into the common view of Pythagoras; the "natural science" (*φυσιολογία*) comes from Anaxagoras, not Pythagoras. But the solemn, pompous mein which Pythagoras and Empedocles have in common is precisely the manner of the shaman—Empedocles the "deathless god" and Pythagoras the "Hyperborean Apollo."

Isocrates in the *Busiris* says in a general way that Pythagoras "was the first to introduce all the other philosophy to the Greeks," but specifically that he "more conspicuously than others attended to sacrifices and temple rites."[30] It is only in the context of *ὁσιότης* that Isocrates thinks of Pythagoras, not along with *ἀστρολογία καὶ λογισμοὶ καὶ γεωμετρία* (23).

In his accounts of the history of philosophy, Aristotle consistently avoids the name of Pythagoras. In the *Protrepticus* he describes Greek natural philosophy as *τὴν περὶ φύσεώς τε καὶ τῆς τοιαύτης ἀληθείας φρόνησιν, οἵαν οἵ τε περὶ Ἀναξαγόραν καὶ Παρμενίδην εἰσηγήσαντο.*[31] These are the living names in the field of physical philosophy; Pythagoras is not included.[32] Theophrastus names Anaximander and Xenophanes as the teachers of Parmenides, and Parmenides as the teacher of Empedocles,[33] where others speak of Pythagoras or Pythagoreans. If we take all these passages together, their silence about Pythagoras the philosopher and mathematician is an extremely remarkable coincidence—if it is that.

The other tradition sets in at just about the time of Aristotle. Aristippus dealt with Pythagoras in a book entitled *Περὶ φυσιολόγων*.[34] Hecataeus of Abdera[35] and Anticlides, who wrote about Alexander the Great,[36] represent Pythagoras as introducing geometry from Egypt. By this time a conception of Pythagoras had become dominant which was obviously quite unknown to the pre-Platonic writers.

[30] *Bus.* 28. It is clear from Herodotus that there was some connection between Pythagoras and Egypt (2.81, 123), and for him too the link was in religious and cultic peculiarities. The *Busiris* was probably influenced by Plato (*Gnomon* 33 [1961] 352).

[31] Arist. fr. 52 = Iam. *Comm. math. sc.* 79.13f. The very fact that Thales and Pythagoras are not named in this passage is an indication of the Aristotelian origin of the wording.

[32] Iamblichus seems to have preserved something of this tradition in another passage (*VP* 166, perhaps from Timaeus; cf. above, ch. II 1, n. 34): Thanks to Pythagoras, he says, the expression Magna Graecia was invented, philosophers, poets, and lawgivers flourished there, rhetoric was born, the *Gnomes* of Epicharmus were much celebrated, *καὶ περὶ τῶν φυσικῶν ὅσοι τινὰ μνείαν πεποίηνται, πρῶτον Ἐμπεδοκλέα καὶ Παρμενίδην τὸν Ἐλεάτην προφερόμενοι τυγχάνουσιν.* Pythagoras himself is not named as a physical philosopher.

[33] Theophr. *Phys. op.* frr. 6, 6a, 3 (on which see *Dox.* 477); cf. below, ch. III 3.

[34] D.L. 8.21 = A162 Giannantoni.

[35] Diod. 1.98.2 = *FGrHist* 264F25.

[36] D.L. 8.11 = F*GrHist* 140F1; cf. below, ch. VI 1.

Every age has its own ideal of Wisdom; and there came a time when the ideal of the Wise Man, who by his own innate powers has achieved a commanding spiritual position and insight, became embodied in the persons of certain great men who seemed to fulfill the highest conceptions of wisdom and power that were attributed to the ecstatic seer and priest of purification . . . We cannot call them philosophers—not even the forerunners of Greek philosophy. More often their point of view was one which the real philosophic impulse toward self-determination and the freedom of the soul consciously and decisively rejected, and continued to reject, though not indeed without occasional wavering and backsliding.

So wrote Erwin Rohde, in reference to figures like Epimenides and Abaris, without including Pythagoras.[37] But the most ancient evidence indicates that it is precisely in this perspective that we must see Pythagoras. To a later age it seemed natural to retroject their own notion of "wisdom" upon the great figures of the past and to impute to them that which from a modern point of view is "science." Only by such reinterpretation could the ancient remain acceptable. This process of the renewal of ancient wisdom took place, apparently, within the Pythagorean tradition, with many a false start and new beginning. For what was most characteristic of Pythagoreanism, from the beginning, was not so much the new as the exotic and archaic. No ancient witness appears to testify that Pythagoras was a scientist; in studying the shadow he cast on later generations, all we can be sure of is the cryptic *πλέον τι εἰδέναι* of the "shaman" and hierophant.

[37] *Psyche* II 90 = 299f Eng. ed. (tr. W. B. Hillis).

III. Philolaus

1. THE SPECIAL POSITION OF PHILOLAUS' BOOK IN THE PYTHAGOREAN TRADITION

It has long been an established principle in the study of the pre-Socratic philosophers that only their original words, so far as they have been preserved, can provide an adequate foundation for interpretation and reconstruction. In any paraphrase lurk potential errors, for alteration of the form is bound to affect the content, whether because of adaptation to modern ways of thinking, or of a polemical bent which results in a reluctance to see the true sense, or of both. The special difficulty in the study of Pythagoreanism comes from the fact that this principle cannot be applied; whether an item of the tradition may be regarded as an authentic pronouncement of a Pythagorean must in each case be decided first on the basis of the indirect testimony. For the mass of writing that was forged[1] and attributed to Pythagoras and his pupils was so vast that, contrary to ordinary methodological principles, in the case of any text purportedly composed by an early Pythagorean, the burden of proof lies with anyone who wishes to maintain its authenticity.

Even in ancient times there was controversy as to whether Pythagoras had written anything. From the third century on, ostensible works of Pythagoras kept appearing in the market, but none of them could hold its place.[2] The opinion was widespread that no such book had been preserved,[3] or even that Pythagoras avoided the written word on principle and recorded his teachings only in the minds of his

[1] To what extent the expression "forgery" is legitimate here is a question we need not go into. In any case, we are talking about writings which, in an unhistorical manner, attribute to Pythagoras and his school material from the schools of Plato, Aristotle, and the Stoa, as well as popular philosophical thought. Cf. Burkert, *Philologus* 1961, and, for a collection of material, Thesleff, *Texts*.

[2] The evidence: Thesleff, *Texts* 155–186, 243–245; van der Waerden, *RE* Supp. X 843–864. On the *Hypomnemata*, see above, ch. I 3, n. 3; on the ἀλειπτικὰ συγγράμματα, above, ch. II 4, n. 111; on the "tripartitum," below nn. 32–33.

[3] Cf. Zeller I 368 n. 2; DK 14.17–19, esp. Posidonius (Galen *Plac. Hipp. et Plat.* 5.6, V 478 K.); also D.L. 1.16, Hieron, *Adv. Ruf.* 3.39f, Claud. Mam. *De statu an.* 2.3.

disciples.[4] There is a Platonic coloration here, and one may suspect that this emphatic declaration served as a pretext for discarding certain Pythagorean writings that were felt to be an embarrassment because of their old-fashioned character.[5] Delatte attempted to refer the various lines of Pythagorean verse, one of which was cited as early as Chrysippus, and most of which eventually found their way into the late compilation called *Carmen aureum*, to a Ἱερὸς λόγος of the fifth century.[6] Rostagni, still more boldly, claimed to have found the "Word of Pythagoras" itself.[7] Heraclides Lembus lists a ἱερὸς λόγος in hexameters in second position in his catalogue of Pythagoras' writings, and, in the manner of an ancient librarian, quotes the first line (D.L. 8.7). The older evidence for this secret document is, however, very fragile. Hecataeus of Abdera reports[8] that Pythagoras brought "the ἱερὸς λόγος," along with mathematics and the doctrine of transmigration, from Egypt. This does not mean, however, that he had seen a book entitled Ἱερὸς λόγος. Like the statements about mathematics and transmigration, that about the ἱερὸς λόγος seems to come from Herodotus,[9] who does use this term in connection with Pythagoreanism. But the context, along with the parallel passages, shows that what he is talking about is the mythical explanation given by the Egyptians, not a "Holy Word" of Pythagoras.[10] A ἱερὸς λόγος is that which is not to be

[4] Plut. *Numa* 22, Nicom. ap. Por. *VP* 57: οὔτε γὰρ αὐτοῦ Πυθαγόρου σύγγραμμα ἦν. (This sentence is not included in the parallel report, Iam. *VP* 252. Rohde, *Q* 140f, assumes that Porphyry has inserted it. But the passage reads smoothly in Porphyry, whereas there is unevenness in Iamblichus: the words πλὴν ὀλίγων πάνυ, p. 135.22, do not go with what immediately precedes them, but with συνεπιλιπεῖν, line 19; and it is just here that Porphyry has the sentence quoted. In Iamblichus there are hints both earlier [ἄρρητον . . . ἐν τοῖς στήθεσιν, lines 19f] and later [p. 136.8] that there were no written works of Pythagoras; but, as Iamblichus repeatedly cites works of Pythagoras, he has every motive to hush this up. Cf. above, ch. II 1, n. 15.)—David *In Porph. Is.*, *CAG* XVIII 2 p. 25.28ff, adds Plato's warning against written texts (*Phdr.* 277e).

[5] Apollonius ap. Iam. *VP* 258, D.L. 8.7, on the Ἱερὸς or Μυστικὸς λόγος. See Gigon, *Ursprung* 124f.

[6] Delatte, *Litt.* 1ff; contra, Theiler, *Gnomon* 2 (1926) 147ff. The material is in Thesleff, *Texts* 158–163. Chrysippus, *SVF* II no. 1000 = *Carm. aur.* 54.

[7] *Il verbo di Pitagora.* He relies especially on Ov. *Met.* 15; but the principal source of that is Empedocles.

[8] *FGrHist* 264F25 = Diod. 1.98.2 (cf. Jacoby IIIa 75f). Pythagoras is named among the sages who visited Egypt at 1.69.4 and 1.96.2. In the latter passage occurs a mention of the ἀναγραφαὶ αἱ ἐν ταῖς ἱεραῖς βίβλοις which is regarded as a strong clue that Hecataeus is the source. There are probably "occasional and not extensive" additions by Diodorus (Jacoby 78.24); but the naming of Pythagoras and of the ἱερὸς λόγος is firmly rooted in the context. Hecataeus could not avoid mentioning Pythagoras (cf. also above, ch. II 2, n. 15).

[9] Hdt. 2.81; cf. above, ch. II 3.—Geometry, Hdt. 2.109, metempsychosis 2.123.

[10] *Pace* Guthrie I 160; Herodotus describes the custom, names the Greek parallels, and remarks in closing that he is not going to reveal the explanation. Cf. ἱερὸς λόγος, 2.62; parallel οὔ μοι ὅσιόν ἐστι λέγειν, 2.61; similarly 2.171, 2.51.

disclosed to the uninitiated, so that a book with such a title is a priori apocryphal. There could have been oral transmission of a sacred poem, but nothing indicates that a comprehensive or authoritative text existed in the early days; in fact, the oscillations of the tradition, for example in the doctrine of metempsychosis, point in the opposite direction.[11] The literary testimony most often cited for Pythagoreanism, obviously the most authentic attainable witness, is the *Katharmoi* of Empedocles.[12] While Anaximander and Parmenides, as well as Empedocles, survived almost exclusively because of the works that bore their names, in the case of Pythagoras what stands at the origin of the tradition is the picture of a "sage," unclear in many details but powerful for that very reason—an outline to be filled in by later generations.

Many works were circulated in the name of various Pythagoreans and have been preserved either in fragments or entire,[13] but there is no longer even any discussion of their authenticity, except for some of the fragments attributed to Philolaus and Archytas. The Philolaus fragments are more important, because they treat of number theory and cosmology, while those attributed to Archytas mostly concern mathematics and acoustics.[14] The question of the genuineness of the

[11] Above, ch. II 3. Even the scholarship of antiquity could not give ancient references for works of Pythagoras, except Heraclitus fr. 129 and Ion fr. 2; and neither of these proves what it is supposed to, in this regard (above, ch. II 3, nn. 63, 52). According to Dicaearchus (Por. *VP* 19) no one had exact information about the teaching of Pythagoras.

[12] Empedocles is simply cited as evidence for Pythagoreanism by Posidonius ap. Sext. Emp. *Math.* 7.92 (fr. 109), 9.127ff (fr. 136–137), Cic. *Rep.* 3.19 (fr. 135ff), Por. *De antr. nymph.* 8 (fr. 120), Hippol. *Ref.* 6.25.1 (fr. 16), Plotinus 4.8.1 (fr. 115). Porphyry says that vegetarianism is prescribed in the philosophy "of Pythagoras and Empedocles" (*Abst.* 1.3), but Theophrastus only includes Empedocles (fr. 128, 139 = Por. *Abst.* 2.21, 27, 31).—Kahn (*AGP* 1960, 28ff) explains the difference between the two books of Empedocles by the conjecture that Emp. wrote the *Katharmoi* after his break with the Pythagoreans, when he no longer felt himself bound by the vow of secrecy. In this case the *Katharmoi* would actually contain, in different form, the *ἱερὸς λόγος*, i.e., the secret doctrine of Pythagoras.

[13] These are listed by Zeller III 2.92ff; Thesleff, *Intr.* 8ff; published by Thesleff, *Texts*; published in part by H. E. Brown, *Philosophorum Pythagoreorum collectionis specimen* (Diss. Chicago, 1941). *Carmen aureum*: P. C. van der Horst (Diss. Leiden, 1932); Diehl, *Anth. lyr. gr.* I 2[3] (Leipzig, 1950) 82ff. Ocellus: Harder. Ecphantus, Diotogenes, Sthenidas: L. Delatte (Liège, 1942).

[14] This is a result of Diels's method of selection. The amount of spurious material, which is not included in DK, far exceeds that of the possibly genuine fragments. Diels separated the two types according to the principle that forgeries are not likely to be found in specialized mathematical and musical material. In principle this seems convincing; no forger works out complicated calculations like those in A16, and the genuineness of the geometrical discovery of A14 is guaranteed by Eudemus (fr. 141). But there are certain grounds for suspicion of B1–3, and B4 must be spurious. Aside from the telltale *εἰδέων πραγματεία*, the assertion that *λογιστικά* helps where *γεωμετρία* fails is nonsensical mathematically. Archytas could determine $\sqrt[3]{2}$ (used in doubling the cube) geometrically, but not arithmetically. Fr. 4 is comprehensible from the point of view of Plato's doctrine of ideal numbers: according to the system of derivation, number is primary as compared to geometrical magnitudes (*ἀριθμητικὴ ἀκριβεστέρα γεωμετρίας*, Arist. *Met.* 982a26ff, etc.). Cf. also below, ch. V 1.

Philolaus fragments has been a matter of controversy for many decades. For A. Boeckh, who devoted a fundamental study to them, they were "in the labyrinthine maze of the traditions about Pythagorean wisdom and the Pythagorean society . . . a point of light whose radiance could, perhaps, brighten this night a little."[15] The authority of Zeller and Diels won acceptance for this view for a good while, though with an important modification.[16] This was true especially in the German-speaking world; among speakers of English, Bywater's condemnation of the fragments, followed, with additional arguments, by Burnet, held sway.[17] Then, since Frank published his elaborate attack against the fragments, the dominant mood has been uncertainty, though scholarly caution has somewhat tipped the balance toward the negative.

In spite of the mountainous bibliography, there does not yet exist a full interpretative study of the Philolaus fragments. But since the days of Bywater the situation has somewhat changed; there are new perspectives, refined methods, and, above all, new source material. A papyrus published in 1893, with excerpts from the history of medicine written by Aristotle's pupil Menon, now makes it certain that there was a book by Philolaus extant in the middle of the fourth century B.C.[18]

The fact that the controversy lives on, in the case of Philolaus, is in itself significant. Writings like those of "Ocellus," "Timaeus of Locri," and a great deal of the material attributed to Archytas have

[15] Boeckh 3.

[16] The athetesis of fr. 21 (below, ch. III 2).

[17] For authenticity: Zeller I 369ff; Diels, DK 44; Reinhardt, *Parm.* 65 (and elsewhere; 248 n. 2 for the genuineness of fr. 20); H. Gomperz, *Hermes* 1932, 155ff (for the genuineness of fr. 22; cf. below, ch. III 2); Scoon, *CP* 1922, *GrPh* 133ff; Rostagni, *Verbo* 47ff; Robin, *Pensée* 60ff; Carcopino, *Bas.* 161ff; Boyancé, *Songe* 99, *Muses* 117 n. 1, 289 n. 2. Against authenticity: V. Rose, *De Aristotelis librorum ordine et auctoritate* (Berlin, 1854) 2; Schaarschmidt (criticized by Zeller, loc. cit.); Rothenbücher; Bywater; A. Döring, *Gesch. d. gr. Philos.* I (Leipzig, 1903) 183f; Burnet, *EGP* 279ff; Tannery, *MSc* III 220–243 (the music fragments); Frank 139ff, 263ff; Howald, *Fs. Sudhoff*; Bollinger 27ff; A. Rivaud, ed. of Pl. *Tim.* (Paris, 1925, Coll. Budé) 24, *REG* 39 (1926) 289ff, *Rev. hist. philos.* 3 (1929) 25; A. Diès, ed. of Pl. *Phlb.* (Paris, 1941, Coll. Budé) xxii; Festugière, *REG* 1945, 1; Heidel, *Maps* 94; Raven, *PyEl* 93ff, in KR 308ff; W. Spoerri, *REA* 57 (1955) 278 n. 5; Cherniss, *Pres.* 37 n. 140, 386f. Criticism of Frank: Mondolfo in ZM 367ff = *RivFil* 15 (1937) 225–245. Doubtful: Rey 183; Wuilleumier 572f; Moreau, *Âme* 145ff; Guthrie I 329–333; Philip 31–32.

[18] A27–28 Supp. Arist. III 1.31, 36; cf. below, ch. III 2.

been convicted as spurious and set aside. The "forger" betrays himself by letting results, or concepts, or terminology of later philosophical thought creep in. (An important criterion here is the evidence of Aristotle that among the Pythagoreans the distinctions had not yet been fully developed between form and matter, sensible and super- or insensible, corporeal and incorporeal.)[19] Another sign of forgery is too close an adherence to the wording of a passage of Plato or Aristotle; even when Plato is dependent on Pythagoreans, he does not copy word for word. According to these criteria the metaphysical fragments of "Archytas," for example, are unquestionably apocryphal,[20] but the case of Philolaus cannot be settled in this way.

It would be convenient if external, linguistic factors could be decisive. Burnet tried to show that a Pythagorean of the fifth century, even in south Italy, would have had to write not Doric but Ionic Greek, the dialect that had become standard for philosophical and scientific writing. The physicians of Cos and Cnidus wrote Ionic, as did Antiochus of Syracuse[21] and Herodotus, deserting in each case the dialects of their home cities.[22] Burnet will admit that Archytas wrote Doric,[23] as Thucydides chose Attic. But the author of the pseudo-Xenophontic *Constitution of the Athenians* was already writing Attic; why would Philolaus have had to restrict himself to Ionic? The word τετρακτύς perhaps points in the direction of Doric.[24] But, above all, we

[19] Above, ch. I 2.

[20] E.g., there is Aristotelian terminology in *Περὶ ἀρχῶν*, p. 19.19 Thesleff: ἁ μὲν μορφώ ἐστι ἁ αἰτία τοῦ τόδε τι εἶμεν, ἁ δὲ ὠσία τὸ ὑποκείμενον, παραδεχόμενον τὰν μορφώ. *Περὶ νοῦ καὶ αἰσθάσιος* (39.3–25 Thesleff) is mostly copied word for word from Plato *Rep.* 509d et seq. The logical writings of ps.-Archytas (pp. 15–19, 21–32 Thesleff) are so obviously Aristotelian that even Themistius supposed they were written not by the ancient Pythagorean but by a later Peripatetic named Archytas (Boeth. *In Cat.* I, Migne 64.162, p. 22.2 Thesleff). Archytas, οὗ μάλιστα καὶ γνήσια λέγεται εἶναι τὰ συγγράμματα (the opinion of Por. *In Ptol.* p. 56) and since he is somewhat later than Philolaus, is treated as more reliable, especially by Frank. It is easy to overlook the fact that more was forged in his name than in that of any other Pythagorean, since the rather extensive fragments of these works are not printed in DK.

[21] *FGrHist* 555F2. In Alcmaeon's fr. 1 are to be found not only a Doric ἔχοντι but an Ionian Κροτωνιήτης. Burnet (*EGP* 282 n. 5) and Thesleff (*Intr.* 80) believe that he wrote in Ionic.

[22] The "three books" attributed to Pythagoras are Ionic (D.L. 8.6ff), as are "Androcydes" (ch. II 4) and "Perictione" (Stob. 4.25.50, 4.28.19; pp. 142–145 Thesleff). It is not at all certain that, as many assume, the so-called *Hypommemata* were Ionic; see Burkert, *Philologus* 1961, 27 n. 3.

[23] *EGP* 283.

[24] The kappa in τετρακτύς is surprising, compared with Ionic χιλιαστύς and Lesbian χελληστύς, but not simply "Doric." Schwyzer (I 597) suggests analogy with τρικτύς, cf. τρικτυαρχέω in Delian inscriptions, τρίκτοια in Sophron fr. 3 Kaibel (cf. LSJ), τρίκτευα in IG II/III² 1126.34 from Delphi. It is doubtful whether the form Ζανὸς πύργος (Simpl. *Cael.* 512.12 – Arist. fr. 204, Procl. *In Eucl.* 90.17; v.l. Ζηνός) can be counted as Doric; see Schwyzer I 577 n. 4. Schwyzer, *Dial.* 696 (Chios).

are told that the physician Acron of Acragas, whose *floruit* was about 430 B.C., wrote in Doric;[25] and it is beyond question that Corax and Tisias spoke and wrote in Doric. All this shows that prose literature in Doric began to appear at least a generation before Archytas. The individual dialect forms, however, are so often inconsistently reproduced in the manuscript tradition that even obviously false forms do not provide a criterion of spuriousness. Hyperdorisms are to be found in Sophron, Theocritus, and Archimedes, and may even have penetrated into popular speech.[26]

Generally speaking, definite proof cannot be offered for the authenticity, but only for the spuriousness, of a book. But the defender of a book's genuineness is not confined to destroying the opponent's arguments, knowing the while that new ones may always appear. What one can and must do is to ask where, in the transmission, the sources of error lie. "Forgeries" are usually pieces of quasi-historical reconstruction; the Pythagorean pseudepigrapha, for example, show what people wanted to be regarded as Pythagorean. We can detect in them a certain tendency of interpretation, a general purpose which is also discernible in the distortions of the doxographical tradition. A genuine fragment must show itself so by standing aloof from this tendency and not being deducible from it. In this way one can establish the presumption that something is probably genuine.

First let us look into the external evidence for Philolaus' book. In various ways, Philolaus' name is connected with the beginnings of written Pythagorean literature.[27] According to a tradition that goes back at least as far as Satyrus, Plato in a letter to Dion commissioned him to buy "from Philolaus" three "Pythagorean books" for the

[25] *Suda* s.v. Ἄκρων: Δωρίδι διαλέκτῳ. There is no reason to think of a pseudepigraphon (*pace* Thesleff, *Texts* 1). The doctors in comedy speak Doric, from Crates (fr. 41 Kock) onward, see Euphron fr. 3, Alexis fr. 142, Menander *Aspis* 439–464: one of them is said to come Σικελᾶς ἀπὸ γᾶς, Epicrates fr. 11.27. The cookbook of Mithaecus (Pl. *Gorg.* 518b) was also Doric (Ath. 7.325f).

[26] Cf. Schwyzer I 185, 719 n. 6. On the dialect of the Pythagorean pseudepigrapha, see A. Matthaei, *De dialecto Pythagoreorum* (Göttingen, 1878); R. Fohalle, "La langue d'un texte 'dorien'" (Aisara) in *Étrennes de linguistique off. à É. Benveniste* (Paris, 1928) 27–49; Delatte, *Pol.* 75ff ("Archytas" Περὶ νόμου καὶ δικαιοσύνης), 127ff ("Hippodamus"); L. Delatte, *Les traités de la royauté d'Ecphante, Diotogène et Sthénidas* (Liège, 1942); É. Wilhelm, *RhM* 70 (1915) 161–223 ("Bryson," "Callicratidas," "Perictione," "Phintys"); Thesleff, *Intr.* 85–92.

[27] Wiersma correctly analyzed the tradition, *Mnemosyne* 1942, 23f. The misunderstanding about the "three books of Philolaus" hung on, from the time of Boeckh, until it made its way into DK, in spite of the express testimony γέγραφε δὲ βιβλίον ἕν (D.L. 8.85). Bywater (26ff) and Schaarschmidt (74ff) capitalized on the contradiction.

stupendous sum of 100 minas; Philolaus had "made these books known" (ἐξήνεγκε).[28] This alleged letter of Plato must actually have existed, to judge by the form of citation;[29] and its purpose is not hard to guess. Ancient publishers frequently preface apocryphal publications with a letter that gives an account of the origin and importance of the writing.[30] Thus there was in circulation, at the time of Satyrus, about 200 B.C., a set of three pseudonymous Pythagorean books, introduced by this letter from "Plato," which named Philolaus, though not as its author. The word ἐξήνεγκε is used of someone who "brings out" or publishes a writing previously unknown, perhaps secret, but in any case already in existence for some time; but it is not used of the author himself.[31] We must get rid of the misconception, which goes back to Boeckh, that Philolaus had written the "three books," and only then will we be able to identify them.

[28] D.L. 3.9: *λέγουσι δέ τινες, ὧν ἐστι καὶ Σάτυρος, ὅτι Δίωνι ἐπέστειλεν (Πλάτων) εἰς Σικελίαν ὠνήσασθαι τρία βιβλία Πυθαγορικὰ παρὰ Φιλολάου μνῶν ἑκατόν.*

D.L. 8.15: *μέχρι τε Φιλολάου οὐκ ἦν τι γνῶναι Πυθαγόρειον δόγμα· οὗτος δὲ μόνος ἐξήνεγκε τὰ διαβόητα τρία βιβλία, ἃ Πλάτων ἐπέστειλεν ἑκατὸν μνῶν ὠνηθῆναι.*

Iam. *VP* 199: *οὐθεὶς οὐθενὶ φαίνεται τῶν Πυθαγορείων ὑπομνημάτων περιτετευχὼς πρὸ τῆς Φιλολάου ἡλικίας· ἀλλ' οὗτος πρῶτος ἐξήνεγκε τὰ θρυλούμενα ταῦτα τρία βιβλία ἃ λέγεται Δίων ὁ Συρακούσιος ἑκατὸν μνῶν πρίασθαι Πλάτωνος κελεύσαντος.*

D.L. 8.84: *παρὰ τούτου (Φιλολάου) Πλάτων ὠνήσασθαι τὰ βιβλία τὰ Πυθαγορικὰ Δίωνι γράφει.* Gell. 3.17.1 "(Platonem) tris Philolai Pythagorici libros decem milibus denarium mercatum." (Denarius = drachma, so that 10,000 denarii = 100 minas.) Cic. *Rep.* 1.16: "(Platonem) Philolai commentarios esse nanctum." Tzetzes ingeniously combined the testimony of Satyrus, Hermippus, and the letter of Lysis to produce a version in which, with Dion's help, Plato bought a book of Philolaus, for 100 minas, from impoverished Pythagorean widows (*Chil.* 10.790ff; 11.39ff). Cf. also the anonymous *Prolegomena to Plato* 27, p. 201 Hermann.

[29] *γράφει* D.L. 8.84. Theiler is doubtful about this (*Gnomon* 2 [1926] 587), as is Beutler (*RE* XVII 2363). Beutler's argument, that the letter would have been famous if it had existed at all, is not persuasive; even the *θρυλούμενα τρία βιβλία* perished almost without a trace.—The ancient sources give various, often contradictory, guesses as to where Plato got so much money. (Onetor ap. D.L. 3.9 says Dionysius gave it to him; Gellius 3.17.1 says it was Dion, according to "certain persons.") Also, how could a Pythagorean be such a clever business man? (Iam. *VP* 199 says Philolaus was in need; certain *ἕτεροι* in D.L. 8.85 said the book was given to Plato by certain Pythagoreans in gratitude for his having procured the release from jail of one of their number.) The letter was there; imaginary circumstances were devised to fit the "fact."

[30] Cf. Burkert, *Philologus* 1961. Examples are the letter of Lysis and the *Hypomnemata* (Delatte, *Litt.* 103ff), the correspondence between Plato and Archytas and Ocellus (Harder 45ff), 2 Macc., Diogenes Antonius, and Dictys Cretensis.

[31] *ἐξενεγκεῖν* is used of Hippasus in the version of the *mathematici*, and in the same passage *ἐξενεχθῆναι* is used of geometry (Iam. *VP* 88 *Comm. math. sc.* p. 77.19). The abbreviated expression "Philolai libros, commentarios" in Cicero and Gellius is irrelevant to this question.

"Pythagoras wrote three books: *On Education*, *On Statesmanship*, and *On Nature*."[32] This so-called tripartitum was available to Heraclides Lembus, who also used Satyrus,[33] and also to the author who put together the account which is at the base of the lives of Pythagoras in Diogenes Laertius and Hesychius.[34] For a time it was regarded as just what it claimed to be: the authoritative and genuine work of Pythagoras.

It was natural enough to invent a correspondence between Plato and Dion about Pythagorean writings, just as the author of the "Ocellus" book represented Archytas as corresponding with Plato. The remarkable thing is that Philolaus was included in the plot, in spite of the chronological difficulties involved. Apparently this was the result of the belief, attested at least since Neanthes, that before Philolaus there were no Pythagorean writings known.[35] No anecdotal or legendary features characterized the publication of Philolaus' book.[36] We have only the simple datum: Philolaus and the first appearance of Pythagorean writings belong together.

We have from Hermippus another story, older than that of the "tripartitum": "He (Philolaus) wrote one book, which according to Hermippus some writer said Plato the philosopher . . . bought from the relatives of Philolaus for forty Alexandrine minas of silver, and from which he copied his *Timaeus*" (D.L. 8.85). In spite of similarities,

[32] D.L. 8.6; cf. Schol. Pl. *Rep.* 600b, and the *Suda* s.v. Pythagoras (from Hesychius).

[33] On the "tripartitum," Diels, *AGP* 1890; Lévy, *Sources* 70ff; Thesleff, *Texts* 170–172. Heraclides Lembus ap. D.L. 8.44 is dependent on the "tripartitum" citation at 8.10.

[34] Cf. nn. 32, 35. (Was it Neanthes?) Further, above, ch. II 1 n. 22. The obvious identification of the "three books" mentioned in the supposed Platonic letter with the "tripartitum" (made by Wilamowitz, *Platon* II 87; Wiersma, *Mnemosyne* 1942, 23) has often been prevented by the prestige of Boeckh. In his desire to fit all the data on Philolaus into a single harmonious picture, he decided that Philolaus had written a work in three books (18ff). This led Diels to the opinion that the two three-book works were in "competition" (*AGP* 1890, 461f). Harder (150) connected the "Plato" letter with Philolaus *Περὶ ψυχᾶς* (fr. 21).

[35] Neanthes *FGrHist* 84F26 = D.L. 8.55: *μέχρι Φιλολάου καὶ Ἐμπεδοκλέους ἐκοινώνουν οἱ Πυθαγορικοὶ τῶν λόγων*; the continuation, with its account of the break between Empedocles and the Pythagoreans, is connected with Timaeus *FGrHist* 566F14 = D.L. 8.54, on the *λογοκλοπία* of Empedocles (cf. *κωλυθῆναι* in Neanthes with *τῶν λόγων ἐκωλύθη μετέχειν* in Timaeus). There is nothing further in this passage about Philolaus, but D.L. 8.15 would be a natural continuation: *οὗτος δὲ μόνος ἐξήνεγκε τὰ διαβόητα τρία βιβλία.* Diogenes (8.15) and Iamblichus (*VP* 199) emphasize that no Pythagorean writings were known before the time of Philolaus. Demetrius of Magnesia attests that he was the first Pythagorean to write a book (D.L. 8.85); see also Euseb. *Adv. Hierocl.* 11, p. 380 Kayser (DK 44A8).

[36] Iamblichus' statement that Philolaus was impoverished (above, n. 29) is related to the Lysis letter (Hercher, *Epistologr. gr.* p. 603) and to the version of the *ἐξενεγκεῖν* of geometry found at Iam. *VP* 89 *Comm. math. sc.* p. 78.1ff.

this report is plainly different from those previously mentioned. Only one book is mentioned, it was written by Philolaus himself, and the price is 40 minas.[37] There is no mention of Dion or a letter from Plato, and the purpose is not the introduction of a forgery, but a charge against Plato, that the *Timaeus* was a plagiarism from Philolaus.

Hermippus, a pupil of Callimachus, is to be dated somewhat earlier than Satyrus, and himself cites a still earlier source; consideration of the content also shows that this story is earlier than the "tripartitum" legend.[38] The latter presupposes, without explanation offered, that one could get Pythagorean books from Philolaus. In this story, a direct connection is alleged between Plato and Philolaus' book, and this very theme is present in a still older piece of evidence. Timon, the sillographer, addressed Plato with the reproach,

> Much silver hast thou spent for one small book,
> From which thou then *Timaeus* learnd'st to write.[39]

This is not to be taken as meaning that Timon was the inventor of the story,[40] for charges of Platonic plagiarism are plentiful and even earlier than this. Alcimus found Plato's philosophy preexisting in Epicharmus; according to Theopompus, Plato imitated Aristippus and Bryson in his

[37] The expression "Alexandrian minas" is an appalling historical blunder (Boeckh 21, Schaarschmidt 77); it is incredible that an Alexandrian writer did not know that his city was founded by Alexander the Great. Perhaps a conversion computation was included and has been lost. (Alexandrian money was famous: D.L. 7.18; Paton and Hicks, *Inscriptions of Cos* 34.)

[38] The "tripartitum" legend drove the other one out: three books instead of one, 100 minas instead of 40, Pythagoras himself the author. Though the mention of the *Timaeus* hints at a work of physical philosophy, in the "tripartitum" ethics is in the foregound, in true Hellenistic fashion (Diels, *AGP* 1890, 462).

[39] *πολλῶν δ' ἀργυρίων ὀλίγην ἠλλάξαο βίβλον,*
ἔνθεν ἀπαρχόμενος τιμαιογραφεῖν ἐδιδάχθης,

Timon fr. 54 (*PPF*) = DK 44A8 = Gell. 3.17.6; also Procl. *In Tim.* 1.1.11 et saep.

[40] As did Rohde, Q 161. Timon cannot be the *συγγραφεύς* mentioned by Hermippus, either, for this word specifically designates a prose writer (Pl. *Phdr.* 278e, cf. 235c), *pace* Schaarschmidt 77. Aristoxenus is regarded as the originator of the allegation (Burnet, *EGP* 279f; Wilamowitz, *Platon* II 87; Geffcken [below, n. 41] 94; Wiersma, *Mnemosyne* 1942, 24; Raven in KR 308; Harder 41). Wehrli (*Aristox.* 67) does not distinguish between the Hermippus tradition and that from Satyrus, and arbitrarily attributes D.L. 8.15 to Aristoxenus fr. 43, as though the latter could have known the *διαβόητα τρία βιβλία*. The fact that a citation from Aristoxenus follows proves nothing, considering Diogenes' mosaic-style methods of composition. Iam. *VP* 199 (DK 14.17) stands between Aristoxenus passages (Iam. *VP* 197f = Aristox. fr. 30; Iam. *VP* 205ff = Aristox. fr. 38; but the citation probably begins at Iam. *VP* 200; cf. DK 58D8). Wilamowitz pointed this out (loc. cit.); but fr. 30 (followed by 31) belongs to the book *Περὶ Πυθαγορικοῦ βίου*, and fr. 38 to the *Πυθαγορικαὶ ἀποφάσεις*. Iamblichus uses a mosaic technique, too. There is some likelihood that the author may be Aristoxenus, but nothing approaching certainty.

dialogues; Aristoxenus stated that the *Republic* was contained *σχεδὸν ὅλη* in the *Ἀντιλογικά* of Protagoras;[41] and anyone who wanted to go back still further could maintain that Plato "stole" the doctrine of immortality from Homer.[42]

It is a mistake to suppose that the originals of Plato's alleged plagiarisms were ad hoc forgeries. Not only the Homeric poems but the books of Protagoras, Aristippus, and Bryson were readily available; and even the Epicharmus fragments cited by Alcimus may be genuine.[43] The charge of plagiarism is like a philological discovery, a clever inspiration that is all the more effective if the similarity is not apparent on the surface. The keen-witted *Kritiker* detects the "theft" which completely deceives the man in the street. Therefore we must suppose that the book of Philolaus, too, upon which the *Timaeus* was said to be based, really did exist, but not that it showed so thorough an agreement as does the "Timaeus Locrus" book. It is not necessary for the similarities to be much closer than those between Homer and Plato, for the malicious assertion to be made.[44]

The conclusion will be, then, that before Timon, in the fourth century, there was in existence a book of Philolaus which could plausibly be brought into connection with the *Timaeus*. The later tradition, and especially its use in the story of the "Tripartitum," shows that this book was regarded as the oldest book by a Pythagorean. What is more, the Menon papyrus proves the availability of a book of Philolaus in the fourth century.[45] According to this witness Philolaus' book contained discussion of medical matters; and such topics take up considerable space in the *Timaeus*.

Scholars have assumed that this book, though relatively early, cannot have been written by Philolaus, but must have been composed by someone else and only later given the prestige of his famous name.[46] But this leads immediately to the question, how famous he was and

[41] On these accusations of plagiarism, see Zeller II 1, 429 n. 7; E. Stemplinger, *Das Plagiat in der griechischen Literatur* (Leipzig, 1912) 25ff; J. Geffcken, "Antiplatonica," *Hermes* 64 (1929) 87–109. Alcimus: D.L. 3.9–17 = *FGrHist* 560F6 = DK 23B1–6; Theopompus: *FGrHist* 115F259 = Ath. 11.508c–d; Aristox. fr. 67 = D.L. 3.37, 57.

[42] Ath. 11.597e (from Herodicus).

[43] Alcimus forged the Epicharmus quotations he used, according to the view of A. Covotti, *I Presocratici* (Naples, 1934) 144. Bywater (26.1) cast doubt on Protagoras' book, though it is in Diogenes' catalogue of his works (9.55).

[44] Therefore it is not necessary to assume that Timon had the "Timaeus Locrus" book or the like, as Proclus thought (above, n. 39).

[45] A27–28. "It must be based on something in writing," says Wilamowitz (*Platon* II 88).

[46] Below, n. 86.

what he was famous for. Practically nothing is known of his life.[47] His home was Croton, or maybe Tarentum, and he spent some time in Thebes—all the rest, what little there is of it, is demonstrable embellishment or simple misunderstanding.[48] The mention of Philolaus in the *Phaedo* provides one fixed point chronologically: he must have been in Thebes before 399, and was therefore a contemporary of Socrates, though perhaps somewhat younger. Another fixed point can be found in the statement of Aristoxenus[49] that the "last Pythagoreans," with whom he was acquainted, had been pupils of Philolaus and Eurytus. Archytas spoke of Eurytus; and Philolaus, Eurytus, and Archytas are frequently associated in the tradition.[50] Apollodorus of

[47] Cf. Wilamowitz, *Platon* II 86f; Frank 294 n. 1 (hypercritical: "timeless, mythical person"); Wuilleumier 566ff. The statement of Raven (KR 312), that "there is abundant information concerning Philolaus in the works of several later writers," is at best misleading.

[48] Croton: D. L. 8.84, Menon (DK 44A27). Tarentum: Iam. *VP* p. 144.11, *In Nic.* 119.1, Vitruvius 1.1.16 (DK 44A6), Claud. Mam. *De statu an.* 2.3 (DK 44B22), and especially Aristoxenus fr. 19 (D.L. 8.46). Wuilleumier (567) tries for a compromise: he was born in Croton, and lived after the catastrophe in Tarentum.—Plut. *De gen.* 13.583a, in the dramatic framework of the dialogue, represents a Pythagorean named Theanor as having come to Thebes for the purpose of making offerings in honor of Lysis. He relates that Lysis, in his day, had escaped the great catastrophe of the Pythagorean society along with Philolaus. As against the report of Aristoxenus, who calls Lysis' companion Archippus (fr. 18, followed by Neanthes *FGrHist* 84F30), Plutarch's novelistic treatment has no value; he has simply put Philolaus in place of the completely unknown Archippus. Olympiodorus (*In Phd.* A I 13, p. 9 Norvin, followed by Schol. Pl. *Phd.* 61d = DK 44A1a) has Philolaus himself traveling to Thebes and says that he escaped along with Hipparchus (= Archippus?). He has apparently combined elements from Plato, Plutarch, Aristoxenus, and the Lysis letter.—The statements about the poverty of Philolaus are inventions to fill out the story (Iam. *VP* 199; above, nn. 29, 36).—According to Synesius (*De dono astrol.* 2, p. 134 Terzaghi; not in DK; followed by Theophylactus, DK I 419.19) Philolaus and Archytas were generals. This seems to go back to a source known to Cicero (*De or.* 3.139) which asserted that Philolaus the philosopher was the teacher of Archytas the general.—The report that Philolaus was killed because he was believed to be aiming at tyranny is based on a misunderstanding of his source by Diogenes Laertius (8.84; Olivieri 30; Wuilleumier 567; Maddalena 335); the source referred to Dion, who was mentioned just before.—The account of a meeting between Plato and Philolaus (D.L. 3.6) could be an invention based on the story of Plato's plagiarism.—Aelian remarks, on the injustice of fame (*VH* 1.23), *ἐν τοῖς Ἕλλησι τοῖς πάλαι μακρᾷ τῇ δόξῃ διέπρεπε Γοργίας ὁ Λεοντῖνος Φιλολάου καὶ Πρωταγόρας Δημοκρίτου, τῇ δὲ σοφίᾳ τοσοῦτον ἐλείποντο, ὅσον ἀνδρῶν παῖδες*. This simply means that Philolaus' fame came late—because of his book.

[49] Fr. 19 = D.L. 8.46; above, ch. II 5.

[50] In this order in the catalogue Iam. *VP* 267, p. 144.11; as immediate pupils of Pythagoras, ibid. 104 (see n. 51); Philolaus and Eurytus in connection with Plato, D.L. 3.6; Eurytus as a pupil of Philolaus in the legend, Iam. *VP* 139, 148 (related to Arist. fr. 193 and to what Hermippus tells of Pythagoras and Calliphon, Joseph. *Ap.* 1.164 = DK 19.2); Philolaus as teacher of Archytas, Cic. *De or.* 3.139 (the report of ps.-Demosth. *Erot.* 46 only mentions Archytas; cf. Harder 43 n.1); Vitr. 1.1.16, Synesius (above, n. 48).

Cyzicus, who must be dated earlier than Epicurus, said that Philolaus was teacher of Democritus.[51] Philolaus is, then, a name that appears in accounts of teacher-student relationships, which is connected with the Pythagoreans of Archytas' circle, of Phlius, and of Thebes, but which is not associated with a body of legend or anecdote. In the *Phaedo*, Plato represents him as having spoken about the prohibition of suicide, but as not having said "anything precise" about the reason for it.[52] Aristotle cites a saying of his,[53] but it has rightly been emphasized that nothing in Plato or Aristotle would give reason to think that he had written a philosophical book;[54] and apparently the ancient commentators on the *Phaedo* could find nothing relevant to their purpose in a book of Philolaus.[55] The meaning of all this is that the tradition of the special role of Philolaus as the first Pythagorean who published writings, and in particular of his special relation to the *Timaeus*, cannot have been spun out of Plato and Aristotle,[56] but must have been based on a fact independent of them—on the existence of a book by Philolaus in the fourth century B.C.; and we know from Menon that there was one. Philolaus' special position resulted from the fact that this book was the first—and for a certain period, perhaps, the only—written exposition of Pythagorean speculation on nature and on number. Naturally, we shall attribute to this book, and to Theophrastus' use of it, the doxographical reports pertaining to Philolaus.

[51] D.L. 9.38 = DK 74.2 = 44A2. Apollodorus (v.l. Apollodotus) is dated from his connection with Hecataeus of Abdera and Nausiphanes, who is reported to have been teacher of Epicurus (Clem. Al. *Strom.* 2.130; DK 73, 74, 75).—A letter purporting to be addressed by Pythagoras' son Telauges to Philolaus, and mentioning Empedocles, was already pronounced spurious by Neanthes (*FGrHist* 84F26 = D.L. 8.55, cf. 8.53, 8.74). Cf. Nicom. *Ench.* 9, p. 252.13: *Φιλόλαος ὁ Πυθαγόρου διάδοχος*.

[52] Pl. *Phd.* 61de. Suicide was regarded as dishonorable in Thebes (Arist. fr. 502).

[53] *E.E.* 1225a30 = DK 44B16; cf. above, ch. II 4, n. 139.

[54] Frank 294, Raven in KR 310, 312.

[55] To what extent that which follows in Plato—the *φρουρά* motif and the thought that a human being is *ἓν τῶν κτημάτων τοῖς θεοῖς* is to be attributed to Philolaus is uncertain. In any case Xenocrates, in his statement that the *φρουρά* is *Τιτανικὴ καὶ εἰς Διόνυσον ἀποκορυφοῦται* (fr. 20; see Heinze 150ff), is dependent on Orphism (Pl. *Crat.* 400c) and not on Philolaus. Nor are the other commentators, led by Olympiodorus (*In Phd.* p. 84f Norvin); therefore there was nothing on this subject in Philolaus' book.—Olympiodorus' manner of citing *acusmata* to explain the phrase *οὐδὲν σαφές* (*In Phd.* A I 13, pp. 8f Norvin) shows that he is thinking of them as allegorical, and therefore he is drawing on a late tradition, not Philolaus. Cf. above, ch. II 4.

[56] To be sure, a book was attributed to Cebes, too—*Cebetis Tabula* (see T. Sinko, "De lineamentis platonicis in Cebetis q.v. tabula," *Eos* 45 [1951] 3–31; he dates it in the 2nd century A.D.). And a whole series of writings, of the most dubious authenticity, are attributed to Simmias (D.L. 2.124f; cf. Hobein, *RE* III A 144–155).

From the time of Speusippus, Xenocrates, and Heraclides, a Platonizing interpretation took the place of what Aristotle gives us as the Pythagorean tradition; and in the succeeding era it held the field almost exclusively. Pythagoreanism no longer seemed meaningful except in this form, so that it was propagated by the Pythagorean pseudepigrapha and taken over by the neo-Pythagoreans. Only on certain special questions did one consult Aristotle; in general, the Platonic "system of derivation" was supreme, and there was even polemic, direct and indirect, against Aristotle. One exception stood contrary to this powerful and, for the most part, unified tradition—Philolaus.

According to the explicit testimony of Aristotle, Plato replaced the opposition of *πέρας* and *ἄπειρον* which had figured in Pythagorean speculation, with the pair *ἕν-ἀόριστος δυάς*, so that the place of older cosmological ideas is taken by a system of ontological derivation, whose leading ideas are in the realm of the categories form-matter and active-passive.[57] This system Speusippus regarded as Pythagorean, and in Aëtius too it is presented under the lemma "Pythagoras." Nothing is said of *πέρας* and *ἄπειρον*. In the enumeration of opinions about the *ἀρχαί*, however, we read: *Φιλόλαος ὁ Πυθαγόρειος τὸ πέρας καὶ τὸ ἄπειρον*.[58] When, later on, the opposites *πέρας* and *ἄπειρον* enter the scene once more, the earliest reference, Nicomachus, adds a quotation from Philolaus.[59] In this fundamentally important line of thought, in which, from the time of Theophrastus, Platonic terminology and content had replaced the pre-Platonic Pythagoreanism that Plato and Aristotle themselves discussed, the name of Philolaus appears as the sole representative of the original Pythagoreanism.[60]

Nor does this observation stand alone. The most fundamental difference between Pythagoreanism and Platonism, according to Aristotle, is *χωρισμός*, the discovery and definition of a supersensible, incorporeal realm of being, of which the Pythagoreans knew nothing. "Theano," Syrianus, and Proclus contradict Aristotle on this point.[61]

[57] Cf. ch. I 2–3.

[58] Aët. 1.3.10 = DK 44A9.

[59] Nicom. *Ar.* 2.18.4, citing Philolaus fr. 2; cf. above, ch. I 3, n. 43.

[60] The difference struck so late an author as Damascius. On the one hand, he knows the doctrine of "Pythagoras," about the Monad and the Indefinite Dyad, transcended by a supreme One (*Princ.* I 91.20 Ruelle; cf. Eudorus, above, ch. I 3, n. 53), but then on the other the Limit-Unlimited doctrine of the Platonic *Philebus*, which is connected with Philolaus fr. 1 (*Princ.* I 101.3; I 111.9 Ruelle). Thus the doctrine of "Pythagoras" and that of the Platonic *Philebus* are seen as contrasted (I 86.20ff; I 98.13ff Ruelle).

[61] Above, ch. I 2, nn. 22–23, I 3, n. 52.

Along with the Platonic development of a hierarchy of being went a terminological differentiation between *ἕν* and *μονάς*, though in two different ways. While Speusippus always called the highest principle, beyond being, *ἕν* and distinguished it from the *μονάς* in the realm of numbers,[62] the "Pythagorean" tradition set up the *μονάς* as the basic principle and relegated the *ἕν* to the realm of number and of sense perception.[63] All shared the objective of establishing clear terminological distinction between the realms of existence. But Theo adds, in the passage in which he deals with the distinction of *ἕν* and *μονάς*, *Ἀρχύτας δὲ καὶ Φιλόλαος ἀδιαφόρως τὸ ἓν καὶ μονάδα καλοῦσι καὶ τὴν μονάδα ἕν*.[64] In Philolaus and Archytas he could find no warrant for the separation of levels of being as the Platonists thought of them. Actually, according to Aristotle the Pythagoreans uttered the words *μονὰς καὶ ἕν* in a single breath.[65] Once more we see Philolaus, with Archytas, standing apart from the Platonizing line of interpretation.

Even more striking is a similar situation in the tradition about the movement of the earth. According to Aristotle, the Pythagoreans believed there was a "central fire," about which revolved, first, an invisible "counter-earth," and then our earth, "as one of the stars," and then in successive paths the moon, the sun, the five planets, and the fixed stars. This astronomical system, which has been celebrated in modern times as an anticipation of Copernicus, disappeared, except for a few traces, in the later Pythagorean tradition. (The ancient debate over the word *ἰλλομένην*, which Plato uses of the earth in the *Timaeus*,[66] kept alive the idea of the earth's moving.) According to the *Hypomnemata* (25) the cosmos has the earth "in the middle," and the same thing is presupposed by the biography in Photius[67] and by

[62] Cf. above, ch. I 3, n. 62, Iam. *Comm. math. sc.* p. 17.15 (for the attribution to Speusippus, see Merlan, *PlNeopl* 96–128). Xenocrates, according to Aëtius 1.7.30 (fr. 15 H.), spoke of the *Monás* as the highest god, and according to Favonius 5.7, of the *ἕν* (*unum*) as the highest principle (fr. 16 H). Perhaps it was not until after Speusippus and Xenocrates that terminological exactitude came to be highly valued.

[63] Above, ch. I 3, n. 30. Also ps.-Archytas p. 47.29 Thesleff.

[64] Theo Sm. 20.19f = DK 44A10 = 47A20.

[65] Arist. fr. 203 (p. 139 Ross) = Alex. *Met.* 39.15: *τὸν νοῦν μονάδα τε καὶ ἓν ἔλεγον*.

[66] Below, ch. IV 2, n. 16 (*Tim.* 40b).—The counter-earth is used in the interpretation of the *Timaeus* by Chalcid. *In Tim.* 122; further, Plut. *Numa* 11, *De an. procr.* 1028b. Por. *VP* 31 is unclear (*ἀντίχθων . . . ὑπὲρ ἡμᾶς*). Nicomachus *Th. ar.* 59.5ff, 82.3, 8 (10 *σφαῖραι*, the moon the third from the bottom), Lydus *Mens.* 4.51 p. 108.5.

[67] Anon. Phot. 439b17ff: under the 8 spheres of the heavenly bodies, the spheres of the 4 elements fire, air, water, and earth (cf. the Stoic teaching, D.L. 7.137, 155, ps.-Arist. *De mundo* 392a32ff). Asclepius (*Met.* 35.19ff) counts the elements below the moon as one sphere and adds the counter-earth as a tenth, in an attempt to reconcile the "10 spheres" with the geocentric system. (Perhaps the same motivation is present in Por. *VP* 31 and *Th. ar.* 59.5ff, 82.3, 8; cf. also Origen, comm. on John, 13.40.266).

Aëtius.[68] Theo quotes as Pythagorean doctrine the verses of Alexander of Ephesus on the harmony of the spheres, in which the earth is expressly said to be at rest in the center of things,[69] and Timaeus Locrus simply replaces Plato's worrisome *ἰλλομένην* with the word *ἱδρυμένα*.[70]

There is even outright polemic against Aristotle's report. In the ancients' way of thinking, diametrically opposed to the modern in this point, the idea of the earth's moving was a false notion which had been scientifically refuted;[71] and this forced upon them the task of showing that the Pythagoreans had not taught any such absurdity, but what was "correct." Thus Simplicius, in explication of the Aristotelian account of the Pythagorean system of the world, writes as follows (and quite similar words may be found in Asclepius and in an anonymous scholium to Aristotle):[72]

> This is the way he understood the Pythagoreans' theory himself. But those of them with more genuine knowledge understand by "central fire" the creative force which, from its mid position, produces life over the whole earth, and keeps warm the parts of it that tend to cool off. . . . And they used to call the earth a "star" because it too, like the stars, is a creator of time; for it is the cause of days and nights. The part of it which is shone upon by the sun makes day, and that which is in the cone produced by its shadow makes night. The Pythagoreans called the moon "counter-earth," as though to call it "the ethereal earth," and because it intercepts the sun's light, which belongs characteristically to the earth.

The expression about "more genuine" Pythagoreans here has been eagerly seized upon, because it seemed reasonable that there should have been a geocentric system as a precursor to the more complicated system described by Aristotle. Some have even asserted, mistakenly, that Simplicius is citing Aristotle,[73] though the contrast is clearly drawn

[68] Aët. 3.13.1: *οἱ μὲν ἄλλοι μένειν τὴν γῆν*, in contrast to Philolaus, Heraclides, Ecphantus, and Democritus; Pythagoras is obviously included among "the others." If, according to Aëtius (2.12.1, 3.14.1; cf. below, ch. IV 1), Pythagoras divides heaven and earth similarly into 5 zones, this too presupposes a geocentric system. Aët. 2.29.4 ascribes the counter-earth to "the Pythagoreans," citing Aristotle (above, ch. I 3, n. 26).

[69] Theo Sm. 138f, 141.12f. On this Alexander, see H. Dahlmann and W. Speyer, "Varronische Studien" II, *Abh.Mainz* 1959.11, 42ff; Burkert, *Philologus* 1961.

[70] 97d, p. 215.7 Thesleff. Proclus cites this in his interpretation of the *Timaeus* (III 138.3ff).

[71] For refutation of the idea that the earth moves, see Arist. *Cael.* 2.14, and esp. Ptol. *Synt.* 1.5, 7.

[72] Simpl. *Cael.* 512.9ff DK 58b37; more briefly Ascl. 35.24ff; Schol. Coisl. p. 504b42ff Brandis. Cf. ch. IV 1.

[73] Duhem 26f; Frank 257; Cornford, *Tim.* 128; Kranz, *RhM* 1957, 121. Contra, Cherniss, *Plato* 562, who suspects Iamblichus is the source (referring to Simpl. *Cael.* 507.12ff). Zeller judged the matter correctly, I 529f.

between the "more genuine" Pythagoreans and Aristotle "himself." Also, there is no question here of the earth's rotating.[74] The "more genuine Pythagoreans" are represented as explaining three expressions: *ἐπὶ τοῦ μέσου πῦρ*, the earth as *ἄστρον*, and the *ἀντίχθων*. These are the distinctive ideas in Aristotle's exposition of the Pythagorean system, where they appear in precisely this order;[75] and the counter claim being made is that these words do not mean what they suggest to the unprejudiced reader. The "central fire," they suggest, is a figurative expression for the life-giving force which, emanating from the center, permeates the universe; the earth is called a "star" because it is the instrument of time—a simple quotation from the *Timaeus*[76]—because the shadow of the earth makes night.[77] Finally, the moon is called "counter-earth" as being *αἰθερία γῆ*—an expression rather frequent in the late tradition, bound up with the notion of astral immortality and the moon-Hades.[78] In a word, the intent of the "more genuine" Pythagoreans is to show that Aristotle basically misunderstood Pythagorean astronomy, that it did not include a moving earth, and that it did not imply anything other than the "normal," geocentric cosmology which had been dominant since the Hellenistic age. Their method is a thoroughgoing allegorical interpretation of Aristotle's words. Just as Aristotle's exposition of the Pythagorean number theory was countered with the "original" words of Theano,[79] so here the attack on Aristotle is armed with the claim of "more genuine" doctrine.

In this way, then, an effort is made to expunge a Pythagorean doctrine of terrestrial movement which was felt as an embarrassment.[80]

[74] Schol. Coisl. 505a3f, on the earth: *τοῦτο δὲ τὸ ἄστρον φερόμενον νύκτα καὶ τὴν ἡμέραν ποιεῖν* . . . was taken as evidence of a moving-earth theory by Boeckh, *KosmSyst* 96; Duhem 89f; van der Waerden *Astr.* 58f (Heath, *Aristarchus* 250, thinks the scholium is based on a misunderstanding). The basis is merely Arist. *Cael.* 293a22f, reinterpreted.

[75] *Cael.* 293a21, 22, 24.

[76] *Tim.* 38c, 40c, 42d. Plutarch declares (*Quaest. Plat.* 1006e) that the passage does not imply any movement of the earth.

[77] Cf. Emp. fr. 48 (though he is not consistent; cf. A30), and the *μαθηματικοί* in Porphyry (Stob. 1.49.61; i.e. professional astronomers; above, ch. I 2, n. 76).

[78] Cf. Cumont, *Symb.* 187 n. 6. The moon as *αἰθερία γῆ*: Por. ap. Procl. *In Tim.* I 147 (presented as a teaching of the Egyptians; at II 48.17 further systematized); Macrob. *Somn. Sc.* 1.11.7, 1.19.10 (a doctrine of "physici"). *Ὀλυμπία γῆ*: Plut. *De def. or.* 13.416e. *οὐρανός* as *ἀντίχθων*: Clem. Al. *Strom.* 5.139. Ps.-Arist. fr. 245 (from a collection of *problemata* whose origin is doubtful) gives a different interpretation of the doctrine that the earth is a star and the moon is an earth, namely that moon and earth consist of the same elements.—Only a part of the moon is *ἀντίχθων*, according to Plut. *De fac.* 29.944c; in Cic. *Tusc.* 1.68, the inhabited southern zone of the earth is *ἀντίχθων*.

[79] Above, ch. I 3, n. 52.

[80] Proclus sharply criticizes Aristotle for understanding the word *ἰλλομένην* in the *Timaeus* as denoting a rotation of the earth (*In Tim.* III 137.7ff).

Aëtius, however, distinguishes Philolaus from the "others" who did not accept any movement of the earth—and he includes Pythagoras here—and the astronomical system he ascribes to Philolaus corresponds exactly to that which Aristotle ascribed to the Pythagoreans.[81] People have always been surprised at this remarkable fact, not noticing that the situation is not unique. Not only in astronomy, but in relation to the first principles and the derivation of the levels of being, Philolaus stands in the doxographical tradition as the representative of the Pythagoreans described by Aristotle, though generally Platonic interpretation has taken the place of what was really Pythagorean. Thus too, the doxographical report that Philolaus spoke of the destruction of the cosmos (*φθορὰ τοῦ κόσμου*) corresponds to Aristotle's statement that the Pythagoreans "without a doubt" taught that the universe had a beginning, whereas the later tradition projects onto Pythagoras the Platonic-Aristotelian eternity of the world.[82] If it is necessary, on historical principles, to abandon the Platonic interpretation of Pythagoreanism and realize that only the reports of Aristotle give reliable indications of the genuine ancient teachings, it follows that in the doxographical tradition the Philolaus items are all that remains of what is genuine. And, since these very probably go back to Theophrastus (we recall that Menon, another of Aristotle's pupils, cited Philolaus), there must have been a book of Philolaus available in the fourth century that contained pre-Platonic Pythagorean doctrine.

The agreement of the Philolaus tradition with Aristotle's accounts of Pythagorean doctrine has attracted the attention of many.[83] It can be explained in either of two ways: either Aristotle gets his information about Pythagorean doctrine at least partly from Philolaus' book,

[81] Aët. 2.7.7, 3.11.3, 3.13.2 (= DK 44A16, 17, 21); cf. below, ch. IV 3.

[82] Philolaus A 18 = Aët. 2.5.3. This disagrees sharply with ps.-Philolaus fr. 21, but Stobaeus (1.20.1–2) cites the two one after the other. On Arist. *Met.* 1091a13, above ch. I 3, n. 119; on Pythagoras ap. Aët. 2.4.1, above, ch. I 3, n. 117; cf. *τάξιν ἄλυτον*, p. 165.4 Thesleff.

[83] Cf. Zeller I 369 n. 3, *Kl.Schr.* I 140ff; Frank 254, 290f, 318 n. 1, 327f; Bollinger 70ff; Mondolfo in ZM 367ff; Raven, *PyEl* 98ff, KR 309ff. To collect the key points (cf. ch. I 2): *πέρας-ἄπειρον*, Philolaus A9, B1, 2, 6; even and odd, Philolaus B5; *ἀρτιοπέριττον*, Philolaus B5; origin of the *ἕν* and of the world: Philolaus B1, 6, 7; only one world: Arist. fr. 201, Philolaus B17; the role of the number 10: Arist. *Met.* 986a8, Philolaus A13, Theo Sm. 106.10; the astronomical system, ch. IV 3; the role of mathematical music theory: Arist. *Met.* 985b31, Philolaus B6, A26, below, ch. V 2. On the connection of the "breathing world" of Arist. *Met.* 1091a13f, *Phys.* 213b22, fr. 201, with Philolaus A27, above, ch. I 2, n. 47.

or the book was forged on the basis of Aristotle's accounts.[84] In the first case the book must be regarded as genuine, that is, composed by the Pythagorean Philolaus about 400 B.C. or a little earlier. There is no basis on which to argue that the book Aristotle had was a forgery,[85] or that an authentic and ancient Pythagorean book had been attributed to Philolaus.[86] Naturally, the question would still remain, whether all the fragments we have come from this original work, or whether some or all may come from a revised edition, an imitation, or even a forgery made to supply its loss. Nevertheless, the Philolaus tradition, however troubled, would even in this situation bear witness to a source of tremendous importance.[87]

Aristotle never names Philolaus, except when he cites that memorable apophthegm in the *Eudemian Ethics*,[88] and critics have made this their principal argument against the authenticity of the fragments. If Aristotle used this book, they think, his silence is quite "inconceivable."[89] There is a prior question to answer, however: what *were* Aristotle's sources for Pythagorean teachings? Original Pythagorean writings? Oral tradition? Writings of the Platonists?

This last possibility[90] is excluded for most of Aristotle's reports, for, quite unlike the Platonists, he makes a sharp distinction between Pythagoreanism and Platonism. The role of oral tradition ought not to be underestimated,[91] though here too Platonists were the intermediaries

[84] So Bywater (50), following Schaarschmidt (15), and recently esp. Raven, *PyEl* 98ff, KR 309ff. Ch. III, 2, n. 91.

[85] So Frank 290f, 327f, and passim. He tries to show that the forger was Speusippus (331ff); similarly Bollinger 44f. Against this, above, ch. I 2.

[86] Wilamowitz, *Platon* II 93, thinks of an original Pythagorean writing, later than Aristoxenus, which was then later fathered on Philolaus; but this neglects the close connection with the account of Aristotle.

[87] The Philolaus fragments are sometimes cited in this spirit, as a "forgery" which nevertheless includes valuable material; e.g., Ross, *PTI* 160f.

[88] 1225a30 = DK 44B16; above, ch. II 4, n. 139.

[89] The word "inconceivable" is Burnet's (*EGP* 284 n. 2); "almost inconceivable" Raven in KR 310, cf. *PyEl* 100; Schaarschmidt 14, Tannery *MSc* IX 232f; Wilamowitz, *Platon* II 88, 93 ("Aristoteles und Aristoxenos kannten es [Philolaus' book] nicht"); Cherniss, *Pres.* 37 n. 140 ("Aristotle's silence certainly implies that he had not seen the book of Philolaus"), 386f. For a different answer, see Zeller (*KlSchr* I 136–144) and A. Burns, *C&M* 25 (1964) 93–128.

[90] Tannery, *MSc* IX 234, thought that Heraclides was a principal source of Aristotle; Frank nominated Speusippus, along with other Platonists (258f, 290f, 327f); similarly Howald and Bollinger. Speusippus may be the source for the "table of opposites" (above, ch. I 2).

[91] What Archytas told about Eurytus may have been passed on, as a curiosity, in the Academy and the Peripatos (above, ch. I 2, n. 69). On the *Πυθαγορικαὶ ἀκροάσεις* mentioned in connection with Speusippus (*Th. ar.* 82.12), see below, ch. III 2.

through whom he knew Pythagoreans, so that he is hardly likely to have derived from this source the decisive points that he was able to make against the Platonists. He expresses himself in very definite terms: *οἱ μὲν οὖν Πυθαγόρειοι πότερον οὐ ποιοῦσιν ἢ ποιοῦσι γένεσιν οὐδὲν δεῖ διστάζειν· φανερῶς γὰρ λέγουσιν ὡς. . . .* Aristotle is citing the "clear words" of the Pythagoreans in the discussion as to the sense in which one may speak of a *genesis* of number and thus of the world as a whole; and his polemic is aimed against a Platonizing reinterpretation.[92] The fact that he can insist on the exact wording in this way shows that he has a written source, for oral reports are not amenable to such exactitude. The use of written sources is also suggested by the way in which Aristotle cites Pythagorean technical terms, distinguishes their statements from their "assumptions," and sometimes even states that this or that question remains unanswered.[93] We cannot suppose, however, that the book or books he read claimed to be written by "so-called Pythagoreans" (*καλούμενοι Πυθαγόρειοι*). The name of an individual must have been attached to each. To put it bluntly, even if we disregard the problem of Philolaus, the "inconceivable" is just what happened. Aristotle based his account of Pythagoreanism on at least one book, whose author he never named.

The Anonymus is not Archytas;[94] what Aristotle cites from him indicates an advanced degree of abstraction, though the evidence is far from plentiful. In addition to his books on the Pythagoreans, Aristotle wrote three *περὶ τῆς Ἀρχυτείου φιλοσοφίας*, and this may be taken to show that he distinguished clearly between Archytas and general

[92] *Met.* 1091a13ff, above, ch. I 2, n. 41. Zeller pointed out the significance of the passage (*KlSchr* I 138ff). Schaarschmidt (82ff) denied that Aristotle had written sources for Pythagoreanism.

[93] *χροιά*, *Sens.* 439a30 (above, ch. I 3, n. 96), *μίμησις*, *Met.* 987b11 (above ch. I 2, n. 82), *ἐξ ὧν γὰρ ὑποτίθενται καὶ λέγουσιν . . .*, *Met.* 990a14, . . . *ἀπορεῖν ἐοίκασιν*, *Met.* 1080b20.

[94] *Pace* Frank 361: "Archytas seems to have been the only Pythagorean from whose writings Aristotle learned about Pythagorean philosophy." This is repeated by Wuilleumier, 577. Cf. also Frank 135f, with n. 387, and pp. 77, 196. Aristotle's citations of Archytas, though, do not yield much: *Pol.* 1340b26 (DK 47A10) *Ἀρχύτου πλαταγή*, a child's toy (Frank, 339, rightly compares Pl. *Leg.* 653d); *Rhet.* 1412a12 (DK 47A12) *ταὐτὸν εἶναι διαιτητὴν καὶ βωμόν*, an apophthegm that is reminiscent of the *acusmata*. The "definitions" of Archytas (above, ch. I 2, n. 104) take us a bit further. At Arist. fr. 199 = Theo Sm. 22.5ff, *συμφέρεται δὲ τούτοις καὶ Ἀρχύτας* (DK 47A21), is obviously an addition by Theo (who on another occasion connects Archytas with Philolaus; above, n. 64). Frank states this, though with hesitation (257 n. 2), only to affirm later (361) that Aristotle "cites as the special doctrine of Archytas (fr. 199 Rose) a view which in another passage (*Met.* 996a18) he presents as general Pythagorean."—Philosophically interesting are Archytas A23 (ch. I, 2, n. 106), A23a (I 3, n. 156), and A24 (Eudemus fr. 65).

Pythagoreanism.[95] Archytas on occasion mentioned earlier Pythagoreans,[96] but he was an independent scholar and surely was not concerned to write as detailed a doxography as would have been necessary if Aristotle were to get all his facts about Pythagorean philosophy from it.

The only name that is closely connected, in Aristotle's reports, with the philosophy of the "so-called Pythagoreans," is that of Eurytus; but he seems himself to have essayed an extension of the number theory. Now, from Aristoxenus on down, the tradition constantly associates Philolaus and Eurytus.[97] Philolaus was the teacher of the Pythagoreans from Thebes and Phlius whom Plato knew. So we see the skimpy biographical evidence on Philolaus converging with what can be deduced from the doxography about his book; namely, that it was to Aristotle, in a. way, an authoritative exposition of Pythagorean number theory.

Philolaus does not appear as a rebel against the traditional obligation which Pythagoreans and, to a certain extent, Platonists felt, to recognize that all doctrine originating in their own minds was only extension and confirmation of the ancient wisdom of Pythagoras.[98] At least his pupils, and Plato too, must have understood Philolaus' work in this way. It is only from this point of view that we can understand Aristotle's strange uncertainty about the chronology of the Pythagoreans, as well as his silence about the originator of the system. Subsequently the Platonic view of Pythagoras became dominant, and that book was no longer an accurate presentation of Pythagoreanism, as it was now understood. Its doctrines now appeared to be the private *δόξα* of the author, Philolaus.

The thesis, on the other hand, that the book of Philolaus was a forgery that took its materials from Aristotle takes no account of the general development of the Pythagorean tradition. It is simply not true that the picture of Pythagoreanism standard in late antiquity was derived in its essentials from Aristotle's exposition.[99] The Platonic

[95] Cf. P. Moraux, *Les listes anciennes des ouvrages d'Aristote* (Louvain, 1951) 106, 201, 301 (DK 47A13). Frank (n. 335) considers referring to this book Philop. *Aet. mundi* p. 522.20 (on the *πέμπτον σῶμα*). Some kind of connection between Archytas and the *Timaeus* is indicated by the title *Ἐκ τοῦ Τιμαίου καὶ τῶν Ἀρχυτείων* (no. 94 in D.L. 5.25, no. 85 in the anonymous catalogue); cf. above, ch. I 4, nn. 4–5, I 3, n. 166.

[96] Cf. fr. 1 and the mention of Eurytus (above ch. I 2, n. 69).

[97] Above, n. 50.

[98] Above, ch. I 4, n. 36.

[99] The view of Schaarschmidt, p. 15.

interpretation is dominant, and the evidence of Aristotle was ignored, distorted, or rejected.[100] The "forgeries" were not intended to deceive philologers, but to provide a cachet of age-old wisdom for a doctrine adapted to the needs of the writers' own times. This is the reason why the pseudo-Pythagorean writings take Platonism as their basis. If the Philolaus reports alone, along with some material from Archytas, form an exception to this general trend, then their agreement with Aristotle is not a reason to suspect them, but quite the opposite: an indirect proof of their authenticity. Of course, only a keen and attentive study of all the details, especially in the directly quoted fragments, can enable us to decide, in each case, what reason there may be to believe that they are authentic or spurious. But the external tradition itself shows that the situation of Philolaus is different from that of the great mass of would-be Pythagorean authors. A book made him famous; it was still in existence in the fourth century; and it showed traces, at least, of ancient, pre-Platonic Pythagoreanism.

2. THE SPURIOUS AND THE GENUINE IN THE PHILOLAUS FRAGMENTS

If any genuine fragments of Philolaus' book have been preserved, they are the most important, because the only original, documents of early Pythagorean philosophy. This statement can easily lead to a double misunderstanding. One expects to find bedrock, so to speak, forgetting that Philolaus was a contemporary of Socrates and Democritus, Gorgias and Diogenes of Apollonia, and that he wrote and taught not only after Parmenides, but later than Zeno, Melissus, Anaxagoras, Empedocles, and Protagoras.[1] Again, one is likely to expect to find thoughts of striking and unique originality,[2] overlooking the fact that in the second half of the fifth century, thinkers were much more concerned with assessing the many original but contradictory assever-

[100] It was modern critical method, especially the work of Zeller, that first set Aristotle's testimony, clearly distinguished from the Platonizing and neo-Pythagorean conception of Pythagoras, in a position of authority which, as is easy to forget, it certainly did not enjoy in the ancient world.

[1] Diels, *Hermes* 1893, 417ff, even considered the etymology *φλέγμα–φλέγειν* in Menon's excerpt (A27–28) as borrowed from Prodicus (fr. 4); cf., per contra, Fredrich 37 n. 1, Olivieri 29f, 45f.—Frank (304) states it is impossible to suppose that Plato harked back, in the *Philebus*, to a book "almost 100 years old"; but at the time of the composition of the *Philebus* Philolaus' book was scarcely more than 50 years old.

[2] Bywater (29) does not see in the fragments an "original effort of mind," and athetizes them on the basis of this "test of excellence." But maybe what was original in Pythagoreanism was not philosophical expression.

ations of their predecessors—comparing, reconciling, adapting. It was no longer a day of lonely prophets but one of far-reaching debate. A statesman like Melissus of Samos might take part in it, or a poet like Ion of Chios; physicians, too, were beginning to formulate in written terms the scientific basis of their art. A book by a Pythagorean, in this period, cannot have been so much like an erratic boulder as a link in a long chain of tradition.

We are inclined to see in Philolaus nothing but the transmitter of older Pythagorean teachings, and in fact all indications are that the Pythagoreans themselves saw nothing more in his book than an expression of the wisdom of Pythagoras. But before Philolaus there was no written exposition of Pythagoreanism;[3] and the transition from oral to written teaching is much more than a matter of externals; it means a fundamental transformation, of content as well as of form. An orally transmitted doctrine, in spite of all the special training of the memory in ancient times, notably among the Pythagoreans, must always be enmeshed in the very fabric of life, always exposed to psychological forces that can mold and transform it. No matter how persistently one holds to a traditional knowledge, even if the basic attitude and intentions remain unaltered, changes of detail will appear. As needs change, from time to time, different aspects will move into prominence. Oral teaching is always directed toward specific learners, and therefore its nature is determined by the listener as well as the speaker. In particular, the form of oral transmission will alter, because it must each time be impressive—except that poetic form guarantees a certain consistency. On the other hand, a written work is free from the restraint of a specific situation of speaking and hearing. It makes a claim to be valid in itself, independent of any special *καιρός*, and not needing any further intervention by the writer. A book in prose, furthermore, renounces the embellishment of verse, because its message is supposed to be dependent entirely on the subject matter, and not at all on any predetermined formal structure. The first prose book to prefigure and exemplify this development was that of Anaximander.[4] It is only when linguistic expression attains this kind of objectivity that scientific or philosophical discourse really becomes possible; from the time of Anaximander, Greek science and philosophy

[3] Above, ch. III 1.

[4] DK 12A7; cf. A1. Pherecydes of Syros, who is also named as the first writer of prose (*Suda* s.v. = DK 7A2), may be dependent on Anaximander (von Fritz, *RE* XIX 2030f).

presents itself in this form, of written exposition.[5] This means—though the fact has scarcely ever been clearly recognized—that, if Pythagorean doctrine was not committed to writing before Philolaus, then there did not exist, before Philolaus, any Pythagorean philosophy, in the Greek sense of the word, but only a different kind of thing: a lore or "wisdom" consisting of disconnected teachings about the world, gods, and human beings, having its foundation in a specific way of life and transmitted in individual maxims. This "wisdom" and way of life were variable in detail and lacked logical foundation or systematic and conceptual coherence; in fact they consisted in our familiar *acusmata*, the doctrine of transmigration, and the *βίος Πυθαγόρειος* in which they were rooted.

If a Pythagorean, in a situation like this, undertook to write *περὶ φύσεως*, this was a *μετάβασις εἰς ἄλλο γένος*. It meant the adoption of a manner of exposition foreign to the Pythagorean tradition, the product of a different kind of development. The tools for such an enterprise—concepts, definitions, modes of argument—come from a non-Pythagorean background. What we should expect to find then, would not be a unitary and original product, but a hybrid eclecticism,[6] a conglomeration of Pythagorean attitudes and borrowed conceptualization. For there is clearly nothing in the Pythagorean teaching itself that would demand written consolidation, or formulation in philosophical and scientific terms.

August Boeckh decided, with relation to the Philolaus *testimonia* as a group, that "the only solution is to recognize all we have as genuine or to reject all of it as spurious."[7] Those who doubt its authenticity have been happy to cite this sentence, for it opens up the possibility of affecting the whole by an attack on a part—if they could loosen one column, the whole structure would collapse.[8] But this is a hasty

[5] When Zeno visited Athens, he brought his *γράμματα* with him, and people gathered around him to get to know his book (Pl. *Parm.* 127c; the report is not likely to be historically accurate, but depicts what must have been a possible occurrence). In his depreciation of the written word, Plato is rebelling against the prevailing fashion, in favor of older ways.

[6] Cf. on Gorgias, ps.-Arist. *MXG* 979a14: *συνθεὶς τὰ ἑτέροις εἰρημένα*. C. Schick, *Arch. glottol. ital.* 40 (1955) 128ff, thinks that the book bearing Philolaus' name presupposes a "tradizione interna della scuola pitagorica," a sort of Doric jargon belonging to the school; but this is not proven by Por. *VP* 53 and Iam. *VP* 241f, any more than by the alleged echoes of it in Parmenides and Zeno (below, ch. III 3).

[7] Boeckh 38.

[8] Schaarschmidt 2f, 79 (citing Boeckh), Bywater 50, Frank 290. No one any longer thinks of applying the same procedure in the case of Archytas, where the spurious material amounts to more than 10 times as much as the little bit that is genuine (see above, ch. III 1 n. 14).

oversimplification, especially since Boeckh's conciliatory assumption that Philolaus wrote a single work in three books has turned out to be based on a misunderstanding. Hermippus speaks of "one book" of Philolaus,[9] and Demetrius of Magnesia gives its title, *Περὶ φύσεως*, and cites the opening line, an established bibliographical custom.[10] This citation is closely related to the long excerpt in Stobaeus, to which he gives the heading *Περὶ κόσμου*, but from which Nicomachus gives a fragment with the label *Φιλόλαος . . . ἐν τῷ πρώτῳ φυσικῷ*.[11] Stobaeus' evidence for the title is not important, for his own section, within which Philolaus is cited, bears the heading *Περὶ κόσμου*. Nicomachus in the *Theologumena* has a further fragment of a *Περὶ φύσεως*,[12] and this title is also mentioned by Theo Smyrnaeus.[13] Proclus[14] and Boethius[15] are both doubtless dependent on Nicomachus. The doxographical

[9] Above, ch. III 1.

[10] D.L. 8.85 = DK 44B1: *τοῦτόν φησι Δημήτριος ἐν Ὁμωνύμοις πρῶτον ἐκδοῦναι τῶν Πυθαγορικῶν ‹βιβλία καὶ ἐπιγράψαι› Περὶ φύσεως, ὧν ἀρχὴ ἥδε*. Diels's supplement can scarcely be right, and his reference to D.L. 3.9 is certainly a mistake (above, ch. III 1, n. 28). Perhaps there is no lacuna, and the object of *ἐκδοῦναι* is the title (cf. Strabo 1, p. 15: *ἡ περὶ τῶν ἀγαθῶν ἐκδοθεῖσα ὑπ' αὐτοῦ πραγματεία*; Plut *Rom.* 8 [*ὁ Πεπαρήθιος Διοκλῆς*] *ὃς δοκεῖ πρῶτος ἐκδοῦναι Ῥώμης κτίσιν*). The indefinite plural *ὧν* does not necessarily mean that more than one book is intended. After all, only one "beginning" is being quoted.—In a similar way, D.L. cites the incipit of books of Diogenes of Apollonia (9.57, 6.81), Pherecydes (1.119), Alcmaeon (8.83). At bottom lies the practice of the library at Alexandria (U. von Wilamowitz-Moellendorff, *Antigonos von Karystos* [Berlin, 1881] 323; Wiersma, *Mnemosyne* 1942, 23).

[11] Stob. 1.21.7 = DK 44B2, 4–7. There was another excerpt in the lost prologue of Stobaeus; the only trace remaining is the marginal note, *Φιλολάου* (Stob. I p. 15.4 App.). Nicom. *Ench.* 9, p. 252.13ff (the title cited); pp. 252.17–253.3 = DK I 409.10–410.3. Evidently the short, very corrupt citation in the *Hagiopolites* is from Nicomachus (M. A. J. H. Vincent, *Notices et extraits des manuscripts* XVI 2 [Paris, 1847] 268 = DK 44B6, I 409.10f: *ὁ Πυθαγορικὸς Φιλόλαος, ἔν τινι πονήματι αὐτοῦ, πρός τινα γυναῖκα Πυθαγορείαν ἐκτιθέμενος*. See also Tzetzes, cited ch. III 1, n. 28.) Further, Nicom. *Ench.* 12, p. 264.3 (= fr. 6, DK I 410.1f); *Ar.* 2.18.4 (= B2, DK I 407.2f) (translated by Boethius in *Ar.* 2.32, p. 126.7ff); Nicom. *Ar.* 2.26.2 (= A24). See next note.

[12] *Th. ar.* 25.17 (= fr. 13); on 74.10 (= A12), see below, nn. 41–44.

[13] 106.10 (DK ad fr. 11; below, n. 169).

[14] Procl. *In Tim.* I 84.4, I 176.28ff (*ἐκ περαινόντων καὶ ἀπείρων* = frr. 1–2; cf. Nicom. *Ar.* 2.18.4), *Theol. Pl.* 3.7, p. 132 Portus. Probably the expressions *δίεσις* and *ἀποτομή* are from Nicomachus (Procl. *In Tim.* II 168.28f; the source is given as *τῶν παλαιῶν τινες*); II 190.7ff (cf. A26, B6); perhaps also the report about the gods of various angles (A14; cf. below, ch. IV 3); cf. *Theol. Pl.* 1.4, p. 9 Portus: *καὶ γὰρ τοὺς ἀριθμοὺς ἀνεῖσαν τοῖς θεοῖς καὶ τὰ σχήματα, καθάπερ λέγουσιν οἱ τὰ ἐκείνων ἱστορεῖν σπουδάζοντες.*—Damascius is probably drawing directly on Proclus at *Princ.* I 111.12 Ruelle (*ἐκ περαινόντων καὶ ἀπείρων*), cf. I 101.3: *ἐν τοῖς περὶ φύσεως*. Also, Damascius may have drawn his information about the dedication of certain geometric figures to various gods (A14) from an account of Proclus which has been lost.

[15] Cf. above, n. 11. Boethius surely derived even details of music theory from Nicomachus, whose musicological writings are only preserved in fragments (*Mus.* 3.5 = A26; 3.8 = n. on B6). Iamblichus, too, can have taken his Philolaus citation from Nicomachus *In Nic.* 7.24 = fr. 3; repeated by Syrianus, at *Met.* 147.17).

reports in Aëtius[16] cohere closely with these citations, suggesting the probability that this book *Περὶ φύσεως* from which Demetrius of Magnesia (time of Cicero), Nicomachus, and Stobaeus quoted was the one that Theophrastus read.

Caution must be the watchword, however. Nicomachus knew of more than one book attributed to Philolaus, if he could cite the "first." Stobaeus has a fragment from a book *Περὶ ψυχῆς*,[17] and both Stobaeus and Proclus mention a title *Φιλολάου Βάκχαι*.[18] Claudianus Mamertus speaks of "many books" that Philolaus wrote, and cites a "third book *περὶ ῥυθμῶν καὶ μέτρων*."[19] And in addition we have a long passage given without title by Stobaeus,[20] and various isolated citations.

What the tradition offers us, then, under the name of Philolaus, is not a unified picture, but a great variety. The first task that suggests itself is to demonstrate its homogeneity, but there seems hardly any prospect for success in that, if only because of the open contradiction between the "one book" of the older tradition and the plurality of books in the later. It is true, though, that from the beginning a certain nucleus seems to be discernible: fragments 1, 2, and 4–7, along with the accounts of Aëtius and Boethius, dealing with ontology, astronomy, and music theory.

We may take as the starting point for a closer examination of the tradition the fact that one of the longer fragments is certainly spurious, namely the paragraph on the world soul (21). Since Zeller pronounced it spurious, no one has seriously defended it, and it is unnecessary to repeat all the arguments.[21] This passage is an example of Hellenistic

[16] A9, 15–21. The "indirect" proof of authenticity applies for these accounts (above, ch. III 1).—Also from the doxographical tradition are the notes in Cens. 18.8 (A22), Macrob. *Somn. Sc.* 1.14.19 (A23), Sext. Emp. *Math.* 7.92 (A29; below, n. 55).

[17] Stob. 1.20.2 = fr. 21.

[18] Frr. 17–19; below, nn. 140–148.

[19] Fr. 22; below, n. 45.

[20] Frr. 11–12; below, nn. 169–185.

[21] Zeller I 369.3, 476.1, *KlSchr* I 145ff; Schaarschmidt 24f; Bywater 40ff; Frank 282ff; Moreau, *Âme* 145ff; Thesleff, *Texts* 150f. A few points may be emphasized, beginning with the Aristotelian doctrine of the eternity of the world *ἐξ αἰῶνος εἰς αἰῶνα*, and the presence of technical terms like *ἐνέργεια* (DK I 418.6), *τῷ γεννήσαντι πατέρι καὶ δημιουργῷ* (I 418.11; cf. Pl. *Tim.* 37c; here even Boeckh is forced to admit the intrusion of late terminology; cf. Frank 290 n. 1), *φύσει διαπνεόμενος* (I 417.14) like the Stoic *πνεῦμα διῆκον* (for such Stoic coloration in later Pythagoreanism, see Cic. *Nat. d.* 1.27, Sext. Emp. *Math.* 9.127). At I 417.14, the meaningless *ἐξ ἀρχιδίου* of the MS tradition should be replaced by *ἐξαρχίδιον* (as in the inscription cited ad loc. in DK, *CIG* 5235.2, SIG[3] 712.1). It is an adjective, formed after *ἐξ ἀρχῆς* as *ἐξαιφνίδιος* after *ἐξαίφνης*, and is

"cosmic piety," the resultant of the hymn-like eloquence of the *Timaeus* and Aristotle's doctrine of the eternity of the universe. Significantly, the fragment has a good deal of wording like that of Ocellus. If Harder is right that it is Ocellus who is derivative,[22] then this fragment must have been composed, at the latest, in the second century B.C., and the one certainly spurious fragment would be the earliest attested.

The decisive question is whether this fragment is closely connected with the rest. Frank tried to prove this,[23] but his arguments do not suffice. There are points of contact with some elements, contradictions of others. In contrast to the hymn-like celebration here of the eternity of the world, *ἐξ αἰῶνος καὶ εἰς αἰῶνα*, where the "creator," as in the Academic tradition, is to be understood as a didactic concept, not a reality,[24] one of the other doxographical notes speaks about the *φθορὰ τοῦ κόσμου*,[25] and expressions like *ἁρμόχθη*, *συνέστα*, *ἤρξατο γίγνεσθαι*, and *ἁρμονία ἐπεγένετο* in the word-for-word citations[26] presuppose an origin for the "order" of our world. The contradiction is evident.

Under the rubric *Περὶ τάξεως τοῦ κόσμου*, Stobaeus[27] first ascribes to Philolaus the cosmic system known from Aristotle: the order is

predicative with *ἀρχάν* in line 13. On the question of the *ἀρχὴ κινήσεως* see Arist. *Cael.* 285b5ff.—Macrobius cites as the doctrine of a *secta* of *Platonici* sentences that show extensive verbal agreement with Philolaus fr. 21 (*Somn. Sc.* 1.11.5f; noted by Capelle, *De luna* 9).—Frank, in accordance with his general theory, must credit this fragment to Speusippus, although this compels him to consider the possibility of "spätere Retuschen" (290). Moreau (*Âme*), attempting to find intermediate steps between Plato and the cosmology of the Stoics, would like to date the fragment in the age of Aristotle (149), making it approximately contemporary with the *De caelo*; but he cannot find convincing parallels to *φύσει διαπνεόμενος* (pp. 136ff).—Rostagni (*Verbo* 53) maintains that at least in content the fragment preserves genuine material, in spite of its late style; one wonders if it is not rather the case that Rostagni's version of Pythagoreanism shows Platonic features.

[22] The parallels are collected by Schaarschmidt (20), Bywater (39f) Harder (95f, 119f), and Beutler, *RE* XVII 2364ff. Harder supported his argument for the priority of "Philolaus" with a faulty combination (above, ch. III 1, n. 34), leading Beutler (*RE* XVII 2363) to deny it; but the smooth style and perspicuous organization of the "Philolaus" fragment make its priority seem likely.

[23] 285ff. Contra, Mondolfo in ZM 376f. It is admitted that there are points of agreement with the second part of A16 (cf. below, n. 37), A17 (below, n. 38), and fr. 22. But it is wrong to interpret *ἀρχὰ ξυναπάντων* in fr. 13 as "world soul"; *ψυχά* (= "life") is explicitly distinguished from it. The emphasis on the *one* world (fr. 17; presupposed in frr. 1, 2, and 6) does not prove anything; this is an old controversy (Aët. 2.1.2–3), and Aristotle ascribes the doctrine that there is but one world expressly to the Pythagoreans (fr. 201); cf. below, n. 26.

[24] Above, ch. I 3, n. 118.

[25] A18; cf. ch. III 1, n. 82.

[26] Frr. 1, 6, 7, 17; cf. Arist. *Met.* 1091a13ff, below, ch. I 3, n. 119.—Frank (288f) equates the eternity of the world in fr. 21 with the eternity of being in fr. 6; cf. below nn. 86–87.

[27] Stob. 1.22.1d Aët. 2.7.7 (missing in ps.-Plut.) DK 44A16.

οὐρανός, then the five planets, sun, moon, earth, counter-earth, and as ἑστία the central fire. But then he goes on:

> τὸ μὲν οὖν ἀνωτάτω μέρος τοῦ περιέχοντος, ἐν ᾧ τὴν εἰλικρίνειαν εἶναι τῶν στοιχείων, Ὄλυμπον καλεῖ, τὰ δὲ ὑπὸ τὴν τοῦ Ὀλύμπου φοράν, ἐν ᾧ τοὺς πέντε πλάνητας μεθ' ἡλίου καὶ σελήνης τετάχθαι, κόσμον· τὸ δ' ὑπὸ τούτοις ὑποσέληνόν τε καὶ περίγειον μέρος, ἐν ᾧ τὰ τῆς φιλομεταβόλου γενέσεως, οὐρανόν· καὶ περὶ μὲν τὰ τεταγμένα τῶν μετεώρων γίνεσθαι τὴν σοφίαν, περὶ δὲ τῶν γενομένων τὴν ἀταξίαν τὴν ἀρετήν, τελείαν μὲν ἐκείνην, ἀτελῆ δὲ ταύτην.

This passage contradicts what precedes, first of all, in terminology. What was οὐρανός there, has to be called Ὄλυμπος, to make the contrast with οὐρανός. But a textual correction will not solve the difficulty.[28] First we have the fiery envelope of the world (περιέχον), but then it becomes στοιχεῖα, in the plural. And if the sublunary realm is to be called περίγειον, the point of view is obviously geocentric; and the express designation of this area as "imperfect" leaves the central fire out of account. Therefore we have here elements of two mutually exclusive systems combined in a single passage.

Attempts at compromise will not do;[29] one of the two parts must be rejected.[30] The designation of the heaven as Ὄλυμπος[31] and the separation of the supralunar and sublunar realms are commonly thought of as genuinely early Pythagorean,[32] and for this reason Wiersma decided

[28] Diels, DK I 403.17, replaces the manuscript reading with [οὐρανόν] ‹μετὰ τὴν τῶν ἀπλανῶν σφαῖραν›, referring to Arist. fr. 203 = Alex. *Met.* 39.1.

[29] Boeckh (101) found the contradiction "unbegreiflich," but tried to smooth over the difficulty, as did Zeller, I 548 n. 1. Immisch (72 n. 1) speculated on a "development" in Philolaus' thought. Schaarschmidt (28ff) and Bywater (38f) thought no confusion was below this forger. Frank (279f) ignored the contradiction.

[30] Or, of course, both.

[31] Ὄλυμπος ἔσχατος, Parmenides fr. 11.2 (also γάλα οὐράνιον); Ὀλύμπιος κόσμος *Hebd.* 2; Ὄλυμπος, Empedocles fr. 44 (cf. Soph. *OC* 1655). These three testimonia add up to a certain probability that the expression was Pythagorean in origin. Nothing more need be assumed as basis than an *acusma* like that about the Isles of the Blest (above, ch. II 4; below, ch. IV 4). It was natural to identify the mythical Olympus with the sunny sky, and this took place long before Pythagoras: the θεοὶ Ὀλύμπιοι are at the same time the οὐρανίωνες (Pl. *Epin.* 977b: cf. below, n. 36).

[32] Alcmaeon A1 (from A12 and fr. 2). Heraclitus spoke of the impurity of the moon's abode (D.L. 9.10, Aët. 2.28.6, Hippol. *Ref.* 1.4.3 = Emp. A62). For the moon as μέση, middle and harmonic juncture of the regions of the cosmos, see *Hebd.* 2. This doctrine is regarded as early Pythagorean by Kranz, for example (*NGG* 1938, 141). But the idea of two cosmic realms could only have become widely influential after the scientific astronomy of Eudoxus; the Pythagorean *acusma* about the planets as the "hounds of Persephone" points in a quite different direction (below, ch. IV 1). The doctrine of two worlds became popular after the work of the Old Academy and Aristotle; then it becomes common, in the Pythagorean pseudepigrapha, e.g. "Philolaus" fr. 21, Ocellus 37, *Hypomn.* 26, Anon. Phot. 439b29f; see also Epiphanius *De fide* 9.12 (*Dox.* 590.11).

the second part was genuine.[33] But the stronger arguments are on the other side. The Pythagorean origin of the ideas of the second part is only a conjecture; it is through Platonism that they had their influence. On the other hand, the cosmology of the first part is guaranteed Pythagorean by the testimony of Aristotle, while later Pythagoreans tried to get rid of it.[34] The idea that the highest heaven consists of the εἰλικρίνεια τῶν στοιχείων comes from interpretation of the *Timaeus*.[35] The threefold division of the cosmos corresponds to ideas of the Platonists,[36] and the expression φιλομετάβολος γένεσις shows in both style and content a relationship to the fragment on the world soul.[37] Thus the second part bears the mark of Platonism, while the first corresponds to the Pythagorean doctrines attested by Aristotle, so that there is no question of the second part being authentic; this question rises only in the case of the first. Finally, it is likely a priori that if one part is a

[33] *Mnemosyne* 1942, 25. The first part is explained as the product of an error by Theophrastus, who, he thinks, worked the later system into his interpretation of Philolaus.

[34] Above, ch. III 1.

[35] *Tim.* 32bff, Por. ap. Philop. *Aet. mundi* 13.15 and Cyril. *Adv. Iul.* 2.47e, Procl. *In Tim* II 43.20ff: πάντων αἱ ἀκρότητες make up the substance of the οὐρανός (II 49.15; εἰλικρινὲς πῦρ ἐν οὐρανῷ, II 44.1), schol. Pl. *Phd.* 109b p. 235 Hermann (from Ruhnken; missing in Greene's ed.): λέγει ἐν τῷ Τιμαίῳ τὸν οὐρανὸν τῶν τεσσάρων εἶναι στοιχείων, ἀφ' ἑκάστου τοῦ καθαρωτάτου . . .

[36] The *Timaeus* separates terrestrial γένεσις from the regularity of the world of the stars just as it does the ποικιλία of the planets from the ταὐτοῦ φορά of the fixed stars. The resulting tripartition was emphasized by the ancient commentators (Adrastus ap. Theo Sm. 148.13ff; Anon. Phot. 439b17ff also gives a special status to the ἀπλανές superior to the planets; cf. Immisch 72ff). Xenocrates, somewhat differently, brought the threefold division of the world (sublunary region, region of stars, or heaven, and highest heaven, or what is beyond the heaven) into connection with divine powers and psychic functions (frr. 5, 15, 18 H., and "what the Delphians say," Plut. *Quaest. conv.* 745a; cf. Heinze 75ff). Heraclides (fr. 95) divides the cosmos into the realm of Zeus, that of Poseidon (the spheres of the planets down to the sun), and that of Hades (from the moon on down). Arcesilaus distinguishes three classes of gods, "Olympios, Astra, Titanios," which last group probably corresponds to the ὑποσέληνοι δαίμονες in Xenocrates fr. 15 (Tert. *Ad nat.* 2.2; Varro, following Antiochus, ap. Aug. *De civ. D.* 7.6). Pl. *Epin.* 977b clearly uses κόσμος, Ὄλυμπος, and οὐρανός as synonymous. At 986c the word κόσμος connected in an emphatic way with the paths of the stars (cf. Arist. *Met.* 1063a15), and at 987b the heaven of the fixed stars is said to be κόσμος in the truest sense (cf. Ach. *Is.* p. 36.5, where the *Timaeus* is referred to). At 976d the οὐρανός is celebrated as the provider of time and therefore of true ἐπιστήμη; it is contrasted with imperfect ἀρετή, which is possible even without λόγος (977 c-d). Here are the elements for the theory attributed to Philolaus; it was natural to combine these points with the threefold division of the world and the trinity of its appellations.—One also could read into a verse of Homer the idea that Ὄλυμπος was above the οὐρανός (*Il.* 1.497; cf. Stob. 1.22.2, ps.-Plut. *V. Hom.* 95, Delatte, *Litt.* 126).

[37] On this point even Boeckh admits post-Platonic terminology (100). Cf. fr. 21 (I 418.3), where the words γενέσιος καὶ μεταβολᾶς are used to characterize the sublunary sphere.

later addition, corresponding to later cosmological views, it should follow the other.[38]

Thus there are, in the Philolaus fragments, at least two different strata, one that has its origin in the Hellenistic apocrypha and another, sometimes contradictory, which agrees with the reports of Aristotle. Whether and to what extent these fragments contain original material from the fifth century B.C. is a problem we can only approach by careful study of the directly quoted fragments. But, as a preliminary, let us eliminate whatever has nothing to do with this central question.

Speusippus' book *Περὶ Πυθαγορικῶν ἀριθμῶν* contained thoughts of Speusippus which, according to the testimony of Aristotle, are not Pythagorean.[39] The quotation from the book is introduced by a detailed account of his sources: *Σπεύσιππος . . . ἐκ τῶν ἐξαιρέτως σπουδασθεισῶν ἀεὶ Πυθαγορικῶν ἀκροάσεων, μάλιστα δὲ τῶν Φιλολάου συγγραμμάτων, βιβλίδιόν τι συντάξας γλαφυρόν. . . .* It is unlikely, considering the way it is phrased, that this reference to Philolaus comes from Speusippus himself.[40] A reference of this type would be almost unique in ancient literature.

[38] The citation of Ocellus at Aët. 2.25.13 is probably an addition of Stobaeus. Cf. Diels, *Dox.* 100.1, Harder 39. Also in A16 = Aët. 2.7.7, Stobaeus may have made additions on his own. (The parallel citation of the ps.-Plutarch is missing here.)—The second part of A16 was rejected by Heinze (74 n.1) and Döring (*AGP* 1892, 517f).—A17 (Aët 2.4.15) presents problems similar to those brought by A16. In the sentence *τὸ δὲ ἡγεμονικὸν ἐν τῷ μεσαιτάτῳ πυρί, ὅπερ τρόπεως δίκην προὔπεβάλετο τῆς τοῦ παντὸς σφαίρας ὁ δημιουργὸς θεός*, the word *ἡγεμονικόν* may have been inserted by a doxographer to correspond to the rubric *ποῦ ἔχει τὸ ἡγεμονικόν (ὁ κόσμος)*; But *δημιουργός* too, used in this sense, is impossible before Plato (C. M. A. van den Oudenrijn, *Demiourgos*, Diss. Utrecht, 1951; C. J. Classen, *C&M* 23 [1962] 1–22). Is this another case of a later term creeping in (Boeckh 96f, Wilamowitz, *Platon* II 89)? Or has there been a reinterpretation to introduce conformity with the predilections of the "more genuine" Pythagoreans (above, ch. III 1, n. 72)? The sentence follows A18 in Stobaeus. On the simile of the laying of a keel, cf. Pl. *Leg.* 803a–b.

[39] Speusippus fr. 4 = *Th. ar.* 82.10ff = DK 44A13; above, ch. I 3. It is hard to decide whether Iamblichus had the Speusippus passage from Nicomachus (cf. above, ch. II 1, n. 4), or whether he actually had Speusippus' book. He used Speusippus in writing *Comm. math. sc.* (Merlan, *PlNeopl* 96–128).

[40] Frank (140f, 310, 332) and Cherniss (*Pres.* 390) take it as self-evident that Speusippus referred to Philolaus; this is denied by Wilamowitz (*Platon* II 88) and Theiler (*Gnomon* 7 [1931] 351f). On ancient ways of citing sources, see E. Stemplinger, *Das Plagiat in der griechischen Literatur* (Leipzig 1912) 177ff. For example, Apollonius of Perga, in *Conica* books 1 and 4, mentions his predecessors in a prefatory letter, but this is hardly likely to have been the case with Speusippus. On the form of expression, cf. Nicomachus (Por. *VP* 20 = Iam. *VP* 30) on the *ἀκρόασις* of Pythagoras, and *ἐσπουδάσθη* used of the number theory by Moderatus (Por. *VP* 48). *Ἐξαιρέτως* is only attested late (Plutarch, Arrian). Theo Smyrnaeus says (106.10f) that Philolaus had a good deal to say about the number 10 *ἐν τῷ περὶ φύσεως*, and this could have led to the source attribution.

Further, Nicomachus[41] ascribes to Philolaus a number system which represents a development of being in numerical stages: 1, point; 2, line; 3, plane surface; 4, solid; 5, *ποιότης καὶ χρῶσις*; 6, animation; 7, *νοῦς καὶ ὑγίεια καὶ τὸ ὑπ' αὐτοῦ λεγόμενον φῶς*; 8, *ἔρως, φιλία, μῆτις, ἐπίνοια*. This scheme contradicts the report of Aristotle, according to which the Pythagoreans called the plane surface *χροιά*, that is, they were not able to distinguish, even in terminology, between "surface" and "color."[42] In general, this gradation of being, and in particular the order of geometrical forms, is Platonic and not Pythagorean. In fact, Plutarch cites the very system here attributed to Philolaus, as Platonic.[43] Nicomachus and Proclus[44] made abundant use of this Platonic.[43] Nicomachus and Proclus[44] made abundant use of this "Pythagorean" scheme, which may also have been included in a *ἱερὸς λόγος* attributed to Pythagoras.

Claudianus Mamertus' report of a doctrine on the immateriality of the soul comes from a neo-Pythagorean source, where it was right next to a spurious Archytas fragment;[45] the contradiction of Philolaus' fragment 14 (in one *σῶμα-σῆμα*, in the other "love" of the soul for the body) proves that they cannot both be genuine.[46]

[41] *Th. ar.* 74.10 = DK 44A12.

[42] *Sens.* 439a30; cf. above, ch. I 3, n. 96.

[43] *Quaest. Plat.* 3.1.1002a: from *μονάς* and *ἄπειρος δυάς* come, in order, *στιγμαί, γραμμαί, ἐπιφάνειαι, βάθη καὶ σώματα, σωμάτων ποιότητες*. The introductory sentence, *οὐ γὰρ ποιεῖ μονὰς ἀριθμόν, ἂν μὴ τῆς ἀπείρου δυάδος ἅψηται*, is reminiscent of Speusippus (above, ch. I 3, n. 66).—Somewhat different is the account in *De E* (390 c–d) where 5 is equated with animation. Philo must have known a similar system, since he equates 5 with *αἴσθησις* (*Qu. in Exod.* 2.97).

[44] Six is *ψύχωσις*, *Th. ar.* 44.1ff, 52.5ff (ch. II 3, n. 110). At 63.25ff, 5 = *ποιότης καὶ χροιὰ καὶ φῶς* (so that this idea is differently placed here) *μετὰ τὰ σωματικὰ μεγέθη τριχῇ διαστάντα*, 6 = *ψύχωσις*, *ἕξις ζωτική*, 7 = *τελείωσις, διανόησις*.—Procl. In *Tim.* II 270.5ff: *Πυθαγορείων λόγος, ἀνὰ λόγον ταττόντων σημείῳ μὲν μονάδα, γραμμῇ δὲ δυάδα, τῷ δὲ ἐπιπέδῳ τριάδα, τῷ δὲ σώματι τετράδα, τῷ δὲ πεποιωμένῳ τὴν πεντάδα, τῷ δὲ ἐψυχωμένῳ τὴν ἑξάδα, τῷ δὲ νοερῷ τὴν ἑπτάδα*. This is repeated at III 328.13ff. At II 271.18, 7 is *τὸ κατὰ νόον φῶς* (a trace of Doric dialect). This "light" motif is to be interpreted in the context of Neoplatonism (contra Mondolfo in *ZM* 381, who tries to find pre-Socratic parallels). Delatte (*Litt.* 201) attributes the Proclus passages to the Doric *Ἱερὸς λόγος* of "Pythagoras." Doubts are expressed by Thesleff, *Texts* 166 n. 1. Frank (314ff) justly emphasized the Platonic character of this sytem, though many of his interpretations are arbitrary. This comes out even more clearly in his analogous reconstruction of Speusippus' system (239ff; criticized by Mondolfo in *ZM* 379ff; refutation of one important point, relating to the treatment of the soul, by Merlan, *PlNeopl* 128 n.).

[45] Philolaus fr. 22, ps.-Archytas p. 47.9 Thesleff, ap. Claud. Mam. *De statu an.* 2.3 Cf. F. Bömer, *Der lateinische Neuplatonismus und Neupythagoreismus und Claudianus Mamertus in Sprache und Philosophie* (Leipzig, 1936) 143ff. H. Gomperz accepted it as genuine, *Hermes* 1932, 156; but the emphasis on *ἀθάνατος καὶ ἀσώματος ἁρμονία* here is responsive to the objections of Plato to the soul-harmony doctrine in *Phd.* 85c.

[46] The contradiction is stressed by Mondolfo in *ZM* 377. Of course both fragments could be spurious.

Individual apophthegms, in the tradition, are always problematical. The sentence cited by Clement about the punishment and burial of the soul in the body is based on *παλαιοὶ θεολόγοι τε καὶ μάντεις*, but this, and the close correspondence with Aristotle's *Protrepticus*, are in themselves cause for suspicion.[47] It was only by a misunderstanding that Boeckh thought this saying of Philolaus was cited in Plato's *Gorgias*.[48] It seemed so natural to reconstruct Philolaus' doctrine from the passage in the *Phaedo* that names him, that nothing more than this seems to lie behind the explicit statement of Athenagoras about the *φρουρά*.[49] The saying of "Philolaus" in praise of number (fr. 23) is

[47] The word *θεολογία* is first attested at Pl. *Rep.* 379a; *θεολόγος* is common in Aristotle. Cf. Jaeger, *Theol.* 4; Vlastos, *PhilosQ* 1952, 102 n. 22. The latter points out that parallel formations like *μετεωρολόγος*, *φυσιολόγος*, *μυθολόγος* are common, and that *ἀμφὶ θεῶν λόγος* occurs in Emp. fr. 131. V. Goldschmidt, "Theologia," *REG* 63 (1950) 20–42, ignores the Philolaus fragment. With fr. 14, *μαρτυρέονται δέ μοι οἱ παλαιοὶ θεολόγοι τε καὶ μάντιες*, compare Philo *Op.* 100: *μαρτυρεῖ δέ μου τῷ λόγῳ Φιλόλαος*, and esp. Arist. fr. 60: *καθάπερ φασὶν οἱ τὰς τελετὰς λέγοντες . . . τοῦτο γὰρ θείως οἱ ἀρχαιότεροι λέγουσι* (Iam. *Protr.* 47.23), or in Latin, "ut interdum veteres illi sive vates sive in sacris initiisque tradendis divinae mentis interpretes . . . aliquid vidisse videantur" (Cic. *Hortensius* fr. 95 Mueller = Aug. *C. Iul.* 4.15.78). Likewise, with fr. 14: *ὡς διά τινας τιμωρίας*, Arist.: "ob aliqua scelera" (Cic.) or *διδόναι τὴν ψυχὴν τιμωρίαν καὶ ζῆν ἡμᾶς ἐπὶ κολάσει μεγάλων τινῶν ἁμαρτημάτων* (Iam. *Protr.* 47.25f); fr. 14: *ἃ ψυχὰ τῷ σώματι συνέζευκται*; Arist.: *ἡ σύζευξις . . . πρὸς τὸ σῶμα τῆς ψυχῆς* (Iam. *Protr.* 48.2f). By "priests of the mysteries" Aristotle clearly means the Orphics. But at Pl. *Crat.* 400c the expression *σῶμα-σῆμα* is distinguished from their view (Wilamowitz, *GldH* II 199; Thomas 51; Dodds, *Irr.* 169 n. 87); and we may suppose that, if it is not Orphic, it is likely to be Pythagorean. The ostensible Philolaus, however, contaminates the two and leaves us to guess whether he regards Pythagoras as an "ancient seer." The *σῶμα-σῆμα* idea was familiar in later times (Cic. *Rep.* 6.14, Macrob. *Somn. Sc.* 1.10.9f, Serv. *Aen.* 6.127, 439).—Clearchus, fr. 38, quotes a similar expression from the Pythagorean Euxitheus; but we do not know whether this was a historical character (Wellmann, *RE* VI 1539; Wehrli 59).

[48] At Pl. *Gorg.* 493a–b, Socrates–Plato cites a *σοφός* (= *ὁ πρὸς ἐμὲ λέγων*, 493b), who teaches that life is death and the body a grave, and bases this doctrine on allegorical interpretation of what a *μυθολογῶν κομψὸς ἀνήρ, ἴσως Σικελός τις ἢ Ἰταλικός* had said. The "mythologer" and the exegete must be distinguished from each other, though this is often forgotten. (The wrong interpretation is in Boeckh 183ff, Bywater 47ff, Schaarschmidt 6f, Carcopino, *Bas.* 285ff, Thomas 52f, Long 74f. Even Wilamowitz, *Platon* II 89, does not draw the line sharply enough between the "mythologer" and the interpretation. For the correct solution, Frank 298ff, Wuilleumier 571, Linforth, *UCPCP* 1944, Dodds, *Irr.* 225 n. 5, *Gorg.* 296ff.) The "mythologer" spoke of Hades and the punishment of the uninitiated, of carrying water in a sieve—a teaching of the mysteries widely known in the 5th century (Polygnotus' painting in the *lesche* of the Cnidians at Delphi: Paus. 10.31.9, 11; south Italian vases, Méautis 78f; A. Rumpf, *Misc. Ac. Ber.* II 2 [1950] 41f; cf. Wuilleumier 298f, 552; an allusion at Pl. *Rep.* 363c). The myth may have formed part of an Orphic *katabasis*; that the author was "perhaps a Sicilian or Italiote" is phrased like a conjecture, but alludes to the Pythagoreans (cf. above, ch. II 3; *Σικελὸς κομψὸς ἀνήρ* is an allusion to Timocreon fr. 4 D [Wilamowitz, *Platon* II 89]). That the one who offered the interpretation was himself a Sicilian or Italiote (as Frank seeks to show, 90, 299, n. 219) does not follow; so that it is hard to support the conjecture that Philolaus is intended. Carcopino, *Bas.* 287f, finds the same "tics d'expression" here as in fr. 17 and A13 *Th. ar.* 81.15; but the basis for such an assertion is too small.

as spurious as its neighbor attributed to Hippasus.[50] Through a misunderstanding of Philo, Philolaus is credited with a seemingly monotheistic saying which Lydus ascribes to Onetor.[51] Philolaus equated the number 7 with Athena, the "motherless," and this agrees with Aristotle.[52] The thought that the number 2 is "the consort of Cronus" (Rhea) may belong to the same tradition.[53] There is no reason to suspect the authenticity of the apophthegm passed along by Plutarch,[54] according to which geometry is the *ἀρχὴ καὶ μητρόπολις τῶν ἄλλων μαθημάτων*, or the thought that the *κριτήριον* is the *λόγος ἀπὸ τῶν μαθημάτων περιγινόμενος*, though the terminology is influenced by the more abstract thought of later times, reflected in the doxographers.[55] There is no occasion to attribute to Philolaus the definition of *ἁρμονία* as *πολυμιγέων ἕνωσις καὶ δίχα φρονεόντων συμφρόνησις*.[56]

We cannot regard the word *σοφός* (cf. above, ch. I 3, n. 157) or the use of allegorical interpretation as a sure indication of Pythagoreanism. There are, however, some indications of Pythagorean influence: "Hades" is located in man's lifetime, as in Empedocles (cf. ch. II 3, n. 80; IV 4); *μνήμη* and *πίστις* are thought to hold together the forces of the *ψυχή* (cf. ch. II 4). The background to be assumed is a writing similar to the one introduced to us by the papyrus from Derveni.

[49] Fr. 15 = Athenag. 6; Pl. *Phd.* 61d ff; cf. above, ch. III 1, n. 55; Frank 295ff.

[50] Fr. 23, and Hippasus DK 18.11; the two citations are neighbors at Iam. *In Nic.* 10.20off, and, later, Syrian. *Met.* 123.6ff, 142.21ff. There was no writing of Hippasus (D.L. 8.84). The style is imitative of the *Timaeus*.

[51] Fr. 20 = Philo *Op.* 100, Lydus *Mens.* 2.12. P. Boyancé has shown that Lydus is not dependent on Philo (*REG* 76 [1963] 91). The form of the name Onetor is suggested by the MS reading *ὁ νήτωρ* recorded by Thesleff, *Texts* 140.21 n., though he himself prefers *Ὀνήτας*.

[52] Alex. *Met.* 39.3ff = Arist. fr. 203, with the same rationale: *οὔτε γεννᾷ τινα τῶν ἐν τῇ δεκάδι ἀριθμῶν ὁ ἑπτὰ οὔτε γεννᾶται ὑπό τινος αὐτῶν . . . ἀμήτωρ . . . παρθένος*.

[53] Fr. 20a. Xenocrates, too (fr. 15 = Aët. 1.7.30) associates *Δυάς* and *μήτηρ θεῶν*. Doubtless Rhea is here associated with *ῥέω*, and this is how she comes to provide a name for the *ἀέναος ὕλη* (Xenocrates fr. 28), which becomes *ἀόριστος δυάς*. But Rhea also meets us in the *acusmata* (above, ch. II 4, n. 32).

[54] A7a = Plut. *Quaest. conv.* 8.2.1.718e, where the MSS have *φίλαον*. It is significant that, correctly in consideration of the history of mathematics, geometry is placed first; in the Platonic hierarchy arithmetic stands first (cf. above, ch. III 1, n. 14).

[55] The passage printed in DK 44A29 is misleading. Posidonius (ap. Sext. Emp. *Math.* 7.92ff) is discussing opinions on the *κριτήριον*. Anaxagoras named *λόγος* (7.91), the Pythagoreans, more explicitly, the *λόγος* that comes from *μαθήματα* "as Philolaus also said." The thought is added (connective *τε*) that the Pythagoreans spoke of a relationship of this *λόγος* with the entirety of nature; and in the next sections (92–109, not in DK) this idea is developed at length. But the reference for this second idea is Empedocles (fr. 109), not Philolaus; so that only the first part, about learning through *μαθήματα*, belongs to him. The basis is A7a and fr. 4.

[56] Fr. 10, in Nicom. *Ar.* 2.19, Theo Sm. 12.10. As with number, there must have been pithy sayings of the Pythagoreans current about harmony; cf. "Panaces" ap. Aristid. Quint. 1 p. 3 M. and above, n. 50. *Ἕνωσις* sounds post-Platonic (cf. the definition of *μίξις* in Arist. *Gen. corr.* 328b22.) Boeckh (61) first attributed the fragment to Philolaus.

The decisive group of fragments, in which Philolaus speaks about being which is "limiting" and "unlimited," as well as about number and harmony, has received very little attention; and, in particular, the opponents of their authenticity have not attempted any thorough analysis.[57] Yet only such an analysis can determine whether what we have is a post-Aristotelian "forgery" or a pre-Socratic composition. In the former case, it should be relatively easy to explain it in the categories of Platonic, Aristotelian, and post-Aristotelian thought; in the latter, we should expect that the familiar concepts and formulae of later times would prove inadequate, and that an approach would be possible only through the special characteristics of other pre-Socratic thinkers.

'Α φύσις δ' ἐν τῷ κόσμῳ[58] *ἁρμόχθη ἐξ ἀπείρων τε καὶ περαινόντων, καὶ ὅλος ‹ὁ› κόσμος καὶ τὰ ἐν αὐτῷ πάντα (B 1).*

'Ανάγκα τὰ ἐόντα εἶμεν πάντα ἢ περαίνοντα ἢ ἄπειρα ἢ περαίνοντά τε καὶ ἄπειρα· ἄπειρα δὲ μόνον ‹ἢ περαίνοντα μόνον?› οὔ κα εἴη. *ἐπεὶ τοίνυν φαίνεται οὔτ' ἐκ περαινόντων πάντων ἐόντα οὔτ' ἐξ ἀπείρων πάντων, δῆλον τᾷρα ὅτι ἐκ περαινόντων τε καὶ ἀπείρων ὅ τε κόσμος καὶ τὰ ἐν αὐτῷ συναρμόχθη . . . (B 2). Περὶ δὲ φύσιος καὶ ἁρμονίας ὧδε ἔχει*[59]*. ἁ μὲν ἐστὼ τῶν πραγμάτων ἀίδιος ἔσσα καὶ αὐτὰ μὲν ἁ φύσις θείαν τε*[60] *καὶ οὐκ ἀνθρωπίνην ἐνδέχεται γνῶσιν πλήν*[61] *γα ἢ ὅτι οὐχ οἷόν τ' ἦν οὐδενὶ τῶν ἐόντων καὶ γιγνωσκομένων ὑφ' ἁμῶν*

[57] The most extensive is that of Rothenbücher (66ff); but his conclusion, that the whole complex is "absurd and not Pythagorean," reveals his failure to understand it.—Bywater only cites some verbal reminiscences of pseudo-Pythagorica. Frank (302ff) paraphrases. Scoon (*GrPh* 133ff) treats the matter more fully; see also A. Burns, *C&M* 25 (1964) 93–128.

[58] Heidel's conjecture *ἁ φύσις δὲ τῶ κόσμω* (*AJP* 1907, 79) is mistaken; cf. Anaxagoras fr. 8: *τὰ ἐν τῷ ἑνὶ κόσμῳ*; Diogenes of Apollonia fr. 2: *τὰ ἐν τῷδε τῷ κόσμῳ ἐόντα* (similarly Hippoc. *Nat. hom.* 7, VI 50 L.); *Hebd.* 1: "mundi forma sic omnis ornata erat eorumque quae insunt singulorum" (*τοῦ κόσμου ἡ ἰδέη ὅλου καὶ τῶν ἐν αὐτῷ ἐόντων ἑκάστων*, Pfeiffer, *Sterngl.* 31.2); *ἁ φύσις τῶ κόσμω* would in itself be suspicious (the other way around in Eur. fr. 910: *ἀθανάτου φύσεως κόσμον . . .*); cf. also Kahn 228f. On *φύσις* see D. Holwerda, *Commentatio de vocis quae est φύσις vi atque usu praesertim in Graecitate Aristotele anteriore* (Diss. Groningen, 1955). The use of *φύσις* to mean the totality of *ἐόντα* is common in the time of Philolaus (as Euripides in the passage cited above).—On *ἁρμόχθη* cf. Emp. fr. 107: *ἐκ τούτων γὰρ πάντα πεπήγασιν ἁρμοσθέντα*, and the Derveni papyrus, col. 17.

[59] Cf. Hippoc. *Morb.* init.: *περὶ τῆς ἱρῆς νούσου καλεομένης ὧδ' ἔχει.*

[60] Sic MS F; *γα* Diels, *θεία ἐντί* Badham, Wachsmuth. For *τε . . . καὶ οὐ* in antitheses cf. Hdt. 8.81, Soph. *El.* 885, J. D. Denniston, *The Greek Particles* (Oxford, 1954²) 513.

[61] MSS *πλέον*, retained by Diels and Scoon (354), with the thought that it modifies *γνῶσιν* ("no more . . . than"); but one would expect *ἐπὶ πλέον* or *πλέονα*. The conjecture of Badham, followed by Wachsmuth and Rostagni (*Verbo* 50 n. 2), is supported by the fact that *πλὴν ἢ* is a common combination (e.g. Hdt. 2.111, Ar. *Nub.* 361).

γεγενῆσθαι[62] *μὴ ὑπαρχούσας τᾶς ἐστοῦς τῶν πραγμάτων, ἐξ ὧν συνέστα*[63] *ὁ κόσμος, καὶ τῶν περαινόντων καὶ τῶν ἀπείρων. ἐπεὶ δὲ ταὶ ἀρχαὶ ὑπᾶρχον οὐχ ὁμοῖαι οὐδ' ὁμόφυλοι ἔσσαι, ἤδη ἀδύνατον ἦς κα αὐταῖς κοσμηθῆναι, εἰ μὴ ἁρμονία ἐπεγένετο, ᾧτινιῶν ἄδε τρόπῳ ἐγένετο. τὰ μὲν ὁμοῖα καὶ ὁμόφυλα ἁρμονίας οὐδὲν ἐπεδέοντο, τὰ δὲ ἀνόμοια μηδὲ ὁμόφυλα μηδὲ ἰσοταχῆ*[64] *ἀνάγκα τὰ τοιαῦτα ἁρμονίᾳ συγκεκλεῖσθαι, αἰ*[65] *μέλλοντι ἐν κόσμῳ κατέχεσθαι . . . (B6).*

The *φύσις* in the cosmos has been put together harmoniously from unlimited and limiting (constituents), both the whole cosmos and all the things in it (fr. 1).

Existing things must be, all of them, either limiting, or unlimited, or both limiting and unlimited; but they would not be unlimited only ‹or limiting only(?)›. Since, however, they are clearly neither made of limiting (constituents) only nor of unlimited only, it is therefore obvious that from both limiting and unlimited (constituents) the cosmos and the things in it were harmoniously put together . . . (fr. 2). This is the situation about *φύσις* and harmony: the being (*ἐστώ*) of things, which is eternal, and *φύσις* itself admit of

[62] F, the only authoritative codex, has *γεγνέσθαι*. Mullach altered *οὐθενί* to *οὐθέν*, Usener and Diels to *οὐδὲν τῶν ἐόντων καὶ γιγνωσκόμενον . . . γα γενέσθαι*. The objection to this is that the passive is never expressed elsewhere, I believe, by *γίγνεσθαι* and the present passive participle. (NT Rev. 16.10 has a comparable construction, though with the perfect ppl.: *ἐγένετο . . . ἐσκοτωμένη*. Elsewhere *γίγνεσθαι* is used with the active ppl.; cf. Blass–Debrunner–Funk, *A Greek Grammar of the New Testament* . . . [Chicago, 1961] § 359.) Therefore the MS reading is to be retained (with Rostagni, *Verbo* 50 n. 2, and Scoon, *GrPh* 354ff). Being (*ἐστώ*) is a condition not of knowledge, but of the origin of this world of ours. On the conjecture *γεγενῆσθαι*, cf. the perf. pass. inf. a little further on, and Antiphon fr. 58, *τό γεγενῆσθαι*, for an unalterable situation; aside from that, it avoids the undesirable *γε*. One might also think of a Doric *γενέσθην*. On the *γιγνωσκόμενα* cf. Hippoc. *Vet. med.* 15: *τὰ γιγνωσκόμενα* are equivalent to the known facts of the world of experience; Democritus fr. 165: *ἄνθρωπός ἐστιν ὃ πάντες ἴδμεν. . . .*

[63] On *συνέστη* cf. Eur. fr. 910, Diogenes of Apollonia fr. 2, the Derveni papyrus col. 17.3, Arist. *Met.* 990a26, *ὑπάρχειν* and *ὁμόφυλα* Hippoc. *Nat. hom.* 3, VI 38 L.; cf. Democritus A135, DK II 115.2.

[64] The MS reading could be understood by comparison with Leucippus (cf. below, n. 101), but all the editors emend. Heidel (*AJP* 1907, 78) proposed *ἰσοταγῆ*, which was accepted by DK, but this word is late attested elsewhere. From actual pre-Socratic vocabulary a possibility is *ἰσοπαλῆ* (Meineke). Cf. below, n. 74.

[65] The MS reading is *τὰ τοιαῦτα ἁρμονίαις συγκεκλεῖσθαι, ἢ εἰ* . . . The usual reading is *τᾷ τοιαύτᾳ ἁρμονίᾳ*, and Meineke further altered *ἢ εἰ* to *αἰ* and Diels changed it to *οἵᾳ*; but *εἰ μέλλει* is so common a combination that it ought not to be altered (cf. Democr. fr. 264, Pl. *Rep.* 376b, 416d, and passim; similarly Eur. *Or.* 292, fr. 924). On the other hand *τᾷ τοιαύτᾳ . . . οἵᾳ* is an unusual and harsh locution; *ὁ τοιοῦτος* refers to a characterization previously mentioned or known (see Kühner–Gerth II 1. 630f), rather than to a following *οἷος*. Anyway, "such a harmony" would be an odd phrase; the author is not talking about any particular organization. Thus it seems best to keep *τὰ τοιαῦτα*, which can refer to *ἀνόμοια* etc., and to punctuate before *ἀνάγκη*, even though this creates an asyndeton. (A sentence is frequently introduced by *ἀνάγκη*, as Democr. fr. 253, 277, Antiphon fr. 44 [DK II 354, col. I line 15], Zeno fr. 1, Melissus fr. 7, Philolaus fr. 2.)—For *συγκεκλεῖσθαι* see Emp. fr. 27: *ἁρμονίης πυκινῷ κρύφῳ ἐστήρικται.*

divine but not of human knowledge, except that it was impossible for any of the things that exist and are recognized by us to come to be if there were not the being (ἐστώ) of the things of which the cosmos was composed, both the limiting and the unlimited. And since these beginnings (ἀρχαί) were not alike or of the same kind, it would have been impossible for them to be put together harmoniously if harmony had not supervened—however it was that it came to be. It is not things that are alike and of the same kind that need harmony, but things unlike and different and of unequal speed; such things must be bonded together by harmony, if they are to be held together in a cosmos . . . (fr. 6).

What is striking here is the really heavy-handed insistence with which the leading ideas—"limiting and unlimited," "cosmos," "harmony"—are repeated. But far from giving cause for suspicion,[66] this cautious procedure makes the impression of a genuine effort of thought, anxious to keep hold of the important points, and not seeking to make a display with carefully learned or borrowed fustian. This is the typical style of the pre-Socratics; "the amount of tedious repetition, even in Anaxagoras, is incredible."[67] The very opposite of this is the high-flown style, derived from the *Timaeus*, of a piece of writing like that on the world soul (A21). Here one notices similarities of language to Anaxagoras, Euripides, Empedocles, Diogenes of Apollonia, and the Hippocratic writings. Then there is the surprising δέ in the sentence that is quoted by Demetrius of Magnesia as the first in the book. A δέ in the first sentence, connecting it with the title, is found in several prose works of the fifth century;[68] and this feature also guarantees the title Περὶ φύσιος. Wilamowitz said of the style of the fragments, "Ihre Sprache ist entschieden altertümlich, macht den besten Eindruck."[69]

[66] Frank 305: "This sterile repetition of the same ideas, while the argument makes no progress at all, this continual manipulation of the same phrases, is intolerable; it betrays a second-rate mind."

[67] Diels, *SBBln* 1891, 580 n. 2, on Anaxagoras fr. 6; cf. Diels, *Parm.* 23ff on *naive Iteration* (referring to Parm. fr. 8.33f); also Melissus fr. 2, Diog. Apol. frr. 2, 5, Democr. fr. 191.

[68] Her. fr. 1, Ion fr. 1, ps.-Xen. *Ath. Pol.* 1, Hippoc. *Mul.* 1, *Nat. mul.* 1, *Foet. exsect.* 1, *Septim.* 1, *Gland.* 1, Xen. *Ap.* 1, *Oec.* 1.1, Critias fr. 46. First came the author and title (e.g., Φιλόλαος Κροτωνιάτας περὶ φύσιος ὧδε λέγει·). In a continuous text ὧδε can be followed by a clause introduced by δέ as Hippoc. *Aer.* 13. Cf. K. Gelzer, *Die Schrift vom Staate der Athener* (Berlin, 1937) 100f. Boeckh mistakenly concluded from the δέ that fr. 1 could not be the beginning of the book (45f). This usage, it is true, was often imitated in later times, e.g. by Pollux (E. Bethe, *Hermes* 72 [1937] 240).

[69] Wilamowitz, *Platon* II 90.

The author's basic idea is the division of ἐόντα or πράγματα[70] into ἄπειρα and περαίνοντα; the breach which this opens up in the cosmos is healed by a ἁρμονία. There are obvious similarities with what Aristotle reports, but also characteristic differences. In the first place we have περαίνοντα ("limiting [things]," or "that which limits") instead of πέρας ("the limit") or τὸ πεπερασμένον ("the limited"). Though it has a model in the not infrequent expression περαίνειν πρός τι ("border on something"), still this absolute use is notable. But it is comprehensible, or even necessary; a "limited" presupposes something else which is "limiting" or "limit-setting." The necessary complement to what is unlimited is the existence of "things" that "make limits," which bear the principle of limitation as an active power.[71] Now in one of Aristotle's dialogues, in a clearly "Pythagorizing" passage dealing with the perfection of musical harmony, he uses the phrase ἡ περαίνουσα φύσις for the nature of an odd number.[72] Hesychius s.v. πέραινον (i.e. περαῖνον) says οἱ Πυθαγορικοὶ τοὺς περιττοὺς ἀριθμούς, and formulations like this are found in Nicomachus.[73] The subject of this passage of Philolaus is not primarily numbers, but "things," and the antithesis περαίνοντα καὶ ἄπειρα is used in a broader and more nearly original sense than in the definition given by Hesychius.

Significantly, the "principles," in spite of the occurrence of the suspicious word ἀρχαί,[74] are never designated by an abstract expression like τὸ πέρας, τὸ πεπερασμένον, τὸ ἄπειρον, or, as in the formulation of Aristotle cited above, ἡ περαίνουσα φύσις—or, as in the *Philebus*,[75]

[70] πρᾶγμα in the general sense is equivalent to ὄν elsewhere: Democr. fr. 164 (cf. Hdt. 2.52), Gorg. *Hel.* 17, 18, *Pal.* 24, Epicharm. fr. 3, Euenus fr. 8 Diehl (cf. Theog. 472). Obviously χρῆμα is the older expression, and is always used by Anaxagoras and Melissus (Aët. 1.3.5 gives the gloss Anaxagoras fr. 1, χρήματα λέγων τὰ πράγματα); cf. Diels, *SBBln* 1884, 350 n. 1.

[71] This is emphasized by Reinhardt, *Parm.* 65 n. 1, and Scoon, *GrPh* 140ff; Melissus fr. 5: περαίνειν πρός τι, Hippoc. *Vict.* 1.10, Pl. *Meno* 76a: σχῆμα εἰς . . . ὃ τὸ στερεὸν περαίνει.—Aristotle deals, in his own system, with ἄπειρον and πεπερασμένον (*Phys.* 202b30ff), as had Zeno, fr. 3.

[72] Fr. 47; cf. above, ch. I 3, n. 161. The conjecture περισσῆς, still printed by Ross, is to be rejected because of the Hesychius passage.

[73] Odd numbers are περαινούσης οὐσίας, *Th. ar.* 62.15; square numbers made with odd γνώμονες (above, ch. I 2, n. 27) are ὡρισμένοι τε καὶ περαίνοντες, Nicom. *Ar.* 2.18.3 (where the citation of Philolaus fr. 2 follows in the next section).

[74] A similar plural Hippoc. *Morb.* VII 542, 582, 590 L. (Keus 12ff); cf. also Pl. *Nem.* 1.8. It is significant that in Philolaus a past tense is used (differently from ps.-Archytas 19.5 Thesleff; ἀνάγκα δύο ἀρχὰς εἶμεν τῶν ὄντων). It would be convenient to emend ἐπεὶ δέ τε (sic F) ἀρχὰν ὑπᾶρχον οὐχ ὁμοῖα οὐδ ὁμόφυλα ἐόντα, ἤδη ἀδύνατον ἦς καὶ αὐτοῖς (sic F) κοσμηθῆναι . . . (on ἀρχάν see Anaxagoras frr. 6, 12). But ἐόντα could not be changed to ἔσσαι by an ordinary slip.

[75] Pl. *Phlb.* 24c, cf. 18a. For ancient connection of Philolaus and the *Philebus*, see above, ch. I 4, n. 14.

ἡ τοῦ ἀπείρου φύσις, though this is the regular practice in the later tradition.[76] "Limit" and "unlimitedness" are not isolated as entities in themselves, congealed into an abstract substantive or hypostasized as intangible substance, but they are thought of as scattered or deployed, so to speak, in individual things, περαίνοντα or ἄπειρα. This is a basic difference between the thought of the pre-Socratics and that of the Platonic and Aristotelian schools, a difference brought out especially by Cherniss.[77] By a word like θερμόν or ψυχρόν, for example, a pre-Socratic thinker does not mean an abstract quality, or an οὐσία, but quite concretely the sum of particular things characterized by the word. Only in Plato's dialectic was the foundation laid for separating qualities and quantities from objects and regarding them αὐτὰ καθ' αὑτά. For later ages these distinctions came to seem self-evident. By saying not ἀνάγκα δύο ἀρχὰς εἶμεν τῶν ἐόντων, τό τε πέρας καὶ τὸ ἄπειρον,[78] but ἀνάγκα τὰ ἐόντα εἶμεν πάντα ἢ περαίνοντα ἢ ἄπειρα ἢ περαίνοντά τε καὶ ἄπειρα, Philolaus shows his affinity, in expression and thought, with the pre-Socratics and his difference from all that is Platonic and post-Platonic. From this point of view a forgery is unlikely, since what is involved is not the kind of technical terms or verbal flourishes that are easy to imitate, but the way in which the mind goes about trying to understand reality.

The division of "things" into "limiting" and "unlimited" is of course more abstract than an analysis in terms like "warm," "cold," "dry," and "moist," where reference is made to areas of immediate sense perception. There is no indication what kind of specific experience is implied in the idea of "limit" and "unlimited." The author does give what looks like a hint, in the continuation of fragment 2: δηλοῖ δὲ καὶ τὰ ἐν τοῖς ἔργοις. τὰ μὲν γὰρ αὐτῶν ἐκ περαινόντων περαίνοντι, τὰ δ' ἐκ περαινόντων τε καὶ ἀπείρων περαίνοντί τε καὶ οὐ περαίνοντι, τὰ δ' ἐξ ἀπείρων ἄπειρα φανέονται. But this sentence is couched in quite general terms, and it is impossible to tell what specific sense the word ἔργα has;[79] all we learn is that the "limiting" and the "nonlimiting"

[76] Aët. 1.3.10 (DK 44A9), Damasc. *Princ.* I 101.3 Ruelle, Arist. passim.

[77] *Pres.* 375ff, *JHI* 342ff.

[78] Cf. ps.-Archytas 19.5 Thesleff (above, n. 74).

[79] Boeckh (50) translated "Bauwerke," Diels (*Vorsokratiker*[1]) "Äcker," Scoon (*GrPh* 138) advanced the idea that ἄπειρα are mountain meadows without definite boundaries. Newbold (186ff) essayed a geometrical answer, Becker (*QSt* 4, 154f) an arithmetical (ἔργον is the product; "odd" × "odd" "odd," etc.). Heidel (*AJP* 1907, 77; *Proc.* 1913, 730) has "nothing but the bald assertion that observation or knowledge confirms . . ." Similarly, W. Nestle (*Philologus* 67 [1908] 544f). Cf. Anaxagoras B7: μὴ εἰδέναι . . . μήτε λόγῳ μήτε ἔργῳ. Frank (307 n. 1) points out that expressions like δηλοῖ τὸ ἔργον occur in

that "things" do is dependent on the "limit-forming" and "unlimited" constituents that go into their makeup.

It is very easy to understand "limit" as a formative principle, and "unlimitedness" as a material principle. But in considering a question of authenticity we must differentiate between philosophical interpretation, which seeks to understand the author better than he understood himself, and philological interpretation, whose first duty is to understand and place historically what the author put down. The thought that god gives form to formless matter, by imposing "limits," περατοῦν τὴν ὕλην ἄπειρον οὖσαν, is characteristic of the Platonists;[80] that it does not go back to the Pythagoreans, and that it was not an accomplishment of theirs, as is often thought, to conceive for the first time, in explicit terms, of a formal principle, follows from the exposition of Aristotle, according to which the form-matter dichotomy is not applicable to their number theory. If the situation were different in the Philolaus fragments, this would be a serious cause for suspicion; but any such interpretation is impossible. Boeckh equated the "limit" with the One, the "unlimited" with the Indefinite Dyad, so that the difference between Philolaus and the Platonic system disappeared.[81] On this basis, Frank was able to maintain[82] that Philolaus agreed with Plato, against the testimony of Aristotle. But the text of Philolaus is against this. The One is itself a ἁρμοχθέν, and is therefore not simply περαῖνον.[83] After all, "limiting (things)" (περαίνοντα) and "unlimited (things)" (ἄπειρα) stand side by side, both in the plural, and there is no indication that in the process of "harmonization" (ἁρμόζειν) the "unlimited" disappears;[84] nor is there any suggestion of the one affecting the other, in the way that "form" must affect "matter."

The ideas of ἐστώ ("being") and ἁρμονία complicate the matter

Aristotle (*Mete.* 349b35, cf. *Met.* 1086b5); but they are also in use earlier, as Hippoc. *Flat.* 5: πρὸς αὐτὰ τὰ ἔργα . . . πορευθεὶς ἐπιδείξω . . . On the idea, cf. Hippoc. *Nat. puer.* VII 498 L.: ὅσ' ἀπὸ πυκνῶν ἐγένετο πυκνά ἐστι καὶ ὅσα ἀπὸ ὑγρῶν ὑγρά.

[80] Plut. *Quaest. conv.* 8.2.3.719c. Cf. the construction of elements in the *Timaeus* and in general the system of the One and the Indefinite Dyad.

[81] Boeckh 54f, followed by DK I 407 n.: "Grenze (Form) und Unbegrenztheit (Stoff)"; I 409, in the translation, "diese Prinzipien (*1 und 2*)." Contra, above, ch. I 2.

[82] 268, 308 n. 1, 309 n. 1; accepted by Cherniss, *Pres.* 391. The fact that the octave, which is also called ἁρμονία, shows the ratio 2:1, has nothing to do with the Indefinite Dyad.

[83] Fr. 7: τὸ πρῶτον ἁρμοσθέν, τὸ ἕν, ἐν τῷ μέσῳ τᾶς σφαίρας ἑστία καλεῖται. Though τὸ ἕν looks suspiciously like dittography, the combination of the πρῶτον ἁρμοσθέν with the πρῶτον ἓν συστάν in Aristotle (*Met.* 1080b20, cf. 1091a12ff) still stands quite apart from the problem of authenticity.

[84] As in Plutarch (above, n. 80) and in Scoon's interpretation (below, n. 85).

still further. We cannot simplify it by equating ἐστώ with ἄπειρον and ἁρμονία with περαῖνον. The ἐστὼ τῶν πραγμάτων . . . καὶ τῶν περαινόντων καὶ τῶν ἀπείρων is prior to the differentiation of "limiting" and "unlimited," and belongs to one of these as much as the other, and "harmony," when it "comes," applies to both of the "beginnings, which are not alike nor of the same kind." Nor can ἐστώ be thought of as ὕλη or ἁρμονία as formal cause.[85] Ἐστώ is not ἄποιος ὕλη, but already differentiated into "limiting" and "unlimited," and "harmony" comes along later; it would not be necessary at all, if it had not been for the antithesis of "limiting" and "unlimited." There is no hint here of a division of being into form and matter, which must always be thought of in relation to each other—nothing that goes beyond the pre-Socratic picture of the cosmos as the proper ordering of something previously chaotic.

The word ἐστώ is not to be understood as a material principle in the Platonic and Aristotelian manner. The only thing that is said of it is that ὑπάρχει. Being is. This is the fundamental position of the Eleatics, and it is from this point of view, rather than that of later, abstract conceptualization that the sense comes clear. Since Parmenides, all philosophers had spoken of an eternal being,[86] and the Sophist Antiphon used the word ἀειεστώ.[87] This concept of being had been the subject of heated controversy since the time of Zeno, Melissus, and Gorgias.[88]

[85] As by Scoon, *CP* 1922, 354, *GrPh* 139ff, esp. 145 n. 30, who equates the unknowable ἐστώ with ἄπειρον and πέρας with number, the principle of knowability, and ἁρμονία. The ἄπειρα, he thinks, are rudiments, which the ἁρμονία has not yet fully grasped. Howald was able to cite Scoon in order to bolster his argument that the fragments are infected with Platonism (p. 67; cf. Burnet, *EGP* 285 n. 3). A suspicious resemblance between the unknowable ἐστώ and the Aristotelian ἄποιος ὕλη, ἄγνωστος καθ' αὑτήν, is seen by Schaarschmidt 66, Bywater 34f, Frank 308, and Raven in KR 310f. But for Plato and Aristotle ὕλη is unknowable in principle, and not the object of some higher, divine knowledge.—In ps.-Archytas p. 19.19 Thesleff, ὠσία-ἐστώ is certainly the Aristotelian material principle, contrasted with μορφώ.—Rostagni, *Verbo* 49ff, interprets ἐστώ as "essentia," ideal Being, supernatural "Unity." This, too, is conceived from a Platonic point of view. The contradictions among the interpretations show that Platonic principles are not applicable to Philolaus 6.

[86] Melissus frr. 1, 7, 4, Empedocles fr. 17.3 = 26.12, Diogenes of Apollonia frr. 7, 8 (ἀήρ as ἀίδιον καὶ ἀθάνατον σῶμα, which does not deny the perishability of the cosmos, A10), Gorg. fr. 3 §§68f (hypothetical: εἰ ἀίδιον τὸ ὄν).—Eur. fr. 910; cf. Anaxagoras fr. 17.

[87] Antiphon fr. 22 = Harpocr. s.v. (It is explained as τὴν ἀιδιότητα καὶ τὸ ἐπὶ τῶν αὐτῶν ἀεὶ ἑστάναι.) On the formation of the word, cf. εὐεστώ, Aesch. *Sept.* 187, *Ag.* 647, 929, Hdt. 1.85, Democr. fr. 2c; ἀπεστώ, Hdt. 9.85; συνεστώ, Hdt. 6.128. It is obviously an Ionic formation; Pl. *Crat.* 401c shows that the Doric is ὠσία or ἐσσία. Iam. *VP* 162 cites ἐστώ as one of Pythagoras' neologisms, along with κόσμος, φιλοσοφία, τετρακτύς. Ps.-Archytas, pp. 19.5–20.17 Thesleff, used ἐστώ alternatively with ὠσία.—On αὐτὰ μὲν ἁ φύσις in Philolaus fr. 6, cf. κατ' αὐτό in Parmenides fr. 8.58; similarly, Emp. fr. 17.34.

[88] On the relation of Gorgias to the Eleatic dialectic of Being, cf. W. Bröcker, *Hermes* 86 (1958) 425–440.

There was no agreement as to what attributes it had, whether it existed at all, or whether it was expressible. But Philolaus pushes this controversy aside, entrusting its solution to a divine intelligence.[89] He is clear that there are things that are known to us (ἐόντα καὶ γιγνωσκόμενα), and that their origin would have been impossible if—as Gorgias thought—"nothing" existed. We cannot go beyond this; nevertheless the world, as it is given us, can be understood, as a harmony of opposites.[90]

Like the question of being, that of the origin of harmony and its "arrival" is avoided, with the explicit phrase ᾧτινιῶν ἅδε τρόπῳ ἐγένετο. It is simply a necessary prerequisite for the cosmos, just as ἐστώ is prerequisite to the πράγματα determined by the opposites. Raven thinks he sees here the clearest proof that the fragments are a post-Aristotelian forgery: "It is surprising . . . to find the author . . . expressing perplexity about what seems to have been the most important constituent in his whole cosmology." Aristotle speaks of the ἀπορεῖν of the Pythagoreans in a very similar context, in relation to the formation of the first One. Raven's argument is that if the Pythagoreans had themselves discovered the gap ("omission") in their system, they would have filled it up; if it is noticed and left, a forger mindlessly copied Aristotle.[91] The assumption here is that it is a primary concern of a philosopher to have pat answers to all possible questions, and a fully consistent system. In fact, this is rather the concern of the doxographer, but in a real effort to get at truth there is always an awkward "remainder" that will not fit in, or perhaps an unfilled gap. Scoon aptly mentions that the Νοῦς of Anaxagoras

[89] Cf. Xenophanes fr. 36, Alcmaeon fr. 1, Her. frr. 78–79, Democr. fr. 8, Hippoc. *Vict.* 1.1, and Epimenides fr. 11. Somewhat more advanced and skeptical is Ecphantus DK 51.1: μὴ εἶναι ἀληθινὴν τῶν ὄντων λαβεῖν γνῶσιν, ὁρίζειν δὲ ὡς νομίζειν.

[90] It is a basic idea of Heraclitus that ἁρμονία presupposes an opposition (frr. 8, 51; cf. A22). It is also found, later, in the Pythagorean pseudepigrapha: ps.-Archytas p. 20.4 Thesleff: τὰ δ' ἐναντία συναρμογᾶς τινος δεῖται καὶ ἑνώσιος (Bywater 33).—Nicom. Ar. 1.6.3: πᾶν δὲ ἡρμοσμένον ἐξ ἐναντίων πάντως ἥρμοσται καὶ ὄντων γε· οὔτε γὰρ τὰ μὴ ὄντα ἁρμοσθῆναι οἷά τε οὔτε τὰ ὄντα μέν, ὅμοια δὲ ἀλλήλοις, οὔτε τὰ διαφέροντα μέν, ἄλογα δὲ πρὸς ἄλληλα. This amounts to a kind of paraphrase of Philolaus 6.—Syrian. *Met.* 165.33f asserts that Philolaus taught that τὸν θεὸν πέρας καὶ ἀπειρίαν ὑποστῆσαι, which assumes a highest principle over and above the pair of opposites (followed by Boeckh, 53f, 148). Zeller (I 480 n. 1) finds in Pl. *Phlb.* 23c corroboration for this, although Proclus (*Theol. Pl.* 3.7 p. 132) makes it clear that only Limit and Unlimitedness go back to Philolaus, whereas God as a highest principle is Platonic. The One above the pair of opposites, which Eudorus ascribes to the Pythagoreans (above, ch. I 3, n. 45) is so important in Neoplatonism (cf. Dodds, *CQ* 1928; Procl. *In Tim.* I 176.9f) that this in itself rouses suspicion of Syrianus' statement. Philolaus fr. 8, ἓν ἀρχὰ πάντων, need not be understood in this sense; it can be interpreted as referring to the πρῶτον ἁρμοσθέν (fr. 7).

[91] Raven, *PyEl* 98f, Raven in KR 310. Arist. *Met.* 1080b20 (cf. 1091a12ff): ὅπως δὲ τὸ πρῶτον ἓν συνίστη ἔχον μέγεθος, ἀπορεῖν ἐοίκασιν.

presents an analogous problem.[92] According to the commentator of the Derveni papyrus (col. 17), Zeus, the divine Aër, brought ἁρμονία. Philolaus avoids mentioning the divine. How harmony came to the world is passed over just like the problem of ἐστώ; the world is there, and it is in order.

The question of ἄπειρον and πέρας was much canvassed in Greek philosophy. Anaximander takes the ἄπειρον as his starting point, Parmenides declared that "Being" was perfect and therefore limited (fr. 8.42ff), Zeno dealt with the antinomy of πεπερασμένον and ἄπειρον under the assumption of the existence of plurality (fr. 3), and Melissus decided that being is ἄπειρον (frr. 2–4). Philolaus takes a stand on the question, saying that περαίνοντα and ἄπειρα exist side by side, brought together by the agency of harmony. As others, faced with the multiplicity of the world, picked certain leading ideas as central—finding the basic outline of reality in pairs of opposites like warm and cold, wet and dry, or like Parmenides analyzing the whole world of appearance as the mutual influence of light and night—in the same way Philolaus takes the one feature of limitation as having determined the nature of the cosmos.

But how can one speak of a plurality of ἄπειρα? The air can be called unlimited, or space, or Melissus' Being,[93] but none of these has a plural. One can speak of some things as "unlimited in number" (ἄπειρα τὸ πλῆθος),[94] but περαίνοντα is not the opposite of this. Now Anaxagoras, following Zeno's lead, spoke of an ἄπειρον of the small, of a never ending process of division.[95] Divisibility, dependent on the "empty space in the middle," plays a role in the Pythagorean equation of even number with unlimitedness.[96] If the ἄπειρα πράγματα are thought of in the context of endless divisibility, the plural is comprehensible; the opposite, the περαῦνον, is, then—an ἄτομον. Then the pair περαίνοντα and ἄπειρα, both of which are called ἐόντα, would correspond to the Leucippan δέν and μηδέν—material atoms and the "empty" interstices, which do yet "exist." This close relationship between the Pythagorean and the atomists, no matter how surprising it seems from the point of view of the Platonic interpretation of Pythagoreanism, is actually corroborated by both internal and external evidence. It is not only that Leucippus, like Philolaus, belongs in the Eleatic tradition and scarcely can be outdone by the Pythagoreans' number theory in "forcible abstraction."[97] Aristotle remarked that the atomists, too, in a certain way made things out to be numbers. The "void" of the atomists is not a single, endless space, but the plurality of interstices which make divisibility and plurality possible.[98] The cosmos grows by taking in material from outside.[99] It is obviously the Pythagoreans who inspired the atomists to see in the motes in a sunbeam an indication of the nature of the soul-atoms.[100] We even have the testimony of a contemporary that Democritus studied with a Pythagorean; thus Philolaus and Leucippus are thrown close together.[101] To be sure, Philolaus maintains a distance from the atomists; he does not speculate further about being, but looks for relationships in our given, familiar world and finds them in the ordering function of number.[102]

Before that, though, he seeks to prove his basic doctrine; and here Frank thought he had found an infallible sign of post-Platonic forgery.[103] But the refutation of this was already at hand; Reinhardt had shown that

[92] Scoon, *GrPh* 143f.

[93] Anaxagoras fr. 1, Melissus fr. 6.

[94] Anaxagoras fr. 1, Zeno fr. 3.

[95] Anaxagoras frr. 1 and 3 expressed the idea of the infinitely small; Zeno fr. 3 (cf. fr. 1) used the ὄντα ἄπειρα which result from the assumption of plurality, polemically, as a reductio ad absurdum.

[96] Above, ch. I 2, n. 31.—Aristotle argues, at *Phys.* 204a8ff, against the idea of the ἄπειρον being οὐσία. An οὐσία as such cannot be divisible into a plurality of infinities, πολλὰ δ' ἄπειρα εἶναι τὸ αὐτὸ ἀδύνατον (204a25f), and yet it is a ποσόν. ὥστε ἀτόπως ἂν ἀποφαίνοιντο οἱ λέγοντες οὕτως ὥσπερ οἱ Πυθαγόρειοί φασιν· ἅμα γὰρ οὐσίαν ποιοῦσι τὸ ἄπειρον καὶ μερίζουσι (204a32ff). In this connection, Aristotle associates ἄπειρον with ἄρτιον (204a31). Ross relates Aristotle's criticism to the Pythagorean identification of ἄπειρον with even, i.e. divisible number, but the criticism is more telling, couched in general terms, if the Pythagoreans actually spoke of a plurality of ἄπειρα—as Philolaus did.

[97] Von Fritz, *Philosophie und sprachlicher Ausdruck* (New York, 1938) 15; Arist. *Cael.* 303a8; above, ch. I 2, n. 70.

[98] KR 408.

[99] D.L. 9.32 = Leucippus A1; above, ch. I 2, n. 51; Democr. A40.

[100] Arist. *De an.* 404a1 et seq.; above, ch. I 3, n. 130.

[101] D.L. 9.38: πάντως μέντοι τῶν Πυθαγορικῶν τινος ἀκοῦσαί φησιν αὐτὸν Γλαῦκος ὁ Ῥηγῖνος. Duris (*FGrHist* 76F23) said his teacher was Pythagoras' son Arimnestus; Apollodorus of Cyzicus (above, ch. III 1, n. 51) mentioned Philolaus. For Leucippus and Philolaus as pupils of Pythagoras, see Iam. *VP* 104. On Democritus' Πυθαγόρης see above, ch. II 6, n. 26; on the Pythagores coins of Abdera, above, ch. II 2, n. 2. Frank argued for the dependence of the Pythagorean philosophy on Democritus; but the relationship may be a whole generation earlier. Surely there was an atomist influence on some later Pythagoreans. (On Ecphantus, see above ch. I 2, n. 74; W. Kranz in *Convivium: Festschr. K. Ziegler* [Stuttgart, 1954] 26ff. There could be a relationship between Ecphantus and the cosmology of the papyrus from Derveni; see Burkert, *A&A* 1968, 99.)

[102] On the other hand, Democritus denied that there could ever be φύσιν μίαν ἐξ ἐκείνων (sc. τῶν ἀρχῶν), κομιδῇ γὰρ εὐήθες εἶναι τὸ δύο ἢ τὰ πλέονα γενέσθαι ἂν ποτε ἕν. Arist. fr. 208 = Democritus A37.

[103] 304f: "... ein ganz entwickeltes dialektisches Bewusstsein, wie es vor Plato ... kaum möglich ist."

what Philolaus presents is an imitation of an Eleatic proof:[104] first the possible cases are enumerated, then one after another is eliminated, until the actual one remains. The method of the "contrary-to-fact" inference, too—"this would not be possible, unless . . ."—was known in the fifth century.[105] The author of the book *On the Nature of Man*, in particular, assures himself of the basis of his anthropology with a three-stage proof. The preconditions of all genesis are, first, a plurality of *ἐόντα*, secondly, their homogeneity, and thirdly, that they are mixed in the correct proportions.[106] Philolaus' conclusion as to the preconditions of all development is similar, though more abstract: being in general, then the antithesis of limiting and unlimited, and finally harmony, which is a numerical relationship.

Philolaus also has an epistemological argument: *ἀρχὰν γὰρ οὐδὲ τὸ γνωσούμενον ἐσσεῖται πάντων ἀπείρων ἐόντων* (B3).[107] *καὶ πάντα γα μὰν τὰ γιγνωσκόμενα ἀριθμὸν ἔχοντι· οὐ γὰρ οἷόν τε οὐδὲν οὔτε νοηθῆμεν οὔτε γνωσθῆμεν ἄνευ τούτου* (B4). Here scholars have thought they found, if anywhere, the kind of anachronism that proves forgery.[108] The unknowability of the Unlimited is also Platonic-Aristotelian doctrine, and number as the basis of all knowledge is emphasized

[104] Reinhardt, *Parm.* 65; cf. Mondolfo, ZM 378. Compare also the "three ways" of Parmenides, fr. 6. Gorgias, in particular, used the method of elimination. In the account given by Sextus (*Math.* 7.66, 68, DK 82B3), a three-member scheme appears twice (*ἤτοι τὸ ὂν ἔστιν ἢ τὸ μὴ ὂν ἢ καὶ τὸ ὂν ἔστι καὶ τὸ μὴ ὄν . . . ἤτοι ἀίδιον . . . ἢ γενητὸν καὶ ἀίδιον ἅμα καὶ γενητόν*). But the threefold division does not appear in ps.-Arist. *MXG*, which W. Bröcker believes to be more reliable than Sextus' (*Hermes* 86 [1958] 425–440, esp. 428f, 433f).—Arist. *Cael.* 274a30ff: *ἀνάγκη δὴ σῶμα πᾶν ἤτοι ἄπειρον εἶναι ἢ πεπερασμένον* (cited by Bywater 52) also lacks the characteristic threefold division.

[105] Diogenes of Apollonia fr. 3: *οὐ γὰρ ἂν . . . οἷόν τε ἦν οὕτω δεδάσθαι ἄνευ νοήσιος . . .* (cf. fr. 2; Anaxagoras fr. 12, and still earlier, Heraclitus fr. 23; Zeno, frr. 1–3, and Melissus, frr. 6–7, use hypothetical propositions). Cf. also the method of proof in Hippo fr. 1.

[106] Hippoc. *Nat. hom.* 3 (according to Anon. Lond. XIX, written by Polybus the son-in-law of Hippocrates; i.e., about 400 B.C.): *πρῶτον μὲν οὖν ἀνάγκη τὴν γένεσιν γίνεσθαι μὴ ἀφ' ἑνός. πῶς γὰρ ἕν γ' ἐόν τι γεννήσειεν, εἰ μή τινι μιχθείη; ἔπειτα οὐδ' ἐὰν μὴ ὁμόφυλα ἐόντα μίσγηται . . . οὐδ' ἂν ταῦτα ἡμῖν συντελέοιτο· καὶ πάλιν, εἰ μὴ τὸ θερμὸν τῷ ψυχρῷ καὶ τὸ ξηρὸν τῷ ὑγρῷ μετρίως πρὸς ἄλληλα ἕξει καὶ ἴσως, . . . ἡ γένεσις οὐκ ἂν γένοιτο. . . .*

[107] *τὸ γνωσούμενον* must be passive (differently from Pl. *Crat.* 440b, where *τὸ γνωσούμενον* and *τὸ γνωσθησόμενον* are paired against one another). The passive use of the future middle is common, though rare in verbs with deponent futures (Kühner–Gerth II 1.114ff). Yet Antiphon the Sophist used the expression *τὸ ὀψόμενον* along with *ὄψις, ὀφθαλμοί, ὀπτήρ* (DK 87B7), where it must mean the "object of the act of seeing." Thus the remarkable expression of Philolaus has its parallel in a 5th-century author, and only there.—On *γε μήν* see Denniston, *The Greek Particles* (Oxford, 1954²) 347ff.

[108] Schaarschmidt 66, Bywater 35. The use of *γνωσθῆμεν* with *νοηθῆμεν* shows that *νοεῖν* no longer means "grasp, recognize," but "think," as in Democr. fr. 143. Cf. von Fritz, *CP* 40 (1945) 223–242, 41 (1946) 12–34.

especially in the *Epinomis*.[109] Raven thinks it a very important finding that Aristotle gives no hint of this indispensable epistemological underpinning of the Pythagorean number theory.[110] But in a passage of Iamblichus derived from Aristotle we read that "everything is revealed through numbers," and Alexander, using Aristotle's book on the Pythagoreans, says that in the Pythagorean view, *μήτε γὰρ δύνασθαί τι τῶν ὄντων χωρὶς ἀριθμοῦ εἶναι μήτε γνωρίζεσθαι ὅλως, τοὺς δὲ ἀριθμοὺς καὶ χωρὶς τῶν ἄλλων γιγνώσκεσθαι.* It is clear that the basic idea of the Platonic "system of derivation" has had an influence on Alexander's wording, that is, the distinction of gradations of being according to whether one can be thought of without the other. But this does not necessarily imply the idea of "knowability"; and Alexander adds the words *μήτε γνωρίζεσθαι* as an afterthought. Thus it seems that Aristotle did not completely ignore the Pythagoreans' theory of knowledge, even if, not being much interested in epistemology, he failed to mention it in the *Metaphysics*. The question of knowledge, once again, stems from the Eleatics: *οὔτε γὰρ ἂν γνοίης τό γε μὴ ἐόν . . . οὐ γὰρ φατὸν οὐδὲ νοητὸν ἔστιν ὅπως οὐκ ἔστι.* So said Parmenides (frr. 2.7, 8.8); and from his time on, the problem of knowledge was a frequent topic of discussion in the fifth century.[111] The thought, then, that knowledge is only possible through number, and that it is not only *ἄπειρα* that exist, but that they must be joined together with *περαίοντα* through the agency of a harmony, is far from impossible in this century.

"Everything that is known has number." An example of this is the numerical relationship of the musical intervals: "The extent of a harmony is a fourth and a fifth. . . . The fourth is one and one third (3:4), the fifth one and a half (2:3), the octave a doubling (1:2)"

[109] Arist. *Rhet.* 1408b27f: *ἄγνωστον τὸ ἄπειρον* (in relation to the theory of rhythm, where Pythagorean influence is a possibility; cf. Pl. *Phlb.* 17d). For a later formulation, see Simpl. *Cael.* 608.23ff: *εἰ ἄπειρα ὄντως ἦν, παντελῶς ἦν ἄγνωστα· ἡ γὰρ γνῶσις ὁρίζει καὶ περατοῖ τὸ γνωσθέν.* In Philolaus, however, knowledge is not entirely on the side of Limit, but is rather a matter of the harmonious coexistence of *ἄπειρα* and *περαίοντα* (fr. 6; fr. 11 is different).—In ps.-Pl. *Epin.* 976d et seq. number is praised as the basis of all knowledge; reference is made to the *τέχναι* (977d–e), music (978a), celestial movements (977a–b), and *φύσις* in general (977d et seq., in obscure hints that obviously relate to the "system of derivation"). It is unlikely that Philolaus fr. 4 was derived from the *Epinomis* passage, as Theiler thought (*Gnomon* 1931, 351f)—the simple and jejune from the nuanced and stylish!

[110] Raven, *PyEl* 99, KR 310f.—Iam. *Comm. math. sc.* 25; above, ch. I 2, nn. 112–113. —Alex. *Met.* 40.12 = Arist. fr. 203 (on Arist. *Met.* 985b27).

[111] See Melissus fr. 8 on sense perception, Gorg. fr. 3 on the unknowability of the *ὄν*. Anaxagoras (fr. 12) and Diogenes of Apollonia (fr. 8) emphasize that *νοῦς* or *ἀήρ* "knows everything."

(fr. 6). "Harmony" here (*ἁρμονία*) has the special musical sense "scale an octave long,"[112] but the essence of musical harmony is the same as of that harmony which holds the world together. Of the directly quoted fragments the ordering of the cosmos by number is only alluded to in the sentence about the One in the middle of the sphere of the universe (fr. 7); but we may infer from Aristotle's account that there was also application of numerical and musical relationships to the cosmos.

The Hippocratic writings illustrate how some thought of the numerical and musical ratios as bearing on the life of man. For example *Regimen* defines its basic question as the determination of the right relation between nourishment and activity:

> If indeed . . . it were possible to discover for the constitution of each individual a due proportion of food to exercise, with no inaccuracy either of excess or of defect, this would mean, precisely, the key to health for human beings.[113]

Health, in "precise" terms, is a numerical ratio; whoever knows the numbers has found all he needs.

The numerical ratios have a more special role in embryology. The growth and health of the embryo depend on whether he finds the right harmony—and this is expressed in musical terms:

> If, on changing position, they achieve a correct attunement, which has three harmonic intervals, the fourth, the fifth, and the octave, they live and grow. . . . But if they do not achieve the attunement, and the low do not harmonize with the high in the interval of the fourth, of the fifth, or in the octave, then the failure of one makes the whole scale of no value. . . .[114]

The treatise *On Seven-Month Babies* is more precise still: the same numbers determine the course of the embryo's development and the course of illnesses: *θεωρεῖν δὲ χρὴ οὕτως τριάσι τε καὶ τετράσι, ταῖς μὲν τριάσι συνημμέναις, δύο δὲ παρὰ δύο διεξευγμέναις.*[115] All the odd numbers are important, and of the even ones, the 14th, 28th, and 42nd days. *οὗτος γὰρ ὁ ὅρος τίθεται τῷ τῆς ἁρμονίης λόγῳ πρός τινων καὶ ὁ ἀρτιφυής*[116] *τε καὶ τέλειος ἀριθμός· δι' ἣν δὲ αἰτίην, μακρότερον ἂν εἴη ἐπὶ τοῦ παρόντος διεξελθεῖν.* From this it appears that this book is based on some specific source. In what follows, the doctrine is applied to seven-month and nine-month children; the fact that eight-month babies do not live is thought to be due to the imperfection inherent in an even number.

Similar doctrine is found in the treatise *Περὶ σαρκῶν*: *τὸ παιδίον ἑπτάμηνον [γόνον] γενόμενον λόγῳ γεγένηται καὶ ζῇ καὶ λόγον ἔχει τοιοῦτον καὶ ἀριθμὸν ἀτρεκέα ἐς τὰς ἑβδομάδας . . . ἐννέα δὲ μηνῶν καὶ δέκα ἡμερέων γόνος γίνεται καὶ ζῇ, καὶ ἔχει τὸν ἀριθμὸν ἀτρεκέα ἐς τὰς ἑβδομάδας.*[117] The periods calculated are 280 days (=40×7) and 210 days (=30×7). Other books as well[118] go into the difference of even and odd numbers; even numbers are the weaker, more likely to bring misfortune.

The beginnings of numerology, including the special role of the odd numbers, are older than Pythagoras;[119] but, when significant numbers are thought of specifically as *λόγος ἁρμονίας* and also defined in musical terms, the Pythagorean theory of the numerical relation of musical notes must be part of the background. In addition, we find the same archaic expressions for the "fourth" and the "fifth" as in Philolaus, namely *συλλαβή* and *δι' ὀξειῶν*, as well as his expression *ἀριθμὸν ἔχειν*. There seems to be a direct connection here; and the Hippocratic treatises, by their forms of expression, are clearly dependent on a more detailed treatment of these matters than their own. Lucian attests that Philolaus called the tetractys *ὑγιείας ἀρχή*,[120] and this is to be taken quite

[112] See below, ch. V 2.

[113] Hippoc. *Vict.* 1.2; cf. Iam. *VP* 163, 244 (probably from Aristoxenus; DK 58D1): the Pythagoreans desiderated *συμμετρία πόνων τε καὶ σίτων καὶ ἀναπαύσεως.*

[114] Hippoc. *Vict.* 1.8. The correction *συλλαβήν, δι' ὀξειῶν, διὰ πασέων*, for the MS reading *συλλήβδην διεξιὸν διὰ πασέων*, was proposed by J. Bernays (*Heraclitea I*, Diss. Bonn, 1848, p. 18; cf. Littré VII, p. LIV) and, independently, Delatte (*Mél. Thomas* 160ff; cf. DK 22C1). On *De victu* (*Περὶ διαίτης*, *Regimen*) cf. H. Diller, *Hermes* 87 (1959) 39–56.

[115] Hippoc. *Septim.* 9, VII 448 L. The MSS have *συνεζευγμέναις*, but there is obviously a reference to the musical terms *συνημμένων* and *διεζευγμένων*. "Continued" triads make up the series 1, 2, 3; 3, 4, 5; 5, 6, 7 . . . Thus he means the odd numbers 1, 3, 5, 7, etc. Correspondingly the arrangement of "tetrads" gives the series 1, 4, 7; 8, 11, 14; 15, 18, 21, etc. (Delatte, *Mél. Thomas* 165f).

[116] "Perfect" (wrongly "even" LSJ); cf. *ἀπαρτιλογία*, Hdt. 7.29, Pollux 2.120, Harpocr. s.v. = Lysias fr. 28 Baiter–Sauppe, *ἀντὶ τοῦ ἀπηρτισμένος καὶ πλήρης ἀριθμός.*

[117] 19, VIII 612 L.; K. Deichgräber, *Hippokrates über Entstehung und Aufbau des menschlichen Körpers* (Leipzig, 1935) 20.

[118] *Epid.* 3.17.10, III 132 L.: *οἱ δὲ παροξυσμοὶ . . . ἐν ἀρτίῃσιν μᾶλλον. Acut.* II 250–252 L.

[119] Below, ch. VI 4.

[120] Philolaus A11 = Lucian *Laps.* 5. It is not clear, either from the Philolaus testimonia or from the hints of the Hippocratic books, precisely what calculations they made. According to late sources, by virtue of the tetractys, from the one tetrad of harmonious numbers 6, 8, 9, 12, or from the other one 6, 9, 12, 18—whose sums were, respectively, 35 and 45—one arrived, by multiplying these figures by the "perfect" number 6, at the total of 210 days for the *partus minor* (7 months) and 270 for the *partus major* (9 months). (Cf. Varro ap. Cens. 9, *Th. ar.* 51, 63, Anatolius p. 8 Heiberg, Aristid. Quint. 3 p. 142, Macrob. *Somn. Sc.* 1.6.15ff, Procl. *In Remp.* II 34, etc.) Delatte (*Mél. Thomas* 171) believes

literally: the "tetrad" of harmonic numbers determines growth and health. So we perceive in the Hippocratic corpus reflections of Pythagorean doctrines, which were probably in written form; and the most likely source is the book of Philolaus.

We can learn something from these reflections about the application and the meaning of Pythagorean number theory. But Philolaus is concerned with still other relations between numbers and reality: "Now number has two special types, odd (*περισσόν*) and even (*ἄρτιον*), and a third that comes of the mixture of these two, the even-odd (*ἀρτιοπέριττον*). And each of these two has many forms, which each thing individually reveals" (fr. 5).[121] It is merely a truism to say that there are odd and even numbers,[122] but just as the cosmos contains *περαίνοντα* and *ἄπειρα* side by side but transcended by a third, harmony, so here we have an antithesis of two members, and a third above them. Without any doubt, going by the data of Aristotle and a lexicographical note in Hesychius,[123] the odd numbers are the *περαίνοντα* and the even the *ἄπειρα*. The *ἀρτιοπέριττον*, made from the mixture of the two, is the *πρῶτον ἁρμοσθέν*, that is, the One.[124] The basic outline of reality,

that the allusions in the Hippocratic writings prove that this calculation is early Pythagorean. But different numbers, which are hard to bring into connection with music, are given by Empedocles (A83, fr. 153a; cf. A75, fr. 69) and Hippo (A16). Hippoc. *Nutr.* 42 calculates four possible series for arriving at the duration of pregnancy; and we find still other constructions in Diocles, Strato (frr. 97–98 W.), and Aristid. Quint. 3, p. 151f. —Empedocles spoke of a *λόγος μίξεως* in relation to the origin of the bones, etc. (A78).

[121] Fr. 5. In this sense *εἶδος* is pre-Platonic; cf. Democr. fr. 11: *γνώμης δὲ δύο εἰσὶν ἰδέαι, ἡ μὲν γνησίη, ἡ δὲ σκοτίη* (on the history of the word, Taylor, *VarSocr* 178–267; Wilamowitz, *Platon* II 249ff). On *πολλαὶ μορφαί*, cf. Eur. fr. 839, *Alc.* 1159, etc. The terms *περιττόν* and *ἄρτιον* are found as *εἴδη τοῦ ἀριθμοῦ* also at *Divis. Arist.* ch. 64 Mutschmann. The final words are somewhat disquieting; *αὐταυτό* is Doric, and has the sense of *ἑαυτό* (Archytas frr. 2, 3; Epicharm. fr. 172 Kaibel = DK 23B4; Sophron fr. 19 Kaibel; an inscription from Heraclea, Bechtel, *GrD* II 403, 255f); but it does not mean *αὐτό*. Euripides has a number of expressions like *αὐτὸ σημανεῖ* (*Hel.* 151, *Andr.* 265, *Phoen.* 623, *Bacch.* 976).

[122] Cf. below ch. VI 4.

[123] Above, n. 73; ch. I 2, n. 24.

[124] In later terminology a number is even-odd if is even and each of its halves is an odd number (as early as Arist. fr. 47; see Nicom. *Ar.* 1.9.1; see below, ch. VI 2), so that "even-odd" is a subdivision of "even." If one understands *ἀρτιοπέριττον* in this sense in Philolaus (Newbold 185), his neat classification is spoiled. This inspired Becker (*QSt* 4 [1936] 155ff) and Junge (*DtMath* 351, *C&M* 1958, 64) to interpret *ἄρτιον* as *ἀρτιάκις ἄρτιον μόνον* (numbers of the type 2^n); but this is unexampled. The only choice left is to follow the indication of Aristotle (above, ch. I 2, n. 39) and understand *ἀρτιοπέριττον* as meaning the One. This fits in with the statement that "*both* *εἴδη*" have "many forms," implying that the third does not, and it also comports with the fact that in fr. 7 the One is characterized as *ἁρμοσθέν*, which implies that it carries an antithesis within itself. To be sure, this is unsatisfactory from a mathematical point of view (Becker 159), but not on that account non-Pythagorean.

suggested in the words *περαίνοντα καὶ ἄπειρα*, signifies from the beginning that "everything has number." Things "have" even number, insofar as they contain *ἄπειρα*; they "have" odd, insofar as they consist of *περαίνοντα*.[125] Both elements are bound together in "harmony," just as the harmony of music always consists in the connection of even and odd numbers.[126]

Aristotle's usual formulation is that, according to Pythagorean doctrine, things "are numbers" or "consist of numbers,"[127] while Philolaus speaks of them as "having number" (*ἀριθμὸν ἔχειν*). But Aristotle also uses this expression in a passage clearly under Pythagorean influence, at the beginning of the book *On the Heavens*: "For, as the Pythagoreans say, the whole and everything in it is comprehended in the number three; for end and middle and beginning [taken together] have the number of the whole (*τὸν ἀριθμὸν ἔχειν τὸν τοῦ παντός*), and this is that of the triad."[128] This is a remarkable place to find the word "number." One would expect something like *λόγος*, and only after that, the introduction of the concept of "number." Beginning, middle, and end, considered together, "have" or "contain" or "comprise" the idea of a "whole" or "totality," and since they are three in number they "have" three. A point must be raised here that has been well made by others:[129] that the Greek word *ἀριθμός* is not completely equivalent to the modern concept of "number." *Ἀριθμός* is always a whole number, and tied up with the actual procedure of counting. Thus it is closely connected with things, and in fact is itself a thing, or at least an ordering of things. *Ἀριθμός* means a numerically arranged system, or its parts. Isocrates represents the mythical king Busiris as establishing the castes of priest, artisan, and warrior in Egypt: *ἅπαντας δὲ τοὺς ἀριθμοὺς περιλαβών, ἐξ ὧν ἄριστ' ἄν τις τὰ κοινὰ διοικήσειεν, ἀεὶ τοῖς αὐτοῖς τὰς αὐτὰς πράξεις μεταχειρίζεσθαι προσέταξεν . . .*[130]

[125] Cf. Arist. *Phys.* 203a10: *τὸ ἄπειρον εἶναι τὸ ἄρτιον· τοῦτο γὰρ ἐναπολαμβανόμενον καὶ ὑπὸ τοῦ περιττοῦ περαινόμενον παρέχειν τοῖς οὖσι τὴν ἀπειρίαν.*

[126] Below, ch. V 1.

[127] Above, ch. I 2, n. 15.

[128] *Cael.* 268a10ff. On the expression *ἀριθμὸν ἔχειν* cf. also Pl. *Parm.* 153b; above, n. 117; below, n. 131.

[129] Stenzel *ZG* 24ff; Becker, *ZwU* 21f. The latter renders *ἀριθμός* as "geordnete Mannigfaltigkeit" ("ordered plurality"), and compares such expressions as "couple," "dozen," and "score." Brunschvicg, *Étapes* 34, showed that in pre-scientific thought number is a thing; and Mondolfo, *Inf.* 200, emphasizes that the converse—things are numbers—is but a short step from this.

[130] Isoc. *Bus.* 16. Cf. Hippoc. *Acut.* II 228 L., where *τοὺς δ' ἀριθμοὺς ἑκάστου τῶν νοσημάτων . . . φράζειν* means the subclassification of diseases into their special forms, describing their *πολυσχιδίη* or manifold diversity.

Busiris took "all numbers," that is all the classes, in their state as numbered and ordered groups that would be useful in the government of society. *'Αριθμός* stands for that which is counted. A sentence in the Hippocratic treatise *On Generation* is to be understood in the same sense: Injured or lame parents may have healthy offspring, *ἔχει γὰρ τὸν ἀριθμὸν πάντα τὸ πεπηρωμένον τῷ ὑγιεῖ*.[131] He is alluding to the *τέσσαρες ἰδέαι* of the Hippocratic doctrine of humors; if these four principles are present in full strength, the child gets what it needs. "The crippled has the whole number," that is, all that is necessary, in its numerical order, "as well as the healthy." One may also speak, in the same sense, of *τὰ μέρεα ἔχειν πάντα*.[132]

A significant aspect is the "aristocratic" sound of the word *ἀριθμός*. It is only what is important that "counts;" only the fully competent, effective warrior is *ἐναρίθμιος, μετ' ἀνδρῶν ἀριθμῷ*,[133] in contrast to the *δῆμος ἀπείρων* (*Il.* 24.776). To ask about the "number" means to ask about the essential. Each of the pre-Socratic philosophers, in bold simplification, selects a certain phenomenon or a single aspect of the world's multifarious reality—water, air, fire, the warm and the cold—and each thinks he has got hold of the one most significant thing. The categories of "having" and "being" are not yet strictly separated. The "is" is not taken as indicating a logical classification, but as a statement of what is essential. The book *On the Nature of Man* develops the doctrine of four humors with reference to the views of predecessors who saw the essence of life in one of the humors—blood or phlegm or bile. And the theory is expressed in the form, "Man is blood" (or phlegm, or bile); they proceed from the observation that at the time of death the body loses its blood, or phlegm, or bile. "Each man is, they thought, that which they saw him lose as he died."[134] On the other hand, one may say that the soul "has" a mixture of fire and water,[135] and this means nothing else than that the soul "is" this mixture.

In a similar way, Philolaus tries to get at what is fundamental in the cosmos by pointing out the antithesis of "Limiting" and "Unlimited," and the presence of number and harmony. "Everything has

[131] Hippoc. *Genit.* 11, VII 484 L. Cf. Ov. *Met.* 7.126: "perque suos intus numeros componitur infans," and also the common Latin phrase *omnibus numeris absolutus*.

[132] Hippoc. *Vict.* 1.7.

[133] *Il.* 2.202, *Od.* 11.449, etc., cited by Plato at *Phlb.* 17e; cf. also Eur. fr. 519, and the famous oracular response to Aegium or Megara (Deinias *FGrHist* 306F6 = *A. P.* 14.73).

[134] Hippoc. *Nat. hom.* 6: *τοῦτο δὲ ἕκαστον αὐτῶν ἐνόμισαν εἶναι τὸν ἄνθρωπον, ὅ τι καθαιρόμενον εἶδον αὐτὸν ἀποθνήσκοντα.*

[135] Hippoc. *Vict.* 1.7: *ψυχὴ πυρὸς καὶ ὕδατος σύγκρησιν ἔχουσα.*

number" means about the same as "everything is, basically, number." Aristotle, in the context of his effort to build a systematic ontology, would naturally choose this kind of formulation; at the same time, under the influence of the Platonic theory of numbers, the role of number would naturally be stressed, and ideas of limit, unlimited, and harmony would recede into the background.[136] Philolaus, however, explicitly refused to make any pronouncement about "Being" (*ἐστώ*) and is for that very reason more free to follow up the many relationships of numbers—the meaning of *ἀριθμὸν ἔχειν*.

The correct approach to the Philolaus fragments is not through the categories of Aristotelian or Platonic thought. To be sure, they agree, as far as content is concerned, with the accounts of Aristotle; but the manner of thinking and the style point rather to the pre-Socratic period, or, to put it more precisely, to that era, in the second half of the fifth century, of the coexistence of Eleatic dialectic and Ionic *φυσιολογία*. Thus the idea of a post-Aristotelian forgery is out of the question, and the fragments may be regarded as remnants of that book which the Pythagorean Philolaus wrote not very long before 400 B.C.

The concepts with which he operates—*ἐόντα, κόσμος, ἄπειρον*, as well as *φύσει καὶ οὐ νόμῳ* (fr. 9)—are borrowed from contemporary philosophy. It is not in this area that one would find the originality of the Pythagorean who rises to take his part in this discussion, which has been going on for so long. Most important, the question of Being is bracketed out, and the effort is to establish relationships, by means of the ideas of number and harmony. This step could have led from natural philosophy to natural science and was a necessary step, if one was to make any progress, in avoiding the Eleatic *ἀπορίαι* about Being. But Philolaus clearly did not recognize its significance. If his goal had been exact science, he would have been investigating the many facets of the concept of number, and specific problems would have taken the place of generalizations.[137] For Philolaus, philosophical ideas and specific items of scientific knowledge seem to have been no more than a means of expressing and illuminating a preexisting picture, of a world consisting of a pair of basic opposites, informed by harmony and defined by number. In Aristotle's exposition the mythical background becomes clearer: the antithesis of "limiting" and "unlimited,"

[136] Cf. above, ch. I 2.

[137] His pupil Archytas seems to have been much concerned with special problems in mathematics and acoustics.

of "odd" and "even," is also that of "male" and "female," and their conjunction is "marriage" (*γάμος*). The whole range of observation and experience is compressed, with resolute abbreviation, into a formula which singles out as essential only something which is not observable. What is to be shown is not what the world is, but that the world is "orderly," and in all its parts and aspects ruled by harmonious combination—a postulate that is present also in pre-scientific interpretations of the world.

So that everything may fit neatly into the cosmic pattern, the fire which envelops the world[138] must have as its counterpart the "central fire" alluded to in the expressions *τὸ πρῶτον ἁρμοσθέν, τὸ ἕν, ἐν τῷ μέσῳ τᾶς σφαίρας ἑστία καλεῖται* (fr. 7). Whether or not the related idea of the movement of the earth is possible in the fifth century is a question to be studied in the light of the history of astronomy.[139]

Stobaeus has a passage on the structure of the cosmos, in the chapter he entitles *Περὶ σχημάτων*, purporting to come from a book called *Βάκχαι*.[140] It lacks the usual tinge of dialect; and this is as surprising as its mysterious title. *ὁ κόσμος εἷς ἐστιν,*[141] *ἤρξατο δὲ γίγνεσθαι ἄχρι*[142] *τοῦ μέσου καὶ ἀπὸ τοῦ μέσου εἰς τὸ ἄνω διὰ τῶν αὐτῶν τοῖς κάτω,* ‹*καὶ*›[143] *ἔστι τὰ ἄνω τοῦ μέσου ὑπεναντίως κείμενα τοῖς κάτω. τοῖς γὰρ κάτω τὸ κατωτάτω μέρος*[144] *ἐστὶν ὥσπερ τὸ ἀνωτάτω καὶ τὰ ἄλλα ὡσαύτως· πρὸς γὰρ τὸ μέσον κατὰ ταὐτά ἐστιν ἑκάτερα, ὅσα μὴ μετενήνεκται.* The cosmos develops from the center out, in each direction equally.

[138] A16; cf. above, ch. I 2, nn. 3–4; above, n. 27.

[139] Below, ch. IV 3. Anatolius has a noteworthy comment about the Pythagoreans (p. 30 = *Th. ar.* 6.11ff): *περὶ τὸ μέσον τῶν τεσσάρων στοιχείων κεῖσθαί τινα ἐναδικὸν διάπυρον κύβον, οὗ τὴν μεσότητα τῆς θέσεως καὶ Ὅμηρον εἰδέναι λέγοντα . . .* (*Il.* 8.16). The point of view is geocentric; but Philolaus too identified the *ἑστία* in the middle with the *ἕν* and the cube with *γεωμετρικὴ ἁρμονία, ἀπὸ τοῦ κατὰ τὰ τρία διαστήματα ἡρμόσθαι ἰσάκις ἴσα ἰσάκις* (A24 = Nicom. *Ar.* 2.26.2). Was the central fire, the *πρῶτον ἁρμοσθέν*, thought of as a cube?

[140] Fr. 17 = Stob. 1.15.7.

[141] Arist. fr. 201: *τὸν μὲν οὐρανὸν εἶναι ἕνα . . .*

[142] The conjecture *ἀπό* for the difficult *ἄχρι*, usually accepted since the time of Meineke, brings with it an awkward repetition. Perhaps *ἄχρι* could be understood in relation to a primary phase of cosmic development, "as far as the middle," i.e. until the middle is formed (*τὸ πρῶτον ἁρμοσθέν*).

[143] ‹*καί*› Wachsmuth, *ἔστι* ‹*γάρ*› Diels.

[144] *μέγα* MSS, *μέρος* Wachsmuth (n.). Diels reads *τοῖς γὰρ κατωτάτω τὰ μέσα ἐστὶν ὥσπερ τ. ἀ.*, which does away completely with the reversal of direction (*μετενήνεκται*). On *μέρος*, cf. Pl. *Phd.* 112e2.—In the next clause, the principal manuscript, F, has *τῷ ἀνωτάτω*, Diels *τὰ ἀνωτάτω*.

"What is above is that which is over against the middle,[145] from the point of view of those that are below; for to those below the lowest part is like the highest, and similarly for the rest; for both (upper and lower) have the same relationship to the middle, except insofar as their positions are reversed." The author is trying to express the idea of the relativity of "above" and "below" in the world, but keeps using these terms because he cannot free himself from the idea of an absolute up and down. Plato dealt with this problem in a much more sophisticated way, in the *Timaeus*; and the doxographers record that "Pythagoras," Plato, and Aristotle recognize no up and down in the cosmos.[146] The only other laborious exposition like this is in the *On Sevens* of the Hippocratic corpus—a fifth-century document:[147] *ἴσοι δὲ τὸν ἀριθμὸν ὅμοιοί τε τὴν ἰδέην οἱ ὑπὸ τῇ γῇ κόσμοι τοῖσιν ὑπὲρ γῆς . . . ὥστε τοῖσι κάτω τάδε μὲν τὰ ἄνω κάτω εἶναι, τὰ δὲ κάτω ἄνω.* Clearly the Philolaus fragment is to be classed with this pre-Platonic passage rather than with the easy mastery of thought and expression in the *Timaeus*; and therefore, even if the dialect difference has been smoothed out in the course of transmission, it may be regarded as authentic. And incidentally, the idea of the gradual development of the universe is another feature which would comport ill with post-Aristotelian forgery.[148]

Another fragment cited by Nicomachus from the book *Περὶ φύσεως* also gives the impression of being old:[149] *κεφαλὰ μὲν νόου, καρδία δὲ ψυχᾶς καὶ αἰσθήσιος, ὀμφαλὸς δὲ ῥιζώσιος καὶ ἀναφύσιος τοῦ πρώτου, αἰδοῖον δὲ σπέρματος καταβολᾶς τε καὶ γεννήσιος· ἐγκέφαλος δὲ* ‹*σαμαίνει*› *τὰν ἀνθρώπου ἀρχάν, καρδία δὲ τὰν ζῷου, ὀμφαλὸς δὲ τὰν φυτοῦ, αἰδοῖον δὲ τὰν ξυναπάντων· πάντα γὰρ ἀπὸ σπέρματος καὶ θάλλοντι καὶ βλαστάνοντι.* Of course, scholars have attempted to identify portions of this with

[145] For *ὑπεναντίως* with the genitive, cf. Hdt. 3.80, 7.153. *τοῖς κάτω* is dative of relation, as in the next clause. In the translation of DK, the construction of the genitive *τοῦ μέσου* is not clear: "denn was oben liegt von der Mitte aus, verhält sich zu dem, was unten liegt, entgegengesetzt."

[146] Pl. *Tim.* 62d. Bywater (52f) stigmatized the Philolaus book as a copy of the *Timaeus*. —Aët. 2.10.1.—On *διὰ τῶν αὐτῶν*, cf. Epicharm. fr. 1, Reinhardt, *Parm.* 123.

[147] *Hebd.* 2; on its date, see below, ch. III 3, n. 63. The MS reading (according to Roscher) is . . . *τοῖσι κάτω τὰ δὲ μέντοι ἄνω κάτω εἶναι* (corr. Boll.).

[148] Of the other citations of the *Bacchae*, fr. 18 (where only the subject heading *περὶ ἡλίου* is preserved) could be brought into connection with A19 (below, ch. IV 3), and fr. 19 (a mere allusion) with A14 (both are cited by Proclus). Can *Βάκχαι* be a late, "romanticizing" substitute for the title *Περὶ φύσεως*?

[149] Fr. 13 = Nicom. in *Th. ar.* 25.17.

Plato's three parts of the soul, and their localization in the *Timaeus*, as well as the Aristotelian hierarchy of *θρεπτικόν-γεννητικόν*, *αἰσθητικόν*, *λογικόν*. But such an interpretation demands an incredibly stupid forger[150] with a remarkably profound conception of nature.[151] What are we to make of *ψυχὰ καὶ αἴσθησις* in the heart? The words "soul" and "perception" are obviously not used in their broader signification; and this very fact suggests a pre-Socratic milieu. *Ψυχή* means nothing more than "life,"[152] and *αἴσθησις* goes with it, not as sense perception through the various specialized sense organs,[153] but as the apprehension of stimuli, the faculty of being influenced and reacting.[154] Critias wrote that this kind of *αἰσθάνεσθαι* was closely connected with the *ψυχή* that is, to life;[155] what is dead is *ἀναίσθητον*.

Democritus uses metaphors from botany in speaking of the importance of the navel;[156] the expression *σπέρματος καταβολά* occurs in the passage about Philolaus in Menon's history of medicine (A27); both Empedocles and Diogenes of Apollonia know the hierarchy of plant-animal-man;[157] and the distinction between man and beast by the criterion of *νοῦς* may be derived from Alcmaeon. The tendency to think in terms of parallels and antitheses, which appears here in Philolaus, is unquestionably ancient. The thought of microcosm and macrocosm is also relevant in this context, insofar as the types of living beings are brought into relationship with the organs of the body.[158] In these correspondences, once more, we find the orderly arrangement and "harmony" of the cosmos, here divided into four and to that extent defined by number.[159] Thus this fragment fits into the general point of view shown in the others, even though there is nothing in it about Limit and Unlimited.

There is perhaps even less of this in the best-attested passage, that from Menon (A27–28). Here the subjects are heat and cold, the causes of disease, blood, bile, and phlegm—all treated quite in the manner of a physician writing in the tradition of natural philosophy. Living beings, we learn, develop from "the warm," for the semen is warm as well as the uterus. After birth the infant draws in air and expels it again "as though it were a loan." This is how it cools itself—*ψυχή* is explained from *ψύχειν* as *φλέγμα* is derived from *φλέγειν*. Whether or not this is dependent on Prodicus, a relation which would give us a terminus post quem, is matter for controversy.[160] The closest relationship of the passage seems to be, once again, with Diogenes of Apollonia.[161] The necessarily eclectic nature of Philolaus' book is especially clear in this instance. To be sure, a relationship to Pythagorean cosmology has been alleged here. Just as, according to Aristotle, the cosmos breathes in the "unlimited" void, as the One, after its origin, sucks in the Unlimited, so the newly born, warm living creature breathes in the cool air; here again microcosm and macrocosm are set parallel.[162] Still, in order for the analogy to be really striking, the general subject would have to be embryology, whereas here we have a completely formed infant making its first contact with the air. Perhaps even to a Pythagorean it was more important to have knowledge of a broad range of subjects than to have a perfect system.

[150] At Pl. *Tim.* 69d–70e, the three parts of the soul are localized in the head, the breast, and the belly. Bywater (44ff) thought that the Aristotelian tripartition of the soul and its functions was mechanically grafted onto the scheme of the *Timaeus*; but this involves the oddity of assigning the *αἴσθησις*, since it is the middle member, to the heart. And how does the *ψυχή* come to be associated with it there?—Olerud, too, finds Plato's tripartite soul in Philolaus (74f). Can anyone equate *ψυχὴ καὶ αἴσθησις* with *θυμός*?

[151] Schaarschmidt 12f. Frank (320ff) tries by arbitrary combinations to establish a connection between this and A12 (*ψύχωσις ἐν ἑξάδι*—cf. above, n. 41). The gradation seen in fr. 13 is inconsistent with the scheme of derivation in A12.

[152] Cf. Diogenes of Apollonia fr. 4: *ψυχὴ καὶ νόησις* ("life and intelligence"), *Hebd.* 10, 13, Hippoc. *Vict.* 1.7. Frank (321) translates correctly "Leben." Cf. also Wilamowitz, *Platon* II 91: "das Körperliche wiegt so stark vor, dass die Einheit des Seelenlebens, eigentlich die Seele als solche, überhaupt noch nicht erfasst ist."

[153] If this were the case, the natural thing would be to localize sense perception in the head. Cf. Arist. fr. 95.

[154] On the pre-Socratic use of *αἴσθησις*, see H. Langerbeck, *Δόξις ἐπιρρυσμίη* (Berlin, 1935) 44ff. It means "sensation [Empfindung]," but is not used as a general term for the aggregate of the five senses (cf. Democr. fr. 11, Diogenes of Apollonia fr. 5). In non-philosophical language *αἰσθάνεσθαι* means "perceive" or "comprehend" (Soph. *Aj.* 553; cf. Thrasymachus fr. 1), or "perceive" and "feel" (a misfortune, for example, as at Soph. *OT* 424). Plants, too (Protagoras ap. Pl. *Tht.* 167c), and even lifeless things have *αἴσθησις* (cf. Hippoc. *Morb.* VI 386 L., *Vet. med.* 15). According to Hippoc. *Morb.* VI 392f L., heart and diaphragm have nothing to do with thinking; and yet they *αἰσθάνονται μάλιστα* i.e. "feel" [emotions] most poignantly." The philosophical concept of *αἴσθησις* is worked out in Plato's *Theaetetus* as something new (152c, 160e, 186d–e).

[155] DK 88A23: *τὸ αἰσθάνεσθαι τῆς ψυχῆς οἰκειότατον*. Cf. Hippo A11.

[156] Fr. 148: ... *πεῖσμα καὶ κλῆμα τῷ γεννωμένῳ καρπῷ καὶ μέλλοντι* ... Bywater refers to Chrysippus (*SVF* II fr. 806 = Plut. *Stoic. rep.* 41.1052f: the embryo is *καθάπερ φυτόν*).

[157] Empedocles: Sharastani, DK I 358f, n.; Diogenes of Apollonia: A19 §44 (distinction with regard to *φρονεῖν*). Cf. "plant and animal," Diogenes fr. 2, and on the differentiation of man and animal, Alcmaeon fr. 1a, Archelaus DK 60A4 §6, Eur. *Tro.* 671f. The hierarchy plant–animal–man–god is known in the east from Sumerian times (vase from Uruk, *ANEP* no. 502) to Iranian (J. Duchesne-Guillemin, *East and West* 13 [1962] 200).

[158] Olerud 72ff.

[159] In a fourfold division the power of the tetractys is of course at work (above, ch. I 3; II 4; above, n. 120).

[160] Cf. above, n. 1.

[161] Diogenes of Apollonia A28: *γενναῖσθαι μὲν τὰ βρέφη ἄψυχα, ἐν θερμασίᾳ δέ· ὅθεν τὸ ἔμφυτον θερμὸν εὐθέως προχυθέντος τοῦ βρέφους τὸ ψυχρὸν εἰς τὸν πνεύμονα ἐφέλκεσθαι*. This is like Philolaus A27: the living creature arises from the warm in the warm, and *μετὰ ... τὴν ἔκτεξιν εὐθέως ... ἐπισπᾶται τὸ ἐκτὸς πνεῦμα ψυχρὸν ἄν* ... Hippo, A10 also has something quite similar.

[162] Above, ch. I 2, n. 47. The connection with Philolaus A27 was seen by Frank (127f), Mondolfo in *ZM* (170), Olerud (47ff), and Raven (in *KR* 313).

Philolaus could draw on a well established Sicilian and south Italian medical tradition, with names like Democedes, Alcmaeon, Empedocles, and Acron.[163]

A medical milieu is also the source of the doctrine that the *ψυχή*, "soul" or rather "life," is in fact a "harmony" of the bodily functions.[164] It is ascribed by Macrobius to "Pythagoras and Philolaus";[165] this can come from reliable doxographical tradition, but also may be an inference from the *Phaedo*, where this doctrine is discussed. Scholars saw long ago that this is the only point where Echecrates interrupts the account of Socrates' last conversation: *θαυμαστῶς γάρ μου ὁ λόγος οὗτος ἀντιλαμβάνεται καὶ νῦν καὶ ἀεί, τὸ ἁρμονίαν τινὰ ἡμῶν εἶναι τὴν ψυχήν* . . . [166] This shows that, as Plato represents the matter, the soul-harmony doctrine was important for this Pythagorean from Phlius, who was a pupil of Philolaus.[167] If the order of the universe is *ἁρμονία*, then so is that of organic life, and the passage we have cited from the Hippocratic *Regimen* (n.114 above) shows how one could express the idea of "life" in musical terms, too. It is another question how this concept of harmony comports with the old belief in the immortality and transmigration of the soul. One has the impression that Plato, in this passage of the *Phaedo*, was the first to point out an embarrassing implication in the idea of the soul as a harmony.[168]

[163] Ch. III 3. At A27, the thesis is treated as specifically Pythagorean that illness is caused, among other things, by *ὑπερβολαὶ θερμασίας* . . . *καὶ ἔνδειαι*, so that health naturally is dependent on the correct balance. This idea, however, is not only found in Alcmaeon (fr. 4) and Hippo (A11), but became a commonplace in Greek medicine (Keus 58ff, Krämer 366ff). In this passage of Philolaus, however, it is introduced only as an afterthought (A27).

[164] See Wehrli on Dicaearchus fr. 11 (D., as well as Aristoxenus, frr. 118ff kept this doctrine alive); Heraclitus fr. 67a; Hippoc. *Vict.* 1.7f. For the soul as a mixture, Parmenides fr. 16, Zeno A1 §29. The comparison between the soul and music: Pl. *Gorg.* 482b, 525a, *Rep.* 443d, 432a, 441e, et saep.

[165] Macrob. *Somn. Sc.* 1.14.19 = DK 44A23. The lemma may have read *τῶν Πυθαγορείων τινὲς μέν, ὧν ἐστι Φιλόλαος* (like Aët. 2.30.1 = DK 44A20; Aët. 4.2.3 ascribes to Pythagoras Xenocrates' definition of soul [above, ch. I 3, n. 73]). Cf. Plotinus 4.7.8.4 (*οἱ ἀμφὶ Πυθαγόραν*), Philop. *De an.* 70.5.

[166] Pl. *Phd.* 88d. This was brought out by Döring, *AGP* 1892, 525ff; cf. also Burnet, *EGP* 295f. According to Aristotle, *πολλοὶ* . . . *τῶν σοφῶν* teach that the soul is a harmony (*Pol.* 1340b18, *De an.* 407b27). J. Moreau contested the attribution to Philolaus (*La construction de l'idéalisme platonicien* [Paris, 1939] 374ff), suggesting that the doctrine was too "materialistic." The danger is, however, that, coming after Plato, we see Pythagoreanism through idealistic glasses.

[167] Aristox. fr. 19; above, ch. I 4, n. 40.

[168] Cf. also Arist. fr. 45. Wilamowitz, among others (*Platon* II 90), believed that the enlightened Pythagoreans of Phlius no longer believed in immortality. Guthrie (I 310) emphasizes that the prohibition of suicide (Pl. *Phd.* 61d–e) presupposes a belief in immortality. An attempt to reconcile this belief with ideas of the soul as a harmony is made by H. Gomperz, *Hermes* 1932, 156; Rostagni, *Verbo* 101ff; Carcopino, *Bas.* 168;

There remain several passages in praise of number, cited by Stobaeus in his prologue without indication of the title of the book they are taken from.[169] First comes a paean on the number 10, then one on number in general, which makes possible all our knowledge, is at work everywhere, and admits no *ψεῦδος*.[170] Throughout there is manifested an animation that is almost poetic, and a definitely rhetorical conformation of style—in the choice of words like *παντελὴς καὶ παντοεργός*, *σύμφυτον τᾷ τῶ ἀριθμῶ γενεᾷ*, in placement of words (*πᾶσι παντᾷ*, *οὐδενὶ οὐδέν*), in the use of parallelism and chiasmus.[171] The content is simple, sometimes even trivial. Only one sentence, bearing on the theory of knowledge, is obscure, though not incomprehensible. Number "in the soul, in harmony with sense perception, makes everything knowable and mutually agreeable, working like a carpenter's square, fixing and loosing the proportions of things, each for itself separately, those that are unlimited and those that are limiting."[172]

Both in content and in expression there appear many points of contact with pseudepigraphic Pythagorean writings, and especially pseudo-Archytas. Verbal agreement with an expression of Plato's, in the context of the "nuptial number," is suspicious; for even though in this matter Plato is "Pythagorizing," what that means, in his case, is not copying a striking phrase but following a line of thought. The author of the fragment uses twice the phrase *ἡ οὐσία τῶ ἀριθμῶ*, three

Wuilleumier 572; and others. One might refer to the expression *ἁρμονία ἐπεγένετο* (fr. 6), but there is a danger of importing to the interpretation a degree of ideality that, though Platonic, did not belong to the early Pythagoreans. The idea that health consists in proper proportion and the reconciliation of opposites (above, n. 163) is not exactly identical with the soul-harmony doctrine, but also not contradictory to it, since illness does signify a threat, and finally destruction, to life. (Döring, *AGP* 1892 525ff found a contradiction between A27 and A23; contra, Mondolfo in ZM 370.)

[169] Stob. 1 prooem. cor. 3 = frr. 11–12. (In DK I 412.15, Theo Smyrnaeus is erroneously given as the source of fr. 12.) On what follows, cf. Frank 313 n. 1.

[170] MSS: *ψεῦδος δὲ οὐδαμῶς ἐς ἀριθμὸν ἐπιπνεῖ*. Jacobs' emendation *ἐμπίτνει* seems certain.

[171] *τῶ ἀπορουμένω παντὸς καὶ ἀγνοουμένω παντὶ* . . . , *πολέμιον* . . . *τᾷ φύσει τὸ ψεῦδος, ἁ δ' ἀλάθεια οἰκεῖον* . . . *τᾷ τῶ ἀριθμῶ γενεᾷ*.

[172] *καττὰν ψυχάν* cannot mean "der Seele gemäss," as Becker interprets (*QSt* 4.159ff). Cf. ps.-Archytas 38.19 Thesleff: *ἐν ἁμῖν αὐτοῖς κατὰ ψυχὰν γνώσιες τέτταρες* (namely *νοῦς, ἐπιστήμη, δόξα, αἴσθησις*). From them comes truth, when the four *γνώσιες* become *σύμφωνοι*, p. 39.3 Thesleff. "Agreement with sense perception" becomes clearer in ps.-Archytas p. 36.22 Thesleff: *ἐπικρίνει δὲ ὁ νόος τὸν λόγον ὅκα μὲν ποτὶ τὸ νοατὸν ποτιβάλλων, ὅκα δὲ ποτὶ τὸ αἰσθατόν* . . . For example, the *νόος* recognizes the ratio of the octave as 2:1 by use of the *νοατόν*, but on the other hand, *ὅτι συμφωνεῖ* it knows *διὰ τᾶς αἰσθάσιος*. This also implies an allusion to the *λόγοι* in "Philolaus."—*γνωστὰ καὶ ποτάγορα* must be predicative adjectives with *πάντα*, as in Pl. *Rep.* 546b–c, *πάντα προσήγορα καὶ ῥητὰ πρὸς ἄλληλα ἀπέφηναν*; Moderatus (ap. Stob. 1.49.32): *τὴν (ἁρμονίαν) τὰ διαφέροντα* . . . *σύμμετρα καὶ προσήγορα ἀπεργαζομένην*. (Here differing from Becker,

times ἁ φύσις τῷ ἀριθμῷ,[173] and once ἁ τῶ ἀπείρω καὶ ἀνοήτω καὶ ἀλόγω φύσις. Many parallels for all these can be cited from Pythagorean pseudepigrapha, and in particular the heaping up of various concepts to clarify two opposite φύσεις;[174] but both form and content are unexampled in pre-Socratic or pre-Platonic philosophy. Here φύσις is not, as in fragment 1 and fragment 6, an expression for "all that exists," and also not the existence, in accord with natural law, of a particular thing; it is a realm or grade of Being. Precisely this kind of division or gradation of Being was unknown to the Pythagoreans, according to the testimony of Aristotle; they clung to the single, perceptible world and did not speak of immaterial οὐσίαι.[175] Along with this goes the high poetic style of compounds like παντοεργὸς καὶ παντελής, favorites among the imitators of the *Timaeus*,[176] and the propensity for formations in -ικός,[177] which seems to show a direct connection with Aristotle.[178]

On the other hand, there are clear contradictions with the other group of fragments, in spite of their having in common the phrase τῶν πραγμάτων τῶν τε ἀπείρων καὶ τῶν περαινόντων—something that might very easily have been copied.[179] To be sure, fragments 1–7 have an ἐστὼ τῶν πραγμάτων, but this is general, comprehensive "being"; number does not have a peculiar mode of being, and Limiting and Unlimited are not characterized as two separate principles or φύσεις. Harmony, in those fragments, stands over the pair of opposites which first made it necessary, while here the "nature and harmony of number" is set over against the "nature of the unlimited and unintelligent and irrational." "Soul," ψυχή, is here, unequivocally, the comprehensive notion; knowledge takes place "in it," "in agreement with sense perception," in the fully developed sense. It is placed in a relationship of harmonious tension with thought, that is, with number.[180]

Thus fragment 11 shows so many suspicious features that it cannot have been composed in the fifth century B.C., but only after Plato and Aristotle and along with the rest of the Pythagorean pseudepigrapha.[181] But the status of fragments 1–7 is not affected by the rejection of fragment 11, for aside from one easily imitated phrase there is no apparent connection between them, but in fact a definite difference. There is nothing in the manner of transmission to suggest that they should be taken together; in Stobaeus, fragments 11–12 are introduced between "Timaeus of Locri" and Aristotle, Platonically interpreted, on one side, and pseudo-Archytas on the other.[182]

QSt 3.538 n. 5, 4.159ff, who construes: ἁρμόζων αἰσθήσει πάντα ⟨τὰ⟩ ["must be supplied"] γνωστὰ καὶ ποτάγορα ἀλλάλοις | κατὰ γνώμονας | φύσιν ἀπεργάζεται σωμάτων '. . .) The γνώμων (on which see Newbold 179ff, following Boeckh 142ff; Heath, *Math.* I 78f) is the "square" or "set-square" of the carpenter. In mathematics it is the difference between two rectangles, then, more generally, of two similar polygons, or "figured numbers" of this type (above, ch. I 2, n. 27); cf. Iam. *In Nic.* 58.19ff. But the word came to be used in a much broader way, especially since its etymological significance is so obvious. Iam. *In Nic.* 18.7 uses it synonymously with κανών. According to Lydus *Mens.* 1.15 the number 10 is γνώμων by virtue of being πάντα χαρακτηρίζουσα καὶ . . . ὁρίζουσα. According to Nicomachus, ap. *Th. ar.* 79.23, the τεχνικὸς νοῦς used the number 10 as μέτρῳ τῶν ὅλων . . . καὶ ὥσπερ γνώμονι καὶ εὐθυντηρίῳ (cf. *Th. ar.* 81.15 = Philolaus A13: 10 is called πίστις, ὅτι . . . δεκάδι καὶ τῆς αὐτῆς μορίοις περὶ τῶν ὄντων οὐ παρέργως καταλαμβανομένων [-οι MSS, -οις Ast, DeFalco, DK, -ων Becker] πίστιν βεβαίαν ἔχομεν . . .). —At DK I 412.2, Boeckh's conjecture σωματῶν (σωμάτων MS F) is untenable; this verb is attested only in the passive, and late. Heidel's idea, that σῶμα = ἄθροισμα, so that σωματοῦν = ἀθροίζειν, is without parallel (*AJP* 1907, 79). Newbold's suggestion was better: looking for something to correspond to σχίζων, he proposed συνάπτων; συμβάλλων would be possible (cf. Heraclitus fr. 126a), though less probable paleographically.

[173] He also has ἁ τῷ ἀριθμῷ γενεά, which has a poetical or mythical ring, and is also found in Plato (*Phlb.* 66b): τὸ σύμμετρον καὶ καλὸν . . . καὶ πάνθ' ὅσα τῆς γενεᾶς αὖ ταύτης ἐστίν.

[174] Cf. above, ch. I 2, n. 119, esp. ps.-Archytas fr. 1 p. 20.15 Thesleff: τὰν ῥητὰν καὶ λόγον ἔχοισαν . . . , τὰν ἄλογον καὶ ἄρρητον . . . There are similar phrases in Aristotle, above, nn. 72–73. Cf. also Arist. *Met.* 1010a3: ἡ τοῦ ἀορίστου φύσις, "Eurysus" p. 88.11 Thesleff: δύο φύσιες ἐν τῷ ὅλῳ τῷδε . . . ἁ μὲν ῥητὰ καὶ τεταγμένα καὶ λόγον ἔχουσα . . . ἁ δ' ἄρρητος καὶ ἄτακτος καὶ ἄλογος καὶ οὐδεμίαν σύνταξιν ἔχουσα, and above all the Ἱερὸς λόγος of "Pythagoras," Iam. *VP* 146 = 164.9 Thesleff: τὰν ἀριθμῷ οὐσίαν ἀίδιον ἔμμεν ἀρχὰν προμαθεστάταν τῷ παντὸς ὠρανῷ καὶ γᾶς καὶ τᾶς μεταξὺ φύσιος, ἔτι δὲ καὶ θνητῶν (MS θείων, corr. Festugière, *Rév.* I 338 n. 3) καὶ θεῶν καὶ δαιμόνων διαμονᾶς ῥίζαν . . . (Philolaus fr. 11: θείω καὶ οὐρανίω βίω καὶ ἀνθρωπίνω ἀρχά . . .). —Mondolfo, in ZM 373f, sees the suspicious nature of the language, but would like to evade the difficulty by understanding οὐσία as "wealth" and pointing to the pre-Socratic use of ἔργα (Parmenides fr. 10, etc.). Cf., however, the wording of Arist. *Cael.* 298a28 (Frank 313 n. 1): τῶν φύσει λεγομένων τὰ μέν ἐστιν οὐσίαι, τὰ δ' ἔργα καὶ πάθη τούτων . . . with the alleged Philolaus, τὰ ἔργα καὶ τὰν οὐσίαν τῷ ἀριθμῷ.

[175] Above, ch. I 2.

[176] Cf. Hippasus, DK 18.11 (above, n. 50): number is παράδειγμα πρῶτον κοσμοποιίας . . . κριτικὸν κοσμουργοῦ θεοῦ ὄργανον.

[177] A. Amman, *-ικος bei Platon*, Diss. Fribourg, 1953.

[178] Arist. *Met.* 982a23ff lists these criteria of the highest σοφία: τὸ τὰ χαλεπὰ γνῶναι, τὸ ἀκριβές, διδασκαλικόν, ἀρχικώτερον, αὐτῆς ἕνεκεν. Cf. Philolaus: γνωμικὰ γὰρ ἁ φύσις ἁ τῷ ἀριθμῷ καὶ ἡγεμονικὰ καὶ διδασκαλικά . . . (γνωμικός is unparalleled; an easy alteration would be to γνωστικός).

[179] Fr. 11, DK I 412.2f; fr. 6, DK I 409. 1f, cf. frr. 1–2.

[180] By contrast with fr. 13; above, n. 154.

[181] Another possibility would be that someone issued a new, modernized edition of Philolaus' book; but in any case there were several books in circulation under his name (above, n. 11).

[182] Theo's report (106.10) that Philolaus dealt with the number 10 ἐν τῷ περὶ φύσεως could be based on the genuine book, in which he certainly discussed 10, or on the apocryphal fragment.

Along with fragment 11, no doubt, we must give up fragment 12, and its "verteufeltes Lastschiff."[183] This in spite of the fact that the four elements are neither called by the Platonic term στοιχεῖα nor derived from polyhedra. Though it is quite thinkable that Philolaus took over his theory of the elements from Empedocles,[184] the mention of a "fifth" raises doubts.[185] Our conclusion must be that here once more we have a popularizing enumeration of the elements by someone familiar with Platonic and Aristotelian teachings.

Although some dubious material and some that is unquestionably spurious has been transmitted under the name of Philolaus, there is a group of fragments—1–7, 13, and 17—which can only be interpreted on the basis of pre-Socratic ways of thinking, not those of Plato and Aristotle and their followers. Here the proof of authenticity drawn, in an indirect way, from the doxographical tradition, is directly confirmed; there have been preserved for us remains of a book composed by Philolaus in the pre-Platonic period, including both word-for-word fragments and doxographical reports, which advocates that philosophy of Limit and Unlimited, of number and harmony, to which

[183] Wilamowitz, *Platon* II 91. The principal opponents of the authenticity of this fragment have been Howald (64f), Theiler (*Gnomon* 7 [1931] 351f), and Festugière (*REG* 1945, 16 n. 4). Sachs (41ff) was able to treat it as genuine because it does not have to do with the regular polyhedra (cf. above, ch. I 3, n. 116; the inclusion of the reference in Aët. 2.6.5, attributed to Pythagoras, as Philolaus A15 is of course a slender reed).— . . . καὶ ὁ τᾶς σφαίρας ὁλκάς, πέμπτον—the impressive image of the ship is hard to give up. But there is no meaningful way to connect this with τρόπεως δικην in A17 (Wilamowitz, loc. cit.), nor with Arist. *Cael.* 291a11, to which Cherniss refers (*Pres.* 186 n. 177). Burnet, *EGP* 294 1, compares πόντος τῆς ἀνομοιότητος, Pl. *Pol.* 273d, which he thinks should be equated with the ἄπειρον. Richardson, *CQ* 1926, 116ff adduced the ὑποζώματα of Pl. *Rep.* 616b; cf. also the references collected by Diels, DK I 413 n. The word ὁλκάς remains an embarrassment. It is a technical term of the shipping industry, and presupposes something ἕλκον (cf. Wilamowitz, loc. cit.: "das versteht niemand, auch wenn er sich's abzuringen glaubt"). The noun ὁ ὁλκός, however, which Wilamowitz proposes, means "coils," not "covering" or "rounded sphere." (It is only by way of the Latin *volumen* that Wilamowitz can make his way from one meaning to the other. Rostagni [*Verbo* 56ff] agrees with Wilamowitz [referring to *Orph. hymn.* 87.3, where death ψυχὴν θραύει καὶ σώματος ὁλκόν—an almost incomprehensible expression, where Dieterich felt constrained to conjecture ὄγκον; see Wilamowitz, *GldH* II 516 n. 2]; see also Mondolfo, *Inf.* 286.3.) Timpanaro Cardini (1946, 331ff) wishes to interpret ὁλκός as the πνεῦμα drawn in from the sphere (above, ch. I 2, n. 46); but there are no parallels to support this either. The only recourse is to understand ὁλκός as an adjective, with Frank (318 n. 2) and, before him, Mullach (II 6), who conjectured ὁ τᾶς σφαίρας ὁλκὸς ‹αἰθήρ›. The αἰθήρ causes the movement of the heavens and with a συνεφέλκεσθαι causes all movement in the air and on earth (Arist. *Mete.* 341a2; cf. *Th. ar.* 81.19ff). Thus we shall read: καὶ ὁ τᾶς σφαίρας ὁλκόν, πέμπτον.

[184] On the close connection with Empedocles in the theory of the sun, cf. below, ch. IV 3.

[185] According to A16 (first half), it is not a "fifth element" but fire which is the περιέχον. Cf. P. Moraux, *RE* XXIV 1181–1184.

Plato alludes in the *Philebus* and which Aristotle ascribes to the Pythagoreans. This may well be the only written exposition of Pythagorean number theory before Plato.[186] The book is hardly likely to have been a smashing success; perhaps it would seem to us almost as odd as the book *On Sevens*. The author's own intentions and the borrowed philosophical terminology do not always fit harmoniously, so that much seems tedious or awkward, and much unsystematic or "eclectic." Only in the science of Archytas (who was the teacher of Eudoxus) and in the philosophical reinterpretation of Plato did Pythagoreanism attain to a form in which its real influence could develop.

We shall have to test the result reached here by considering the musicological and astronomical doctrines of Philolaus in the framework of the history of those sciences. Here too we shall find that the Philolaus fragments represent a stage before Archytas, Eudoxus, and Plato, and that one cannot, without serious qualification, speak of a specifically Pythagorean science before Philolaus.

3. REFLECTIONS OF PYTHAGOREAN PHILOSOPHY IN THE FIFTH CENTURY B.C.?

If the earliest book of Pythagorean philosophy was not composed until the latter part of the fifth century B.C., the gap between it and the historical Pythagoras is uncomfortably wide. In order to close it, or at least to narrow it, many attempts have been made to date individual doctrines, in the realm of science or natural philosophy, in the early period; in this way Pythagoras himself could be assured of the credit of providing the "germ," or at least the initial impulse, for the later development. Philolaus himself probably thought of his doctrines as merely an explication of the wisdom of Pythagoras.[1] If that were the case, it would affect our interpretation of the Philolaus fragments: we should expect more genuine Pythagorean tradition and less eclecticism. But when the Philolaus fragments are treated as spurious, the search for chronological clues becomes really crucial. There is, in fact, not a single piece of direct evidence to be found. The later tradition, though it uses the name of Pythagoras, is contaminated with Platonism and must be ruled out of court.[2] And the older, reliable

[186] Aside from Archytas, though he seems to have devoted himself more to specific problems, and from Ecphantus, who took his own way.

[1] Above, ch. III 1, n. 98; I 4, n. 36.

[2] Above, ch. I 3.

evidence has to do with the Pythagoras legend and the *acusmata*. It reveals the picture of a shaman-like sage and a *βίος* lived in accord with his precepts,[3] a life in which philosophy or science, as the Greeks understood it, does not necessarily have any part at all. The *acusmatici*, later on, did not recognize the *mathematici* as Pythagoreans.

If there is no direct evidence, can indirect testimony be found, for example reflections of Pythagorean teachings in the works of other philosophers? They may have taken over Pythagorean material, or entered into polemic against it. Since the day of Tannery[4] scholars have been treading this path, with growing confidence. They attempt to discover doctrines of Pythagoras from their influence, as an astronomer sometimes infers the existence of a hitherto unknown star from irregularities in the course of known planets.[5] In this way a tempting chapter of the history of philosophy may be built; erratic boulders and unidentifiable gravel coalesce into a comprehensive structure. The suspected interaction of the Eleatics and Pythagoreans, in particular, becomes a living dialogue. Parmenides, the apostate Pythagorean, sets up his own system in opposition to that of the school; in response, the Pythagoreans revise their theories, only to be subjected to new attacks, by Zeno; this forces them to undertake further revision . . .

This structure, however, rests on a shaky foundation. It is true that there are many points of contact between what Aristotle or the later tradition attributes to Pythagoreanism, and pre-Socratic doctrine; but to conclude immediately that these come from a Pythagorean origin would be to presuppose what ought first to be proved. It is also possible in each case that a later Pythagorean like Philolaus, in an eclectic spirit, borrowed material from others,[6] or that the later tradition is wrongly ascribing to Pythagoras material from a foreign source.[7] Again, what

[3] Above, ch. II 3–5.

[4] Tannery, *HScH* 125, on Pythagoras: "en l'absence de documents authentiques directs . . . nous ne pouvons deviner ses opinions particulières sur le monde que par les traces qu'elles ont pu laisser dans les écrits de ses contemporains ou des penseurs de la génération suivante."—Cherniss (*Pres.* 387, 157 n. 68) considers the "number atomism" deduced from Zeno so certain that he uses it as a basis from which to criticize Aristotle (39f; cf. ch. I 2, n. 66). Raven (KR 236, *PyEl* passim) considers that their relationship to the Eleatics provides our only opportunity to learn any detail or to arrive at any chronological determination for the Pythagoreans; he knows, though, that this method is "hazardously conjectural."

[5] The comparison is made by J. Stenzel, *Metaphysik des Altertums* (Munich, 1931) 46, and Gigon, *Ursprung* 126.

[6] Consider, e.g., Philolaus' theory of the sun (below, ch. IV 3), and the astronomical theories of Alcmaeon (below, ch. IV).

[7] E.g., the doctrine of the 4 elements (above, ch. I 3, n. 113), or the division of the earth into 5 zones (below, ch. IV 1).

appears both in Pythagorean and in other sources may go back to a common source rather than being the result of mutual influence.[8] In each case, proof of an "irregularity in orbit" would be of decisive importance in the evaluation of the indirect source, proof of a change of direction or a distortion of the course of thought and the system which could only be explained on the basis of external influences. But this is a very difficult thing to prove, even in the case of Plato, and seems to lead to nothing but further controversy; for the pre-Socratics, preserved only in sorry fragments, it is practically hopeless.

Unknown quantities keep multiplying, for the nature and characteristics of Pythagoreanism, whose influence and diffusion one is trying to determine, are far from being clearly understood. In order to get any kind of start, one has to take something or other as presupposed, "given." Scholars have frequently regarded it as almost self-evident that the Pythagorean doctrine of Limit, Unlimited, and number must have existed from the day of Pythagoras in some form or other, which in any case was abstract and philosophical; in this way, the only question is to decide what aspects of it Parmenides and Zeno presuppose.[9] But this is the very thing—the existence of these doctrines before the book of Philolaus—which ought to be proven first; and particular attention should be paid to clarifying the nature of a philosophy without written documents. Other scholars, whether consciously or not, proceed on the assumption that Pythagoreanism had certain general characteristics: it was a powerful religious movement, the source of all non-Homeric elements of Greek religiosity;[10] or it is the great synthesis of religion and science, featuring the *cura animarum* as the special concern of philosophy, and celebrating the harmony of man and cosmos.[11] Or, finally, it was the source of exact science among the Greeks, responsible for all significant attainments, especially in the realm of astronomy and mathematics.[12] Each of these hypotheses

[8] Cf. above, ch. III 2, n. 31, on *Ὄλυμπος*, below, ch. VI 2, on even and odd.

[9] Raven proceeds, tacitly, from this premise.

[10] To carry out this idea consistently, Homer himself must, it is thought, have been revised in the light of Pythagorean ideas. The second part of the Nekyia is thought to be Pythagorean (*Od.* 11.565–632: Delatte, *Litt.* 135; *Od.* 11.601ff, and also 576–600: Cumont, *Symb.* 369 n. 5, *Lux* 190); also the Second Nekyia and the "Gate of dreams" (*Od.* 24.11ff, 19.562ff: Carcopino, *Apôtres* 202 n. 94, 208 n. 128); perhaps also *Od.* 19.27f (V. Bérard, *RPh* 45 [1921] 231; Homère, *L'Odyssée* [Paris, 1924–1925] ad loc.). Onomacritus the Orphic (i.e. Pythagorean) is likely to be the forger, thinks Carcopino (*Apôtres* 208 n. 128).

[11] Cf., e.g., Gigon, *Ursprung* 12: ". . . dass mit Xenophanes ein neues Element in der Philosophie auftaucht . . . Und da wir wissen, dass Xenophanes . . . Pythagoras gekannt hat, nennen wir dieses Element Pythagoreisch." 120: "Thema wird nun das Schicksal der Seele und die Erziehung der Seele zu ihrem Schicksale."

[12] Chs. IV–VI.

contains something that is attractive, and perhaps a measure of truth; but as long as they are not defined and supported by direct evidence, they will not serve as the foundation for reconstruction of the system. An epoch in which a unique development in the history of thought took place, like the period of something more than a hundred years between Pythagoras and Plato, surely saw inner transformations even in apparently stable traditions. In the relation of religion and rationalism the center of gravity must have shifted. If we take the legend of Pythagoras, the *acusmata*, and the accounts of the *acusmatici* as seriously as they deserve, we realize that within his school it was only after the day of Pythagoras that the movement from myth to science took place. We also run the danger, influenced by Plato's evaluation, of seeing the φυσιολογία of Anaxagoras, for example, in too rational and nonreligious a light, and disregarding mythical and religious forces which were at work in this area as well.

It is a reasonable guess that thinkers from Magna Graecia would show Pythagorean influences; but only meticulous study of the internal and external evidence can raise this possibility to a probability—to say nothing of certainty.

Most important of the relationships between Pythagoreans and other groups are those with the Eleatics. Geographically these were close to the south Italian centers of the Pythagoreans, and the abstract, immaterial character of their philosophy naturally results in coincidences with a philosophy of number. In addition, the ancient tradition makes Parmenides and Zeno Pythagoreans, or at least pupils of Pythagoreans.[13]

Even in Xenophanes Tannery saw polemic against a Pythagorean teaching. Diogenes Laertius paraphrases his idea of god, ὅλον δὲ ὁρᾶν

[13] The statement of Sotion about the hero shrine that Parmenides dedicated to his teacher Ameinias, the son of Diochaetas, may be derived from a genuine inscription (D.L. 9.21), though it would still be doubtful whether Ameinias was called a Pythagorean. For Parmenides and Zeno as Pythagoreans, see Anon. Phot. 439a35, Strabo 6, p. 252, Procl. *In Parm.* 619.4 Cousin (DK 28A4; Proclus cites Nicomachus as his source, but there is a variant reading "Callimachus," which is taken into the text by Cousin and Stallbaum, and later accepted by Zeller I 680.1; the Latin translation makes the reading "Nicomachus" certain: R. Klibansky, *Plato Latinus* III [London, 1953] xxxii; Callim. 822 Pfeiffer, is among the "delenda"). Cebes *Tab.* 2, Iam. *VP* 267: Parmenides and Melissus in the catalogue of Pythagoreans; cf. schol. p. 150 Deubner, Iam. *VP* 166; cf. ch. II 4, n. 106. Down to imperial times there was a guild of physicians in Elea which traced its history back to Parmenides. The president had the title φώλαρχος, which is reminiscent of the Pythagorean ἄντρα (above, ch. II 3, n. 197). Cf. P. Ebner, *Rassegna storica Salernitana* 23 (1962) 4ff; M. Gigante, *Parola del passato* 19 (1964) 135–137, 450–452; P. Merlan, *AGP* 48 (1966) 267–276; H. Jucker, *MH* 25 (1968) 181–185. The feud between Pythagoreans and Eleatics, which most scholars have believed in since Tannery, is not hinted at in the ancient sources, as Capparelli emphasizes (I 267ff).

καὶ ὅλον ἀκούειν,[14] μὴ μέντοι ἀναπνεῖν.[15] But according to the Pythagoreans the cosmos breathes in the Void, or Unlimited.[16] Xenophanes, Tannery concludes, is rejecting this idea, thus presupposing that it had been stated. If this were right, an essential motif of Pythagorean cosmology would be older than Xenophanes, perhaps suggesting, in the opposition of κόσμος and πνεῦμα the theory of πέρας and ἄπειρον in general.[17]

But this conclusion is anything but certain. Xenophanes is not talking about the cosmos, but about god, and it is a much mooted question in the interpretation of Xenophanes whether his god is to be equated with the universe.[18] But even if this question were to be answered in the affirmative, it would be just as reasonable to suspect an allusion to Anaximenes as one to Pythagoras;[19] in fact it is impossible to prove that he is voicing criticism of any particular person. Xenophanes is reacting against the naive, anthropomorphic conception of the gods. The principal criteria of a living being, along with the ability to see, hear, and apprehend psychologically (νοεῖν), are breath and motion. The god, "as a whole," exercises the former functions, but not the latter. There is no more reason to suppose the denial that he breathes is directed polemically against contemporary philosophers than the denial that he moves about.[20]

Parmenides has been exploited much more as a source for Pythagorean philosophy, though in a different way. Tannery maintained that the *doxa* section of his poem, with the specific statement that its teachings are "deceptive" (fr. 8.52), was a doxography, from a hostile point of view, of Pythagorean cosmology.[21] Later writers claimed that

[14] This much is preserved as a directly quoted fragment (24).

[15] D.L. 9.19 = DK 21A1.

[16] Above, ch. I 2, n. 46.

[17] Tannery, *HScH* 125ff, followed by Burnet, *EGP* 108; DK I 113.26 n.; Rostagni, *Verbo* 25; Rey 134, 207ff; Mondolfo, ZM 314ff. Contra, Rathmann 37f.

[18] The case for a pantheistic interpretation is argued by Zeller, I 656ff, and is presupposed by Tannery, *HScH* 125. Reinhardt (*Parm.* 116ff) believed that world and god were separate for Xenophanes. Aristotle's comments in *Met.* 986b10ff are of course not conclusive.

[19] Burnet, *EGP* 108, refers to Anaximenes fr. 2. The Pythagorean doctrine discussed by Aristotle can be brought into connection with Diogenes of Apollonia (A9, fr. 5), and thus with Philolaus (above, ch. III 2, n. 161). Pl. *Tim.* 33c argues against this theory.

[20] Fr. 26. This is of course directed against the Homeric tales of the gods; though it goes without saying that Homer's gods breathe. Cf. Aesch. Eum. 651, on Zeus: οὐδὲν ἀσθμαίνων μένει.

[21] For the interpretation of the *doxa* section as a doxography, Zeller I 724ff, Diels, *Parm.* 63. Application to the Pythagoreans: Tannery, *HScH* 232ff, Burnet, *EGP* 183ff, Rey 183ff, 347ff, Mondolfo, ZM 326f, *Inf.* 286ff, Cornford, *CQ* 1922, 137, *PlParm* 1ff.

in Parmenides' particular manner of developing his argument there could be seen a reflection of Pythagorean mathematics.[22] Then, finally, Raven sought to explain the Parmenidean predicates of Being as a polemical expression against the Pythagorean doctrine of opposites.[23]

The world of *doxa* is, in Parmenides' poem, the result of opposition and interpenetration of the contrary powers of Fire and Night. The attempt to bring this into relation with the Pythagorean opposition of Limit and Unlimited has led to contradictory results.[24] There is not a single detail that can be shown to be exclusively Pythagorean;[25] there is nothing in Parmenides about number and mathematics, nothing in the realm of *doxa* about Limit and Unlimited, and nothing about the harmony which unites the opposites. Pairs of opposites played a part in Ionian physical philosophy from the time of Anaximander, and also in Alcmaeon. Their reduction into a single opposition is a result of Parmenides' fundamental thesis; as he comprehends Being as a unity, the multiplicity of the world turns out to rest on one basic contradiction.[26] Thus the *doxa* section of the poem is for this very reason a personal achievement of Parmenides—a fact generally accepted since

[22] Gomperz, *GrD* 136; Rey, 189ff; Cornford, *PlParm* 29, *PrSap* 117. Contra: Szabó, *AA* 1955, 67ff; 1956, 109ff; cf. below, ch. VI 1.

[23] Raven, *PyEl* 21ff, KR 274. Similarly, K.-H. Ilting, *ABG* 9 (1964) 103–131, tried to show that *πεῖρας–ἄπειρον* in Parmenides presupposes a Pythagorean doctrine.

[24] Tannery, *HScH* 207 and 235, equated Night with earth (with Arist. *Met.* 986b34 = A24) and with Limit, and Light with Unlimited. On the other hand, Burnet (*EGP* 109, 186f, followed by Rey 208, 273, 372f) equated Night with *πνεῦμα* and Unlimited, but Light with form and Limit. (This is in harmony with the placement of Light and Darkness in the table of opposites, and fits in with the fire doctrine of Hippasus, DK 18.7.) Mondolfo, ZM 327, *Inf.* 345ff, assumes a reversal of positions: the Unlimited was originally a dark *πνεῦμα* until, after Philolaus, it becomes the fiery aether (*περιέχον*, A16); Timpanaro Cardini argues against this (1946, 328ff).

[25] The astronomical system of *στεφάναι* (A37, fr. 12; below, ch. IV 1), carried on by Plato's system of *σφονδύλοι* (*Rep.* 616c; cf. Morrison, *JHS* 1955), comes from the fiery wheels of Anaximander. On the relation of Anaximander and Parmenides, cf. Fränkel, *WF* 186ff. Why Pythagoras is thought to be the intermediary is hard to see (Tannery, *HScH* 237).—On *Ὄλυμπος*, above, ch. III 2, n. 31. *Ἀνάγκη*, absolute necessity conceived of by thought, is a discovery of Parmenides. (It is of course a misleading convention to spell the word with a small alpha at fr. 8.16 and a capital at 8.30 and 10.6.) The later tradition has no indication that *ἀνάγκη* is specifically Pythagorean (Tannery, *HScH* 242ff; above, ch. I 3, nn. 143–150. At Emp. fr. 115 *Ἀναγκη* decides the fate of the soul, but in Pi. fr. 133 it is Persephone, and in the Gold Plates, Moira).—There remains the "right-left theory" of his embryology in fr. 17 (cf. E. Lesky, *AbhMainz* 39ff). The association of right with male and left with female appears in the table of opposites; but this leads nowhere except into a realm of pre-philosophical musing about analogies and "order." See G. E. R. Lloyd, *JHS* 82 (1962) 56–66. On the astronomical discoveries that are ascribed to Parmenides, rivaling those of Pythagoras, see ch. IV 1.

[26] Reinhardt, in particular, has worked out the argument for the origin of the doctrine of opposites in the thought of Parmenides (*Parm.* 71ff, 236ff).

the day of Reinhardt.[27] There is nothing to impel us to insert, between the Ionians and the philosopher of Elea, a specifically Pythagorean doctrine of opposites.[28]

According to the view of Raven, Parmenides arrives at his pronouncements on the *ἐόν* as an alternative to the Pythagorean doctrine of opposites. The *ἐόν* is limited, single, indivisible, and cannot come into being or perish, while in Pythagoreanism Limit stands over against the Unlimited, and in a cosmogonic process the One develops into a Many. Raven himself, however, acknowledges that Parmenides takes his stand, on principle, against any kind of cosmogony,[29] and this takes the force from Raven's argument. For there is no way of showing that Parmenides is aiming at a specific Pythagorean system. To be sure, his preference for *πέρας* is significant, and to a modern the solution of Melissus seems more natural. But this is merely a matter of pre-philosophical valuation; the connection of *πέρας* and *τέλειον* is deeply rooted in Greek language and thought. He explicitly rejects the notion of condensation and rarefaction advanced by Anaximenes.[30] This shows that Pythagoreans were not, at least, his only opponents. In fact, there is nowhere any unequivocal indication that Parmenides' poem presupposes any Pythagorean science or philosophy.

There may be relationships on a different level. The prologue, in which Parmenides depicts, in the present tense, a journey in a horse-drawn chariot beyond the great gate, to a meeting with a divinity and the revelation of truth, is, as has long been recognized, equivalent to a

[27] Reinhardt, *Parm.*, passim; cf. Fränkel, *DPh* 463f, *WF* 179ff; Schwabl, *WS* 1953; Raven, too (*PyEl* 37), argues against connecting the *doxa* section of the poem with Pythagoreanism.

[28] This hypothesis has been advocated by Schwabl, *WS* 1953, 63f, 68f, *AAHG* 9 (1956) 148; Ilting (above, n. 23).

[29] *PyEl* 41f. On the date of the table of opposites, see above, ch. I 2, n. 120. Cornford and Raven believe in a reformation of the Pythagorean philosophy as a result of the critique of Parmenides. Cornford thinks they put forward their "number atomism." Raven thinks of the One as even-odd, which carries the opposites in itself; Parmenides had denied that plurality can emerge from unity. On the priority of the One as even-odd, see above, ch. I 2, n. 52.

[30] Fr. 8.22ff. Cf. Reinhardt, *Parm.* 50; Szabó, *AA* 1953–1954, 247ff. Reinhardt has been the principal opponent of the attribution of Pythagorean material to Parmenides (*Parm.* 231ff; cf. 66: "tatsächlich deutet denn auch nicht ein einziges Wort bei Parmenides auf etwas Fremdes, Aussenstehendes, Nicht-Eleatisches hin"); cf. N. B. Booth, *Phronesis* 2 (1957) 93ff. Fränkel rightly ignores the relationship with the Pythagoreans, in his studies of Parmenides. E. L. Minar, "Parmenides and the World of Seeming," *AJP* 70 (1949) 41–55, sees relationships principally in the personal and the socio-political realms.

journey into the next world.[31] We may compare it with Aristeas or Epimenides, as well as the shamanistic healing of illness and the journey of the dead into the other world found in southern Italian sources, especially the *katabasis* of Pythagoras.[32] Parmenides puts his knowledge into competition with older wisdom. Light and Night represent the realms of life and death. The goddess "sends the souls, now from the visible into the invisible, now back again."[33] This means that the existence of the soul is antecedent to the cycle of life and death, and implies a kind of transmigration. Further, we learn of the *Δαίμων*,

πάντα γὰρ ‹ἦ› στυγεροῖο τόκου καὶ μίξιος ἄρχει.

"Hateful birth and intercourse"—such a phrase cannot be a simple, formulaic allusion to the pains of childbirth;[34] the attitude of disgust and revulsion expressed in the word *στυγέειν*, *στυγερός* is contrary to normal feeling and betrays a remarkable degree of alienation from ordinary ways of thinking.

κλαῦσά τε καὶ κώκυσα ἰδὼν ἀσυνήθεα χῶρον

—this is Empedocles' reaction, as expressed in the *Katharmoi*, to entrance into human life; and a Pythagorean *acusma* calls our birth a punishment.[35] Such a "puritanical" attitude to life, which sees our existence mainly as a burden and a punishment, can scarcely be called anything but Pythagorean, especially in southern Italy. This withdrawal from the world of the senses, and concentration on the solitary "Is," has its prototype in yoga-like exercises in concentration, and in "shamanistic" ecstasy.[36] Parmenides' doctrine of Being is, to a degree, a transposition of the theory of immortality to an entirely new plane. It is not from Pythagorean science or philosophy that this man takes his departure, who himself initiated a new era in Greek philosophy, but from that complex of "shamanistic" prophecy, unusual beliefs about the soul, and puritanical outlook on life to which the story of Pythagoras and the *acusmata* lead us.

[31] Cf. Diels, *Parm.* 9ff (where, at pp. 14f, we already find the word *Schamanismus*); Nestle in ZN 727 n. 1; Morrison, *JHS* 1955; Cornford, *PrSap* 118; Meuli 171f. Cf. above, ch. III 3.

[32] Cf. Burkert, *Phronesis* 1969.

[33] Simpl. *Phys.* 39.19 (DK ad fr. 13); cf. Rohde, *Psyche* II 157f = 372f Eng. ed. (though he takes no account of frr. 12 and A1); Zeller, too, recognizes here at least a relationship with the doctrines of metempsychosis (I 722 n. 2). The fact that perishing is named before coming-to-be shows that death is not an end.

[34] Fr. 12.4. Zeller (I 722 n. 2), opposing M. Ritter, who had seen the correct answer, would like to interpret *στυγερός* here as meaning nothing more than the pains of labor; but the *ὠδῖνες*, though they may be *πικραί* (*Il.* 11.271) are also *ἐραταί* (Pi. *Ol.* 6.43). Besides, the word *στυγερός* surely casts its influence over *μίξις*. Above all, in Greek usage, Death, or Hades, is *στυγερός*; *στυγερὸς τόκος* is parallel to *στυγερὸς Ἀΐδης* (*Il.* 8.368)—life is a death . . .

[35] Empedocles, fr. 118; Iam. *VP* 85. On the concept of "puritanism" introduced by Dodds, see above, ch. II 3. One might also refer to D.L. 9.22 = A1: *γένεσίν τε ἀνθρώπων ἐξ ἡλίου πρῶτον γενέσθαι*—if the text is accurate. Cf. Burnet, *EGP* 190 n. 4; differently Zeller I 718 n. 1. Also see Pl. *Symp.* 190b, and below, ch. IV 3, on the mythical descent of human beings from the sun and the moon. The Pythagorean *acusma* says that the sun and moon are the "Isles of the Blest" (above, ch. IV 4, n. 72).

Some have thought they discovered in Zeno's polemical argumentation still more specific references to Pythagorean philosophy and mathematics. In Plato's view he was defending Parmenides against those who made mock of his doctrine of the one Being, by proving that the assumption of plurality was even more absurd,[37] and it has been thought that the partisans of plurality could only be Pythagoreans.[38] The complex of fundamental problems that makes its first appearance in the paradoxes of Zeno is still live in modern logic and mathematics,[39] and this gives an additional incentive to understand Zeno's arguments as fully as possible and to identify his opponents. To be sure, even the most determined advocate of his relationship to the

[36] Cf. Fränkel *DPh*² 417–420, and above, ch. II 3.

[37] Pl. *Parm.* 128c, not of course a direct quotation from Zeno about his book, but Plato's view of the matter.

[38] Tannery, *HScH* 258ff, and Burnet, *EGP* 314f, who argue that, since the atomists and Empedocles were later than Zeno, only the Pythagoreans could fit the circumstances and that Zeno was not likely to be attacking ordinary common sense, because that is inexpugnable. Plato, however, speaks of others "making fun" of him (*κωμῳδεῖν*), and that does not require a formal philosophical system.—Tannery's thesis was developed further by Cornford, *CQ* 1923, 7f, *PlParm* 56ff, who was supported by others, notably Lee. Cf. Stenzel, *Metaphysik des Altertums* (Berlin, 1929), 45 f; Cherniss, *Pres.* 43 n. 165, 95 n. 401; Mondolfo, *Inf.* 238ff; on the mathematical side, Hasse-Scholz. It was contradicted by Zeller I 752 n. 1; Junge, *Symb. Joach.* 232ff; Heidel, *AJP* 1945, 21ff; Vlastos, *Gnomon* 1953, 31; Fränkel, *WF* 234 n. 1; G. E. L. Owen, *Proc. of the Arist. Soc.* 58 (1957–1958) 199–222; D. J. Furley, *Two Studies in the Greek Atomists* (Princeton, 1967) 44–56; above, ch. I 2, nn. 66–74. For the specifically mathematical aspects, van der Waerden, *MtAnn* 1940–1941; doubts: Booth, *Phronesis* 2 (1957) 1ff, 99ff; mediating: Mau, *Inf.*—Plato and Simplicius speak of a single book of Zeno (Heidel, *AJP* 1945, 22). According to the *Suda* (A2) Zeno wrote, along with other books, one with the title *Πρὸς τοὺς φιλοσόφους*, and this is taken as a reference to Pythagoreans (Burnet, *EGP* 312 n. 2; Lee 8; Joly 31f; cf. Burkert, *Hermes* 1960, 170). Zeno was regarded as the ancestor of skepticism (D. L. 9.72; cf. 9.99); one might say that, just as Sextus Empiricus divided philosophy into logic, physics, and ethics, and wrote books *πρὸς λογικούς*, *πρὸς φυσικούς*, *πρὸς ἠθικούς* (*Math.* 7–11), so Zeno wrote *πρὸς τοὺς φιλοσόφους*, but this is an indication of content, which became a title only in the *Suda*; it has nothing to do with Pythagoreans.

[39] See, for example, Bertrand Russell, *Mysticism and Logic* (London, 1917, repr. 1950) 80ff.

Pythagoreans[40] admits that his arguments against movement and against the idea of space are expressed so generally that it is impossible to relate them to any historical persons. It is only the arguments against plurality that are referred to specifically Pythagorean doctrines, a mathematics of infinitesimals or a "number atomism."

In the directly quoted fragments, Zeno formulates the thesis to be refuted as generally as possible: εἰ πολλά ἐστιν (frr. 1, 3). This is interpreted, "there is a plurality of concrete things . . . each of these concrete bodies is a number, or plurality of units."[41] This way of understanding the phrase is supported by the fact that some sources designate the "plurality," against which Zeno is speaking, more specifically as a πλῆθος ἑνάδων[42]; but, thanks to the thoroughness of Simplicius, we can see precisely how this tradition came to be, and can show that it represents a diversion from Zeno's own thoughts and manner of speaking.

In the second book of the *Metaphysics*, Aristotle discusses, along with other problems, those of ἕν and ὄν, in the tradition of Plato's *Parmenides*. Here he mentions the view that ἕν is identical with the point and remarks that the latter would be, κατὰ τὸ Ζήνωνος ἀξίωμα, "nothing."[43] This is a reference to fragment 2, which we have in Zeno's own words, and nothing else. It is employed in the context of Platonic discussion; and the definition of a point as μονὰς θέσιν ἔχουσα too, belongs to Plato and not to Zeno.[44]

Eudemus, also following along Platonic lines, develops the antinomy of ἕν and πολλά, which he regards as solved by the Aristotelian distinction between δυνάμει and ἐνεργείᾳ.[45] Then, with a φασί, he adds a dictum of Zeno's: καὶ Ζήνωνά φασι λέγειν εἴ τις αὐτῷ τὸ ἓν ἀποδοίη τί

[40] Lee 64.

[41] Cornford, *PlParm* 58; cf. Raven, *PyEl* 71; Tannery, *HScH* 259f. From the point of view of the history of mathematics, below, ch. VI 1 and 3.

[42] Collected by DK 29A21; Lee 14–18, with comm. pp. 24ff (Lee introduces these secondary testimonia first, and only later interprets the verbally quoted fragments). Among others, Burnet refers to this evidence (*EGP* 315; also Raven, *PyEl* 72, who uses the definition of plurality as a "number of units" as a criterion for Pythagoreanism, though he rejects the idea of "number atomism" at 67ff). General skepticism is expressed by Fränkel, *WF* 213.1; Heidel, *AJP* 1945, 23; Booth, *Phronesis* 2 (1957) 1ff (though he speaks of "ones," as does Zeller I 749: "jede Vielzahl eine Anzahl von Einheiten").—Burnet, *EGP* 291 n. 3, found in the word ὄγκος (Zeno A28 = Arist. *Phys.* 239b33ff, the "Stadium") a Pythagorean technical term for the "unit-points" (followed by Rey 197). But ὄγκος is used for a physical "body" generally; cf. Pl. *Tht.* 155a, *Parm.* 164d, *Tim.* 31c, Heraclides frr. 118, 120; frequent in Aristotle. It is an open question whether Aristotle is using his own terminology here or Zeno's.

[43] Arist. *Met.* 1001b7.

[44] Cf. above, ch. I 3. The gradation point–line–plane–solid, which Aristotle mentions in this passage, does not occur in Zeno's own words (frr. 1–2).

[45] Fr. 37a W. = Simpl. *Phys.* 97.7ff.

ποτέ ἐστιν, ἔξειν τὰ ὄντα λέγειν.[46] This is a very loose kind of citation, and not a reference to Zeno's book.[47] Eudemus proceeds to explain the sentence, on his own lines, beginning ἠπόρει δὲ ὡς ἔοικε . . . and by saying "apparently," he shows that what follows is exposition and not quotation. In the realm of perception, he says, there is no ἕν because of the plurality of predicates of each thing—an *aporia* of Plato's[48]—but a point is not a ἕν for Zeno, but a "nothing." Here he is using the same Zenonian argument as Aristotle. Eudemus then follows this line of thought further, and concludes with the answer to Zeno, that ἕν is not ἐνεργείᾳ "many," but is so δυνάμει.

Simplicius states that Zeno's argument, in his book, goes in just the opposite direction. There, as Plato also testifies, he refutes plurality in favor of the Parmenidean unity of Being whereas Eudemus makes him prove that there is no One. Alexander of Aphrodisias, however, continues Simplicius, interpreted the argument set out by Eudemus differently, as a refutation of plurality: if there is no ἕν, there is also no plurality, for plurality is a group of unities, a πλῆθος ἑνάδων.[49] In saying this, Alexander has Eudemus' own words before his eyes, as is proved by an explicit citation; that is, he is trying to bring Eudemus' report into connection with the content of Zeno's work—it is interpretation of interpretation. These comments have no value as original source material for the history of philosophy,[50] especially since Alexander's own source, Eudemus, is not concerned at all with the historical Zeno, but is merely injecting citations of Zeno into his discussion of a Platonic problem. Finally, then, when Philoponus adds to the argument from διχοτομία a further argument of "Zeno" against plurality, that there is no ἕν because of the plurality of predicates, and therefore no plurality as τὸ πλῆθος ἐξ ἑνάδων, he is dependent on Alexander of Aphrodisias,[51] and what little he adds is merely pedantic systematization.

[46] Simpl. *Phys.* 97.12 = DK 29A16.

[47] It is correctly classified in DK (29A16) under the heading "Apophthegmatik"; the source could be some Sophistic or Academic dialogue (like Plato's *Parmenides*). Diels thinks of A29, too, as perhaps coming from some ancient dialogue.

[48] Cf., e.g., *Phlb.* 14d et seq.

[49] Simpl. *Phys.* 99.12 = DK 29A21.

[50] Zeller (I 749.2) and Lee (26) are able to regard Alexander as right because they forget his dependence on Eudemus.

[51] Philop. *Phys.* 42.9ff (80.23ff is merely repetition). For the διχοτομία, we have the original wording (frr. 1, 3), in which he speaks of ἕτερον πρὸς ἕτερον and of πολλά, but never of ἑνάδες. Alexander also interpreted the διχοτομία in his sense (Simpl. *Phys.* 138.5); and Philoponus takes this over, as refutation of the ἕν. The second argument against the ἕν, from the plurality of predicates (where even Socrates makes his appearance as an illustration), cannot be Zeno's; this is acknowledged by Lee (27ff), though he overlooks the connection with Eudemus.

The idea that Zeno conceived the plurality that he wanted to refute as a "plurality of units"[52] is a secondary product of an exposition of Alexander, who combined the historical Zeno with an argument of Eudemus' which was not meant as a historical statement at all. Aristotle and Eudemus, in discussing the equation of point and ἕν, cited a passage of Zeno which had not been written for this purpose. So there is no justification for using this passage as a basis for inferences about a Pythagorean "number atomism."

The origin, meaning, and intention of Zeno's philosophy will remain controversial, because the problems he raised are of so fundamental a nature; therefore we may leave unsettled the question, to what extent Zeno's ἀπορίαι[53] are intended to prove a positive doctrine, like for example the idea of a continuum.[54] The connection of Zeno and Parmenides is crucial; Plato himself made it clear that Zeno's arguments represented the polemical "reverse" to the "obverse" of Parmenides' philosophy of Being.[55] Zeno's target is the naive worldview of "sound common sense" in general.[56] An additional consideration here is that any polemic reshapes its own opponent. The same realization brings recognition of the correct and rejection of the false; the antithesis is determined by the thesis. If one is, rightly, mistrustful of Aristotle even where his aim is "merely" to report the opinions of others, because interpretation, as well as polemic, will always introduce some distortion, then it must be quite a hopeless undertaking to reconstruct an opposition from Zeno's polemic, where

[52] Lee is forced to assume that Zeno spoke of the One in a double sense: both as the Parmenidean Being, and as the Pythagorean atomic units—an unlikely and unprovable complication. Since Zeno's first concern is to prove formally that Being must have size (fr. 2), he is not engaging in polemic against a ready-made doctrine of atomic unities (cf. Vlastos, *Gnomon* 1953, 32).—Sen. *Ep.* 88.44 (DK 29A21) may also come from Eudemus, or from the doxographical tradition which made Zeno out to be a skeptic (above, n. 38).

[53] ἀπορίαι without solution, dialectic proof of the impossibility of both thesis and antithesis: Isoc. 10.3, Pl. *Phdr.* 261d, Arist. *Phys.* 239b10.

[54] That Zeno was aiming to show the continuity of space and time, against a Pythagorean "atomistic" conception, is the belief of Tannery, *HScH* 258ff; Burnet, *EGP* 320; Cherniss, *Pres.* 157; Mau, *Inf.* 15ff, and others. Contra: van der Waerden, *MtAnn* 1940–1941. According to Aristotle, the Platonists introduced ἄτομοι γραμμαί precisely because they wanted to get away from Zeno's paradoxes (*Phys.* 187a1, *Lin. ins.* 968a18 [DK 29A22]; cf. Xenocrates, frr. 41–49; van der Waerden, op. cit. 153; Mondolfo, *Inf.* 238ff). Rey (192ff) supposes there were two opposed Pythagorean theories, one of which advocated continuity and the other discontinuity of space and time, so that, in whatever direction Zeno aimed his blows, he would hit Pythagoreans. Surely this is carrying Pythagoromania ad absurdum!

[55] Pl. *Parm.* 128ab. Cf. W. Kullmann, "Zeno und die Lehre des Parmenides," *Hermes* 86 (1958) 157–172; Szabó, *AA* 1953–1954, 254ff.

[56] Cf. Zeller I 747 n. 1; Fränkel, *WF* 234 n. 1, 221 n. 2, 229 n. 1; Heidel, *AJP* 1945, 20ff.

an opponent is not even named. It cannot be proved that there existed a Pythagorean philosophy or science before Parmenides and Zeno.

The possibility of Pythagorean influences must be considered in relation to many thinkers of the fifth century. Ancient tradition brings Alcmaeon,[57] Epicharmus,[58] and especially Empedocles[59] into connec-

[57] On the form of the name, see DK I 495.39. For Alcmaeon as a Pythagorean, D.L. 8.83, Iam. *VP* 104, 267, Philop. *De an.* 88.11, Sophonias *De an.* 14.31, schol. Pl. *Alc.* 121e, Simpl. *De an.* 32.3. Simplicius emphasizes that Aristotle does not call him a Pythagorean: at *Met.* 986a27ff (on the text, above, ch. I 2, n. 6) a relation between Alcmaeon and the Pythagoreans is seen, in their doctrines of opposites; but this very point sets them up as different from one another. Brotinus, Leon, and Bathyllus, to whom Alcmaeon's book is dedicated, are regarded as Pythagoreans. (On Brotinus, above, ch. II 2; the catalogue, Iam. *VP* p. 144.2 D., names Leon, and at 145.10, "Bathylaus" as Pythagoreans.)—Modern writers emphasize sometimes the originality of Alcmaeon (Heidel, *Medicine* 43; R. A. Stella, "Importanza di Alcmeone nella storia del pensiero greco," *R. Acc. Linc.* 336 [1939] 233–287; Guthrie I 341–359), and sometimes his dependence on the Pythagoreans (Rostagni, *Verbo* 35f; Mondolfo, ZM 620ff; Timpanaro Cardini 118ff, with refs.). Zeller tries to compromise between the two positions (I 596f).—If, as Favorinus says, Alcmaeon was the first to write περὶ φύσεως (D.L. 8.83 = DK A1, cf. A2), he is obviously to be dated earlier than Parmenides.

[58] Epicharmus as a Pythagorean: D. L. 8.78, Iam. *VP* 266; with the support of apocryphal writings, D.L. 8.7 (cf. 78), Plut. *Numa* 8 (DK 23B65), Iam. *VP* 241ff. Ennius, in his *Epicharmus*, deals with Pythagorean doctrines of transmigration (DK 23B47–54). Rostagni, more than anyone else, used Epicharmus in the attempt to reconstruct early Pythagoreanism (*Verbo* 7ff; cf. Mondolfo, ZM 318ff; Zeller had already taken the opposite position, I 607ff; Timpanaro Cardini does not include Epicharmus in her collection). Reinhardt (*Parm.* 118ff) set out the relations to the Eleatics. Epicharmus cited Xenophanes by name (Arist. *Met.* 1010a5 = DK23B15). In considering the fragments in detail, one is faced at every step with the problem of authenticity. What Alcimus quotes may be accepted, tentatively, as genuine (III 1, n. 43; cf. M. Gigante "Epicarmo, Pseudo-Epicarmo e Platone," *Parola del passato* 8 [1953] 161–175). What looks like a theory of ideas we can be sure, from the testimony of Aristotle, is not Pythagorean (fr. 3, spurious according to Diels, DK I 193, and according to Schmid I 644 n. 6, comprehensible "only if Pythagorean"). It is not, however, fundamentally Platonic (see Zeller I 608 n. 6); we can only guess at the comic context from which it may have come. Is it the contrast between φυᾷ ἀγαθός and ἀγαθόν as an acquirable πρᾶγμα? In that case the fragment might be genuine.—On even and odd numbers in fr. 2, see below, ch. VI 2.—Among the entirely or partly apocryphal works, the *Politeia* (DK 23B56–57) was composed, according to Aristoxenus (fr. 45 = DK 23A10) by Chrysogonus the flute player (a contemporary of Alcibiades; cf. Ath. 12.535d). What Clement quotes from it has a Pythagorean ring, but also shows affinity with Platonism. Fr. 57.1: ὁ λόγος ἀνθρώπους κυβερνᾷ κατὰ τρόπον σῴζει τ' ἀεί, fr. 56.2: ζῶμεν ἀριθμῷ καὶ λογισμῷ· ταῦτα γὰρ σῴζει βροτούς. Cf. *Epin.* 976e: θεὸν δ' αὐτὸν (τὸν ἀριθμόν) . . . δόντα ἡμῖν σῴζειν ἡμᾶς. See also the Derveni papyrus, col. 20. on the ἀριθμός of the winds and seasons. On the date of Epicharmus, see Schmid I 1.638f (before 488/487, going by Arist. *Poet.* 1448a33).

[59] On fr. 129, see above, ch. II 3; Alcidamas ap. D.L. 8.56 (above, ch. II 2); Timaeus *FGrHist* 566F14 = D.L. 8.54; Neanthes *FGrHist* 84F26 = D.L. 8.55 (quoting the "Telauges" letter); Hermippus, D. L. 8.56. A spurious line of Empedocles names Telauges as his teacher (fr. 155; cf. Euseb. *Praep. evang.* 10.14.15, Theodoret 2.23, *Suda* s. v. Empedocles); Alcidamas and Timaeus name Pythagoras himself; and the Telauges letter names Hippasus and Brotinus. Theophrastus mentioned only his relation to Parmenides, but

tion with Pythagoras himself, but also Leucippus and Democritus.[60] Hippasus[61] and probably also Hippo[62] were called Pythagoreans. Modern scholars have seen Pythagoreanism in the book *On Sevens*,[63] and in ancient medical writings generally, as well as in Polyclitus,[64]

Simplicius added *καὶ ἔτι μᾶλλον τῶν Πυθαγορείων (ζηλωτής)* (*Phys.* 25.19; Theophr. *Phys. op.* fr. 3, *Dox.* 477, with Diels's note; cf. D.L. 8.55).—Empedocles is sometimes cited, quite simply, as evidence for Pythagoreanism; cf. above, ch. III 1, n. 12.

[60] Above, ch. III 2, n. 101.

[61] Above, ch. II 5.

[62] DK 38. Zeller treated him among the Ionian philosophers (I 333ff), but Aristoxenus seems to have mentioned him as a Pythagorean (fr. 21, with Wehrli's comment p. 54; Iam. *VP* 267). He is connected with Pythagoreanism by number speculation (along with his emphasis on the importance of 7, he mentions 10 as *consummatio*, *τέλειον*, Cens. 7.2 = A16), with Philolaus by the idea of *ψύξις διὰ τῆς ἀναπνοῆς* (A10; above, ch. III 2, n. 161), and with Alcmaeon by his "myelogenic" theory of the origin of semen (A12; cf. Alcmaeon A13, E. Lesky, *Abh Mainz* 1950, 9ff). The epitaph (fr. 2)

Ἵππωνος τόδε σῆμα, τὸν ἀθανατοῖσι θεοῖσιν
ἶσον ἐποίησεν Μοῖρα καταφθίμενον

is branded spurious by Diels (DK) and Wellmann (*RE* s.v. Hippon). Hippo was regarded as *ἄθεος* (A2, 4, 6, 8), and Mullach (I 82) understood the epigram as atheistic: the gods are nothing, just as the dead man is. But Clement, who cites the couplet, understands them as signifying self-deification, and compares Menecrates-Zeus. This puts him in the company of Empedocles, in the Pythagorean milieu of Magna Graecia. (Cf. also, in the Gold Plates, *θεὸς ἐγένου ἐξ ἀνθρώπου*, DK 1B18; similarly 20. Hippo's place of origin is given as Metapontum by Cens. 5.2, Rhegium by Sext. Emp. *PH* 3.30, and Samos by Aristox. fr. 21, Iam. *VP* 267.) Even people like Empedocles could fall into disrepute as *ἀσεβεῖς*: Hippoc. *Morb.* VI 358f L.

[63] Roscher has the credit for calling attention to this remarkable document, whose date and relation to Pythagoreanism are still matter for dispute. In any case, the book was written in Ionia, for Magna Graecia is not included in the "world map" of ch. 11 (Kranz, *NGG* 1938, 152). Roscher's date (time of Anaximander) was accepted by Sarton, *Hist.* I 215f (but see *Intr.* I 97: "Pythagorean treatise"), and Jones, *PhMed.* 9ff. On the other hand, Boll (*KlSchr* 213ff) and Kranz (*NGG* 1938, 138ff) have shown that the "world map," on which Roscher had based his argument, does not prove anything. Kranz separates the writer of the book from the conception of the world that he reproduces, and assigns the latter to the time of Anaximenes (but on *ἀνταύγεια* of the fixed stars see Metrodorus of Chios, DK 70A9 and Philolaus A19). Boll decided for "450 or even later" (224; so Rey 426ff, Mondolfo, ZM 239ff), and this seems the most likely solution. It is not clear whether the author thinks of the earth as spherical (cf. Kahn 84f), but the four elements are present (ch. 10), and the view of the world's structure corresponds to that of Hippoc. *Vict.* The figure of 7 planets plays no part, and this provides a terminus ante quem (below, ch. IV 1), rendering unlikely the attempt to date it around 400 or even later (Wellmann, *Qu. u. Stud. z. Gesch. d. Naturw. u. Med.* 4.1 [1933] 6–10 [370/350]; Heidel, *Maps* 70 [ca. 400]; Festugière, *REG* 1945, 21.7). Roscher's dating would make the treatise pre-Pythagorean; Pythagoreanism was seen in it by Pfeiffer, *Sterngl.* 30ff; Wellmann, loc. cit.; Boyancé, *REA* 1934, 340; and Mondolfo, ZM 323 (cf. also DK I 406f, n.).

[64] DK 40. Cf. Raven, *CQ* 1951. But the idea of mathematical proportionality is not exclusively Pythagorean (below, ch. VI 1–2). The "many numbers" and the *παρὰ μικρόν* of the single fragment of Polyclitus' *Canon* go beyond the radical simplifications of Pythagorean numerology. Vitruvius 3.1.2ff is a better example of Pythagoreanism, but Raven thinks this is based on a Pythagorizing source dependent on Polyclitus, rather than the other way around (151).

Damon,[65] Hippodamus,[66] Theagenes of Rhegium,[67] Iccus of Tarentum,[68] Menestor,[69] Xuthus,[70] and even in Anaximenes,[71] Heraclitus,[72] and Socrates.[73]

We have already mentioned some general aspects of the problems these conjectures raise. To regard the fact that a philosopher came from Magna Graecia, or the close connection of myth and reason, the idea of a "cosmos," the presence of number symbolism, or the concept of

[65] Damon (DK 37) is not directly called a Pythagorean (Frank 2, 161), though Pythoclides, who is brought into connection with him (Schol. Pl. *Alc.* 118c, DK 37A2) is (and at the same time is made the teacher of Agathocles the teacher of Pindar!). This is a shaky foundation for the attempt to derive the whole doctrine of the moral effect of music from Pythagoreanism (Rostagni, *ScrMin* I 135ff; H. Koller, *Die Mimesis in der Antike* [Bern, 1954] 125ff).

[66] DK 39. He was made into a Pythagorean by forgeries in Doric (cf. Delatte, *Pol.* 125ff; Thesleff, *Texts* 93–102). The geometrical and modernistic character of his city plans need not be regarded as Pythagorean (cf. below, ch. VI 1).

[67] DK 8. He was the first to apply allegorical interpretation to Homer. It was a hypothesis of Delatte, often accepted but unproved, that he was a Pythagorean and that the Pythagorizing interpretations of single lines of Homer go back to his time (*Litt.* 114f; most of what is Pythagorean in the Homer scholia comes proximately from Porphyry).

[68] Below, n. 79.

[69] DK 32. He was the first to study the physiology of plants, following Empedocles in certain points (Theophr. *Caus. pl.* 1.21.5 = DK 32.5). Of prime importance was the opposition warm–cold, which is also significant to Philolaus (A27) and Hippo (DK 38A10). He was a Pythagorean from Sybaris according to Iam. *VP* 267. Cf. W. Capelle, *RhM* 104 (1961) 47–69.

[70] DK 33, Arist. *Phys.* 216b26. Simplicius calls him *Πυθαγορικός*, but probably had no other source than this passage of Aristotle.

[71] A. Chiappelli (*AGP* 1 [1888] 582–594) tried to show that Anaximenes was dependent on Pythagoras, but this is improbable on chronological grounds if nothing else. Burnet thought there was influence, but the other way around (*EGP* 78f, 108; see above, n. 19).

[72] If Heraclitus' originality were not so transparent, people would surely make him into a Pythagorean. He speaks of *ἁρμονία* in frr. 8, 10, 51, 54, measure and *λόγος* in the world process (frr. 30–31), and has ideas very similar to that of *σῶμα–σῆμα* (Sext. Emp. *PH* 3.230, Dodds, *Gorg.* 300). See also Gigon, *Ursprung* 198ff. But what we know with utter certainty is that he hurled abuse at Pythagoras (frr. 40, 81, 129).

[73] On the attempt to trace the theory of ideas, via Socrates, back to the Pythagoreans, see above, ch. I 2, n. 82 (versus the testimony of Aristotle). After Grote, as early as his *History of Greece* (vol. IV [London, 1847] 335), had seen a connection between the burning of the Socratic *phrontisterion* in Aristophanes' *Clouds* and the catastrophe of the Pythagoreans in Croton, Taylor (*Var.Socr.* 1ff) interpreted the charge of impiety brought against Socrates as a measure against newly introduced Pythagorean religion, and saw the *phrontisterion* as the site of Pythagorean mysteries: not only initiation and the common meal, but also natural science, astronomy, and mathematics (129ff). Morrison, *CQ* 1958, 203, maintaining that Aristophanes "is suggesting that Socrates' circle . . . was . . . a Pythagorean *synedrion*," thinks this is a really Aristophanic invention, contrary to fact. But the ritual pattern involving meal and initiation may have been familiar to the Athenians from Athenian life, from the *ἑταιρίαι*. A few years later, the "imitation" of mysteries in one of these aroused a scandal. In the *Clouds*, it is Thales who is named as the great geometer (*Nub.* 180). As far as the fire is concerned, the words *παῖε* and *φλέγε* are close together (cf. Pratinas fr. 1.11 Diehl).—It is true, though, that Socrates seems to

harmony,[74] as a clue to the influence of Pythagoreanism is to presuppose that Pythagoreanism, as the most powerful intellectual and spiritual movement in southern Italy, produced a mighty synthesis of religion and science. The danger in this stereotype is obvious from the single fact that one inescapably certain point—because attested by the contemporary Glaucus of Rhegium—namely the relationship between Democritus and certain Pythagoreans, is felt as almost an embarrassment, and in any case gives us no help in the reconstruction of Pythagorean philosophy and science.[75]

The relation of the south Italian medical tradition to Pythagoreanism is well worth careful thought. If the physician Alcmaeon wrote in Croton about 500 B.C., when Pythagoreans formed the ruling group in the city, there must have been some kind of relationship between them. Empedocles is embedded in the medical tradition, physicians in Elea thought of themselves as successors of Parmenides,[76] both Hippo and Philolaus deal with medical topics, and the physician Acron was Philolaus' predecessor in the written use of the Doric dialect.[77] Shamanistic *γοητεία* and medical art, "medicine man" and medicine,

have had something to do with Pythagoreans (above, ch. I 4, nn. 39–40). In the conjuring up of the dead by Socrates in the *Birds* (1553ff), E. Cavaignac sees the earliest testimony for Socrates' connection with Pythagoreanism ("Pythagore et Socrate" *RPh* 33 [1959] 246–248); but the comic poet may well have made Socrates' *ἐπιμέλεια ψυχῆς* into a conjuring up of spirits. On Aeschines' Telauges see ch. II 5.

[74] E.g., the occurrence of the word *ἁρμόζειν* in the treatise *Περὶ ἀρχαίης ἰητρικῆς* has been seen as a "fragile . . . link" with Pythagoreanism (Jones, *PhMed* 44ff, following H. Wanner, *Studien zu Περὶ ἀρχαίης ἰητρικῆς* [Diss. Zürich, 1939, 38f, 54]). Diller has shown (*Hermes* 80 [1952] 393ff, 397ff) that this book probably was influenced by Platonism.—*Hebd.* 2: *ἡ δὲ σελήνη μέση οὖσα συναρμόζει αὐτὴ τἆλλα πάντα* (namely the region of the stars, with the sun and the region of earth) was interpreted by Pfeiffer (*Sterngl.* 119ff, comparing Xenocrates, Heinze pp. 75f; Plut. *De fac.* 945c, *Quaest. conv.* 745a; cf. above, ch. II 4 n. 158) as a reflection of the Pythagorean doctrine of the harmony of the spheres; but the word *μέση* does not have any specific musical significance here (cf. Kranz, *NGG* 1938, 144 n. 2).

[75] Cf. above, ch. III 2, n. 101. In his ethics Democritus emphasizes *μετριότης*, *συμμετρία*, and *ἁρμονίη*, but this is an emphasis common to many Greeks. If Anaxagoras were not unquestionably an Ionian, and labeled a materialist by Plato, Rostagni would have been able to interpret the juxtaposition of *Νοῦς* and the formless mass (*ὁμοῦ πάντα χρήματα ἦν*) in the light of the alleged Pythagorean dualism of "forma e sostanza" (cf. *Verbo* 42). There are also points of contact with Pythagoreanism in the astronomy of Anaxagoras, as well as that of Alcmaeon (the inhabited moon, dark bodies, and the close relation of sun and moon; cf. below, ch. IV 1, 3). Were Anaximander not indubitably older than Pythagoras, he could be more "Pythagorean" than anyone else (the perfect circle, the importance of the number 9, justice in the cosmos).

[76] Above, n. 13.

[77] Above, ch. III 1, n. 25.

do go together, and in Empedocles the two are still combined.[78] A kind of "knowledge" about the arrangement of the cosmos, which is partly a matter of number symbolism, forms part of this amalgam, closely bound up with ritual in the medical prescriptions and procedures which are supposed to lead to recovery. In particular, the transition from the *βίος Πυθαγόρειος* (or *Ὀρφικός*) to a rationally based regimen is only a new approach to the same thing; in each case there is a system of injunctions to abstinence, whose goal is to concentrate, to enhance, and to control the powers of the individual. This is how the trainer Iccus of Tarentum became famous, even before Herodicus of Selymbria;[79] and before him the tradition calls Pythagoras the inventor of a new kind of regimen,[80] and sometimes even calls him a physician.[81]

Of course, one can hardly think of Pythagoras as the only, or even the most important, originator of these trends. The oldest of the famous physicians of Croton, Democedes, who was active in the courts of Polycrates and Darius, is likely to have been approximately contemporary with Pythagoras. His father, Calliphon, came from Cnidus, which shows that the connections between Ionia and Croton were not all due to Pythagoras.[82] Empedocles drew many inspirations from the medical tradition, and in particular from Alcmaeon,[83] and in turn

[78] As Hippoc. *Morb.* 1–2 attacks shamanistic charlatans, *Vet. med.* 20 attacks the *physiologia* of Empedocles. Democedes functioned as a seer in the court of Darius (Hdt. 3.132). Cf. also F. Wehrli, *MH* 8 (1951) 36ff.

[79] DK 25. Also Ael. *NA* 6.1, *VH* 11.3, Lucian *Hist. conscr.* 35 (Wuilleumier 566). The dating before Herodicus of Selymbria comes from Pl. *Prot.* 316d: *Ἴκκος δὲ ὁ Ταραντῖνος καὶ ὁ νῦν ἔτι ὢν οὐδενὸς ἧττων σοφιστὴς Ἡρόδικος ὁ Σελυμβριανός.*

[80] See above, ch. II 4, for Pythagoras as the alleged reformer of the regimen of athletes. On the bean taboo, Diogenes Antonius (who sometimes used good sources) says, speaking of Pythagoras (Por. *VP* 35), *αὐτῷ καὶ τὸ σῶμα ὥσπερ ἐπὶ στάθμῃ τὴν αὐτὴν ἕξιν διεφύλαττεν, οὐ ποτὲ μὲν ὑγιαῖνον, ποτὲ δὲ νοσοῦν, οὐδὲ αὖ ποτὲ μὲν πιαινόμενον καὶ αὐξανόμενον, ποτὲ δὲ λεπτυνόμενον καὶ ἰσχναινόμενον.* Similarly, Aristox. ap. Iam. *VP* 196.

[81] Ael. 4.17 (from Aristotle? cf. above, ch. II 3): *ἐπιστρεφομένου δὲ τὰς πόλεις αὐτοῦ, διέρρει λόγος ὅτι Πυθαγόρας ἀφίκετο οὐ διδάξων ἀλλ' ἰατρεύσων.* For Pythagoras as a doctor see Celsus *Med.*, prooem. 7, D.L. 8.12. For Pythagoreans and medicine, Iam. *VP* 163, 244, 264, Ael. *VH* 9.22; and the *acusma*, Iam. *VP* 82, *τί τὸ σοφώτατον τῶν παρ' ἡμῖν; ἰατρική.*

[82] On Democedes, DK 19, and esp. Hdt. 3.125–137. His father Calliphon is connected with Pythagoras in anecdote (Hermippus ap. Joseph. *Ap.* 1.164, DK 19.2). Milo, too, the father-in-law of Democedes, is connected with Pythagoras (above, ch. II 2).—We know through an inscription of a physician named Somrotidas of Megara Hyblaea, about 550 B.C. (*SEG* 14 [1957] 599).

[83] Cf. Kranz, *Emped.* 59ff; below, ch. V 1, n. 57.

exercised, himself, a strong influence on the medical literature.[84] Philolaus certainly adopted some medical ideas; yet certain of the Hippocratic writings seem to be in some way dependent on him.[85] The book *On Sevens*, composed in Ionia and in its speculations on microcosm and macrocosm dependent on very ancient ideas, looks more like a phenomenon parallel to Pythagoreanism.[86] Though the Hippocratic *Oath* has features in common with Pythagoreanism—the teacher as "father," the injunction to secrecy, the religious horror of suicide, regimen valued more highly than surgery—much of this is simply to be explained as the result of a common background.[87]

The main question is the source, in all these relationships, of the really scientific elements,[88] from Ionic φυσιολογία and Eleatic logic of Being, or from a hypothetical Pythagorean philosophy of nature. And isolated coincidences are not enough to demonstrate the existence of any such system.

Aristotle himself compared Alcmaeon's theory of opposites with the Pythagoreans' "table of opposites," and Parmenides builds the cosmos, in the realm of *doxa*, upon the opposition of fire and night.

[84] Cf. the polemic at Hippoc. *Vet. med.* 20.

[85] Ch. III 2, nn. 113–117.

[86] Above, n. 63. One is reminded of Pythagoreanism by remarks on the healing power of music (ch. 30); but the "7 vowels" (ch. 9) were not yet known in Magna Graecia in the 5th century. On the expression Ὀλύμπιος κόσμος, see above, ch. III 2, n. 31. In place of ἄκριτος κόσμος for the outermost heavenly sphere, Roscher suggested ἄκρητος, which Pfeiffer (*Sterngl.* 33f) combined with Philolaus A16 (see above ch. III 2, n. 35); but ἄκριτος is guaranteed by the equation of *inseparabilis soliditas* with ἄκριτον πάγος in ch. 6 (a gloss of Galen, VIII 637.1 L.; Kranz, *NGG* 1938, 124 n.). Ἄκριτος is "endless," Parm. fr. 6.7, Critias fr. 19.4. The idea of macrocosm and microcosm unites *On Sevens* with *Regimen* (see esp. Kranz, *NGG* 1938), and, more remotely, with Philolaus fr. 13. An Iranian origin for this concept was suggested by A. Götze, "Persische Weisheit in griechischem Gewande," *Zs. f. Indol. u. Iran.* 2 (1923) 60–98, 167–177. See Olerud, passim; J. Duchesne-Guillemin, in *Problemi attuali di scienza e di cultura* 76 (Rome, 1966) 427, has withdrawn the objections he offered in *HTR* 49 (1956) 115–122. Götze (86ff) believed that the Pythagoreans formed an intermediary link between Persia and Greece, as did Olerud (212; at p. 220, however, he envisages the tradition as passing from Persia to the Pythagoreans via Cnidus).

[87] Cf. above, ch. II 4, n. 104. The most striking parallel is the prohibition of suicide, but this makes perfectly good sense in the medical tradition; the physician as preserver of life may not promote death. Classification of the *Oath* as "a Pythagorean document" by Edelstein outruns the evidence.

[88] J. Schumacher, *Antike Medizin* (Berlin, 1963²) 46ff, 81ff, traced the scientific basis of Greek medicine to Pythagoras (the idea of the regularity of nature, the concept of health as a norm and as harmony, the responsibility of the sick person to himself, the importance of the daily regimen). This means, of course, simply deciding a priori that what is scientific is Pythagorean. Jones, *PhMed.* 1ff, thinks that at the least the combination of philosophy and medicine is attributable to Pythagoras, though the unity of medicine and cosmological ideas was present very early in the context of shamanism, and the only question is, what Pythagoras contributed that could be called scientific.

Did the background of these thinkers include a Pythagorean doctrine of Limit and Unlimited? In considering this, let us remember that it is hardly possible to arrange in parallel συστοιχίαι the opposites named by Alcmaeon, which are taken from everyday observation.[89] Does not this fact, in itself, show that his idea is pre-Parmenidean? The "table of opposites," on the other hand, is probably later, and Alcmaeon takes up ideas about opposites which had been a part of the Ionians' thought since the time of Anaximander. Hippasus the Pythagorean appears in the doxographical tradition as a plain monist;[90] Hippo speaks of moist and dry, warm and cold, in the same way as the Ionian physiologers;[91] and even in Philolaus the warm and cold of his physiology do not seem to be integrated into the concept of the Limiting and Unlimited of his cosmology.[92] There is nothing to show that this theory of opposites was already there at the beginning of the fifth century. If we restrict our attention to the evidence, it seems much more likely to be a position taken on the question that rose, in reaction to the work of Parmenides and Melissus, about the limitedness or unlimitedness of Being. Number symbolism is to some extent older, but there are no traces of it in Parmenides, and at the most, very slight traces in Alcmaeon and Empedocles.[93] Before Philolaus, Hippasus seems to be the only one to have dealt with music theory.[94]

[89] According to Aristotle *Met.* 986a22 (and elsewhere), Alcmaeon mentioned white and black, sweet and bitter, good and bad, large and small. Aëtius (5.30.1 = fr. 4) names moist and dry, cold and warm; but these are so common that we cannot rely on this report (cf. Pl. *Soph.* 242d; *Dox.* 223). These ideas appear also, however, at A5 and A9. Timpanaro Cardini also thinks that the table of opposites is later than Alcmaeon (120; cf. above, ch. I 2, n. 120). Mondolfo, ZM 321f, posits an older, more primitive dualism in Pythagoreanism as the source of Alcmaeon; but why should we need to insert an intermediary between the Ionians and Alcmaeon? Warm and cold, dry and moist are already there in Anaximander (A9, 10, 11 §§6f, 17a, 27, 28); cf. Anaximenes A21, Heraclitus fr. 126, Kahn 100ff, 160ff.

[90] Above, ch. II 5. Burnet (*EGP* 109) tried to bolster his equation of Parmenides' Light with Limit by a reference to Hippasus (cf. above, n. 24), but there is no mention of an opposing principle in Hippasus, and ever since Aristotle he has had his place beside Heraclitus.

[91] DK 38A11, cf. A10. At the same time, the life-giving "moist" is for him the real ἀρχή.

[92] Above, ch. III 2, on A27. According to the evidence of Menon, our bodies "consist of the warm," and the cold air is something foreign, borrowed, so that alongside the idea of harmony a monistic thought seems to find a place.

[93] Cf. Alcmaeon A15. On Empedocles' doctrine of the elements, below n. 101. On ἁρμονία Emp. fr. 27.3, 96.4, 23.4. Empedocles speaks, in fr. 96, of a numerically determined proportion in the mixture of the elements, and Mondolfo (ZM) decides that this is an indication of Pythagorean influence. Aristotle, however (*Met.* 1092b8ff), differentiates between number as the λόγος μίξεως, and the Pythagorean and Platonic concept of number (Zeller I 1026).

[94] Cf. ch. V, below.

Alcmaeon had things to say about astronomy, and in particular the "contrary movement" of the planets—not a topic in which a physician would obviously be interested. He also tried to prove the immortality of the soul by its relation to the stars: both were, he thought, in eternal motion. This has so "Pythagorean" a ring that scholars have often suspected a post-Platonic forgery.[95] Still, the astronomical views can be derived from the Ionians,[96] and the proof of immortality, which is not identical with that in Plato's *Phaedrus*, seems to be Alcmaeon's original contribution.[97] If this is so, what Alcmaeon did was to translate pre-scientific, and Pythagorean, material into scientific language; nothing is presupposed that is Pythagorean except the theory of metempsychosis and a connection of immortality with the heavenly bodies, which appears in the *acusmata*.[98]

Empedocles has a perennial interest because of his intermixture of rational explanation of natural phenomena and religious pronouncements like those of a prophet, both emanating from an eloquent and passionate personality. We cannot maintain a neat correlation of the two sides of his nature with his two poems, *On Nature* and *Purifications*;[99] and it has been rightly emphasized that this very fact shows

[95] Wilamowitz, *Platon* I 461, 717; Frank vii; Moreau, *Âme* 155 n. 6. The difficult point about this thesis is that it requires us to suppose Aristotle let himself be deceived by a post-Platonic forgery.

[96] Below, ch. IV 2.

[97] Arist. *De an.* 405a29 = Alcmaeon A12, cf. A1, as well as the expression in fr. 2, "People perish because they cannot join the beginning with the end" (with an allusion, no doubt, to formulae of the mysteries about beginning and end; cf. Pi. fr. 137a, Her. fr. 103, Hippoc. *Vict.* 1.19, *Nutr.* 9, 24, *Loc. hom.* 1, Ocellus 1.14). The equation of ψυχή and movement makes the foundation for the proof of immortality: when life ceases, the creature stops moving. That which is a conclusion from analogy in Alcmaeon becomes in Plato, through the idea of self-movement, a dialectical proof (*Phdr.* 245c).

[98] Cf. above, ch. II 4, and below, ch. IV 4.—Whether Alcmaeon taught transmigration is as controversial as the relationship between the doctrine of immortality and his physiology. (Rostagni, *Verbo* 102ff, distinguishes between an "anima sopranaturale" and a "natura corporea," which were confused, through a "grosso equivoco"; cf. Kerényi 28ff, Stella 278ff [above, n. 57], Guthrie I 355f).

[99] This is emphasized by Long 46ff. A development "vom Mythos zum Logos" is accepted, after J. Bidez, *La biographie d'Empédocle* (Ghent, 1894), 159ff, by Kranz, *Hermes* 70 (1935) 111, *Emped.* passim. On the opposite side, for dating the *Purifications* after the *On Nature* (favored by fr. 131) are Diels (*SBBln* 1898, 396ff), Wilamowitz (*SBBln* 1929, 626ff), and especially Reinhardt (*CP* 45 [1950] 170ff). For the inner unity of the two poems (e.g., fr. 115 has the 4 elements and Strife; fr. 23.11 represents Empedocles as god or nearly so, fr. 15 has immortality, and in the phrase "life, as men call it" an allusion to doctrines of release or salvation), Nestle (*Philologus* 65 [1906] 545ff), Nestle (in ZN I 1007ff), Jaeger (*Theol.* 128ff), Dodds (*Irr.* 145f, with nn.); H. Schwabl (*WS* 69 [1956] 49–56), and esp. Kahn (*AGP* 1960). Long (*AJP* 70 [1949] 142–158) tries to make the matter too neatly systematic.

the mark of a very ancient type, that of the shaman.[100] Thus one might guess that in Pythagoras, too, both aspects were present—but wisdom is not science. If we suppose that the fact that the elements are four in number is related to the symbolic power of the number four,[101] or that the idea of harmony suggests a relationship to the Pythagoreans,[102] still he is not known to be concerned with music theory, or with Limit and Unlimited; and if he speaks of a ratio of mixture of the elements, that does not mean that "things are numbers." His basic ontological idea, the characterization of the elements as material that is qualitatively unalterable, without origin, and imperishable, agrees even in vocabulary with Parmenides; and in astronomy and physiology Empedocles employs an unabashed empiricism, which takes what it needs from any source. The doctrine of transmigration comes from Pythagoreanism, just as some individual cosmological ideas have their roots in myth,[103] but his ontology and natural science are not Pythagorean.

Alcmaeon, Parmenides, Empedocles, Hippasus, Hippo, Philolaus—all of them sprang from the soil of Magna Graecia at a time when the doctrine of Pythagoras was still alive; and all of them, as well as the Ionic book *On Sevens*, had points of contact with Pythagoreanism. But to infer in each case the priority of a developed, systematic Pythagorean philosophy, is petitio principii. If we consider these persons as a group, it is almost impossible to reconstruct as their common background any kind of well-defined scientific and philosophical system. If we look for a dualistic foundation, which could unite Alcmaeon, Parmenides, and to a certain extent Empedocles, with Philolaus, then Hippasus and Hippo must be left out. And even aside from this, the differences are considerable: in Empedocles we have the periodic interchange of opposite world-states, to compare with Philolaus'

[100] Dodds, *Irr.* 146.

[101] Cf. Zeller I 950f; Diels, *Elementum* (Leipzig, 1899) 15; above, ch. II 4, n. 155.—Rostagni (*Verbo* 262f) would like to discover a Pythagorean doctrine of elements; but his references (including Philolaus fr. 12) are all under the influence of the *Timaeus* (above, ch. I 3). To be sure, there is a pre-philosophical conception of "elements" especially in the Indo-Iranian area, in which the cosmos is divided into regions of fire (the stars), air, water, and earth. (Cf. J. Przyluski, "L'influence iranienne en Grèce et dans l'Inde," *Rev. de l'Univ. de Bruxelles* 37 [1932] 283–294; Olerud 136ff). We cannot go into the question here, to what extent something of the kind existed in Greece (Kranz, *Hermes* 70 [1935] 113ff, mentions Orphism, but this is a very uncertain factor).

[102] Above, n. 93; ch. III 2, n. 65.

[103] When Empedocles compares the shape of the world to an egg, we cannot ignore the relationship of this to Orphic cosmogony (A50, Ar. *Av.* 693ff, Nestle in ZN I 980 n. 2).

harmony of opposites in a single world. This is comparable with Alcmaeon's *ἰσονομία* of opposites in the organism; but in Parmenides the opposites are separate and irreconcilable. If we postulate a philosophy of form, or of an advanced degree of abstraction, we might set Philolaus alongside the Eleatics; but the atomists come much closer to belonging in this company than the sensuous world of Empedocles, to say nothing of Hippasus and Hippo. There is no single characteristic trait to unite the astronomical views of Alcmaeon, Parmenides, Empedocles, and Philolaus; in fact there is one point in which the latter agrees rather with the Ionians than with Parmenides and Plato.[104]

If there is a sense in which Pythagoreanism forms part of the background for all this, it is not Pythagoreanism as a fully formed scientific and philosophical structure, but as an inspiration or stimulus of a pre-philosophical sort, in the area of mythic lore and religious aspiration, which each, in his own way and in accordance with his own personality, allowed to influence him in the formulation of his thoughts. This is true not only of Alcmaeon, Parmenides, and Empedocles, but also of Philolaus, whose thoughts of the "limiting," the "unlimited," and of number were helped by the efforts of Plato to a career whose scope could hardly have been foreseen at the beginning.

There is no law of reciprocal interaction in the field of thought, which could make it possible for us, by inference, to fill adequately the gaps in our tradition, as the law of gravitation enables astronomers to calculate the position or movement of an unknown star. In fact, the suspicion persists that the lacuna in the tradition about early Pythagoreanism is not an accident. If we cannot get a clear idea of the philosophy and science of Pythagoras, it is because Plato and Aristotle did not consider him a philosopher. If we cannot find a clue to the philosophy of Limit and Unlimited and their harmony achieved through number, before the day of Philolaus, it is because this doctrine, in this abstract form, was first created as Philolaus worked to formulate anew, with the help of fifth-century *φυσιολογία*, a view of the world that came to him, somehow, from Pythagoras.

[104] In the assumption of a material *περιέχον* outside the heaven (Kahn 234 n. 4).

IV. Astronomy and Pythagoreanism

1. THE STRUCTURE OF THE WORLD AND THE PLANETARY SYSTEM

The Greeks acknowledged, almost too eagerly, that Greek astronomy was based on the accomplishments of the East.[1] Modern scholarship, studying the original Babylonian and Egyptian sources, has made clear how much of the final achievement is due to the Greeks themselves.[2] It is true that Greek astronomers used the observational data which by various routes made their way from the Orient, and especially from Babylonia;[3] but they made an original contribution, without precedent in the East, in the development of a conception of the world's structure from the insights of Greek mathematics—the famous Ptolemaic system, in which the planets circle about the spherical earth, at various distances, enclosed in turn by the sphere of the fixed stars. The risings and settings of the stars were thought of as related to the geometry of the sphere, and the irregularities in the paths of the planets were explained by the combination of mathematically perfect circular movements. The problem of cosmic distances was taken up too, and not without success.[4] From Eudoxus through Hipparchus to Ptolemy, the development of this Greek scientific enterprise can be followed fairly easily, but what came before the first great epoch is, as so often, difficult to make out clearly.

[1] See, for example, (Pl.) *Epin.* 986e, Arist. *Cael.* 292a8, Diod. 2.30.

[2] See esp. Neugebauer, *ExSc* 156. It is important that, contrary to a widespread belief, the Greek planetary system cannot be shown to have been Babylonian; cf. Boll, *RE* VII 2561ff.

[3] According to Sen. *QNat* 7.3.2, Eudoxus was the first to bring from Egypt exact data about the movements of the planets (T15; cf. T12–20 Lasserre). About 500 B.C. Babylonian astronomy was exerting an influence in Egypt, as is shown by a new discovery: R. A. Parker, *A Vienna Demotic Papyrus on Eclipse- and Lunar Omina* (Providence, 1959); van der Waerden, *Anf.* 131–133. Also, cf. the reference to "Egyptian" observations in Arist. *Mete.* 343b10, 28; cf. *Cael.* 292a8; Chaeremon *FGrHist* 618F7. During Alexander's expedition, Callisthenes is supposed to have sent Babylonian data to Greece directly (Simpl. *Cael.* 506.11 = *FGrHist* 124T3; cf. Hipparchus ap. Ptol. *Synt.* 4.11 p. 340 Heiberg). Suspicion is aroused by the late attestation, its novelistic presentation, and the absurd claim that observations had been carried on for 31,000 years in Babylon (Neugebauer, *ExSc* 151).

[4] Hipparchus came fairly close to establishing the distance of the moon correctly, putting it at 33½ times the diameter of the earth (*RE* VIII 1676).

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The earliest connected discussions of astronomical matters are found in the works of Plato, and it is not merely a coincidence that almost all the important astronomers of later times were Platonists.[5] The Greek idea of the general structure of the world is set forth here in all its essential features: the earth is spherical and rests, free of support, at the center of the sphere of the fixed stars; the planets are stationed in concentric paths at varying distances; and their apparent irregularities are explained by mathematical principles. The order of the planets, from the earth at the center, is moon, sun, Venus, Mercury, Mars, Jupiter, Saturn[6]—an order retained by Eudoxus, Callippus, Aristotle, and even Eratosthenes.[7] It is "correct," insofar as the planets are arranged according to how long it takes them to make a circuit through the zodiac, with a longer time corresponding to a greater distance. Since the inner planets Venus and Mercury are, from the geocentric point of view, "isodromous" with the sun,[8] that is, like it they make the circuit of the zodiac in a year, their position in relation to the sun cannot be determined on this principle, and this is what led to the ancient controversies about the order of the planets.

It is known that the facts about the planets came to Greece from Babylon. They were known centuries earlier there, and it is from there and not from Greek mythology that we have the association of the planets with individual gods, which still provides their names.[9] The discovery of the most important data on planetary movement is due to the Babylonians, and in particular the time of their orbits. At least in the case of Saturn, whose orbit takes 29½ years, this would require several generations. The discovery of the planet Mercury, which is difficult to observe, also belongs here; in fact, a fund of exact knowledge about the planets is doubtless the most obvious debt to oriental knowledge in the pre-Platonic period.

Along with knowledge of the planets, the recognition of the spherical shape of the earth and the postulate of perfect circular movements make up the world picture of Greek astronomy, as it is presented by Plato. Ever since the ancient commentators on the *Timaeus* it has been thought that "the" Pythagoreans were the source to which Plato owed his astronomical interest and knowledge, and this is not the least important cause of the high place accorded them in the history of science.

Aristotle, however, records, along with the famous ideas of the harmony of the spheres and the special theories of "some" Pythagoreans about comets and the Milky Way, another peculiar Pythagorean system. Here the earth is one of the planets, and circles, along with a "counter-earth" which is invisible to us, about a "central fire." The doxographers attributed this system to Philolaus, and we may justifiably use the well-established expression, "the system of Philolaus," without prejudice to the question of its date or its relation to the Pythagorean named in the *Phaedo*. For in fact this relation has come into question among modern scholars. The attribution to Philolaus of the system described by Aristotle suggested the reconstruction of an older, more simple, geocentric system, supposedly developed by Pythagoras himself or the early Pythagoreans; the materials for this enterprise were some late references, but, above all,

[5] The Platonism of Eratosthenes and Ptolemy is well known; but Hipparchus, too, shows himself to be in the influence of the *Timaeus* when he teaches "cognationem cum homine siderum animasque nostras partem esse caeli" (Plin. *HN* 2.95).

[6] Perspicuously described at *Rep.* 616e et seq.; *Tim.* 38d emphasizes the number 7 and names Venus and Mercury, but when the idea is taken up again he reverses the order and names Mercury first. This is why Aët. 2.15.4 has Plato putting Venus over Mercury (followed, no doubt, by Eratosthenes in his *Hermes*; Chalcid. 73). *Epin.* 987b has a simple enumeration; here it is also stated that aside from ἑωσφόρος the planets are nameless, though ἐπωνυμίαν εἰλήφασιν θεῶν. Here and later the style of appellation is ὁ (τῆς) Ἀφροδίτης ἀστήρ . . .; the names Φωσφόρος, Στίλβων, Πυρόεις, Φαέθων, Φαίνων do not occur before the Hellenistic period (Heraclides fr. 66 is of doubtful value; cf. Wehrli 82). Not before the first century B.C. is a planet called simply Aphrodite or the like (Cumont, *AC* 1935, 5–43). On Plato's astronomy see the commentaries on the *Timaeus*; Duhem 28ff; Heath, *Aristarchus* 134–189; *Math.* 310ff.

[7] Eudoxus: Procl. *In Tim.* III 62 (cf. Eudox. *Ars astr.* 22) = D9 Lasserre; Xenocrates fr. 17. Callippus–Aristotle: *Met.* 1073b17ff, Procl. loc. cit., *De mundo* 392a23ff. Chrysippus: *SVF* II no. 527. Eratosthenes *Hermes*: Adrastus ap. Theo Sm. 142.7 = Chalcid. 73; "Pythagoras" Anon. Phot. 439b17ff, Cic. *Nat.d.* 2.52f (Stoic praise of cosmic order); *IG* XII 1.913 (100 B.C.); *Pap. Aberdeen* 13 (Aug. 1, A.D. 187). This order of the planets is also to be assumed for Heraclides Ponticus fr. 95 (so that the sun is not central, as Wehrli 92f supposes; the spheres assigned to Pluto are, aside from that of the moon, those of the elements; cf. Anon. Phot. 439b25 and *Hebd.* 1).—On the whole subject, Immisch 69ff, and esp. Boll, *RE* VII 2566ff.

[8] ἰσόδρομοι, *Tim.* 38d, Macrob. *Somn. Sc.* 1.19.4, etc.

[9] See Boll, *RE* VII 2561ff; Gundel, *RE* XX 2025, 2029; Meissner 405ff; van der Waerden, *Anf.* 105, 108, 172. The Babylonian gods are Ishtar, Nabu, Nergal, Marduk, and Ninurta (Aphrodite, Hermes, Ares, Zeus, Cronus). The designation of Saturn as ἡλίου ἀστήρ is Babylonian (Pl. *Epin.* 987c; the text is emended even by Burnet; see, against this, Eudox. *Ars astr.* col. V, Diod. 2.30, Simpl. *Cael.* 495.28, J. Bidez, *RPh* 29 [1905] 319f, Meissner 254). The colors attributed to the planets in Pl. *Rep.* 616e go back to Babylonia (Bidez, *Eos* app. I), in spite of the objections of W. J. W. Koster (*Le mythe de Platon, de Zarathoustra et les Chaldéens* [Leiden, 1951] 66ff), who thinks this is based on independent observation. For example, Jupiter, called λευκότατος by Plato, is simply called "the white star" (Molobabar) in Babylon (Meissner 404).

conjectures based on Parmenides and Plato.[10] Then Frank declared the Philolaic system too bold and advanced even for the end of the fifth century and dated it in the years of Plato's old age.[11] For him, the history of astronomy was one of the most striking proofs of the spuriousness of the Philolaus fragments. These arguments, however, must be reexamined. From the point of view of the history of science, the most important points are the discovery of the spherical shape of the earth, the recognition of the five planets, and the explanation of the apparent irregularities of their courses by means of circular movements.

The history of the exact sciences is of special importance in the history of thought, because, as O. Neugebauer has put it, "the inherent accuracy of the mathematical sciences will penetrate to some extent into purely historical problems."[12] For this reason it is a special methodological advantage that the investigation upon which we are now embarking can be conducted independently, in all important respects, of the conclusions reached in earlier chapters.[13] But it is very important to establish at the outset the extent, and thus also the limits, of the mathematical exactitude to be attained in the history of science.

With great confidence, Frank reconstructs the main stages of Greek astronomy, in an apparently necessary order:[14] development of the understanding of space, of solid geometry and perspective, by Anaxagoras and Democritus; discovery of the sphericity of the earth and the "true movements of the planets" in the form of "geometrically perfect orbits" (28) by the Pythagoreans of Archytas' circle; first mathematical explanation of the movement of the planets, by Eudoxus; discovery of the rotation of the earth on its axis, and finally the "Copernican view of the world" in the system of Philolaus (35)—which thus represents the second step beyond Eudoxus. Similarly, van der Waerden[15] sees in "the logical development of astronomy" the

[10] Gruppe, *KosmSyst* 50f; Boeckh, *KosmSyst* 89; and esp. Martin, *Pyth.* Döring already was calling the theory "generally recognized" in *AGP* 1892, 508; cf., i.a., Burnet *EGP* 110ff, 296ff; Berger, *Erdkunde* 185f; Heath, *Aristarchus* 48ff; van der Waerden, *Astr.* 26ff; Guthrie I 289ff.

[11] Below, ch. IV 3.

[12] *ExSc* 1.

[13] What we shall presuppose, from previous discussion, is a fundamental skepticism of the post-Aristotelian tradition, strongly influenced by Platonism, as well as the relative priority of the *acusmata*—and both of these points are confirmed again and again. We may add a few points of detail, like the dependence of the so-called "more genuine" Pythagoreans on Aristotle (ch. III 1) and the (related) analysis of Philolaus A16 (ch. III 2).

[14] Frank 19ff; cf. 184ff.

[15] *Astr.* 28. Yet, in agreement with the ancient sources, he has the theory of the revolution of the earth on its axis originating in the Philolaic system, while Frank has it preceding the more complicated one.

"strongest proof" that the system of Philolaus was preceded by that geocentric, Pythagorean system which must be assumed. Still and all, in the history of science logical necessity and historical sequence are not always identical. Of course, every forward step in this field depends on a certain group of preliminary studies, but it is just as obvious that there do occur backward steps, significant enough to cause correct answers already found to be given up.[16] And intuitive anticipations occur, too, based on inadequate foundation, so that, in such a case, true progress consists in giving up results which, to hindsight, will be seen as correct. Perhaps the allegedly "Copernican" system of Philolaus will fit into this category.[17]

Here, as always, that evidence must be decisive which gives us a clue to the purely fortuitous aspects of the development; for if we were to rely on logical calculation of the probabilities, we should find a good many alternative possibilities. Above all, an inference about preconditions, though it have almost the certainty of mathematical demonstration, can never, at the same time, prove who is responsible for these earlier achievements. That the astronomy which preceded the Philolaus system was Pythagorean, or that Pythagoras set up an astronomical system of the same or even a higher order than those devised by Anaximander or Parmenides, is nothing but a historical hypothesis, which cannot be corroborated or refuted by the inner logic of the history of science, but must be known from external testimony. For the most part, this question is not even considered; under the influence of the Platonic tradition all mathematical science in early Greece is called Pythagorean.[18] To put the question at all means to remove one of the main supports from under most of the reconstructions.

Late sources attribute to Pythagoras a decisive role in the formation of the Greek view of the world's structure, but these reports are strikingly contradicted by statements about Parmenides and the latter has the oldest authorities on his side. The matter in question is the

[16] As compared with Anaximander, the astronomical views of Xenophanes and Heraclitus are regressive (Heath, *Aristarchus* 54ff); but so are those of Anaximenes, who does not dare to let the earth hang free in space. On the relationship of Aristarchus, Ptolemy, and Copernicus, see below, ch. IV 2, n. 1; ch. IV 3, n.1.

[17] If Aristotle had not happened to mention the Philolaic system, modern scholars would confidently date it in the era of Aristarchus of Samos!

[18] Even Frank clings to the Pythagorean origin of Greek natural science and mathematics, only he dates it in the time of Archytas.

sphericity of the earth and the identity of evening star with morning star, which means the beginning of a true understanding of the planets.

Diogenes Laertius writes of Pythagoras: *τὸν οὐρανὸν πρῶτον ὀνομάσαι κόσμον καὶ τὴν γῆν στρογγύλην· ὡς δὲ Θεόφραστος, Παρμενίδην.*[19] In another passage, which in all likelihood goes back to Theophrastus, he likewise says of Parmenides, *πρῶτος δὲ οὗτος τὴν γῆν ἀπέφαινε σφαιροειδῆ καὶ ἐν μέσῳ κεῖσθαι.*[20] The spherical earth appears again as a doctrine of Pythagoras in the *Hypomnemata* (reported by Diogenes Laertius, 8.25).

At first, the natural assumption seemed to be that Parmenides had learned this fact from Pythagoras and published it,[21] but Frank maintained vigorously that Parmenides could not have known that the earth was spherical; it could be seen from Plato's *Phaedo*, he thought, that at the time of that dialogue this was a quite new discovery. Parmenides had, according to Theophrastus' evidence, merely been the first to use the word *στρογγύλος* for a disc-shaped earth.[22] And recently it has even been contested whether Plato does describe the earth as a sphere in the *Phaedo*.[23]

But the Theophrastus passage cannot be eliminated so easily.[24] We cannot, in fact, ascertain what word Theophrastus used, but it seems sure that in the context he was thinking of a spherical rather than a discoid body,[25] so that the dubious inference from the *Phaedo* cannot be used against the evidence of Theophrastus.[26]

[19] D. L. 8.48 = Theophr. *Phys. op.* fr. 17, *Dox.* 492 = DK 28A44.

[20] D. L. 9.21 = Theophr. *Phys. op.* fr. 6a, *Dox.* 482; on the attribution to Theophrastus, *Dox.* 166ff.

[21] Tannery, *HScH* 236.

[22] Frank 184ff, 198ff; Heidel, *Maps* 70ff, 83ff; Morrison, *JHS* 1955, 64.

[23] T. G. Rosenmeyer, CQ 50 (1956) 193–197; W. M. Calder, *Phronesis* 3 (1958) 121–125; Rosenmeyer, ibid. 4 (1959) 71–72; Morrison, ibid. 101–119. The myth may preserve a bit of older cosmology, but *Phd.* 112e seems to refer to the middle of a sphere. Theophrastus' testimony on Parmenides helps to decide the issue.

[24] Against Frank, P. Friedlaender, *Plato* I (New York, 1958) 386; Mondolfo, "La prima affermazione della sfericità della terra," *Accad. d. scienze di Bologna* 1937 (not available to me; see ZM 339ff); Rehm–Vogel 12; Kahn 115ff; cf. Thomson 110ff.

[25] The word *στρογγύλος* is used with reference to Pythagoras, and here surely means "spherical." The parallel passage D.L. 9.21 has *σφαιροειδής*, which is of course not reliable. (At D. L. 2.1, the earth of Anaximander, the "column drum," is called *σφαιροειδής*, as Frank remarks, 200.) *Στρογγύλος* can mean circular or spherical; it is used of hailstones at Aristoph. *Nub.* 1127. It is not certain what shape is attributed to the earth by Diogenes of Apollonia; it is called *στρογγύλη* in D.L. 9.57.

[26] At Pl. *Phd.* 108e, the word *περιφερής* (*στρογγύλος*, 97e) is used quite casually; the main thing is the assurance that the earth does not need the support of air or the like: Plato is here using another doctrine to hark back, against Anaxagoras (A88), Diogenes of Apollonia (A16a; cf. C2 = Eur. *Tro.* 884), and Democritus (Arist. *Cael.* 294b14f; also Anaximenes A6, 20), to Anaximander and Parmenides.

Not only was the Democritean Bion of Abdera acquainted, about 400, with the mathematical consequences of the spherical shape of the earth,[27] but about 430 B.C. Hippocrates of Chios had projected the celestial circles onto the earth, obviously presupposing its sphericity.[28] A spherical earth is but an easy step from the thesis that it is located in the middle of the universe and "because of equality" did not fall in one direction or the other. This is a thesis Parmenides took over from Anaximander; and if the description of the world of *doxa* is an independent creation of Parmenides, it is not at all far fetched to suppose that, having in mind the perfect "sphere" of Being, he was the first to call the earth on which we live a sphere.[29] Empedocles seems to have followed him.[30] That this theory did not win out, and that Democritus clung to his peculiar idea of a bowl-shaped earth,[31] is one of those retrograde steps that sometimes occur in the history of the natural sciences.

Posidonius found that Parmenides was the *ἀρχηγός* of the division of the earth into five zones, but objected that he had made the torrid zone too broad, so that it extended beyond the tropics.[32] Most scholars have been very skeptical of this datum,[33] but it fits quite well into the Parmenidean dichotomy of Fire and Night: just as, in the heavens, Fire and Night occur both pure and mixed, so on the surface of the

[27] DK 77.1 = D.L. 4.58: *οὗτος πρῶτος εἶπεν εἶναί τινας οἰκήσεις, ἔνθα γίνεσθαι ἐξ μηνῶν τὴν νύκτα καὶ ἐξ τὴν ἡμέραν.* Strabo calls him *ἀστρολόγος* (1, p. 29 = DK 77.2). Wellmann (RE III 486) dates Bion, rightly, before Eudoxus. Bion wrote Ionic and Attic (DK 77.1). We need not decide whether older myths may have contained reminiscences of the long polar nights (cf. the controversies over *Od.* 10.82ff and Hdt. 4.25).

[28] DK 42.5 = Arist. *Mete.* 343a8: the torrid zone between the tropics. On Hippocrates' date, see below, n. 77.

[29] The earth at the center: A44, following Anaximander (below, n. 44); Being compared to a sphere: fr. 8.43; cf. Theophr. *Phys. op.* fr. 6, *Dox.* 482. See Gigon, *Ursprung* 86f, 275.

[30] Kranz, *Emped.* 50: "die Erde nach damaliger, altpythagoreischer Vorstellung als Kugel im Mittelpunkt" (but cf. below, n. 44). Actually, the idea (A56) that the sun is a reflection of the "round" earth (*κυκλοτερής*) implies sphericity, since the sun is not stationed directly over our section of the earth, and the reflection of a disc would appear distorted rather than perfectly round. But it is naturally an open question to what extent Empedocles was preoccupied with such geometrical considerations. He did give up the idea of Hades as a region under the earth (below, ch. IV 4), and this would be consistent with a spherical earth.

[31] A94; Frank 187ff. Oriental tradition may be behind this. According to Babylonian teaching the earth is *σκαφοειδὴς καὶ κοίλη* (Diod. 2.31; Meissner 107ff). Herodotus ignores the sphericity of the earth (e.g., 3.104). On *Hebd.* 2, above, ch. III 3, n. 63.

[32] Posidonius ap. Strabo 2, p. 94 = DK 28A44a; derived from this, Ach. *Is.* p. 67.27 Maass, Aët. 3.11.4 (where, in contradiction of Posidonius, a connection with the *ζῶναι τροπικαί* is claimed—a simplification).

[33] Reinhardt, *Parm.* 147 n. 1, *Kosmos* 361 n. 2; Heidel, *Maps* 76, 80, 91; cf. Rehm–Vogel 11f.

earth there is a mixed zone between the extremes; and, as the "circles" in the heavens were arranged in some kind of symmetry, the southern hemisphere is a symmetrical reverse of the northern. And the very fact that, according to Posidonius, no mathematical astronomy was associated with this, or any projection of the tropics onto the earth, is an encouraging sign.[34] On the other hand, the doxographers ascribe to Pythagoras the precise astronomical division of the earth into five zones, bounded by the tropics and the polar circles,[35] *ἀναλόγως τῇ τοῦ παντὸς σφαίρᾳ*, and trace the corresponding division of the celestial sphere to *Θαλῆς Πυθαγόρας καὶ οἱ ἀπ' αὐτοῦ*,[36] and the determination of the obliquity of the ecliptic, upon which the location of the tropics of course depends, to Pythagoras himself, adding *ἥντινα Οἰνοπίδης ὁ Χῖος ὡς ἰδίαν ἐπίνοιαν σφετερίζεται*.[37] We have a parallel for this last sentence in the words of Aristotle's pupil Eudemus, the best authority for the history of Greek science before the Hellenistic period: *Οἰνοπίδης εὗρε πρῶτος τὴν τοῦ ζῳδιακοῦ διάζωσιν* . . . While Eudemus appears have spoken simply of a discovery of Oenopides,[38] the later tradition maintains that this is a plagiarism from Pythagoras. For both the sphericity of the earth and the division of it into zones, Pythagoras' name stands in competition with Parmenides'; in the second case the more advanced, "correct" formulation of the theory is ascribed to Pythagoras; and in both cases the earlier evidence—that of Theophrastus and Eudemus—has no mention of Pythagoras.

[34] On the problems of Parmenides' celestial system, see below, n. 40. Frank (200 n. 1) wrongly states that Posidonius "attributed to Parmenides the projection of the tropics from the celestial sphere onto the concentric spherical earth." Posidonius' words were *(Παρμενίδην) σχεδόν τι διπλασίαν ἀποφαίνειν τὸ πλάτος τὴν διακεκαυμένην*[. . .] *ὑπερπίπτουσαν ἑκατέρων τῶν τροπικῶν εἰς τὸ ἐκτὸς* . . . According to this the width of the torrid zone would be not 46° 54′ 6″ but about 90°, i.e., for Parmenides it is a "ring" whose width is about a fourth of its circumference; and this has nothing to do with astronomy or tropics (differently Hippocrates, above, n. 28).

[35] Aët. 3.14.1; cf. Mart. Cap. 6.609 (MSS *pytharas* or *pythagoras*, wrongly corrected to *Pytheas* in Dick's edition).

[36] Aët. 2.12.1; cf. 2.23.6: *Πλάτων Πυθαγόρας Ἀριστοτέλης* (on the obliquity of the ecliptic), where the position of the name Pythagoras is a giveaway.

[37] Aët. 2.12.2.

[38] Eudemus fr. 145 = Theo Sm. 198.14f. Diels (DK 41.7) conjectured *λόξωσιν* for *διάζωσιν*. The topic of discussion is the measurement of the angle of the ecliptic; the fact of its obliquity—i.e. of the zodiac—was known since Anaximander (A5) and Cleostratus (DK 6B2); see von Fritz, *RE* XVII 2260f. Of course, Theo gives only a much abbreviated excerpt from Eudemus; but, if his original had corresponded to the report of Aëtius, the name Oenipides would have fallen out rather than Pythagoras.—Aët. 2.32.2 ascribes the 59-year cycle to "Oenopides and Pythagoras," while Ael. *VH* 10.7 and Cens. 19.2 speak only of Oenopides. The mention of Pythagoras is to be explained either from the reproach of plagiarism or as a conclusion from Philolaus A22.

Favorinus records the same rivalry for the discovery that the evening and morning stars are the same.[39] This planet—did he already call it after Aphrodite?—had a special meaning for Parmenides, and he has it circling in the pure *αἰθήρ* above the sun.[40] This double-track tradition about Parmenides and Pythagoras can be understood in the light of a statement of Diogenes Laertius about Parmenides: "He seems to have been the first to discover that the evening star and the morning star are the same, as Favorinus says . . . but some attribute this to Pythagoras; but Callimachus denies that he is the author of the poem." Callimachus denied Pythagoras' authorship of a certain poem which dealt, among other things, with the planet Venus.[41] Actually, Heraclides Lembus, in his list of the writings of Pythagoras, gives first place to a work *περὶ τοῦ ὅλου ἐν ἔπεσιν* (D.L. 8.7). It is likely that this poem dealt not only with the heavenly bodies but with the spherical shape of the earth and its division into zones, borrowing from Parmenides, and certainly also from Empedocles. No one will claim that there was a didactic poem actually written by Pythagoras. Callimachus was right in rejecting the poem, and in doing so joins Theophrastus and Eudemus as a third witness against the "Pythagoras" version of this tradition. We need not be concerned whether the doxography took its reports from the apocryphal poem or perhaps from older statements by Platonists, for

[39] D. L. 9.23 (below, n. 41); Parmenides also Aët. 2.15.7; Pythagoras, Apollodorus *FGrHist* 244F91 and Plin. *HN* 2.37 (dated 612 B.C. as a discovery of Pythagoras). It is almost incredible that so elementary an astronomical fact remained for so long unknown to the Greeks; about 1580 B.C. the Babylonians already knew the time of an orbit of Venus (van der Waerden *Anf.* 49). Wilamowitz, *Hermes* 18 (1883) 416–423, found the identity of morning and evening star referred to in mythology; and Ibycus of Rhegium (approximately a contemporary of Pythagoras; Burnet, *EGP* 191 n. 3) spoke of it (fr. 331 Page).

[40] A40a. In general, it is regarded as an almost hopeless task to reconstruct the celestial system of Parmenides (fr. 12; A37). See Raven, KR 284f; De Vogel, *GP* I 41ff; among older writers, Tannery, *HScH* 238ff; Zeller I 708ff; Burnet, *EGP* 187ff; Gigon, *Ursprung* 276ff; a quite different answer by Morrison, *JHS* 1955, 60ff. Reinhardt (*Parm.* 11ff) is probably right that it was primarily a cosmogonical process (which would be understandable in connection with Anaximander A10 and Empedocles A30, 49, 50); but the result must be our world.

[41] D. L. 9.23, Callim. fr. 442 Pfeiffer; cf. Wilamowitz, *Platon* II 85.2. Pfeiffer thinks of a poem whose authorship was contested between partisans of the two men; but there was only one book by Parmenides (D.L. 1.16 = DK 28A13). Diels (DK I 225, n.) thought that this one didactic poem of Parmenides was circulated under Pythagoras' name, but it is more likely that there was an apocryphal revision labeled "Pythagoras."—D. L. 8.14: *πρῶτόν τε Ἕσπερον καὶ Φωσφόρον τὸν αὐτὸν εἰπεῖν (Πυθαγόραν) ὥς φησι Παρμενίδης*. This has been emended from the time of Casaubon to *οἱ δέ φασι Παρμενίδην*. Diels, DK 28A40a considers *ὥς φησι ⟨καὶ⟩ Παρμενίδης*. Burnet, *EGP* 191 n. 3, defends the MS text. But, if Parmenides had mentioned Pythagoras or even made a clear allusion to him, the ancient scholars, in their search for evidence about Pythagoras would have preserved the verse as they did Empedocles fr. 129.

example in the commentaries on the *Timaeus*;[42] in neither case are the statements reliable.

For there is no corroboration in the astronomical field, more than anywhere else, that Parmenides got his philosophy of nature from a Pythagorean source. The later tradition ascribes to Pythagoras a much more advanced system than Parmenides had.[43] In his astronomy, Parmenides holds quite closely to Anaximander, and Empedocles closely follows him;[44] there is no reason to insert a Pythagorean science.

Commenting on a passage in which Aristotle refers to the professional astronomers for the question of the order and the distances of the stars, Simplicius says, "Anaximander was the first to discover the inquiry into the sizes and distances (of the stars), as Eudemus reports, adding that the Pythagoreans were the first to give their order."[45] Here is ascribed to the Pythagoreans—though not to Pythagoras[46]—a not unimportant contribution to the development of the world model found in Plato. Eudemus is trying to discover, in good Aristotelian fashion, the nature of the progress in science that led to the situation as he knew it; results are what he wants to record.[47] In pursuit of this aim, he attributed to the Pythagoreans the order of the planets known to Plato, Aristotle, and Eudoxus. But in order to estimate the period to which this takes us back, and to understand the relation to the datum about Anaximander,[48] we must survey the theories of the pre-Socratics on the order of the planets.[49]

[42] The attribution to Pythagoras of the invention of the word *κόσμος*, mentioned along with the sphericity of the earth at D.L. 8.48, may well come from Heraclides (above, ch. I 3, n. 151).

[43] Above, nn. 34–35; cf. ch. IV 2.

[44] Above, nn. 29, 40, 58. For the rings of fire and the *στεφάναι*, ch. III 3, n. 25. Empedocles, too, speaks of "wheels," at least in a simile (fr. 46).

[45] Simpl. *Cael.* 471.1 (on Arist. *Cael.* 291a29) = Eudemus fr. 146 = DK12A19: . . . *'Αναξιμάνδρου πρώτου τὸν περὶ μεγεθῶν καὶ ἀποστημάτων λόγον εὑρηκότος, ὡς Εὔδημος ἱστορεῖ τὴν τῆς θέσεως τάξιν εἰς τοὺς Πυθαγορείους πρώτους ἀναφέρων* . . . (The key words *τάξις* and *ἀποστήματα* are from Aristotle; *θέσεως τάξις* is also found in Theo Sm. 181.1).

[46] Eudemus, like Aristotle, does not mention Pythagoras in a scientific context. (On fr. 133, see below, ch. VI 1, n. 62.)

[47] Cf. *πρῶτος* frr. 144, 145, 147, 148; and twice in the fragment cited; similarly, Theophrastus, above, nn. 19–20.

[48] The juxtaposition of the two reports is at first hard to understand, since the calculation of distances and sizes naturally presupposes a certain definite order. This is why Gundel (*RE* XX 2041) rejects the statement about Anaximander, and Wehrli (121) understands Eudemus to be putting the Pythagoreans chronologically ahead of Anaximander. The source of misunderstanding is often an imprecise translation. E.g., Tannery (*Astr.* 126) has ". . . les premiers à spéculer sur la question de l'ordre des sphères," and van der Waerden (*Astr.* 29) ". . . die Untersuchung der Reihenfolge ihrer Lage führt er . . . auf die Pythagoreer . . . zurück." Eudemus has nothing about "speculations" or "investigation"; he is talking about a "finding."

[49] See the material collected by Gundel, *RE* XX 2038–2046.

The fundamental points of that specifically Greek conception of the world's structure are already to be found in Anaximander:[50] the earth remains in place, without support, held by the "equality" of distance in all directions,[51] in the center of the circling rings of fire which are the paths of the stars.[52] Mythical conceptions of the "roots of earth," and of Helios floating back over Ocean to his starting place, have been abandoned;[53] and we find the idea of a perfect circle playing its role in astronomy before Pythagoras.[54] Into this bold outline Anaximander inserts precise figures: the sun is as large as the earth; its "circle" is 27—or 28—times as large as the earth; that of the moon is 18—or 19—times as large; and doubtless the corresponding numbers for the stars would be 9 and 10.[55] The remarkable prominence of the number 9 is a point of contact with Hesiod,[56] and the idea that the stars are closest to the earth, while the sun is furthest away, comes from Iranian teachings about the soul.[57] In this point Anaximenes introduced the essential correction, based on consideration of physical phenomena;

[50] Tannery, *HScH* 90ff; Zeller I 297ff; Burnet, *EGP* 62ff; Diels, *AGP* 1897, 228ff; Heath, *Aristarchus* 31ff; Gigon, *Ursprung* 84ff; Raven, KR 131ff; Kahn 75ff.

[51] A26, A11 §3. Heidel, *Maps* 68f, 151, doubts Aristotle's statement, on the ground that Anaximander's cosmos was not a sphere (below, n. 54). But the argument applies also to circles, rings, or wheels. Aristotle (*Cael.* 295b11ff) is dealing expressly with the *αἰτίαι* for the *μένειν* of the earth. Anaximander must have stated a reason, so that this cannot be simply a false deduction of Aristotle.

[52] A11 §3; A18, A21, A22.

[53] "Roots of the earth," Hes. *Op.* 19, Xenophanes fr. 28; the return of Helios, Mimnermus fr. 10, Stesichorus fr. 185 P.; Anaximenes A7 §6, A14 returns to the older idea. Xenophanes (A33, A41) and Heraclitus (fr. 6) are not interested in mathematical astronomy.

[54] It is hard to decide whether a solid shell of the universe is presupposed—a perfect sphere (as Gigon believes, *Ursprung* 85)—or whether we look out between the "wheels" into the *ἄπειρον* (as Burnet, *EGP* 69). The solid shell is part of the mythic background (*χάλκεος οὐρανός*) and is also found in Anaximenes (*κρυσταλλοειδές*, A14), Empedocles (A30, 51), *Hebd.* 6 (*ἄκριτος πάγος*), Hippoc. *Vict.* 1.10 (*ὁ περιέχων πάγος*). Cf. also *Etym. magn.* s.v. *βηλός*.

[55] A11 §5 (incomplete), A21, A22. The numbers for the stars have been reconstructed since Tannery (*HScH* 94f). The two sets of figures are generally interpreted as an indication of the "thickness" of the rings—which must correspond to the size of the earth, if the sun is the same size as the earth, so that one time the outer, and another time the inner, diameter would be intended, though this involves an error in computation (Raven, KR 136 n. 1).

[56] *Theog.* 722f (cf. the *Odyssey*'s *ἐννῆμαρ φερόμην*). See Nestle, ZN 301 n.; Kranz, *Kosmos* 13f, etc. (9 is an augmentation of 3, which is itself a symbol of plurality; below, ch. VI 4). Diels (*AGP* 1897, 230ff) mentions similar ideas among the shamans. Kahn (94–97) is hardly right in denying these mythical elements in Anaximander.

[57] R. Eisler, *Weltenmantel und Himmelszelt* I (Munich, 1910) 90 n. 3; cf. Boll, *RE* VII 2565; Kranz, *NGG* 1938, 156; Cumont, *Lux* 143; Burkert, *RhM* 1963, 97–134. Diod. 2.30.6 says that the doctrine is Babylonian; probably it is a matter of Babylonian-Iranian syncretism. (The Babylonians had a different doctrine of 3 heavens over one another: Meissner 108.) Kahn (90) follows Diels (*AGP* 1897, 229f) in believing that Anaximander thought the matter out independently, concluding that the largest fire is the highest.

the stars, as distinguished from the sun, do not provide us heat, and are therefore further away (διὰ τὸ μῆκος τῆς ἀποστάσεως).[58]

This makes Eudemus' report on Anaximander comprehensible. The arrangement of the celestial bodies, ἡ τῆς θέσεως τάξις, is wrong, in his account, but still he "discovered" a fundamentally important fact. Not only did he consider the question of the sizes and distances of the heavenly bodies, but, in spite of the arbitrariness of his hypotheses and the incorrectness of his results, he did find the right path toward the answer. The sun is the same size as the earth, although it looks to us about a foot wide,[59] because it is tremendously far off and apparent size decreases as distance increases. True size, apparent size, and distance stand in a definite mathematical relationship. Thus the basic ideas of geometrical proportion and optics have been "discovered," and are then boldly applied to cosmic magnitudes and distances, where any kind of verification is impossible. This is the λόγος which, according to Eudemus, Anaximander "was the first to discover."

What the Pythagoreans could contribute in addition to this was, after the correction made by Anaximenes, the correct order of the planets. The planets do not seem to have been discussed by Anaximander.[60] Of course, morning and evening star had been known for a long time, and people must have noticed long before this that some other bright stars are not always associated with specific constellations, but "wander about." Thus the word itself, πλάνητες (ἀστέρες) may be old.[61] But the further details—that there are five planets, that they have definite orbital periods and thus also regular courses, and that in the character of their movements they are to be compared rather with the sun and moon than with the fixed stars[62]—all this came to the Greeks from Babylon, and later than Anaximander, though before Plato.

Detailed knowledge of the planets cannot be proven for any of the

[58] Anaximenes A7 §6. The sun is found again as the highest star in Parmenides (A40a; doubted by Tannery *HScH* 240, Zeller 714 n. 2; but the directly quoted, short fragments 10–12 do not provide any firm guidance on the question; cf. Heath, *Aristarchus* 74f), Empedocles (A50; cf. above, nn. 29, 40, 44 on the line Anaximander–Parmenides–Empedocles), Leucippus (A1 §33), Metrodorus (DK 70A9), Crates of Mallos (Aët. 2.15.6), and in mysteries of late antiquity (Julian *Or.* 4.148a).

[59] Cf. Arist. *De an.* 428b2, Heraclitus fr. 3.

[60] Cf. Gundel, *RE* XX 2041. The fact that Aët. 2.15.6 also speaks of planets (Anaximander A18) proves nothing, since Metrodorus and Crates of Mallos are also included in the lemma. Diels supplies the word "planets" as part of a conjectural restoration in A11 (Hippol. *Ref.* 1.6.5); but this remains uncertain.

[61] On its formation see Schwyzer I 499. The word is applied not only to wandering stars, but to fevers (πλάνητες πυρετοί).

[62] The natural and unsophisticated way of grouping the heavenly bodies has been—to the present day—sun, moon, and stars (Anaximander, Parmenides frr. 10, 11, *Hebd.* 1.2, 6.1, Anaxagoras fr. 12). In this context one speaks of five planets (οἱ πέντε ἀστέρες,

older pre-Socratics. To be sure, in Anaximenes the basic distinction was drawn between the planets and the fixed stars. The stars are fastened "like nails" in the "ice-like" vault of the sky, though some float on the air like big leaves.[63] Alcmaeon's view may have been similar to this,[64] and Empedocles' certainly was.[65] Anaxagoras supposed there was an indefinite multitude of planets and explained comets as produced by the collision of two of them.[66] The group of five planets is still not canonical, and the planets are still not accepted as equal members in the cosmic hierarchy.

Indication of an important further step is found in Philolaus, but

Arist. *Mete.* 343a31; cf. Aratus 454). It was a scientific achievement not to be underrated to go against appearances, separate the five planets from the fixed stars, and classify them with the sun and moon, so that there are seven planets: ἑπτὰ κύκλοι, Pl. *Tim.* 36d, 38c–d; οἱ ἑπτὰ ἀστέρες, Schol. Arat. p. 429.12, 478.8 Maass; οἱ ἀστέρες οἱ ἑπτά, Dio Cassius 37.18.1; τὰ ἑπτὰ ἄστρα, Hippol. *Ref.* 1.2.2; "septem sidera," Manil. 1.308; Plin. *HN* 2.12; cf. *Hymn. Hom. Ares* 7, Cic. *Rep.* 6.17, etc. Of course this is correct, from the geocentric point of view.

[63] A7 = Hippol. *Ref.* 1.7.4: ἥλιον καὶ σελήνην καὶ τὰ ἄλλα ἄστρα πάντα πύρινα ὄντα ἐποχεῖσθαι τῷ ἀέρι διὰ πλάτος. Cf. A15, but A14 = Aët. 2.14.3: ἥλων δίκην καταπεπηγέναι τὰ ἄστρα τῷ κρυσταλλοειδεῖ. The contradiction can be removed by understanding the first of the sentences quoted of the planets, and the second of the fixed stars. A slight alteration in the text of the next clause in Aëtius (2.14.4) would yield this sense: ἔνια (MSS ἔνιοι) δὲ πέταλα εἶναι πύρινα ὥσπερ ζωγραφήματα (Heath, *Aristarchus* 42). Ach. *Is.* p. 40.20 Maass, however, has τινὲς δὲ in the corresponding clause; the source of the trouble must be older. Cf. Gundel, *RE* XX 2042; Guthrie I 135–137.

[64] A4 = Aët. 2.16.3 may originally have applied only to the sun, since Alcmaeon's name appears only as an afterthought. In A12 ἀστέρες are named along with the οὐρανός, but it is not really necessary to interpret this as indicating a distinction between planets and fixed stars. In any case, Alcmaeon would not have been the first to speak of the planets (as Gigon says, *Ursprung* 150), but following Anaximenes, with whom the doctrine of the "flat" sun also brings him into contact (A4; cf. Anaximenes A15).

[65] A54: τοὺς μὲν ἀπλανεῖς ἀστέρας συνδεδέσθαι τῷ κρυστάλλῳ, τοὺς δὲ πλάνητας ἀνεῖσθαι. The word πλάνητες cannot refer to the sun and moon alone; Empedocles had special theories about them.

[66] A1 §9: τοὺς δὲ κομήτας σύνοδον πλανήτων φλόγας ἀφιέντων. This sounds as though he were thinking of a regular collision; in that case, Anaxagoras would have had to station all the planets in the same region. The comet's tail was explained as "flame," with the help of the analogy of flint. Aristotle (*Mete.* 342b25) speaks of σύμφασις, "apparent conjunction"; but, since he includes Anaxagoras and Democritus together (cf. below, n. 74), he may have obscured the difference between them.—Anaxagoras was the first to make neighbors of the sun and moon (Eudemus fr. 147; cf. above, n. 58).—It may remain an open question whether the sentence in *Hebd.* 2, lines 64ff, τὰ τοίνυν ἄστρα τὰ οὐράνια ἑπτὰ ἐόντα τάξιν ἔχει τῆς τῶν ὡρέων ἐκδοχῆς, refers to the 7 planets, as Boll contended (*NJb* 1913 = *KlSchr* 220ff), following the commentary of ps.-Galen (*CMG* XI 2.1 p. 35). Kranz agreed with him (*NGG* 1938, 142, *Kosmos* 32 n. 8) as did Gundel (*RE* XX 2040) and Roscher (*Hebd.* p. 134 n. 191; but he rejects it *SBLpz* 1919.5, 65ff). The dating of *Hebd.* depends precisely on the question whether it shows knowledge of the planets and the spherical shape of the earth. In my opinion the thesis that the planets were known to the author falls with correct interpretation of the phrase ὡρέων ἐκδοχή, which must mean "the succession of the seasons" (cf. *Hebd.* 4, 14). The constellations named in the next paragraph are connected with this (Arcturus, the Pleiades, the Hyades, Orion, and the Dog), but the 7 planets are not (except, of course,

also in Democritus. He has been severely criticized for rejecting the sphericity of the earth, and consequently his entire astronomical system has been called backward and almost primitive.[67] But a book title like *Ἐκπετάσματα* should make us prick up our ears; if Diels is right about its meaning, the subject was a projection of the armillary sphere on a plane surface, or in other words, mathematical astronomy.[68] Democritus was well informed in both mathematics and astronomy and wrote a whole book *περὶ τῶν πλανήτων*.[69] His series of heavenly bodies, from the earth as center, was moon, Venus, sun, planets, fixed stars; and the planets themselves were put at various distances from the earth.[70] The reason for this arrangement is that in the "whirl" the stars nearer the earth lose ground relatively to the fixed stars. Thus the planets are integrated into the world system, with different distances according to their different "speed;" Democritus knows that there are a number of planets, and that they have definite orbital periods, in some cases longer than those of the sun. The special position of Venus points to influence from Babylon, where the trinity of sun, moon, and Venus is attested very early.[71] It is incredible that Democritus should not have known the five familiar planets.

the sun). Perhaps the words *ἑπτὰ ἐόντα* allude to the fact that certain constellations have 7 stars (the Bear, Pleiades, Orion, and, according to Pherecydes, the Hyades; *FGrHist* 3F90, Hippias DK 86B13); Varro in his *Hebdomades* (Gell. 3.10.2) mentioned the seven-group of the Septentriones and the Pleiades even before the planets. (For seven-groups of fixed stars in Babylon, see Meissner 407f.) *Ζεὺς αἰγίοχος* (*Hebd.* 6.2, line 17) cannot mean the planet Jupiter (as Kranz thinks, *NGG* 1938, 125 n. 2; cf. *Kosmos* 32 n. 8), since this manner of referring to the planets is only attested for a much later period (above, n. 6; Cumont's conclusion could only be said to be "corrected"—Kranz, *NGG* 1938, 126—if there were no other possible interpretation). There remains Boll's idea that it means the sky, because mention is made of its "change of color," i.e. the alternation of day-sky and night-sky (*AbhMü* 1918, 25; cf. below, ch. IV 3, nn. 27.64).

[67] The "childish character" of Democritus' astronomy is spoken of by Burnet, *EGP* 339; i.e., he was not a Pythagorean! (Cf. above, n. 31).

[68] DK 68B11q; also the n. II 141, with ref. to Ptol. *Geogr.* 7.7.

[69] Fr. 5b. In comparing Democritus with the allegedly more progressive "Pythagorean" astronomy of Plato, one must bear in mind that for Plato we have detailed expositions in his surviving books, but for Democritus only scattered and incomplete citations. If one matches the doxographical accounts of Plato with those of Democritus, the latter comes out fully as well (compare [/] Aët. 2.13.4/12, 2.15.3/4, 2.20.5/7, 2.25.6/9, 2.29.6/30.3); an outstanding difference is that he gives physical causes rather than mathematical descriptions (cf. 2.16.1/7, 2.23.6/7). In his explanation of the Milky Way, Democritus (unlike Aristotle) has the right answer (Aët. 3.1.6).

[70] A86, A40 §4; Lucr. 5.621ff = Democritus A88.

[71] Shamash, Shin, Ishtar, represented as a group as early as the Naramsin stele; see Cumont, *AC* 1935, 10 n. 7; Meissner 18ff, 402ff. It is not entirely impossible that Parmenides is subject to oriental influence in the important position he gives to Venus (cf. above, n. 40); but Democritus is certainly not following Parmenides. In fact, he differs from Leucippus as to the position to be assigned the sun (above, n. 58).

Seneca, though, says that Democritus did not "yet" have an adequate knowledge of the courses of the five planets. "Democritus quoque . . . suspicari se ait plures stellas esse, quae currant, sed nec numerum illarum posuit nec nomina, nondum comprehensis quinque siderum cursibus."[72] Yet this does not mean that Democritus "knew neither the number nor the names of the planets."[73] According to Seneca's report, Democritus expressed the suspicion that there were "more" planets, which surely means "more" in comparison to those he did know and describe. Seneca, or his source, sees in this belief—in which Democritus was correct, as we now know—a deficiency or shortcoming; the astronomy of later antiquity thought it had attained final results, and no longer reckoned with undiscovered stars.[74]

Eudemus must have found Democritus' arrangement of the planets incorrect, too, because of the special position of Venus. The "correct" one was, however, included in the system of Philolaus; for the sequence of the ten "divine bodies" was, in the unanimous testimony of Aristotle and the doxographers,[75] central fire, counter-earth, earth, moon, sun, five planets, heaven of the fixed stars. If we consider only the portion between earth and heaven, this is the order accepted by Eudoxus, Plato, and Aristotle. In addition, this is the system which Aristotle ascribes simply to "the Pythagoreans;" so nothing seems in the way of the assumption that Eudemus meant the same Pythagoreans, and his report that "the Pythagoreans" had established the order of the planets referred precisely to the system of Philolaus.

One would hardly suggest that Philolaus was dependent on Democritus;[76] rather, there must have been a common source, someone who, in the interval between Anaxagoras and the time of Philolaus and Democritus, introduced to Greece, from Babylon, detailed knowledge about the planets. In fact, this very epoch is that of the efflorescence of professional Greek mathematics and astronomy. The floruit of

[72] *QNat* 7.3.2 = Democritus A92.

[73] Frank 202, followed by Gundel, *RE* XX 2040; more cautiously Cumont, *AC* 1935, 9f: Did Democritus intentionally ignore the divine names of the planets?

[74] See Zeller I 1107 n. 2, and Heath, *Aristarchus* 128. Democritus explained comets as a phenomenon of reflection between planets or a planet and a fixed star (A92, to which Arrian ap. Stob. I p. 229.5 Wachsmuth and Posidonius ap. Schol. Arat. 1091 should be added; cf. n. 66).

[75] Arist. fr. 203, Philolaus A16. Gundel (*RE* XX 2100f) erroneously takes the system described at Plut. *De an. procr.* 1028b as that of Philolaus.

[76] Apollodorus of Cyzicus made him Democritus' teacher. Cf. above, ch. III 2, n. 101.

Hippocrates of Chios was about 430 B.C.[77] It is true that our only direct information about his planetary theories concerns an exceptional case, that of "the" comet; but in this is implied that he discussed the behavior of the planets in detail. They move within the tropics, that is, in the area of the ecliptic; and they "stay behind" the fixed stars, each along his peculiar orbit (*ὑπολελειμμένον ὅλον τὸν ἑαυτοῦ κύκλον*, Arist. *Mete.* 343a6), that is, they have definite periods. Heavenly and terrestrial events affect each other—"the" comet sucks up moisture. But the spherical plan of the celestial movements is already there, in some detail: the parallel circles, oblique to the horizon and cut by it at various angles.[78] Hippocrates probably was influenced by his countryman Oenopides, who had much to say about "circles" and "inclinations."[79]

June 27, 432 B.C. is the date of the summer solstice observed by Meton, who was also "astronomer and geometer."[80] We know that

[77] DK 42; on his theory of comets, Arist. *Mete.* 342b36ff = DK 42.5 (Olympiodorus on this passage, DK 42, speaks of the "sixth planet," showing that he assumes a theory of 5 planets for Hippocrates; but this is not probatory). It does not seem to have been noticed that Aristotle's refutation provides an astronomically precise dating for Hippocrates. At *Mete.* 343b2ff he brings into evidence against him a comet observed in the year 427/426 B.C., which showed characteristics that Hippocrates and his pupils had declared impossible (343b7: *αὐτοὶ τῶν ἀδυνάτων εἶναί φασιν*). According to this, Hippocrates must have published his theory before 427. This chronological datum is very important for the history of mathematics (contra Frank's late placement, "kaum vor 400," 81, 227; cf. also Burkert, *Philologus* 1959, 193 n. 1).—Hippocrates is just once called *Πυθαγορικός* in a late source (Schol. Arat. p. 546.21 Maass), but Aristotle distinguishes him from the Pythagoreans (below, n. 111).

[78] The way the problem is posed, whether the segment of the parallel circle above the horizon or that below is greater (Arist. *Mete.* 343a11, 18), corresponds to the method followed for the quadrature of the lune (Eudemus fr. 140; below, ch. VI 1).

[79] DK 41.2. He was a somewhat younger contemporary of Anaxagoras, named together with Eudoxus by Hecataeus of Abdera, *FGrHist* 264F25 = Diod. 1.98, and also *CCAG* VIII 3, p. 95.17 (a fuller version of the text given at DK 41.1a), where one reads, *πρῶτος . . . τὰς ἀστρολογικὰς μεθόδους ἐξήνεγκεν εἰς γραφήν*. The date given, "end of the Peloponnesian War," is inexact; he is connected with Zeno, Herodotus, and Gorgias. He was probably older than Meton. Cf. also von Fritz, *RE* XVII 2258–2272. Schiaparelli (*Vorl.* 15) interpreted his 59-year "great year" as comprehending the cycles of all the planets (2 × Saturn, 5 × Jupiter, 31 × Mars, 59 × sun, Venus, and Mercury, 729 × moon; the 59-year cycle of Saturn was known in Babylon: van der Waerden, *Hermes* 1952, 135.3, 139). Tannery (*Astr.* 143) also interprets Philolaus A22, which obviously is connected with Oenopides, in the same way. But the tradition brings Oenopides' "great year" with that of Meton (DK 41.9), and that has nothing to do with the planets and is only concerned to coordinate the courses of the sun and moon. This is doubtless how Oenopides' "great year" is to be interpreted (von Fritz, *RE* XVII 2262f). Oenopides is never directly called a Pythagorean; there is a point of contact in the theory of the Milky Way (below, n. 116), as well as a later charge of plagiarism (above, n. 37), and a remarkable mélange of myth and science, though this is not necessarily a criterion of Pythagoreanism.

[80] Schol. Ar. *Av.* 997. On Meton, see Heath, *Aristarchus* 293–295; Kubitschek's *RE* article (XV 1458–1466) is unsatisfactory.

he had some contact with Babylon, however it was established, for his nineteen-year cycle had been used there since 499.[81] Meton is named several times along with Eudoxus.[82] He seems to have been familiar not only with the idea, but with the graduation, of the zodiac.[83] In the Babylonian manner, he designated the ascension of various fixed stars as weather signs, and Democritus followed him in this.[84] Tzetzes ascribes to Meton a doctrine of the "Great Year" and of the destruction of the world when all the planets meet in the sign of Aquarius.[85] This may be merely a confusion of Meton's nineteen-year cycle with the World Year;[86] and from a single rather indefinite and general note we cannot tell for certain in what sense Meton dealt with the "distances" of the stars.[87] The fact remains that at the epoch of Meton and Hippocrates we can ascertain the presence of not only empirical astronomical data borrowed from Babylon, but also a precise, geometrical conception of a spherical universe. From this time on, the general public was interested in astronomical topics.[88] This suggests that by then a fairly accurate

[81] Van der Waerden, *Anf.* 112. There was one intercalation, which does not fit into the system, in 386 B.C.

[82] Columella 1, praef. 32, 9.14.12; Schol. Basileios pp. 196.24–197.2 Pasquali.

[83] Columella 9.14.12; van der Waerden, *Anf.* 170; cf. ch. IV 2.

[84] Used in [Geminus] p. 212.3 Manit., Ptolemaeus, *Phaseis*. On the close relationship of this literature with Babylon, see C. Bezold and F. Boll, "Reflexe astrologischer Keilinschriften bei griechischen Schriftstellern," *SBHeid* 1911.7; F. Boll and A. Rehm, "Griechische Kalender, III," *SBHeid* 1913.3; Democr. fr. 14.

[85] *Chil.* 10.534ff, 12.219ff, 12.283ff. The statement that Meton was the first to write a book on astronomy (12.128, 214) has to compete with the same statement applied to Oenopides (above, n. 79).

[86] Cf. the confrontation Schol. Arat. p. 478.5. But before Berossus (fr. 37 Schnabel = Sen. *QNat* 3.29.1 = *FGrHist* 680F21), not only did Plato know of the destruction of the world by fire and water (*Tim.* 22c; the "great year" 39d), but Philolaus also did (A18): *Φιλόλαος διττὴν εἶναι τὴν φθορὰν τοῦ κόσμου, τὸ μὲν ἐξ οὐρανοῦ πυρὸς ῥυέντος, τὸ δὲ ἐξ ὕδατος σεληνιακοῦ, περιστροφῇ τοῦ ἀέρος ἀποχυθέντος· καὶ τούτων εἶναι τὰς ἀναθυμιάσεις τροφὰς τοῦ κόσμου*. This comes under the title, *πόθεν τρέφεται ὁ κόσμος*. The connection of *φθορὰ τοῦ κόσμου* and *τροφὴ τοῦ κόσμου* can only be understood to mean that the catastrophe happens to only a part of the world, doubtless the region of the earth (Zeller I 549f). Parmenides and Heraclitus had taught that the stars are nourished by *ἀναθυμιάσεις* (Aët. 2.17.4).—The catastrophe idea has nothing to do with Philolaus' "great year," A22; for the latter takes a period of only 59 years. See further, on the "great year," van der Waerden, *Hermes* 1952, *Anf.* 116–119. There is much controversy over the evidence of Heraclitus, frr. 66, 100, A13; cf. Reinhardt, *Hermes* 77 (1942) 1–27, 225–248; Kirk, *Heraclitus* 300–305, 359–361; M. Marcovich, *RE* Supp. X 297–303.

[87] Schol. Basileios ed. Pasquali (*GGN* 1910), 196.24–197.2.

[88] Cf. Eur. fr. 861 (below, Ch. IV 2, n. 47), Gorg. *Hel.* 13, Pl. *Gorg.* 451c, Xen. *Mem.* 4.7.5. Here Socrates is said not to have wished to spend time on hairsplitting subtleties about the movements of the planets or the like; this is of course a gibe at the Academy, but, in the light of the other evidence, ought not to be considered an anachronism.

notion of the planets and their periods, as well as their assignment to various gods, had become familiar to the Greeks.

Probably the historical situation had its importance. From 500 to 479 B.C., a state of war prevailed between the Persian Empire and nearly all the Greeks and made peaceful contacts difficult, if not completely impossible.[89] This seems to have been the only period of any considerable extent during which connections between Greece and the Orient were almost completely severed. In this era of isolation the most characteristic endeavors of the Greeks were reaching their maturity; at its beginning, Parmenides was active. Then, as normal conditions once more returned[90] and new relationships to the Orient became possible, the Greeks were able, thanks to the progress made in the meantime, to evaluate and select in a way quite different from that of Anaximander's time.

The opinion is widespread that Pythagoras himself, who is supposed to have traveled in the East,[91] brought this astronomical knowledge back to Greece with him and passed it on through his school. In fact, he is thought of as the most important link in the transmission of oriental science to the Greeks. More cautious scholars are more likely to speak not of Pythagoras, but of the early Pythagoreans, who are supposed to be the only Greeks before Philolaus to have any advanced astronomical knowledge.

This opinion leaves unanswered the question why we can detect no influence of such knowledge, even in relation to the planets. In Parmenides and Empedocles, at least, one would have expected to find some traces; and if Oenopides "stole" the determination of the obliquity of the ecliptic from Pythagoras, why did he not also take his knowledge of the planets? Since there is no apparent reason why the doxography should report the views of Democritus in more detail than those of earlier thinkers, we are driven to the assumption that the planets did not really play so important a role with them. The Pythagoreans' astronomic erudition would be, then, a secret doctrine with no effect on others—a buried treasure. For Democritus' astronomy

[89] Themistocles was probably not the only Greek who learned Persian in those days. The Democritus story represents Magi and Chaldaeans coming with Xerxes to Abdera (D.L. 9.34). On the other hand, Eudoxus was later prevented by political reasons from traveling in Persia, and went instead—recommended by Agesilaus—to rebellious Egypt (above, n. 3).

[90] As Democedes was active at the court of Darius I before the Persian Wars, so was Ctesias under Darius II and Artaxerxes II.

[91] Above, ch. II 2, nn. 15–16.

cannot in any case be derived from Pythagoreanism. There is not a trace in the Pythagorean tradition of the special position of Venus in his system, which points unmistakably toward Babylon. The conventional view involves a remarkable double development; what the Pythagoreans had long ago brought from Babylon had to be fetched, a second time, from the same source.

But there is no good reason to assume a mysterious, secret pre-Philolaus astronomy, belonging to Pythagoras or the Pythagoreans, aside from one dubious inference from Parmenides and some even more dubious late reports. Scholars have seized upon the assertion of the "more genuine" Pythagoreans, that the central fire is a force in the interior of the earth, named *Ἑστία*.[92] In fact, this epithet is applied to the earth a few times in the fifth century,[93] and Empedocles spoke of fires beneath the earth.[94] Here, it is thought, we have traces of a geocentric system belonging to the early Pythagoreans, which we should postulate anyway and which displays a suitable mixture of myth and science. Nevertheless, the basis of the reconstruction, the report of the "genuine Pythagoreans," is an artificial reinterpretation of the reports of Aristotle, and without independent value as a source. The only other point is the name of *Ἑστία*, but this is comprehensible as an expression of the central location of the earth, without the idea of a central fire.[95] And since we may not regard every point of contact between mythology and *φυσιολογία* as a priori Pythagorean—tradition even associates Euripides and Anaxagoras as among those who called the earth *ἑστία*[96]—the conclusion that Pythagoreans are behind this

[92] Above, ch. III 1, n. 72. The idea of a central fire within the earth was developed, as the original Pythagorean theory, by H. Richardson (CQ 20 [1926] 113–133), following Burnet, *EGP* 297f, then also by Wiersma, *Mnemosyne* 1942, 23ff. One piece of evidence is *Th. ar.* 6.11ff (cf. above, ch. III 2, n. 139). A theory of fire in the earth is well attested for the Stoic Archedemus (*SVF* III 264).

[93] Soph. fr. 558 N. = 615 Pearson, Eur. fr. 944; cf. Procl. *In Eucl.* 173.18f. At Pl. *Phdr.* 246, too, *Ἑστία* is used of the earth (Dercyllides in Theo Sm. 200.7f, Macrob. *Sat.* 1.23.8). Cleanthes attacked Aristarchus of Samos as *κινοῦντα τοῦ κόσμου τὴν ἑστίαν* (Plut. *De fac.* 923a).

[94] Frr. 52, 62. This is natural enough for a Sicilian, with Etna before his eyes. At Parmenides A37, a conjecture of Diels introduces the idea of fire beneath the earth; but, uncertain as the whole world system of Parmenides is, this must remain in doubt (above, n. 40).

[95] Anat. p. 30 = *Th. ar.* 6.15f . . . *κατηκολουθηκέναι τοῖς Πυθαγορείοις οἵ τε περὶ Ἐμπεδοκλέα καὶ Παρμενίδην καὶ σχεδὸν οἱ πλεῖστοι τῶν πάλαι σοφῶν φάμενοι τὴν μοναδικὴν φύσιν ἑστίας τρόπον ἐν μέσῳ ἱδρῦσθαι* . . . Here the subject of discussion is the central position of the earth, in Parmenides and Empedocles, not a fiery center for the earth, such as is attributed to the Pythagoreans. The hearth is the center of domestic life, and the newborn child is carried around it (*Ἀμφιδρόμια*; cf. Nilsson I 95 n. 8; Hestia *μέσῳ οἴκῳ Hymn. Hom. Aphrod.* 30). Fire burns on the hearth, not in it.

[96] Anat. p. 30 (*Th. ar.* 6.18ff) = Eur. fr. 944 = Anaxagoras A20b.

development is quite shaky. And there is no proof of special astronomical knowledge, going back before Anaxagoras.

At the same time, the effort continues to attribute to the Pythagoreans before Plato, and even before Philolaus, a different, geocentric planetary system which became dominant in the later Hellenistic period. In this, the sun is in the middle of the seven planets, flanked on each side by three of them—Venus, Mercury, and the moon in the direction of the central earth, and Mars, Jupiter, and Saturn in the direction of the heaven of the fixed stars. This arrangement of the planets attained a canonical position in astrology, and still determines the order of the days of the week. It cannot be documented earlier than Archimedes,[97] yet it was attributed to Pythagoras and connected with a system of the harmony of the spheres, in which the sun, both in its position and its function, is μέση.[98] This coincidence, so gratifying to every Pythagorean heart, and in general the "solar theology" that pervades the theory, insured that the age and the originality of the system would not go without defenders.[99] An independent testimony seems to corroborate this. Ptolemy says that the "older mathematicians" (παλαιότεροι) put the sun in the middle, whereas "some later ones" (ἔνιοι τῶν μετὰ ταῦτα) put all the planets above the sun. Since this is precisely the conception of Plato, Eudoxus, and Aristotle, the conclusion is drawn that the "older" mathematicians in question are pre-Eudoxan, and therefore Pythagorean, scientists.[100]

[97] Archimedes: Macrob. *Somn. Sc.* 1.19.2, 2.3.13 (differently Hippol. *Ref.* 4.8 = Archimedes ed. Heiberg II 552ff; this is unreliable, cf. Boyancé *Songe* 61, *REG* 1952, 345f; Cumont, *Lux* 179 n. 1). Archimedes combined this with calculations of distance that were criticized by Platonists: Macrob. *Somn. Sc.* 2.3.14 (cf. the *mathematici*, Cic. *Div.* 2.91).—Hipparchus: Tannery, *Astr.* 127.—Nechepso-Petosiris (Plin. *HN* 2.88); doubtless the source of its attribution to Egypt (Dio Cassius 37.19.2), and also the Chaldaeans (i.e. astrologers; Macrob. *Somn. Sc.* 1.19.2).—Cic. *Rep.* 6.17, Philo *V. Mos.* 2.103, *Quis rer. div. heres* 224, *Qu. in Exod.* 2.75, Geminus 1.24ff, Cleomedes 1.3, Manilius 1.811ff (thus also, probably, Posidonius; Boyancé, *Songe* 63, vs. Reinhardt, *Kosmos* 131f; differently Chrysippus, see above, n. 7); Plin. *HN* 2.32–41 (thus also Varro), Or. Chald. cited Procl. *In Tim.* III 63.22. Hultsch (*RE* II 1833) still wanted to refer this system to the ancient Babylonians; Cumont (*Lux* 144) suggested "Magusaioi" of the 5th century B.C. But there is no indication in the abundant original sources on Babylonian astronomy that any such planetary sequence was known there (Boll, *RE* VII 2561ff; Neugebauer, *ExSc* 168ff).

[98] Plin. *HN* 2.84, Cens. 13.3, Theo Sm. 138.9ff = Chalcid. 72f. On this interpretation of the harmony of the spheres, see Burkert, *Philologus* 1961; it is obviously dependent on Eratosthenes.—This planetary system is contaminated with that of Philolaus by Plutarch, *De an. procr.* 1028b (above, n. 75).

[99] Boyancé, *Songe* 78ff.

[100] Ptol. *Synt.* 9.1. The conclusion that Pythagoreans are meant: Boll, *RE* VII 2568.34; Boyancé, *Songe* 62f; van der Waerden, *Astr.* 34ff, *RE* XXIII 2.1809.

The evidence of Eudemus makes considerable difficulty for this view;[101] he can only be referring to the arrangement of the planets accepted by Aristotle, whose discovery he traces back to the Pythagoreans, for Simplicius is citing Eudemus merely to help explain Aristotle. If there had been two different Pythagorean planetary systems, Eudemus would surely have mentioned the fact; and he could not have spoken of a discovery, but of a dilemma, since both systems are worth about the same from a geocentric point of view. As it is, there is not a trace of the second system, mentioned above, before Archimedes;[102] and it seems clearly to be later in origin. Beginning with the naive idea of "sun, moon, and stars," one's first step in advance is to recognize five of the stars as a special group—the planets—to distinguish them from the fixed stars and class them, to a certain extent, along with the sun and moon.[103] One sees the expression of such a development in the systems of Philolaus, Plato, and Eudoxus: moon, sun, five planets, fixed stars. Only when the close connection of the "seven" has come to seem a matter of course, would one appreciate the symmetry of the other arrangement, in which the insignificant planet Saturn counterbalances the old familiar moon.

The conclusion drawn from Ptolemy does not hold up. When he compares "old" and "newer" observations,[104] he means by "old" astronomers those from the third century on, as far as Hipparchus, ca. 150 B.C. (who himself figures as the most important of the "ancients"),[105] by contrast with the "moderns" of the last hundred years before Ptolemy. Eudoxus and Callippus are never cited in the *Almagest;* the astronomy of the fourth century has already sunk from sight. Aristarchus, to be sure, assigned the sun to a central position; but Plato's great prestige kept his own system alive, even among professional astronomers.[106] The expression "some later ones," then, refers to astronomers later than Hipparchus, who, doubtless because of their orthodox Platonism, championed the older system. If even the

[101] Above, n. 45.

[102] On Heraclides Ponticus, see above, n. 7.

[103] Above, n. 62.

[104] *Synt.* 9.7 (II 262.11, 264.14 Heiberg), 9.10 (II 283.10), 10.9 (II 352.4), 11.3/4 (II 386.17, 391.18), 11.7/8 (II 419.11, 425.6).

[105] *Synt.* 3.1 (I 191.17); cf. 4.2 (I 270.1: οἱ ἔτι παλαιότεροι = astronomers before Hipparchus); 4.9 (I 328.5). The παλαιοί in the *Apotelesmatica* of Ptolemy are Nechepso-Petosiris (*RE* XXIII 1837.49ff). Cf. also n. 97.

[106] Hesitation of the μαθηματικοί as to the order of the planets: Aët. 2.15.5, Theo Sm. p. 143.1 = Chalcid. 73. The way in which Ptolemy presents the arguments for each solution shows that the discussion was lively. Cf. also the criticism directed by certain *Platonici* against Archimedes (Macrob. *Somn. Sc.* 2.3.14).

brilliant Eudoxus had fallen into oblivion, there can be very small profit in looking to Ptolemy for evidence on pre-Platonic Pythagoreanism.

Nor can internal indications of "solar theology" or the harmony of the spheres prove that this late-attested system belonged to the early Pythagoreans. Boyancé is able to cite a number of rather early references to the identification of Apollo and the sun, and to the connection between the sun and the harmony of the cosmos.[107] But the interrelation of myth and the interpretation of the natural world is not Pythagorean alone; all Greek cosmology had to make its peace with the existing myths; and the thought of cosmic harmony is by no means tied to any particular planetary system.[108] We shall not have to alter the conclusion drawn from the passage of Eudemus: the arrangement of the planets found in Plato and in the Philolaic system, and no other, was known to Pythagoreans before Plato. We may conclude that this went back to the acceptance of Babylonian information in the time of Meton, just before that of Philolaus, and not to a more ancient Pythagorean tradition.

The *acusma* which says that the planets are the "hounds of Persephone" may well be older.[109] Most nearly comparable to this is the Babylonian designation of the planets as "rams."[110] But we need not suppose a direct dependence; the background is that of naive observation of the skies: the stars that catch attention by their independent movements are thought of as living beings. These Pythagoreans, therefore, are looking for the realm of Persephone in the skies; it is also known that the sun and moon are the "Isles of the Blest." As well as this fits in, it does show that sun and moon were not reckoned among the planets. When one notices that certain stars "wander about" in the skies, the idea of mathematical arrangement and regularity does not immediately spring to mind. There is here a kind of "astronomy,"

[107] Heraclitus fr. 100 (cf. also Anaximenes A14, Xenophanes A42, Kahn 104ff), Oenopides DK 41.7 (Helios = Apollo Loxias), Pl. *Crat.* 405 c–d, Scythinus DK 22C3.1 = Plut. *Pyth. or.* 402a (the constellation Lyra that of Apollo, the sunray as plectrum; on the date of Scythinus see *FGrHist* 13), Cleanthes *SVF* I 499, 502 (sunray as plectrum).

[108] In Hippoc. *Vict.* 4.89 we find *ἡλίου δὲ ἡ μέση* (*περίοδος*), namely between stars and moon (also 1.10: *πάντων ἐπικρατεῖται*). The planets thus have no role here. On the harmony of the spheres, see below, ch. IV 4.

[109] Por. *VP* 41 (above, ch. II 4); Gundel, *RE* XX 2022.36.

[110] "Bibbu"; cf. Meissner 404. Also comparable is the designation of the planets as "generals of Ahriman" in Iran; but it is not attested till late (A. Christensen, *Die Iranier* (Munich, 1933) 229. Rougier (1ff) would like to show that in comparison with oriental ideas the Pythagorean astronomy represented a definite forward step; but he overlooks this *acusma*.

or we might better say, certain notions about the heavenly bodies which prevailed among Pythagoreans and were peculiar to them—for there is nothing else in Greece like this—but they had nothing to do with scientific Greek astronomy. There was a pre-scientific area within Pythagoreanism itself. And this confirms that there was not a continuous transmission of advanced astronomical knowledge from the day of Pythagoras, but that scientific elements were added only later on, in the course of the general development of Greek science.

There is only a single passage in Aristotle to indicate that the Philolaic system was not the only one in vogue among the Pythagoreans, stating that "some" of them believed, as did Hippocrates of Chios and his pupil Aeschylus, that "the" comet was a planet,[111] a view also shared by Diogenes of Apollonia.[112] In the system of Philolaus, the importance of the perfect number 10 leaves no opportunity for any such intrusive planet. It is difficult to interpret chronologically the "similarity" of the Pythagorean theory to that of Hippocrates. The latter gives a complicated explanation of the origin of the comet's tail—a phenomenon of reflection, he thinks, which happens only under certain conditions, which is why the comet is so seldom visible.[113] The Pythagoreans do not go into the question of the tail; they only say that this "planet" is only seldom visible and does not rise far above the horizon.[114] One might conjecture that the simpler theory is the older; and in fact Aristotle mentions it first. But this makes it difficult to see why the comet is not accounted for in the system of Philolaus. It would also be possible that the Pythagoreans in question should be dated later than Philolaus, and later than the refutation of Hippocrates' theory by the comet of 427/426 B.C. (above, n. 77). Their concern was not to set up a bold new scientific hypothesis, but merely to suggest a possibility.

With relation to the Milky Way, too, Aristotle knows of different theories held by Pythagoreans. Some see in it the path of a star that was driven from its course in the catastrophe caused by Phaethon, others,

[111] Arist. *Mete.* 342b30ff, followed by Aët. 3.2.1, ps.-Galen *Phil. hist.* 75, Schol. Arat. p. 545 Maass.

[112] A15: *ἀστέρας εἶναι τοὺς κομήτας.*

[113] The reflection of light is a favorite idea of the *φυσικοί* in the 5th century B.C. Cf. n. 115 and below, ch. IV 3, n. 29.

[114] The reference to Mercury at 342b32ff, being couched in direct discourse in the midst of the indirect, is probably an explanatory note added by Aristotle.

a route once followed by the sun.[115] Philolaus (A18) spoke of fire flowing from the sky. A doxographical entry attributes the second answer to Oenopides and adds that the sun changed its course out of disgust at the meal of Thyestes.[116] The conclusion has been drawn from this that Aristotle thought of Oenopides as a Pythagorean;[117] but in that case Democritus' pupil Metrodorus of Chios, who held the same view,[118] would also have to be a Pythagorean. A "good idea," once expressed, keeps getting repeated. It is entirely possible that Metrodorus and the Pythagoreans were both dependent on Oenopides; we know that Philolaus took over the idea of the "Great Year" from him.[119] In any case, there is no trace, before Philolaus and Oenopides, of a highly developed Pythagorean astronomy.

2. THE THEORY OF PLANETARY MOVEMENTS

From Plato to Kepler, astronomy was dominated by the assumption that all movements of celestial bodies were to be explained by the combination of "perfect," uniform, circular movements. On the basis of this assumption there are two approximately correct explanations of celestial movements, a heliocentric explanation like that of Aristarchus or Copernicus, and a geocentric system like that of Ptolemy, with epicycles and eccenters. From a mathematical or descriptive point of view, the two systems are simply equivalent;[1] the decisive advance was Kepler's recognition that the orbits in question were ellipses rather than circles.

[115] Arist. *Mete.* 345a14ff, and Aët. 3.1.2 (DK 58B37c). The latter also includes under the lemma *Πυθαγόρειοι* the explanation of the Milky Way as a phenomenon of reflection which goes back to Hippocrates (Arist. *Mete.* 345b10ff, DK 42.6); and he does the same thing, at 3.2.1, with Hippocrates' theory of comets. The error doubtless comes from the fact that a mathematician was automatically regarded as a Pythagorean (cf. above, n. 77.)

[116] Ach. *Is.* 55.18 Maass = DK 41.10. Elsewhere it is stated that the stars reversed their courses because of the meal of Thyestes; i.e., they formerly rose in the west and set in the east (Eur. *El.* 726ff, *Or.* 1001ff, Pl. *Pol.* 268e).

[117] Von Fritz, *RE* XVII 2259; Boyancé, *Songe* 96, *REG* 1952, 348.

[118] Aët. 3.1.3 = DK 70A13: *διά τὴν πάροδον (γίνεσθαι τὸν γαλαξίαν)· τοῦτον γὰρ εἶναι τὸν ἡλιακὸν κύκλον.*

[119] Above, n. 79.

[1] See Tannery, *Astr.* 101; Neugebauer, *ExSc* 122ff, 183. Copernicus knew that his system was no more adequate, descriptively, than that of epicycles and eccenters (ed. Soc. Cop. Thorunensis, Thorn 1873, 5.14), but he cited the "prima principia de motus aequalitate," which the epicycles violate. The "Copernican revolution" mainly affects physics (aside from man's feeling toward the world). Newton's mechanics is possible only in a heliocentric system. In the modern theory of relativity, this difference between geocentric and heliocentric has vanished again. See also J. Mittelstrass, *Die Rettung der Phänomene* (Berlin, 1962) 140–221.

Eudoxus of Cnidus, taking his departure from this same assumption of uniform circular motions, had devised a completely different solution,[2] and his system of concentric spheres, modified by Menaechmus and Callippus, was adopted by Aristotle. It is hopelessly inferior to the system of epicycles and can be proven fallacious by many observations.[3] Eudoxus, however, was not just some crackpot theorist; he was one of the most brilliant mathematicians of ancient times.[4] The fact that he devised this system at all, and that it could hold its place for at least a generation, is only comprehensible if the system of epicycles is a later invention. We do not know its author, but in any case Apollonius of Perga was using it about 200 B.C.[5] As we can readily understand, it drove the Eudoxan system so completely from the field that the latter is never mentioned by Ptolemy, and it is only the exigencies of commenting on the works of Aristotle that have preserved the knowledge of it for us. Eudoxus' solution was incomplete and was superseded, but, as the first attempt to explain the movements of the planets in a mathematical way, it was an astounding achievement. Eudemus attests explicitly that Eudoxus was "the first" to apply himself to such "hypotheses."[6]

Thus both external attestation and internal evidence provide a refutation, in advance, of all the attempts, ancient[7] and modern,[8] to

[2] The standard study is G. V. Schiaparelli, *Le sfere omocentriche di Eudosso, di Callippo e di Aristotele* (Milan, 1875; Ger. tr. by W. Horn: Leipzig, 1877). See esp. Heath, *Aristarchus* 190ff and, for an attempt to reconstruct the method of Eudoxus himself, Becker, *MD* 80ff. For Menaechmus and Callippus, see Dercyllides ap. Theo Sm. 201.25.

[3] In the first place, the changes in size and brightness of the planets made it obvious that there were changes in their distance from the earth, which would not be possible according to Eudoxus' system (cf. Arist. fr. 211, Sosigenes ap. Simpl. *Cael.* 505.10ff, Heath, *Aristarchus* 208ff, Neugebauer, *ExSc* 154).

[4] Eudoxus' theory of proportion (Euclid, book 5) is highly esteemed by modern mathematicians; see Hasse–Scholz 13ff; Becker, *MD* 104ff; van der Waerden, *SA* 187–189.

[5] Ptol. *Synt.* 12.1 = Apollonius fr. 59 Heiberg. Cf. Heath, *Aristarchus* 266f; van der Waerden, *SA* 238; Neugebauer, *ExSc* 155.

[6] Fr. 148. Sosigenes, ap. Simpl. *Cael.* 504.17ff, attests that until the time of Autolycus (a little earlier than Euclid) no explanation had been found for the differing distances of the planets. Cf. also Simpl. *Cael.* 32.16–27.

[7] It seemed self-evident to the Platonist that Plato knew all the correct answers, at least to any important question. See Dercyllides ap. Theo Sm. 201.7ff, Adrastus ibid. 188.25ff (the only controversy was whether we should employ eccenters or epicycles; and Apollonius had already shown that these amount to the same thing mathematically; above, n. 5). Iamblichus, on the other hand (Procl. *In Tim.* III 65.7), and Proclus following him (*In Tim.* III 96.27ff, 146.14ff) reject the complicated tangle of circles altogether; they are only interested in the metaphysical answers, not in a mathematical description of phenomena.

[8] A recent champion of this view is van der Waerden, *Astr.* 37ff; it is extended by Becker, *RhM* 97 (1954) 89–92. The main text is *Tim.* 38 c–d: the god created sun, moon, and planets as "instruments of time"; *ἔθηκεν εἰς τὰς περιφοράς, ἃς ἡ θατέρου*

find that the system of epicycles is alluded to as early as Plato, and therefore during the lifetime of Eudoxus.[9] Plato's text can in fact be understood without the epicycle hypothesis. To be sure, the ancient

περίοδος ἥειν, ἑπτὰ οὔσας ὄντα ἑπτά, σελήνην μὲν εἰς τὸν περὶ γῆν πρῶτον, ἥλιον δὲ εἰς τὸν δεύτερον ὑπὲρ γῆς, ἑωσφόρων δὲ καὶ τὸν ἱερὸν Ἑρμοῦ λεγόμενον εἰς [τὸν] τάχει μὲν ἰσόδρομον ἡλίῳ κύκλον ἰόντας, τὴν δὲ ἐναντίαν εἰληχότας αὐτῷ δύναμιν. [τόν], though securely attested, was deleted by Burnet. (MS y and older editions have τούς). Taylor (*Tim.* 196) and Cornford (*Tim.* 105 n. 2) make εἰς govern an understood κύκλους, and van der Waerden translates (*Astr.* 45) (the god put Venus and Mercury) "in 'circles' which themselves go in circles" ("in solche 'Kreise,' die einen Kreis gehen"), and calls this "a very pregnant expression for epicycles" (46). Becker saw that the expression is not at all pregnant—the decisive point, the introduction of new circles, cannot depend on a word to be supplied!—and therefore corrected Plato's text to read εἰς [τὸν] τάχει μὲν ἰσόδρομον ἡλίῳ κύκλον, ‹κύκλους δὲ› ἰόντας τὴν [δὲ] ἐναντίαν εἰληχότας αὐτῷ δύναμιν. The manuscript text, however, can be understood without alteration, though there is a slight anacoluthon. Instead of the phrase we should expect, εἰς τὸν τρίτον τε καὶ τέταρτον κύκλον, attention is centered on the description of the peculiar orbits of Venus and Mercury, using the two participles ἰόντας and εἰληχότας, and κύκλον is taken ἀπὸ κοινοῦ both with εἰς τὸν and as inner object to ἰόντας. "(The god put) the morning star and the one that is called sacred to Hermes into the circle that in speed is equal to that of the sun—this is the circle they travel; but they are endowed with the power contrary to the sun's." It is true that the puzzling ἐναντία δύναμις would have a precise sense in the epicycle theory (the epicycle of the sun revolves to the right, that of Venus and Mercury to the left; Adrastus ap. Theo Sm. 175.13ff, Chalc. 109, van der Waerden, *Astr.* 46; the phrase is discussed by Duhem 58f, Heath, *Aristarchus* 165ff, *Math.* I 311, Taylor, *Tim.* 196f). But in Eudoxus' system, too, the fourth sphere of a planet moves contrary to the third, whereas the sun has only 3 spheres. The expression can also be understood in a quite general sense, like the phrase κατὰ τἀναντία ἀλλήλοις ἰέναι, which was used of all the planets a little earlier (36d; Cornford, *Tim.* 80ff, 106ff).—It is impossible to be sure that the sentence exhibits a terminological differentiation of περιφορά and κύκλος as main circle and epicycle (as van der Waerden, *Astr.* 46); at 36c κύκλος, φορά, and περιφορά are completely synonymous.—Another argument is made from the "breadth" of the "whorls" in *Rep.* 616e (van der Waerden, *Astr.* 42ff). The text and interpretation were very much debated in antiquity (Procl. *In Remp.* II 227ff). Dercyllides seems to be the one who first interpreted the whorls in the light of the theory of epicycles or eccenters (Theo Sm. 201.7ff; cf. above, n. 7). Dercyllides was a Platonist who wrote 11 books on Plato's philosophy (Simpl. *Phys.* 247.30ff), after Hermodorus and Eudemus (ibid.) but apparently earlier than Thrasyllus (Albinus 4, p. 149 Hermann; see *RE* V 242). The refutation of this version depends not so much on the fact that the series of widths "does not, to be sure, yet" agree (van der Waerden, *Astr.* 44), as on the fact that the widest ring is that of the heaven of the fixed stars, which cannot have anything to do with epicycles. The simplest and most obvious interpretation is still that in a tentative way the distances of the planets from each other are set to correspond to their differences: the largest interval is that of the sphere of the fixed stars; Mars, Jupiter, and Saturn form a group, as do Venus and Mercury. Jupiter and Saturn are most similar, as can be seen, e.g., in Frank's sketch (27). See also Tannery, *Astr.* 327, *MSc* VII 49ff; Heath, *Aristarchus* 111, 156.

[9] Van der Waerden, *Astr.* 48 tries the expedient of supposing that the theory of epicycles was first devised for the sun, Venus, and Mercury; but his own theory of the spindle whorls will not fit this. And can we imagine that Eudoxus, instead of taking the natural step of extending the theory of epicycles, proposed his own very different scheme, applying it, against all common sense, to Venus and Mercury? In fact the ἱπποπέδη (the figure-eight curve which is the product of Eudoxus' theory) was surely devised to explain the phenomena of "overtaking and being overtaken" observed in Venus and Mercury. See also below, n. 61.

tradition boldly asserted that Pythagoras himself was the inventor of epicycles, or eccenters[10]—one of the most horrendous examples of anachronism in the construction of a science for Pythagoras. The system of epicycles was worked out by professional Greek astronomers between Autolycus (fl. ca. 310 B.C.) and Apollonius of Perga and is wholly irrelevant to the question of pre-Platonic Pythagoreanism.[11]

In Plato's *Laws* astronomy is endowed, more than any other science, with a religious function. The study of astronomy does not lead to atheism, as was thought in the day of Anaxagoras, but is "dear to the god" (821a). To be sure, its conclusions contradict popular opinion; it is false that sun, moon, and stars "never follow the same path," and the appellation "planets" is wrong:[12] τὴν αὐτὴν γὰρ αὐτῶν ὁδὸν ἕκαστον καὶ οὐ πολλὰς ἀλλὰ μίαν ἀεὶ κύκλῳ διεξέρχεται, φαίνεται δὲ πολλὰς

[10] Adrastus ap. Theo Sm. 150.12ff (cf. Chalc. 77f): ἡ δὲ ποικίλη τῆς φορᾶς τῶν πλανωμένων φαντασία γίνεται διὰ τὸ κατ' ἰδίων τινῶν κύκλων καὶ ἐν ἰδίαις σφαίραις ἐνδεδεμένα καὶ δι' ἐκείνων κινούμενα δοκεῖν ἡμῖν φέρεσθαι διὰ τῶν ζῳδίων, καθὰ πρῶτος ἐνόησε Πυθαγόρας, τῇ κατὰ ταὐτὰ τεταγμένῃ ἁπλῇ καὶ ὁμαλῇ φορᾷ κατὰ συμβεβηκὸς ἐπιγινομένης τινὸς ποικίλης καὶ ἀνωμάλου κινήσεως. "The apparent variety in the movements of the planets results from the fact that, though in reality they are bound to certain circular paths of their own and certain spheres of their own, they seem to us to follow a course through the zodiac, as Pythagoras was the first to notice; thus to their regular, simple, and uniform movement there is added, accidentally, a kind of variegated and nonuniform movement." This unquestionably refers to the theory of epicycles and eccenters (cf. Theo Sm. 152.2ff, 154.12ff); and it can be seen from the parallel report in Chalcidius that the whole thing, not merely the zodiac, is intended to be claimed for Pythagoras. Heath (*Aristarchus* 50f) translates too loosely, so that the reference to the epicycle system disappears and he is led to accept the passage as historical evidence.—Procl. *Hypotyp.* 1.34: καὶ τοῖς κλεινοῖς *Πυθαγορείοις*, ὡς ἐκ τῆς ἱστορίας παρειλήφαμεν, αἱ τῶν ἐκκέντρων καὶ ἐπικύκλων ὑποθέσεις ἥρεσκον ὡς ἁπλούστεραι τῶν ἄλλων ἁπασῶν—δεῖν γὰρ ἐπ' ἐκείνων καὶ αὐτὸν παρακελεύεσθαι τὸν Πυθαγόραν ζητεῖν ἐξ ἐλαχίστων καὶ ἁπλουστάτων ὑποθέσεων δεικνύναι τὰ ζητούμενα . . . Cf. Procl. *In Remp.* II 230.2ff. Iam. *VP* 31: δι' ὃν (sc. *Πυθαγόραν*) περὶ . . . ἐκκεντροτήτων καὶ ἐπικύκλων . . . ὀρθή τις καὶ ἐοικυῖα τοῖς οὖσι παρεισῆλθεν ἔννοια . . . According to Simpl. *Cael.* 507.12ff, Nicomachus "and after him, Iamblichus" attributed to Pythagoras only the theory of eccenters. Geminus *Is.* 1.19ff only attributes to the Pythagoreans, directly, the postulate of uniform circular movement; but the proper solution is here too, of course, the theory of epicycles and eccenters.

[11] Of course van der Waerden (*Astr.* 37ff) traces the epicycle theory allegedly present in Plato to Pythagoreans. "Pythagoreans . . . contemporaries of Plato or a little later," was the verdict of Schiaparelli (*Vorl.* 60ff; similarly Duhem 433). Such are the expedients used to rescue some bit of credibility for the testimonia about Pythagoras the scientist. But after Plato and Archytas there was no longer any specifically "Pythagorean" astronomy, but only the general Greek professional astronomy. It may have been Heraclides Ponticus who first postulated epicycles, for Venus and Mercury, though the interpretation of his fr. 109 is controversial (for epicycles: Schiaparelli, *Vorl.* 52ff; Heath, *Aristarchus* 255ff, Duhem 406ff; differently Frank 211f, followed by Wehrli and van der Waerden, *Astr.* 62ff). Perhaps Heraclides referred to Pythagoras (above, ch. I 3, n. 77), though it may not have been till after Hipparchus that Pythagoras was given credit for this discovery (cf., on Aët. 4.13.9, below, ch. VI 1).

[12] In later times it became a commonplace that the word "planet" is a misnomer. See Plin. *HN* 2.12.

φερόμενον . . . (822a). Twice more Plato returns to this judgement, so important and so satisfying to him, which, as he thought, refuted the mechanistic interpretation of the world and which makes it possible "to come, with λόγος, to the aid of the ancient Law" (890d). Of all kinds of movement, the one most closely related to mind (νοῦς) is uniform, circular movement; it is perfect (898ab), and the heavenly bodies move in conformity with it, not in any "crazy and disorderly" way (897d). "If they did not have souls, if they did not possess intelligence, they would never, in so exact a way, hold to such wondrous calculations" (967b). Plato represents the Athenian as saying that he heard this doctrine "neither as a young man nor very long ago" (821e), and we may apply this to Plato himself, for in the *Republic* he was still of the opinion that real exactitude was impossible in the physical world, so that the true astronomer should not depend on sense perception but busy himself with purely ideal magnitudes and movements.[13] In the *Timaeus*, too, Plato speaks without hesitation of the "wandering" of the planets.[14] Thus it is easy to suppose that, only toward the end of his life, Plato learned and welcomed enthusiastically a new theory of planetary motion which attributed the apparent irregularities to uniform circular movements.[15]

If we cast about for a planetary theory that Plato might have known, not only is the epicycle theory ineligible, but also the movement of the earth, or any system like Copernicus'.[16] The only possibility, aside from the system of Eudoxus, would be some kind of aprioristic

[13] *Rep.* 530a–b: *(ὁ τῷ ὄντι ἀστρονομικὸς) οὐκ ἄτοπον, οἶει, ἡγήσεται τὸν νομίζοντα γίγνεσθαί τε ταῦτα ἀεὶ ὡσαύτως καὶ οὐδαμῇ οὐδὲν παραπλάττειν, σῶμά τε ἔχοντα καὶ ὁρώμενα . . .*

[14] *πλάναι . . . πεποικιλμέναι δὲ θαυμαστῶς* (39d), *τὰ τρεπόμενα καὶ πλάνην τοιαύτην ἴσχοντα* (40b); but in the *Laws* (821d) the "normal" conception of the planets is a *βλασφημεῖν*. Simplicius, e.g., noticed the difference between *Timaeus* and *Laws* (*Cael.* 489.5ff). To be sure, the expressions *ἐναντία δύναμις* (*Tim.* 38d) and *ἐπανακυκλήσεις καὶ προχωρήσεις* (*Tim.* 40c) could be applied to the Eudoxan system (above, n. 8; Cornford, *Tim.* 135f).

[15] Frank's argument (202) is correct, up to this point.

[16] Rotation of the earth was seen at this point by Schiaparelli, *Vorl.* 39ff, Burnet, *EGP* 305 n. 2, and Heiberg 51.9, among others. Van der Waerden's nominee is the Philolaic system (*Astr.* 55). But neither of these systems explains the one essential question, that of the *πλάναι* of the planets. Therefore Gruppe supposed that Plato knew the heliocentric system (*KosmSyst* 151ff; more cautiously Burnet, *ThPl* 348; C. Ritter, *SBHeid* 1919 no. 19, pp. 54ff), a supposition neither provable nor likely. Certain other passages have been cited in this connection. The word *ἰλλομένην* at *Tim.* 40b was taken by Aristotle (*Cael.* 293b30ff, 296a26ff; perhaps following Heraclides, as Cherniss thinks, *Plato* 546ff) as referring to the revolution of the earth on its axis (cf. Taylor, *Tim.* 226ff; Heath, *Aristarchus* 174ff). The most ingenious solution is that of Cornford, that the earth turns in a direction opposite to that of the fixed stars, thus canceling out its movement and appearing at rest (*Tim.* 120ff; Cherniss agrees, *Plato* 554ff, with further elaboration;

postulate, completely regardless of observed phenomena, to be ascribed to the Pythagoreans.

For to suggest, as Frank does, that the "discovery of the true movements of the planets" was due to the Pythagoreans of Archytas' circle,[17] is in itself absurd. This discovery is supposed to consist of the thesis that the planets "describe geometrically perfect circular orbits according to strictly mathematical laws" (28; cf. 201). What gave the

G. S. Claghorn, *Aristotle's Critism of Plato's Timaeus* [The Hague, 1954] 74ff; van der Waerden, *Astr.* 57; Kranz, *RhM* 1957, 114ff; cf. the *ἀνελίττουσαι σφαῖραι* of Aristotle).—At *Epin.* 987b, the sphere of the fixed stars is called the true *κόσμος, ὃς ἐνάντιος ἐκείνοις σύμπασιν* (the planets) *πορεύεται, ‹οὐκ› ἄγων τοὺς ἄλλους, ὥς γε ἀνθρώποις φαίνοιτ' ἂν ὀλίγα τούτων εἰδόσιν.* Whether one accepts Burnet's conjecture (*‹οὐκ›*) or with Taylor, *Tim.* 169f, apprehends the passage as ironic, the passage would mean that the movement of the fixed star firmament which carries the planets along with it, is rejected, i.e. the earth must move (Schiaparelli, *Vorl.* 40ff; cf. J. Harward's notes [Cambridge, 1928]; and Taylor, loc. cit.). This is supposed to mean rejection of Eudoxus' system (characterizing the most brilliant astronomer of the time as a "person who knows little of these matters"!). But at 978dff, the movement of the heavens is accepted as self-evident (Cornford. *Tim.* 91 n. 1; also Heath, *Aristarchus* 184f); the text is meant seriously, without *‹οὐκ›*. The stress is on the word *ἀνθρώποις*; our human knowledge is basically inadequate; the preceding context runs, *βεβαίως δὴ διισχυρίζομαι τό γε τοσοῦτον*, and the following, *ὅσα δὲ ἱκανῶς ἴσμεν, ἀνάγκη λέγειν καὶ λέγομεν· ἡ γὰρ ὄντως οὖσα σοφία ταύτῃ πῃ φαίνεται.* —Critias 121c: Zeus summons the gods *εἰς τὴν τιμιωτάτην αὐτῶν οἴκησιν, ἣ δὴ κατὰ μέσον παντὸς τοῦ κόσμου βεβηκυῖα καθορᾷ πάντα* . . . here the text breaks off. Frank (207, 217f) read this as an allusion to the central fire, and therefore to the system of Philolaus. Certainty is not possible, since the sentence is incomplete (Cherniss, *Plato* 564). Clearchus fr. 8, in a mythical context, seems to speak of a central *'Εστία* outside the earth.—Theophr. *Phys. op.* fr. 22, *Dox.* 494 = Plut. *Quaest. Plat.* 8.1.1006c (cf. *Numa* 11): *τῷ Πλάτωνι πρεσβυτέρῳ γενομένῳ μεταμέλειν ὡς οὐ προσήκουσαν ἀποδόντι τῇ γῇ τὴν μέσην χώραν τοῦ παντός.* Add Arist. *Cael.* 293a27ff: *πολλοῖς δ' ἂν καὶ ἑτέροις συνδόξειε μὴ δεῖν τῇ γῇ τὴν τοῦ μέσου χώραν ἀποδιδόναι τὸ πιστὸν οὐκ ἐκ τῶν φαινομένων ἀθροῦσιν ἀλλὰ μᾶλλον ἐκ τῶν λόγων· τῷ γὰρ τιμιωτάτῳ οἴονται προσήκειν τὴν τιμιωτάτην ὑπάρχειν χώραν.* Speusippus fr. 41 = Theophr. *Met.* 11a22: *σπάνιόν τι τὸ τίμιον ποιεῖ τὸ περὶ τὴν τοῦ μέσου χώραν.* Frank bases on this his thesis that the Philolaic system belongs to the Old Academy (207ff); cf. Schiaparelli, *Vorl.* 36f; Burnet, *ThPl* 347f; Mondolfo, *Inf.* 451ff; van der Waerden, *Astr.* 55. The passage of Theophrastus is rejected as a misunderstanding or arbitrary interpretation by Boeckh, *KosmSyst* 144ff; Tannery, *MSc* IX 234; Cherniss, *Plato* 561ff; Cornford, *Tim.* 125ff, and others. It is certain that Theophrastus is not referring to any of Plato's writings, but to a *δευτέρα φροντίς* which was obviously made orally. Unless, like Cherniss, one entirely disbelieves in oral teaching by Plato, the impression remains that the testimony of Aristotle and Theophrastus is mutually corroborative, i.e. some members of the Old Academy defended the Philolaic system. It is just as forced to read the doctrine of a central fire at the center of the earth (see ch. III 1, n. 92) into the passage of Aristotle (as Cornford does, *Tim.* 126ff) as it is to regard his report as purely hypothetical ("if they were to proceed from a priori principles," Cherniss, *Plato* 560; against this is *οἴονται* at 293a30; Mondolfo, *Inf.* 453). It would still be possible that Theophrastus transferred to Plato an opinion that belonged to his pupils (Heath, *Aristarchus* 185ff). Anyhow, all we learn about the Philolaic system is a terminus ante quem (contra Frank 207, 278ff; van der Waerden, *Astr.* 55). Plato does not adopt a system because it is new, but because, when he thought the matter over again (*μεταμέλειν*), different aspects seemed important. Cf. the gloomy thoughts about the evil world soul in *Leg.* 896e et seq. and 906a.

[17] Frank 26ff, 201ff; following him, Moreau, *Âme* 76f, Gundel, *RE* XX 2053f.

planets their name, what drew the attention of observers to them at all, was the fact that they do not, like the fixed stars, follow perfect, uniform circular orbits, but that, compared to the latter, they "wander about." It would also have seemed noteworthy that they do not, like the sun, move regularly from west to east through the zodiac (the sun is not originally a "planet" but the paradigm of cosmic order, *ἡ τοῦ χρόνου τάξις*).[18] The first important lesson learned in Greece about the planets, namely the identity of morning and evening star,[19] means simply that Venus stands now to the right and now to the left of the sun, moves away from it for a certain distance and then back, overtakes the sun and is overtaken in its turn.[20] The stoppings, the retrograde movements, and the alterations in velocity were inherent in the very discovery of the planets; they were known from the beginning. A theory of the planets that takes no account of these matters explains nothing at all.[21]

Now Plato makes it a matter of reproach against the Pythagoreans that, at least in music theory, they value empirical data too highly, and he surely gives no indication that the ideal astronomy demanded in the *Republic* had actually been developed among the Pythagoreans.[22] Aristotle says the Pythagoreans do not go beyond what is subject to sense perception.[23] If this is so, it does not seem likely that they would have set up a postulate that contradicted all the evidence of the senses, without seeing any possibility of solution.

Geminus, on whom Proclus is obviously dependent, writes,[24]

[18] Cf. Anaximander fr. 1, Parmenides fr. 1.11ff, Heraclitus fr. 94; O. Gigon, Fondation Hardt *Entretiens* I (Geneva, 1952) 137f.

[19] Above, ch. IV 1, n. 39.

[20] Pl. *Tim.* 38d.

[21] Frank finds that with their postulate of uniform movement the Pythagoreans had "grasped the problem of the planets in its entirety, and had, in the essentials, also solved it, but it does not seem that at that time they had yet explained all those pauses, retrograde movements, alterations of velocity, and distortions of their orbital paths" (34f). In that case, however, what have they explained?—Moreau (*Âme* 77) finds that the postulate of circular motion is "le principe même de l'astronomie mathématique," and suggests that application to a specific system is not the main thing. But what is a principle that cannot be applied?

[22] *Rep.* 530f; cf. ch. V 1.

[23] *Met.* 989b29.

[24] *Hypotyp.* 1.34 (cited above, n. 10). Compare the following phrases (Geminus is given first each time): *προέτειναν οὕτω/παρακελεύεσθαι τὸν Πυθαγόραν—ὑπέθεντο ἐγκυκλίους καὶ ὁμαλὰς . . . κινήσεις/ἐξ ἐλαχίστων καὶ ἁπλουστάτων κινήσεων—ἀποδοθείη τὰ φαινόμενα/δεικνύναι τὰ ζητούμενα—τὴν γὰρ τοιαύτην ἀταξίαν οὐ προσεδέξαντο πρὸς τὰ θεῖα καὶ αἰώνια/πρέπειν γοῦν τὰς τοιαύτας (κινήσεις) τοῖς θείοις σώμασι μᾶλλον.* When Proclus adds to this, *ὡς ἐκ τῆς ἱστορίας παρειλήφαμεν*, he is not thinking of Eudemus (as van der Waerden thinks, *Astr.* 48), but Geminus himself, whom Tannery long ago recognized to be the most important source for Proclus in matters of the history of mathematics.

"The Pythagoreans were the first to approach such questions, and they assumed that the motions of the sun, moon, and planets are circular and uniform." (*Is.* 1.19, tr. Heath). But the "stationary points" of the planets are also explicitly mentioned, and in new formulation the problem of the Pythagoreans is, *προέτειναν οὕτω, πῶς ἂν δι' ἐγκυκλίων καὶ ὁμαλῶν κινήσεων ἀποδοθείη τὰ φαινόμενα* (*Is.* 1.21). Thus the Pythagoreans seek not arbitrary postulates, but the explanation of phenomena; Geminus undoubtedly, like Adrastus and later writers, considers the system of epicycles and eccenters to be Pythagorean.

Simplicius has a different account:[25]

> Eudoxus of Cnidus, as Eudemus recounts in the second book of his *History of Astronomy* and as Sosigenes repeats on the authority of Eudemus, is said to have been the first of the Greeks to deal with this type of hypothesis.[26] For Plato, Sosigenes says, set this problem for students of astronomy: "By the assumption of what uniform and ordered motions can the apparent motions of the planets be accounted for?"

Whether the statement about Plato forms part of the citation from Eudemus and deserves any credence is a controversial question; the anecdotal style and the fact that Simplicius names Sosigenes twice may well rouse suspicion.[27] But the sentence about Eudoxus, which is certainly taken from Eudemus, is itself enough to refute the assertion of Geminus. For what the latter attributes to the Pythagoreans—that they "were the first to devote themselves to such inquiries and based their account on uniform circular movements"—is attributed by Eudemus, in almost the same words, to Eudoxus. He was "the first to make use of such hypotheses," namely "what uniform, orderly, circular movements one must assume, in order to save the phenomena." What doubt can there be that Geminus, in diverging from Aristotle's pupil Eudemus, is citing a late fiction, just as the epicycle theory he cites is later in origin than the theory of Eudoxus? The explanation of the movements of the planets, and thus the outstanding achievement of mathematical astronomy, begins not with the Pythagoreans but with Eudoxus.

But it is new knowledge of which Plato speaks in the *Laws*, not a

[25] Simpl. *Cael.* 488.18ff = Eudemus fr. 148, tr. Cohen and Drabkin.

[26] Namely (Simpl. *Cael.* 488.16), *τίνων ὑποτεθέντων δι' ὁμαλῶν καὶ τεταγμένων καὶ ἐγκυκλίων κινήσεων δυνηθήσεται διασωθῆναι τὰ . . . φαινόμενα.*

[27] The sentence about Plato was deleted by Hultsch (*RE* VI 939) as a conjecture of Sosigenes; it is defended by Friedlaender, *Plato* I 353 n. 15.

postulate without a solution. Just as, in the *Timaeus*, he mentions the "overtaking and being overtaken," the "retrogradations and advances" of the planets,[28] he cannot, in his later book, regard them as either nonexistent or unimportant; instead, he emphasizes the "exactitude" with which the heavenly bodies follow the "calculations" (967b); this is "not easy to understand, but not extremely hard, either" (821e). Since the matter is "proven," it is to be incorporated into the educational system (822c). *Μαθήματα* are "prerequisite" to instruction in astronomy (967e).[29] The mathematical theory of the planets which Plato knows in the *Laws* can therefore only be that of Eudoxus.[30] There is no chronological impediment to this conclusion;[31] in content the allusions in the *Laws* are compatible with Eudoxus' system,[32] and there is plenty of evidence for a relationship between Eudoxus

[28] Above, n. 14. Also, the word *μιμήματα* (*Tim.* 40d) obviously refers to a mechanical model to illustrate the matter (Rivaud, *Rev. hist. philos.* 2 [1928] 4ff; Taylor, *Tim.* 244f).

[29] Cf. also *Epin.* 990a–c.

[30] That Plato was referring to Eudoxus seemed self-evident to Wilamowitz, *Platon* I 502; Bidez, *Eos* 150; W. Schadewaldt, "Eudoxos von Knidos und die Lehre vom unbewegten Beweger," *Satura: Festschr. Weinreich* (Baden-Baden, 1952) 105 n. 5; not to Lasserre (181–182, 270).

[31] On the chronology of Eudoxus, see Jacoby, *Apollodor* 314ff; von Fritz, *Philologus* 85 (1930) 478–481; G. de Santillana, *Isis* 32 (1940) 248–262; Merlan, *Studies in Epicurus and Aristotle* (Wiesbaden, 1960) 98ff; Lasserre 137–139. Even with the generally accepted later dating (ca. 395–342 B.C.), not only *Leg.* 821 but the *Timaeus* could well be referring to Eudoxus (above, n. 8; see also O. Raith, *Philologus* 111 [1967] 33). Lasserre's assertion that "ein System kreisförmiger, regelmässiger Plantenbahnen nicht erst von Eudoxos . . . ausgearbeitet wurde" (270) is not based on any evidence and contradicts Eudemus.

[32] Plato's statement that each planet traverses "the same path, not in many but in one only" (822a) has been understood to exclude not only the *ἕλιξ* of *Tim.* 39a (the overlapping of daily movement and regression in relation to the zodiac) but also the system of Eudoxus, in which 3 or 4 circular movements overlap (Burnet, *ThPl* 347; Taylor, *Tim.* 210f, 231; van der Waerden, *Astr.* 55, and others; this is the reason for the efforts to prove that Plato believed in rotation of the earth or a heliocentric system; see above, n. 16). Of course the theory of epicycles and eccenters would be affected in just the same way; and, since the Copernican system does not enter into the question, we would have to renounce even speculating what Plato meant precisely (Taylor, *Tim.* 232). The tradition of antiquity thought of both the epicycle system and that of Eudoxus as responsive to the Pythagorean postulate. Theo Smyrnaeus says (181.4ff), *τὰ ἐν αὐταῖς (ταῖς σφαίραις) ἄστρα τῇ τούτων ἁπλῇ καὶ ὁμαλῇ κινήσει φερόμενα κατὰ συμβεβηκὸς αὐτὰ δοκεῖν συνθέτους καὶ ἀνωμάλους καὶ ποικίλας τινας ποιεῖσθαι φοράς.* This evaluation is of course correct. The naive conception which Plato rejects is characterized by formulations like *οὐδέποτε τὴν αὐτὴν ὁδὸν ἰέναι* (821b), *οὐδέποτε ἰόντας εἰς τὸν αὐτὸν δρόμον ἀλλὰ πάντῃ πλανωμένους* (821c), *πλανᾶταί ποτε* (822a), *πολλὰς (ὁδοὺς διεξέρχεται)* (822a). This is the original idea, that the planets do not have fixed courses, but move at random; they are "wandering" stars, "rams," "hounds of Persephone." In Eudoxus' system, as with the epicycles, a planet does not move faster or slower from time to time, nor sometimes forward and sometimes back, but with a uniform velocity and direction, in a circle which itself is moving, but just as regularly and in a perfectly circular course. Thus the planet does have a specific, uniform path, governed by a uniform mathematical law and repeating itself with mathematical regularity. This is what Plato means.

and Plato.[33] The astronomical conception which became the basis of the world view of Plato and Aristotle and indeed dominated people's ideas of the world and their attitude to it until the time of Galileo—the differentiation of the eternal and inalterable order of the heavens from the chance and confusion of terrestrial events—comes not from Pythagoreans but from Eudoxus. Plato did not, in a dogmatic spirit, adopt a certain early Pythagorean system; he was au courant with the scientific activity of his time, and was capable of appropriating its most recent results for his own use.

Eudoxus is listed as a pupil of Archytas, in geometry,[34] and is included by Diogenes Laertius among the Pythagoreans. With respect to the indisputably original achievements for which Eudoxus became famous in his lifetime,[35] his supposed membership in the school has little meaning. We know little about Archytas' astronomy.[36] Circular movement was for him the "natural movement," which contains the "proportion of equality"; this is why circles and round bodies occur in nature—and this is applied, in the passage we have, to organic nature.[37] But Archytas was concerned with the analysis of curves produced by a moving body,[38] and it is tempting to suspect the influence of Archytas, the "inventor of mechanics" (D.L. 8.83), in the celebration of the marvels of the circle in the Aristotelian *Mechanics* (847b15–848a19), and the reduction of mechanical problems to the relationships of larger and smaller circles.[39] He had exerted an influence on Plato's thought as early as the *Gorgias*.[40] Eudoxus went on from there; but the admiration of the perfect circle takes us back to a much earlier period. Alcmaeon spoke of the imperfect circle, of the failure to join beginning and end which is the cause of a man's death (fr. 2),

[33] See D.L. 8.86ff; Jaeger, *Arist.* 15 (= 16f Eng. ed.); Lasserre 141–142, 254–256.

[34] D.L. 8.86 Callimachus is cited here, though perhaps only for the fact that he studied medicine with Philistion (see Pfeiffer on Callim. fr. 429).

[35] D.L. 8.88 mentions a decree in his honor by his native city of Cnidus, and records (8.91) that people called him *Ἔνδοξος*.

[36] There is a vague allusion in Hor. *Carm.* 1.28; Prop. 4.1.77. An isolated sentence of Philoponus (*Aet. mundi* 522.20 Rabe) about the *πέμπτον σῶμα τὸ ὑπ' Ἀριστοτέλους καὶ Ἀρχύτου εἰσαγόμενον* may be from Aristotle's *Archyteia* (above, ch. III, 1 n. 95).

[37] Archytas A23a = (Arist.) *Pr.* 915a25: *διὰ τί τὰ μόρια τῶν φυτῶν καὶ ζῴων, ὅσα μὴ ὀργανικά, πάντα περιφερῆ; . . . πότερον, ὡς Ἀρχύτας ἔλεγε, διὰ τὸ ἐν τῇ κινήσει τῇ φυσικῇ ἐνεῖναι τὴν τοῦ ἴσου ἀναλογίαν (κινεῖσθαι γὰρ ἀνάλογον πάντα), ταύτην δὲ μόνην εἰς αὐτὴν ἀνακάμπτειν, ὥστε κύκλους ποιεῖν καὶ στροργύλα, ὅταν ἐγγένηται;* Cf. Frank 378f.

[38] Above, ch. I 3, nn. 98–99.

[39] F. Krafft in *Antiquitas Graeco-Romana ac tempora nostra: Acta congr. intern. habiti Brunae 1966* (Prague, 1968) 537–539; *Dynamische und statische Betrachtungsweise in der antiken Mechanik* (Wiesbaden, 1970).

[40] Above, ch. I 3, n. 156.

as well as of the eternal circular movement of the divine stars (A12). A Pythagorean *acusma* says that circle and sphere are the most beautiful shapes.[41] But even before Pythagoras, in Anaximander, the marvelous properties of the circle keep the earth in equilibrium; the cycles of day and year are even older ideas; and Homer himself speaks of the "sacred circle."[42] Thus in the postulate of uniform circular movement, which formed Eudoxus' point of departure for the solution of his problem, there is a reminiscence of more ancient speculations. Thinking of the *acusma* and the evidence of Archytas, we may be justified in seeing a Pythagorean inspiration here. But Eudemus' testimony precludes us from supposing that Pythagoreans earlier than Eudoxus had succeeded in applying this concept to the apparent irregularities in the planetary orbits and thus become the founders of mathematical astronomy. Pre-scientific inspiration is in a sense the matrix of science, but the two should be neither confused nor equated.

Except for certain stationary points and retrogradations of the planets, the sun, the moon, and the planets seem to make the circuit of the zodiac, each in its respective period, traveling from west to east. This difference from the uniform east-to-west movement of the fixed stars was interpreted in antiquity in two ways. Some of the older natural philosophers spoke of the planets as "getting left behind" (*ὑπολείπεσθαι*) by the fixed stars in the all-embracing cosmic revolution —for example, Anaxagoras, Hippocrates of Chios, and Democritus.[43] Others thought of them as having a contrary movement of their own from west to east, in spite of which they are carried along with the general movement of the heavens, like ants crawling the wrong way on a potter's wheel.[44] This theory of contrary movement is found in Alcmaeon,[45] Oenopides,[46] and Euripides.[47] Plato rated this theory very

[41] Above, ch. II 4, nn. 18, 23. Cf. Pl. *Tim.* 33b, Arist. *Cael.* 286b10ff, *Hypomn.* 25, Ocellus 1.15, etc.; also Empedocles fr. 27.4 = 28.2.

[42] *Il.* 18.504. Cf. the circle of slabs around the shaft graves at Mycenae. Also, above ch. I 3, nn. 144–145.

[43] Anaxagoras A78, Hippoc. DK 42.5 = Arist. *Mete.* 343a5ff, Democritus: Aët. 2.16.1 and A88 = Lucr. 5.621ff; Geminus *Is.* 12.14ff.

[44] This comparison is made by Cleomedes (1.3) and others.

[45] A4 = Aët. 2.16.3. "Contrary movement" is given as the doctrine of the *mathematici*, meaning professional astronomers, with the later addition, *συνομολογεῖ καὶ Ἀλκμαίων* (above, ch. IV 1, n. 64).

[46] DK 41.7 = Macrob. *Sat.* 1.17.31. Here the reference is only to the sun.

[47] Fr. 861: *δείξας γὰρ ἄστρων τὴν ἐναντίαν ὁδόν* . . . (perhaps words of Atreus from the *Thyestes*).

high and thought it was only the one that explained correctly the velocities of the planets: Saturn is not the most rapid of them because he is left behind least by the fixed stars, but the slowest, because he is least able to free himself from their influence.[48]

The theory of contrary movement is regarded as specifically Pythagorean and as a great advance over the Ionian theory. This is inferred from Plato and Alcmaeon, whose evidence seems to take us back to Pythagoras himself.[49] But the relation of the two theories is more complex.

Oenopides was an Ionian, a pupil of Anaxagoras;[50] so that we hardly ought to speak of a fundamental opposition between Pythagorean and Ionian astronomy. And, if Euripides could allude in a play to the "contrary course" of the stars, we may be sure that the theory was also known to Democritus; but he saw no occasion to adopt this "advanced" view.

Actually there is no difference, from a purely descriptive point of view, whether one speaks of "getting left behind" or of "contrary movement," as long as attention is confined to explaining the apparent fact, that is the seeming displacement in the zodiac, which is inclined with relation to the celestial equator. Hippocrates of Chios,[51] and thus doubtless the other advocates of the retardation theory, did not maintain that this *ὑπολείπεσθαι* took place on parallel circles to the celestial equator. No one can deny that the sun moves further to the north in the summer and to the south in the winter. Naturally Anaxagoras and Democritus knew about the movement of the sun—and the planets—in the zodiac.[52]

The dicovery that was crucial for the description of the movements of the planets was that of the zodiac. According to reliable tradition its twelve signs were introduced to Greece by Cleostratus of Tenedos, a pupil of Anaximander,[53] after Anaximander himself had spoken of the

[48] *Tim.* 38e, *Leg.* 822a et seq.; cf. *Rep.* 617a, *Epin.* 987b.

[49] Tannery, *HScH* 214; Burnet, *EGP* 110f (Pythagoras himself); Heath, *Aristarchus* 50; van der Waerden, *Astr.* 27.

[50] Above, ch. IV 1, n. 79.

[51] Arist. *Mete.* 343a8 (about the comet in Hippocrates' theory): *ὑπολείπεσθαι δ' αὐτὸν καὶ πρὸς ἄρκτον καὶ πρὸς νότον* . . . Geminus *Is.* 12.19 objects, against this theory, that the *ὑπολείπεσθαι* would have to take place in parallel circles.

[52] For the solstices there had to be a secondary cause, along with the vortex. Anaxagoras found it in winds (A72), following Anaximander (A27) and Anaximenes (A15). Democritus' answer to this problem is not clearly reported (A89).

[53] DK 6. The principal evidence is that of Plin. *HN* 2.31 = DK 6B2. See Kroll, *RE* Supp. IV 912f.

oblique wheel of the sun.[54] According to a controversial report, Meton knew of the measurement of the ecliptic in degrees.[55]

The oldest Babylonian text yet known that refers to the "signs" of the zodiac, not to constellations, is a horoscope from the year 410 B.C.[56] Van der Waerden takes expressions like "at the end of Pisces," which occur some decades earlier, as evidence for the introduction of the twelve signs before that date, in place of the ancient names of constellations.[57] For even centuries earlier than this, people had marked out the "path of the moon" in the sky and already discovered that the sun, too, and the five planets, travel the same route.[58] In expressions about the "path" of the sun or moon is reflected the conception that is natural for the unsophisticated observer, that the sun and moon, and basically also the planets, make their way from constellation to constellation in an eastward direction. One disregards, for the moment, the daily movement of the fixed stars—which is easy if one's observations are made at approximately the same time of night—and discovers the peculiar movements of the other stars. They seem like living creatures of a higher kind than ours, which move freely through the heavens—whether one calls them "rams," "hounds of Persephone," or gods.

The conception of the independent movement of sun, moon, and planets, their heavenly journey from west to east, is accordingly earlier than the vortex theory of Anaxagoras and Democritus and is rooted in naive observation and interpretation of the skies. Anaxagoras, Hippocrates, and Democritus do not give less than this; what they want to do is to contribute more. In place of unexplained, spontaneous movement they posit a universal force, which works with the power of necessity. As the cosmos is divested of its anthropomorphic features, the stars too must become things, governed by the *ἀνάγκη φύσεως*.

[54] Anaximander A5, 22. Cf. Tannery, *HScH* 96f. On the relation of this to the report on Oenopides, see above, ch. IV 1, n. 38.

[55] Columella 9.14.12: Hipparchus placed solstices and equinoxes in the first degree of the appropriate zodiacal sign, but the author follows Eudoxus and Meton, who put them in the eighth ("Eudoxi et Metonis antiquorumque fastus astrologorum"). Cf. Lasserre, *Eudoxos* 222–224, though his citation of the Babylonians is out of date. Van der Waerden, *Anf.* 171, uses the Meton reference to establish a terminus ante quem for dating the Babylonian "system B."—Eur. fr. 755 N., from the *Hypsipyle*, alludes to the zodiac.

[56] A. Sachs, *J. of Cuneiform Stud.* 6 (1952) 49; Neugebauer, *ExSc* 187; van der Waerden, *Anf.* 247.

[57] *Anf.* 124–125 (differently Neugebauer, *ExSc* 140); cf. 98–99, 132, on texts of the 6th century which still are concerned with stars rather than "signs." Van der Waerden dates the Babylonian "system A" of planetary calculation to the time of Darius I (*Archive for the Hist. of the Exact Sciences* 5 [1968] 70–78; cf. *Anf.* 171; above, n. 55.)

[58] The series Mul-Apin, van der Waerden, *Anf.* 77–79.

The retardation of the planets answers the attempt to replace arbitrary movement with movement according to natural necessity. The difference between the two theories, then, of retardation and of contrary movement, comes to a difference between physical explanation and mere description, whether naive or mathematical. The transition from one to the other is not simply a forward step; rather, the line of development turns back, on a higher level, to an earlier stage. The rash conjectures of the Ionians on physical matters led into a blind alley. The peculiarities of the planetary movements could not be explained by the force of the cosmic whirl, and still, as people were learning, the phenomena in question were regular and periodic. The only way open to progress in astronomy was to abandon physical explanations based on the necessary laws of movement and to adopt purely mathematical description. The result was the Greek mathematical theory of planetary motion, a tremendous achievement. It was not possible, however, to find one's way back from its complexities to simple physical laws; so that, from Aristotle's time on, the two-world theory was dominant, regarding the realm of the heavens as wholly different from and foreign to ours. Only with Galileo and Newton did astronomy once more, from the heliocentric standpoint, align itself with physics. Plato thought it was an inescapable conclusion that the orderly movement of the stars is due to beings with souls;[59] it is a voluntary, chosen order. Here sophisticated Greek science harks back to the pre-scientific way of thinking and comes to rest in it.

If Alcmaeon thought the stars were divine, he must have ascribed to them a movement of their own; there is no advanced astronomical theory involved here, as against Anaximander or Cleostratus—in general it is obvious that Alcmaeon's astronomical views are dependent on the Ionians.[60] Thus Alcmaeon's statement is no evidence for scientific advances attributable to Pythagoras. It was of course possible for Eudoxus to take his departure from the idea of contrary movement,[61] and, following out the principle in a mathematical way, to lay the basis of the descriptive, though no longer physical, astronomy of the Greeks.

[59] Pl *Leg.* 888e, 967a. Dercyllides ap. Theo Sm. 201.20 has the phrase *κίνησις προαιρετική*.

[60] The sun is "flat": Anaximenes A7 §4, A14–15; Alcmaeon A4. The moon is "boat-shaped": Heraclitus A12, Antiphon fr. 28 (cf. Frank 188 n. 1); Alcmaeon A4.

[61] In the Zodiac, there were to be seen two opposite circular movements, whose axes are oblique to one another. Their overlapping produces the *ἕλιξ* of Pl. *Tim.* 39 a–b. Archytas may have developed the idea this far. Eudoxus may have wondered what the result would be if the two movements were equally rapid, thus being led to his *ἱπποπέδη*.

Alexander has a report, in the context of discussion of the harmony of the spheres, about the Pythagoreans' doctrine of the planets. This is taken from Aristotle's special book on the Pythagoreans. They thought, he says, that the distances between the planets, and therefore their velocities and the tones they make, are according to harmonic ratios, and that those which move at the greatest distance move the most rapidly (*κινεῖσθαι δὲ τάχιστα μὲν τὰ τὸ μέγιστον διάστημα κινούμενα*).[62] Thus for these Pythagoreans Saturn is fastest, next to the sphere of the fixed stars, and the moon is the slowest—the very view which Plato ridicules as an absurd blunder. We cannot assume that Alexander has made a mistake, for he repeats his statement with the notation, *ὡς προείρηκε* (*Ἀριστοτέλης*). And in fact this conception fits the idea of harmony of the spheres better than the one Plato favors; in the latter the sphere of the fixed stars, which moves fastest, is followed by Saturn, the "slowest" of the heavenly bodies, whereas in the former, retardation provides for a progressive gradation—a necessity for the connection with the musical scale.[63] Far from simply taking over a Pythagorean system, Plato hits the Pythagoreans, as well as Democritus, with his criticism. There is one way out, as always: to posit a Pythagorean astronomy anterior to or foreign to Philolaus, and claim that Plato is dependent on this; but there is no foundation for such an assumption. In any case, the idea of contrary motion is not unanimously held, even among the Pythagoreans.

Plato's astronomy is not a copy of any Pythagorean system. To be sure, the idea of the harmony of the spheres is Pythagorean, as is, according to Eudemus, his planetary order, though here the only difference from Democritus is in the position of Venus. There is probably also a reflection of Pythagoreanism in his admiration of the circle. Beyond this, Plato takes his place in the lively discussion of his time, which could look back to almost a century and a half of Greek astronomical speculation—an Ionian tradition, Anaximander to Cleostratus and Anaxagoras to Oenopides. Plato takes a polemical stance toward Anaxagoras and Democritus, because for him not physical causality but logico-mathematical relationship is decisive. The earth's ability to hang free in space, and also the movement of the planets, are comprehensible on grounds of mathematical order,

[62] Alex. *Met.* 40.7 = Arist. fr. 203; 41.5: *κινεῖσθαι κατὰ ἀναλογίαν τῶν διαστημάτων, ὡς προείρηκε*. Cf. Arist. *Cael.* 290b21, on the music of the spheres, *τὰς ταχυτῆτας ἐκ τῶν ἀποστάσεων ἔχειν τοὺς τῶν συμφωνιῶν λόγους*.

[63] This is why Burnet attributes the "contrary movement" to Pythagoras himself (*EGP* 110f), but the developed system of the harmony of the spheres to a later generation (306f). Cf. below, ch. IV 4.

without any extraneous causation involved. Perhaps on this point Plato learned from Archytas. With regard to planetary movement, what he cared about was the mathematical theory, and this accomplishment must be credited to Eudoxus.

3. THE COSMOS OF PHILOLAUS

Copernicus says that he got the stimulus for his revolutionary cosmological system from ancient sources, and in this connection he twice names Philolaus.[1] And, since the time was far from past when everything new was taken as a rediscovery of something ancient, the Copernican system was current for a time under the name of *astronomia Pythagorica* or *Philolaica*[2]—an error long ago laid to rest. Nevertheless, the association with Copernicus still exerts a dominant influence in the discussion of the Philolaus fragments; so subtly developed an astronomical system cannot have been devised in the fifth century, it is thought, and certainly not by the Philolaus who, as Plato said, provided no "clear" rationale for his teachings.[3]

The doctrine in question, according to the consistent testimony of Aristotle and the doxographers,[4] was that our earth is "one of the

[1] Ed. Soc. Cop. Thorunensis (Thorn, 1873) 17, 6ff ("Philolaus Pythagoricus, mathematicus non vulgaris"), and in a passage deleted from the definitive edition, p. 34 n. See also the letter of dedication to Pope Paul III, 6.6ff. His sources were "Plutarch," i.e. Aëtius 3.13.1–3 (DK 44A21) and the report about "Nicetus," i.e. Hicetas, in Cic. *Acad. pr.* 2.123 (DK 50.1).

[2] Cf. Martin, Philolaus 128f, Schiaparelli, *Vorl.* 17ff.

[3] Van der Waerden, *Astr.* 49ff, 54. Before him Frank, especially, had sought to prove the impossibility of the system of Philolaus in the 5th century B.C. (35, 207ff). He was followed by Rehm–Vogel (47), Gundel (*RE* XX 2056f), and others. Wiersma, *Mnemosyne* 1942, 25ff, tried to prove that Philolaus did not advocate this "system of Philolaus" (above, ch. III 2, n. 33). Its origin is attributed conjecturally to Hicetas (DK 50) by Wiersma and van der Waerden.

[4] Arist. *Cael.* 293a18ff, 986a8ff, frr. 203, 204; Aët. 2.29.4 = Arist. fr. 16 Walzer; also Aët. 2.7.7, 3.11.3, 3.13.1 = Philolaus A16, 17, 21. On the relation of the "Pythagoreans" of Aristotle to Philolaus, above, ch. III 1. Aristotle and Aëtius agree not only in substance but in actual wording, so that it is not only permissible but mandatory to integrate the material included in one line of tradition but not the other into the total picture. (The "plus" in Aristotle: the creation of night and day, *Cael.* 293a22; the proportionality of distances and velocities, fr. 203, *Cael.* 290b21; the absence of parallaxes, *Cael.* 293b25ff; the counter-earth and lunar eclipses, *Cael.* 293b23, Aët. 2.29.4. The "plus" in the Philolaus doxography: fire as *περιέχον*, A16; the sun as a burning-glass, A19; the moon inhabited, A20; movement of the earth in the same direction as sun and moon, in an oblique circle; world catastrophes, A.18. Mondolfo, *Inf.* 286.2, 340ff, sees a difference in the fact that in Philolaus the world is surrounded by fire, but in Aristotle's report by *ἄπειρον, κενόν* [above, ch. I 2]; but this is a contradiction only if, like Mondolfo, one supposes that the surrounding fire is infinite in extent, and there is no good reason to do so.)

stars" and along with the moon, the sun, five planets, and an invisible "counter-earth" revolved about a "central fire." The earth a planet! This seems to anticipate Copernicus' momentous discovery, and one involuntarily regards the Philolaic system as an attempt to explain, in as clear a way as possible, certain specific astronomical observations.

Nevertheless, this interpretation leads from one difficulty to another. The system of Philolaus would be truly Copernican if it could provide an explanation for the apparent irregularities in the orbits of the planets. Frank thinks it does: "In this 'Pythagorean' or 'Philolaic' system . . . the apparent retrograde movements and pauses find their explanation . . . in quite the same way as in our modern heliocentric system."[5] For this reason, he thinks, the system must be later than Eudoxus and Heraclides Ponticus. The fact remains, however, that it was not the sun which the Pythagoreans thought occupied the central position, but the central fire; and this, like the counter-earth, is always invisible to us "because the earth is in the way."[6] It follows that the earth's circular orbit and its rotation on its own axis must be combined in such a way that the earth always exposes to the central fire the side opposite the one on which we live. This is inconsistent with the main point of the system of Aristarchus or Copernicus; for in the latter, the earth's rotation on its axis explains the daily rising and setting of the stars, and its annual revolution about the sun explains the changing seasons and the convolutions of the planetary orbits. If these two movements are inseparably connected with each other in the system of Philolaus, they can only explain one astronomical fact. Aristotle says explicitly that the earth "by its circular movement about the center creates day and night" (*Cael.* 293a22ff). Thus the period of revolution about the central fire, but relatively to the sun, is one day. This excludes the "Copernican" idea of explaining the distortions in the orbits of the planets as distortions of perspective due to the movement of the earth. The planets would have to turn retrograde in the course of a single night—unless there are in fact no appreciable parallaxes! This is the point of view taken by the Pythagoreans, according to Aristotle. Even on the geocentric hypothesis, they thought, the astronomer is not making his observation from the center, but is one

[5] Frank 38f, cf. 35ff, 207ff. Similarly Gundel, *RE* XX 2056: "their effort is to solve the (planetary) problems that arise in a mathematical way." More cautiously, Rehm–Vogel, 47. Naturally those who are experts in the subject matter, like Martin, Heath, Schiaparelli, and van der Waerden do not commit this error.

[6] Invisibility of the counter-earth because of the ἐπιπρόσθησις τῆς γῆς Arist. *Cael.* 293b22, Simpl. *Cael.* 511.34f, quoting Arist. fr. 204.

earth radius away from it; "there is no reason not to suppose, they think, that celestial phenomena are the same, even though we are not at the center," but on the earth which revolves about the central fire (*Cael.* 293b25ff). In other words, this Pythagorean system, which expressly denies a parallax as the result of the earth's movement, cannot provide any theory of planetary movement and has no intention of doing so. In this respect it is to be classified with pre-Eudoxan astronomy, in which the capricious prancings of the planets were simply taken to be inexplicable. Frank does not hesitate to assert that the central fire and the daily revolution of the earth around it are "speculative and mythical reinterpretations" of a truly Copernican system, made by the "philosophers of the Academy," and he therefore explains the traditional account as the corruption of an unattested "Pythagorean system of scientific astronomy,"[7] thus himself abandoning the realm of historical reality in favor of arbitrary speculation.

The effect of the earth's movement in Philolaus' system is the same as that of simple axial rotation. One might see in this a magnificently bold and fruitful idea: the daily rising and setting of sun, moon, and stars, central facts of human life, become illusion, caused by an unobserved movement of the earth, though it always must appear to be fixed and at rest—a triumph of thought over mere appearance. The earth moves each day, from west to east, "in the same way" as the sun and moon,[8] in an "oblique circle." Its path is therefore in the plane of the equator, that of the planets in that of the ecliptic. The planets, including the sun and moon, likewise move from west to east, but much more slowly; their angular velocity is less in proportion to their distance from the central fire.[9] If one can forget about the unexplained irregularities in the courses of the planets, this makes a system of impressive symmetry. Each celestial body has one and only one circular

[7] 207, 208.1. For the central fire Frank substitutes "den (ideellen) Mittelpunkt des ganzen Planetensystems" (35; cf. 207), about which the earth circles "in einem Jahr?" (208). This last item is taken over by Rehm–Vogel (47), but without Frank's question mark. Frank's citation of Heraclides misses its mark (below, n. 19; Wehrli 97), and his reconstruction is unsatisfactory astronomically. As long as the course of the sun remains outside that of the earth (see Frank's drawing, p. 36), there can be no adequate explanation of the phenomena. On the "ideal center" Heath says (*Aristarchus* 278), speaking of Heraclides, "It is inadmissible to suppose that, in Heraclides' time, any one could have assumed that the place in the centre of the universe was occupied by nothing and that both the sun and the earth revolved about an ideal point."

[8] ὁμοιοτρόπως, Philolaus A21. Elsewhere the expression "oblique circle" is always used of the ecliptic (Andrissi 7f).

[9] Schiaparelli, *Vorl.* 14; Martin, *Philolaus* 155.

course to complete[10] - provided, that is, that the sphere of the fixed stars is stationary. But this very proviso cannot be allowed, according to the explicit testimony of the sources! Ten bodies "dance a roundelay" about the sky;[11] nothing is stationary but the central fire, the ἑστία in their midst. However the movement of the fixed stars is imagined, they must carry the rest of the stars with them, in order to conform to observed appearances.[12] Thus the astronomical significance and special advantages of the Philolaic system have vanished. None of the attempts to devise a precise interpretation of the movement of the heaven of the fixed stars is successful.[13] As a matter of fact the confusion becomes complete only when we include the testimony of Alexander, so often overlooked, that the stars move with a velocity proportional to their distances from each other—the further away, the faster they move.[14] This means that the sphere of the fixed stars is the fastest of all, and there is no difference from the "whirl" of Democritus. There is no point in discussing the various possibilities of this system; one can only analyze motion in relation to something stable; and the point of reference in the system of Philolaus is the eternally invisible central fire.[15] Proceeding in this fashion, one can conjecturally attribute any kind of motions to the heavenly bodies, so long as they all participate

[10] This is why it is brought into connection with Pl. *Leg.* 821f (above, ch. IV 2, n. 32).

[11] Arist. *Met.* 986a10; τὰ φερόμενα κατὰ τὸν οὐρανὸν δέκα μὲν εἶναί φασιν. Arist. fr. 203 = Alex. *Met.* 41.3: τὰ δέκα τὰ κινούμενα σώματα, cf. 38.23; Philolaus A16: περὶ τοῦτο δέκα σώματα θεῖα χορεύειν.

[12] If the fixed stars moved around the axis of the ecliptic, the constellations would change position relatively to the poles and the equator; the polestar would not remain such. If the fixed stars had a course around the pole and the planets did not, the courses of the planets would be altered, with respect to the constellations, and the zodiac would no longer be the zone of planetary movement.

[13] Boeckh (118) thought of the precession of the equinoxes, and this was taken up by Gomperz (*GrD* I 93 = I 114 and 544 Eng. ed.), though Boeckh later vacillated (*KosmSyst* 93). Knowledge of the precession is not attested before Hipparchus, and is not likely. Van der Waerden, *Astr.* 54, thinks of a rotation of the sphere of the fixed stars through the ecliptic in a period of 18½ years, which is supposed to explain the retardation of the nodes of the moon. But the author of the system does not seem to have any exact knowledge about lunar eclipses (below, n. 31, and above, n. 12). Schiaparelli cut the Gordian knot by simply rejecting the evidence and postulating "absolute motionlessness of the heaven of the fixed stars" (*Vorl.* 11, 14f). Tannery, too, says (*HScH* 246) "l'essence même du système est l'immobilité de la sphère des fixes." See also G. B. Burch, *Osiris* 11 (1954) 267–294, esp. 282–285.

[14] Above, ch. IV 2, n. 62.

[15] According to the Pythagoreans, the motive power of the cosmos is localized in the central fire: Simpl. *Phys.* 1354.2, 1355.3 (probably from Alexander, and hence ultimately Aristotle).

in them in such a way that their relative displacements conform to observed facts;[16] but this is not what is usually meant by astronomy.

Things are different with the astronomical doctrines ascribed to Hicetas, Ecphantus, and Heraclides Ponticus. Though there is controversy about many details,[17] this much is clear, that Hicetas and Heraclides believed that the outer heaven was at rest and emphasized that from this hypothesis the phenomena could be explained just as well as if it were thought of as rotating.[18] Heraclides stated explicitly that the apparent position of a planet with relation to the heaven was determined by the straight line from the earth, that is, the eye of the observer, to the planet.[19] Thus he was dealing with the basic ideas of perspective and of projective geometry, the differentiation of true and apparent position, or true and apparent movement—in other words, this is mathematical astronomy. It is relevant here that Philip of

[16] On the idea of an unnoticeable, undetectable motion, cf. Martin, *Philolaus* 140ff, 155, who refers to Ptol. *Synt.* 1.7 p. 24 Heiberg; Schol. Arat. p. 91.27 Maass; Schiaparelli, *Vorl.* 13.18. The end result is about the same in the more complicated solution of Andrissi (who is followed by Timpanaro Cardini 1946, 325). He posits two motions of the earth, which run counter to each other and cancel each other out in their astronomical effect, (Cf. above, ch. IV 2, n. 16, on *Tim.* 40b).

[17] Hicetas, DK 50. (According to Heath, *Aristarchus* 189, he is to be identified with Hicetas of Syracuse, the tyrant of Leontini, friend of Dion and rival of Dionysius II. Cf. Plut. *Dion* 58, *Timoleon* 1–32.) Ecphantus, DK 51. Heraclides, frr. 104–110. We cannot enter into the controversies about the astronomy of Heraclides (see above, ch. IV 2, n. 11). In Tannery's view (*MSc* VII 249ff, IX 232ff, 253ff) Hicetas and Ecphantus are both characters in a dialogue of Heraclides (agreeing: Frank 138 and nn. 402ff; disagreeing: DK I 441 n; Mondolfo, ZM 349; van der Waerden, *Astr.* 55; cf. Wehrli 96). The conjecture is based on the naming of Ecphantus along with Heraclides in Aët. 3.12.3 (= DK 51.5 = Heraclides fr. 104) and the similarity of some kind of atomic theory (DK 51.1–4, Heraclides frr. 118–121). Theophrastus' reference to Hicetas (*Phys. op.* fr. 18 = DK 50.1) is not a positive disproof of this (contra DK; cf. above, ch. I 3); but the counter-earth, advocated by Hicetas, is never mentioned by Heraclides.—The system of Hicetas, as described by Cicero, is impossible. Not only the fixed stars but also sun and moon are to be stationary (DK 50.1). Tannery thinks Cicero made a mistake (*MSc* IX 234f), as does Duhem (23); Frank thinks Hicetas exaggerated for didactic purposes (n. 402); van der Waerden tries for a solution by slightly altering the translation (55). See also Wehrli, *Herakleides* 95.

[18] Hicetas, DK 50.1: "eadem effici omnia quae si stante terra caelum moveretur"; Heraclides fr. 108: σῴζεσθαι τὰ φαινόμενα τοῦ μὲν οὐρανοῦ καὶ τῶν ἄστρων ἠρεμούντων, τῆς δὲ γῆς . . . κινουμένης. Rotation of the earth on its axis is attested for Ecphantus and Heraclides (DK 51.5, Heraclides fr. 104–108); on the connection with Arist. *Cael.* 293b30ff, 296a26ff, above, ch. IV 2, n. 16—Schiaparelli, *Vorl.* 50f, and van der Waerden, *Astr.* 58f, are wrong in concluding from Schol. Coisl. Arist. p. 505a3 that the "more genuine" Pythagoreans taught that the earth rotates; the sentence in question is a citation of Arist. *Cael.* 293a22; cf. above, ch. III 1, n. 74.

[19] Fr. 109: ". . . solem et lunam et luciferum et omnes planetas, ubi eorum quisque sit, una linea a puncto terrae per punctum stellae exeunte demonstrari . . ."

Opus, who was mainly interested in astronomy, wrote a book entitled *Ὀπτικά*.[20]

There is nothing of this sort in Philolaus, only an invisible central fire, an equally invisible counter-earth, unobservable movements of the earth and the stars—mythology in scientific clothing, rather than an effort, in accord with scientific method, to "save the phenomena." This is just what Aristotle says of the Pythagoreans: "not seeking accounts and reasons to expain the phenomena, but forcing the phenomena and trying to fit them into arguments and opinions of their own."[21] The system of Philolaus is not a scientific astronomy,[22] and there is no call to set it late chronologically on grounds of its sophisticated and advanced nature.

There are details among the astronomical teachings of Philolaus that point toward the fifth century, and in particular his remarkable theory of the sun. The sun is a "glass-like" body which receives light and warmth from the "ethereal fire" which it then "strains through" certain "narrow interstices," so as to shine on the earth.[23] His basic

[20] *Suda* s.v. *φιλόσοφος*.

[21] *Cael.* 293a25, tr. Guthrie. Speculation and concentration on the world apprehended by the senses are two interpenetrating tendencies in pre-Socratic philosophy. It was only after Plato had made clear the dichotomy between immaterial and material that mathematics, become independent, could set about from a new basis to "save the phenomena."

[22] Wilamowitz, *Platon* II 93: "Ich fürchte, es ist von Astronomie dabei wenig zu rühmen."

[23] Aët. 2.20.12 = DK 44A19. Also Ach. *Is.* 19, a passage that Diels did not include in the *Vorsokratiker* because he thought Achilles was directly dependent on ps.-Plutarch (*Dox.* 22ff). This thesis is not tenable, and it was withdrawn (orally) by Diels himself; see Pasquali, *GGN* 1910, 221f. Achilles draws from a more detailed source, as the following parallels show:

Aët. 2.20.12 (from Stob. and Plut., supplementing each other; DK 44A19):

ὑαλοειδῆ τὸν ἥλιον, δεχόμενον μὲν τοῦ ἐν τῷ κόσμῳ πυρὸς τὴν ἀνταύγειαν,

διηθοῦντα δὲ πρὸς ἡμᾶς τό τε φῶς καὶ τὴν ἀλέαν,
ὥστε τρόπον τινὰ διττοὺς ἡλίους γίνεσθαι
τό τε ἐν τῷ οὐρανῷ πυρῶδες καὶ τὸ ἀπ' αὐτοῦ πυροειδὲς κατὰ τὸ ἐσοπτροειδές,
εἰ μή τις καὶ τρίτον λέξει τὴν ἀπὸ τοῦ ἐνόπτρου κατ' ἀνάκλασιν διασπειρομένην πρὸς ἡμᾶς αὐγήν. καὶ γὰρ ταύτην προσονομάζομεν ἥλιον οἱονεὶ εἴδωλον εἰδώλου.

Ach. *Is.* 19 p. 46.13ff Maass:

(τὸν ἥλιον) τὸ πυρῶδες καὶ διαυγὲς λαμβάνοντα ἄνωθεν ἀπὸ τοῦ αἰθερίου πυρός
πρὸς ἡμᾶς πέμπειν τὴν αὐγὴν διά τινων ἀραιωμάτων,
ὥστε κατ' αὐτὸν τρισσὸν εἶναι τὸν ἥλιον,
τὸ μὲν ἀπὸ τοῦ αἰθερίου πυρός, τὸ δὲ ἀπ' ἐκείνου πεμπόμενον ἐπὶ τὸν ὑελοειδῆ ὑπ' αὐτοῦ λεγόμενον ἥλιον,
τὸ δὲ ἀπὸ τοῦ τοιούτου ἡλίου πρὸς ἡμᾶς πεμπόμενον.

idea is obviously that of the burning-glass, which became familiar precisely in the late fifth century; Gorgias speaks of its *πόροι*, through which fire penetrates.[24] Diogenes of Apollonia seems to have had a quite similar idea: the sun is a body "like pumice stone," on which "rays from the aether concentrate."[25] Here again, the aether is the source of the sun's fire, the visible sun only a transmitter and amplifier, and the passages or "pores" of the pumice correspond to the "narrow interstices" of Philolaus. There are also other points of agreement between Diogenes and Philolaus.[26] But Empedocles as well had a peculiar theory of the sun, and, in spite of various contradictions in the evidence, it is clear that it was similar to that of Philolaus.[27] Circling about the earth, he thought, were a fiery and a dark hemisphere. The fiery one is the source of the sun's light and heat, whereas the sun we see is a phenomenon of reflection, or maybe a burning-lens. Though it is only by conjecture, at best, that a chronological order can be seen in this complex of solar theories,[28] it is at least certain that they all belong closely together. Obviously the realization that the moon has no light of its own[29] was exerting an influence in impelling people

Aëtius and Achilles supplement one another. The "mirror" mentioned by Aëtius must be an error, as *διηθεῖν* shows; so that the report of Achilles is the more reliable (Boeckh 127; Martin, *Philolaus* 135f; cf. also Tannery, *HScH* 237f; Heath, *Aristarchus* 117). For *πῦρ περιέχον* see Philolaus A16. Cf. also Guthrie I 285f.

[24] Gorg. fr. 5: *ἀπιέναι τὸ πῦρ διὰ τῶν πόρων*. Cf. Diels, *SBBln* 1884, 343ff, Theophr. *De igne* 73f (the burning-glass and burning-mirror classed together under the heading of *ἀνάκλασις*). Ar. *Nub.* 767–769.

[25] Diogenes of Apollonia A13 = Aët. 2.20.10: *κισηροειδῆ τὸν ἥλιον, εἰς ὃν ἀπὸ τοῦ αἰθέρος ἀκτῖνες ἀποστηρίζονται*. Cf. A12 and A14, on the stars and the moon. Epicurus fr. 343 Usener = Ach. *Is.* p. 46.18 Maass: *κισηροειδῆ (τὸν ἥλιον) . . . ἐκ πυρὸς διὰ τρημάτων τινῶν τὸ φῶς ἐκπέμποντα*.—The *αἰθήρ* is different from the air, according to Anaxagoras (A70, cf. frr. 1, 2, 12); it is warm, and surrounds the earth. Diogenes of Apollonia also speaks of the *θερμότης τοῦ αἰθέρος* (Ach. *Is.* p. 40.9 Maass).

[26] Cf. above, ch. IV 1, n. 112; below, n. 36.

[27] Frr. 44, A30, A56; cf. Burnet, *EGP* 238f; G. Kafka, *Philologus* 78 (1923) 212ff (p. 213: the introduction of glass in this period); Kranz, *Hermes* 73 (1938) 103, and *Emped.* 50. A56 is very close to A30 in the description of the two hemispheres, but seems to say that the sun is in the dark half of the sky. Kafka and Kranz mention the burning-glass; but it is striking that all the testimonia speak of the role of the earth in the *ἀνάκλασις* phenomenon. Can Empedocles be thinking of reflection, after all—the sun as a sort of mirror-image of earth in the sky?

[28] The fact that the doctrine of Philolaus is relatively clearest is due to Achilles' omission of the corresponding theories of Empedocles and Diogenes. Philolaus was probably younger than Empedocles. Burnet (*EGP* 298) and Heath (*Aristarchus* 90f) believed that Empedocles' theory of the sun was earlier, Zeller (I 982, 1027) the reverse.

[29] Anaximenes A16 = Eudemus fr. 145 (Tannery, *HScH* 216ff, can hardly be right in questioning the authenticity of this; cf. Gigon, *Ursprung* 108), Anaxagoras A76, Empedocles frr. 43, 46, A30; on Parmenides (frr. 15, A42) see Jaeger, *RhM* 100 (1957) 42ff.—Ion, DK 36A7, called the moon *ὑελοειδές*.

to look for foreign sources of the sunlight and starlight as well.[30] Even if it were not certain that the historical Philolaus lived in the late fifth century, we should have had to assign the theory of the sun in the Philolaic system to that period.

The report of Aristotle and Philip of Opus, that the higher frequency of lunar than of solar eclipses was explained by the presence of the counter-earth, and perhaps also other earth-like bodies in space, takes us into a similar context.[31] Sometimes the earth, and sometimes the counter-earth, shuts off the sun's light from the moon.[32] This astronomical use of the counter-earth has often been played off against Aristotle's ironical statement that the counter-earth was only invented to bring up the number of celestial bodies to ten;[33] but from an astronomical point of view, this explanation of lunar eclipses is unsatisfactory and betrays a lack of exact information.[34] In any case it is not Pythagorean in origin. Anaxagoras had invisible, dark bodies circling the earth below the moon, responsible, along with the earth's shadow, for eclipses.[35] Diogenes of Apollonia followed him,[36] and, earlier, even Anaximenes had assumed such "earth-like" bodies.[37] Once more a feature of the Philolaus system leads us into the ambit of fifth-century φυσιολογία; it is impossible to separate "Ionian" and "Pythagorean" astronomy.

Even the most exciting idea of the Philolaic system, the movement of the earth, may not be unexampled. Leucippus declared: τὴν γῆν

[30] "Men saw reflected light everywhere," says Burnet (*EGP* 239, writing on Empedocles); cf. Metrodorus of Chios, DK 70A9 (the fixed stars illuminated by the sun; also τὴν τῶν ἄστρων ἀνταύγειαν, *Hebd.* 1.2), and Hippocrates (above, ch. IV 1, nn. 113, 115).

[31] Solar eclipses are equally numerous, but always visible from only part of the earth's surface.

[32] Philip of Opus (from his book Περὶ ἐκλείψεως σελήνης; cf. *Suda* s.v. φιλόσοφος; Martin, *Philolaus* 150) and Aristotle (fr. 16 W., not in Rose) = Aët. 2.29.4: ἀντιφράξει τοτὲ μὲν τῆς γῆς, τοτὲ δὲ τῆς ἀντίχθονος; cf. Arist. *Cael.* 293b24.

[33] Against Aristotle (*Met.* 986a8), Burnet (*EGP* 305) alleges that the counter-earth was "a hypothesis intended to account for the phenomena," and Cherniss (*Pres.* 199) says that Aristotle in another passage shows that "he is aware of the baseless captiousness of his former criticism."

[34] Emphasized by Martin, *Philolaus* 150. It has been conjectured that the "dark bodies" are introduced to explain the rare phenomenon of the sun being visible during an eclipse of the moon, on the opposite horizon (Cleomedes 2.6; Heath, *Aristarchus* 79f; Boll, *RE* VI 2351; actually, it is a phenomenon of refraction). But in that case the body casting the shadow would have to be above the earth, not below it as the Pythagorean counter-earth is; and the Pythagoreans would have spoilt the special point of their hypothesis. That they had this very uncommon phenomenon in mind is, however, unlikely.

[35] Anaxagoras A42 §§6, 9, A77 = Theophr. *Phys. op.* fr. 19, *Dox.* 493.

[36] Diogenes of Apollonia A12.

[37] Anaximenes A7, 14.

ὀχεῖσθαι περὶ τὸ μέσον διατομένην.[38] Here, too, it must be a question of unobservable motion. The earth takes part in the vortical whirl, though more slowly than the bodies dancing about it. Anaxagoras also had the earth taking part in the cosmic whirl, if, as he says, the stars are stones thrown off from it.[39] In Democritus' view, the earth at first "wandered about," then later became dense and heavy, and settled to rest.[40] Like Leucippus, he thought the celestial pole originally stood at the zenith; then later the earth tilted toward the south, so that it became the "north pole." This means, in principle, the explanation of a celestial phenomenon by alteration of the position of of the earth.[41]

The first, intuitive conception of a theoretical hypothesis is a different thing from its development into a precise and verifiable theory. Parmenides can hardly have had more reason than his postulate of symmetry for supposing the earth to be spherical; and the first thought of the movement of the earth was also probably not an attempt to explain specific phenomena. It may have sprung, simply, from the wish to assign the earth a position as a fully-fledged member of the cosmic process. Only when people, making a new start, tried to discover physical laws to explain movement, did those arguments emerge which, from Aristotle to Ptolemy, seemed to refute the hypothesis of a moving earth.

The principle of cosmic uniformity lies behind the expression in Aristotle that the earth is "one of the stars."[42] But this leads immediately into a thicket of mythological connections. The converse of

[38] Leucippus A1 §30. Schiaparelli objected to this, that the fixed stars also do move (§33; Schiaparelli, *Vorl.* 21 n. 37); but compare the same inconsistency in the system of Philolaus.—On the other hand, Anaximander A26 = Eudemus fr. 145, ἡ γῆ μετέωρος· καὶ κινεῖται περὶ τὸ τοῦ κόσμου μέσον, is probably corrupt (καὶ ἀκίνητος?). See DK, n.; Wehrli 120f. It is a misunderstanding according to Zeller I 303 n. 1; Kahn 54f; but accepted by Burnet, *EGP* 66 n. 3; Heidel, *CP* 1906, 279ff; Taylor, *Tim.* 164; Mondolfo, *Inf.* 311 n. 1.

[39] Burnet, *EGP* 269 n. 2, citing Anaxagoras A71 and fr. 16.

[40] Democritus A95: κατ' ἀρχὰς μὲν πλάζεσθαι τὴν γῆν . . .

[41] Leucippus A27, Democritus A96 (cf. Anaxagoras A1 §9). Kranz (*RhM* 100 [1957] 122ff, *Emped.* 50) concluded from fr. 48 that Empedocles believed in rotation of the earth. Plato (*Symp.* 190b) says that sun, moon, and earth are all round and in motion (Kranz, *ABG* 2.46 n. 29), and at *Crat.* 397d says that sun, moon, earth, stars, and sky are "all together" eternally in movement (Boyancé, *REG* 1941, 146); this might be a matter of direct Pythagorean influence. Arist. *Cael.* 293a18, τῶν πλείστων ἐπὶ τοῦ μέσου κεῖσθαι λεγόντων (τὴν γῆν), ὅσοι τὸν ὅλον οὐρανὸν πεπερασμένον εἶναί φασιν, leaves all possibilities open for those who believe the heaven to be infinite (Anaxagoras, Leucippus, Democritus?).

[42] *Cael.* 293a22; cf. Philolaus A16, 21.

this statement, that the stars are a kind of earth, is ascribed by Heraclides to the Pythagoreans.[43] Philolaus taught that the moon "is inhabited all around, as the earth is in our zone, by creatures and plants that are larger and more beautiful, for living creatures on the moon are fifteen times as strong, and eliminate no excrement. Their day is proportionately longer."[44] Herodorus of Heraclea, in the fifth century, wrote that "women on the moon lay eggs, and their offspring are fifteen times as large as we are."[45] Herodorus presupposes the story that Helen, who was born from an egg, had fallen from the moon;[46] a similar story was told of the Nemean Lion.[47] As support for his theory that the moon was an inhabited "earth," Anaxagoras cited not only the observation of a fallen meteorite, but the story of the Nemean Lion.[48] The Pythagorean *acusma* that the sun and moon are the "Isles of the Blest" belongs in this context.[49]

[43] Heraclides frr. 113 (113a: *ταῦτα δὲ τὰ δόγματα ἐν τοῖς Ὀρφικοῖς φέρεται* may be an addition of the doxographer, as Tannery thinks, *MSc* IX 221ff), 114, 115 (every star a world to itself with its own atmosphere).

[44] Philolaus A20, *τῶν Πυθαγορείων τινὲς μέν, ὧν ἐστι Φιλόλαος* . . . An error of calculation is involved in the statement that on the moon a day is 15 times as long as a day on earth. This is easy because the word "day" is ambiguous (12 or 24 hours). During a lunation the moon rotates once on its own axis, so that its "day" is 15 earth days (i.e. 15 × 24 hours!), and its "night" 15 earth nights. (Martin, *Philolaus* 145ff, imagines a kind of axial rotation, too complicated to describe here, in order to eliminate the error; but the Philolaic system in general does not rest on mathematical subtleties.)

[45] Herodorus, *FGrHist* 31F21 = Ath. 2.57f (also cf. fr. 4). This has been seen as relevant to Philolaus since Martin, *Philolaus* 144; cf. DK I 404 n.; Capelle 3; M. Detienne, *RHR* 158 (1960) 25ff. There is no way of knowing whether Herodorus is drawing on Philolaus or on older tradition.

[46] Neocles of Croton (otherwise unknown; a Pythagorean?) ap. Ath. 2.57; cf. Jacoby's notes; Eustathius p. 1488. 19ff (on *Od.* 4.121); cf. M. Detienne, "La légende pythagoricienne d'Hélène," *RHR* 152 (1957) 129–152.

[47] Herodorus *FGrHist* 31F4 = Tatian 27; Epimenides DK 3B2.

[48] A77. This may be a target of Democritus' charge of plagiarism (D.L. 9.34). Philolaus is thus not the first to speak of the moon as inhabited (pace Gundel, *RE* XVI 77f); cf. also *Orph. frag.* 91. Xenophanes A47 remains uncertain (see DK, n. ad loc., and Gigon, *Ursprung* 172ff). The stories of children of the moon are of a somewhat different character. Musaeus was a son of Selene (Pl. *Rep.* 364e, Hermesianax 2.15 Diehl, etc.), and a verse attributed to Epimenides runs, *καὶ γὰρ ἐγὼ γένος εἰμὶ Σελήνης ἠυκόμοιο* . . . (DK 3B2, attributed to Musaeus by Diels, DK I 33 n.; O. Kern, *Religion der Griechen* II [Berlin, 1935] 175, conjectures that Epimenides introduced Musaeus speaking; but in any case the words *καὶ γὰρ ἐγὼ* . . . show that there were other members of the moon family. For the prophet's reference to his own divine origin and experience, cf. Empedocles frr. 115.13, 117.1)

[49] Above, ch. IV 1, n. 109. This could be connected with the legend of Helen (who ends up in Elysium) as well as with Musaeus and Epimenides. In each case are found both the origin from a higher realm and the return to it.—Plato (*Symp.* 190b) speaks of the human race as descended from sun, moon, and earth. Cf. Capelle 3; Cumont, *Symb.* 182ff.—The relation between the moon and the isle of the Hyperboreans, as depicted by Hecataeus of Abdera, is interesting (*FGrHist* 264F7 = DK 73B5 = Diod. 2.47.5): *καὶ τὴν σελήνην ἐκ ταύτης* . . . *φαίνεσθαι παντελῶς ὀλίγον ἀπέχουσαν τῆς γῆς καί τινας ἐξοχὰς γεώδεις ἔχουσαν*. A journey to the moon was featured in the novel of the Pythagorizing Antonius Diogenes, parodied by Lucian *Ver. hist.* 1.11ff.

No one can give us information about the inhabitants of the moon and the stars except someone who is one of them or who can make contact with them; and the idea of a "journey through the skies" brings us back once more into the world of "shamanism." Certain details seem to confirm that this is a real connection: the fact that that other world is "greater and more beautiful" is part of the experience of ecstasy,[50] and freedom from excretory function suggests an existence apart from the body.[51]

But the counter-earth as well fits into this picture. The same Herodorus asserted that vultures are not native to our earth, *εἶναι τοὺς γῦπας ἀφ' ἑτέρας γῆς ἀδήλου ἡμῖν*.[52] This cannot mean the moon, which is anything but invisible. But the Pythagorean counter-earth is a *γῆ ἄλλη, ἡμῖν ἄδηλος*,[53] so that the *ἀντίχθων* surprisingly turns up in fifth-century literature, in a purely mythical context. The vultures of which Herodorus speaks are rationalization of the griffins—*γῦπες* for *γρῦπες*—which live along the road to the land of the Hyperboreans, at the entrance to the world beyond, "Gripes Hyperborei, quos . . . generat mundus alter."[54] Theopompus represents Silenus as telling of an infinitely great "continent outside this world of ours," whose inhabitants are twice as large and twice as long-lived as we. All their laws are exactly opposite to ours. And, when these creatures once wished to come and visit us, they only got as far as the Hyperboreans and turned back.[55] The theme of a "counter-world" where everything

[50] Cf. Pl. *Phd.* 109b, Plut. *De gen.* 590c, *De sera* 563f.

[51] D.L. 8.19, of Pythagoras: *οὐδέποτ' ἐγνώσθη οὔτε διαχωρῶν* . . . Lucian *Ver. hist.* 1.23, Ctesias *FGrHist* 688F45 §44 = Phot. *Bibl.* 48b12 (Rohde, *Rom.* 206 n. 4, 288 n. 2). Along with this goes the absence of normal nourishment; Lucian's moon folk subsist on odors (cf. Megasthenes ap. Strabo 15, p. 711, Plin. *HN* 7.25, Plut. *De fac.* 938, scarcely fortuitous in the context of moon dwellers), as do the dead (Lucian *Charon* 22, Heraclitus fr. 98). Perhaps the origin of this idea is in the practice of the shamans, who (in Scythia) achieve ecstasy by inhaling certain fumes (Hdt. 1.202, 4.75; Meuli 121ff; Schol. Lucian p. 20.13). Aristotle tells us that, according to the Pythagoreans, *τρέφεσθαι* . . . *ἔνια ζῷα ταῖς ὀσμαῖς* (*Sens.* 445a16 = DK 58B43); and this could be interpreted as applying to the inhabitants of the moon. Aristotle himself (*Gen. an.* 761b21) assumes that there are inhabitants on the moon (in a similar context to [Pl.] *Epin.* 984d), and mentions the "vultures" of the "other earth" (below, n. 52).

[52] *FGrHist* 31F22 = Arist. *Hist. an.* 563a7. Jacoby, in his note, mentions the moon but is dubious.

[53] Arist. *Cael.* 293a23, b22.

[54] Apul. *Met.* 11.24.3. In Lucian *Ver. hist.* 1.11 the moon men ride on vultures (*ἱππόγυποι*). Here moon and counter-earth have coalesced (cf. above, ch. III 1, n. 78).

[55] *FGrHist* 115F75 = Ael. *VH* 3.18. Rohde (*Rom.* 219ff) sees in this an attempt to outdo Plato's Atlantis (*καταντικρὺ ἤπειρος*, Pl. *Tim.* 25a); and there is clearly also a connection with the *Phaedo* (above, n. 50), in that the "true earth" is larger and more beautiful. This does not mean, however, that Theopompus is arbitrarily spinning out Plato's statements; rather, he is drawing from the same tradition.—The "true continent" outside our world, beyond the Ocean, appears also in Plut. *De fac.* 941b, in the Pythagorizing final myth, as well as Schol. Pl. *Alc.* 122a; cf. Iambulus in Diod. 2.56.

is the opposite of what we know, is widespread in folklore and has its effect on travel literature as well;[56] it is especially common to represent the realm of the dead in this manner.[57] Thus the Pythagorean counter-earth, which is naturally thought of as inhabited,[58] has a real meaning in the world of myth, and its position in the astronomical system is an expression of its character. Clearly there is a background for this, in the tradition of shamanistic narrative, similar to that which lies behind the story of the inhabited moon. The legend of Pythagoras and the doctrine of transmigration, as well as the *acusma* about the moon and sun as the Isles of the Blest, presuppose just such a shamanistic outlook, so that we are probably justified in calling the whole complex Pythagorean—perhaps various features of it actually originated with Pythagoras himself, and perhaps he was just the most conspicuous member in a more comprehensive chain of tradition.

The system of Philolaus, which we found intractable to analysis as an expression of scientific astronomy, now takes its place beside, perhaps even before, Herodorus of Heraclea, that is, in the second half of the fifth century B.C.—precisely the time of the Pythagorean Philolaus of whom Plato speaks. It emerges from the same kind of concern as that of Herodorus, the interpreter of myth: ancient lore, transmitted *ἐν μύθου σχήματι*, is newly formulated in the terms of contemporary natural philosophy or *φυσιολογία*. Thus shamanistic myths take concrete form as specific components of the world, and the earth is relegated to equality with them as "one of the stars." An "eccentric" attitude toward things, so to speak, a devaluation of earthly existence in comparison with "purer" worlds,[59] is doubtless as strong a motive force here as the requirement of symmetry which makes correlatives of *Ὄλυμπος* and *Ζανὸς φυλακή*, the "Limiting" which is united in harmony with the "Unlimited."

[56] Hdt. 2.35ff: *Αἰγύπτιοι . . . τὰ πολλὰ πάντα ἔμπαλιν τοῖσι ἄλλοισι ἀνθρώποισι ἐστήσαντο ἤθεά τε καὶ νόμους.*

[57] L. Lévy-Bruhl, *The "Soul" of the Primitive*, tr. L. A. Clare (New York: Praeger, 1966) 303f. See also Pi. fr. 129: day for us is night for them, etc.

[58] Philolaus A17, *τοὺς ἐν ἐκείνῃ . . .* G. B. Burch maintains, *Osiris* 11 (1954) 286–289, that the counter-earth is necessary to preserve the equilibrium of the universe; but, if that is so, the inhabited moon would necessitate a counter-moon (288, n. 96).

[59] Cf. the *acusma τί ἀληθέστατον λέγεται; ὅτι πονηροὶ οἱ ἄνθρωποι*, and *ἀγαθὸν οἱ πόνοι . . . ἐπὶ κολάσει γὰρ ἐλθόντας δεῖ κολασθῆναι* (Iam. *VP* 82, 85; above, ch. II 4, n. 14).

Proclus and, after him, Damascius[60] report that Philolaus "dedicated" certain geometrical figures to particular gods—the angle of the triangle to Cronus, Hades, Ares, and Dionysus, the angle of the square to Rhea, Demeter, and Hestia, and the angle of the dodecagon to Zeus. Damascius adds that the semicircle was sacred to the Dioscuri. One would quickly reject this late testimony, if it were not corroborated by a very ancient piece of evidence. Eudoxus mentions that in the Pythagorean doctrine the angle of a triangle belongs to Hades, Dionysus, and Ares, and that of the square to Rhea, Aphrodite, Demeter, Hestia, and Hera, that of the dodecagon to Zeus, and that of the 56-angled figure (the *hekkaipentekontagonion*) to the baneful Typhon.[61] This remarkable doctrine is thus attested for pre-Platonic Pythagoreans by a contemporary of Plato. Scholars from Boeckh to Zeller scarcely knew what to make of it,[62] till Tannery, Newbold, Boll, and Olivieri pointed out the connection with astrology. According to an astrological procedure often repeated, with certain variations, triangles and squares are inscribed in the zodiac and are then associated with elements and planets.[63] There are four *τρίγωνα* and three *τετράγωνα*; a triangle spans four signs, a square three. This seems to explain the striking connection of three goddesses with the square, and of four gods with the triangle, in Philolaus (even though this precise correspondence is not attested in the Eudoxus passage). The dodecagon, which corresponds to Zeus, is the whole zodiac with its twelve signs. Half the

[60] Philolaus A14 = Procl. *In Eucl.* 130.8, 166.25, 173.11, 174.12. Proclus alludes elsewhere to the Pythagorean connection of *σχήματα* and gods (*Theol. Pl.* 1.4 p. 9; *In Parm.* p. 647, 924 Cousin [Paris, 1864]), so that Damascius (*Princ.* II 127.7 Ruelle) may have derived his additional data about the circle and the semicircle from a more detailed exposition by Proclus which is now lost. In any case the sentence *καὶ μήποτε* ("perhaps") *ὡς καθόλου εἰπεῖν τὸ μὲν περιφερὲς κοινὸν σχῆμά ἐστιν πάντων τῶν νοερῶν θεῶν ἢ νοερόν, τὰ δὲ εὐθύγραμμα ἴδια ἑκάστων* is Neoplatonic interpretation, probably by Damascius himself.

[61] Eudoxus fr. 84 Gisinger = 293 Lasserre = Plut. *De Is. et Os.* 30.363a. Schaarschmidt (43ff) maintained, consistently with his general argument, that the Eudoxus fragment was spurious. But Boll (*Sphaera* 472–478) traced other similar indications of astral theology to Eudoxus.

[62] Boeckh 152ff, Zeller I 499 n. 1.

[63] Tannery, *AGP* 2 (1899) 379–386 = *MSc* VII 131–139; Newbold 198ff; Boll unfortunately never worked out in detail the hints he gave at *NJb* 1908, 119 (= *KlSchr* 19f, repeated 382). Olivieri 30ff. The *τρίγωνα* are associated with the 4 seasons and the 4 elements, though the goddesses named by Philolaus cannot be related with them directly, nor with the zodiacal signs. Olivieri tried to carry the interpretation out in detail. Boll pointed out the connection of the planet Jupiter with the dodecagon. Newbold hazarded an astronomical interpretation of the connection between Typhon and the 56-angled figure (207ff). Cf. the 28-day month (Plut. *De Is. et Os.* 367f).

signs of the zodiac become visible in a single night, and this suggests the semicircle, which is sacred to the Dioscuri. These sons of Zeus live and die on alternate days.[64] The orbital period of Jupiter is almost exactly twelve years (11.86), so that in a single year it traverses, approximately, one sign. *Dodeketerides* built on this are ascribed to Zoroaster and Orpheus; van der Waerden relates them to the "primitive zodiacal astrology" developed in Babylon in the sixth century B.C.[65] Frank insisted that no traces of astrology can be detected in Greece before the time of Plato's old age;[66] yet not only Eudoxus[67] but Ctesias as well speak of the Chaldaeans' divination by the stars,[68] and reflections of astrological ideas have been detected in the Hippocratic *Regimen*.[69] If the zodiac was introduced to the Greeks from Babylon before 500,[70] and the planets about 440, one would infer the presence of the related astrological conceptions even without specific evidence. In the mélange of myth and *φυσιολογία* which Philolaus' astronomy proves to be, we also find the first traces of astrology; though here the Babylonian tables are replaced by the idea of the angle—the graphic, geometrical, that is to say, the specifically "Greek" element.

4. HARMONY OF THE SPHERES AND ASTRAL IMMORTALITY

A very famous and impressive image, uniting the disciplines of music and astronomy, is that of the "music of the spheres." It is

[64] *Od.* 11.303f; cf. Delatte, *Litt.* 115f; Carcopino, *Bas.* 358f, and esp. Cumont, *Symb.* 74ff, *Lux* 192ff; (Pl.) *Axioch.* 371; Iam. *VP* 155; Eust. p. 1686.30, 410.18. At Sext. Emp. *Math.* 9.37 the Dioscuri are related to the hemispheres of day and night (cf. Empedocles, above, n. 27). The doctrine of the 2 halves of the heaven was ascribed to Pythagoras by Lact. Plac. *Theb.* 4.527.—The sky is called Zeus (Empedocles fr. 142.1, Democr. fr. 30; cf. Critias fr. 25 and later Aratus 1ff, Ennius fr. sc. 345 Vahlen, Macrob. *Somn. Sc.* 1.17.14). He is so as fire (Empedocles fr. 6.2), air (Diogenes of Apollonia A8, Eur. *Tro.* 886); *Διὸς αἰθήρ*, Hdt. 7.8γ, Eur. fr. 839; also see *Hebd.* 6, above, ch. IV 1, n. 66.

[65] *Anf.* 243.

[66] Frank 281f.

[67] Fr. 343 Lasserre = Cic. *Div.* 2.87: "Chaldaeis in praedictione et in notatione cuiusque vitae ex natali die minime esse credendum." Reports of dubious authenticity are given by ps.-Arist. fr. 32 = D.L. 2.45 and Gell. *NA* 15.20.2.

[68] *FGrHist* 688F1 §§23–25.

[69] W. Capelle, "Alteste Spuren der Astrologie bei den Griechen," *Hermes* 60 (1925) 373–395.

[70] Above, ch. IV 2, nn. 53–55 R. Böker, *RE* XXIII 876, on the ground of a dubious conjecture in Plin. *HN* 2.31, ascribes to Cleostratus the zodiacal trigona.

attested by Aristotle,[1] and indirectly by Plato,[2] as being a doctrine of the Pythagoreans, who interpreted the Sirens of mythology as the makers of this cosmic music.[3] Scholars have long thought that the music of the spheres had no organic connection with the Philolaic system.[4] It is difficult to relate the ten revolving celestial bodies to music.[5] The source of the whole idea, it is thought, is rather the association of the ancient and proverbial "seven-stringed" lyre[6] with the later, but no less well known, idea that the planets are seven in number.[7] The general nature of the Pythagorean musical theory is rightly cited: the coherence of number and sound. Harmonic intervals

[1] *Cael.* 290b12ff ("the Pythagoreans" named at 291a8), fr. 203. Archytas fr. 1, Plut. *Mus.* 1147a. The expression "harmony of the spheres" is inappropriate, strictly speaking, as applied to the time before Eudoxus, for then one spoke of bodies, wheels, rings, circles in the sky, but not yet of spheres (Burnet, *EGP* 110).

[2] *Rep.* 530d, *Crat.* 405c; also *Rep.* 617b, in the myth of Er. On the *Timaeus*, see below, n. 19.

[3] On the *acusma* of the tetractys, see above, ch. II 4, n. 154. Pl. *Rep.* 617b. On the Siren idea, see E. Buschor, *Die Musen des Jenseits* (Munich, 1944); on the Pythagorean interpretation, Delatte, *Litt.* 132f, 260f. In the Temple of Bel at Palmyra (early Empire) the Sirens are depicted next to the planetary gods; this is interpreted by L. Curtius, *RM* 50 (1935) 348–353, as having to do with the music of the spheres. On the other hand, later Pythagorean tradition spoke of the Muses in relation to the harmony of the spheres and thought of the Sirens as representing sensuous, worldly music; see Clem. Al. *Strom.* 1.48.6, Por. *VP* 39, Demophilus 23 (Mullach I 486), and P. Courcelle, *REA* 46 (1944) 73ff (vs. Cumont, *Symb.* 329f).—"Pythagoras" himself says, Schol. *Od.* 1.371 p. 172.6 Thesleff, *ἔξω γενόμενος τοῦ σώματος ἀκήκοα ἐμμελοῦς ἁρμονίας*; and Nicomachus also records that Pythagoras was able to hear the harmony of the spheres (Por. *VP* 30 = Iam. *VP* 66f, with a mistaken interpretation of Empedocles fr. 129).

[4] Martin, *Pyth.* 110ff; Tannery, *Astr.* 327; Zeller I 540 n. 2; van der Waerden, *Astr.* 29. Tannery (cf. *MSc.* VII 158f) puts the harmony of the spheres later than Philolaus, though usually the Philolaic system is thought of as a later development. It is not important that the surviving fragments of Philolaus do not refer unambiguously to the cosmic music; the doxographers would hardly have had occasion to include this under the rubrics *περὶ τάξεως ἀστέρων* (A16), *περὶ ἡλίου* (A19), or *περὶ γῆς* (A21). We do find the expression *χορεύειν* in A16; and Aristid. Quint. p. 145 M. = 119.27 W.-I. finds in the words *ἄστρων χορός* an allusion to *τὴν τῶν πλανήτων ἐμμελῆ κίνησιν*.

[5] Though before the time of Timotheus, Histiaeus of Colophon had introduced a 10-string lyre (Nicom. *Exc.* 274.4 Jan).

[6] Sarcophagus of Haghia Triada, *Hymn. Hom. Merc.* 51, (Terpander) fr. 4.4, Pi. *Pyth.* 2.70, *Nem.* 5.24, Bacchyl. 20b2, Ion of Chios fr. 6.3 Diehl, Eur. *Alc.* 446, *Ion* 881, etc. See Deubner, *AM* 54 (1929) 194–200. It is quite uncertain how these 7 strings were tuned; below, ch. V 2, nn. 34ff.

[7] Varro Atacinus fr. 14 Morel (Gramm. Lat. VI 60), Philo *Op.* 126, Nicom. *Ench.* 3 (cf. *Th. ar.* 71.15ff), Lucian *Astrol.* 10, "Orpheus" *Orph. frag.* T58a Serv. *Aen.* 6.645, Dio Cass. 37.18, Lydus *Mens.* 2.3. There do not seem to be any older references than these on the number 7 as a link between music and astronomy. Both are omitted from *On Sevens*, as Roscher emphasizes. The 7 analogy as point of departure for the whole theory of cosmic music is accepted, among others, by Jan, *Philologus* 1893, 15f; Wilamowitz, "Die Harmonie der Sphären," *Reden aus der Kriegszeit* III 6 (Berlin, 1915) 15; and Gundel, *RE* XX 2056.

correspond to harmonic relationships of distance and velocity;[8] and since a musical tone—as distinguished from a mere noise—implies a uniform motion, one can infer from all this a Pythagorean system of astronomy, in which the planets—all seven of them, long known—circle about the earth in uniform movements, at various distances from one another. This Pythagorean astronomy, to be seen in Plato, is thought to go back earlier than Philolaus.[9] If his system belongs to the fifth century, then "Pythagorean astronomy" had reached unexampled heights even earlier; and, if the latter cannot be dated earlier than Archytas, then "Philolaus" becomes a forgery—one produced, however, before the time of Aristotle.

It is striking that Aristotle and Alexander of Aphrodisias, the latter citing Aristotle's now lost book on the Pythagoreans, make no distinction between the system of Philolaus and the assumed, very different Pythagorean astronomy.[10] When one considers also that before Eudoxus there was no mathematical theory of the planets based on the concept of uniform circular movement, one cannot help suspecting that it is a mistake to assume the existence of an earlier mathematical astronomy that served as basis for the idea of the music of the spheres. Actually, the view, put forward again and again as virtually self-evident, that the thought of cosmic music was an inference from some kind of scientific knowledge,[11] is a misconception.

None of the explanations of the celestial gamut known in the tradition of later antiquity has any claim to be authentic. Plato speaks in the *Republic* of "a *harmonia*" of eight tones, and in his *Hermes* Eratosthenes presented this as a scale one octave long.[12] Thus the oldest sources speak not of seven but of eight notes, as the Pythagoreans too, in Aristotle's account, include the sphere of the fixed stars in the cosmic orchestra.[13] The scale of the music of the spheres which in later times was most widely known and was attributed to Pythagoras himself is actually a botched version of Eratosthenes' exposition,

[8] Arist. *Cael.* 290b21, fr. 203.

[9] Cf. Frank 30ff; van der Waerden, *Astr.* 26, 29ff; Junge, *C&M* 1947–1948, 183ff.

[10] Alex. *Met.* 41.2ff = Arist. fr. 203 expressly relates the harmony of the spheres to the "10 moving bodies."

[11] According to Tannery, *Astr.* 328, the harmony of the spheres was deduced from (or "followed," *découlée*) the idea of the kinship of the sciences; according to Junge, *C&M* 1947–1948, 185ff, from the discovery that the planets have fixed orbital periods.

[12] Pl. *Rep.* 617b, Eratosthenes fr. 15 Powell = Theo Sm. 105.15, Anat. p. 38 (*Th. ar.* 75.6); cf. Theo Sm. 142.7ff = Chalcid. 73.

[13] Arist. *Cael.* 290b18: τοσούτων τὸ πλῆθος ἄστρων . . . φερομένων, cited by Zeller I 541.2; Heath, *Aristarchus* 108.

created by the desire to build cosmic measurements into it.[14] An attempt in Cicero's *Somnium Scipionis*[15] to combine eight spheres with seven notes is a patchwork compromise. Nicomachus is at least more consistent, in eliminating the movement of the fixed stars and basing the system on the individual movements of the planets in the zodiac, so that the moon being fastest, is credited with the highest note; but in both respects he contradicts the reports of Aristotle.[16] The most lucid and consistent system of cosmic harmony, employing only φθόγγοι ἑστῶτες, makes no claim to be derived from the ancient tradition.[17]

Alexander intimates that Aristotle himself knew no detailed exposition of the Pythagorean cosmic harmony. The distances of the heavenly bodies from one another ("intervals"), Alexander explains, have a certain ἀναλογία; their velocities correspond to these distances, and the tones to the velocities. Thus the tones themselves have a "harmonic" relationship, determined by number: "Thus, the

[14] Burkert, *Philologus* 1961, 31ff.

[15] Cic. *Rep.* 6.18. Venus and Mercury seem to have the same tone ("in quibus eadem vis est duorum"), as also in Ptolemy (below, n. 17); cf. Macrob. *Somn. Sc.* 2.4.9 (Boethius is wrong, *Mus.* 1.27). Boyancé, *Songe* 111f, interprets this expression as meaning that the moon and the fixed stars make an octave (cf. Por. *In Ptol.* 104.7, on the octave: δύναμίς ἐστιν ἡ αὐτή; similarly, Arist. *Pr.* 19.14, 19.18); contradicted by O. Seel, *PhW* 58 (1938) 491ff. There remains in any case the question how the sun can have a different tone from its two ἰσόδρομοι. At Plut. *De an procr.* 1029b, sun, Venus, and Mercury are obviously taken together, so that there remain only πέντε διαστήματα.

[16] Nicom. *Ench.* 3 p. 241 Jan, and following him Boeth. *Mus.* 1.20, 27. This is held to be the original view by Jan, *Philologus* 1893, 17f, and van der Waerden, *Astr.* 36f. Frank, too (31), thinks the moon had the fastest movement. An apparent argument for this is that here Hypate, taken literally, is the "highest" note (Saturn); but the names of the notes are derived from the way the instrument was held. In defense of his position that the ancient Pythagoreans derived the harmony of the spheres from the independent movement of the planets in the zodiac, van der Waerden (*Astr.* 36f) cites Arist. fr. 203; but he quotes only a part of it and overlooks the fact that, later on, more rapid movement is attributed to the more distant planets. Against Nicomachus' statement that the *synemmenon* system was original, see below, ch. V 2. The *Excerpta ex Nicomacho* emphasize that, differently from Nicomachus, "the first" (οἱ δὲ δὴ πρῶτοι) made Saturn the Nete and the moon Hypate (3 p. 272.9 Jan).

[17] Mentioned at Plut. *De an. procr.* 32.1029b (ibid. 1029a–b, a somewhat different system based on the 5 tetrachords); Ptolemy, Canopus inscription (*Op. min.* ed. Heiberg, II 154f; authenticity contested by Jan, *Philologus* 1893, 35ff; Hoeg, *Gnomon* 6 [1930] 657ff; defended by Düring 1934, 280ff; cf. van der Waerden, *RE* XXIII 1818–1823; the corresponding exposition in the *Harmonics* of Ptolemy is lost); *Th. ar.* 75.8ff = *Exc. Neap.* 2, p. 412 Jan (*Mus. scr. gr.*), with the superscription Πτολεμαίου μουσικά; Anon. Bellerm. §84. The notes are, transcribed, A B c a b d' e' a' b', corresponding to the series 8 9 12 16 18 21⅓ 24 32 36. Since 21⅓ (found in Ptolemy, Canopus inscr.) is not a whole number, most have simply substituted 21, which is musically wrong. The Canopus inscription distinguishes earth (8 = A) from air (9 = B), and in compensation gives Venus and Mercury only a single note (16 = a); the others count 8 celestial spheres.—Since this system is older than Plutarch, it could go back to the mathematician Hypsicles, who, according to Ach. *Is.* 16 p. 43.9 Maass, wrote on the harmony of the spheres.

distance of the sun from the earth being, say, double the distance of the moon . . . they considered that there was some arithmetical ratio in the case of the other planets as well . . ."[18] Alexander introduces his exposition with φέρε εἰπεῖν, that is, "supposing," or "say, for example." The figures given belong to a hypothetical case, cited for clarity's sake, and do not belong to a traditional account. If Aristotle had credited the Pythagoreans with an unambiguously described scale, Alexander would not have used a fictitious example.

Nor does the structure of the world soul in Plato's *Timaeus* signify a system of cosmic harmony, in which each planet had a note corresponding to its "distance." The seven planetary orbits are derived by sixfold division of the inner sphere of the heavens, which is dominated by the nature of the "other." This division is made "according to" three powers each of the numbers 2 and 3, which had previously been developed; but the game he plays with 6 and 7 seems to suggest that one ought not simply to identify the 7 numbers with the 7 planets and 7 notes.[19] To be sure, musical theory is involved,[20] but Plato says expressly that the movement of the world soul takes place ἄνευ φθόγγου καὶ ἠχῆς (37b); the mythic image of the Sirens, from the *Republic*, is translated into the realm of the immaterial and abstract. At the same time the naive thought of the Pythagoreans reported by Aristotle,[21]

[18] Alex. *Met.* 40.3ff = Arist. fr. 203, tr. Heath. Heath saw the significance of φέρε εἰπεῖν (*Aristarchus* 111 n. 2). Tannery, too (*Astr.* 327) believed that all detailed expositions of the celestial music were late.

[19] *Tim.* 36d: σχίσας ἐξαχῇ ἑπτὰ κύκλους ἀνίσους κατὰ τὴν τοῦ διπλασίου καὶ τριπλασίου διάστασιν ἑκάστην, οὐσῶν ἑκατέρων τριῶν, depending on 35b–c. On the contradictory ancient attempts to relate the 7 numbers of the *Timaeus* with magnitudes, velocities, and intervals of the planets, cf. Plut. *De an. procr.* 31.1028a, Chalc. 96, Macrob. *Somn. Sc.* 2.3.14f, Procl. *In Tim.* II 212, Taylor, *Tim.* 161ff. Like Zeller (II 1. 779 n.), Boeckh (*KlSchr* III 168), and Duhem (53), Taylor comes to the conclusion that "Timaeus is giving us an estimate of the radii of the planetary orbits in terms of the diameter of the moon's orbit" (163). Heath (*Aristarchus* 163) and Cornford (*Tim.* 79) are rightly dubious of this. Like all musical relationships (cf. 36a–b), all numerical relationships are ultimately derivable from the numbers given; the basic rules of the "system of derivation" (above ch. I 1) are presupposed. Junge (*C&M* 1947–1948, 183ff) attempts an interpretation that makes sense astronomically: we have "an expression of the orbital periods of all the planets by means of the numbers 2 and 3 and their multiples." For example, the moon takes 27 days, the sun 729 (= 27^2) half-days, Saturn 27 years, Mars 2, and Jupiter 12 years. It is likely that Plato's thoughts were running in this direction (Junge refers to *Rep.* 588a); but the basic idea is more important than any specific result—the problem is more important than the solution.

[20] This was denied by Rivaud, *Rev. hist. philos.* 3 (1929) 16ff (so that he found it necessary to brand not only the Philolaus fragments but those of Archytas post-Platonic forgeries; he seems to overlook, on pp. 6f, that Plato himself, at *Rep.* 530, expressly mentions Pythagorean musical theory). The harmony of the spheres, he thinks, is merely crude interpretation of the *Timaeus*. (Similarly Moreau, *Âme* 55.)

[21] *Cael.* 290b15ff.

that the tremendous bodies of the stars could not move without making a noise, is rejected. In the *Republic* Plato demanded, in explicit polemic against the Pythagoreans, that the harmony of numbers be regarded in itself, and without reference to audible sounds, and he carries out this program himself in the *Timaeus*.[22] Thus the relationship between astronomy and music is traced to its ultimate, immaterial principle. The numbers in their order are the basic principle of the cosmos, which is at the same time "beauty" and reflection of "the Good." Therefore the basic constituents of matter, the elements, are also derived from the process of geometrical arrangement. All this, as Aristotle and Plato make clear, is not Pythagorean but Platonic.

The contradictory nature of the later tradition, too, sometimes using seven,[23] sometimes eight,[24] and occasionally three musical notes,[25] and even connecting the nine Muses with the sphere of the All,[26] shows that the idea of cosmic music is not bound to any particular astronomical system. This concept has nothing to do with mathematical or musical theory, but comes from a deeper root; and this is why it was able to outlive even the Ptolemaic cosmology. Of course this carries the implication that any conclusion about certain astronomical theories of the Pythagoreans, on the basis of the harmony of the spheres, is hazardous in principle; we need not suppose that any detailed system formed its basis.

There are also other conceptions of cosmic music, quite different though related. One need not speak of planets, or of a many-storied universe. No matter how things are divided, the correctly attuned ear will hear music.[27] The ancient lyre had only four strings, "in imitation

[22] *Rep.* 531c; cf. below, ch. V 1.

[23] Above, n. 7. A connection was also seen with the 7 vowels: Nicom. *Exc.* 6 p. 276 Jan; Schol. Dion. Thr. p. 197.33ff Hilgart; the Gnostic Marcus ap. Hippol. *Ref.* 6.48; Nicom. *Th. ar.* 71.13ff; Lydus *Mens.* 2.3; *CIG* II 1 no. 2895; cf. F. Dornseiff, *Das Alphabet in Mystik und Magie* (Leipzig, 1925²) 82f.

[24] Plato and Eratosthenes, above, n. 12. The arithmological sources (Theo Sm., Anatolius) associate the music of the spheres with the passage of Eratosthenes in their discussion of the number 8, not 7.

[25] In the λόγος of the Delphians, Plut. *Quaest. conv.* 745a. Burnet (*EGP* 110) and Kranz (*Philologus* 1938, 437) thought that Pythagoras associated the 3 "rings" of Anaximander (stars, moon, and sun) with the basic concords of the fourth, the fifth, and the octave.

[26] The eighth Muse, Urania, fits well; and the ninth was either *concinentia maxima*—Calliope as προφερεστάτη ἁπασέων (Macrob. *Somn. Sc.* 2.3.1f)—or was assigned "the region of earth" (Plut. *De an. Procr.* 32.1029d, *Quaest. conv.* 746a). Others looked to the counter-earth for help (Por. *VP* 31.)

[27] We may mention also the association of astrological configurations with the basic musical concords (Plut. *De an. Procr.* 31.1028d–e, Ptol. *Harm.* 3.9). In *Harm.* 3, Ptolemy presents a goodly sample of such ingenious fooleries. Cf. also Phld. *Mus.* p. 100 Kemke.

of the cosmic music, which consists of four elements."[28] The four seasons correspond in turn to the elements, and stand in concord with one another: spring makes a fourth along with fall, a fifth with winter, an octave with summer, "as they say Pythagoras taught."[29] All the sources for this are late, and the part about the elements is consciously derived from the *Timaeus*.[30] But, if Scythinus and Cleanthes called the sun the "plectrum" of the cosmos, this fits better with the rhythm of the seasons, which does depend on the course of the sun, than with the planetary scale.[31] A ἁρμονία of the seasons is spoken of by Euripides, and then also by Plato.[32] Such ideas are traced to oriental beginnings.[33] The idea of a cosmic music, and specifically one connected with the changing seasons, is widespread; for example, the five notes of Chinese music correspond to five elements and to the seasons.[34] Here one should not think of direct influence, one way or the other, but of a parallel development due to similar psychological impulsions; and for this very reason one seems here to be closer to the root of the idea of cosmic music than in the context of the planetary scale.

[28] Boeth. *Mus.* 1.20 p. 206 Friedlein, from Nicomachus. Cf. Clem. Al. *Protr.* 1.5.1: fire as νήτη.

[29] Aristid. Quint. 3, p. 144f M.: summer is 4, autumn 6, winter 12, spring 8; also Plut. *De an. procr.* 31.1028f ("Chaldeans"); somewhat differently *Hymn. Orph.* 34.10ff (winter is Hypate, summer Nete, spring the Doric tone: Mese?); a hint only, *Hymn. Orph.* 8.9. O. Neugebauer, *AJP* 63 (1942) 455–488, mistakenly tries to derive a precise astronomical sense from the passage of Plutarch and to present the actual differences of the astronomical seasons as harmonic numbers. He overlooks the explanation in Aristid. Quint. and in addition has to assume a confusion of autumn and summer. Also, all this speculation obviously takes its departure from the contrast of summer and winter.

[30] Aristid. Quint. 3, pp. 144f M. Plato himself speaks of ἀναλογία as the δεσμός of the elements (*Tim.* 31b–c), and derives from that the fact that they are 4 in number; thus he knows of a sort of "harmony of the elements"; but the polyhedra cannot be brought into direct connection with this. What Plato cares about is not a definite and accurate account, but intimations as to the basic structure of reality.

[31] Above, ch. IV 1, n. 107.

[32] Eur. fr. 943 (thought spurious by Lobeck), Pl. *Symp.* 188a, *Phlb.* 26a, where Plato seems to be taking up Pythagorean themes.

[33] Plut. *De an. procr.* 31 speaks of "Chaldeans." Diodorus reports, of Hermes–Thoth: *λύραν τε νευρίνην ποιῆσαι τρίχορδον μιμησάμενον τὰς κατ' ἐνιαυτὸν ὥρας· τρεῖς γὰρ αὐτὸν ὑποστήσασθαι φθόγγους, ὀξὺν βαρὺν καὶ μέσον, ὀξὺν μὲν ἀπὸ τοῦ θέρους, βαρὺν δὲ ἀπὸ τοῦ χειμῶνος, μέσον δὲ ἀπὸ τοῦ ἔαρος* (1.16.1; Hecataeus of Abdera, *FGrHist* 264F25 [?]; cf. W. Spoerri, *Späthellenistische Berichte über Welt, Kultur und Götter* [Basel, 1959] 164ff). When Eratosthenes, in his *Hermes*, represented this god as inventing the harmony of the spheres, he was obviously combining Egyptian elements with Greek-Pythagorean material. G. A. Keller (*Eratosthenes und die alexandrinische Sterndichtung* [Diss. Zürich, 1946] 95ff, 98ff) emphatically denies any Egyptian influence on Eratosthenes. But the 3-stringed lyre is Egyptian (Keller, 99). Hermes as inventor of the lyre is Greek, but Thoth as inventor of astronomy is Egyptian (Pl. *Phdr.* 274c).

[34] See E. M. von Hornbostel, "Tonart und Ethos," *Festschr. J. Wolf* (Berlin, 1929), I cannot verify Hornbostel's suggestion of similar ideas among the Arabs. On the Chinese, see M. Granet, *La pensée chinoise* (Paris, 1934) 209ff.

Before the beginning of reflective thought, man feels, in various contexts, an involvement. He unconsciously arranges the multiplicity of phenomena into a restricted number of schemata. It is the business of reflection, when it begins, to raise these transitory insights into the realm of consciousness, to name them, and to assimilate them to one another. This is how the world becomes comprehensible. In myth and ritual man tries to make these realizations present and clear, to assure himself that, in spite of all confusion and all the immediate threats of his environment, everything is "in order." It is in such a pre-scientific conception of order that the idea of cosmic music has its roots; and number speculation springs from the same soil.

But relationships that usually have their effect unconsciously, or only enter consciousness as the result of slow and patient reflection, become immediate, overwhelming experience in ecstasy. The soul that in ecstasy, or dream, or trance, travels to heaven, hears there the music of the universe, and its mysterious structure immediately becomes clear to him. The incomparable and supernatural sound is part of the same thing as the incomparable beauty and colorfulness of other worlds.[35] If Pythagoras was something like a shaman, who in ecstasy made contact with worlds "beyond," then the tradition that he personally heard the heavenly music surely preserves something of truth.[36] When we look beyond the facade of analysis and explication of the harmony of the spheres, what we find is neither empirical nor mathematical science, but eschatology. In the religion of Zarathustra, the paradise to which the soul ascends is called *garo demana*, "House of Psalms." It was related of Pythagoras that in his dying hour he asked that the monochord be played: "Souls cannot ascend without music."[37]

Attention has often been called to the fact that changes were forced upon Greek beliefs about the fate of the dead, not least by the developing understanding of astronomy.[38] If the earth is a sphere, inhabited

[35] Ch. IV 3, n. 50.

[36] Above, n. 3. Cf. also Pl. *Rep.* 617b, Nechepso–Petosiris fr. 1 Riese = Vett. Val. 6 prooem. p. 241 Kroll: *ἔδοξε δέ μοι πάννυχον πρὸς ἀέρα . . . καί μοί τις ἐξήχησεν οὐρανοῦ βοή*; Cic. *Rep.* 6.18, Plut. *De gen.* 590c, *De fac.* 944ab, *Corp. Herm.* 1.25.

[37] Aristid. Quint. 3, p. 116 M. = p. 97 W.-I.; Varro on the lyre of Orpheus in the Vergil scholium edited by J. J. Savage, *TAPA* 56 (1925) 235; cf. A. D. Nock, *CR* 41 (1927) 169–171, 43 (1929) 60.

[38] Often emphasized by Cumont (*After-life* 91ff, *Or. rel.* 114ff, 270f, *Symb.* passim, *Lux* esp. 142ff). Also see Capelle; Pfeiffer, *Sternkl.* 113ff; Carcopino, *Bas.* 266ff; Nilsson II 470ff, *Op.* III 250–265; van der Waerden, *Anf.* 204–252.

all around and limited in size, there is no longer any place for the subterranean Hades of Homer or for the "Isles of the Blest" in the distant Occident. In general, the idea finally prevailed that the Beyond is in the realm of the stars, that man's soul came from the skies and will return there some day. With many variations in detail, these concepts are dominant from the late Hellenistic period on, both in works of literature and in the art and the inscriptions found on gravestones.[39] Looking for the decisive turning point in this development, modern scholarship, following ancient tradition, has seen oriental influences at work—Babylonian and Iranian—and, notably, the influence of Pythagoreanism.[40]

To be sure, Rougier's attempt to establish the belief in an immortality among the stars as a deduction from an astronomical discovery of Pythagoras, making the religious idea a logical conclusion from scientific knowledge, is not convincing in the light of the history of either religion or astronomy.[41] It is a dangerous oversimplification to

[39] Cf. esp. Cic. *Somn. Sc.*, the lost *Consolatio*, and *Hort.* fr. 97 Müller; Vergil *Aen.* 6 (with Norden, Vergil VI, 23ff); Plut. *De sera* 563ff, *De gen.* 589ff, *De fac.* 942ff. Tomb inscriptions are collected by Capelle 33ff; Rougier 108 n. 1; R. Lattimore, *Themes in Greek and Latin Epitaphs* (1942) 31–43; P. Lambrechts, *Hommages Déonna* (Brussels, 1957) 322ff. None of these is complete. On grave symbolism, Cumont, *Symb.* passim; On the representation of ascent and descent of the soul in Mithraism, Cumont, *Or. rel.* 114ff, 145f, 290 n. 69.—On the likely role of Posidonius in this connection, see Reinhardt, *Kosmos* 276ff, 308ff, *RE* XXII 778–791.

[40] Cumont sees the origin in a fusion of Babylonian astral religion and Persian dualism (*After-Life* 95, *Or. rel.* 272 n. 91, *Symb.* 264, *Lux* 143ff). See also Pfeiffer, *Sterngl.* 113ff. Bidez, *Eos* 9ff, 98f, believes that the origin was Babylonian but that it was transmitted by the Pythagoreans, and van der Waerden's solution is similar. With Rougier, Boyancé (*REG* 1952, 314f) warns against overhasty derivation from the Orient, and emphasizes the role of the Pythagoreans, even of Pythagoras himself (349), but also that of more ancient beliefs and later of Platonism. Nilsson takes a similar line, *Op.* III 250ff.

[41] According to Rougier, the origin of the idea is not the religious imagination of the easterners, but "la révolution astronomique de Pythagore" (21ff), i.e., the discovery of mathematical physics. Pythagoras' discovery of the contrary movement of the sun through the zodiac meant that the movement of one of the so-called planets had been seen as perfectly circular, and the mathematical arrangement of celestial phenomena recognized. As "conséquences religieuses" (42ff) of this came the doctrines of the presence of souls in the stars, and of their divinity, of the relationship between the stars and the soul, because of their eternal movement, of the dualism of heavenly order and terrestrial confusion; and the reflections of these doctrines in Alcmaeon prove, Rougier thinks, that their originator was Pythagoras. To this it may be replied, first, that the sun is not a planet, for the unsophisticated observer, but the paradigm of cosmic order (above, ch. IV 2, n. 18). Its movement with respect to the zodiac was already known to Cleostratus, to say nothing of the Babylonians, and the concept of independent movement of the planets, is, in its basis, older than the Ionian vortex theory (above, ch. IV 2). In all this, then, there was nothing to discover that might have meant an "astronomical revolution." Rougier stopped short, rightly, of ascribing to the Pythagoreans a mathematical theory of the actual movements of the planets. He recognizes the originality of Eudoxus (26ff), and does not inject the epicycle theory into the discussion, though he does put forward

represent "the Homeric religion" as followed by a "Pythagorean revolution." In people's beliefs about the afterlife, there are present from the very beginning a large number of overlapping and contradictory themes;[42] even in Homer, the idea of the musty "House of Hades," hateful to the gods, coexists with that of Elysium and with the apotheosis of Heracles.[43] The mystery cults bring man the hope of escaping death and joining the gods; and it is an easy step from this to the doctrine that man is of divine descent and returns at death to his place of origin.[44] The association of gods and sky is primeval and seems self-evident.[45] Artistic representations from the archaic period show the journey of the deified dead into the Beyond. A team of winged horses may provide escort "to Heaven,"[46] whereas sea creatures —Tritons, Nereids, dolphins—point to the Isles of the Blest, beyond

the vague suggestion that the Pythagoreans may have tried the same kind of explanation for the other planets as for the sun (22); but here he fails to notice that the passage of Theo he cites (150.12ff) anachronistically introduces the epicycles. Rougier overlooked an important piece of evidence, the *acusma* about the planets as "hounds of Persephone," which does provide a connection between Pythagorean teaching and the oriental ideas alleged to be so radically different (above, ch. IV 1, n. 109). For argument against Rougier, see also Cumont, *Symb.* 116 n. 3, *Lux* 147 n. 1.—Rougier republished his 1933 monograph almost unchanged in 1959 (*La religion astrale des Pythagoriciens*), but omitted the references, so that the older work has been cited here.

[42] A Maori can depict the dead as descending into the underworld, while at the same time maintaining that they live in the tenth heaven, says L. Lévy-Bruhl, *The "Soul" of the Primitive* 175. Thus in the Trophonius ritual Timarchus travels into the depths and is by virtue of this translated to the stars (Plut. *De gen.* 22.590b; cf. Vergil *Aen.* 6, and below, n. 46.)

[43] The Elysium idea itself shows the blending of lightning worship and the myth of the Isles of the Blest (Burkert, *Glotta* 39 [1961] 208ff). The apotheosis of Heracles is doubtless very old, though the verses about it at *Od.* 11.601ff and Hes. fr. 25.26–33 M.–W. were athetized in antiquity.

[44] For τὸν λοιπὸν χρόνον μετὰ θεῶν διάγειν as a goal in the mysteries, see Pl. *Phd.* 81a. Also the Gold Plates: θεὸς ἐγένου ἐξ ἀνθρώπου (DK 1B18, 20) and καὶ γὰρ ἐγὼν ὑμῶν γένος ὄλβιον εὔχομαι εἶναι (DK 1B18.3, 19.3; cf. 17.7, 17.6, 17a3.)

[45] θεοὶ οὐρανίωνες, cf. the relationship of root and idea in *Zeus, deus, dies.* On *Olympus*, above, ch. III 2, n. 31. Dionysus, too, is brought into connection with the stars: ἰὼ πῦρ πνεόντων χόραγ' ἄστρων, Soph. *Ant.* 1147; cf. Diod. 1.11: *Εὔμολπος μὲν ἐν τοῖς βακχικοῖς ἔπεσί φησιν· ἀστροφαῆ Διόνυσον ἐν ἀκτίνεσσι πυρωπόν.* Musaeus, the father of Eumolpus, is a son of Selene (above, ch. IV 3, n. 48).

[46] Etruscan gems (Furtwängler, *Gemmen* I pl. 7, nos. 1, 2, 3, ca. 600 B.C.) were compared with the *Phaedrus* myth by Delatte, *Litt.* 74f; the iconographic motif of the team of winged animals was, however, taken from the Orient, and originally does not have sepulchral connotations; see the examples published by J. Boardman, *Antike Kunst* 10 (1967) pl. 2; but it acquired these by the 6th century B.C.; see, on the bronze chariot of Monteleone in New York, R. Hampe and E. Simon, *Griechische Sagen in der frühen etruskischen Kunst* (Mainz, 1964) 53–67; generally, Meuli, Bachofen VII 498f; F. Matz, *Gnomon* 33 (1961) 63–65. On the throne of Amyclae, Hyacinth and Polyboea were represented "going to heaven" (time of Croesus: Paus. 3.19.4), although at the same time Hyacinth receives his death offerings through a door in the altar (Paus. 3.19.3). Cf. also Wuilleumier 357, 547ff.

the Ocean. The journey to the Beyond is outside of all geography. In any case, the notion of divine origin and of return to heaven was widespread, at least in germ, before the time of Pythagoras. Another element present is the divinity of the stars, which inevitably gained in significance through contact with Babylon.[47] Then there is the popular belief in a connection between man and star,[48] which had taken on a special form in Egypt,[49] and the general belief in ghosts—that is, in the spirits of the dead and their proclivity to roam about through the atmosphere, especially at night, so that in a mysterious way they have a special relationship to the moon.[50] And, finally, there is the Iranian conception of the soul's making a journey to heaven.[51] The position of Pythagoras and the Pythagoreans in this complex tangle is a question that cannot be settled by general considerations, but only by analysis of the evidence.

The idea of astral immortality is attested a good number of times for the fifth century B.C. Aristophanes alludes to the doctrine that people become stars after death;[52] Alcmaeon put the eternal motion

[47] Cf. Wilamowitz, *GldH* I 253ff = I 248ff, 2nd ed.; Nilsson, *Op.* III 31–39. Aristotle says (*Met.* 1074b1) that the *ἀρχαῖοι καὶ παμπάλαιοι* taught the divinity of the stars *ἐν μύθου σχήματι*, but worship of the stars was generally taken as the sign of a barbarian (Ar. *Pax* 406, Pl. *Crat.* 397d). For a certain example of oriental influence on the Pythagoreans, see above, ch. II 4, n. 47. Aeschylus calls the stars (*Ag.* 6) *λαμπροὶ δυνάσται ἐμπρέποντες αἰθέρι.*

[48] Examples from American Indians to Australian aborigines in J. G. Frazer, *The Golden Bough* IV (London, 1911) 64ff; Capelle 19ff. There are two versions. Either the soul of the dead becomes a star, and a shooting star is a soul hastening to its rebirth, or the living person "has" his star, which at his death is extinguished in the form of a shooting star. The former version is still influential in Plato's *Phaedrus*, as Stenzel showed (*KlSchr* 1ff; he compares *Rep.* 621b and Plut. *De gen.* 22.591c–d). The latter is the opinion of the *volgus* according to Plin. *HN*2.28; cf. Euseb. Migne 86.1.453, Lucian's Lychnopolis, *Ver. hist.* 1.29, Boll, *ZNTW* 18 (1917) 40ff, Eur. fr. 971. The metamorphosis of the dead into a star is often mentioned in sepulchral verse (not in prose inscriptions; poetic form allows more liberty): Peek, nos. 1097, 1829, 1776, 648; *Arch. eph.* 1953–1954, 2, 290–296.

[49] The dead person lives on in the sky, in the retinue of the sun bark (Plut. *De Is. et Os.* 21; Kees 42, 87ff).—Greek catasterisms go back to a very early period; this is attested at least for the Bear and Orion (Hes. frr. 148, 163), though these are exceptional cases.

[50] Cf. Capelle 3ff; Cumont, *Symb.* chs. II–III. It is not clear how far back the identification of Hecate and the moon-goddess goes (Norden, Vergil VI 23f, and Kerényi 79f find it as early as *Hymn. Hom. Cer.* 52), or that of the *Gorgoneion* with the moon (Meuli, Bachofen VII 497f).

[51] Above, ch. IV 1, n. 57.

[52] Ar. *Pax* 832–837: the slave asks Trygaeus, after his return from the sky, whether it is true that the dead become stars, as people say. Trygaeus replies that it is indeed true, and that Ion of Chios, who composed a poem on the morning star, was, after his death, greeted in the sky as *ἀοῖος ἀστήρ*. This need not be based on anything more than Ion's poem; and for the claim that Ion was a "Pythagorean poet" (Cumont, *After-Life* 95; Capelle 24f), the fact that he mentions Pythagoras in 2 of the 119 fragments (in Blumenthal's collection) is a slight basis (above, ch. II 3, nn. 13, 51).

of the "divine" stars together with that of the soul to make a proof of immortality;[53] and Euripides represents Helen as translated to the "palace of Zeus" beyond the starry sky.[54] The idea is often expressed that man consists of body and soul, and in death the body returns to the earth, and the soul to the heavenly aether whence it came; in the epigram for those who fell at Potidaea (432 B.C.) this view acquires a quasi-official recognition.[55] It occurs not only in Euripides, but in Epicharmus; and this might seem to point directly toward a Pythagorean origin. But in this respect as in others Euripides is connected closely, in the tradition, with Anaxagoras.[56] The latter spoke of an *aetherius calor* in the embryo—life comes from the "warm" aether, then.[57] For Diogenes of Apollonia the soul was a part of the "divine" air and thus a *μόριον θεοῦ*.[58] Anaxagoras, too, regarded the soul as immortal,[59] and that is why these two are ranged with Pythagoras. Medical writers reflect the same belief;[60] but even in Leucippus and

[53] Above, ch. III 3, n. 97. The etymologies *θεός : θεῖν* and *αἰθήρ : αἰεὶ θεῖν* (Pl. *Crat.* 397c, 410b) probably related to something earlier—perhaps Alcmaeon, perhaps even the Ionians (below, n. 60). If the Pythagoreans, in their interpretation of the motes in a sunbeam, saw motion as the essential characteristic of soul (Arist. *De an.* 404a16ff; cf. above, ch. II 4, n. 138), both Anaxagoras and Democritus had the same idea (Arist. *De an.* 403b20ff).

[54] Eur. *Or.* 1683–1690.

[55] *IG* I² 945 = Peek no. 20 line 5: *αἰθὲρ μὲμ φσυχὰς ὑπεδέχσατο, σό[ματα δὲ χθόν]*; Epicharm. DK 23B9: *συνεκρίθη καὶ διεκρίθη κἀπῆλθεν ὅθεν ἦλθεν πάλιν· γᾶ μὲν εἰς γᾶν, πνεῦμα δ' ἄνω*, cf. fr. 22; Eur. *Supp.* 531ff: *ἐάσατ' . . . ὅθεν ἕκαστον ἐς τὸ φῶς ἀφίκατο, ἐνταῦθ' ἀπελθεῖν· πνεῦμα μὲν πρὸς αἰθέρα, τὸ σῶμα δ' ἐς γῆν*, Eur. *Erechtheus* fr. 65.71f Austin: *ψυχαὶ μὲν οὖν τῶνδ' οὐβεβᾶσ' Ἄιδην πάρα, εἰς δ' αἰθέρ' αὐτῶν πνεῦμ' ἐγὼ κατῴκισα.* Cf. *Supp.* 1140, *Hel.* 1013ff (soul and aether immortal), frr. 839, 877, 971 (also Rohde, *Psyche* II 255f = 435f, Eng. ed.) The same conception on other tomb inscriptions: *IG* II/III² 11466 = Peek no. 1755 (early 4th century B.C.), 12599 (Peek, no. 1759, 3rd century B.C.); Peek nos. 1760–1761.—Hippoc. *Hebd.* 52 = *Aph.* 8.12 (VIII 672 L.).

[56] Eur. fr. 839 = Anaxagoras A112, lines 8ff:

χωρεῖ δ' ὀπίσω
τὰ μὲν ἐκ γαίας φυντ' εἰς γαῖαν,
τὰ δ' ἀπ' αἰθερίου βλαστόντα γονῆς
εἰς οὐράνιον πάλιν ἦλθε πόλον·
θνῄσκει δ' οὐδὲν τῶν γιγνομένων,
διακρινόμενον δ' ἄλλο πρὸς ἄλλου
μορφὴν ἑτέραν ἀπέδειξεν.

[57] Anaxagoras A109. W. H. Friedrich finds the origin of the whole idea in Anaxagoras (*Philologus* 97 [1948] 281f).

[58] Diogenes of Apollonia A20, 19 §42.

[59] Aët. 4.7.1.

[60] E.g. Hippoc. *Carn.* 2 (DK 64C3): an *ἀθάνατον θερμόν* which "thinks, sees, hears, and knows everything"; most of it has "pushed out into the outermost circumference, and this, I think, is what the ancients named *αἰθήρ*."—The concept of a "world soul" (which van der Waerden, for example, *Astr.* 25, supposes to be specifically Pythagorean, in contrast to the Ionian physics) is implied in such doctrines. According to Plato (*Crat.* 412d–e) "many people" believe in a *διεξιόν* which penetrates the whole world, sometimes

Democritus the soul is "fiery," like the sun and the moon,[61] so that Lucretius, carrying on their materialist tradition, could borrow expressions from Euripides.[62] The soul was "fire" according to Parmenides and Hippasus,[63] but Heraclitus too is said to have called it *scintilla stellaris essentiae*.[64] And, as early as Anaximenes, the ψυχή was related to the "divine" air, which surrounds and supports the cosmos.[65]

That the human soul has a very close relationship to the sky and the stars, and even that it comes from heaven and returns to it, is thus a generally held belief in Ionian φυσιολογία, at least from the time of Heraclitus and Anaxagoras.[66] In the garb of φυσιολογία, and in "materialist" phraseology, what starts as a μῦθος continues to exert its influence sometimes with more emphasis on salvation of the soul, and sometimes with more on the general thought of microcosm and macrocosm: man is made of portions of the cosmos, and in death like returns to like.[67]

If this whole development were to be traced back to Pythagoras, one would have to ascribe to his teaching a maximum of influence and a minimum of definite content; for the testimonia are extremely disparate. The divinity of the stars, which was known to Epicharmus,[68]

called sun, sometimes θερμόν, sometimes νοῦς. From the circle of Diogenes come expressions like that of Ar. *Nub.* 229ff; and Pl. *Crat.* 396c (the Muse *Οὐρανία* is ὀρῶσα τὰ ἄνω, ὅθεν δὴ καί φασιν . . . τὸν καθαρὸν νοῦν παραγίνεσθαι οἱ μετεωρολόγοι) is not simply "Pythagorean," either (pace Boyancé, *REG* 1941, 156f; Joly 36f).—A "divine" θερμόν as ψυχῆς δύναμις, ἀνάλογον οὖσα τῷ τῶν ἄστρων στοιχείῳ, is recognized by Aristotle (*Gen. an.* 736b29ff, on which see F. Solmsen, *JHS* 77 [1957] 119–123; H. A. T. Reiche, *Empedocles' Mixture, Eudoxan Astronomy, and Aristotle's Connate Pneuma* [Amsterdam, 1960] 97ff; Arist. Περὶ φιλοσοφίας fr. 27 W. = Cic. *Acad.* 1.26).

[61] Leucippus A28, Democr. A 101–102; cf. D.L. 9.44.

[62] Lucr. 2.991ff, after Eur. fr. 839.

[63] DK 18.9, 28A45; cf. 46a–b.

[64] Heraclitus A15: the νοῦς is related to the περιέχον; A16 §129ff, cf. A17: the return of the soul πρὸς τὸ ὁμογενές. Stoic interpretation and systematization seems to be involved here; Heraclitus spoke of the "death" of the soul (fr. 36), or at least of certain souls (cf. Kirk in KR 205ff).

[65] Anaximenes fr. 2, and J. Longrigg, *Phronesis* 9 (1964) 1–4. A θεῖον περιέχον (the ἄπειρον) is already present in Anaximander.

[66] "There is nothing Pythagorean here," says Nestle (ZN I 610 n. 1) of the Epicharmus fragments concerned. On the other hand, Pfeiffer (*Sterngl.* 114) avers that Heraclitus owed "wichtige Grundzüge seiner Weltanschauung" to Pythagoras. It seems to be Plato's adverse criticism of him that prevents scholars from advancing the same thesis in relation to Anaxagoras.

[67] Already in the Rig Veda; see H. v. Glasenapp, *Die Religionen Indiens* (Stuttgart, 1943) 84; Olerud 153. Plato, *Tim.* 63c, purposely restricts this "theory of dissolution" to the material.

[68] Menander fr. 614 Körte: ὁ μὲν Ἐπίχαρμος τοὺς θεοὺς εἶναι λέγει ἀνέμους ὕδωρ γῆν ἥλιον πῦρ ἀστέρας. To be sure, this sounds less like ancient piety than incipient skepticism; since one no longer believes in the mythical gods, one restricts himself to what is visible.

and the analogical conclusion of Alcmaeon are as far from the idea that the soul is made of heavenly fire as is the uncomplicated notion that souls become stars. To be sure, these various contradictory ideas are all at one time or another ascribed to Pythagoras.[69] But in Pindar's exposition of metempsychosis there is no more trace of astral motifs than in Empedocles' theory of the fallen *daimon* exiled to earth, into the "cave" (Hades he placed in the realm of air).[70] In the monuments of southern Italian eschatology, there is no reliable, early evidence for belief in astral immortality.[71]

There remains to consider the *acusma* which asks the question, "What are the Isles of the Blest?" and answers "The sun and moon."[72] This places the Beyond in the orderly cosmos; it represents the same desire for stability that forges a theology of the soul out of myths about the soul and puts ritual taboos together into a "way of life"

[69] *Hypomn.* 27: The sun, moon, and stars are gods by virtue of the θερμόν working in them. The soul is an ἀπόσπασμα αἰθέρος, immortal (28) and invisible (30). After death it floats in the air until Hermes conducts the pure souls ἐπὶ τὸν Ὕψιστον (on this expression see Cumont, *Or. rel.* 58, 273 n. 93; *RE* s.v. Hypsistos; Cook, *Zeus* II 876ff; Zeller III 2.106, n. 2, is hardly right in seeing Jewish influence here).—Varro in *Comm. Bern. Luc.* 9.9 p. 291 Usener: "Pythagoras dixit animas in stellas converti virorum fortium" (Varro *Antiq. rer. div.* fr. 25b Agahd; but see Reinhardt, *RE* XXII 589).—Plutarch gives an exposition of astral immortality in the dialogue on the *daimonion* of Socrates, whose setting is a Pythagorean circle in Thebes (above, n. 39).—On the moon as abode of souls see below, n. 75; on the Milky Way and the gates of heaven, below, nn. 90, 94.

[70] Cf. above, n. 38, and ch. II 3, n. 80. ἄντρον, Emp. fr. 120; in fr. 6 Ἀιδωνεύς is air, Hera earth; cf. the Derveni papyrus, col. 18.7. On the reinterpretation of "Hades," Cumont, *Symb.* 35ff, *Lux* 189ff, Nilsson II 228ff. Possible solutions: Hades could be the night half of the heavenly hemisphere (above, ch. IV 3, n. 64); it could be the moon and the sublunary region (below, nn. 89, 93); or it could be life on earth (Lucr. 3.978–1023; cf. the full treatment of Carcopino, *Bas.* 264ff.—The differentiation of sublunary from translunary regions is ascribed to Empedocles (A 62), as well as Alcmaeon and Heraclitus (cf. above, ch. III 2, n. 32), but we cannot assume that Empedocles held the later doctrine of the eternal, perfect order of the celestial realm; his cosmos is perishable, destined to enter into the perfect unity of the "Sphere."

[71] The Brindisi disc, which depicts the journey of the heroized dead in the zodiac (Wuilleumier 544ff, pl. 45, with refs.), is hard to date. Wuilleumier decides on a period before the Roman conquest, i.e. the 3rd century B.C. at latest. It shares with other discs from Tarentum a good many motifs, but not the zodiac.—The vases of Canosa (3rd century B.C.) which Bachofen made famous have no unambiguously astral symbolism (Meuli, Bachofen VII 493ff).—The traces of astral immortality seen in the Gold Plates are uncertain (αὐτὰρ ἐμοὶ γένος οὐράνιον, DK 1B17.7, from Petelia; ἱμερτοῦ δ' ἐπέβαν στεφάνου, DK 1B18.7, from Thurii, see Carcopino, *Bas.* 314f, and Diels, DK I 16 n.; A. Dieterich, *Kl. Schr.* 95 compared *Orph. Arg.* 761, but in the next verse Persephone is called χθονία βασίλεια).—The puzzling conclusion, ἔριφος εἰς γάλα ἔπεσον, was interpreted as an allusion to the Milky Way by A. Dieterich, *De hymnis Orphicis* (Marburg, 1891) 35ff = *Kl. Schr.* 95ff; more emphatically by Carcopino, *Bas.* 311ff, Wuilleumier 547f. Contra, S. Reinach, *Cultes, mythes et religions* II² (Paris, 1909) 125ff; K. Wyss, *Die Milch im Kultus der Griechen und Römer* (Giessen, 1914) 53ff. The word ἔριφος points to Dionysus (cf. above, n. 45).

[72] Iam *VP* 82 (above, ch. II 4).

(*βίος*). Height and depth, fall and ascent do not become dominant ideas in the theology of the soul until the realm of the stars is taken in to become part of the picture. Related ideas are that the Great and Little Bear are the hands of Rhea, the Pleiades the lyre of the Muses, the planets the hounds of Persephone, an earthquake is a conspiratorial meeting of the dead, and that the purpose of thunder is to frighten those being punished in Tartarus. In this context it is taken for granted that Hades is beneath the earth, and all the allusions to a Pythagorean *katabasis* give the same impression.[73] The conclusion that astronomy makes an underground Hades impossible was not drawn by the Pythagoreans. The Isles of the Blest are not a part of Hades; they are far removed from the land of the dead. The soul goes there, finally, as to its last resting-place, where it listens, no doubt, to the "harmony" of the "Sirens."[74] This separation of the Isles of the Blest from the realm of the dead is doubtless ancient, and presumably the earliest form of the school's doctrine, and therefore the total picture, along with the *acusmata*, makes consistent sense. Of course it is wholly pre-astronomical, as is shown even by the simple association of "sun and moon," as though they were islands in the same sea. There is no hint of the multi-storied universe which is standard in the later tradition.[75] Here, once more, we find that the *acusmata* represent a strand of tradition independent of the later tradition, and also of Empedocles and Plato,[76] and evince a Pythagoreanism still innocent of the scientific view of the world.

Even in Plato's eschatological myths, in which, according to the usual opinion, he does most "Pythagorizing," astronomical motifs only gradually make their appearance, showing that he is not repro-

[73] Above, ch. II 3. Carcopino, *Bas.* 272 n. 4, sees here a backsliding into the primitive, by the *acusmatici*.

[74] Above, nn. 3, 37. There may be some significance in the fact that Homer (*Od.* 12.52, 167) speaks unmistakably of two Sirens.

[75] The moon is very often called the Isle of the Blest (references in Capelle 10ff; Cumont, *Symb.* 177ff; Nock, *AJA* 1946, 142f), e.g. Castor of Rhodes (1st century B.C., *FGrHist* 250F16 Plut. *Quaest. Rom.* 76.282a, Plut. *De fac.* 29, Por. ap. Stob. 1.49.61, Serv. *Aen.* 6.640, 887). However, the moon is usually merely an intermediate stop, and the sun is the next higher stage (Plut. *De fac.* 29.944c, *Amat.* 20.766b, Por. ap. Stob. 1.49.55). Only in *Comm. Bern. Luc.* 9.9 (Posidonius according to Reinhardt, *RE* XXII 589f, 780) are sun and moon on an equal basis: the soul returns "in suam sedem, hoc est in solis globum ac lunae." Reinhardt (*Kosmos* 312 n. 2) doubted the antiquity of the Pythagorean *acusma*.

[76] It is difficult to be sure about the relation of the *acusma* to Alcmaeon, Empedocles, and Philolaus; one gets the impression that they are harking back to still more ancient strata—Alcmaeon to the divinity of the stars, Empedocles to the noncosmological myth of salvation, Philolaus to shamanistic lore about the inhabited moon.

ducing an already completed system. In the *Gorgias*, the Beyond is not brought into any kind of relation to the structure of the cosmos; the only question is the ethical one, of "what kind of man one ought to be." The myth of the *Phaedo* interprets our life as an existence in the deeps, by contrast with that much more precious and wondrous life on high; but in carrying this out, it only describes "the earth," in its amazing size and remarkable structure.[77] The image of the Cave expresses the same conception of life, but it does not have to do with the cosmos, but with the *νοητὸς τόπος* out beyond it.[78] In the concluding myth of the *Republic*, the image of the spindle with its complicated spindle whorl revolving in the lap of Necessity, the system of the planets is depicted, but it has no relationship to the previously described "paths of the souls." When this description presents us with two doorways, in sky and earth, with the good souls traveling through the heaven and the bad ones through the earth, the general conception presupposed is the pre-scientific one of the flat earth, the hemispherical heavenly shell, and the dread Underworld. The souls making their way along the heavenly path naturally find themselves in the company of the stars, and many have the impression that the stars enter and leave the sky by two doors.[79] An indication of Pythagorean influence on Plato is the identification of the road "up" with that "to the right."[80] But the adjustment of scientific and mythical views of the world was not yet attained.

It is in the *Phaedrus* that the fate of the soul and the movement of the heavens are for the first time brought into a really intimate connection. The winged soul-horses, following the gods in their journey to the heavens, striving, at the zenith of the heavenly vault, for the vision of the realm beyond the heavens, are borne along in the revolution of the universe, and brought back, by a circular course, to their starting point.[81] The fall and reascent of the soul are given their most grandiose

[77] Pl. *Phd.* 108d.

[78] Pl. *Rep.* 517b. We need not raise the question here of how much myth and ritual may lie at the base of the simile of the Cave. (Empedocles has the word *ἄντρον* in fr. 120.)

[79] On the idea of the two heavenly gates, see Cumont, *Symb.* 40ff; Meissner 110. On Egyptian details (the sun bark, ships of the dead, the gate between upper and lower world), see Kees 64f, 67ff, 84f.

[80] Pl. *Rep.* 614c; cf. Arist. *Cael.* 284b6ff.

[81] Pl. *Phdr.* 246b: *πάντα οὐρανὸν περιπολεῖ*, 246c: *μετεωροπορεῖ καὶ πάντα τὸν κόσμον διοικεῖ*, 247a: *μακάριαι θέαι καὶ διέξοδοι ἐντὸς οὐρανοῦ* (*διέξοδος* is the technical term for the periodic celestial movements: Hdt. 2.24, Eur. *Andr.* 1086; *Hebd.* 1 had *ἔξοδος*), 247a: *ἄκραν ἐπὶ τὴν ὑπουράνιον ἀφῖδα πορεύονται πρὸς ἄνατες*, 247c: *αὐτὰς περιάγει ἡ περιφορά*, 247d: *ἕως ἂν κύκλῳ ἡ περιφορὰ εἰς ταὐτὸν περιενέγκῃ· ἐν δὲ τῇ περιόδῳ . . .*

expression in the *Phaedrus*. Then comes the *Timaeus*: the Demiurge creates as many souls as there are stars, for each star a soul, puts each into its star "as in a wagon," and thus shows it "the nature of the universe." Then it must leave its star, to be incarnated on earth; then after a period of trial on earth or another planet it may claim the promised return to its *σύννομος ἀστήρ*.[82] Though we hear echoes of folklore motifs,[83] at the same time the relation of soul and celestial movement has become very close indeed: the form of the world soul is the moving principle of the cosmos (36b). The same basic ideas are developed in the *Laws*, without any mythical garb and with an earnest claim to embody the truth. The soul, as the principle of self-movement, is primary as against any kind of corporeality; soul reveals itself in the celestial movements, and the scientifically proven regularity and perfection of these circular movements shows that the soul of the universe is intelligent and good.[84] Then the *Epinomis* repeats the same line of thought, in a systematizing way: the stars are gods, and astronomy is worship.[85]

Plato's students interpreted and systematized their master's myths, and seem, in this process, to have developed the final form of "astral immortality." Little is known about Xenocrates in this connection.[86] It is a tempting conjecture, though not directly attested, that Crantor, in his influential book *On Grief*, presented the thought in a popular way.[87] We can, however, get some impression of the exposition of Heraclides Ponticus, who also worked on special problems of astronomy.[88] He related the vision of a certain Empedotimus.[89] Pluto and

On the debate over astrological influences in the *Phaedrus*, see K. Kerényi, "Astrologia Platonica," *ARW* 22 (1923–1924) 245ff; Bidez, *Eos* 60ff; J. Kerschensteiner, *Platon und der Orient* (Stuttgart, 1945) 183ff; W. J. W. Koster, *Le mythe de Platon, de Zarathoustra et des Chaldéens* (Leiden, 1951) 4ff.—Even as late as the *Phaedrus*, the influence of the pre-scientific idea of the world's structure is felt; the wicked are punished *ὑπὸ γῆς* (249a). But at the same time the universe, as a perfect sphere, should have no up and down, and therefore no zenith.

[82] *Tim.* 41d et seq.

[83] Namely, the identification of soul and star (Stenzel, *KlSchr.* 8; above, n. 48).

[84] Pl. *Leg.* 891c, 967a; see also ch. IV 2.

[85] *θεοσέβεια Epin.* 981d.

[86] The myths of Plutarch are suspected of containing material from Xenocrates (Heinze 123ff). For the distinction of cosmic regions, and the stars as gods, see Xenocrates frr. 5, 15.

[87] Boyancé, *REG* 1952, 337ff. Earlier reconstructions of Crantor's *consolatio* are critically analyzed by R. Kassel, *Untersuchungen zur griechischen Konsolationsliteratur* (Munich, 1958) 35f, 68f.

[88] See above, ch. IV 2, n. 11; ch. IV 3, n. 17.

[89] Frr. 93–94. Probably the name is a deliberate combination of parts of "Empedocles" and of "Hermotimus." In addition to Wehrli's notes, consult Wilamowitz, *GldH* II 533ff; Bidez, *Eos* 52ff. Ancient writers considered him a historical personality like Epimenides, Pythagoras, and Empedocles.

Persephone appeared to him, freed him from the usual restrictions of human eyesight, and revealed to him the truth about the nature and fate of the soul. The cosmos, he saw, is divided into three realms. The spheres of the moon and the elements below it belong to Pluto and comprise the heavenly Hades. The Milky Way, conceived, as in Aristotle, as an atmospheric phenomenon below the region of the stars,[90] is the pathway for these souls; from there they sink to earth, and later they return to it. The soul is a "light," *αἰθέριον, οὐράνιον σῶμα*. The heavenly Hades is not a place to stay forever; above it are the spheres of the planets, the realm of Poseidon, and highest of all is the heaven of the fixed stars, which belongs to Zeus. There must be a connection between the three realms and the three "gates" which Empedotimus sees in the region of the zodiac. One is the entrance Heracles used at his deification.[91] Surely, for every soul the ultimate goal is to follow this road into the company of the gods.

The decisive influence was that of the myths of Plato, Heraclides, and the other Platonists;[92] and especially important were the concepts of the sublunary Hades[93] and the Milky Way as dwellings of the souls.[94]

[90] Boyancé, *REG* 1952, 335 n. 7. The sun is thought of as being below the planets, not in their midst (above, ch. IV 1, n. 7). The Milky Way was known to the Babylonians as a road between earth and heaven (Meissner 111).

[91] Fr. 94. The gates are in the sign of Scorpio, between Leo and Cancer, and between Aquarius and Pisces. Wehrli thinks (92) that this detail is hardly correct, because "it is hard to imagine the zodiac as a path to the subsolar region." But the zodiac is not a hollow ring; it pervades all the spheres, and even the moon travels along it. The road toward incarnation passes through Cancer to Leo, according to Macrobius (*Somn. Sc.* 1.12.4, 7), with the sun moving downward, and the first gate is between these two signs. Pisces and Aquarius are the suitable signs for the realm of Poseidon; and the third gate, that of Zeus, lies directly opposite, in the middle of the sign itself—no longer a transitional stage. The journey depicted on the disc of Brindisi (above, n. 71) goes to Scorpio. The division of the circle into portions of 105°, 105°, and 150° (7 : 7 : 10) may have seemed especially "harmonic" to Heraclides (7 is "the rational diameter of 5" in *Rep.* 546c).

[92] This has also been put forth by Boyancé, *REG* 1952, 321ff.

[93] E.g., Plut. *De fac.* 28.943c, Sext. Emp. *Math.* 9.72f, Cornutus 5; see above, n. 75.

[94] Numenius and Cronius ap. Por. *De antr. nymph.* 28, Macrob. *Somn. Sc.* 1.12, Procl. *In Remp.* II 129, Schol. *Od.* 13.103 (the discrepancy between this and Porphyry is probably a mistake of the scholiast rather than a reflection of independent tradition, as Delatte assumes, *Litt.* 130). The testimonia are collected by Capelle 39f and Leeman 147f. See also Delatte, *Litt.* 129ff. Where the Zodiac crosses the Milky Way, he says, in the sign of Cancer and Capricorn, there are two gates, one "for men" in the north, through which souls descend for rebirth, and one "for gods" in the south, through which souls return to their place of origin. Below the Milky Way begins the realm of Hades; the exposition is ostensibly an explanation of the Homeric lines about the gate of the sun (*Od.* 24.12f) and about the cave of the Nymphs (*Od.* 13.102ff). The only part of this that is explicitly attributed to Pythagoras is that the souls which are punished with reincarnation gather in the Milky Way; and this is brought into connection with the fact that the infant's first food is milk. On the Milky Way as a habitation of souls see also Cic. *Rep.* 6.13, 16; Gundel, *RE* VII 563–566; cf. above, n. 90.

Probably Heraclides mentioned Pythagoras, too, in his *Empedotimus*,[95] but a simple inference from the *Timaeus* may be sufficient cause why the details of astral immortality in later tradition were attributed to Pythagoras. These testimonia do not have pre-Platonic content. Of course the interpretation of the Milky Way could be combined with the naive identification of soul and star,[96] but the Pythagoreans of Aristotle's accounts had a different explanation of the Milky Way.[97] The idea of two heavenly gates is simpler and therefore earlier than the picture drawn by Heraclides, but they belong primarily to a different realm.[98]

Thus the idea of astral immortality only gradually developed into a system apparently built on a scientific basis. At the inception stands shamanistic "knowledge" of cosmos and soul, in the most ancient stratum of the Pythagorean tradition, the *acusmata*. Later on, scientific discoveries became current. The knowledge that the earth is spherical did away with the subterranean Hades—from the time of Empedocles. Recognition of the orderly character of the movements of the planets confirmed—from the time of Eudoxus—the contrast of celestial order and earthly imperfection. This was the path that led to that synthesis of astronomy and religion which we find in the later Plato, in Heraclides, Aristotle, and Xenocrates; we cannot simply call it "Pythagorean." This doctrine then, taking its departure from Plato and Aristotle, finally became canonical. The agreement of science and religion, emphasized by the Stoics, obviously made a tremendous impression on the Romans. That it was, basically, a hasty oversimplification remained unnoticed until, from the time of the Renaissance, Greek natural science was carried forward by new methods.

[95] Julian writes (ap. *Suda* s.v. *Ἐμπεδότιμος*, Heraclides fr. 92): *ἡμεῖς δὲ Ἐμπεδοτίμῳ καὶ Πυθαγόρᾳ πιστεύοντες οἷς τε ἐκεῖθεν λαβὼν Ἡρακλείδης ὁ Ποντικὸς ἔφη* . . . But Julian is not citing at first hand (Wehrli 91). Wilamowitz says of "Pythagoras'" teaching about the Milky Way, "Pythagoras has taken the credit here that belongs to Heraclides" (*GldH* II 535 n. 1).

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V. Pythagorean Musical Theory

1. SPECULATION, EXPERIMENTATION, AND FICTION

It is a striking paradox that music, which is the most spontaneous expression of psychic activity, at the same time admits, or rather even challenges, the most rigorous mathematical analysis. There are two systems by which the division of the tonal continuum may be described, and the distinction between natural and tempered tuning follows from them. One can think of the interval in a spatial metaphor, and equal intervals as representing equal distances or lines; then greater intervals are made up of the sums of smaller ones. The standard unit is the "tone" (or "step"), the difference between a fourth and a fifth; it can be subdivided at will. A fourth, then, comprehends $2\frac{1}{2}$ whole tones, a fifth $3\frac{1}{2}$, and an octave six. The usual divisions of the tetrachord in classical Greek music are, in the diatonic genus or scale, semitone, whole tone, whole tone; in the chromatic, semitone, semitone, tone and a half; in the enharmonic, quarter-tone, quarter-tone, ditone. The image of a line and its divisions is especially natural for us, because of our familiarity with the piano keyboard, and of our system of musical notation; but the Greeks used this image, too, as is shown even by the word they use for "interval," *διάστημα*.[1]

A different system results from the recognition that the harmonic intervals can be expressed as simple numerical ratios. These can easily be illustrated by the length of vibrating strings or sounding pipes. It has been known for a long time that pitch depends on the rate of vibration; and simple, whole-number ratios of frequencies result in the musical concords. The ratio for the octave is 2 : 1, for the fifth, 3 : 2, for the fourth, 4 : 3. Addition of intervals results in multiplication, and subtraction results in division, of the numerical ratios; to halve an interval means the extraction of a square root. In the terminology of modern mathematics, the intervals, thought of as lengths of line,

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correspond to the logarithms of the respective ratios.[2] To this extent the two descriptive systems are equally accurate and can be converted into each other.[3] In antiquity this mathematical schematism was unknown, and there was no adjustment of the two systems to each other. In particular, irrationality was excluded from the theory of musical proportion.[4] The line-segment idea is connected with the name of Aristoxenus, the theory of musical proportion with that of the Pythagoreans.[5] The two schools, Aristoxenians and Pythagoreans, stood in a relation of hostility: the latter accused Aristoxenus' followers

[2] So, for example, fifth + fourth = octave, i.e. $(3:2) \times (4:3) = 2:1$; fifth − fourth = whole tone, i.e., $(3:2):(4:3) = 9:8$. The "semitone" that remains in the diatonic tetrachord, when two whole tones are subtracted from the fourth, is $(4:3):(9:8):(9:8) = 256:243$, the so-called *leimma*, somewhat smaller than a tempered semitone ($\sqrt{9:8}$) (Pl. *Tim.* 36b). Ancient theoreticians usually went through a somewhat more laborious procedure, involving a step-by-step extension of the proportions; e.g., the fifth is $3:2 = 9:6$; the fourth is $4:3 = 8:6$; so that the whole tone is $9:8$ (Theo Sm. 67.16ff, etc.). The fundamental importance of the concept of linear distance is shown by the fact that one speaks of *συντιθέναι* and *ἀφαιρεῖν* of the proportions (Eucl. *Sect. can.* passim). The correspondence of addition and multiplication, and of subtraction and division, was the point of departure for the development by Napier of the method of calculation by logarithms. "Log-arithm" means "ratio-number" (Tannery, *MSc* III 71).

[3] For the method of conversion, and comparison of the values, especially, of Aristoxenus and Archytas, see Tannery *MSc* III 98ff. Of course, equal division of the interval-"line" leads to ratios of vibration frequency that are almost exclusively irrational ("tempered tuning"), whereas frequency ratios using whole numbers lead to transcendental division of the interval-"line" ("natural tuning"), making impossible the establishment of a common unit and therefore precluding modulation.

[4] According to the Pythagorean music theory it is impossible to halve the octave, fifth, fourth, or whole tone, and the tempered halftone simply does not exist. (In the Aristoxenian system one could speak of *ἄλογα διαστήματα*, and the tempered halftone was regarded as "rational"; Cleonides *Is.* pp. 196.27, 200.2ff Menge.) Experience with the monochord compelled recognition that the range of tones is as much a continuum as a divided line; but the experiment was branded inaccurate (Theo Sm. 70.14ff). It was admitted that the movements that produced tones could take place *ἀλόγως πρὸς ἄλληλα* (Adrastus ap. Theo Sm. 50.13ff), but the result, one said, was noises (*ψόφοι*), not musical tones. The acoustic discovery that the number of air impulses determines the pitch (below, n. 41) seemed to exclude irrationality: we are told that *κινήσεις πυκνότεραι* or *ἀραιότεραι* consist of "parts" and therefore must be related like whole numbers, *Sect. can.* prooem. 158.7ff Menge. This overlooks the fact that it is not the number of vibrations itself which is in question, but the number per time unit. The interpolation of the arithmetic and harmonic means leads directly to approximative values for the square root (Tannery, *MSc* III 83f), but it is unlikely that the study of irrationality stemmed from musical theory (as Tannery thinks, *MSc* III 83ff; Mau, *Inf.* 16); the connection is not recognized or presented in any extant source.

[5] Didymus wrote *περὶ διαφορᾶς τῆς Πυθαγορείου μουσικῆς πρὸς τὴν Ἀριστοξένειον* (Por. *In Ptol.* 5.11f, 25.5f, cf. 3.13f; fragments: 26.6ff, 27.17ff, 107.15ff; it was used without recognition by Ptolemy—see the criticism, *Harm.* p. 14.1ff, of what Por. reports at 107.15ff). Ptolemaïs of Cyrene wrote a *Πυθαγορικὴ τῆς μουσικῆς στοιχείωσις* (Por. *In Ptol.* 22.23ff, 25.9ff, 114.5ff; cited by Didymus, see Düring 1934, 157). For exposition of the Pythagorean theory see esp. Ptol. *Harm.* 1.2, 5ff, 9ff, Por. *In Ptol.* 1.2ff.

of imprecision, and were accused, in their turn, of using arbitrary hypotheses and contradicting the clear testimony of the musical ear.[6]

In the one passage where he explicitly names *οἱ Πυθαγόρειοι*, Plato credits the Pythagoreans with a mathematical theory of music;[7] this is one of the few fixed points in the reconstruction of Pythagoreanism before Plato. As early as Xenocrates, the crucial discovery was attributed to Pythagoras himself; and though this testimony is treated with great reserve,[8] still it is generally regarded as established that the first natural law to be formulated mathematically—the relation between pitch and the length of a vibrating string—was a discovery of the Pythagorean school. Let us try, however, to answer somewhat more precisely the question, to what extent, and from what time, there was anything in this area truly analogous to modern science. For in this field, as in others, it may be that a different kind of speculation preceded real science.

In the *Republic*, when he discusses the necessary subjects for the education of the "Guardians," Plato takes up music after astronomy; they are "sister sciences, as the Pythagoreans say and we agree": *ἀδελφαί τινες αἱ ἐπιστῆμαι . . . ὡς οἵ τε Πυθαγόρειοί φασι καὶ ἡμεῖς . . . συγχωροῦμεν* (530d). Socrates will follow them, he says,[9] but they do not fulfill adequately the basic requirement, to push forward beyond the sensible world to true Being and thus to really exact knowledge: "they waste their time like the astronomers, measuring audible concords and sounds against one another" (531a).

Glaucon, the interlocutor, follows this up with a depiction of the activities of the musicologists, bending over their strings and arguing whether it is possible to detect still another difference of pitch, or whether the "smallest interval, the unit of measurement" has been reached.[10] These "worthy men," says Socrates mockingly, subject the strings to a painful inquisition, stretching them on the rack, with pegs. But, he continues, these are not the musical theorists he meant, but

[6] According to Ptolemaïs of Cyrene (Por. *In Ptol.* 23.25ff) the Pythagoreans stated that, if the finding of theory (*λόγος*) contradicted that of the senses (*αἴσθησις*), the latter was false; "This is the canonists' way of thinking, carried in absurdum" (Düring 1934, 144).

[7] *Rep.* 530d et seq.; also Arist. *Met.* 985b31 et saep., Eudemus fr. 142.

[8] Xenocrates fr. 9; cf. above, ch. I 3, n. 68.

[9] *ἐκείνων πευσόμεθα*, 530d. On the passage as a whole, see Frank 150ff; E. Moutsopoulos, *La musique dans l'oeuvre de Platon* (Paris, 1959) 48ff; Richter 54ff.

[10] 531a: *τὸ σμικρότατον διάστημα, ᾧ μετρητέον.*

"those of whom we were just saying that we would consult them about harmony"—that is, the Pythagoreans (531b). They, too, however, fall into an error similar to that of the astronomers:

> They try to find the numerical properties hidden in these audible consonances, but they do not rise to the level of formulating problems and investigate which numbers are consonant and which not, and why.[11]

Thus Plato distinguishes two schools of musical theorists. One group try to find the smallest interval, as a unit of measurement, proceed in a purely empirical way, and rely on their musical sense. They must conceive of the tone continuum as a line, since they are seeking the basic unit, and are therefore the predecessors of Aristoxenus.[12] Their method of study is, to Plato, not even worth discussing; but for the Pythagoreans, who are looking for the "numbers" in the musical concords, he has appreciation to mix with his criticism. He will follow them in detail, though they are still too closely wedded to the empirical. What Plato desiderates is not an analysis of audible music but pure number theory, above and beyond experience. In the *Timaeus*, Plato carried out this program, at least by way of suggestion, using a series of numbers derived from the ultimate principles, which arrayed themselves in a scale without audible sound, the numerically harmonic structural pattern of the world, the "world soul."[13] Succeeding ages

[11] 531b. *πρόβλημα* is a technical term in mathematics; see Oenopides DK 41.12 (*τίνος ὄντος τί ἐστιν*), Pl. *Rep.* 530b: *προβλήμασιν . . . χρώμενοι ὥσπερ γεωμετρίαν οὕτω καὶ ἀστρονομίαν μέτιμεν.*

[12] Among non-Pythagorean music theorists we can name the following: Lasus of Hermione, the teacher of Pindar, was the first to write *περὶ μουσικῆς* (*Suda* s.v., Mart. Cap. 9.936; see Schmid–Stählin I 1, 544ff). Epigonus (Aristox. *Harm.* 1 p. 3.21, Philochorus *FGrHist* 328F23 = Ath. 14.637f, Ath. 4.183d, Por. *In Ptol.* 3.4) constructed an instrument with 40 strings (Pollux 4.59). Stratonicus, according to Phaenias fr. 32 W., was the first to make a *διάγραμμα* (on his chronology, see above, ch. II 5, n. 53). Then there are also Eratocles (Aristox. 1 p. 5.9ff, Por. *In Ptol.* 3.5), Agenor (Aristox. *Harm.* 2 p. 37, Isoc. *Ep.* 8.1, Por. *In Ptol.* 3.5), and Pythagoras of Zacynthus (Ath. 14.637bff, Aristox. *Harm.* 2 p. 36f). In all these the combination of practical and theoretical music is characteristic. On the other hand, Damon (DK 37) is only cited for the ethical and pedagogical value of music. That Aristotle, too, knew two schools of musical theorists is shown by his expressions *οἱ κατὰ τοὺς ἀριθμοὺς ἁρμονικοί* (*Top.* 107a15) and *ἡ ἐν τοῖς μαθήμασιν ἁρμονική* (*Met.* 997b21).

[13] The relationship of the *Timaeus* and the *Republic* on this point was emphasized by Frank, 13ff, 181ff. See also Rivaud, *Rev. Hist. Philos.* 3 (1929) 1ff. On the interpretation of the number series, see Frank 163f, and more recently B. Kytzler, "Die Weltseele und der musikalische Raum", *Hermes* 87 (1959) 393–413. After the "One" (which does not count as a number, Euclid 7 defs. 1–2) come the 2 first numbers 2 and 3 (exemplars of "even" and "odd"; Adrastus ap. Theo Sm. 94.12ff). They are raised to the third power

regarded the construction of the world soul in the *Timaeus* as one of the most illustrious examples of Plato's "Pythagorean" wisdom; but his own words, in the *Republic*, show that he went beyond the teachings of the Pythagoreans in an independent way.

Frank tries to determine the character and chronological position of Pythagorean musical theory from Plato's pronouncement: he "rejects their comparative measurements as the most contemptible empiricism."[14] "What these scholars are striving for is, in a word, natural science and physics on a mathematical basis, quite in our modern sense" (172). It was Plato who first introduced a priori speculations into music theory; and this is another reason, he thinks, to regard the Philolaus fragments as spurious. The time of origin of Pythagorean music theory is "established beyond question" (159) by Plato: Glaucon's misunderstanding showed (Frank thought) that the Pythagorean theory was practically unknown in Athens in Socrates' time, and had therefore only emerged about 400 B.C., in the circle of Archytas. Of course, the numerical ratios that make the basic concordant intervals must have been known, on an empirical basis, for a long time—"every maker of instruments had to know these numerical formulae" (11); but it was only the theory of proportion, on one hand,[15] and on the other the recognition of the nature of sound as air vibrations, that made possible the origin of Pythagorean musical theory.

Van der Waerden paints a different picture, on the basis of a much more even-handed study of the sources. Pythagoras himself, he thinks, may be credited with the arithmetical manipulation of the basic harmonic ratios, which had been known for a long time, and also with the recognition that sound is derived from movements of air.[16] But even in Archytas, in his view, considerations of number theory, rather

because it is the three-dimensional physical world which is to be constructed (cf. *τρὶς αὐξηθείς Rep.* 546c, Arist. *Pol.* 1316a7f, Pl. *Tim.* 32b [in a proportion with 2 mean proportionals, the outer members are cubes], *Leg.* 894a and the succession line–plane–solid [above, ch. I 1], *Epin.* 991a, and the ancient commentators on the *Timaeus*, Adrastus ap. Theo Sm. 65.1ff, 95.21ff, and Procl. *In Tim.* II 170). The further rule, to supplement the resulting *ἐπίτριτα* with *ἐπόγδοα* (36b), cannot be explained by a priori considerations, but only from the needs of music.

[14] Frank 152, cf. 13; 161, 172.

[15] Frank's statement that "the proportion theory of the intervals was first worked out by Eudoxus" (160) is not supported by Theo Sm. 61.11ff (= Archytas A19a); Eudoxus made the theory of proportion applicable to irrational relationships (cf., e.g., Becker, *MD* 102ff; van der Waerden, *EW* 309ff = *SA* 187ff) and this has nothing to do with music theory (above, n. 4); the bases of calculation by ratios are much older; cf. ch. VI 1.

[16] *Hermes* 1943, 179, 192. Van der Waerden rightly emphasizes the age of the *acusmata* tradition. (Above, ch. II 4, n. 157 on the tetractys.)

than empirically exact measurements, were most important; and the monochord was probably not invented until after Archytas' time. Thus the course of the development would be, from the everyday experience of the maker of instruments to the theories of the Pythagoreans, and then to the experiments that can really be called scientific. Van der Waerden did not take account, however, of the fact that alongside, and prior to, the Pythagorean theory of music there was also a non-Pythagorean music theory and a non-Pythagorean natural science.

The question of what observations lay behind the discovery of the numerical relations of musical intervals, and of when this happened, is harder to answer than it first seems. The often expressed opinion, that these numbers had been known "from time immemorial" in the daily work of the makers of musical instruments,[17] does not comport well with the nature of Greek instruments. The most common stringed instruments have strings of equal length and no finger board, though flageolet tones seem to have been recognized.[18] In the triangular harp the tension of the strings and their thickness played some role, but we do not know just what.[19] In a wind instrument with finger holes, that is, the *aulos*, the distances between the holes do not correspond directly or accurately to the ratios of the intervals;[20] actually the holes were simply bored at equal distances. The syrinx was not used in the music of the classical period.[21] And the monochord with a movable bridge, the *κανών*, the only "instrument" on which Pythagorean musical

[17] So Frank 11f, 161; Tannery, *MSc* III 241; Kranz, *Philologus* 1938, 437; van der Waerden, *Hermes* 1943, 172.

[18] *διάληψις* Arist. *Pr.* 19.12 cf. 23; Theo Sm. 59.22; Jan, *Musici scriptores graeci* p. 84 n.

[19] A reference to the *τρίγωνα ψαλτήρια τῆς ἴσης ἐπιτάσεως γινομένης* (!) Arist. *Probl.* 19.23. M. Schmidt, *Zur Entstehung und Terminologie der elementaren Mathematik* (Leipzig, 1914²) 108, derives the discovery of the numerical relationships from this "Egyptian harp"; similarly Farmer ap. E. Wellesz (ed.), *New Oxford History of Music* I (Oxford, 1957) 275. Cf. Ael. ap. Por. *In Ptol.* 34.29ff.

[20] Ptol. *Harm.* 1.8, p. 17.2ff; 2.12, p. 66.31: flutemakers (*οἱ τὰ ἔμπνευστα ὄργανα κατασκευαζόμενοι*) proceed in a completely empirical fashion. The assertion that they placed the finger holes with reference to the numerical ratios is a product of the scholarly imagination (Arist. *Pr.* 19.23: *καὶ οἱ αὐλοτρῦπαι οὕτω λαμβάνουσιν*, Ael. ap. Por. *In Ptol.* 34.21ff, Theo Sm. 61.2, Nicom. *Ench.* 6 p. 248.15, 10 p. 255.4ff, Por. *In Ptol.* 119.22ff). Contra, Aristox. *Harm.* 2 p. 37.25ff; K. Schlesinger, *The Greek Aulos* (London, 1939).

[21] It only came into common use as a musical instrument with the water organ (Por. *In Ptol.* 119.28f, *ὕδρας*). References to experiments with the syrinx: Arist. *Pr.* 19.23, Theo Sm. 60.6f, Cens. 10.10f, Nicom. *Ench.* 6 p. 248.15, Por. *In Ptol.* 119.14ff. The remoteness of the theorists from practical music is shown in the nonsensical statement that the pipes had to be equal in thickness, and that wider tubes would produce higher tones (Ael. ap. Por. *In Ptol.* 34.11ff).

theory can be demonstrated with any approach to exactitude, is an artificial device for experimentation, the time of whose invention is controversial.[22]

An extensive and richly attested tradition makes Pythagoras himself the discoverer of the numerical ratios; but the observations and experiments attributed to him are impossible, physically. One story is that in passing a smithy he recognized, to his surprise, that the sounds made by the hammers exemplified the intervals of fourth, fifth, and octave. He ascertained that the only difference among the hammers was their weight, and found that their weights were related in the ratios 4 : 3, 3 : 2, and 2 : 1. The law presupposed here, that the vibration and sound of a metallic body are directly proportional to their volume and weight, is false.[23] The story goes on that Pythagoras hung weights corresponding to these hammers from equally long strings, and found, on plucking them, that the same intervals were produced.

[22] The crucial point is the *Sectio canonis* (*Κατατομὴ κανόνος*), transmitted under the name of Euclid (ed. H. Menge). Euclid's authorship is contested (Tannery, *MSc* III 213ff; Menge xxxvii et seq., hesitantly Düring 1934, 177), and in addition Tannery claimed that the only passage to mention the *κανών* (props. 19–20) is an addition made in the time of Eratosthenes. In these propositions the diatonic scale is constructed, but in prop. 17 the *λιχανός* is 2 whole tones below the Mese, which presupposes the enharmonic scale. But aside from the fact that 19–20 may have displaced an older, enharmonic *sectio canonis*, the juxtaposition of enharmonic and diatonic is comprehensible; The enharmonic is basic to musical practice, and hence comes the name *λιχανός* = f; but as the basis for *μεταβολαί* one needs the diatonic *ἀμετάβολον σύστημα*, which is built up in props. 19–20, and here we do not find the word *λιχανός* but only *διάτονος* = g. That the illustrative figure gives *λιχανός* means nothing, in view of the uncertain transmission of such drawings. Van der Waerden (*Hermes* 1943, 172ff, 177) therefore puts the invention of the *κανών* after 300 B.C.; but, since neither Tannery's analysis nor the conclusion drawn from it is certain, we may go back further than that. Aristotle never mentions the *κανών*; for him music belongs to arithmetic, just as optics does to geometry (*An. post.* 75b14ff, 76a9ff, 22ff, *Met.* 1078a14ff), while the *κανών* presents a combination of music and geometrical line-division. We should probably be justified in taking this as a terminus post quem. H. Koller dates the invention of the *κανών* in the 5th century (*Glotta* 38 [1959] 66ff), but his derivation of logic and epistemology from music theory is pure construction. Duris speaks of the *κανών* (*FGrHist* 76F23); see also Philodemus (*Mus.* p. 100 K.) and Varro (Gell. 16.18.4). Its invention is attributed to Pythagoras by D. L. 8.12, Gaudentius 11 p. 341.12ff Jan, Boeth. *Mus.* 1.11; cf. Aristid. Quint. 3 p. 116 M., Procl. *In Tim.* II 174.23, Por. *In Ptol.* 120.17ff. We can disregard here the other experimental devices that Ptolemy describes (*Harm.* 2.2, 2.12f, 3.1). See also H. Oppel, *ΚΑΝΩΝ* (Leipzig, 1937; *Philologus* Supp. 30.4). Late texts on the "Pythagorean *kanon*" have been edited by A. Stamm, *Tres canones harmonici* (Diss. Strassburg, 1881).

[23] The physical impossibility of the alleged experiments of Pythagoras was shown by M. Mersenne, *Questions harmoniques* (Paris, 1634) 166 (Schuhl, *Essai* 262.2; Capparelli II 627). See also van der Waerden, *Hermes* 1943, 170ff; H. Oppermann, "Eine Pythagoraslegende," *Bonn. Jb.* 130 (1925) 284–301. For the narrations, see Nicom. *Ench.* 6 p. 245ff = Iam. *VP* 115ff, abbreviated Iam. *In Nic.* 121.13ff, most vividly Macrob. *Somn. Sc.* 2.1.9ff; also Gaudentius 11 p. 340 Jan, Boeth. *Mus.* 1.10, Isid. *Et.* 3.16.1 (cf. Zeller I 508.1).

But again, the proposition that the frequency of vibration of a string is proportional to its tension is false.[24]

Our oldest attestation for this tradition is that of Nicomachus and Adrastus. It is definitely earlier than Ptolemy, who rejects the weight experiment, since it could only bring *διαβολαί* to the correct theory.[25] Thus the inauthenticity of the experiments had already been recognized and used in polemic against the Pythagorean musical theory. It is unlikely that Adrastus and Ptolemy are dependent on Nicomachus,[26] for the exegesis of the *Timaeus* reflected in Adrastus and Macrobius is based on earlier material. In fact, Xenocrates attributed the discovery of the musical ratios to Pythagoras himself (above, n. 8); and it is as unlikely that he omitted to mention the way the discovery was made as it is that he mentioned the *κανών* (above, n. 22). Should we suppose that here again we have a falsified tradition about Pythagoras emerging from the Old Academy?

In any case the legend, in spite of its physical impossibilities, does make a certain kind of sense. The mythical inventors of smithcraft, the Idaean Dactyls, were regarded not only as wizards and founders of mystic rites, but also as the inventors of music. They were mentioned along with the Curetes and Corybantes, but also with Orpheus, and even with Pythagoras.[27] Music magic is found throughout the mystery cults[28] and takes on a special character among the Pythagoreans. Because of this ritual and magical background, we should take seriously Aristoxenus' reports about the Pythagoreans' musical *κάθαρσις*;[29] and Plato's conception of music belongs in this context.[30] The *acusma* which states that the sound of bronze when struck is the

[24] Nicomachus, Iamblichus, Gaudentius, Macrobius, Boethius, as cited in the preceding note; without the story of the hammers, Adrastus ap. Theo Sm. 57.4 = Chalcid. 45, Cens. 10 (Varro?), Por. *In Ptol.* 119.29ff, cf. Theo Sm. 60.7f; 66.21ff, Aristid. Quint. 3 p. 113 M. For an attempt to explain why such an experiment fails to work, see Ptol. *Harm.* 1.8 p. 17.7ff. In reality, the pitch or frequency is proportional to the square root of the tension; e.g., 4 times as much weight produces an octave; but this law does not seem to have been discovered in ancient times (Tannery, *MSc* III 440). The *κανών* is mentioned last in Porphyry, Gaudentius, and Boethius; and, in Nicom. *Ench.* 10, it is treated without any mention of the discovery of musical ratios.

[25] Ptol. *Harm.* p. 16.32ff.

[26] Adrastus was about contemporary with Nicomachus. He does not tell the impressive story of the smithy.

[27] See Kern, *RE* IV 2018ff; B. Hemberg, *Eranos* 50 (1952) 41–59; esp. Ephorus *FGrHist* 70F104 = Diod. 5.64, Plut. *Mus.* 5, Clem. Al. *Strom.* 1.73.1, Solinus 11.6. Terpander was regarded as the descendant of the Dactyls, Schol. T. *Il.* 22.391, Pythagoras as an initiate of the Dactyls, Por. *VP* 17.

[28] Boyancé, *Muses* passim.

[29] Aristox. frr. 26, 121; also Iam. *VP* 64ff, 110ff, Aristid. Quint. 2 p. 110 M., Schol. T *Il.* 22.391, Por. *VP* 30, 32f, Iam. *VP* 163f, 224. See also Zeller I 406.2; Boyancé, *Muses* 93ff, Rostagni, *ScrMin* I 135ff. The music of the monochord accompanied the death of Pythagoras, Aristid. Quint. 2 p. 116 M.

[30] *Tim.* 47d; Boyancé, *Muses* 173.

voice of a *daimon*[31] makes the transition, in the Pythagorean milieu, between music and metal-working. The claim that Pythagoras discovered the basic law of acoustics in a smithy is a rationalization—physically false—of the tradition that Pythagoras knew the secret of magical music which was discovered by the mythical blacksmiths.[32]

A report about Hippasus, based on Aristoxenus, is of a different kind: "Hippasus prepared four bronze discs in such a way that their diameters were equal, while the thickness of one was $\frac{4}{3}$ that of the second, $\frac{3}{2}$ that of the third, and double that of the fourth; when struck, they made concordant intervals."[33] This experiment is "correct," in terms of the physical principles involved. With free-swinging circular metal plates of the same diameter, the vibration frequencies are directly proportional to their thickness.[34] Therefore we must regard as authentic the statement that Hippasus knew and studied the numerical ratios of the basic concords.[35]

Another report associates Lasus of Hermione with Hippasus:

> *Λᾶσος δὲ ὁ Ἑρμιονεύς, ὥς φασι, καὶ οἱ περὶ τὸν Μεταποντῖνον Ἵππασον Πυθαγορικὸν ἄνδρα συνέπεσθαι τῶν κινήσεων τὰ τάχη καὶ τὰς βραδυτῆτας, δι' ὧν αἱ συμφωνίαι <. . .> ἐν ἀριθμοῖς ἡγούμενος λόγους τοιούτους ἐλάμβανεν ἐπ' ἀγγείων. ἴσων γὰρ ὄντων καὶ ὁμοίων πάντων τῶν ἀγγείων τὸ μὲν κενὸν ἐάσας, τὸ δὲ ἥμισυ ὑγροῦ πληρώσας ἐψόφει ἑκατέρῳ, καὶ αὐτῷ ἡ διὰ πασῶν ἀπεδίδοτο συμφωνία . . .*

The text can scarcely be sound,[36] but Lasus seems to be the subject throughout, as well as in the succeeding passage about the subdivision

[31] Above, ch. II 4, n. 34.

[32] Burnet, *EGP* 106f, said of these stories, "Their absurdity is their chief merit. They are not stories which any Greek mathematician could possibly have invented, but popular tales bearing witness to the existence of a real tradition that Pythagoras was the author of this momentous discovery." But the content of the tradition changed as it was rationalized.

[33] Aristox. fr. 90 = Schol. Pl. *Phd.* 108d = DK 18.12; on the text, see above, ch. II 5, n. 71. Also Euseb., Migne 24.746, Zenobius II 91, etc.; cf. O. Crusius, *Philologus* 52 (1893) 514ff. The ratio of the discs' thickness was therefore 12 : 9 : 8 : 6. Experiments with discs are mentioned by Theo Sm. 57.7, Nicom. *Ench.* 6 p. 248.15, Por. *In Ptol.* 120.13ff, and (critically) Ptol. *Harm.* p. 17.18. According to Aristoxenus the musician Glaucus of Rhegium played on the discs of Hippasus. There were also other comparable cymbal-like instruments. A Diocles, who may be the Pythagorean of Phlius (Aristox. fr. 19), in the 5th century discovered *τὴν ἐν τοῖς ὀξυβάφοις ἁρμονίαν ἐν ὀστρακίνοις ἀγγείοις* (*Suda* s.v. Diokles; cf. Crusius, cited above).

[34] *Handbuch der Physik* 8 (Berlin, 1927) 232; H. Gomperz, *PhSt* 57f; van der Waerden, *RE* XXIV 279.

[35] It is uncertain how much of what Eubulides reports about Hippasus goes back to Hippasus himself (DK 18.14).

[36] Theo Sm. 59.7ff. The source is neither Thrasyllus, who is copied in 47.18–49.5, nor Adrastus, whose report (cf. 49.6, *φησί* in 50.4, 50.5–12 = Por. *In Ptol.* 7.24–8.5, 50.22–51.4 = Por. *In Ptol.* 96.2–6) apparently extends at least as far as 57.10 (= Chalcid. 45) and possibly to 59.3, and is then taken up again at 61.17: *ἐπανέλθωμεν δὲ ἐπὶ τὰ ὑπὸ*

(διάληψις) of strings. The experiment with vessels filled in varying degrees is no more workable, with ordinary blows, than the one with the hammers or the weights. The effect is to be obtained by producing a vibration in the column of air in the interior of the vessel, preferably by resonance. Actually, the Aristotelian *problema*, in describing a similar experiment only speaks of ἠχώ, echo;[37] and the resonance of hollow vessels was used in the Greek theater.[38] Thus it would be possible that Lasus and Hippasus, in studying phenomena of resonance, learned the numerical laws, and that a subsequent restatement in somewhat cruder form transformed their action to one of striking the vessels. Confidence is inspired by the fact that the next section discusses experiments with strings.[39]

Lasus of Hermione, who became prominent in the time of the Peisistratidae (Hdt. 7.6), was a close contemporary of Pythagoras. He is never called a Pythagorean, but was doubtless among the earliest Greek musicologists.[40] What distinguished the Pythagoreans was apparently not a special knowledge, inaccessible to others. Rather, something which may well have lost its interest for professional musicians came to be prized among them as a fundamental insight into the nature of reality. The wondrous potency of music, which moves the world and compels the spirit, captured in the net of number—this was a cardinal element of the secret of the universe revealed to the wise Pythagoras.

τοῦ Ἀδράστου παραδεδομένα (cf. Schönberger 29 n. 1). The fact that the passage does not come from the prevalent tradition of "Pythagorean" exegesis of the *Timaeus* makes it all the more valuable.—συνέπεσθαι . . . τὰ τάχη . . . ἐν ἀριθμοῖς can scarcely be right; συνέπεσθαι requires an object. Schmid (I 1.545 n. 2) conjectures συνέπεσθαι ‹τῷ τῶν κινουμένων πάχει›; Schönberger argues against the assumption of a lacuna (26ff). He is doubtless right in taking καὶ οἱ περὶ τὸν Μεταποντῖνον Ἵππασον Πυθαγορικὸν ἄνδρα as an addition breaking the context: the sequel contains singular forms. One might assume a rather long lacuna, in which a new subject had been introduced (Eudoxus, according to Jan, *Musici scriptores graeci* 131f; "Pythagoras" would be another possibility). But in consideration of the context, Lasus can only have been cited for an experiment to prove the numerical ratios. δίσκοι καὶ ἀγγεῖα are often mentioned together (Theo Sm. 57.7, Ptol. *Harm.* p. 17.18, Por. *In Ptol.* 120.8ff). F. Lasserre thinks experiments by Lasus with the flute may be intended (*Plutarque De la musique* [Olten, 1954] 35ff).

[37] Arist. *Pr.* 19.50. ἀγγεῖον and ἠχώ go together for Aristotle (*De an.* 419b25). Bonitz, *Index aristotelicus* s.v. ἠχώ, considers the word as equivalent to ἦχος in *Pr.* 19.50 and 19.42; but in 19.42 also the subject is surely resonance, in spite of the uncertainty of the text.

[38] ἠχεῖα. See Arist. *Pr.* 11.8–9; Heiberg 80; Tannery mentions them in connection with the report on Lasus and Hippasus (*MSc* III 241ff).

[39] 59.21ff; the experiment with weights is introduced later, with the words οἱ δέ, 60.7).

[40] Above, n. 12. Tannery, *MSc* III 241f, drew the conclusion that the significance of Pythagoras in the development of musical theory was less than is generally supposed.

There is no necessary connection between the discovery of the musical ratios and the knowledge of the nature of sound, of vibration or wave movement of the air. A nearly correct description is found in the *Sectio canonis* of Euclid, the Aristotelian *Problemata*, and in the *De audibilibus*:[41] sound is a very rapid sequence of πληγαὶ ἀέρος, which are disseminated as a result of the condensation and rarefaction of the air; the idea of the sound wave is attested as early as the Stoa.[42] A closer succession of πληγαί makes a higher tone; and the concords are correctly explained by the simple coincidence of πληγαί, where the ratios are simple.[43] There was no attempt to measure the frequencies of vibration experimentally.[44]

A much less fully developed theory is found in Plato and Aristotle. Here too we find πληγαί and φορά, and rapid movement makes higher pitch; but rapidity of propagation is confused with frequency, so that higher tones are said to come to the hearer sooner than lower ones.[45] This conception is found in the Archytas fragments.[46] Clearly there

[41] See, in general, Schönberger 26–40 (E. Graf, *Die Theorie der Akustik im griechischen Altertum*, Progr. Gumbinnen, 1894, is unsatisfactory). On the authorship of *De audibilibus* (possibly Strato), see Zeller, II 2.95n.; Düring 1934, 169f; Wehrli, *Straton* 73f.—*Sect. can.* prooem.: πάντες οἱ φθόγγοι γίνονται πληγῆς τινος γινομένης . . . αἱ μὲν πυκνότεραι (κινήσεις) ὀξυτέρους ποιοῦσι τοὺς φθόγγους, αἱ δὲ ἀραιότεραι βαθυτέρους. (Arist.) *Aud.* 800a4: (τὸν ἀέρα) κινεῖσθαι . . . συστελλόμενον καὶ ἐκτεινόμενον . . . 803b29: ἀεὶ γὰρ ὁ ἕτερος ἀὴρ τὸν ἕτερον κινῶν . . . *Pr.* 11.6.899b3: ἡ μὲν γὰρ φωνὴ γίνεται ἡ συνεχὴς ἀέρος ὠθουμένου ὑπ' ἀέρος . . .

[42] Aët. 4.19.4: ἐπειδὰν δὲ πληγῇ (ὁ ἀήρ), κυματοῦται κατὰ κύκλους ὀρθούς (the comparison is with a wave in the water), καὶ αὕτη μὲν κυκλικῶς κινεῖται, ὁ δ' ἀὴρ σφαιρικῶς.

[43] (Arist.) *Aud.* 803b40: περισυγκαταλαμβάνεσθαι τοὺς ἑτέρους ἤχους ὑπὸ τῶν ἑτέρων . . ., πλεονάκις . . . ἐν πάσαις ταῖς συμφωνίαις ὑπὸ τῶν ὀξυτέρων φθόγγων αἱ τοῦ ἀέρος γίνονται πληγαί, *Pr.* 19.39 (cf. 42), 921a22, Nicom. ap. Boeth. *Mus.* 1.31. Cf. van der Waerden, *Hermes* 1943, 194ff.

[44] There is no way of verifying Capparelli's idea (II 639) that the Ἀρχύτου πλαταγή was a contrivance to measure vibration frequencies (DK 47A10). Could it have been a kind of ratchet wheel, with a flexible pawl engaging a toothed wheel?

[45] ἐναρμόνιος φορά, *Rep.* 530d, *Tim.* 47d. At *Tim.* 67b we find the correlation between faster movement and higher tone, as between slower movement and lower tone. At *Tim.* 80a–b there is an attempt to explain why in spite of this we hear concords (the slower tone finally catches up with the faster: καταλαμβάνει). Arist. *Sens.* 448a19ff sets it forth as a problem, λέγουσι . . . ὅτι οὐχ ἅμα μὲν ἀφικνοῦνται οἱ ψόφοι. This is cautiously denied *De an.* 420a31ff, and more determinedly, from the point of view of the audibility of the συμφωνίαι, Theophr. fr. 89 Por. *In Ptol.* 63.19ff; cf. also Arist. *Pr.* 11.6, 16, 20, 21, 62, 19.37, 39, 42.

[46] Archytas and Eudoxus ap. Theo Sm. 61.11ff DK 47A19a D64 Lasserre: . . . τὴν μὲν ταχεῖαν κίνησιν ὀξεῖαν εἶναι ἅτε πλήττουσαν συνεχὲς καὶ ὠκύτερον κεντοῦσαν τὸν ἀέρα, τὴν δὲ βραδεῖαν βαρεῖαν . . .; Ptol. *Harm.* p. 30.9 says of Archytas: μάλιστα τῶν Πυθαγορείων ἐπιμεληθεὶς μουσικῆς. The calculation of the types of scale made by Archytas (A16) and the proof from the properties of number transmitted by Boethius (A19) may be regarded as authentic; but there are grave doubts about fr. 1 (cf. also ch. III 1, nn. 14, 20). Here Archytas first praises his predecessors (like Hippoc.

was no thought of vibration numbers, for Archytas apparently assigned the smaller number to the high tone and the larger to the low.[47]

A certain Heraclides attributes the most advanced, and relatively most correct theory of acoustics, the idea of tone as a multiplicity of

Vict. 1.1, *Hebd.* 53); half of the fragment is taken up with their findings, in indirect discourse. Thus Archytas appears in the role of a mere transmitter of Pythagorean wisdom, as in the Ocellus fiction (D.L. 8.80). The word *μαθήματα* is used as a technical term, and goes beyond the usage of Plato; *τοὶ περὶ τὰ μαθήματα* seem to have fulfilled the demands of the *Epinomis*. It is odd that alongside the quadrivium—*περὶ γαμετρίας καὶ ἀριθμῶν καὶ σφαιρικᾶς καὶ οὐχ ἥκιστα περὶ μωσικῆς*, there stands, independently, *περί τε δὴ τᾶς τῶν ἄστρων ταχυτᾶτος καὶ ἐπιτολᾶν καὶ δυσίων*. What is this if it is not the content of *σφαιρικά*? One sentence, DK I 432.7f, corresponds exactly to Pl. *Rep.* 530d, the single passage in which Plato cites Pythagoreans by name. What a stroke of luck, if the source of this particular passage has been preserved! But Plato is speaking here (and cf. *Crat.* 405c) only of the relation of astronomy and music (harmony of the spheres), whereas Archytas obviously is intending to include all the *μαθήματα*. Especially suspicious is *περὶ γὰρ ἀδελφεὰ τὰ τῶ ὄντος πρώτιστα δύο εἴδεα τὰν ἀναστροφὰν ἔχει* (DK I 432.8f). This apparently means *πλῆθος* (*ποσόν*) and *μέγεθος* (*πηλίκον*), exactly corresponding to the scheme from which Nicomachus derives the quadrivium (*Ar.* 1.2f; *δύο εἴδη*, ibid. p. 4.20). This kind of diaeresis of being is not even attested for the late Plato and Aristotle. Archytas, however, speaks so briefly and allusively that one has to rely on Nicomachus as commentary. On the other hand, not only does the enumeration of *σημεῖα* in the second part make a good impression, but precisely the lack of clarity about the concept of speed (rapidity of propagation or frequency of vibration) leads to the thought that Theophrastus' polemic was directed against Archytas (fr. 89). Archytas combines the idea of "high" tone with that of "strong" tone, which "can be heard further" (*πόρσωθεν κ' ἀκούσαιμες*, DK I 434.13, cf. 433.15ff, 434.3f, but also Arist. *Pr.* 19.37, 920b25ff) against which Theophrastus maintains that an equal amount, though a different kind, of force is necessary for a lower tone (Por. *In Ptol.* 63.1ff, 63.20: *εἰ, ὥς φασιν, καὶ πορρωτέρω ἀκούεται ὁ ὀξύτερος φθόγγος τῷ πορρωτέρω διὰ τὴν τῆς κινήσεως ὀξύτητα διικνεῖσθαι*). Still, Theophrastus says of his opponents that they judged music not *αἰσθήσει* but *τοῖς τῶν νοητῶν ἀριθμῶν λόγοις* (Por. *In Ptol.* 62.2f), while the Pythagoreans are too empiricist for Plato (*Rep.* 531c). Cf. Aristoxenus' comment on his opponents' musical theory: *τὴν μὲν αἴσθησιν ἐκκλίνοντες . . . νοητὰς δὲ κατασκευάζοντες αἰτίας*, *Harm.* p. 32 M. This makes it seem that Theophrastus is arguing against Pythagorean musical theory transformed from a Platonic point of view, or on the basis of a Platonic treatment. In fact the vagueness of the *Timaeus* on the concept of velocity exercised a continuing influence in the commentaries on the dialogue; see Adrastus ap. Theo Sm. 50.5ff = Por. *In Ptol.* 7.24ff, and Ael. ap. Por. *In Ptol.* 33.19ff.—In any case Frank's statement is wrong, that Archytas "understood the proportions of the intervals . . . as the relation of the vibration-numbers" (12, cf. 174ff).

[47] At Archytas A16 the greater number is associated with the lower tone. To be sure, Ptolemy always proceeds in such a way, in accordance with his canon experiments; he could have converted the table of Archytas to correspond to his own system. In the *Timaeus* there is no unambiguous indication how numbers and tones are connected (cf. Kytzler, *Hermes* 87 [1959] 395ff); thus the problem of the authenticity of Philolaus A26 is not affected. But, since most experimental observations must lead to the association of low tone and large number, this may—in line with the Archytas passage in Ptolemy—be regarded as original (otherwise, Düring 1934, 162 n. 2, who cites Schönberger; cf. the next note.) On the other hand, Arist. fr. 47 (Plut. *Mus.* 1139c) assigns the number 12 to the Nete, 6 to the Hypate (*Pr.* 19.23, 919b1: *διπλασία ἡ νήτη τῆς ὑπάτης*), and Theophr. fr. 89 says of the higher tone, *πλείονας ἀριθμοὺς κεκινῆσθαι* (Por. *In Ptol.* 62.14f, 63.19f). Similarly, *Pr.* 19.35, cf. also Plut. *De an. procr.* 1021e; in the contrary sense, *Pr.* 19.12, 23, 50; detailed treatment of the opposed principles of arrangement, Nicom. *Ench.* 10, p. 254.

immeasurably rapid atmospheric impulses, to Pythagoras himself; since people identified him with Heraclides Ponticus,[48] and since he also cites Xenocrates, this attribution has found credence. As a consequence, the opening sentence of the *Sectio canonis* becomes a quotation of Pythagoras,[49] and the confusion of rapidity of propagation with vibration frequency, in Archytas, Eudoxus, and Plato, becomes a remarkable distortion of the long-known correct answer,[50] an error into which all the leading minds of their age fell. However, the identification of this Heraclides with Heraclides of Pontus is untenable.[51] Porphyry is copying out some rather late compendium, containing a popular exposition of the way the inventor may have proceeded.[52] Thus the whole is given as the reasoning of Pythagoras; Xenocrates is cited at the beginning as a weighty authority, but scarcely more than the one sentence can come from him.[53] This account has no more

[48] *Ἡρακλείδης ἐν τῇ μουσικῇ εἰσαγωγῇ*, Por. *In Ptol.* 30.2–31.21, 32.23–33.4. Zeller vacillated (II 1.1036 n. 1 for the identification, but I 509 n., against it), and Heinze (6 n. 2) opposed the identification; but Jan, *Musici scriptores graeci* 135ff, Schönberger 118ff, and Düring 1934, 154ff argued in detail for it. Jan and Düring see in Heraclides Ponticus the discoverer of tonal vibration, Schönberger (113ff) and van der Waerden (*Hermes* 1943, 192) in Pythagoras. Cf. below, n. 51.

[49] Heraclides: *ἀνελθὼν ἐπὶ τὴν γένεσιν τῆς φωνῆς ἔφη ὡς "εἰ (ὡσεὶ* all texts; *ὡς* = *ὅτι* = quotation mark) *μέλλει τι ἐκ τῆς ἰσότητος σύμφωνον ἀκουσθήσεσθαι, κίνησιν δεῖ τινα γενέσθαι."* *Sect. can.* prooem.: *. . . εἰ ἄρα μέλλει τι ἀκούσεσθαι, πληγὴν καὶ κίνησιν πρότερον δεῖ γενέσθαι.* It is assumed by Düring (1934, 155f) and van der Waerden (*Hermes* 1943, 192) that Pythagoras is being cited here.

[50] So expressly Schönberger 38f.

[51] Wehrli, *Herakleides* 113, gives the following arguments for a negative verdict: (1) Contradiction of Heraclides fr. 122. (2) Heraclides Ponticus can hardly be citing Xenocrates, who was more probably younger than he (already in Heinze); Heraclides did not hesitate to attribute important material to Pythagoras, on his own responsibility (cf. Burkert, *Hermes* 1960, 159ff). (3) "The detailed, systematic structure of the theory" goes even beyond Aristotle. We may add: (4) *καταλαμβάνειν*, "grasp," "recognize" (Por. *In Ptol.* 32.24), and *ἀκατάληπτος* (31.19), "unknowable," are technical terms devised by the Stoic Zeno (*SVF* I 60). (5) A title in the form *εἰσαγωγή* cannot be cited before Chrysippus; and we may suppose that the pretentious Heraclides of Pontus did not busy himself with schoolbooks. But this Heraclides' *εἰσαγωγή* is pedantic, long-winded, and characterless. Compare the passages (Arist.) *Aud.* 803b34ff: *αἱ δὲ πληγαὶ γίνονται . . . πολλαὶ καὶ κεχωρισμέναι, διὰ δὲ μικρότητα τοῦ μεταξὺ χρόνου τῆς ἀκοῆς οὐ δυναμένης συναισθάνεσθαι τὰς διαλείψεις μία καὶ συνεχὴς ἡμῖν ἡ φωνὴ φαίνεται*, and Heraclides 31.13: *ἑκάστη τῶν χορδῶν πλείους προίεται φθόγγους* (an awkward expression) *. . . ἀνὰ μέσον τῶν κατὰ φθόγγους πληγῶν σιωπαὶ ἂν εἶησαν . . . ἡ δὲ ἀκοὴ τῶν μὲν σιγῶν οὐ συναισθάνεται διὰ τό . . . τὰ διαστήματα μικρὰ ὄντα καὶ ἀκατάληπτα τυγχάνειν.* The laborious proof of the *Sectio canonis* that the vibrations must be related as whole numbers (above, n. 4) is replaced here by the simple assertion (30.8) *ἡ δὲ κίνησις οὐκ ἄνευ ἀριθμοῦ γίνεται.*

[52] 33.3: *καὶ οὕτως ἄν τις ἐπιδείξειεν . . .*

[53] The *φησί* that keeps recurring in the Heraclides fragment is not referring to Pythagoras (as Heinze 7), but to Heraclides; it is a sign that we have an epitome, and serves the function of a new quotation mark. Düring 1934, 155ff, thinks Didymus was the intermediary source.

historical value than the doxographical paragraph, in which Pythagoras is put alongside Plato and Aristotle in the explanation of the nature of *φωνή*,[54] or the report of Adrastus, who attributes to Pythagoreans the Aristotelian theory of sound, using Aristotelian terms.[55]

There were theories of sound and acoustics among the pre-Socratics quite outside the Pythagorean ambit. A connection of air movement, resistance, and tone was obvious to anyone who considered the human voice or a wind instrument. Alcmaeon discovered the auditory canal and the eardrum, and spoke of the "echo" in the interior of the ear, though instead of air he spoke of the "void."[56] Empedocles built onto what Alcmaeon had achieved, speaking of the "movement" and the "blows" of the air.[57] This has been taken as an indication of the Pythagorean origin of this doctrine;[58] but scholars have overlooked the same explanation of sound from movement of air and *πλῆξις* in Anaxagoras,[59] and no attention has been paid to a statement about Archelaus, the pupil of Anaxagoras, who was active in Athens about 440 B.C.: *πρῶτος δὲ εἶπε φωνῆς γένεσιν τὴν τοῦ ἀέρος πλῆξιν*.[60] This report, about one of the less prominent pre-Socratics, is important for that very reason. In his case, the doxography apparently found represented for the first time a formulation that was later considered correct. It follows that Archelaus must have gone beyond Anaxagoras and Empedocles, in a way not made completely clear; but it can hardly mean anything more than that he gave more exact information about the connection of

[54] Aët. 4.20.1: *Πυθαγόρας Πλάτων 'Αριστοτέλης ἀσώματον (τὴν φωνήν)*.

[55] Adrastus ap. Theo Sm. 50.6f = Por. *In Ptol.* 7.22ff: *ψόφος δὴ πλῆξις ἀέρος κεκωλυμένου θρύπτεσθαι*, cf. Arist. *De an.* 419b4ff, esp. b10ff, *πληγή*, 420a8 *ὅταν δὲ κωλυθῇ θρύπτεσθαι (ὁ ἀήρ)*, *ἡ τούτου κίνησις ψόφος*. What is worked out in detail in Aristotle becomes a pithy definition for the "Pythagoreans."

[56] Alcmaeon A5: *ἀκούειν . . . τοῖς ὠσίν, διότι κενὸν ἐν αὐτοῖς ἐνυπάρχει· τοῦτο γὰρ ἠχεῖν*. Cf. A6.

[57] Emp. A86 = Theophr. *Sens.* 9: *τὴν δ' ἀκοὴν ἀπὸ τῶν ἔσωθεν γίνεσθαι ψόφων· ὅταν γὰρ ὁ ἀὴρ ὑπὸ τῆς φωνῆς κινηθῇ, ἠχεῖν ἔντος . . . κινούμενον δὲ παίειν τὸν ἀέρα πρὸς τὰ στερεὰ καὶ ποιεῖν ἦχον*.

[58] Schönberger 31f, van der Waerden, *Hermes* 1943, 192.

[59] Anaxagoras A106: *τὴν φωνὴν γίνεσθαι πνεύματος ἀντιπεσόντος στερεμνίῳ ἀέρι, τῇ δ' ὑποστροφῇ τῆς πλήξεως μέχρι τῶν ἀκοῶν προσενεχθέντος*. Diogenes of Apollonia also follows Empedocles, A21: *(τὴν ἀκοὴν γίνεσθαι) τοῦ ἐν τῇ κεφαλῇ ἀέρος ὑπὸ τῆς φωνῆς τυπτομένου καὶ κινουμένου*. According to Theophrastus (*Sens.* 59), the explanation of *φωνή* as *κίνησις τοῦ ἀέρος* is the answer of "earlier" thinkers generally. He names Empedocles, Anaxagoras, and Democritus. Democritus' theory of sound was complicated by atomistic ideas (A135), but in any case, *τὴν φωνὴν εἶναι πυκνουμένου τοῦ ἀέρος καὶ μετὰ βίας εἰσιόντος* (§55).

[60] DK 60A1 = D.L. 2.17.—Theo (59.9) ascribes to Lasus and Hippasus the explanation of sound by "rapidity and slowness of movement" (above, n. 36); but this may be the interpretation of the source, as the emphasis is on *ἐν ἀριθμοῖς*.

"blow" and tone, and accordingly the influence of the velocity and strength of the "blow" on the pitch and intensity of the tone.

The acoustical theory found in Archytas, Eudoxus, and Plato is therefore not a Pythagorean theory, but belongs in the general context of Ionian *φυσιολογία*. Aristophanes, too, shows that speculation about the origin of tones was in the air in the fifth century, when in the *Clouds* he represents Socrates as investigating scientifically the buzzing of a gnat and explaining it, ingeniously, by the principle of the trumpet.[61]

If neither the recognition of the simplest numerical laws of music nor physical theory on the nature of sound is exclusively Pythagorean, then there only remains, as that which, in Plato's eyes, distinguished the Pythagoreans, the fundamental emphasis on number as such, which led to development of the mathematical theory of music quite beyond the requirements of actual practice.[62] Later presentations of Pythagorean musical theory tried to derive as much as possible from a priori considerations, and to refer as seldom as possible to experience and experiment. Even the basic facts are—apparently—derived from speculation, and everything else is derived from calculation of ratios. The basic principle, at least from the time of the *Sectio canonis* and the Aristotelian *Problemata*, was that musical intervals are expressed in the form of "superparticular" or "multiple" proportions.[63] The reasons for the preferred position of the superparticular proportions are not immediately obvious. It is based partly on the fact that all these

[61] Ar. *Nub.* 156ff: *κοῖλον πρὸς στενῷ προσκείμενον τὸν πρωκτὸν ἠχεῖν ὑπὸ βίας τοῦ πνεύματος*. This carries a verbal reminiscence of Democritus A 135, above, n. 59: *στενόν-πυκνούμενον, βία*. Perhaps the ultimate source is Leucippus, from whom Diogenes also borrowed (Theophr. *Phys. op.* fr. 2).

[62] Performing musicians work, according to Pl. *Phlb.* 56a, *οὐ μέτρῳ ἀλλὰ μελέτης στοχασμῷ*. Arist. fr. 52 = Iam. *Comm. math. sc.* p. 80.15ff: *οἱ μὲν γὰρ τὰς ἀποδείξεις καὶ τοὺς συλλογισμοὺς διωρισμένοι περὶ συμφωνίας καὶ τῶν ἄλλων τῶν τοιούτων ὥσπερ οἱ κατὰ φιλοσοφίαν σκοπεῖν εἰώθασιν, οὐδενὸς δὲ κοινωνοῦσι τῶν ἔργων, ἀλλὰ κἂν τυγχάνωσιν αὐτῶν δυνάμενοί τι χειρουργεῖν, ὅταν μάθωσι τὰς ἀποδείξεις, ὥσπερ ἐπίτηδες εὐθὺς αὐτὰ χεῖρον ποιοῦσιν*.

[63] *ἐπιμόριοι λόγοι* are proportions of the type (n + 1): n; in Greek they are called *ἐπίτριτος*, *ἐπιτέταρτος*, and so on. Like the *λόγοι πολλαπλάσιοι (διπλάσιος, τριπλάσιος*, etc.), they are therefore "expressed in one word," *ἑνὶ ὀνόματι πρὸς ἀλλήλους λεγόμενοι*, *Sect. can.* prooem. p. 160.1ff, and this is regarded as adequate basis for the postulate. Cf. Arist. *Pr.* 19.34, 41. The arbitrariness of the principle is obvious in the problem of the eleventh (octave + fourth = 8 : 3), which belongs among the concords but is not recognized as such by the Pythagoreans. This was seen by Ptolemy (*Harm.* 1.6), who therefore gave up the whole postulate. Actually, the important thing with the concords is small whole numbers rather than "a single name." See also Iam. *In Nic.* 120.18ff, Boeth. *Mus.* 2.18–20. 5.8–9.

proportions were designated in Greek by a single word;[64] but at the same time every ἐπιμόριος λόγος represents the connection of an odd and an even number, and thus exemplifies the harmony of Limit and Unlimited. So Pythagorean musical theory is intimately related to numerical cosmology, and the importance of superparticular proportion comes from its relation to number speculation in general.

The only empirical observations presupposed by the *Sectio canonis* (10–12) for the derivation of the basic concords are that an octave consists of a fourth and a fifth, and that, while a double octave is consonant, a double fifth or double fourth is not. From these facts are derived the mathematical proportions 2 : 1, 3 : 2, and 4 : 3. A proposition first proved by Archytas plays a central part in this,[65] so that the whole procedure is credited to him, though Ptolemy, who briefly recapitulates the proof, speaks of "the Pythagoreans" generally.[66]

In any case Archytas "devoted most attention, among the Pythagoreans, to music" (Ptol. *Harm.* p. 30.9). Frank tried to show that "there is not a trace, in Archytas, of the kind of a priori numerical speculation" that was introduced into music theory by Plato (166). In fact, only seven of the nine calculations of Archytas (A16) show superparticular ratios; and in one point Winnington-Ingram has shown an interesting connection between Archytas' calculation and musical practice.[67] Tannery had argued that the postulate of "superparticular or multiple proportions" was not early Pythagorean,[68] and Frank maintained that Archytas' only concern was "to determine by exact measurement the string lengths corresponding to the tones in question."[69]

[64] Above, n. 63; cf. ch. VI 1.

[65] A19; cf. *Sect. can.* 3; cf. ch. VI 2.

[66] Ptol. *Harm.* 1.5, p. 12.8–27, referred to Archytas by van der Waerden, *Hermes* 1943, 170; *RE* XXIV 279. Ptolemy advertises (pp. 11.20–12.7) a "more speculative" (λογικώτερον) derivation of the basic concords. The "best" ratio, 2 : 1, corresponds to the "most beautiful" concord, and the "first two" superparticular proportions correspond to the "first two" concords, the fourth and the fifth. Van der Waerden (*Hermes* 1943, 168, 198) extracts from this an axiomatic theory which he attributes to the Pythagoreans before Archytas.

[67] *CQ* 26 (1932) 195–208; cf. Düring 1934, 251ff; van der Waerden, *Hermes* 1943, 185. The interval Parhypate–Hypate (28 : 27) of Archytas, with the Hyperhypate, which is one whole tone lower (9 : 8), yields the pleasing interval of the diminished third (7 : 6). This very interval, Parhypate–Hyperhypate frequently occurs in the fragment of the music for Euripides' *Orestes*; so Archytas apparently took as his point of departure the practical application of enharmonics.

[68] *MSc* III 76f. He stressed that it was not held to by either Archytas or Plato.

[69] Frank 266. At p. 157, Frank cites the ἀναμετροῦντες in *Rep.* 531a; but against this van der Waerden correctly emphasizes (*Hermes* 1943, 176f) that in Plato's view φθόγγοι and συμφωνίαι are measured "against one another" (ἀλλήλοις), which sounds more like the calculation of ratios than empirical measurement of the length of strings. One number "measures" the other.

It is hard to estimate, at this late date, how close Archytas was to actual musical practice. That he assigns the same pitch to the Parhypate in all three genera corresponds to the uniformity of the name, and comports with the method of notation.[70] But it contradicts the evidence of the senses, according to Ptolemy;[71] and Aristoxenus is witness for Ptolemy against Archytas in this point. Was Archytas depending more on nomenclature than on the ear? In any case, proportions like 243 : 224, or even 32 : 27 must have been derived by calculation rather than exact measurement; and he must have based his calculations on certain postulates. Even the fundamental idea that intervals are expressible in proportions of whole numbers cannot be verified by measurement. And quite exact measurement on the monochord (if Archytas used this instrument at all; see above, n. 22) is required to decide whether a double tone is 81 : 64 or 80 : 64 = 5 : 4.

Ptolemy says of Archytas that he begins with the postulate of superparticular proportions, but then abandons his own principle.[72] Of course it would be possible that Ptolemy ascribed to Archytas, wrongly, the postulate which later came to be regarded as self-evident in Pythagorean theory; but Archytas' tenet that the superparticular proportion is indivisible (A19) shows that this proportion played an essential part in his music theory, for it is only important there. It means that the octave, fifth, fourth, and whole tone cannot be divided into equal parts, so that the arithmetic and harmonic mean take the place of the geometric in music theory.[73] And we have evidence that the problem of means was important to Archytas.[74] Thus it is not a coincidence that in Archytas' table seven of nine proportions are superparticular. How the exceptions came about remains a question.

Van der Waerden is right, then, in seeing Archytas' work as

[70] Düring 1934, 253; Winnington-Ingram, *CQ* 50 (1956) 179; Vogel I 84. Vogel ascribes to Archytas' calculations a basic significance in Greek music (47–57, 83–93). But the oldest music theorists, who were only concerned with the enharmonic genus (Aristox. *Harm.* p. 2 M.), must have been active before the day of Archytas, since he takes all the 3 genera into consideration.

[71] Ptol. *Harm.* p. 30.15 (cf. 32.3ff): ἀπάδων δὲ σαφῶς τῶν ἄντικρυς ἤδη ταῖς αἰσθήσεσιν ὡμολογημένων.

[72] Ptol. *Harm.* p. 32.1ff (cf. 30.13ff): παρὰ μὲν δὴ τὴν πρόθεσιν . . . αὐτῷ συνεστάθη τὸ . . . χρωματικὸν τετράχορδον.

[73] Dividing the octave according to the arithmetical and harmonic means gives the famous series 6 8 9 12 (e a b e′; cf. Arist. fr. 47), the τελειοτάτη ἁρμονία which Iam. *In Nic.* 118.19ff tells us was brought by Pythagoras from Babylon.

[74] On the "harmonic mean" see Hippasus and Archytas, DK 18.15, Philolaus A24, and below, ch. VI 2. Archytas' fr. 2 offers no impediment. For the application of the means in music, see also *Tim.* 36a, *Epin.* 990c, which points back to Archytas again. For the attempt to derive the table of Archytas (A16) from the doctrine of means, see van der Waerden, *Hermes* 1943, 184ff (following Tannery, *MSc* III 105); Vogel I 52–53.

belonging mainly to the development of the mathematical theory of music.[75] How successful he was in this, mathematically, may be discussed later.[76] Archytas' comparison of the *συμφωνίαι* as to the degree of harmony or euphony they show, ends up in mere arithmology.[77] In any case, Archytas was not conducting investigations in empirical physics, involving exact measurements; nor was he a practicing musician. Rather, some observed facts and some speculative a priori postulates are manipulated in a logical way, but do not coalesce into a complete system. Plato's criticism of "the Pythagoreans" also hits Archytas. The numerical series he sets up is intended to correspond to actual, audible music, but the solution is not satisfying in every respect. The principles involved are still not fully applied, and, on the other hand, agreement with musical practice is not complete. Plato's complaint does not mean that the Pythagoreans did not employ any kind of speculation in their discussion of music, but that they hastily combined *λόγος* and *αἴσθησις*, in their concern about the *ἀκουόμεναι συμφωνίαι*, and therefore were forced to compromise in both fields.

2. NUMBER SYMBOLISM AND CALCULATION OF PROPORTIONS IN PHILOLAUS

"Entaché d'absurdités mathématiques et d'erreurs pratiquement énormes"—this is the judgment of such an expert as Tannery[1] on the musical system transmitted to us under the name of Philolaus. Nevertheless, Tannery, whose thesis was later developed more fully by Frank,[2] relied mainly on a different argument to prove the spuriousness of the fragments in question. This was, that the calculation of a diatonic scale to which Plato refers in the *Timaeus* was Plato's own work, not something borrowed from earlier Pythagorean calculations.[3] From this it followed, in his view, that since the Philolaus fragments,

[75] Van der Waerden, *Hermes* 1943, 166ff.

[76] Cf. below, ch. VI 2.

[77] Archytas A17 = Por. *In Ptol.* 107.15ff; the intermediate source is Didymus. Ptolemy (*Harm.* 1.6, p. 14.1ff) argues against this method, without naming names. The idea of *ὅμοια* and *ἀνόμοια* reminds of the *ὁμοιοῦντες καὶ ἀνομοιοῦντες* of Pl. *Rep.* 546b (the "nuptial number"), where the concept of *πυθμήν* also occurs.

[1] *MSc* III 223.

[2] Frank 263–277.

[3] Tannery concludes, from the contradiction between Plato and Archytas, that there had not been in existence any earlier Pythagorean calculation, especially since Archytas (fr. 1) says he is following—not correcting—his predecessors (above, ch. V 1, n. 46). Frank develops the idea of the contradiction between Pythagorean "empiricism" and Platonic "apriorism" (above, ch. V 1, n. 13).

unlike Archytas, agree with the *Timaeus*, they must be a post-Platonic forgery.[4]

The scale of the *Timaeus*, which has only equal whole-tone intervals in the ratio 9 : 8, and therefore pure fourths and fifths, but no thirds,[5] is in Frank's opinion "a purely speculative figment," which has "scarcely anything in common with the scales of real Greek music."[6] Ptolemy is differently minded. He introduces the *διάτονον διτονιαῖον* explicitly in relation to practicing musicians, who find it especially "useful"; it is quite common in the tuning of the lyre.[7] Actually such tuning is most easily accomplished by the use of fourths and fifths.[8] Ptolemy adds that musicians, without regard to accurate theory, speak of the "half-tone" between Hypate and Parhypate;[9] and this corresponds exactly to the conception of Aristoxenus, who certainly did not take his description of the diatonic genus from the *Timaeus*. In fact the word "diatonic," *διάτονος*, means precisely that this scale is constructed *διὰ τόνου*, tone by tone; the alphabetic series of note designations, too, follows the diatonic series. The conception that the diatonic tetrachord consists of two equal tones plus a "remainder" is therefore the earliest, rooted in the practice of musical performers.[10] Any

[4] The following is a table, for ready comparison, of the various calculations of the diatonic tetrachord:

	Philolaus A26, fr. 6	Archytas A16	*Tim.* 36a–b
Mese (a)			
	> 9 : 8	9 : 8	9 : 8
Lichanos (g)			
	> 9 : 8	8 : 7	9 : 8
Parhypate (f)			
	>256 : 243	28 : 27	256 : 243
Hypate (e)			

[5] The scale of the *Timaeus* passed, by way of Boethius, into the musical theory of the Middle Ages. Archytas knows the major third 5 : 4 in the enharmonic genus (but not in the diatonic). This is called by Frank "the decisive step in acoustics, upon which the entire physical analysis of the scale depends to the present day" (167). The arrangement of our scale is 9 : 8, 10 : 9, 16 : 15, and here (9 : 8) × (10 : 9) = (5 : 4); but, since Greek music does not use triads, the third scarcely plays any part in it.

[6] Frank 13; cf. 181ff, 268.

[7] Ptol. *Harm.* p. 39.14ff; cf. p. 43.19ff, p. 80.16ff.

[8] "Our piano tuners still follow this procedure," says van der Waerden (*Hermes* 1943, 190), rightly referring to Aristox. *Harm.* p. 55 M.; also see *Sect. can.* 17.

[9] Ptol. *Harm.* p. 39.18f.

[10] Van der Waerden, *Hermes* 1943, 190.3, refers also to Aristid. Quint. p. 22 M. This passage was traced to Damon by R. Schäfke, *Aristides Quintilianus von der Musik* (Berlin, 1937); but the attribution proves to be untenable (R. Wagner, *Arch. f. Musikforsch.* 4 [1939] 316ff).—Ptol. *Harm.* p. 39.18ff emphasizes that the *διάτονον διτονιαῖον* was not significantly different from his own values for the diatonic genus (9 : 8, 10 : 9, 16 : 15); on the other hand, Frank maintains (166) that the ditone (9 : 8) × (9 : 8) gives "a dissonance quite intolerable to our ear." It seems to me more likely that Ptolemy tested this out than that Frank did; or are we to suppose that Ptolemy was tone-deaf?

calculation taking its departure from here must necessarily lead to the figures found in the *Timaeus*, and only the search for "superparticular" ratios, along perhaps with keener observation, could discover a difference between the two "whole" tones. This in itself is enough to destroy the basis for Frank's rejection of these fragments.

We can get further by consideration of the inconsistency in the system of Archytas which Ptolemy mentions.[11] According to Archytas, the pattern of the chromatic genus is,

Mese (a)

$>$ 32 : 27

Lichanos (g flat)

$>$ 243 : 224

Parhypate (f)

$>$ 28 : 27

Hypate (e)

In the effort to explain these remarkable figures, scholars have pointed out that the two lower intervals together make a whole tone,[12] or that the interval Lichanos–Paramese is a pure fourth.[13] But the rationale offered by Ptolemy has not been much noticed: "Archytas obtains the second tone in the chromatic genus (g flat) . . . with the help of the tone that occupies the same position in the diatonic genus (g); for, he says, the second highest tone in the chromatic genus stands in the ratio of 256 : 243 to the corresponding tone in the diatonic genus."[14] This explanation is so odd that we can not attribute it to an intermediary source or to Ptolemy himself, even aside from the plain *φησὶ γὰρ ('Αρχύτας)*. Who could have got the idea, instead of using the obvious relationships pointed out by modern scholars, of introducing a calculation so complicated and based upon a different genus, the diatonic? We have no alternative but to recognize the derivation Ptolemy gives as that of Archytas. He found the highest interval in the chromatic tetrachord not by harmonic division and not by reference to the natural concords, but by the extrinsic addition of two previously known values, that of the diatonic whole tone (9 : 8) and the ratio 256 : 243—the "remainder" when two whole tones are subtracted from a fourth.[15] Thus Archytas is presupposing two things: a music theory which builds its scale by the addition and subtraction of intervals, and a calculation of the diatonic scale by the numerical values found in the *Timaeus*.

[11] A16 Ptol. *Harm.* 1.13.

[12] Düring 1934, 255; Vogel I 52. (243 : 224) × (28 : 27) = 9 : 8. Still, the series 6 : 5 (minor third), 15 : 14, 28 : 27 would be more "beautiful"; here the lower tones taken together make a whole tone (15 : 14) × (28 : 27) = 10 : 9 (Tannery, *MSc* III 234f).

[13] Van der Waerden, *Hermes* 1943, 184f.

[14] Ptol. *Harm.* p. 31.2ff.

This music theory may be identified with the one ironically rejected by Plato, which sought to identify the smallest interval, as a standard of measurement; and we may conclude from Plato, as Frank does, that this theory was better known than the Pythagorean. The Aristoxenian conception of the tonal continuum is in any case primary; both the language of professional musicians and the beginnings of musical theory are couched in its terms. According to this approach, the diatonic tetrachord has the structure tone, tone, semitone; the chromatic, tone and a half, semitone, semitone; and the enharmonic, ditone, diesis, diesis.[16]

But beyond this, Archytas presupposes a calculation of the scale by ratios—a diatonic tetrachord with the intervals 9 : 8, 9 : 8, and 256 : 243. Since his own picture of the diatonic tetrachord is different from this,[17] it can hardly be his own invention; but, since musical theory based on numerical ratio is a hallmark of Pythagoreanism, Archytas must have borrowed the value 256 : 243 from a Pythagorean predecessor. This much we can infer from Archytas quite without reference to the Philolaus problem; but since the ratio 256 : 243 does appear among the Philolaus testimonia, Archytas must be regarded as the most important witness to the authenticity of the Philolaus fragments dealing with music.[18]

This is really the point where difficult problems begin. First of all, the remarkable terminology of the directly quoted fragment (fr. 6)

[15] (4 : 3) : (9 : 8) : (9 : 8) = 256 : 243. This "remainder" (*λεῖπων*, Pl. *Tim.* 36b) is usually called *λεῖμμα*: Plut. *Procr.* 1017f, Adrastus ap. Theo Sm. 67.6 et saep., Cens. 10.7, Ptol. *Harm.* p. 23.2ff et saep., Por. *In Ptol.* 129ff, 154.22, Nicom. *Exc.* 2 p. 267.9, etc.

[16] Above, ch. V 1; Pl. *Rep.* 530f.

[17] Archytas' figures are 9 : 8, 8 : 7, and 28 : 27, all superparticular ratios. If Archytas is adopting older calculations, it seems that he may have arrived at his results by a process of trial and error. Since, in the diatonic genus, the older calculation led to the awkward 256 : 243, he took 8 : 7 for one of the intervals instead of 9 : 8, leaving the remainder of 28 : 27. In the enharmonic genus, he adopted the figures 80 : 64 = 5 : 4, instead of (9 : 8) × (9 : 8) = 81 : 64; so that he had the remainder of 28 : 27 and, in between, 36 : 35. In the chromatic tetrachord the correction was not so easy; instead of 32 : 27 he had to choose 30 : 25 = 6 : 5 (minor third) or 35 : 30 = 7 : 6 (diminished third).

[18] Frank did not wholly miss this; on the ratio 256 : 243 he writes (271), ". . . numbers which play no role at all in Archytas; implicitly, they are included between the chromatic g and the diatonic g." The contradiction of the two clauses apparently did not draw Frank's attention.

ἁρμονία, δι ὀξειᾶν, συλλαβή in place of the usual διὰ πασῶν, διὰ πέντε, διὰ τεττάρων; and δίεσις instead of λεῖμμα for the "smaller semitone";[19] and above all Trite instead of Paramese for the tone which is separated by a fourth from the Nete (c′), and by a whole tone from the Mese (a)—namely b. Are these expressions, in this application, ancient and genuine, or are they unfortunate, artificial archaisms?

Some are unquestionably early technical terms of professional musicians: ἁρμονία (a scale one octave long),[20] συλλαβή, fourth (from the manner of holding the lyre strings in the hand),[21] and δι' ὀξειῶν, fifth (the play in the strings that brings up the tetrachord to an octave).[22] In particular, συλλαβή and δι' ὀξειῶν occur in an obviously Pythagorean passage of the Hippocratic book *De victu*.[23] Furthermore, the use of these terms by the Pythagoreans is attested by Theophrastus: οἱ μὲν Πυθαγόρειοι τὴν μὲν διὰ τεσσάρων συμφωνίαν συλλαβὴν ἐκάλουν, τὴν δὲ διὰ πέντε δι' ὀξειᾶν, τὴν δὲ διὰ πασῶν τῷ συστήματι, ὡς καὶ Θεόφραστος ἔφη, ἔθεντο ἁρμονίαν.[24] Then it is frequently mentioned in the *Timaeus*

[19] δίεσις is mentioned repeatedly by Aristotle as a smallest interval (*An. post.* 84b39, *Met.* 1016b18ff, 1053a12, b35, etc.). In Aristoxenus a δίεσις is a third tone or quarter tone (*Harm.* p. 46 M.); in any case it is smaller than a semitone.

[20] On the following cf. Boeckh 65ff, Frank 273ff (though they fail to take account of Hippoc. *Vict.* 1.8). On ἁρμονία see Arist. fr. 47 (the ἁρμονία consists of the numbers 6, 8, 9, 12), Pl. *Phlb.* 17d, Aristox. 2 p. 36 M., Aristid. Quint. 2 p. 91 M.: διὰ πασῶν, ὃ καὶ καλοῦμεν ἁρμονίαν. Similarly also Arist. *Pr.* 19.44, 47. Aristoxenus states (*Harm.* 2, p. 45 M.) that the ancient scales only extended for 1 octave. Van der Waerden, *Hermes* 1943, 176.2, correctly prefers the translation "scale" (rather than "octave").—Frank's assertion (268) that the appellation ἁρμονία for the octave presupposed Plato's philosophy of number, including the One and the Indefinite Dyad, is wrong; the word is old, and the ratio 1 : 2 is a physical fact. (On ἁρμονία see also H. Koller, *MH* 16 [1959] 238–248).

[21] Frank (273) finds an indication of late date in the term συλλαβή. There is presupposed, he thinks, the "Democritean" simile comparing tone and letter, interval and syllable, σύστημα and word (cf. Frank 167ff; Burkert, *Philologus* 1959, 177.2). This is the way Aelian explains the expression at Por. *In Ptol.* 96.30 (cf. Nicom. *Ench.* 9. p. 252.6f, Procl. *In Remp.* I 231.1, Olympiod. *In Phd.* p. 169.16ff Norvin): but Aelian goes on, κατὰ δὲ τοὺς ὀργανικοὺς λυρικοὺς συλλαβὴ εἴρηται ἀπὸ τοῦ λυρικοῦ σχήματος τῆς χειρός, ἐπειδὴ ἐν τῇ ἑπταχόρδῳ χρήσει ἡ πρώτη σύλληψις τῶν δακτύλων κατὰ τὸ διὰ τεσσάρων ἐγένετο σύμφωνον (Por. *In Ptol.* 97.2ff). This is to be understood quite concretely, and the reference to the practical use of the lyre is to be preferred to the tortuous explanation from the letter simile.

[22] The expression is found at Arist. *Pr.* 19.34, 41. cf. Düring 1934, 179: "doubtless primarily the interval that filled out the original tetrachord . . . to an octave, progressing from lower to higher." Thus one plays διὰ τεττάρων, δι' ὀξειῶν, διὰ πασῶν.

[23] *Vict.* 1.8; cf. above, ch. III 2, n. 114.

[24] Acl. ap. Por. *In Ptol.* 96.21ff. One might ask whether Theophrastus mentioned ἁρμονία alone, or also the two other terms; but, in any case, he was speaking of striking Pythagorean language, and all 3 glosses are also cited in Nicom. *Ench.* 9, p. 252.5ff (quoting Philolaus fr. 6) and Aristid. Quint. 1 p. 17 M.: παρὰ μέντοι τοῖς παλαιοῖς τὸ μὲν διὰ τεσσάρων ἐκαλεῖτο συλλαβή, τὸ δὲ διὰ πέντε δι' ὀξειῶν, τὸ δὲ διὰ πασῶν ἁρμονία. Also see Hesych. s.v. δι' ὀξειᾶν (from Diogenianus): παρὰ τοῖς Πυθαγορικοῖς λέγεται. Notice the Doric form in Hesychius and Aelian. Hesychius s.v. πέραινον also seems to refer to Philolaus (above, ch. III 2). Frank 276 concedes that Theophrastus is the terminus ante quem for the fragment of Philolaus.

commentaries that the interval 256 : 243, usually called λεῖμμα, had been called δίεσις by the Pythagoreans.[25] These are, then, ancient technical terms of professional musicians whose use by Pythagoreans is early attested. Still, the question might be raised, whether Theophrastus was thinking of Philolaus' book, or some other work, in which case some later forger made sophisticated use of the lexicographical glosses.

The situation of Philolaus' Trite is more complicated, and indeed leads to problems of the origin of the Greek musical system which are mainly unsolved, and, given the present state of the source material, insoluble. Boeckh saw here a point "so deeply recondite that it cannot be attributed to any ordinary composer of pseudepigrapha" (70), while Tannery thought, "Il est à craindre que l'auteur n'ait trop archaïsé";[26] and Frank speaks of a "commonplace which can be found in almost any popular writer on music of late antiquity" (275). The problem is that of the seven-stringed lyre.

According to the ordinary system of Greek music, the basic tetrachord (e–a) is continued by another, either *synemmenon* (a–d′) or *diezeugmenon* (b–e′). In one case the Trite is b flat, in the other c′, but in Philolaus it is b, a whole tone from the Mese and a fourth away from the Nete. It is no wonder that, according to Nicomachus, many accused Philolaus of an error.[27] This very fact, however, makes it improbable that some forger has injected an artificial archaism; the purpose of an artificial patina is to arouse confidence, not mistrust. So, before deciding that this is a mistake stemming from sheer stupidity, we should try to interpret the name in a way that makes sense.

It is certain that the lyre, for a long time, had seven strings,[28] and that the number of strings was gradually increased in the fifth and fourth centuries B.C., but next to nothing is known about how these seven strings were tuned. From the time of the Aristotelian *Problemata* the theory is attested that the seven-stringed lyre embodied the *synemmenon* system, and that that of the *diezeugmenon* was introduced later; Nicomachus attributes this step to Pythagoras.[29] To this extent we can

[25] Theo Sm. 55.11, 56.18, Chalcid. 45, Macrob. *Somn. Sc.* 2.1.23, Procl. *In Tim.* II 168.28, Boeth. *Mus.* 2.28.

[26] Tannery, *MSc* III 240.

[27] Nicom. *Ench.* p. 253.12: οἱ δὲ τοῦτο (i.e., the explanation of Nicomachus, below, n. 30) μὴ συνιέντες αἰτιῶνται ὡς οὐκ ὄντος δυνατοῦ ἐν ἐπιτρίτῳ λόγῳ εἶναι τρίτην ἀπὸ νήτης.

[28] Cf. above, ch. IV 4, n. 6.

[29] It is presupposed in the way the question is put at Ar. *Pr.* 19.7, 47 that the Nete was missing in the ancient heptachord (on the text, see below, n. 36); cf. also section 25. Nicom. *Ench.* 3 p. 241; 5 p. 244.14ff, cf. 7 p. 249.20ff, 11 p. 257.17ff, Boeth. *Mus.* 1.20. The "ancient" heptachord *synemmenon* (e f g a b flat c′ d′) is clearly described in

accede to Frank's use of the word "commonplace"; but this does not explain Philolaus' Trite. In addition, the whole theory seems suspicious. It is hardly credible that the most impressive of the musical intervals, the octave, was not used in the early systems. The expression *διὰ πασῶν* says in itself that "all the strings" together make an octave, and not a seventh. An eight-stringed lyre is scarcely mentioned; it follows from this that the seven-stringed lyre comprehended an octave, and this means that one note of the scale was missing.

Nicomachus makes a twofold, and very tortuous, explanation of Philolaus' enigmatic Trite:[30]

> By Trite he means here the Paramese in the heptachord, before the insertion of the dividing tone in the octachord. (1) This [i.e. the Trite = Paramese] was separated from the Paranete (d) by an undivided interval of one and a half tones.[31] The inserted string took away a whole tone from this, and the remaining semitone was left in the *diezeugmenon* between the Paramese and the Trite. Understandably, then, the old Trite was separated by a fourth from the Nete (b–e′), an interval which now is closed by the Paramese (b) instead of the other . . . (2) Others say, quite persuasively, that the inserted tone was not between Mese and Trite but between Trite and Paranete, and that it was now called Trite, while the old Trite became Paramese in the *diezeugmenon*.

In both the interpretations that Nicomachus gives, the scale presupposed is obviously e f g a b d′ e′.[32] The only difficulty rises from

Nicomachus, and the transition to the octachord (e f g a b c′ d′ e′) is described in the words *Πυθαγόρας παρενέθηκεν ὄγδοόν τινα φθόγγον μεταξὺ μέσης καὶ παραμέσης*, p. 244.22f; *μεταξὺ μέσης καὶ τρίτης*, p. 253.15; *τὸν ὄγδοον φθόγγον . . . μεταξὺ μέσης καὶ τῆς ἀρχαίας τρίτης παρενέθηκαν*, p. 257.18f. It is not specifically mentioned that the whole upper tetrachord is thus changed in pitch by one tone. The names of the notes remain the same, except for the differentiation of Trite and Paramese; perhaps Nicomachus' ideas come more from calculation than from hearing. Another interpretation is proposed by J. Chailley, "L'hexatonique grec d'après Nicomaque," *REG* 69 (1956) 73–100. He postulates an original defective heptachord e f g a c′ (Paramese = Trite) d′ e′ (pp. 73ff), and finds this in Nicomachus (77ff). He thinks Nicomachus kept jumping back and forth between a (never described) defective heptachord and the *synemmenon* heptachord which he described in detail: "In order to preserve the octave and so that the Mese might not be distant by a fourth from the two ends," Pythagoras inserted the new note between Mese and Paramese.

[30] Nicom. *Ench.* 9 p. 253.4ff.

[31] The MSS have *ἡμιτόνιον*. Meibom emended to *‹τρι›ημιτόνιον*. This expansion is necessary, because it is essential to what follows (one tone is taken away and a half tone remains over), and also to the epithet *ἀσύνθετον*, which is incomprehensible if applied to a semitone (especially since only the diatonic is in question here). Vogel (II 771.) rejects the correction, and ignores the succeeding context.

[32] Cf. Jan, *Musici scriptores graeci* 81 n.

Nicomachus' dependence on the "insertion of the dividing tone" as a transition from the *synemmenon* to the *diezeugmenon*, which can contribute nothing but confusion, in this context.[33] The clue to the correct interpretation lies in recognizing the existence of an earlier, defective scale with only seven notes but the range of an octave, in which one note was missing in the upper tetrachord, the later Trite, c′. Now we have, in fact, further traces of such a scale. The question is posed in the Aristotelian *Problemata*, referring to the *synemmenon*: "Why did the ancients, in their seven-stringed scales (*ἁρμονίαι*) retain the Hypate, and not the Nete?"[34] But the answer offers the consideration that both notes, Hypate and Nete, were present, and that it was rather the Trite which was omitted, taking us back to the scale e f g a b d′ e′.[35] The same interpretation is to be given the statement that Terpander "removed the Trite and added the Nete";[36] and the *σπονδειάζων τρόπος* of Olympus, in which the Trite is supposed to have been lacking, belongs in this context.[37]

The history of Greek scales is to a great extent still obscure, and in

[33] The first explanation is very puzzling. Nicomachus, as the second explanation shows, is thinking of an interpolation between Mese and Paramese (= Trite), which comports with his own system (above, n. 29); but it is hard to see how he could say that the insertion of an interval could "take away" anything from the interval Paramese–Paranete.

[34] (Arist.) *Pr.* 19.7; the names of the notes are given as in the *σύστημα τέλειον*. Nete (= e′) is lacking in the *synemmenon* system.

[35] Jan, *Musici scriptores graeci* 81 n., and Chailley (n. 29 above) 96f believe that the *synemmenon* Trite (b flat) was named as lacking, so that the series was e f g a c′ d′ e′; but since the Nete named surely belongs to the *diezeugmenon* system, we are dealing, in the question as well as in the answer (*ἀμφοτέρας γὰρ κατέλιπον, τὴν δὲ τρίτην ἐξῆρουν*), with *diezeugmenon* notes, i.e., the Trite *diezeugmenon* (c′) is missing.

[36] (Arist.) *Pr.* 19.32: *ἑπτὰ ἦσαν αἱ χορδαὶ τὸ ἀρχαῖον, εἶτ' ἐξελὼν τὴν τρίτην Τέρπανδρος τὴν νήτην προσέθηκε*. The *synemmenon* system is again taken as primary (*τὸ ἀρχαῖον*), but the name Nete is taken from the *diezeugmenon* system, and therefore also the Trite; once more, c′ is omitted, and the series is e f g a b d′ e′. Plutarch (*Mus.* 28.1140f) also speaks of the invention of the "Doric Nete" (e′) by Terpander.—Another defective system is presupposed at *Pr.* 19.47, in which were lacking "what is now called the Paramese and the whole-tone interval," that is, b. Chailley (loc. cit. above, n. 35) puts this down as evidence for his scale e f g a c′ d′ e′, but apparently without reading the following context: *ἐχρῶντο δὲ μέσῃ τῇ ἐσχάτῃ τοῦ ἐπὶ τὸ ὀξὺ πυκνοῦ*, i.e. next above the Mese (a) came a *πυκνόν*, certainly not a *τριημιτόνιον ἀσύνθετον* as in Chailley's theory. The initial sentence, at 922b5, *ἢ οὐ τὴν ὑπάτην . . . ἀφῄρουν*, is obviously corrupt; it could be corrected with Gevaert to *οὐ τὴν νήτην . . .* (i.e. perhaps e f g flat a b flat c′ flat e′), or to *οὐ τὴν νήτην μόνην* (i.e. the *synemmenon* system e f g a b flat c′ d′), with Jan.

[37] Plut. *Mus.* 19, looking back to 11 (= Aristox. fr. 83). The reconstruction of the *σπονδειάζων τρόπος* is especially complicated, since there seems to be a tone assumed as standing between c′ and d′. Cf. Tannery, *MSc* III 299ff; R. Winnington-Ingram, "The Spondeian Scale," *CQ* 22 (1928) 83–91 (he accepts e f♯ a b c♯, as also for Philolaus, maintaining that e f g a b d′ e′ is later reconstruction intended to explain Philolaus; p. 87). See also Vogel II 9–38.

particular we shall not go into the question of the relation of the number of strings to the structure of the scale.[38] But we may regard it as certain that "defective" scales are very old, that the upper tetrachord was in the early days not completely developed, but that from a quite early date the ἁρμονίαι had the range of an octave—διὰ πασῶν.[39] There are significant traces of a scale in which the later Trite (c′) was not included, and among them we may count the fragment of Philolaus. For it is incredible that a forger drew the conclusion, from learned comments about the "missing" Trite, that the Paramese was originally τρίτη, expecting his readers to draw the same conclusion; he would gain nothing but criticism for all his trouble. These considerations, along with the archaic terminology, allow us to regard Philolaus' fragment 6 as one of the oldest pieces of evidence for Greek music.

We still have to consider the "mathematical absurdities" reported by Boethius. This seems on the face of it a very late source; but since Boethius is virtually translating Nicomachus, and since the latter quotes directly Philolaus' fragment 2 and a part of fragment 6, one must suppose that the long word-for-word quotation, fragment 6, and the details reported by Boethius come from the same line of tradition and stand or fall together.[40]

The question is that of the finer division of the tone. Philolaus divides the whole tone into two unequal parts, *diesis*, 256 : 243, and *apotome*; the difference between the two is called *komma*. According

[38] It is certain that the player on the lyre (*kithara*) did not have a special string for each tone he could play. (The principal evidence is that of Pherecrates ap. Plut. *Mus.* 1141d; cf. Düring, *Eranos* 43 [1945] 176–197; Winnington-Ingram, *Lustrum* 3 [1958] 26f, cites further literature.) It is not clear, however, how they produced the alteration of tone; the *kithara* has no fingerboard. Taking account of special characteristics of the Greek musical notation, C. Sachs, and, following him, O. Gombosi, *Tonarten und Stimmungen der antiken Musik* (Copenhagen, 1939), have tried to reconstruct the string arrangement of the ancient lyre. According to them the 5-stringed lyre (common in artistic representations till the 5th century B.C.) had the strings e a b d′ e′, and here, according to Gombosi (42.8), is where Philolaus fits into the picture. The 7-stringed lyre had, according to Gombosi (43ff) e g a b c′ d′ e′; or rather, including the Hyperhypate, d e g a b d′ e′ (Düring, *Eranos* 1945, 192f). But see Winnington-Ingram, *CQ* 50 (1956) 169–186, and *Lustrum* 3 (1958) 15. The epigram of Ion (fr. 6 Diehl, cf. above, n. 28) speaks of intervals of a fourth on the 7-stringed lyre.

[39] Ancient theory, from the time of Aristoxenus, denied that there had been any such development; "the ancients," they said, had known all the notes, and only refrained, on aesthetic grounds, from using some of them. (Plut. *Mus.* 19, harking back to 11 = Aristox. fr. 83: οἱ παλαιοὶ οὐ δι' ἄγνοιαν ἀπείχοντο τῆς τρίτης . . . ἀλλὰ δῆλον ὅτι τὸ τοῦ ἤθους κάλλος . . .).

[40] Above, ch. III 2.

to this, the *apotome* would be 2187 : 2048, and the *komma* 531441 : 524288—pure frivolity.[41] Philolaus' treatment is different:[42]

> He establishes as the basis of tone the number which first makes the cube of the first odd number and was highly honored among the Pythagoreans [i.e., 27] . . . a number which is separated by a whole tone from 24.[43] . . . From this, then, Philolaus makes two parts, one, which is larger than half, and which he calls *apotome*, and another, which is smaller than half, which he calls *diesis*; later it was called "the smaller semitone." The difference between these parts he calls *komma*. And, first, he thinks that the *diesis* consists of 13 units, because this is the difference between 256 and 243, and because this same number, i.e. 13, is made up of a 9, a 3, and a 1. Of these the one assumes its station as the point, the three that of the first odd-number line, and the nine that of the first odd square number. Thus when he has for these reasons established 13 as the *diesis*, the so-called semitone, he concludes that the remainder of 27, which consists of 14 units, is the *apotome*. But since the difference between 13 and 14 is 1, he decides that 1 is to be established as the *komma*. But he makes the whole tone consist of 27 units because 27 is the difference between 216 and 243, which are distant from each other by a whole tone.

Philolaus finds a special meaning in the numbers 1, 3, 9, 13, and 27, to the detriment of their mathematical sense. Intervals are actually determined by numerical ratios, and here a single number is meaningless; but Philolaus is deciding on the inner meaning of single numbers. He seems to have calculated a diatonic tetrachord correctly, working from the ratios 4 : 3 for the fourth and 9 : 8 for the whole tone to reach the numbers 192 (Mese), 216 (Lichanos), 243 (Parhypate), and 256 (Hypate);[44] but then, for him, the *diesis* (Parhypate–Hypate)

[41] The *apotome* is calculated at Timaeus Locrus, p. 211.6 Thesleff, and called μεῖζον ἀμετόνιον, both *apotome* and *komma* in Procl. *In Tim.* II 180ff (ὡς οἱ παλαιοί φασιν, p. 184.2) and Boeth. *Mus.* 2.30 p. 263f; 3.4 p. 275f; cf. 2.31 p. 267; 3.9 p. 281ff; 3.12 p. 286ff; 3.15 p. 295ff.—Frank's statement (272) is nonsense, "the author's use of the expression *apotome* for the irrational gives him away," on the ground that according to Eudemus "Theaetetus was the first to devise the term *apotome* and introduce it into harmonics." In his treatment of complicated irrational magnitudes, Theaetetus used the term *apotome* for magnitudes of the form $\sqrt{a}-\sqrt{b}$, and derived it from the harmonic mean (sic; cf. ch. VI 2), and this has nothing to do with music. The musical *apotome* has still less to do with irrationality, either when correctly calculated or in the incorrect version of Philolaus.

[42] Philolaus A26 = Boeth. *Mus.* 3.5 p. 276.15.

[43] 27 : 24 = 9 : 8, a whole tone.

[44] This calculation is found, e.g., at Theo Sm. 86.15ff. 216 : 192 = 243 : 216 = 9 : 8.

"consists of" the number 13, and the whole tone of 27, because these are the differences between the numbers occurring in the respective ratios; and the fact that a whole tone may also be expressed by the ratio 27 : 24 (= 9 : 8) seems to him a confirmation—in spite of the totally different role played by 27 in this case. Philolaus is not thinking of the mathematical uses of numbers, but they are to him entities in themselves, and he combines them by addition and subtraction, where the proper procedure is the multiplication and division of ratios. Of course this leads him out of the realm of commonly understandable numerical relationships. He seems to assume that, because 256 : 243 is a *diesis* and 243 : 216 a whole tone, the *diesis* also extends from 243 to 230 or from 229 to 216, and the *apotome* from 243 to 229 or 230 to 216, as though 256 : 243, 243 : 230, and 229 : 216 were the same ratio!

Tannery thought, quite understandably, that one could scarcely ascribe such mathematical absurdities even to a composer of pseudepigrapha.[45] According to Frank (271) this kind of thing cannot have been written by "any mathematician, and least of all by a Pythagorean; their principal achievement was in the discovery of the mathematical principles which Philolaus so disgracefully betrays." All the same, Frank does not hesitate to attribute all this to Speusippus, so that one cannot help wondering how the latter could gain admission to the Academy so *ἀγεωμέτρητος*.

Boeckh's expedient, to suggest that Boethius was responsible for the errors,[46] will not work; of all extant authors, Boethius has the most detailed—and correct—exposition of the *apotome* and *komma*. Nor can the error be removed by the correction of one or another of the numbers;[47] though this be madness, yet there is method in it. The basic flaw is that again and again difference takes the place of

[45] Tannery, *MSc* III 223: ". . . qu'on peut à peine attribuer même à un faussaire."

[46] Boeckh 79f. He refers to Proclus, who names Philolaus in the context of the scale of the *Timaeus*. Procl. *In Tim.* II 180f proves that the *leimma* is not a full semitone and therefore calculates the *apotome*—with the correct figures; above, n. 41.—In the calculation of the scale of the *Timaeus* Proclus finds 34 *ὅροι*, and Timaeus Locrus 36, since he inserts two *apotomai*. Proclus rejects this, but carries through the calculation, concluding, *δέδεικται μὲν οὖν ἐκ τῶν Φιλολάου τὸ πλῆθος τῶν παρὰ τῷ Τιμαίῳ γραφέντων ὅρων* (II 190.8). It was not Proclus himself, then, but earlier commentators on the *Timaeus*, who inserted *apotomai* with citation of the authority of Philolaus. It cannot be determined whether they found in him the correct calculation or merely the concept of *apotome* (the difference of whole tone and *diesis*). In any case, Proclus did not find his calculation in Philolaus; he had worked it out correctly in advance.

[47] Diels' sentence, DK I 405 n., "The *apotome* has $17\frac{3}{8}$ units, not 14" (based on a misunderstanding of Boeckh, p. 79), gives an "emendation" worthy of Philolaus. But, aside from that, fractions are not allowed in Greek calculation of proportions (Procl. *In Tim.* II 184.6 excuses himself for taking this liberty).

proportion; in place of the calculation of proportions, the idea of addable and subtractable lines takes the center of attention. This impression is strengthened also by the separate report that Philolaus used the expression *ὑπεροχή* with relation to all the intervals.[48]

Similar numerological games are to be found in the discussions of the *Timaeus* by Plutarch and Adrastus.[49] The number 13 is the essence of the *leimma*,[50] the number 27 "is" the tone;[51] and here Plutarch cites the Pythagoreans as his authority. There is doubtless a close connection between this and the fragment of Philolaus, and the reliability of Boethius is confirmed; the only question is, whether the commentators on the *Timaeus* had an authentic book by Philolaus or whether the Philolaus book was written, pseudonymously, on the basis of the *Timaeus* commentaries.[52]

But Philolaus goes even further, as Boethius elsewhere reports.[53] After the *diesis* has been correctly defined as the measure by which a fourth exceeds two whole tones, and the *komma* as the excess of the whole tone over the sum of two *dieses*, comes the statement, "schisma est dimidium commatis, diaschisma vero dimidium diescos." The

[48] Philolaus A25 = Por. *In Ptol.* 91.4ff.

[49] Plut. (*De an. procr.* 1027e) knows, of course, that in music theory the important thing is proportion, but he also thinks that the consideration of the individual numbers is a *θεωρία χάριν ἔχουσα οὐκ ἀφιλόσοφον*. Thus he finds a deep meaning in any combination of the numbers given in the *Timaeus*.

[50] Plut. *De an. procr.* 1012.1017f: *τὰ δὲ τρισκαίδεκα λεῖμμα, καθάπερ Πλάτων, (οἱ Πυθαγορικοὶ ἐκάλουν).*—Theo Sm. 69.3 (probably based on Adrastus, who certainly is behind the section ending at 66.18 [*φησίν*, 66.12], and also is named in 73.16ff) calculates the *leimma* at 486 : 512, and comments, *τινὲς δέ φασι μὴ ὀρθῶς εἰλῆφθαι τούτους τοὺς ἀριθμούς· τὴν γὰρ ὑπεροχὴν . . . μὴ γίνεσθαι ιγ′, ὅσα Πλάτων εἴρηκε δεῖν ἔχειν τὸ λεῖμμα. οὐδὲν δὲ κωλύει καὶ ἐφ' ἑτέρων ἀριθμῶν τὸν αὐτὸν εὑρίσκειν λόγον . . . οὐ γὰρ ἀριθμὸν ὡρισμένον ἔλαβεν ὁ Πλάτων, ἀλλὰ λόγον ἀριθμοῦ*. Here the error is recognized, but Theo himself falls victim to it (though clumsiness of expression is partly to blame): *τὸ διάστημα τὸ τῶν σνς′* (256) *πρὸς σμγ′* (243), *τουτέστι τὰ ιγ′* . . . (69.12f). At 69.15 the semitone is equated with the ratio 17 : 16, and correspondingly we read at 70.16f, *ἐν μὲν ἀριθμῷ οὐκ ἀεὶ εἰς ἴσα τέμνεται τὸ ἐπόγδοον διάστημα*—as though 18 : 17 were the same as 17 : 16; then the *ὑπεροχή* 27 is divided into 14 and 13 (70.18), and this is supposed to show that the tone 243 : 216 is not evenly divisible. So in Nicom. *Exc.* p. 269f, the series tone, *leimma*, tone is calculated with the numbers 216, 243, 256, and 288; it is pointed out that 13 (256 − 243) is not the half of either 32 (288 − 256) or of 27 (243 − 216), so that the tone is not evenly divisible. Similarly, in Macrob. *Somn. Sc.* 2.1.22, the tone 9 : 8 is said to be indivisible because 9 cannot be bisected. All of this is not far behind Philolaus in mathematical absurdity; but deficiency of mathematical talent cannot serve as a chronological criterion. (Cf. n. 47.)

[51] Plut. *De an. procr.* 14.1018e: *οἱ Πυθαγορικοὶ τὸν τόνον ἐν τούτῳ τῷ ἀριθμῷ* (sc. 27) *τάττουσι.* 1019a *οἷς* (with the numbers 13 and 27) *τὰ μελῳδούμενα μετροῦσιν εὐσήμως οἱ μαθηματικοὶ διαστήματα, τὸ μὲν δίεσιν, τὸ δὲ τόνον καλοῦντες.*

[52] The latter solution is adopted by Tannery, *MSc* III 240f, citing the passages of Plutarch.

[53] Boeth. *Mus.* 3.8 p. 278.11 (DK I 410.4ff).

shock of this is less in the hairsplitting procedure of dividing even the *komma* once more than in the nonchalance with which this process of bisection is introduced. If the *komma* can be bisected, surely a whole tone may; but it is a basic tenet of Pythagorean musical theory that neither the whole tone, the octave, nor in general any of the "superparticular ratios" can be divided into two equal parts.[54] To be sure, Philolaus realized that the *diesis* 256 : 243 is smaller than a semitone; but when the calculation of proportions is abandoned, bisection comes in by the back door. In Tannery's opinion, to allow this is to disqualify oneself as a Pythagorean.[55]

Once more, however, there is a significance underlying the error. The *apotome* is necessary for the construction of the chromatic tetrachord; in this case the series semitone, semitone, tone and a half is defined more precisely as *diesis*, *apotome*, *diesis* plus a whole tone; and this formulation is obviously presupposed by Archytas.[56] The bisection of the *diesis*, however, has to do with the enharmonic genus, whose structure, rather than quarter-tone, quarter-tone, ditone become *diaschisma*, *diaschisma*, ditone.[57] This makes it seem that Philolaus dealt with all three genera, taking the usual, "Aristoxenian" conception as his point of departure. It is to some extent restated in terms of proportion, and correspondingly corrected, but this is not carried through, and the numbers are treated as addable magnitudes. The idea of lines and distances replaces the theory of proportion, just as number symbolism smothers mathematics.

The coincidences with commentaries on the *Timaeus*, which were to be found in the faulty determination of the *diesis* (above, n. 50), do not extend to the further subdivision of the *diesis* and *komma*. There was no necessity, from the point of view of the *Timaeus*, to go into the chromatic and enharmonic genera; but the *Timaeus* commentaries never fail to state that the whole tone cannot be bisected[58]—not even by subterfuge. Thus we are precluded from deriving the Philolaus

[54] For the impossibility of dividing evenly a superparticular ratio, and in particular the whole tone, see Archytas A19, *Sect. can.* 3, 16, 18, Ptol. *Harm.* 1.10, Nicom. *Exc.* 2, p. 267ff, Procl. *In Tim.* II 179.8ff, Gaudentius 13, Boeth. *Mus.* 3.1ff; above, ch. V 1, n. 4.

[55] Tannery, *MSc* III 233. The naiveté of Leucippus is analogous, in declaring that the atoms are indivisible because of their smallness (DK II 97.26, 67A13).

[56] Above, n. 14. Proclus, *In Tim.* II 188.21, says that the *apotome* does not belong to the diatonic genus.

[57] Tannery, *MSc* III 224f.

[58] Cf. above, nn. 50, 54. For the indivisibility of the *leimma*, see *Sect. can.* 18.

testimonia from the *Timaeus* tradition; and Philolaus stands quite alone in the later tradition with his statements about the *schisma* and *diaschisma*. But Archytas was the first to give a proof of the basic tenet of music theory which Philolaus violates. If we consider this in connection with the fact that Archytas presupposes the calculation of the *diesis* at 256 : 243, and the structure of the chromatic tetrachord in the manner of Philolaus, it follows that the material introduced by Boethius must have belonged to Pythagorean musicology before Archytas. The very inadequacy of the mathematics of Philolaus' system, which takes no account of the accomplishments of Archytas, becomes a proof of its authenticity. Thus the results of our examination of the philosophical fragments and the astronomical system are corroborated in a third area: there have been preserved, from a book of the Pythagorean Philolaus, written toward the end of the fifth century B.C., some authentic fragments, partly in the original wording and partly as reported by others.

If these Philolaus testimonia are genuine, the conception of Pythagoreanism which Frank and Tannery take as their point of departure will at any rate have to be corrected. If a Pythagorean of the fifth century B.C. could fall into mathematical inconsistencies, and even commit gross errors, then the nature of pre-Platonic, or rather, perhaps, pre-Archytean Pythagoreanism cannot have lain in exact mathematics or in natural science, but in the interpretation of the world with the help of numbers thought of as symbols. In fact the practical significance of Pythagorean musical theory is minimal. Only the basic facts can be established by observation, and it was impossible to transpose the values established by calculation back into audible tones. From this point of view it is not surprising that Philolaus' results for the *apotome* and *komma* were mistaken. The attraction and the significance of this theory lie not in the theory itself but in the orderly, rational pattern that it reveals. Order and pattern, however, which the human spirit craves, are to be found not only in the form of conceptual rigor and neatly logical structure, but, at an earlier level, in richness of mutual allusiveness and interconnection, where things fit together "symbolically." Thus the interrelation of number and music can be conceived, earlier than any mathematically oriented natural science and quite apart from it, as an aspect of the universal orderliness of the cosmos. In the Chinese culture, where the recognition of basic musical relationships is developed into an ingenious and intricately varied numerical

structure, mathematical precision is purposely avoided.[59] Not from Philolaus alone does it become clear that the important thing in Pythagorean musical theory was not the function of the proportion but the meaningful numbers. Van der Waerden draws attention to the tetractys, which has its roots in the ancient stratum of the *acusmata* tradition.[60] The "Fourness" which is the "harmony" in which the Sirens sing, suggests the numbers 1, 2, 3, 4, which group themselves into the fundamental concords 2 : 1, 3 : 2, and 4 : 3, and thus comprehend the orderliness not only of music but of the universe; and the sum of these four numbers is 10, the "perfect" number. The tradition of the *acusmata* is independent of Philolaus,[61] and leads back, past him, to the oldest stratum of Pythagoreanism; and the idea of the music of the cosmos is also of great antiquity. According to a report of Eudemus, the Pythagoreans emphasized that the fourth, the fifth, and the octave are comprised in the number 9, because $2+3+4=9$; and here, too, it is clearly number as such, not proportion, that is the significant thing.[62] Not only Hippasus but Archytas as well classified the intervals by use of individual numbers.[63] The earliest Pythagorean musical theory is not founded on mathematics or on experimental physics, but on "reverence" for certain numbers in their roles in music and cosmology; and this situation is never completely abandoned. On this basis, according to our information, Hippasus made certain experiments, and Philolaus, in his effort to express Pythagorean lore in the form of Ionian *φυσιολογία*, made individual statements about the numerical structure of ordinary music, showing a truly remarkable mixture of calculation and numerical symbolism, in which its "sense" is more important than its accuracy. Only in Archytas does a real mathematical number theory grow out of this, and, in its application to music, a certain sort of natural science —though this is nearly incapable of further development or progress, since its effort is to discern unalterable order in what already exists.

[59] Cf. below, ch. VI 4.
[60] *Hermes* 1943, 178f.
[61] On astral immortality, above, ch. IV 4.
[62] Eudemus fr. 142 = DK 58B18.
[63] Hippasus DK 18.14 (but see ch. V 1, n. 35), Archytas A17 (ch. V 1, n. 77).

VI. Pythagorean Number Theory and Greek Mathematics

1. DID THE PYTHAGOREANS LAY THE FOUNDATIONS OF GREEK MATHEMATICS?

As pre-Greek mathematics has been rediscovered in Egyptian papyri and Babylonian clay tablets, a clearer light has been thrown on the outstanding achievement of the Greeks in the development of pure mathematics. The Babylonians, in particular, had made considerable progress in the accumulation of detailed knowledge, in practical calculation, and in the solution of even rather complicated problems in arithmetic; beyond question, the Greeks had much to learn from them. But it was always single problems they were concerned with, making use of certain "recipes," without any theoretical explanation or even an attempt at proof; "we cannot even be certain that the Babylonians formulated theorems in general terms."[1] Some of the "recipes" or formulas are inexact,[2] but this did not matter as long as they provided a practically useful approximation. Only with the advent of Greek geometry do we find the demand for generalized and stringent proof, for a deductive system based on axioms and postulates.[3] This is the system presented to us in the *Elements* of Euclid, a model which until the nineteenth century seemed not to require any essential improvement. All later achievements, including those of the Indians and the Arabs,[4] build on the foundations laid by the Greeks.

The very importance and influence of Euclid, however, makes the study of early Greek mathematics more difficult. What is preserved for us is, as so often happens, the final accomplishment, and by its very greatness it obstructs our vision of the earlier development. It

[1] Becker, *Grdl.* 22; cf. *MD* 11: "Sammlung von Rezepten"; Neugebauer, *ExSc* 48: "Babylonian mathematics never transgressed the threshold of prescientific thought"; 146: "Greek mathematics of the Euuclidean style is a strictly Greek development."
[2] For example, the Babylonian formulas for the volume of a frustum of cone and pyramid are wrong (though the former is still used today by woodsmen for calculating the volume of a tree trunk): van der Waerden, *SA* 75f.
[3] On this see the important paper of Kurt von Fritz, *ABG* 1955.
[4] Neugebauer, *ExSc* 166ff.

virtually doomed its predecessors to extinction;[5] pre-Euclidean mathematics has to be reconstructed piecemeal from isolated fragments and allusions. The most important are, first, the mathematical passages in Plato and Aristotle,[6] and, secondly, the few fragments that have been preserved from the ἀριθμητικὴ ἱστορία and the γεωμετρικὴ ἱστορία of Aristotle's pupil Eudemus.[7] The "catalogue of geometers" in Proclus' commentary on Euclid is thought, with good reason, to be derived in the main from Eudemus.[8] In the van of geometers who are known for their specific accomplishments stands Hippocrates of Chios, who was active about 430 B.C., in Athens.[9] His methods for the quadrature of lunes, which Eudemus describes in detail, show a well-developed style in setting forth a proof and a considerable stock of geometrical theorems, some of a rather complicated kind.[10] As the first to write Στοιχεῖα, "arguments set in line," he had a clear idea of the deductive nature of mathematics.[11] A contemporary of Hippocrates was Theodorus of Cyrene, who discussed irrational quantities,[12] and a generation later came Theaetetus of Athens[13] and Archytas of Tarentum.[14] Still a little later was Eudoxus of Cnidus, who proved himself the most brilliant Greek mathematician before Archimedes, no less by his work on the theory of proportion and his rigorous application of the method of exhaustion than by his epoch-making theory of the planets.[15] Eudoxus was a younger contemporary of Plato. This was a period of especially rich development in Greek mathematics, which reached its high point in the following century, the time of Euclid, Archimedes, and Apollonius.

As early as Hippocrates of Chios the special character of Greek mathematics was fully in evidence. But how did the geometry of Hippocrates come to be? The usual view, held almost unanimously, is that its origin is to be sought among the Pythagoreans, if not in Pythagoras himself. Of course, the unreliability of the later tradition about Pythagoras, and the well-known inclination to attribute later achievements to Pythagoras, have not been left out of account; but scholars have felt that the demands of the critical conscience were adequately satisfied by speaking of Pythagoreans instead of Pythagoras.[16] In accounts of the history of mathematics, "Pythagorean mathematics" is treated as a self-contained unit between Thales and

[5] The two small, extant astronomical writings of Autolycus of Pitane (ed. J. Mogenet, Louvain 1950; see also Neugebauer, *ExSc* 225f) are somewhat earlier than Euclid.

[6] C. Mugler, *Platon et la recherche mathématique de son époque* (Paris, 1948), on which see Cherniss, *Rev. of Metaphysics* 4 (1951) 395–425; R. S. Brumbaugh, *Plato's Mathematical Imagination* (Bloomington, 1954); A. Wedberg, *Plato's Philosophy of Mathematics* (Stockholm, 1955; only treats purely philosophical problems); T. Heath, *Mathematics in Aristotle* (Oxford, 1949).

[7] Frr. 133–142, along with the fragment preserved in Arabic, below, ch. VI 2, n. 8.

[8] Fr. 133 = Procl. *In Eucl.* 64.16–68.4. The most important indication of its derivation from Eudemus is Proclus' remark (68.4), οἱ μὲν οὖν τὰς ἱστορίας ἀναγράψαντες μέχρι τούτου προάγουσι τὴν τῆς ἐπιστήμης ταύτης τελείωσιν (i.e., until the time of Plato and Philippus of Opus); Eudemus wrote before Euclid.

[9] Above, ch. IV 1, n. 77.

[10] Eudemus fr. 140; edited with commentary by F. Rudio, *Der Bericht des Simplicius über die Quadraturen des Antiphon und des Hippokrates* (Leipzig, 1907). On the importance of Hippocrates, von Fritz, *Philologus* 1932, 47ff. Hippocrates knew, for example, the Pythagorean theorem in the generalized form applying to scalene triangles. There are 5 lunes that can be squared, and Hippocrates squared 3 of them (Heath, *Math.* I 199f). Aristotle accuses him of a fallacy (*Phys.* 185a16, *Soph. el.* 171b12), which consisted, according to Simplicius (*Phys.* 67.4ff, 69.23ff), in supposing that he had squared "every" lune because he had succeeded in doing so in the cases in which the outer arc is greater than, equal to, or less than a semicircle, and that, further, he had hereby also solved the problem of squaring the circle, because in another special case he could square the sum of a lune and a circle. Historians of mathematics hesitate to attribute such a fallacy to Hippocrates, in view of the sophistication of his mathematical procedure (Heath, *Math.* I 198f, 196 n. 1; von Fritz, *Philologus* 1932, 49ff; cf. *ABG* 1955, 93 n. 174; Wehrli, *Eudemos* 117). The goal of Hippocrates, however, was surely the squaring of the circle. A difficult question, hardly ever asked, is how much Eudemus is modernizing in his report and making Hippocrates' proofs seem more sophisticated than they were (as a modern historian of mathematics unhesitatingly uses algebraic notation even in discussing the Greeks).

[11] See Burkert, *Philologus* 1959, 193ff; below, n. 98.

[12] DK 43, with the addition of Iam. *Comm. math. sc.* p. 77.25. Theodorus appears in the catalogue of Pythagoreans (Iam. *VP* p. 146.8). For confirmation, reference has been made to Pl. *Tht.* 145c, according to which Theodorus taught the quadrivium; but the same source says expressly that he was a pupil of Protagoras, who "veered off" into mathematics (164e; cf. 161b, 162a, 183b). Even if we take the Pythagorean catalogue seriously (see ch. II 1, n. 40), there remains the question whether Theodorus owed his mathematical prowess any more to his Pythagoreanism than Dicon did his Olympic victory (p. 146.4 Deubner; *RE* XI 74).

[13] See esp. Sachs, and von Fritz, *RE* V A 1353–1363. The most important evidence, outside of Plato, is a fragment of Eudemus preserved in Arabic (below, ch. VI 2, n. 82).

[14] His principal accomplishment was the first solution of the problem of doubling the cube; Eudemus fr. 141 = DK 47A14.

[15] Cf. Heath, *Math.* I 322ff; van der Waerden, *SA* 179ff; Lasserre.

[16] Zeller, *Vortr. u. Abh.* 38f: "It was the Pythagoreans whose successful efforts in the mathematical sciences first ensured these studies a place among the Greeks"; I 405: "among the sciences, the Pythagoreans devoted themselves especially . . . to mathematics, which owes to them its first significant treatment"; and, with this, n. 2: ". . . that Pythagoras gave the first impulse to the momentous development of mathematics in his school." Cornford says, *CQ* 1923, 5, "Pythagoras was the discoverer of the world of mathematics"; Burnet, *ThPl* 38: "it is . . . difficult to reject the tradition that makes Pythagoras the true founder of mathematical science"; Hasse-Scholz 5: "the Pythagoreans laid the foundations of Greek mathematics"; Becker, *MD* 12: Pythagoras was "unquestionably the founder of a great school of mathematics" and creator of "mathematics as pure *theoria*"; and Frank, *AJP* 61 [1940] 48: "Mathematics as a science came into the world, which was the accomplishment mainly of the Pythagoreans."

Hippocrates, and is dated, at least by implication, in the first half of the fifth century B.C.

To be sure, the general belief in "Pythagorean mathematics" has not gone without criticism. Tannery more than once expressed himself skeptically on the matter,[17] and the mathematicians Junge and Vogt subjected the "geometry of Pythagoras" to a critical examination[18] which had the notable result of redating the discovery of incommensurability to the end of the fifth century. Though this chronological revision was disputed, the attribution of this discovery to Pythagoras himself, made by Proclus, has not since been seriously defended. Eva Sachs's book had as its objective to "get rid of the myth of Pythagoras the mathematician" (p. VI); and she was able to show, with relation to the special problem of the regular polyhedra, how the tradition of Pythagoras' treatment of it had been derived from the *Timaeus*, obscuring the contribution of Theaetetus.[19] This result has been widely accepted, though not enough attention has been paid to the more general implications, and in particular those resulting from analysis of the "catalogue of geometers." Somewhat later Heidel made a fresh start on the whole question of the Pythagoreans and Greek mathematics and worked out in a more adequate way the development of non-Pythagorean, Ionian mathematics.

No other branch of history offers such temptations to conjectural reconstruction as does the history of mathematics. In mathematics every detail has its fixed and unalterable place in a nexus of relations, so that it is often possible, on the basis of a brief and casual remark, to reconstruct a complicated theory.[20] It is not surprising, then, that the gap in the history of mathematics which was opened up by critical study of the evidence about Pythagoras has been filled by a whole succession of conjectural supplements. They have been based mainly on the analysis of Euclid's *Elements*, regarded increasingly as a collection rather than an original work, and, secondly, on conclusions drawn from the argumentation of the Eleatics, especially Zeno, in which

[17] *HScH* 381ff, *MSc* II 200f.

[18] Junge, *Symb. Joach.*; H. Vogt, "Die Geometrie des Pythagoras," *Bibl. Math.* III 9 (1908/1909) 15–54; "Die Entdeckungsgeschichte des Irrationalen," *Bibl. Math.* III 10 (1909–1910) 97–155.

[19] Above, ch. I 3, n. 116.

[20] The standard example is the reconstruction of the planetary system of Eudoxus (above, ch. IV 2, n. 2).

scholars have found a specifically mathematical sense. Tannery was a pioneer in both these lines of research;[21] and the most influential reconstructions have been those of Becker[22] and van der Waerden.[23] Becker's results, especially, have come to be looked upon as a fixed point in the history of Pythagorean science.[24]

We have already mentioned the general problems involved in this kind of reconstruction, in the history of science. The fact of a logical inference can never determine precisely the time or place of the inference, or the person who made it; no matter how exactly it may be determined what propositions Hippocrates of Chios assumes as proven, this will not increase the likelihood that Pythagoras or certain Pythagoreans discovered and proved these propositions. This is the point at which the critical analysis of historical evidence must supplant logical inference.[25] Another consideration cuts even deeper: a mathematically impeccable reconstruction is valid only to the extent that it can be known with certainty that mathematical logic was at work in the original formulation. In dealing with the beginnings of mathematics, when mathematical logic was first being developed, it cannot be regarded as certain a priori that a mathematician respected the presuppositions and drew the conclusions that mathematical logic would have dictated. Of course logic has been inherent in human thinking from time immemorial, and perhaps especially in Greek thinking. But abstract mathematics, making use of proof, was an invention of the Greeks, and not part of their original mental equipment. The manner of its development cannot be determined by logical inference, since this presupposes the decisive point. In the history of science the old maxim is still valid, that we must not ascribe to thinkers, especially those of early times, "either the principles of their consequences or the consequences of their principles."[26]

[21] For the argument based on the Eleatics, see above, ch. III 3. In his evaluation of "Pythagorean arithmetic," Tannery renounced his earlier skepticism when he became aware of Archytas A19 (1905: *MSc* III 244ff), and decided for "l'existence, dès le temps d'Archytas, d'Éléments arithmétiques développés sous la forme que nous nommons euclidienne" (249).

[22] *QSt* 3; cf. below, VI 2, nn. 46ff.

[23] *MtAnn* 1947–1949, cf. below, ch. VI 2, nn. 98ff.

[24] Thus Reidemeister (16ff) pushes all the evidence to one side and finds the true "pythagoräische (sic) Arithmetik" in Becker's reconstruction.

[25] Above, ch. IV 1, nn. 14–16.

[26] "Ni les principes de leurs conséquences ni les conséquences de leurs principes," as formulated in the 18th century by Charles Batteux (Mondolfo, *Inf.* 306), and cited repeatedly by Tannery.

What is the origin of the firmly rooted conviction that Pythagoreanism was the source of Greek mathematics? This question is easy to answer: it came from the educational tradition. Everyone comes upon the name of Pythagoras for the first time in school mathematics; and this has been true from the earliest stages of the Western cultural tradition. None of the ancient textbooks which formed the basis of the medieval curriculum forgets Pythagoras. He is the companion of Arithmetica in Martianus Capella;[27] and according to Isidore he was the first, among the Greeks, to sketch out the doctrine of number, which was then set forth in detail by Nicomachus.[28] This takes us back to the origin of this tradition; Nicomachus, who is himself called a Pythagorean, begins his *Arithmetic*, which was much used as a schoolbook, with praise of the Master. Boethius' *Arithmetic*, drawn largely from Nicomachus, also names Pythagoras in its first line. Likewise, Gerbert of Aurillac mentions the name of Pythagoras several times in his geometry;[29] and among the patrons of their *ars geometriae* medieval Freemasons include Pythagoras.[30] The *Ars geometriae* bearing the name of Boethius, though obviously not composed before the High Middle Ages, even presents an early version of the Arabic numerals as an invention of the "Pythagorici," and describes the method of calculating with these *apices* on an abacus, called *mensa Pythagorea*—perhaps the most striking of the anachronisms in which the Pythagorean tradition is so rich.[31] Finally, the early modern period derived the astronomy of Copernicus and Galileo from Pythagoras.[32]

The general belief in the Pythagorean origin of mathematics thus stems from the Neoplatonic and neo-Pythagorean scholastic tradition of late antiquity. In evaluating this it is worth bearing in mind that according to an earnestly meant statement of Iamblichus, even the

[27] Mart. Cap. 7.729, 8.803.

[28] Isid. *Et.* 3.2.

[29] Heath, *Math.* I 366.

[30] M. Ghyka, *Le nombre d'or, rites et rhythmes pythagoriciens dans le développement de la civilisation occidentale* I (Paris, 1931) 66.

[31] Boeth. *Geom.* p. 396f Friedlein. The numerals in question are the so-called Gobar numerals (1 to 9, but no zero). See K. Menninger, *Zahlwort und Ziffer* II² (Göttingen, 1958) 231ff and 132ff. The problem of the Boethian geometry, and especially the *mensa Pythagorica*, has not been completely solved; see M. Manitius, *Lat. Lit. d. Mittelalters* I (Munich, 1911) 27f; Heiberg 49; Schanz–Hosius, *Röm. Lit.-Gesch.* IV 2 (Munich, 1920) 153f. The literature is cited by Capparelli (I 51ff), who may well be the only one to believe that Pythagoras himself invented the Arabic numerals. See M. Folkerts, *"Boethius," Geometrie II: Ein mathematisches Lehrbuch des Mittelalters* (Wiesbaden, 1969).

[32] Above, ch. IV 3, n. 2.

problem of squaring the circle was solved by Pythagoreans.[33] The earlier tradition is much more sparse, but it does take us back to the threshold of the Hellenistic period. It is repeatedly stated that geometry was imported from Egypt; Herodotus speaks of the Nile floods and the continuing necessity to resurvey the land,[34] and Aristotle of the leisure (σχολή) of the Egyptian priests, which enabled them to engage in speculation (θεωρία).[35] Now, as early as Hecataeus of Abdera, Pythagoras is represented as bringing to Greece from Egypt, along with the doctrine of metempsychosis, *τὰ κατὰ γεωμετρίαν θεωρήματα καὶ τὰ περὶ τοὺς ἀριθμούς*;[36] and not much later Anticlides the historian of Alexander wrote that Pythagoras brought geometry to perfection, after it had been invented by the Egyptian king Moeris.[37] Hermesianax of Colophon regards Pythagoras as primarily the coryphaeus of geometrical astronomy,[38] and Callimachus alludes, though in an ambiguous way, to Pythagoras' discoveries in geometry.[39] To be sure, the notable development of Egyptian mathematics is legendary,[40] and it is remarkable that so remote a writer as Anticlides should be concerned to testify to the mathematical achievements of Pythagoras.

[33] Iam. ap. Simpl. *Categ.* 192.16ff, citing a certain Sextus. Perhaps the opinion of "certain persons" belongs here (Alex. Aphrod. ap. Simpl. *Phys.* 58.25ff) that the problem of squaring the circle would be solved when a *τετράγωνος ἀριθμός* became at the same time *κυκλικός*. (On this idea, Nicom. *Ar.* 2.17.7, Theo Sm. 38, Iam. *In Nic.* 61, 94f, Procl. *In Eucl.* 150f).

[34] Hdt. 2.109; cf. Strabo 17, p. 787, Diod. 1.69, 81, Iam. *VP* 158, Hero *Geom.* 2 p. 176 Heiberg. Invention of mathematics by Theuth: Pl. *Phdr.* 274c. Egyptian *ἁρπεδονάπται*, "rope-stretchers," are mentioned by Democritus, fr. 299—an interesting, and indeed famous word, but attested in a fragment of dubious authenticity. Cf. Heath, *Math.* I 121f.

[35] Arist. *Met.* 981b23.

[36] Diod. 1.98.2 = *FGrHist* 264F25 (cf. Schwartz, *RE* V 670f).

[37] D.L. 8.11 = *FGrHist* 140F1 (Müller, *FHG* I 212 had mistakenly taken this clause with the preceding citation of Timaeus, *FGrHist* 566F17): *καὶ γεωμετρίαν ἐπὶ πέρας ἀγαγεῖν* (sc. *Πυθαγόραν*), *Μοίριδος πρώτου εὑρόντος τὰς ἀρχὰς τῶν στοιχείων αὐτῆς ὥς φησιν Ἀντικλείδης ἐν δευτέρῳ περὶ Ἀλεξάνδρου*. The whole sentence must come from Anticlides, for Diogenes Laertius had no other reason to cite him except for his mention of Pythagoras.

[38] Hermesianax fr. 2.85ff Diehl:

> . . . *Πυθαγόρην ἑλίκων κομψὰ γεωμετρίης*
> *εὑράμενον καὶ κύκλον, ὅσον περιβάλλεται αἰθήρ,*
> *βαιῇ ἐνὶ σφαίρῃ πάντ' ἀποπλασσάμενον.*

(So Powell: *ἀποτασσάμενον* MSS, *ἀπομασσάμενον* Hemsterhuis, Diehl, *ἀπομαξάμενον* Kaibel.)

[39] Below, n. 106.

[40] Modern scholarship rates the Babylonian influence on Greek mathematics much higher (see Heath, *Math.* I 122ff; Becker, *MD* 9; van der Waerden, *SA* 15ff, 35f). On the other hand, the Babylonian tradition is scarcely mentioned in the Greek literature (aside from Iam. *In Nic.* 118.23; below, ch. VI 2, n. 89). But we learn that Pythagoras learned *τὰ περὶ ἀριθμούς τε καὶ λογισμούς* from the Phoenicians (Por. *VP* 6, Iam. *VP* 158).

Neither Herodotus nor Isocrates sees any occasion to mention Pythagoras in connection with Egyptian geometry, though they do so in speaking of Egyptian burial rites and *ἁγιστεῖαι*.[41] Was there a significant change in the image of Pythagoras between Isocrates and the epoch of Hecataeus of Abdera and Anticlides—that is, in the period of the Old Academy? The way the tradition about Pythagoras expanded can be seen in a sentence of Aëtius, appended to his report of Hipparchus' theory of vision: "Some also credit Pythagoras with a share in this doctrine, as being the chief authority in mathematics (*βεβαιωτὴν τῶν μαθημάτων*)."[42] Here tradition is not being transmitted, but manufactured, on the basis of the dogma that it was Pythagoras who established the mathematical sciences.

Modern scholarship bases its judgment about Pythagoras the mathematician on certain other pieces of evidence that appear to be more reliable. Tannery's point of departure was the crucial question of when "Pythagorean geometry" was committed to writing. In Iamblichus we read, *ἐκαλεῖτο δὲ ἡ γεωμετρία πρὸς Πυθαγόρου ἱστορία*,[43] and Tannery translated this, "geometry was called 'the tradition according to Pythagoras.'"[44] In consideration of the context in Iamblichus, he interpreted this to mean that before Hippocrates of Chios there was published a treatise on geometry with the title *The Tradition according to Pythagoras*. This Pythagorean textbook was the cornerstone of Tannery's reconstruction, and it has continued to play a part right down to the present day.[45] It owes its existence, however, to an obvious mistake in translation; it is impossible to take *πρὸς Πυθαγόρου ἱστορία* together, and the meaning must be "geometry was called *ἱστορία* by Pythagoras." Thus the topic under discussion is Pythagoras' use of words, and not anything about a book. Hölk long ago found the surprising explanation of this report. Heraclitus wrote, "Pythagoras son of Mnesarchus practiced inquiry (*ἱστορίην*) most of all men" (fr. 129). This sentence is given by Diogenes Laertius as proof that Pythagoras left writings, and thus it has been a link in the Pythagorean tradition. It was only natural for a later Pythagorean to draw the conclusion: Pythagoras was neither a historian nor a geographer, but *βεβαιωτὴς τῶν μαθημάτων*; therefore Heraclitus must mean *γεωμετρία*. Thus the whole sentence is an erroneous philological inference from a sentence of Heraclitus.[46] Even if this explanation were not allowed as more than a possibility, there remains no firm basis for the belief that Pythagoras was a geometer, and in any case no attestation of his having written anything.

The chief testimony for Pythagoras as a mathematician, always cited in the literature,[47] is in the "catalogue of geometers" given by Proclus, whose principal source is rightly thought to be Eudemus.[48] "Pythagoras turned its (geometry's) philosophy into a form of liberal education, seeking its first principles (*ἀρχάς*) from a higher source (*ἄνωθεν*) and hunting out its laws by a nonmaterialistic and intellectual procedure (*ἀύλως καὶ νοερῶς*) . . ." The weight of this pronouncement is enhanced by the prestige of Eudemus as a pupil of Aristotle, as well as by the undeniable fact that the special character of Greek mathematics consists precisely in its theoretical structure, as distinguished from the oriental "recipes." To be sure, the passage that follows, ascribing to Pythagoras the discovery of irrationality and of the "cosmic bodies," is less often accepted;[49] but even in the sentence quoted there are suspicious features. Does the phrase *ἀύλως καὶ νοερῶς* seem more like the phrase of an early Peripatetic, or like a favorite theme of all Neoplatonists, and especially Proclus?[50] And does Aristotle not say expressly, of the Pythagoreans, "they apply their propositions to bodies"—bringing out the distinction, in this regard, between them and all genuine

[41] Cf. above, ch. II 3.

[42] Aët. 4.13.10 (cf. above, ch. I 2, n. 76).

[43] Iam. *VP* 89 = *Comm. math. sc.* p. 78.5; on the connection between the two parallel versions, see above, ch. II 5. The sentence cited here is obviously an interpolation in the Aristotelian context; cf. below ch. VI 3, n. 59.

[44] "Tradition touchant Pythagore," Tannery, *Géom.* 81f; "une géometrie, la *Tradition suivant Pythagore*," *HScH* 124; *MSc* VII 110f.

[45] Rey 227ff; Michel 81, 174f; van der Waerden, *SA* 116f; Szabó, *Maia* 10 (1958) 106ff. Heath had the correct translation (*Math.* I 166), as did Frenkian, *Maia* 11 (1959) 243–245.

[46] Hölk 8f. Thus it is not so that "it is utterly inconceivable how . . . anyone could have thought of inventing anything of the sort" (von Fritz, *SBMü* 1960, 20).—Ps.-Hippoc. *Ep.* 22 uses the phrases *ἱστορίη γεωμετρικὴ* (1) and *ἡ τῆς γεωμετρίης ἱστορίη* (2), along with the word *ἀρίθμησις*; thus he knows the connection of *γεωμετρίη* and *ἱστορίη*, and uses it as an element of Ionic coloring.—Von Fritz (*SBMü* 1960, 20) interprets the word *ἱστορίη* as reflecting the fact that Pythagoras collected specific pieces of mathematical knowledge from oriental sources; and Frenkian's interpretation is similar.

[47] E.g. Heath, *Aristarchus* 46, *Math.* I 141; Rey, 216; Michel passim, esp. 168ff; Morrison, *CQ* 1956, 153; Becker, *Grdl.* 22, *MD* 12. A cautious attitude is expressed by van der Waerden (*SA* 90f; but cf. 100), by Wehrli (*Eudemos* 114; but at 115 the sentence cited is interpreted as deriving from Eudemus), by Reidemeister 18ff, Heidel (*AJP* 1940, 16ff), and von Fritz (*RE* XXIV 198—but no mention is made that the sentence comes from Iamblichus). Timpanaro Cardini (I 30 n.) cites Sachs (whom Frank follows, 363 n. 209), but without reason given concludes, "ritengo che sia da sostenere la derivazione da Eudemo"; Iam. *Comm. math. sc.* is ignored.

[48] Procl. *In Eucl.* 65.16 = Eudemus fr. 133 (cf. above, n. 8) = DK 14.6a.

[49] Above, nn. 18–19.

[50] *ἄυλος* is attested once by Aristotle, *νοερός* twice, but neither as adverb. Cf. Procl. *In Eucl.* 9.13, 63.5, 137.21, et saep.

Platonists?[51] And does not Eudemus, as far as we know from other fragments, always speak of Pythagoreans, never of Pythagoras, just as Aristotle himself does in philosophical or scientific contexts?[52] A lucky coincidence turns suspicion into certainty: the sentence in question is taken word for word from Iamblichus' *De communi mathematica scientia*—a work that Proclus copies sometimes by the page in his commentary on Euclid.[53] Iamblichus is concerned with "Pythagorean mathematics."[54] He mentions (ch. 21) that the origins of mathematics lie before Pythagoras, in the work of Egyptians, Assyrians, and Chaldaeans. The distinctive aspect of Pythagoras' work[55] is not only in new discoveries, but above all in the "purity, subtlety, and exactitude"[56] of his method, and in the way it purifies the soul and leads on to the highest principles and a realm of pure, immaterial Being. Iamblichus admits that Pythagoras and his pupils did not write any of this down, and therefore it is necessary to reconstruct, with considerable effort, "what they would probably have said if one of them could have taught his doctrine publicly."[57] But Iamblichus has no doubts about his Neoplatonic theme: "If we are to pursue mathematics in the Pythagorean manner, we must follow its upward path, full of divinity, which brings purification and perfection." *ὅτι τοίνυν οὐδὲ εἰκῇ Πυθαγόρας τὴν περὶ τὰ μαθήματα φιλοσοφίαν εἰς σχῆμα παιδείας ἐλευθερίου μετέστησε, καὶ τῷ τε πλήθει τῶν δεικνυμένων προῆγεν αὐτὰ καὶ τῇ τῶν ἀποδείξεων ἀκριβείᾳ, τῆς τε ἀναγκαίας χρήσεως πρὸς τὸν βίον περιττότερον αὐτὰ ἤσκησεν, ἐντεῦθεν ῥᾴδιον καταμαθεῖν.* This is the beginning of a new chapter,[58] in which the significance of mathematics is discussed, both

[51] Arist. *Met.* 1083b18; on the other hand, Plato *Rep.* 521d, Plut. *Quaest. conv.* 8.2.1. For the "Pythagorean" saying *σχᾶμα καὶ βᾶμα*, see Procl. *In Eucl.* 84.15ff, Iam. in the text above, and cf. above, ch. I 2. Plato's direct criticism of the Pythagoreans is also to be considered; above, ch. V 1.

[52] Eudemus frr. 60, 88, 136, 137, 142, 146. For Aristotle, ch. I 2.

[53] See the references in N. Festa's edition of *Comm. math. sc.* (Teubner 1891).

[54] *Comm. math. sc.* p. 66.9f: *ἐπεὶ δὲ τῆς Πυθαγορείου μὲν μαθηματικῆς προηγουμένως ἀντιποιούμεθα . . .*

[55] *Comm. math. sc.* p. 67.6f (cf. the table of contents 6.21f): *τὴν . . . ἰδιότητα αὐτοῦ τῆς μαθηματικῆς . . .*

[56] *Comm. math. sc.* p. 67.22f: *καὶ μὴν ἀποδείξεών γε καθαρότητι λεπτότητί τε καὶ ἀκριβείᾳ . . .*

[57] *Comm. math. sc.* p. 68.11ff: *. . . δεῖ τοιόνδε τι ποιεῖν· ἀπὸ σμικρῶν αἰθυγμάτων ὁρμωμένους σωματοποιεῖν ἀεὶ τὰ τοιαῦτα καὶ συναύξειν, εἰς ἀρχάς τε αὐτὰ ἀνάγειν τὰς προσηκούσας καὶ τὰ παραλειπόμενα ἀναπληροῦν, στοχάζεσθαί τε κατὰ τὸ δυνατὸν τῆς ἐκείνων γνώμης, τίνα ἂν εἶπον, εἰ ἐνεχώρει τινὰ αὐτῶν διδάσκειν . . .*

[58] *Comm. math. sc.* 23, p. 70.1ff; *ἐλευθέριος παιδεία* points to the succeeding context (p. 70.15ff), *προῆγεν* and *ἀκρίβεια* to the preceding (p. 67.3ff). Thus it is out of the question that Iamblichus just happened to quote Eudemus in the transitional sentence.

for practical life and in itself. This may be based in part on Aristotle;[59] but, if anything in it is original with Iamblichus, it is the chapter division and the transitional formulae.[60] In the succeeding passage there is nothing about Pythagoras and the Pythagoreans except Iamblichus' introduction and his concluding sentence, "It was natural, then, that for all these reasons the Pythagoreans honored the study of mathematics."[61]

Therefore the often cited sentence about Pythagoras in the "catalogue of geometers" is not from Eudemus, but is a formulation of Iamblichus, as had been recognized over sixty years ago.[62] Thus its "authority" is precisely reversed; if, in a context whose significant parts are obviously derived from Eudemus, the passage dealing specifically with Pythagoras has been supplemented with material from Iamblichus, this is an indication that there had been a gap to fill, and that Eudemus did not give enough information about Pythagoras or even none at all. As far as concerns the specific discoveries attributed to Pythagoras, the "cosmic bodies" are regarded by Proclus as the apex, or the quintessence, of all geometry;[63] and the discovery of irrationality had, long before Proclus, been interpreted in a Platonic sense and bound up with the thrilling story about mathematical secrecy and its betrayal and the ensuing divine punishment.[64] Proclus has merely attributed to

[59] Merlan, *PlNeopl* 141ff, referred *Comm. math. sc.* 23 to Aristotle's *Protrepticus*. This was accepted by Festugière, *Rev. philos.* 81 (1956) 117–127, but rejected by I. Düring, *Aristotle's Protrepticus* (Göteborg, 1961) 209.

[60] Deubner showed that the division into chapters and the prefixed summaries were the work of Iamblichus himself (*SBBln* 1935, 689f). Merlan (*PlNeopl* 127) is inclined to attribute the mention of Pythagoras to Aristotle, though he admits (126) the likelihood of some reworking by Iamblichus, precisely at the beginning and end; Festugière (126; see preceding note) also expresses doubts. In the parallel passage to which Merlan refers, however (Arist. *Protr.* fr. 11 W. = Iam. *Protr.* p. 51.7ff), the mention of Pythagoras is another addition by Iamblichus (cf. Burkert, *Hermes* 1960, 166ff).

[61] *Comm. math. sc.* p. 73.17; the passage between (pp. 70.7–73.17) is mostly put in the first plural.

[62] Vogt, *Bibl. math.* 1908–1909, 31f; Sachs 30ff (1917). A refutation of this has never been attempted and could scarcely succeed; the general denial by Friedlaender (*Platon* I [Berlin, 1928] 108 n. 3) was omitted from the second edition (I² [1954] 331 n. 15; Eng. ed. 353 n. 15).

[63] Procl. *In Eucl.* 70.24: *περὶ τῶν κοσμικῶν σχημάτων ἐστὶν ὁ σύμπας τῷ γεωμέτρῃ λόγος . . . τελευτῶν . . . εἰς τὴν ποικιλίαν τῆς τούτων συστάσεως* (cf. 65.20: *τὴν τῶν κοσμικῶν σχημάτων σύστασιν*).—The terminology is therefore that of Proclus. Naturally the latter was affected in this judgment by the *Timaeus*, which for him shared with the Chaldean Oracles the honor of being the most important book in the world (Marinus *V. Procl.* 38).

[64] Cf. below, ch. VI 3. There is no occasion to reject the reading *ἀλόγων* in favor of the weakly attested *ἀναλόγων*, as von Fritz does (*MtAnn* 1945, 245 n. 13). (Friedlein refers not to any manuscript but to "alii," named by E. F. August in his edition of Euclid [Berlin, 1826].)

Pythagoras the two most famous and, from the Platonic point of view, most significant achievements of geometry, supplying an illustration to accompany the generally phrased sentence from Iamblichus. Nothing is left of the supposed testimony of Eudemus to the achievement of Pythagoras in the foundation of mathematics.

Another testimony, supposedly coming from Aristotle himself, is cited, scarcely less often, as evidence for Pythagoras as a mathematician. It is given as follows in the collections of Aristotle's fragments: "After these men, chronologically, Pythagoras, the son of Mnesarchus, first worked at mathematics and numbers, but at some later period he also indulged in miracle-mongering like that of Pherecydes."[65] This comes from the *Historiae mirabiles* of Apollonius, who has just finished dealing, successively, with the miracles performed by Epimenides, Aristeas, Hermotimus, Abaris, and Pherecydes. For Pherecydes, Theopompus is his source.[66] The legendary material which follows, about Pythagoras, is certainly derived from Aristotle; but naturally the transitional sentence between Pherecydes and Pythagoras, which also separates the two sources Theopompus and Aristotle, is supplied by the compiler, either Apollonius or his source Bolus. It corresponds to the Hellenistic conception of Pythagoras as *βεβαιωτὴς τῶν μαθημάτων*, but does not give us any information about Aristotle's view of the matter; in his extant works he never connects Pythagoras with *μαθήματα*. It would be superfluous to set this out in such detail, if it were not that the supposed fragment of Aristotle has been cited, even in very recent studies, as a basis for the assessment of Pythagoras' contribution.[67]

The decisive passage is the introduction to Aristotle's chapter on the Pythagoreans in the *Metaphysics*: "Contemporaneously with these philosophers (Leucippus and Democritus) and before them, the so-called Pythagoreans were the first to take up mathematics; they advanced this study, and having been brought up in it they thought its principles were the principles of all things."[68] Taken in isolation, this passage

[65] Arist. fr. 191 = Ap. *H.m.* 6 = DK 14.7.

[66] *FGrHist* 115F70; cf. ch. II 3.

[67] Heath, *Math.* I 66; Burnet, *EGP* 97; Ciaceri II 94 (who ingeniously combines this with Heraclitus fr. 129; first Ionian *ἱστορίη*, then Italian *κακοτεχνίη*); Sarton, *Hist.* 203f; Cameron 25; Raven, *PyEl* 2; Mondolfo in ZM 354; van der Waerden, *Astr.* 7; Morrison, *CQ* 1956, 138; De Vogel, *GP* I 10, no. 22; Stapleton, *Osiris* 1958, 44. The correct interpretation: Heidel, *AJP* 1940, 8; Philip 23.

[68] Arist. *Met.* 985b23ff = DK 58B4. The reading *πρῶτον*, which Christ adopted from *recentiores*, has against it the authority of the tradition (*πρῶτοι* is also found in the second version of Alexander of Aphrodisias, on p. 37.12 Hayduck and Ascl. *Met.* 35.30; also Bekker, Diels, Ross, Jaeger).—*πρῶτοι* goes with *ἁψάμενοι*; cf. *ἐξ ἀρχῆς ἁψάμενοι*, 984a28; *Gen. corr.* 320b34.—The sentence is cited, for example, by Zeller I 405 n. 2; Heath, *Math.* I 66; van der Waerden, *Hermes* 1943, 163, and often. We may add a passage from a lost writing of Aristotle (Iam. *Comm. math. sc.* p. 78.8ff; cf. above, ch. I 2, n. 112, and on the text, below, ch. VI 3, n. 1): *οἱ δὲ Πυθαγόρειοι διατρίψαντες ἐν τοῖς μαθήμασι καὶ τό τε ἀκριβὲς τῶν λόγων ἀγαπήσαντες, ὅτι μόνα εἶχεν ἀποδείξεις ὧν μετεχειρίζοντο ἄνθρωποι, καὶ ὁμολογούμενα ὁρῶντες τὰ περὶ τὴν ἁρμονίαν ὅτι δι' ἀριθμῶν, ὅλως αἴτια τῶν ὄντων ταῦτα ᾠήθησαν εἶναι καὶ τὰς τούτων ἀρχάς . . .*

[69] Above, ch. I 2, nn. 99–100.

[70] Below, n. 83.

[71] Cf. the constantly repeated *πρῶτος* and similar expressions, *Met.* 983b6, b20, b29, 984a27, b18ff, b23, 985a8, a29f, 986b21.

[72] Cf. *Met.* 986b24, on Xenophanes, and especially 987a29ff, on Plato (with the same ingressive aorist *ᾠήθη* 987b19, as 985b26). On *ἐντραφέντες*, cf. Pl. *Tim.* 19d. The balance of the sentence, projected in the use of *τε-καί* is disturbed by the intrusion of the psychological point of view and the interpolation of *ἐντραφέντες* . . . instead of, for example, something like *ταῦτά τε προήγαγον καὶ τὰς τούτων ἀρχὰς τῶν ὄντων ἀρχὰς ἔθεσαν.*

could mean that the foundations of mathematics were laid by the Pythagoreans, if they were the first to "take up and advance" it. Still there is a chronological problem: What about Thales? Aristotle is speaking of "Pythagoreans," and he dates them in relation to the atomists.[69] Thales, however, who is, in Aristotle's view, the founder of philosophy (*Met.* 983b20), was regarded as the patron saint of mathematics even in the fifth century,[70] and Eudemus ascribes to him some special discoveries.

In order to understand why Aristotle passed over Thales in the passage cited, one must consider the function of the passage in its context. The objective of the first book of the *Metaphysics* is to demonstrate that there are four first principles. The discoveries of earlier thinkers, in spite of their many imperfections in detail, are all seen as moving, in the developing history of thought, in the direction of this conclusion. Whoever was "first" to bring forth a new view is carefully recorded.[71] The "first" of the Pythagoreans is to be seen from this angle: they "first took up" mathematics; this means that they were first to see the relevance of the "principles of mathematics" to the general question of "principles." The question is not who invented mathematics, but who connected mathematics with philosophy. From this point of view, there was no occasion to name Thales; his mathematics had no connection with his doctrine about water.

Inextricably entwined with this, in the passage of Aristotle, is the psychological question of how the Pythagoreans came to their system, which seemed as odd to him as it did to others.[72] His answer is that they devoted themselves so intensively to mathematics and became so closely identified with it that they saw nothing in the whole

world but numbers.[73] Thus for Aristotle the Pythagoreans' number theory is a by-product of their mathematical studies. This view has found adherents,[74] but is by no means self-evident. There is plenty of number speculation which does not presuppose mathematics; and we know from Aristotle's own words that nonmathematical associations were influential in forming the Pythagorean numerical cosmology.[75] That it was mathematics of the deductive type which preceded and produced the number philosophy is a psychological conjecture of Aristotle, which the historian is not obliged to accept. We may credit Aristotle's statement that there were certain Pythagoreans who achieved success in mathematics[76] and that there did exist a Pythagorean philosophy of number. But as far as our information from Aristotle goes, it remains an open question when the mathematical achievements of these Pythagoreans occurred and how they are to be placed in the general development of Greek mathematics. His concern was the history, not of mathematics, but of philosophy.

A statement by Aristotle's pupil Aristoxenus is obviously influenced by his master: "Pythagoras seems to have honored, most of all, the study of numbers, and to have advanced it in withdrawing it from the use of merchants and tradesmen, likening all things to numbers . . ."[77] The "advancement" of mathematics, and number philosophy, is here moved back from the Pythagoreans to Pythagoras, and also seen in the light of the Platonic demand that λογιστική should not be carried on

[73] The same basic idea is expressed in a somewhat more friendly spirit in the parallel passage cited above, n. 68: the Pythagoreans like the exactness of mathematics, see its application in music, and by this route arrive at their number theory.

[74] Zeller, *Vortr. u. Abh.* 39: "From these mathematical and scientific studies they proceeded to develop a set of beliefs about the nature of things in general"; Sarton, *Hist.* 204: "In order to develop a mystical theory of numbers it was necessary first of all to obtain a sufficient knowledge of them"—as though elementary calculation did not provide plenty of material for numerical speculation, especially considering the existence of Babylonian techniques of computation.

[75] *Met.* 985b29: ὅτι τὸ μὲν τοιονδὶ τῶν ἀριθμῶν πάθος δικαιοσύνη, τὸ δὲ τοιονδὶ ψυχὴ καὶ νοῦς, ἕτερον δὲ καιρός . . . See below, ch. VI 4.

[76] This is confirmed by the details Eudemus gives; and Hippasus may also be recalled in this connection (below, ch. VI 3).

[77] Aristox. fr. 23 = DK 58B2, according to Stob. 1 prooem. 6 ἐκ τῶν Ἀριστοξένου περὶ ἀριθμητικῆς. Wehrli (54) doubts the title—Stobaeus' whole introduction is περὶ ἀριθμοῦ—and opines that "in the existing formulation, of an elementary introduction to numerical concepts, fr. 23 is not from Aristoxenus." One has the impression of an abbreviated excerpt; but the introductory sentence, with its meticulous formulation, looks like an exact quotation. The name of Pythagoras is cited with an air of introducing a new element. Wehrli's assignment of the fragment to the book Περὶ Πυθαγόρου καὶ τῶν γνωρίμων αὐτοῦ is conjectural. Both Frank (260 n. 1) and Wehrli (54) point out the relation to Aristotle, as well as the connection between Aristoxenus' further statement, οἱ δὲ ἐκ τῶν θείων περιφορῶν ἐπινοηθῆναι (sc. φασὶν τὸν ἀριθμόν) with *Epin.* 978c.

ἰδιωτικῶς . . . οὐκ ὠνῆς οὐδὲ πράσεως χάριν ὡς ἐμπόρους ἢ καπήλους μελετῶντας.[78] It is worth noting that the subject is "numbers," that is, arithmetic, and not geometry. The fact that Aristoxenus uses the word "honor" (τιμᾶν) shows that he was familiar with the nonmathematical aspects of the Pythagoreans' use of numbers. We see the elements from which the picture of the Great Master is taking shape. The way in which the name of Pythagoras could acquire a truly mythical force in such a context is illustrated by another report of Aristoxenus: "He was also the first to introduce weights and measures among the Greeks."[79] In an egregious anachronism—as if the Homeric heroes were still unable to measure or weigh anything!—Pythagoras is classed with Prometheus and Palamedes.[80]

The apparently ancient reports of the importance of Pythagoras and his pupils in laying the foundations of mathematics crumble on touch, and what we can get hold of is not authentic testimony by the efforts of latecomers to paper over a crack, which they obviously found surprising, by the use of various kinds of reconstruction and reinterpretation. On the other hand, there are ancient and unassailable indications of a Greek mathematics antedating Pythagoras and quite outside his sphere of influence.

To be sure, the tradition about Thales has been the subject of vigorous controversy;[81] but in any case it was already firmly established in the fifth century B.C.[82] Before the entrance of the "Thinking-shop" in Aristophanes' *Clouds* stand statues of Astronomy and Geometry. The mention of geometry leads to that of the world map (γῆς περίοδος), which points unmistakably to the tradition of Ionic ἱστορία going back at least to Anaximander (*Nub.* 201–216). Socrates is described by his pupil as scattering ashes over a table and bending a cooking spit into

[78] Pl. *Rep.* 525c.

[79] Fr. 24 = D.L. 8.14.

[80] Palamedes: Aesch. fr. 303 M., Soph. fr. 432 P., Gorg. *Pal.* 30; Pheidon of Argos: *RE* XIX 1943; Prometheus: Aesch. *PV* 459, and also Pl. *Phlb.* 16c, where the inventor of the philosophy of number (above, ch. I 4) is called a "Prometheus."

[81] See Tannery, *HScH* 54ff, *Géom.* 89ff; Heath, *Math.* I 128ff; van der Waerden, *SA* 85ff; esp. von Fritz, *ABG* 1955, 77ff, and Becker, *MD* 37ff. Guthrie (I 52ff, 217ff), von Fritz, Becker, and Wehrli (*Eudemos* 115) have returned to a more positive evaluation of the tradition about Thales, while the extreme skepticism of Burnet, *EGP* 45f, and Schuhl (*Essai* 175ff) has been taken up again by D. R. Dicks, "Thales," *CQ* 53 (1959) 294–309. Dicks, however, ignores the arguments of von Fritz and Becker, as well as the passages in Aristophanes. On Thales' alleged prediction of an eclipse of the sun, see Neugebauer *ExSc* 142.

[82] See B. Snell, "Die Nachrichten über die Lehren des Thales und die Anfänge der griechischen Philosophie- und Literaturgeschichte," *Philologus* 96 (1944) 170–182; he determined that the intermediary source was Hippias (D.L. 1.24 = DK 86B7).

shape to serve as a pair of compasses (διαβήτης), meaning of course that he was going to make a geometrical diagram—and under cover of this activity he steals a coat. "Why do we go on admiring old Thales?" cries Strepsiades (τί δῆτ' ἐκεῖνον τὸν Θαλῆ θαυμάζομεν;). This shows that for the Athenian public Thales and geometry belong together; and this is also evident from another passage in Aristophanes. Meton arrives in Cloudcuckooland; "I want to survey the air for you—γεωμετρῆσαι βούλομαι τὸν ἀέρα ὑμῖν." He will apply his ruler, "so that the circle may become a square," which will look like a star . . . All Pisthetaerus can say is "The man's a Thales" (ἄνθρωπος Θαλῆς).[83] Eudemus gives detailed reports about mathematical propositions, proofs, and constructions that Thales was supposed to have discovered,[84] he distinguishes between tradition and deductions of his own,[85] even recording an archaic locution used by Thales.[86] This implies that there was a book, available to him or his authority, ascribed to Thales. The book in question must be that *On the Solstice and the Equinox*.[87] Whatever the situation may be with regard to authenticity, there obviously existed, in the sixth century, Ionic technical writings on problems of astronomy and the calendar;[88] already with them, geometrical concepts—circles and angles—seem to take the place of

[83] Ar. *Av.* 995ff, 1009. These references to Thales are not included in DK. Cf. B. Gladigow, *Hermes* 96 (1968) 264–275; he connects the compasses with Thales' measurements of distance (A20).

[84] DK 11A20; Eudemus fr. 134 = Procl. *In Eucl.* 352.14 (the congruence of triangles which have one side and two corresponding angles equal); fr. 135 = Procl. *In Eucl.* 299.1 (equality of the vertically opposite angle); probably from the same source, Procl. *In Eucl.* 157.10 (bisection of the circle by the diameter) and 250.20 (the equality of the base angles in an isosceles triangle).—Hieronymus of Rhodes fr. 40 W. = D. L. 1.27 (with Plin. *HN* 36.82, Plut. *Conv. sept. sap.* 147a, DK 11A21): measurement of the height of a pyramid.—The attribution to Thales of the theorem that the angle in a semicircle is a right angle is problematic (Pamphila ap. D.L. 1.24f; Heath, *Math.* I 134ff). It is closely connected with the theorem on the sum of the angles of a triangle, for which, according to Eudemus fr. 136, the Pythagoreans were the first to find the proof.

[85] Fr. 134: τὴν γὰρ τῶν ἐν θαλάττῃ πλοίων ἀπόστασιν δι' οὗ τρόπου φασὶν αὐτὸν δεικνύναι, τούτῳ (τῷ θεωρήματι) προσχρῆσθαι . . . ἀναγκαῖον. From this it appears that what he found in the tradition was a specific method for determining distances (φασίν), and Eudemus himself decides what are the underlying mathematical propositions.

[86] Procl. *In Eucl.* 251.1 (equality of the base angles): ἀρχαικώτερον δὲ τὰς ἴσας (γωνίας) ὁμοίας προσειρηκέναι, cf. Becker, *QSt* 4, 151f; DK I 486.36ff (*Nachträge*); von Fritz, *ABG* 1959, 48f.

[87] Περὶ τροπῆς καὶ ἰσημερίας, D.L. 1.23, *Suda* s.v. Thales; taken by D.L., probably through a misunderstanding, as two works. Eudemus (fr. 145; cf. 144) did in fact ascribe the discovery of the inequality of the seasons to Thales. Gigon (*Ursprung* 55) thought of the *Ναυτικὴ ἀστρολογία*, but the word γωνία will not fit into a hexameter poem (DK 11B1).

[88] Thales' rivals are Phocus of Samos (D.L. 1.23) and Mandrolytus of Priene (Apul. *Flor.* 18 = DK 11A19); and in the same tradition are Matricetas of Methymna and Cleostratus of Tenedos (DK 6).

Babylonian calculation. The Babylonians do not use the concept of the angle.[89] Becker has shown that all of the propositions attributed to Thales can be derived simply from considerations of symmetry,[90] and von Fritz points out that the method assumed, that of superposition (ἐφαρμόζειν), was consciously avoided in later Greek geometry.[91] With Thales the point is still a graphic or perceptible "showing" (δεικνύναι).[92] But in this perspicuity itself there is a new element by contrast with the Babylonian "recipes." It is in the perceptible figure that mathematical propositions become clear in all their generality and necessity: Greek geometry begins to take form.

Thales' name was the only one to remain familiar;[93] aside from him we can discern the specifically Greek innovation in geometrical thought only through its effects. The world of Anaximander "is constructed on severely mathematical ratios";[94] it is "essentially geometrical."[95] Cosmology is subject to mathematical logic; the earth must hang free in the middle because it is equidistant, in every direction, from the "wheels" of the stars. His statements about the size and distance of the heavenly bodies, in which geometry extends its purview to the whole universe, presuppose at least some knowledge about geometrical proportion in the correlation of distance with true and apparent magnitudes.[96] Anaximander relies on the accuracy of geometry in matters beyond the range of any kind of verification—in its application

[89] Babylonian mathematics has a measure of inclination (*šaqal*, = Egyptian *śqt*, corresponding to the cotangent function) but not the concept of an angle (Becker, *Grdl.* 27, *MD* 10, 37).

[90] *MD* 37ff.

[91] *ABG* 1955, 76ff, *ABG* 1959 passim.

[92] See Á. Szabó, "*ΔΕΙΚΝΥΜΙ* als mathematischer Terminus für 'beweisen'," *Maia* 10 (1958) 106–131.

[93] It is hard to make out anything about the brother of Stesichorus mentioned as a geometer by Hippias (Procl. *In Eucl.* 65.12 = DK 86B12 = Eudemus fr. 133, the source of Hero *Deff.* 136.1 p. 108.12 Heiberg and *Suda* s.v. Stesichorus). The correct form of the name is probably *Μαμέρτιος* (as Hero; the *Suda* has *Μαμερτῖνος*, the MSS of Proclus *Αμερι'*, apogr. *'Αμέριστος* corr. *Μάμερκος*); *Μαμέρτιον* is the name of a city in Bruttium near Rhegium (Oldfather, *RE* XIV 952). Thus combinations with *Marmakos* the alleged father (D.L. 8.1) or *Mamerkos* the son of Pythagoras (Plut. *Aem. Paul.* 1, *Numa* 8; Festus p. 22 L.; M. Detienne *RHR* 152 [1957] 142) are built on sand, especially since Stesichorus is generally agreed to have lived in the first half of the 6th century.

[94] Jaeger *Paideia* I 157 (Eng. tr.).

[95] Heidel *AJP* 1940, 30; Kahn 81: "Anaximander is ancestor . . . also to the geometric philosophy usually associated with the name of Pythagoras"; cf. 77ff, 92ff, The assertion in the *Suda* that "in general, (Anaximander) drew up the blueprint for geometry" (s.v. Anaximandros = DK 12A2: . . . ἰσημερίαν εὗρε καὶ τροπὰς καὶ ὡρολογεῖα . . . γνώμονά τε εἰσήγαγε καὶ ὅλως γεωμετρίας ὑποτύπωσιν ἔδειξεν) is a cruder version of Favorinus ap. D. L. 2.1f: εὑρεν δὲ καὶ γνώμονα . . . τροπάς τε καὶ ἰσημερίας σημαίνοντα καὶ ὡροσκοπεῖα κατεσκεύασε καὶ γῆς καὶ θαλάσσης περίμετρον πρῶτος ἔγραψεν.

[96] Above ch. IV 1.

to cosmic proportions and also in contradiction to appearance, which suggests that the sun is about as large in diameter as the width of a human foot.

The concept of geometrical similarity is also the precondition for Anaximander's attempt to construct a map of the world. And geometrical thought is still in the saddle in the work of Hecataeus, mocked by Herodotus for representing the earth as circular (quartered by the Nile and the Danube) and depicting less well known countries like Libya as built of rectangles and squares.[97]

In the time of Polycrates, the architect Eupalinus of Megara showed the same kind of bold and confident geometrical thinking. The water supply for the city of Samos was routed through a mountain, in a conduit about a kilometer long, a technological accomplishment attested by Herodotus (3.60). The historian does not mention, however, the fact which appeared when the tunnel was rediscovered in 1882, that the tunnel had been dug from the two ends simultaneously,[98] the two galleries meeting, with slight inaccuracy, in the middle. Even though this undertaking did not require complicated geometry, nothing more in fact than relatively simple measurements, the daring character of the enterprise remains. The geometer relied on his στοιχεῖα even where the facts could not be directly checked, and the accuracy of his plan could only be tested after years of work. When Hezekiah had a similar conduit constructed for Jerusalem, about 700 B.C., the direction was checked and corrected by means of a series of vertical test shafts, so that the channel when completed ran a zigzag course.[99]

We know the names and accomplishments of a whole series of contemporary technologists.[100] They point to a phenomenon only scantily attested in the literary tradition, which contributed to the development of mathematics, namely the striking advance in technology, to be seen above all in temple architecture from the end of the

[97] See Jacoby's note on *FGrHist* 1F36 (northern Libya, Hdt. 4.168ff; Thrace frr. 146–183; Scythia frr. 184–190) Hdt. 4.36, 2.16, 33f. On his dependence on Anaximander, Kahn 82ff.

[98] E. Fabricius, *MDAI* 9 (1884) 165–192; Bürchner, *RE* I A 2189 s.v. Samos; W. Kastenbein, *AA* 1960, 178–198; J. Goodfield and S. Toulmin, *Isis* 56 (1965) 46–55. Goodfield and Toulmin argue persuasively that the direction of the tunnel was regulated by means of a row of stakes running across the mountain, against van der Waerden, who cited Hero's *Dioptra* 15 (*SA* 102–105). The poles set in line would probably have been called στοιχεῖα (cf. στοιχίζω, περιστοιχίζω, διαστοιχίζομαι).

[99] Van der Waerden, *SA* 103.

[100] See Sarton, *Hist.* 188ff: Theodorus and Rhoecus of Samos (inventors of the hollow casting of bronze), Chersiphron of Cnossus and his son Metagenes (movement of columns; see Vitr. *De arch.* 10.2.11f), Mandrocles of Samos (first bridge of ships, 514 B.C.).

seventh century. Certain technical terms of geometry clearly are borrowed from architecture, like γωνία,[101] ὀρθὴ γωνία, and τετράγωνον,[102] as well as the geometrical tools κανών, γνώμων, and διαβήτης.[103] As early as Theognis and Simonides we find these concepts or words used as symbolic of impeccable truthfulness and accuracy.[104] Is this elevation of ideas, from the realm of craftsmanship to the plane of the symbolic, Pythagorean? Pythagoras of Samos came from a milieu in which technological and geometrical thinking was in its heyday. It is unthinkable that he took no notice of it; but there is no way of knowing whether he made any contribution to it himself. The career of Greek geometry began before Pythagoras' time, and there is no warrant for supposing that every trace of mathematical and geometrical thought in early Greece is *eo ipso* Pythagorean.[105]

[101] γωνία used of buildings, Hdt. 1.51 et saep.; *IG* XII 7.1; γωνιαῖοι λίθοι and the like in building accounts, *IG* I² 372.19, 161, 313.86, 373.80. See A. Debrunner, *IF* 60 (1949) 38–46. It is not entirely clear whether the vowel in γωνία shows compensatory lengthening or the lengthened grade (as Schwyzer thinks, I 358). If it is compensatory lengthening, Debrunner would like to refer to the Pythagoreans of southern Italy, though he must admit that for this area the compensatory lengthening to ω is not firmly attested. We can regard it as certain that architecture before Pythagoras did not get along without the term τετράγωνος (πλαίσιον is only the brick mold); and perhaps one ought not to dismiss as readily as Debrunner Hesychius' gloss γῶνορ, γωνία. Λάκωνες.—According to Proclus *In Remp.* II 26.18 and Hero *Deff.* 15, the Pythagoreans called the angle not γωνία but γλωχίς (elsewhere used of the feathering of an arrow: τρίγλωχιν ὀιστόν, *Il.* 5.393, 11.507, Soph. *Trach.* 681). There are examples of τρίγλωχις, "3-cornered," Pi. fr. 322, Callim. fr. 1.36; of τετράγλωχις, "4-cornered," Leonidas *A. P.* 6.334.3; and an expression ἰσογλώχινι τριγώνῳ, Nonnus *Dion.* 6.23. This evidence would seem to show that γλωχίς is a poetical term, used as a substitute for the unmetrical γωνία. If ancient Pythagoreans had in fact used it, the whole development of Greek geometry was unaffected; Procl. *In Eucl.* 130.8 etc. = Philolaus A14 uses γωνία as casually as Eudemus fr. 141 = Archytas A14.

[102] The restriction of this word's sense—from "quadrangle" to "rectangle," and usually "square"—is a development resulting from the usage of masons. For τετράγωνον in the context of architecture, see Hdt. 1.178, 181, 2.124; *IG* I² 313.101.

[103] According to Plin. *HN* 7.198, the inventions of Theodorus of Samos included "norma, libella, tornus, clavis." Technologists in particular had connections in the Orient. Not only did they build bridges of ships for Darius and Xerxes, but there were Greek masons at the court of Cyrus I (G. M. A. Richter, *AJA* 50 [1946] 15–30).

[104] Theog. 805f: τόρνου καὶ στάθμης καὶ γνώμονος ἄνδρα θεωρὸν εὐθύτερον χρὴ (ἔ)μεν, Κύρνε, φυλασσόμενον (cf. 543); Simonides 542.3 Page: ἄνδρ' ἀγαθὸν μὲν ἀλαθέως γενέσθαι χαλεπόν, χερσίν τε καὶ ποσὶν καὶ νόῳ τετράγωνον, ἄνευ ψόγου τετυγμένον (like a building stone with which the supervisor or foreman can find no fault. Cf. the "cornerstone" of Pss. 117.22, Isa. 28.16, Matt. 21.42, Acts 4.11). A dedicatory inscription from the neighborhood of Sybaris (*Bull. Epigr.* 1967, no. 697, 6th century B.C.) has a noteworthy expression about a votive offering Fίσο(μ) μᾶκός τε πάχος τε (a cube?). A pot by the Cage Painter (Louvre G 318; E. Pottier, *Vases antiques du Louvre* III [1922] pl. 135; *ARV*² 348.3; dated 480/470 B.C.) has a scene representing instruction in geometry: a boy with a pair of compasses, and a γνώμων hanging on the wall.

[105] Fränkel drew attention to the thought pattern of the "geometric mean" in Heraclitus—man is to god as ape is to man, etc.—and thinks there may be Pythagorean influence here (*AJP* 59 [1938] 309–337, in German *WF* 253–283, esp. p. 265 n. 5; similarly Minar, *CP* 34 [1939] 337–340). The idea of proportion, however, antedates Pythagoras.

Callimachus makes play with the crossing of the traditions about Thales and Pythagoras in the well-known passage from the *Iambi*, in which an Arcadian is to give a bowl "to the wisest." First he comes to Thales, who is "scraping the earth and drawing the figure which the Phrygian Euphorbus discovered, the first of men to draw triangles and oblique figures and the curved spiral, who taught us to abstain from animal food . . ."[106] Thales is concerned with geometrical propositions discovered by "Phrygian Euphorbus," which naturally means Pythagoras.[107] This has been misunderstood to mean that some kind of tradition actually ascribed such discoveries to Eurphorbus,[108] or that Pythagoras, as a youthful prodigy, founded Greek mathematics during Thales' lifetime.[109] This makes the chronological inconsistency, to which Callimachus so wittily alludes, disappear. He consciously ascribes to Thales a knowledge that he could not have had, chronologically, but then, with the help of the doctrine of metempsychosis, makes the impossible possible. Pythagoras introduced geometry from Egypt to Greece and brought it to perfection; yet before Pythagoras' time Thales was already famous as a geometer. Therefore Pythagoras must have made his discoveries in an earlier incarnation.

From about the middle of the fifth century, it is clear that mathematics is a center of intellectual interest. Almost all the important thinkers are concerned with mathematical questions. Anaxagoras, Hippias, Antiphon, and Hippocrates of Chios worked on the squaring of the circle; this was a problem so widely known that it could be

[106] Callim. fr. 191.58–62 Pfeiffer. The words *τὸ σχῆμα* ("the figure") may hold an allusion to the Pythagorean theorem (Pfeiffer and Howald in the Artemis edition, 1955, p. 320f). *τρίγωνα καὶ σκαληνά* ("triangles and oblique figures") reminds of Eudemus frr. 136–137 (below, ch. VI 3). In the first part of verse 61 (*καὶ κύκλον ἕλ[ικα]*) Diodorus (10.6.4) has the unmetrical *κύκλον ἑπταμήκη*, *POxy.* VII 1011 (p. 32; cf. p. 71), Hunt reads *ἐπ* with *λ* written above *π*; according to Pfeiffer *ἐπ* is "satis certa." Pfeiffer considers *ἔπαγε* or *ἔταμε*, Maas suggests *ἔπλασε*. Diels's conjecture *ἕλικα* (DK) has an important point in its favor: *ἕλιξ* is a technical term for the apparently "spiral" movement of the planets which results from the overlapping of the revolution of the earth and the planets' own movement (Pl. *Tim.* 39a, Hermesianax fr. 2.86f Diehl, Ptol. *A. P.* 9.577); and the description of the zodiac was ascribed to Pythagoras, in rivalry with Oenopides. As the word *νησεύειν* shows, Callimachus is not speaking of mathematics alone, but is giving, in a concise form, a general characterization of Pythagoras' teaching—and therefore could certainly not leave out astronomy (cf. lines 54f, on the astronomy of Thales). On the text of line 62, see H. Lloyd-Jones, *CR* 17 (1967) 125–127.

[107] Above, ch. II 3.

[108] Delatte, *Vie* 157f, makes Euphorbus, "Good-Shepherd," a Phrygian culture hero; Rostagni agrees, *Verbo* 120 n. 1, 240 n. 1.

[109] Lévy, *Sources* 41f.

used in comedy.[110] Hippocrates of Chios was pioneer on the line of thought which Archytas followed up in solving the problem of doubling the cube.[111] Protagoras tried to refute the geometers on principle by maintaining that their postulates never fit reality: there is no visible line touching a circle at only one point.[112] His disciple Theodorus dealt with irrationality.[113] Democritus did a good deal of work on mathematical problems.[114] The statues of Geometry and Astronomy at the entrance of Socrates' "Thinking-shop" are a clear indication that before the end of the fifth century the branches of mathematics, as such, had a firm place in the curriculum of the Sophists' program of higher education.[115] When Xenophon represents Socrates as criticizing excessive enthusiasm about geometry, astronomy, *λογισμοί*, and medicine, when they are carried beyond the needs of practicality, this is not simply an anachronistic criticism of Plato.[116] Hippias, as Plato tells us more than once, taught *λογισμοὺς καὶ ἀστρονομίαν καὶ γεωμετρίαν καὶ μουσικήν*.[117] And all these special fields are also attested for Democritus.[118] It follows that the quadrivium is not merely a Pythagorean import brought by Plato from Italy. If, as in the later tradition, it is to be regarded as Pythagorean,[119] its influence must have

[110] See Heath I 233ff; Becker, *MD* 93ff; Anaxagoras A38; Hippias fr. 21 (on which, Becker, *MD* 95ff, van der Waerden, *SA* 146, 191; Heath, *Math.* I 182, 226ff); Antiphon fr. 13; Hippocrates (above, n. 10); Ar. *Av.* 1005.

[111] DK 42.4; cf. 47A14.

[112] DK 80B7 = Arist. *Met.* 997b32. Probably Democritus treated the same problem in his book (B11l) *Περὶ διαφορῆς γνώμης ἢ περὶ ψαύσιος κύκλου καὶ σφαίρης*.

[113] DK 43.4 = Pl. *Tht.* 147d; cf. above, n. 12; below, ch. VI 3.

[114] Democritus' importance in the history of mathematics was expounded by Frank (esp. 82), not without a certain amount of exaggeration. Archimedes gives him credit for discovering how to determine the volume of a cone (DK II 174 n.). The fact that he is not included in Proclus' catalogue of geometers may be due to the prejudice of the Platonists (as van der Waerden thinks, *SA* 91). Cf. the preceding note, and on the quadrivium, below, n. 118; on irrationality, below ch. VI 3.

[115] Ar. *Nub.* 200ff. Strepsiades must also learn *μουσική*; he can already calculate interest.

[116] Xen. *Mem.* 4.7.2ff.

[117] Pl. *Prot.* 318d–e, *Hp. ma.* 285b, *Hp. mi.* 366c, 368e.

[118] Geometry: B11l, m, n, p; arithmetic: the title *ἀριθμοί*, B110; astronomy: B11q–B14; music: B15c–B26a. Frank (10 and n. 23) sees in Democritus the founder of the quadrivium; but Hippias was about the same age, and the systematic association of the four subjects cannot be shown to be present in Democritus.

[119] Nicom. *Ar.* 1.3, with citations of "Androcydes" and Archytas (fr. 1); a citation of Pythagoras *Th. ar.* 21.7 (from the book *περὶ θεῶν*); "Cleinias," ibid.; ps.-Archytas p. 6.11 Thesleff. The quadrivium is also Pythagorean for Proclus, *In Eucl.* 35.21ff; but Theo (16ff) speaks only of Plato. On Theodorus, above, n. 12. Morrison, *CQ* 1958, 203ff, recognizes that "higher mathematics, astronomy, geometry, and the theory of numbers were all the subject of lectures at Athens at the close of the fifth century" (213), but assumes, for this reason, that Hippias and Theodorus "brought Pythagorean mathematics to Athens" (203).

extended to Hippias, Democritus, and the Aristophanic Socrates. It was the cultural influence of the Academy that brought the system of the "four fields" to their position of special prominence;[120] the only unequivocally Pythagorean element is the arithmetization of music theory and, to a degree, the elevation of number theory ("arithmetic") to an independent branch alongside geometry.[121] All the rest had been generally known and could assume their place in the scheme of higher education without the help of any esoteric or Pythagorean influence. This is the way Isocrates is looking at the matter when he distinguishes between the traditional pair, γραμματική and μουσική, and the subjects that had emerged in his own time, namely γεωμετρία, ἀστρολογία, and διάλογοι ἐριστικοί.[122] Here he is alluding to the educational system of the Academy.

Along with the appearance of the quadrivium comes the semantic development, in which the word μαθήματα, "fields of study," is reduced to the branches of "mathematical" study, while μαθηματική is restricted to mathematics proper (including astronomy). This usage is not consistently followed by Plato and Isocrates until their later works,[123] to be taken up then in the *Epinomis* and in the works of Aristotle. In general Plato still uses μάθημα in its original, broader sense. Thus it seems that the word "mathematics" became fixed only in the time of the Old Academy. The question of the extent to which Pythagoreans anticipated Plato, in the treatment of the four branches and the development of the concept of the μαθήματα, depends on the problem of the genuineness of the long fragment of Archytas.[124]

In any case Greek geometry assumed its final form in the context of the Old Academy. After Plato had placed an especially high valuation on mathematics, and had fixed its position as a discipline of pure thought (so that Protagoras' objections became irrelevant), there came

[120] Plato treats the four μαθήματα, already adding solid geometry, in *Rep.* 524dff; see also *Leg.* 747a, *Epin.* 990cff, 991e. On the development from Plato on, see Merlan, *PlNeopl* 78ff; he does not go into the pre-Platonic period.

[121] Below, ch. VI 2.

[122] Isoc. *Panath.* 26; cf. *Antid.* 261, 264f., 266f, *Bus.* 23; on the relationship to Plato, Jaeger, *Paideia* III 147 (217 Ger. ed.); K. Ries, *Isokrates und Platon im Ringen um die Philosophia* (Diss. Munich, 1959).

[123] Pl. *Leg.* 817e, Isoc. *Antid.* 261, 10, *Panath.* 27.

[124] Above, ch. V 1, n. 46; on the *mathematici*, above, ch. II 5. For a Pythagorean origin of the μαθήματα concept, B. Snell, *Die Ausdrücke für den Begriff des Wissens in der vorplatonischen Philosophie* (Berlin, 1924) 77ff; Heath, *Math.* I 11; von Fritz, *SBMü* 1960, 20f.

an unprecedented development of these studies.[125] Aristotle writes, "Those who concern themselves with geometry and calculation and the other sciences have from small beginnings made by now such progress in a very short time as no other field has made in any of the arts."[126] But even in the age of Anaxagoras, Protagoras, and Hippocrates of Chios, we notice that mathematicians are no longer concerned merely with individual problems, but are working on fundamental ones. The problems of squaring the circle and doubling the cube are not soluble by ordinary geometrical means, with the use of ruler and compass; and the fact of irrationality can only be deduced, or "shown" in a logical argument, never made immediately perceptible. Here geometry freed itself from its bondage to the needs of practicality. For the needs of everyday life, there were already plenty of approximative values to use in the calculations involving roots and the dimensions of the circle. In the geometry of the fifth century, however, it was obviously recognized that these were merely approximations, and that there is a basic difference between these and any exact solution resting on proof; it was understood that the task of expressing a magnitude like $\sqrt{2}$ in whole numbers, or expressing the length of the diagonal in terms of that of the sides of a square, was not only "not yet" solved, but in principle insoluble. Mathematical logic and deductive proof go beyond what is perceptible, and this is what carried Greek geometry far beyond its predecessors, no matter how suggestive, in the oriental cultures.

If a basic influence was exerted by Pythagoras or by Pythagoreans on Greek mathematics, this would have had to take place in the period between Anaximander, Cleostratus, and Hecataeus, on the one hand, and Anaxagoras, Oenopides, and Hippocrates of Chios on the other. What ensued was, in the first place, the development of detail and, in the second, a metaphysical and logical undergirding in which the

[125] *Acad. Ind. Herc.* p. 15ff Mekler (cf. below, ch. VI 3, n. 86); Procl. *In Eucl.* 66.8ff = Eudemus fr. 133, where no fewer than 11 mathematicians between Plato and Euclid are named—and we may add Polemarchus and Helicon of Cyzicus, Amphinomus, Bion of Abdera, Aristotherus, Autolycus, and a certain Aristaeus. Even if Plato was not a professional mathematician, his philosophy seems to have provided the decisive breakthrough in the establishment of mathematics, and at the same time to have directed the attention of philosophers, even more than had been the case, toward mathematics. See above n. 6. On the motto ἀγεωμέτρητος μηδεὶς εἰσίτω, see H. D. Saffrey, *REG* 81 (1968) 67–87.

[126] Fr. 53 = Iam. *Comm. math. sc.* p. 83.13ff (with Aristotelian origin guaranteed by Cic. *Tusc.* 3.69): τοσοῦτον δὲ νῦν προεληλύθασιν ἐκ μικρῶν ἀφορμῶν ἐν ἐλαχίστῳ χρόνῳ ζητοῦντες οἵ τε περὶ τὴν γεωμετρίαν καὶ τοὺς λόγους καὶ τὰς ἄλλας παιδείας, ὅσον οὐδὲν ἕτερον γένος ἐν οὐδεμιᾷ τῶν τεχνῶν.

Pythagoreans, according to the testimony of Aristotle, had no part: *τὰ γοῦν θεωρήματα προσέπτωκεν τοῖς σώμασιν ὡς ἐξ ἐκείνων ὄντων τῶν ἀριθμῶν.*[127] In that early period there does seem to be a certain gap, coinciding with the turbulent period of the Persian Wars. There were, nevertheless, direct connections between the older Ionic school and Anaxagoras, and, as Anaximander's pupil Cleostratus was an astronomer, so was Oenopides of Chios, the pupil of Anaxagoras and compatriot of Hippocrates. According to Eudemus, Oenopides worked out, for the first time, certain elementary geometrical constructions "since he believed that this would be useful for astronomy."[128] The significance of these apparently simple accomplishments seems to lie in the fact that in them geometry consciously restricts itself to the tools of compass and ruler.[129] Oenopides is still not doing mathematics for its own sake; but in the demand for precise construction, and the concept of the *πρόβλημα*,[130] he represents a definite forward step in the direction of "pure" theory. Hippocrates, too, was concerned with astronomy,[131] but he also wrote a book called *Στοιχεῖα*. If there were more reliable evidence about the discovery of pure theory by Pythagoras than the *philosophia* anecdote of Heraclides and the supposed Eudemus,[132] then one would have to recognize the true accomplishment of the Pythagorean school through its influence here in Oenopides and Hippocrates, and one might speak with perfect justification of the Pythagorean basis of Greek mathematics. But there is another solution.

The attempt at purely logical argumentation, a systematic progression from one thought to another, and the advancement of proofs and conclusions in conscious contradiction to the evidence of the senses make their first appearance in Parmenides. Kurt von Fritz has shown how *νοεῖν*, which previously meant an intuitive comprehension, first became logical "thinking" in Parmenides.[133] "They were led to

[127] Arist. Met. 1083b18.

[128] Oenopides, DK 41.13 = Procl. *In Eucl.* 283.4 (constructing a perpendicular to a straight line through a given point); DK 41.14 = Procl. *In Eucl.* 333.1 = Eudemus fr. 138 (to construct an angle equal to a given angle at a given point on a straight line); the first notice also obviously comes from Eudemus.—*CCAG* VIII 3, p. 95.12: *πρῶτος . . . τὰς ἀστρολογικὰς μεθόδους ἐξήνεγκεν εἰς γραφήν.*

[129] Von Fritz, *RE* XVII 2266. Every surveyor has a simple instrument for measuring right angles (basically the *γνώμων*). Oenopides in fact uses the expression *κατὰ γνώμονα* for "vertical" (DK 41.13), but he constructs the angle without mechanical help.

[130] DK 41.12.

[131] Cf. above, ch. IV 1, n. 77. On *στοιχεῖα*, Burkert, *Philologus* 1959.

[132] On Heraclides, Burkert, *Hermes* 1960, 159ff; on Eudemus fr. 133, above, n. 62.

[133] Von Fritz, *CP* 40 (1945) 223–242, 41 (1946) 12–34, esp. 1945, 241.

transcend sense perception and to disregard it on the ground that 'one ought to follow the argument'"—this is Aristotle's characterization of the Eleatics,[134] and *ἀκολουθεῖν τῷ λόγῳ* is the distinctive feature of Greek mathematics as early as Hippocrates. "La mathématique grecque restera plus ou moins éléatique jusqu'au bout" is Abel Rey's way of putting the matter (148), and more recently Árpád Szabó, in a number of publications, has shown the dependence of Greek mathematics on the thought of the Eleatics.[135] The connection of geometry, and especially that of Hippocrates of Chios, with the logic of the Eleatics is obvious. The Eleatic thought pattern of the differentiation of cases dominates Hippocrates' treatment of the problem of the quadrature of lunes; the exterior arc of the lune is either greater or less than a semicircle, or equal to it, so that all possible cases are exhausted.[136] Zeno's methods of proof, the reductio ad absurdum and the regressus in infinitum, are basic to all the proofs about irrationality.[137] In the logic of the Eleatics we find the factor that brought about the advancement of mathematics, in the hiatus between Anaximander and Anaxagoras; there is a similar development in natural philosophy between the early Ionians and the post-Parmenidean thinkers. Anaxagoras, Empedocles, and Leucippus, each in his own fashion, achieved an advance by the combination of Ionian beginnings with Eleatic ontology and patterns of thought. There followed then the playful intellectual diversions of the Sophists. That which had emerged from everyday activity and bold speculation in the sixth century, was discussed, sifted, and gradually brought into a logical system. In the process, natural philosophy drifted into a dilemma between eclectisism and skepticism, but geometry took on its typical Greek form.

A contrary interpretation would have Parmenides deriving his style of argumentation from Pythagorean mathematics: "The method of

[134] *Gen. corr.* 325a13; cf. *Cael.* 298b22: the Eleatics were the first to believe in a Being uncreated and immobile, though not (as in Plato) distinguished from the perceptible.

[135] *AA* 1955, 67ff; "Wie ist die Mathematik zu einer deduktiven Wissenschaft geworden?" *AA* 1956, 109–152: "*Δείκνυμι* als mathematischer Terminus für 'beweisen,'" *Maia* 10 (1958) 117ff; "Die Grundlagen in der frühgriechischen Mathematik," *StudIt* 30 (1958) 1: "the historically necessary prerequisite for the earliest Greek mathematical science" is "the Eleatic philosophy." All the same, Szabó does not free himself from the communis opinio on Pythagorean mathematics, but accepts Becker's "doctrine of even and odd," and places the Pythagoreans, as "the first representatives of deductive mathematics" (*Maia* 1958, 130), between Parmenides and Hippocrates. Were the Pythagoreans the only ones who could learn mathematical logic from Parmenides?

[136] Above, n. 10; cf. Reinhardt, *Parm.* 35ff, 64ff; above, ch. III 2, n. 104.

[137] Cf. below, ch. VI 2, n. 47; VI 3, n. 81.

reasoning he imported into philosophy is the method of geometry."[138] Certainly; but does Parmenides "import" someone else's results? The price at which the Pythagorean origin of mathematics is saved, in this interpretation, is tremendous. Are we to suppose that Parmenides merely applied a previously developed method to a new concept, τὸ ἐόν? Only if Parmenides himself gave some indication of the fact, or if other testimony gave a clear indication of the existence of Pythagorean mathematics before Parmenides, would this thesis be acceptable. It would greatly diminish the originality and the basic importance of Parmenides in the growth of Greek philosophy, which has been generally recognized since the work of Reinhardt. In Parmenides himself there is not a word that points toward the field of mathematics—what a contrast with Plato and Aristotle! Further, if it can be shown that Zeno, in his manner of argument, holds close to the ideas of his teacher,[139] and if Zeno also, in his manner of expression, shows no dependence on mathematics, then there is no occasion to interpolate a "Pythagorean mathematics" between the two.[140] In discussing Being, Parmenides discovered the independence of thought; and deductive mathematics as well as logic took rise from this beginning; from the point of view of the development of thought, ontology is prior to the formal schematism.

Greek mathematics did not emerge from the revelation of a Wise Man, and not in the secret precinct of a sect founded for the purpose, but in close connection with the development of the rational Greek view of the world. Taking its departure from "measurement," geometry becomes a component of natural philosophy, φυσιολογία. In one of these realms as well as the other, Thales and Anaximander, Parmenides and Zeno are the important names, and even Oenopides and Hippocrates are φυσικοί in their astronomy. Then, earlier than other fields, geometry and astronomy become the domain of specialists, because their increasing complexity demanded a specifically mathematical talent, which is not dependent on one's origin or one's membership in any kind of school. At the same time, the Sophists brought about a division, since the exactness of mathematical results was more and more obviously in contrast with the uncertainty of φυσιολογία. Thus in Plato's time mathematics was already the model science, which even skepticism had to take seriously. Individual Pythagoreans had some part in this development, but in its "essence" mathematics is not Pythagorean but Greek.

[138] Cornford, *PrSap* 117; cf. *PlParm* 29; Rey has the same judgment, 191; cf. Cherniss, *JHI* 1951, 336: "The example of mathematical proof he [Parmenides] learned from them [sc. the Pythagoreans] may have determined the rigorously deductive form of his argument and may have been the origin of his confidence in this method." See also T. B. L. Webster, *Greek Art and Literature* 700–530 B.C. (London, 1959) 91ff. But there is no proof given that deductive geometrical proof was characteristic of the Pythagoreans before Parmenides. See above, n. 92.

[139] Above, ch. III 3, n. 55.

[140] Though Szabó does so, once more, in *AA* 1955, 83ff. That Zeno speaks of adding, taking away, and προέχειν does not imply special mathematics; and the role of bisection is common enough. There is a problem of proportionality in the millet-seed argument (A29), but this of of dubious authenticity, and in any case is not from Zeno's book (cf. above, ch. III 3, n. 47).

2. PYTHAGOREAN ARITHMETIC

That Pythagoreans were much concerned with numbers is established fact, but it is a question, in what sense this activity can be called mathematics; for the paradigmatic form of Greek mathematics, as a deductive system based on axioms, is geometry. On the other hand, there is one remarkable type of arithmetic that appears exclusively in the Pythagorean tradition, in which numbers are represented by figures made with counters or pebbles, ψῆφοι.[1] Aristotle knows of "triangular numbers";[2] and the "perfect" number 10, in its deployment in the form of the "tetractys," was certainly presented as a triangular number long before Aristotle.[3] And what at first seems merely a game does lead to arithmetical combinations that are by no means trivial. For example, if the odd numbers, when added successively in a pebble figure, make a square each time, this means discovery of the rule for the series of square numbers; and, if in the ἑτερομήκεις ἀριθμοί, constructed in similar fashion from the even numbers, one recognizes the triangular numbers, doubled, then he has the formula for the sum of triangular numbers, a special case in the arithmetic series.[4]

[1] Above, ch. I 2, n. 27. Ψῆφοι provide the simplest tool for practical calculation, in the abacus (on which see M. Lang, *Hesperia* 26 [1957] 271–287). It may be mentioned also that numbers have been indicated on dice, from very early times, by point figures (see Daremberg–Saglio s.v. *tessera*). These figures, which make the difference between gain and loss, are naturally looked upon with a certain emotion and come to appear as independent beings. In the substantive ἡ οἴνη is preserved the Indo-European root for "one"; and the substantive ἡ μονάς was also used in dice games (Pollux 7.204).

[2] *Met.* 1092b11: οἱ τοὺς ἀριθμοὺς ἄγοντες εἰς τὰ σχήματα τρίγωνον καὶ τετράγωνον.

[3] Above, ch. I 3, n. 120.

[4] ἑτερομήκεις ἀριθμοί are numbers of the form $n \times (n+1)$ (see Theo Sm. 26.21ff, Nicom. *Ar.* 1.19.19, 2.17.1, Iam. *In Nic.* 74.19ff). From the figure it can be seen that $1+2+3+4 \ldots + n = \frac{1}{2}(n+1) \times n$. In itself the word ἑτερομήκης must mean "having unequal sides" in general (cf. ἑτεραλκής, ἑτεροκλινής, ἑτεροπαχής, ἑτεροπλατής,

From the ψῆφος diagram can also be derived immediately the formula "of Pythagoras," which can be traced back as far as Anatolius, for calculating the rational sides of a right triangle, beginning with an odd number;[5] and this very fact has been rightly regarded as showing that this is an element of ancient tradition. This of course presupposes the knowledge of the "Pythagorean theorem," that most famous of all features of Pythagorean mathematics, connected in the tradition with the proverbial sacrifice of an ox. There has been much controversy over this tradition. Proclus, in his curiously twisted sentence, does not in any case cite Eudemus.[6] The principal testimony consists of two verses, cited several times but not known in their original context, from a certain Apollodorus, known as ὁ λογιστικός or ὁ ἀριθμητικός, who may be the same as the Democritean from Cyzicus, and in that case would be datable to the fourth century B.C.[7] For him, already, the γράμμα of Pythagoras and his offering of cattle are "famous." The fact that the offering of cattle featured in the story is in flagrant contradiction with Pythagorean vegetarianism ought rather to be considered an indication of antiquity than the reverse.[8]

What Neugebauer first suggested as a possibility in 1928 has since then grown into a certainty—namely, that the "Pythagorean theorem" had been used routinely for centuries in Babylon, and was therefore obviously not a discovery of the Greeks.[9] It must have been introduced as a piece of Babylonian arithmetical technique. It is possible that Pythagoras was the intermediary; but, in view of the multiplicity of contacts between Greece and the Orient in the sixth century, the "fame" of Pythagoras can hardly be explained on this one ground. There is no testimony that he gave a strict proof of the theorem, and this cannot be made to seem probable.[10] The suspicion remains that the theorem had more than a mathematical significance in Pythagoras' school, and that the numbers involved seemed in a cryptic way meaningful. The formula of "Pythagoras" points in this direction, as it belongs to the context of the pebble figures, like the form of the tradition that only mentions the triangle with the sides 3, 4, and 5.[11] In fact, this fits especially well with the kind of number speculation we learn of from Aristotle, where 3 is male, 4 is female, and 5, which mysteriously unites them in the Pythagorean triangle, is "marriage."[12] Plato's "nuptial number" obviously presupposes this interpretation.[13] What we are considering here, then, is not Pythagorean geometry, but arithmetic, developed by speculative interpretation of Babylonian formulas.

ἑτερόπλευρος). The restriction of its sense (cf. Iam. and Nicom. in the passages cited) indicates, as is the case with τετράγωνον (above, ch. VI 1, n. 102), that the term is rather old (Tannery, *MSc* II 97). The general term for "rectangular" in later times was προμήκης, coined, according to D.L. 3.24, by Plato (at *Tht.* 147e it is used interchangeably with ἑτερομήκης). See further *Rep.* 546c, *Tim.* 54a; ἑτερομήκης in the general sense: Xen. *Eq.* 7.14, Arist. *De an.* 413a17, *Cat.* 11a10, Eucl. I, def. 22.

[5] Procl. *In Eucl.* 428.7ff, Hero *Geom.* 8 Heiberg IV p. 218; both are cited in the scholium to Euclid, p. 215.27; cf. 213.19ff. On the relation of the formula to the "gnomon" arrangement of pebbles, see Heath, *Math.* 81; Heath, *Eucl.* I 358ff; Becker, *MD* 53; von Fritz, *AnnMath* 1945, 252. The corresponding formula for even numbers, which is ascribed to Plato, is hard to derive from the pebble diagram.

[6] The fullest treatment of the problem is Heath, *Eucl.* I 350ff. Proclus' sentence (*In Eucl.* 426.6 = DK 58B19) runs: τῶν μὲν ἱστορεῖν τὰ ἀρχαῖα βουλομένων ἀκούοντας τὸ θεώρημα τοῦτο εἰς Πυθαγόραν ἀναπεμπόντων ἐστὶν εὑρεῖν καὶ βουθύτην λεγόντων αὐτὸν ἐπὶ τῇ εὑρέσει. The sentence is regarded as corrupt by DK and KR (no. 281); one expects an accusative object after ἔστιν εὑρεῖν. The scholium to Eucl., p. 213.1, simplifies Proclus: οἱ ἀρχαῖοι τὸ θεώρημα τοῦτο εἰς Πυθαγόραν ἀναπέμπουσιν. Vogt, in particular (*Bibl. Math.* 1908–1909, 16ff), has brought out emphatically that Proclus is not citing Eudemus.

[7] ἡνίκα Πυθαγόρης τὸ περικλεὲς εὕρετο γράμμα κεῖν' ἐφ' ὅτῳ κλεινὴν ἤγαγε βουθυσίην. D.L. 8.12, 1.25 (ὁ λογιστικός), Ath. 10.418f (ὁ ἀριθμητικός; in line 2, κλεινὸς ἐφ' ᾧ), Plut. *Non posse* 11.1094b (v.l. Ἀπολλόδοτος; line 2, κεῖνος ἐφ' ᾧ λαμπρὴν ἤγετο). Probably from D.L., *A.P.* 7.119; *RE* s.v. Apollodoros (68). On Apollodorus of Cyzicus, see above, ch. III 1, n. 51; DK 74. Eudoxus had founded his school in Cyzicus, and his pupils Helicon and Polemarchus came from there; but the Democritean Bion of Abdera (DK 77) was also a mathematical astronomer, so that the Democritean Apollodorus might well have been an ἀριθμητικός. The verses do not say which "famous" figure of Pythagoras is in question, but D.L. and Ath. call it "the Pythagorean theorem," and Plutarch presupposes the same interpretation in his varying version (on the application of planes, see the passage cited and *Quaest. conv.* 8.2.4).

[8] Above, ch. II 4, n. 110. Cic. *Nat. d.* 3.88 is dubious about the sacrifice of an ox; he passes over the geometrical problem with the neutral phrase "in geometria quiddam." — One cannot simply brand the tradition "impossible," as van der Waerden does (*SA* 100).—Pamphila, ap. D.L. 1.24, seems to be transferring the Pythagoras story to Thales.

[9] Neugebauer, *NGG*, math.-ph. Kl., 1928, 46–48; *QSt* 3 (1936) 257; *ExSc* 35ff; cf. Becker, *MD* 10, 55; van der Waerden, *SA* 76ff; Stapleton, *Osiris* 1958, 12ff. A table ("Plimpton 322") with "Pythagorean numbers" was published by O. Neugebauer and A. Sachs, *Mathematical Cuneiform Texts* (1945) 38–41. The "Pythagorean theorem" was also known in India (Apastamba-Sulva-Sutra; cf. Heath, *Eucl.* I 352ff, *Math.* I 145ff; Becker, *MD* 55ff)—though Greek influence is not impossible here—and in China (Becker, *MD* 56; cf. below ch. VI 4, n. 38).

[10] The classic "windmill proof" comes from Euclid, but more primitive proofs are possible; see Heath, *Eucl.* I 352ff, *Math.* I 147ff; Becker, *MD* 55ff. It is pure guesswork to suggest that Pythagoreans tried anything of the sort.

[11] Vitr. 9 praef. (mentioning the sacrifice of an ox), Nicom. ap. *Th. ar.* 50.21ff, Iam. *VP* 130f, cf. 179 (and Delatte, *Pol.* 59ff), Alex. *Met.* 75.27ff (from Aristotle?).

[12] Above, ch. I 2, nn. 26, 62; cf. also Plut. *De Is. et Os.* 56 (3 is Osiris, 4 is Isis, and 5 is Horus), Greg. Naz. *Ep.* 198, Philo *Vit. cont.* 65, Schol.D. *Il.* 19.119.

[13] ἐπίτριτος πυθμὴν πεμπάδι συζυγείς, *Rep.* 546c; brought into connection with the "Pythagorean triangle" in Iamblichus, Alexander (above, n. 11), Aristid. Quint. 3 p. 151f M. Cf. also below, ch. VI 4.

Based on the "Pythagorean theorem," there appear the Pythagoreans' "side numbers" and "diagonal numbers,"[14] a series that provides increasingly accurate approximations to the value of $\sqrt{2}$. Plato apparently knew about this calculation.[15] The manner of constructing the series can be perceived from a simple jigsaw-puzzle procedure;[16] and the theoretical basis for the procedure is given in the tradition in the form, "The unit, as the origin of all things, is both the side and the diagonal of the square."[17] This does not in any way presuppose a concept of irrationality.[18]

Further reconstruction of Pythagorean arithmetic depends on the question, to what extent the detailed expositions in Theo, Nicomachus, and Iamblichus may be regarded as evidence for early Pythagorean mathematics. They are usually treated, in modern accounts, as "Pythagorean arithmetic," before Hippocrates of Chios,[19] and no attempt is made to separate more ancient and more recent material. It is inadmissible, however, to avoid the question of the form of this "Pythagorean arithmetic," for this is the crucial question in a consideration of its scientific character. The compilations from later antiquity offer a goodly number of arithmetical facts, but without any attempt at general proof of any of them. The rule is illustrated by a few examples, and this passes as verification. Of course, such incomplete inductive procedure leads to errors.[20] The books of Euclid which are devoted to arithmetical matters are quite different; general and rigorous proof, *more geometrico*, is the rule, the numbers being represented by line segments. Is the inductive, proofless form ancient and primitive,[21]

[14] Theo Sm. 42.10ff, Procl. *In Remp.* II 24.16ff; cf. Heath, *Math.* I 91ff, *Eucl.* I 398ff; van der Waerden, *SA* 126f; Becker, *MD* 67f, 73f.

[15] The number 7 is *διάμετρος ῥητὸς πεμπάδος* (i.e. $\sqrt{50}$): *Rep.* 546c.

[16] Side plus diagonal gives a new side; 2 sides plus a diagonal give a new diagonal. Van der Waerden gives a geometrical proof (*SA* 127), but the adjoining figure is still more perspicuous: from 3 isosceles right triangles a new one is made.

[17] Theo Sm. 43.5ff.

[18] *διὸ καὶ οἱ Πυθαγόρειοι ἐθάρρησαν τῇ μεθόδῳ*, Proclus writes—(*In Remp.* II 25.9ff), because the inaccuracies cancel each other out—the approximations to $\sqrt{2}$ are alternately too great and too small. This does not sound as though the impossibility, in principle, of attaining an exact result, was seen as important. In any case, this method can be discovered by induction, without depending on Euclid 2.10, which is given in the late sources.

[19] Heath, *Math.* 65–117; van der Waerden, *SA* 92–127; Becker, *MD* 40ff; cf. von Fritz, *Gnomon* 30 (1958) 82ff; van der Waerden, *RE* XXIV 280–285.

[20] For example, the conjecture is erroneous (Iam. *In Nic.* 33.20) that there was 1 perfect number in each category (units, tens, hundreds, thousands, etc.), and that they ended alternately in 6 or 8 (Nicom. *Ar.* 1.16.3). They only worked out the first 4 perfect numbers (6, 28, 496, 8128); the next turns out to be 33,550,336 (Heath, *Math.* I 74f).

[21] As van der Waerden thinks, *SA* 97, *RE* XXIV 283f.

or is it the product of decadence—a dilute, popularizing selection from what had been a rigorous mathematical system?[22]

There did exist a continuous arithmetical tradition which bypassed Euclid. Aristotle knows of the *ψῆφοι* numbers, which reappear in Nicomachus and Theo, whereas Euclid represents the numbers as lines; Aristotle speaks of triangular numbers,[23] Euclid only of "plane" (that is, rectangular) and "solid" numbers. Certain intermediate stages of this extra-Euclidean tradition can be discerned, beginning with Plato's immediate pupils: Speusippus wrote *Περὶ Πυθαγορικῶν ἀριθμῶν*—but he also included non-Pythagorean material;[24] Xenocrates, too, wrote books entitled *Τῶν περὶ τὰ μαθήματα βιβλία ς'*, *Περὶ ἀριθμῶν*, *Ἀριθμῶν θεωρία* (D. L. 4.13); and Philip of Opus wrote *Ἀριθμητικά*, *Περὶ πολυγώνων ἀριθμῶν*, *Κυκλιακά*, *Μεσότητας*.[25] Later on, Hypsicles too wrote on polygonal numbers.[26]

Certain advances beyond the pre-Aristotelian stage are clearly discernible: Aristotle speaks only of triangular and square numbers, and the expansion to polygonal numbers probably does not antedate Philip of Opus. It is more important that there is no trace, before Aristotle, of "perfect" numbers in the sense of Euclid.[27] For the Pythagoreans, as Aristotle knew them, the number 10 is *τέλειον*, and neither 6 nor 28 plays any role. Nor does 28 occupy any significant position in the later tradition.[28] Euclid's definition of "perfect number" is too abstract to catch the imagination of the lovers of symbolism. It is foreshadowed in Plato's *Laws*, where the number of citizens is to be 5,040 because this is a number with a particularly large number of

[22] So Heath, *Math.* I 98f; Capparelli II 451 (with a ref. to the fact that Euclid too was simplified in the Middle Ages).

[23] Above, n. 2.—Eucl. 7 defs. 17–18.

[24] Speusippus fr. 4; cf. above, ch. I 3, n. 106.

[25] *Suda* s.v. *φιλόσοφος*.

[26] Diophantus ed. Tannery I 470.27, 472.20.

[27] A perfect number is one which is equal to the sum of its divisors, including 1; e.g., 6 is divisible by 1, 2, and 3, and $1 + 2 + 3 = 6$; and 28 is divisible by 1, 2, 4, 7, and 14, which add up to 28. The matter is treated in detail by Theo Sm. 45.9ff, Nicom. *Ar.* 1.16, Iam. *In Nic.* 32.20ff; cf. Heath, *Math.* I 74ff; Becker *MD* 47ff; van der Waerden, *SA* 97f (also including "friendly numbers"); above, n.20.

[28] The perfect number 28 is mentioned in Diocles and Strato (frr. 97–98 Wehrli), Nicom. ap. *Th. ar.* 59.14ff, 66.13, Philo *Op.* 101 (from Posidonius?); Nicom. *Exc.* p. 275.16 (cf. 280f) interprets the 28 tones of the tone system with reference to the *Timaeus*, not to the "perfect number." The difference between the two ideas of a "perfect number" is noted by Nicomachus ap. *Th. ar.* 44.15 and Theo Sm. 46.12ff.—The Pythagoras riddle in *A. P.* 14.1 gives 28 pupils of Pythagoras, just as, according to Ath. 1.4e, the Academy had 28 members. Tannery emphasized that the Euclidean perfect number was not the same as the early Pythagorean (*HScH* 382).

divisors;[29] and in the context of the "generation" of numbers in Plato's system of derivation, it was inevitable that the question of the divisibility of larger numbers come to the fore. But the Euclidean "perfect number" is not a feature of early Pythagoreanism.

There are variations in the terminology, too. For Plato and Aristotle 2 is an even number,[30] for Aristotle and Euclid a prime number;[31] but in the later "Pythagorean" sources it will not fit the definition of an even number[32] or that of a prime[33]—in fact it is not regarded as a number at all, any more than 1, but constitutes, along with the latter, the first principle of number.[34] This is sometimes regarded as ancient, because it is illogical;[35] but it might better be regarded as a development from the Platonic system of derivation, in which the numbers have their origin in ἕν and δυάς.[36] On the other hand, the later sources show a more developed logic in the subdivision of even numbers; while in Plato and Euclid[37] ἄρτια περιττάκις (δύο τρίς) and περιττά ἀρτιάκις (τρία δίς) are defined side by side, the later writers distinguish ἀρτιοπέρισσος from περισσάρτιος in such a way that the two concepts are mutually exclusive.[38] The conception of the ἀρτιοπέριττον is quite different in the Pythagoreans; here it is one that has a part in both terms.[39] As early as Aristotle a change is evident to the more developed mathematical terminology; in a passage where he is "Pythagorizing" on his own, he calls the number 6 ἀρτιοπέριττον.[40]

As far as concerns the method employing deduction and proof, the later tradition shows, in one respect, a decline, giving Euclid's

[29] Pl. *Leg.* 737e et seq. The number 5,040 is divisible by all numbers from 1 to 12 except 11. The meaning of ἀριθμὸς τέλειος in *Rep.* 546 is a moot question (cf. also ch. VI 4, n. 76).

[30] Pl. *Parm.* 143e, Arist. *Top.* 157a39.

[31] Arist. *Top.* 157a39, Eucl. 7 def. 12.

[32] Cf. Nicom. *Ar.* 1.7.3f.

[33] Nicom. *Ar.* 1.11 and Iam. *In Nic.* 26.18ff present the prime numbers as a subdivision of the odd; and Theo Sm. 23.14ff calls them περισσάκις περισσοί, but 2, because of its special position, is also called περισσοειδής (24.7).

[34] Nicom. ap. *Th. ar.* 9.1ff.

[35] Heath, *Math.* I 71; Becker, *MD* 46.

[36] Cf. above, ch. I 1, I 3, and esp. Theo Sm. 20.5ff.

[37] Pl. *Parm.* 143e, Eucl. 7 defs. 8–11. Because of the mathematical equivalence of the two concepts, some have athetized def. 10 (cf. the controversy as early as Iam. *In Nic.* 20.11ff). In any case, the agreement with Plato is noteworthy.

[38] Nicom. *Ar.* 1.8–11, Theo Sm. 23ff, Iam. *In Nic.* 20.14ff: ἀρτιοπέρισσος is a number whose half is odd, $2 \times (2n + 1)$; περισσάρτιος is one whose half is in turn even, $2^m \times (2n + 1)$; Euclid finds it necessary to express this in a clumsy circumlocution: ἀρτιάκις τε ἄρτιος καὶ ἀρτιάκις περισσός (9.34).

[39] Above, ch. I 2, n. 39. On Philolaus fr. 5, see above, ch. III 2, n. 124.

[40] Arist. fr. 47; above, ch. I 3, n. 161.

rule for "perfect numbers" without Euclid's proof, but, on the other hand, Speusippus obviously proceeded in no less inductive a manner than his successors.[41] Hypsicles was a professional mathematician, but some of his propositions were not furnished with proofs before Diophantus.[42] The inductive, proofless kind of arithmetic, which lasted till long after Plato, may be regarded, along with a few basic rules, as belonging to the early Pythagoreans.

This is confirmed by the amazingly close agreement of this inductive arithmetic with Babylonian techniques of calculation. There too, by use of rules which are unproven, series are developed and made available for practical use in tabular form: series of squares, of cubes, and even of "Pythagorean" numbers. In fact, when so unusual a series as that of the form $n^2 (n + 1)$ makes its appearance, with the name παραμηκεπίπεδοι, in the Pythagorean tradition, we cannot but think of direct influence.[43] It is possible that this Babylonian import was introduced by Pythagoras himself; but it was altered in the process, at least insofar as, in Aristoxenus' words, the numbers were withdrawn from the use of merchants and "honored" for themselves.[44] The graphic procedure with ψῆφοι makes it possible to formulate impressively generalizations about numbers; but it also "reveals" each fact without deducing one from the other in an abstract chain of reasoning. It is the element of the unforeseeable which gives number games their appearance of something profound and secret. The "occult" charm of mathematics comes from the fact that the human mind forgets its own way of proceeding and loses sight of its own preconceptions; for alert mathematical analysis, that which fills the naive mind with amazement is seen as tautologous and therefore self-evident. What we find among the Pythagoreans is amazement and "reverence" for certain numbers and their properties and interrelations. "Even" and "odd" are united in "marriage"; and to them this means that cosmic forces are at work. A scheme of proof could hardly be anything but annoying because it would show the result as the logical consequence of the preconceptions, and reduce it to banality.

Even a game may be regarded legitimately as a kind of mathematics;[45]

[41] Stressed by Tannery, *HScH* 387.

[42] Diophantus ed. Tannery, I 470.27 (cf. 472.20): ἀπεδείχθη τὸ παρὰ Ὑψικλεῖ ἐν ὅρῳ λεγόμενον.

[43] O. Neugebauer, *QSt* 4 (1938) 181–192, on Nicom. *Ar.* 2.16.3. Cf. above, n. 9; van der Waerden, *SA* 45.

[44] Fr. 23; above, ch. VI 1, n. 77.

[45] K. Lorenz, *Arithmetik und Logik als Spiele*, Diss. Kiel, 1961.

the axiomatic-deductive form is not the only one possible. But the distinctive achievement of Greek geometry is that, for the first time, it created such an axiomatic and deductive system. Seen in the context of this Greek geometry, Pythagorean arithmetic is an intrusive, quasi-primitive element.

By analysis of Euclid, Oskar Becker has reconstructed a set of theorems which has been widely heralded as proof of the existence of a deductive Pythagorean arithmetic,[46] namely the "doctrine of odd and even," as developed in Euclid 9.21-34. These propositions stand isolated in Euclid, a trivial appendage to the sophisticated number theory of books 7–9, which culminates in the proof that there are infinitely many prime numbers (9.20). The propositions about the even and the odd are only once applied by Euclid, in the proposition about perfect numbers (9.36), and in a proof of irrationality, given as an appendix, which Aristotle already knew.[47] On the other hand, the basic importance of the even-odd antithesis for Pythagorean cosmology is well known; and since a fragment of Epicharmus uses the antithesis of odd and even, the conclusion has been drawn that the Pythagorean odd-even doctrine was already so well developed and so widely known about 500 B.C. that it would be recognized in a comic allusion.[48] The fact that most of the propositions can be proved with simple pebble figures gives an important piece of confirmatory evidence; so the "doctrine of odd and even" is regarded as the best-attested element of very early Pythagorean science, in the form of deductive mathematics.

Yet this structure, apparently so firm, has fissures that must lead inevitably to its collapse. It is true that the Pythagoreans were concerned with even and odd numbers; but this is far from proving that they set forth the propositions found in Euclid, in sober mathematical formulation, and provided them with proofs, or that the propositions

[46] Becker, *QSt* 3, 533ff, repeated *Grdl.* 37ff, *MD* 47ff. Agreeing, among others, van der Waerden, *MtAnn* 1949, 127ff, *SA* 108f; Reidemeister 31ff; Szabó (above, ch. VI 1, n. 135); von Fritz, *RE* XXIV 202.

[47] Eucl. 10 App. 27 Heiberg; Arist. *An. pr.* 41a26, 50a37. It is proved that $\sqrt{2}$ would have to be even and odd at the same time, and is therefore not expressible in numbers. Cf. Heath, *Math.* I 91, *Eucl.* III 1; Burnet, *EGP* 105; van der Waerden, *SA* 110; Becker, *MD* 51f. Mondolfo, *Inf.* 241, asserts simply that the proof had been "attribuita da Aristotele all'antico pitagorismo"; but this is baseless (cf. n. 109). Becker, *QSt* 3, 547, finds a hint of this proof in Pl. *Leg.* 819d, "if these words have a precise sense" (meaning a precise mathematical sense). Cf. also below, n. 109.

[48] Epicharm. fr. 2, cited by Becker *QSt* 3, 537; Mondolfo in ZM 318ff; von Fritz, *RE* XXIV 204.

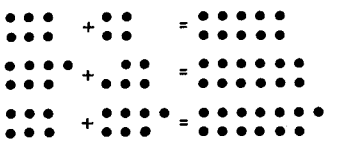
about irrationality and perfect numbers belonged with them from the beginning.

Becker himself assumes that Euclid reworked the Pythagorean doctrine, and reconstructs simpler proofs with pebble figures. But this transformation has deep consequences. In Euclid, the theory unrolls in systematic fashion, and one proposition presupposes the preceding, in the strictly deductive manner, whereas a proof with ψῆφοι is essentially inductive and pictorial. According to the principle of perfect induction, it can be regarded as probatory, but it does not presuppose other propositions; every set of facts is evident in itself. There is no need for a systematic structure, which is of the essence of deductive mathematics. Of course the Pythagoreans knew that odd plus odd makes even, and that odd plus even gives odd—they demonstrated this with their pebble diagrams[49]—but they did not deduce one proposition from the other. They saw, directly, that the "male" odd number showed itself dominant in association with the "female" even number.

The propositions about perfect numbers and about the irrational are on a quite different level. To be sure, Becker can derive from a pebble figure an important lemma for the proposition about the Euclidean perfect numbers,[50] but to go further than this requires an abundant use of modern algebraic notation. Here is presupposed a whole chain of logical conclusions based on very precise concepts of divisibility, prime numbers, and factors, which is totally inconceivable without a written system of στοιχεῖα and a quasi-algebraic method of representing numbers by general symbols.[51] In addition, it is probable on other grounds that the Euclidean idea of a "perfect number" was first developed in the Academy (above, nn. 27–28). It is not part of ancient Pythagorean speculation about the odd and even.

[49] Becker imagines that they represented even numbers by equal numbers of white and black pebbles; but an arrangement in two rows is even more striking (cf. Pl. *Euthyphro* 12d, where even number is defined as ἰσοσκελής and odd as σκαληνός):

Euclid 9.21: even + even = even
Euclid 9.22: odd + odd = even
even + odd = odd



(But see also above, ch. I 2, n. 31. The "power" of the odd number depends on the "one" that it has in its "middle.")—In Euclid, 9.22 presupposes 9.21.

[50] Namely, the summation formula $1 + 1 + 2 + 4 + 8 \ldots + 2^n = 2^{n+1}$ (*QSt* 3, 539ff).

[51] Archytas, like Euclid, represents numbers by line segments (A19).

The same sort of thing is true for the proof of irrationality. Here too, Becker tries to reconstruct a proof with pebble figures:[52] "Given . . . $a^2 = 2b^2$ . . . Think of the number a^2 laid out with pebbles once and b^2 twice . . ." Now, there is no question of the Pythagoreans merely "thinking of" their ψῆφοι; they held them in their hands and arranged them in visible patterns, as is obvious from the testimonia about Eurytus.[53] If one tries, however, following the spirit of Becker's reconstruction, to represent the numbers a^2 and b^2 with counters, one quickly realizes that this is impossible; there are no whole numbers which will exemplify the formula $a^2 = 2b^2$. Becker demands, however, that the Pythagoreans proceed, undaunted by this experience, to "think of" the unrealizable condition as met, so as to go on drawing further conclusions from this contrary-to-fact postulate, until the logical contradiction springs to view (b would have to be even and odd at the same time)—the contradiction which "proves" the absurdity of the merely "thought of" postulate, which from the beginning flouted common sense. Such a train of thought has nothing in common any more with pebble figures and is compelling or meaningful only where the demands of strict mathematical method are already recognized; simpler souls will be satisfied with the inductive discovery that there are no pebble diagrams to be devised which satisfy the equation $a^2 = 2b^2$. The arithmetic of ψῆφοι and the theory of the irrational are mutually exclusive.[54] What is more, we learn from Aristotle that the Pythagoreans considered 1 as both even and odd, and from Theo of Smyrna that the Pythagoreans employed the unit as both the side and the diagonal of a square.[55] The axiom basic to the arithmetical proof of irrationality, that every number is either even or odd, *tertium non datur*, simply does not exist, in this form, for Pythagorean speculation. This gives us all the more reason to suspect that irrationality had originally no part in it and was rather discovered in the field of geometry, where the existence of the irrational is immediately obvious.[56]

[52] *MD* 51; cf. *QSt* 3, 544f.

[53] Above, ch. I 2, n. 69.

[54] Reidemeister (52) says of the irrational: "It cannot be illustrated, only thought and inferred." Becker's illustrations, consisting of empty squares, are quite different from other pebble diagrams (cf. *MD* 52 with 40ff).

[55] Above, ch. I 2, n. 39; VI 2, n. 17.

[56] Von Fritz, too (*AnnMath* 1945, 255), thinks that the irrational was first discovered in geometry; see below, ch. VI 3. For Plato, the expression ἄλογοι ὥσπερ γραμμαί is already proverbial (*Rep.* 534d); and ἀσύμμετρος is also a term coming from geometry. If it is ἄρρητος (*Rep.* 546c), "not expressible (in numbers)," this presupposes the existence of the corresponding object. True to its ontological basis, Greek mathematics, unlike the modern, only develops concepts for "existing" magnitudes.

Only the propositions about the perfect numbers and the proof of irrationality give mathematical substance to the "doctrine of odd and even"; without them all we have is a remnant of very elementary and disconnected theorems. Becker himself believes that Euclid recast the proofs; but how simple it would have been for a pre-Euclidean mathematician to put the propositions about the odd and even into a systematic paradigm of deduction! The proof of irrationality and the proposition on perfect numbers presuppose this reformulation; they do not belong to the oldest stratum of Pythagorean arithmetic. The "doctrine of odd and even" only later became part of deductive mathematics, influenced by the *mos geometricus*.

There remains the division of numbers into odd and even, generally, in which one might, in spite of all, see the beginning of number theory; it is confidently attributed to Pythagoras himself.[57] But this very point can be refuted by philological means. In all Pythagorean speculation the odd number is more highly valued; it is what "sets the limit," is the male element, and stands in the "column of the good." In all this is preserved, as shown both by anthropological parallels from folklore and linguistic observation, an ancient and widespread piece of number lore.[58] But in the Greek language the even number is "well-structured," ἄρτιος, and the odd is "excessive," περιττός; in normal usage ἄρτιος is unequivocally the one with favorable connotation, and περιττός is negative, representing a transgression of the norm.[59] Thus Greek terminology for even and odd is in its tendency diametrically opposite to the Pythagoreans' numerical theory. This terminology is easily comprehensible from everyday use: dividing in half has from time immemorial had an important role in practical life, and it is no surprise that the words for "half" are built on a special root, which is not true of either "third" or "fourth."[60] In Greek lands it was often necessary to match two semi-choruses in a ritual dance, or to match pairs in an *agon*—in Plato's view this is the sort of example upon which

[57] Heath, *Math.* I 70: "The distinction between odd (περισσός) and even (ἄρτιος) doubtless goes back to Pythagoras." Rostagni, *Verbo* 27: this distinction, "nel suo valore scientifico, tutti sanno essere d'origine pitagorica" (though one might wonder if it has "scientific value" in the Pythagorean tradition). See also Sarton, *Hist.* 204.

[58] Above, ch. I 2; below, ch. VI 4.

[59] On ἄρτιος see Solon 3.39 Diehl, Theognis 154, 946; on περισσός, Empedocles fr. 13. Line 18 of Cleanthes' hymn to Zeus is significant: σὺ καὶ τὰ περισσὰ ἐπίστασαι ἄρτια θεῖναι.

[60] ἥμισυς is Indo-European in root and meaning.

children should practice calculation.[61] Either the procedure is successful, because the number is "symmetrically constructed," or one person is left over—*περιττεύει*.[62] It did not need a Pythagoras to produce the insight that the problem "comes out right" with 2, 4, 6, 8, or 10 persons, but not with 3, 5, 7, or 9. Just as in Latin, English, and German,[63] the Greek distinction between even and odd grew out of elementary, everyday calculations; but then, among the Pythagoreans, it took on a different and unpredictable significance.[64]

The term *λόγος*, in its mathematical sense of "relation, ratio, proportion," has been attributed by von Fritz to the Pythagoreans, and, conjecturally, to Pythagoras himself.[65] Its origin lies, he thinks, in Pythagorean musical theory: the *λόγος*, a "Mitteiling von etwas Wesentlichem an einem Gegenstand," "the communication of something essential about a thing" (81), is, in relation to music, the numerical ratio present. If one knows this, he knows the nature (*Wesen*) of the interval, and can reproduce it.[66] And similar reasoning holds in the realm of geometry; the ratio 3:4:5 determines the shape of a right

[61] Pl. *Leg.* 819b.

[62] Cf. Hes. fr. 278 M.–W. (from the *Melampodia*): *μύριοί εἰσιν ἀριθμόν, ἀτὰρ μέτρον γε μέδιμνος, εἷς δὲ περισσεύει* (i.e. the bushel measure holds 9,999 figs).

[63] Neither *par* and *impar* nor *gerade* and *ungerade* are borrowed from Greek. English *odd* comes from the Old Norse, and originally designates the third or other "odd" man in a council, whose vote decides in case of a tie (*NED* VII 58ff).

[64] Epicharmus plays with the ideas of odd and even (DK 23B2 = fr. 170.7ff Kaibel = Alcimus *FGrHist* 560F6 = D.L. 3.11; cf. above, ch. III 3, n. 58; the authenticity of the fragment was denied by Wilamowitz, among others, *Platon* II 28.2): *‹αἰ› ποτ ἀριθμόν τις περισσόν, αἰ δὲ λῇς, ποτ ἄρτιον ποτθέμεν λῇ ψᾶφον ἢ καὶ τᾶν ὑπαρχουσᾶν λαβεῖν, ἢ δοκεῖ κά τοί γ' ‹ἔθ'› ὡύτὸς εἶμεν;* . . . Luckily, in this case we know the comic context in which the words occurred. A debtor is trying to free himself of his debts by using the claim that he is no longer the same person; we know from Chrysippus (ap. Plut. *Comm. not.* 1083a; cf. Pl. *Tht.* 152d–e) that this *περὶ αὐξήσεως λόγος* did occur in Epicharmus. When something is added or subtracted, what is left is no longer the same thing; all of us are constantly gaining weight . . . and so on. The proposition is proved from two examples, involving number and size respectively. This restriction is not specifically Pythagorean, but essential to the argument, which is only valid with relation to quantity. The further subdivision of numbers into odd and even seems less essential (Rostagni, *Verbo* 28f), and therefore an indication of external, presumably Pythagorean, influence; but, for the idea to be clear enough for the comic stage, it was necessary that the general concepts be replaced by something more specific. This is why the idea of measure is expressed from the start as *παχυαῖον* and the idea of number as even/odd. By the addition of a single pebble a number changes from odd to even or the reverse; it is precisely the notion of odd and even which makes it clear that the very smallest alteration signifies a fundamental change. Even if one were to assume that there is a relation between this passage and the Pythagoreans, the "number theory" presupposed would be the inductive and graphic, rather than the deductive. See also von Fritz, *RE* XXIV 203–205.

[65] *ABG* 1955, 81ff (cf. *RE* XXIV 199).

[66] Though not in a practical way; cf. Aristotle, cited above ch. V 1, n. 62.

triangle, and with its help one can reproduce the right angle. Thus the *λόγος* would be "the group or bundle of numbers that lie hidden in a thing, by use of which it can be not only described but reproduced" (83).

Now a distinctive feature of the Greeks' calculation of proportions, and in particular of Pythagorean musical theory, is the occurrence of certain terms like *ἐπίτριτος λόγος*, *ἐπόγδοος λόγος*, and the more general *ἐπιμόριος λόγος*, which from a German or an English point of view seem odd. Their sense is quickly evident; *ἐπίτριτον*, for example, "a third in addition," means $1 + \frac{1}{3}$ or $\frac{4}{3}$. But the question remains, what made these ratios so important that they alone, in the Greek language, have special names, while a fraction like $\frac{3}{5}$, unlike $\frac{6}{5}$ (*ἐπίπεμπτον*), can only be expressed in a cumbersome circumlocution. The answer is simple: they were terms used in the calculation of interest. Whoever lends money expects to get his principal back and a specified fraction of it in addition. This could be *ἐπίτριτον* ($\frac{4}{3}$, or $33\frac{1}{3}\%$),[67] *ἐπίπεμπτον* ($\frac{6}{5}$, or 20%),[68] more usually *ἔφεκτον* ($\frac{7}{6}$, or $16\frac{2}{3}\%$),[69] sometimes *ἐπόγδοον* ($\frac{9}{8}$, or $12\frac{1}{2}\%$),[70] at the lowest *ἐπιδέκατον τόκον*[71] (interest of $\frac{1}{10}$; this was what the gods received).[72] It is certain that the practice of loaning money at interest went back before the time of Solon; and, though there is no direct evidence, it can hardly be doubted that the expressions mentioned were in use that early—long before the day of Pythagoras.[73] Thus when terms like this turn up in a musicological context—*ἐπίτριτον* and *ἐπόγδοον* in Philolaus (fr. 6) and the general *ἐπιμόριος λόγος* in Archytas (A19)—they are borrowed from everyday speech.

But the calculation of interest is in fact called *λογίζεσθαι*: *λογίσωμαι*

[67] Xen. *Vect.* 3.9, Isaeus fr. 79 Sauppe, Arist. *Rhet.* 1411a17.

[68] Xen. *Vect.* 3.9.

[69] Demosth. 34.23.

[70] Demosth. 50.17.

[71] *IG* I² 377.12 (434/432 B.C.); cf. *IG* VII 4263 = *SIG*³ 544 (Oropus, 3rd century B.C.): whoever lends the city money at this rate becomes *πρόξενος καὶ εὐεργέτης*; *IG* IX 161a19, Arist. *Rhet.* 1411a17, *Oec.* 1346b32.

[72] *τοῦ θεοῦ τὸ ἐπιδέκατον*: *IG* I² 45.25 (446/441 B.C.), 39.35 (446 B.C.), Xen. *Hell.* 1.7.10 et saep. *ἐπιδέκατον* means here simply "a tenth," as *ἐπίπεμπτον* means "a fifth" (Ar. fr. 201 = Harpocr. s.v. *ἐπίπεμπτον*, with further refs.; also *IG* VII 3073.1 = *SIG*³ 972.1); but it is precisely the tenth or fifth that must be added, "extra," because the other party lays claim to it. In the Sagras battle, the Locrians outbid the Crotoniates by vowing a ninth—*ἐνατεύειν* instead of *δεκατεύειν* (Justin 20.3.3).

[73] That proportion was used at this time is also shown by the expression *ἑκτημόροι* (on which see D. Lotze, "Hektemoroi und vorsolonisches Schuldrecht," *Philologus* 102 [1958] 1–12).

τοὺς τόκους, says Strepsiades.[74] The officials who calculate the interest on temple loans are called λογισταί,[75] and the calculation itself is λόγος.[76] Heraclitus writes: "The turnings of fire: first sea, and of sea half is earth, half burner . . . (Earth) is dispersed as sea, and is measured to the same proportion as it was before it became earth" (καὶ μετρέεται εἰς τὸν αὐτὸν λόγον ὁκοῖος πρόσθεν ἦν ἢ γενέσθαι γῆν).[77] Here the mathematical sense of the word is clearly present. Later one said κατὰ τὸν αὐτὸν λόγον,[78] or simply ἀνὰ λόγον "in the (ascending) sequence of calculation." Thus ἐπίτριτος λόγος means "calculating at $33\frac{1}{3}\%$," as λόγον διδόναι means "render account," and ὧδ' ἔχει λόγος "this is the state of the account." The sense "calculation" comes from the basic sense of the root λεγ- almost more directly than the sense "word."[79] But Pythagoras has nothing to do with all this.

The connection of proportion and music, resulting in the equation of διάστημα with λόγος, remains to the credit of the Pythagoreans; and in one aspect of the theory of proportion, the doctrine of "means," Pythagorean influence is a possibility. The three μεσότητες, the arithmetic, geometric, and harmonic means, are generally regarded in the tradition as a discovery of Pythagoras.[80] The fact that all three means have a role in the *Timaeus*[81] could rouse suspicions about the tradition involving Pythagoras himself. But Theaetetus already knew the system of the three means, and used it, in a rather forced manner, as the point of departure for his classification of irrational lines;[82] thus the

[74] Ar. *Nub.* 20. λογίζεσθαι refers to the elementary techniques of calculation learned at school, whose culmination and most difficult part was the calculation of interest (Hdt. 2.16, 36).

[75] *IG* I² 324.1, 72 (426/422 B.C.).

[76] Pi. *Ol.* 10.11.

[77] DK 22B31. The supplement ‹γῆ›, rejected by Reinhardt (*Hermes* 77 [1942] 16), is accepted by KR no. 221.

[78] *IG* I² 76.9 (423/422 B.C.); κατὰ λόγον τῆς τεταγμένης ἀποφορῆς, Hdt. 2.109; λόγον διδόναι, Hdt. 3.142f; *IG* I² 91.25 (decree of Callias). Cf. Hdt. 1.134, 2.13, 14, 68.

[79] *Od.* 4.451: λέκτο δ' ἀριθμόν. Cf. esp. καταλέγειν, κατάλογος.

[80] Theo Sm. 113ff, Nicom. *Ar.* 2.21ff, Iam. *In Nic.* 100.15ff.

[81] The geometric mean as δεσμός, 31c–d; the arithmetic and harmonic means, 36a, cf. *Epin.* 991a; Sachs 130ff.

[82] This is attested by Eudemus in a fragment preserved only in an Arabic version (overlooked by Wehrli): Pappus, *Comm. on Euclid X*, ed. G. Junge and W. Thomson (Cambridge, 1930; earlier ed. by Woepcke, Paris, 1855) 1.1 p. 63: "it was nevertheless Theaetetus who distinguished the powers (i.e. the squares) which are commensurable in length, from those which are incommensurable (i.e. in length) [this is from Pl. *Tht.* 147d et seq.] and who divided the more generally known irrational lines according to the different means, assigning the medial line to geometry, the binomial to arithmetic, and

means are presumably older, and they are closely related to Pythagorean music theory. The name of the "harmonic mean" is to be explained directly from the latter; the Mese is the harmonic mean of the octave Nete-Hypate.[83] In fact, it is attested that Archytas and Hippasus introduced the term ἁρμονικὴ μεσότης in place of the older ὑπεναντία μεσότης[84] and the series 6, 8, 9, 12 is presupposed in the experiment which Aristoxenus ascribes to Hippasus.[85] If this third and most complicated of the means was known, then the two others surely were.[86] It would be conceivable that the harmonic mean was discovered in the context of Pythagorean music theory;[87] but it also has another use: along with the arithmetic mean it provides increasingly closer approximations of the square root;[88] and, if, as Iamblichus says, the "most perfect proportion," 12 : 9 = 8 : 6, was introduced by Pythagoras from

the *apotome* to harmony, as is stated by Eudemus, the Peripatetic." Also, 2.17 p. 138: "those who have written concerning these things declare that the Athenian Theaetetus assumed two lines commensurable in square and proved that if he took between them a line in ratio according to geometric proportion, then the line named the medial was produced, but that if he took (the line) according to harmonic proportion, then the apotome was produced." On the concepts medial, binomial, and apotome, see Euclid 10; on the connection with the three means, Junge–Thomson 17. This fragment of Eudemus is important because its content is independent of the *Theaetetus* of Plato and is thus reliable evidence for the existence and importance of the mathematician Theaetetus. It was cited by Sachs (135, 177), Frank (272; cf. above ch. V 2, n. 41), and von Fritz (*RE* V A 1354f).

[83] I.e., if one assigns the larger number to the Nete (above, ch. V 1 n. 47): 6 is Hypate, 12 is Nete, 8 (the harmonic mean) is Mese, and 9 (the arithmetic mean) is Paramese.

[84] Iam. *In Nic.* 100.22 = DK 18.15: ἡ ποτὲ μὲν ὑπεναντία λεγομένη . . . ὑπὸ δὲ τῶν περὶ Ἀρχύταν αὖθις καὶ Ἵππασον ἁρμονικὴ μετακληθεῖσα. Also Archytas fr. 2, DK I 435.20f: τρίτα δ' ὑπεναντία (μέση), ἂν καλέοντι ἁρμονικάν; I 436.8f: ἁ δ' ὑπεναντία, ἂν καλοῦμεν ἁρμονικάν. In two other passages Hippasus and Archytas are named together, in reference to the doctrine of means (Iam. *In Nic.* 113.16ff, 116.1ff). Tannery concluded from this (*MSc* II 190, *HScH* 394) that Archytas must have cited Hippasus. Actually, Iamblichus states in the two later passages, in contradiction to his own earlier account, that Hippasus and Archytas had already discovered the fourth, fifth, and sixth means (cf. n. 92); but it is possible that he himself misunderstood his source.—Philolaus, too, knew the harmonic mean (A24).

[85] Above, ch. V 1, nn. 33–34.

[86] Hippocrates of Chios extends the idea of the geometric mean to that of the "two mean proportionals" (not a : x = x : b, but a : x = x : y = y : b), in order to solve the problem of the duplication of the cube; thus, even before his time, the geometrical mean was known and used for "doubling the square," i.e., for determining square roots.

[87] Music theory soon led, inevitably, to the series 6, 8, 9, 12. It is obvious that 9 is the "mean" between 6 and 12; but from a musical point of view it is just as clear that, as the octave (6 : 12) is divided by 9 into fifth and fourth, so it is divided by 8 into fourth and fifth—ὑπεναντίως (cf. Philolaus fr. 6). Thus the expression ὑπεναντία μεσότης was meaningful in music too.

[88] Cf. C. Müller, *QSt* 2 (1932) 281ff; Becker, *MD* 65f.

Babylon,[89] we may be on the track of something genuine, namely that the arithmetic and harmonic mean were used in Babylonian calculating technique to find the square root.[90] If so, a rule used in Babylonian calculation has been transposed into Pythagorean number speculation, just like the rule involved in the "Pythagorean theorem."[91] The original name itself, *ὑπεναντία μεσότης* ("subcontrary"), is comprehensible from its use as a tool in calculation; one forms first the "numerical mean," then its appropriate "reversal." Here too, then, Pythagorean musical theory is an outgrowth of practical methods of calculation. The further development of the doctrine of means only begins with Eudoxus;[92] the accomplishment of the Pythagoreans around and after Hippasus is rather in the application and interpretation of known methods than in the foundation of a theory of numbers.

An unquestionable piece of scientific Pythagorean arithmetic is Archytas' proof that a superparticular proportion cannot be divided into equal parts by a mean proportional.[93] In contrast with the inductive

[89] Iam. *In Nic.* 118.23f: *εὕρημα δ' αὐτήν φασιν εἶναι Βαβυλωνίων καὶ διὰ Πυθαγόρου πρώτου εἰς Ἕλληνας ἐλθεῖν.* The proportion in question is, greater number: arithmetic mean = harmonic mean : smaller number, or, greater number : harmonic mean = arithmetic mean : smaller number (also, therefore, greater number × smaller number = arithmetic mean × harmonic mean). The Babylonians did not know the concept of proportion (Becker, *Arch. f. Mus.-W.* 1957, 156); but the "mean" may very well have been employed in calculation.

[90] Becker, *MD* 65.

[91] Above, n. 9.

[92] He discovered the fourth, fifth and sixth means: Eudemus fr. 133 = Procl. *In Eucl.* 67.5ff = D22 Lasserre; Iam. *In Nic.* 101.1ff (above, n. 84); Nicom. *Ar.* 2.28 and Iam. *In Nic.* 113.16ff only call the arithmetic, geometric, and harmonic means "ancient." Philip of Opus also wrote *Μεσότητας* (*Suda* s.v. *φιλόσοφος*). The seventh to tenth means were discovered, according to Iam. *In Nic.* 116.4ff, by the *Πυθαγορικοί* Myonides and Euphranor, not only later than Eudoxus but also later than Eratosthenes; not all Pythagorean arithmetic is pre-Platonic. It does not seem possible to discover any further detail about Myonides and Euphranor (though a Euphranor wrote a *Περὶ αὐλῶν*, Ath. 4.184e; cf. also Nicom. *Ar.* 2.28).—Thymaridas, whose *ἐπάνθημα* is mentioned at Iam. *In Nic.* 62.18ff, has been placed "in the time of Plato or somewhat earlier" (Tannery, *MSc* I 106ff, II 192ff; *HScH* 396; also Becker, *QSt* 4, 165f; *MD* 43f), because the proposition in question can be proved by a (complicated) pebble figure; but for the chronology this does not mean any more than a mere possibility. Heath, *Math.* I 94, lays down that the terminology used about the *ἐπάνθημα* by Iamblichus agrees with that of Diophantus (probably 3rd century A.D.); and Diels (DK I 447 n.) emphasized that the definition of the *μονάς* as *περαίνουσα ποσότης*, which is ascribed to the same Thymaridas by Iam. *In Nic.* 11.2, cannot be dated earlier than Plato. Thymaridas is named as a pupil of Pythagoras at Iam. *VP* 104, and edifying anecdotes about him are given at Iam. *VP* 145 (after the "tripartitum" D.L. 8.9 ?) and 239; in the catalogue of Pythagoreans (p. 145.5) he is listed as a Parian. How tradition and legend got connected with a specific mathematical problem remains a mystery.

[93] Archytas A19 = Boeth. *Mus.* 3.11. See also ch. V.

pebble games, a general proposition is here proved deductively, and the numbers, represented by letters, are obviously thought of as line segments, as in Euclid.[94] A number of arithmetical concepts, like proportion, divisibility, the smallest numbers in a given ratio, relative prime, and mean proportional, are confidently handled; the structure follows conventional order: statement of what is "given," statement of theorem, proof;[95] and the method of proof is the reductio ad absurdum. Above all, Archytas presupposes a whole series of arithmetical propositions and expressly cites an auxiliary theorem.[96] Tannery, who called attention to the proof of Archytas transmitted by Boethius, concluded from this that Archytas must have had a kind of *Elements of Arithmetic*,[97] and van der Waerden undertook to reconstruct, systematically, this number theory presupposed by Archytas, coming to the conclusion that in all essentials the material of Euclid's seventh and eighth books must already have been in existence; book 7, he thought, "existed in written form before 400 B.C." and "had been taken over by Euclid without significant alteration."[98] Book 8 was the work of Archytas himself.[99] If this were correct, we should have an imposing edifice of Pythagorean arithmetic of an entirely different kind from what Speusippus, Aristotle, Theo, and Nicomachus lead us to expect.

It is impossible to discuss in detail here the analysis of Euclid and the problem of evaluating his work as a mathematician;[100] but we may

[94] Unlike Euclid, the author designates with letters not the ends of line segments ("line AB"), but the segments themselves; DE means the number D plus the number E. This is not, however, an ancient feature (as maintained by Tannery, and van der Waerden, *MtAnn* 1947–1949, 134), but the method followed by Boethius (also *Mus.* 4.2, in the translation of *Sect. can.* 3). Every exposition in the history of mathematics "modernizes."

[95] To be sure, the structure can be the work of the intermediate source.

[96] Archytas A19: ". . . qui enim sunt minimi in eadem proportione, quibuslibet aliis numeris, hi primi ad se invicem sunt," corresponding to Eucl. 7.22.

[97] Tannery, *MSc* III 249; cf. Heath, *Math.* I 90; *Eucl.* II 295; Becker, *MD* 44f.

[98] *MtAnn* 1947–1949, 146.

[99] Ibid. 149; cf. *SA* 153ff, *RE* XXIV 284–285.

[100] Van der Waerden, *SA* 197: "Euclid is by no means a great mathematician," though he is "the greatest schoolmaster known in the history of mathematics" (196). This judgment calls for contradiction; the *Elements* does constitute a scientific achievement, rather than merely a schoolbook. And it is based in part on a mistranslation. In the scholium to Euclid, p. 654.8 Heiberg, one reads, after the account of the discovery of the regular solids by the Pythagoreans and Theaetetus, *Εὐκλείδου δὲ ἐπιγράφεται καὶ τοῦτο τὸ βιβλίον διὰ τὸ στοιχειώδη τάξιν ἐπιτεθεικέναι*, "This book too [sc. 13] bears Euclid's name, because he gave it the order of *στοιχεῖα*" (see Burkert, *Philologus* 1959, 189ff). Cf. Schol. Eucl. p. 73.4 (after Proclus 69.4ff), *συνήγαγεν εἰς στοιχείωσιν, τάξιν αὐτοῖς καὶ ἀποδείξεις ἀκριβεστέρας ἐπιθείς* . . . Van der Waerden, however, translates (*SA* 173), "This book also carries Euclid's name, because he embodied it in the *Elements*," as though he had taken it over from an earlier author without alteration or addition.

refer once more to the large number of mathematicians who were active in the brief period between Plato and Euclid.[101] Several of these were concerned, precisely, with writing or rewriting στοιχεῖα.[102] The miracle of Pythagorean arithmetic would indeed be amazing, if in spite of this, Euclid had taken over "without significant alteration" expositions that were over a hundred years old.

Here we are confronted again with a basic problem of mathematical history: to be sure, it can be shown with perfect exactitude what propositions Archytas must have been presupposing as proven—but only on the presupposition, on our part, that Archytas built up a complete theory of number, in accordance with the demands of Euclidean precision. Even the suggestion that Archytas was concerned to prove a proposition in number theory is a dubious formulation. Boethius cites the proof in the context of music theory; and among the works of Euclid, it is found in the *Sectio canonis*, not in the number theory of the *Elements*. There is no question that for Archytas the musical application of the theorem—the indivisibility of the intervals of the octave, the fifth, the fourth, and the whole tone—was of primary importance. To what extent he was able to refer to a previously existing arithmetic, and to what extent he understood and fulfilled the demands of a complete, deductive system, remains to be established.

Scholars have laid emphasis on the fact that Archytas' proof was identical with the one found in the *Sectio canonis* (3).[103] Boethius, however, characterized Archytas' proof as "nimium fluxa" and replaced it with another, consisting of a literal translation of the *Sectio canonis*.[104] Thus the tradition which Boethius was following emphasized not so much the agreement as the difference between Archytas' proof and that of the *Sectio canonis* and found Archytas' inadequate. In order to understand Boethius' criticism, we must compare the two proofs in detail.

The first step they have in common: for the given superparticular proportion a : b is substituted that of the two smallest numbers in

[101] Above, ch. VI 1, n. 125.

[102] Eudemus at fr. 133 = Procl. *In Eucl.* 66.20 names Leon, at 67.14f Theudius, and at 67.20ff Hermotimus.

[103] Van der Waerden, *Hermes* 1943, 169, and *SA* 153, concludes from this that like Euclid 8, the *Sectio canonis* mainly derives from Archytas (i.e., without the last 2 sentences; above, ch. V 1, n. 22), while Frank sees in the musical theory of Archytas the sharpest kind of contrast to the "Platonically" influenced *Sectio canonis* (above, ch. V 1).

[104] Boeth. *Mus.* 4.2 p. 303.19ff. The whole passage, *Sect. can.* 1–9, is translated by Boethius at *Mus.* 4.2.

the same ratio; m : n = a : b. The proof continues, in the *Sectio canonis*, as follows: (1) m and n are by definition, as the smallest numbers in the given ratio, prime to each other.[105] (2) According to the definition of the superparticular proportion, the difference between m and n is a common divisor of m and n. (3) Thus m − n = 1. (4) Accordingly there is no mean proportional between m and n, (5) and by extension there is none between any numbers that stand in the same ratio, a : b.[106] Archytas, for his part, first establishes step (2), mistakenly introducing, in the statement of the reason, the concept "minimi."[107] Then he sets up proposition (3), as an assertion and proves it in detail, by reductio ad absurdum of the opposite. This is a basically superfluous repetition,[108] made necessary by his view that 1 is not a number.[109] The only crucial statement is (1), which is cited in full. After another repetition,[110] steps (4) and (5) follow without detailed rationale; the necessary, and not quite evident, auxiliary theorem is not cited.[111]

The repetitions here could be regarded as no more than a stylistic defect, but the fact that necessary and nonnecessary presuppositions are not distinguished strictly is more disturbing; and for this reason we can hardly suppose that it is only some copyist's fault that the necessary auxiliary theorem is not cited. A particularly arresting formulation is found in the citation, in Euclid, of another lemma: "qui enim sunt minimi in eadem proportione quibuslibet aliis numeris, hi primi ad se invicem sunt, et solam differentiam retinent unitatem."[112] Tannery excised the last five words as "absurd,"[113] and the more recent paraphrases leave them out;[114] but the criticism of Boethius, that is, that

[105] This depends on Eucl. 7.22.

[106] Citing Eucl. 8.8.

[107] DK I 429.35: "quoniam igitur sunt minimi in eadem proportione C, DE et sunt superparticulares . . ." Only their superparticularity is necessary for the conclusion.

[108] DK I 429.36: "DE numerus C numerum parte una sua eiusque transcendit; sit haec D"; then 429.38: "si enim est numerus D et pars est eius qui est DE, metietur D numerus DE numerum, quocirca et E numerum metietur; quo fit, ut C quoque metiatur; utrumque igitur, C et DE numeros metietur . . ."

[109] DK I 429.37f: "dico quoniam [ὅτι] D [i.e., m — n] non erit numerus sed unitas." The proof of irrationality in Eucl. 10 App. 27 proceeds in a quite similar manner.

[110] DK I 430.5: "igitur DE numerus C numerum unitate transcendit."

[111] I.e., Eucl. 8.8; cf. n. 106. In the list of propositions presupposed by Archytas which van der Waerden sets up (*SA* 111), the auxiliary proposition Eucl. 8.8 is one of the most important links.

[112] DK I 430.3; cf. Eucl. 7.22.

[113] *MSc* III 248. (He thinks this is an interpolation by Boethius; but shall we suppose he made interpolations only to argue against them?)

[114] E.g. Heath, *Math.* I 215; van der Waerden, *SA* 111.

of his Greek source—confused as it is—takes this very sentence as its point of departure.[115] The difference of the "smallest numbers in a given ratio" is not of course 1 in every case, but only in the superparticular ratio. Did Archytas perhaps, as he previously introduced the idea of "minimi" when dealing with the superparticular ratios (above, n. 107), think of superparticulars when dealing with "minimi" and so formulate a conclusion which was only applicable to the former?[116] In any case, athetesis is no suitable way to get rid of the problem. This is to create the illusion of a perfect mathematical clarity and obstruct our view of a mathematics still feeling its way along an uncertain path.

Archytas' achievement is not to be underestimated. A start was made on the way to a general theory of numbers, such as we find in Euclid; but it is certain that the perfection of Euclidean form had not yet been reached, and that the similar proof in the *Sectio canonis* was regarded as a basic improvement. Archytas' number theory grew out of his music theory, and a complete, systematic structure is not to be expected. Was Archytas the first, using the methods of proof suggested by a highly developed geometry, to create a deductive theory of number of which Philolaus still had no suspicion?[117] The name "number theory," *ἀριθμῶν θεωρία*,[118] appears to come from Xenocrates, following Plato in his demand for a "pure," logical and deductive treatment of numbers, above and beyond the realm of sense impression—*θέα τῆς τῶν ἀριθμῶν φύσεως*.[119] Thus the number theory stemming from Archytas—like,

[115] Boeth. *Mus.* 3.11 p. 286.7ff (no longer included in DK): "Et secundum Archytae quidem rationem idcirco in superparticulari nullus medius terminus cadit, qui aequaliter dividat proportionem, quoniam minimi in eadem proportione sola differunt unitate, quasi vero non etiam in multiplici proportione minimi eandem unitatis differentiam sortiantur, cum plures videamus esse multiplices praeter eos, qui in radicibus [*πυθμένες* = minimi, Archytas A17] collocati sunt, inter quos medius terminus scindens aequaliter eandem proportionem possit aptari. Sed haec, qui arithmeticos numeros diligenter inspexerit, facilius intellegit. Addendum vero est, id ita evenire, ut Archytas putat, in sola superparticulari proportione; non autem universaliter est dicendum." Perhaps Boethius did not himself understand his "reprehensio" (285.8). The concluding sentence is correct; what Archytas (apparently at least) expressed in general terms, that the "differentia" of the "minimi" is one, is valid only for the superparticular proportion.

[116] It would also be possible that an intermediate source transposing Archytas' proof into the later form created the confusion; or that archaic terminology was misunderstood, and Archytas meant, with a phrase like *ὑπέχειν τὴν μονάδος διαφοράν*, the "common divisor 1" (the common divisor is found by alternate subtraction).

[117] Above, ch. V 2. But the title *'Αριθμοί* is also found in Democritus (B11 o).

[118] The programmatic title of a work of Xenocrates (D.L. 4.13).

[119] Pl. *Rep.* 525c; on the subject in general, aside from *Rep.* 522c et seq., *Phlb.* 56d, *Tht.* 195d, *Epin.* 990c. When Plato, at *Rep.* 525c, cites *οἱ περὶ ταῦτα δεινοί*, the "professionals," for their axiom that the monad is indivisible and fractions are not allowed in the calculation of proportions, he may well mean the same Pythagoreans he speaks of at

in a sense, Pythagorean philosophy in general—achieved its final form in Platonism. Before Archytas there were number games accompanied by the "interpretation" of, and "reverence" for, number.

3. PYTHAGOREAN GEOMETRY AND MATHEMATICAL SECRETS

A passage of Iamblichus, to whose Aristotelian provenance we have already drawn attention, reads as follows:[1]

οἱ δὲ Πυθαγόρειοι διατρίψαντες ἐν τοῖς μαθήμασι καὶ τό τε ἀκριβὲς τῶν λόγων ἀγαπήσαντες, ὅτι μόνα εἶχεν ἀποδείξεις ὧν μετεχειρίζοντο ἄνθρωποι, καὶ ὁμολογούμενα ὁρῶντες [ἔνισον] τὰ περὶ τὴν ἁρμονίαν, ὅτι δι'ἀριθμῶν, [καὶ τὰ περὶ τὴν ὄψιν μαθήματα διὰ <δια>γραμμάτων], ὅλως αἴτια τῶν ὄντων ταῦτα ᾠήθησαν εἶναι καὶ τὰς τούτων ἀρχάς· ὥστε τῷ βουλομένῳ θεωρεῖν τὰ ὄντα πῶς ἔχει, εἰς ταῦτα βλεπτέον εἶναι, τοὺς ἀριθμοὺς [καὶ τὰ γεωμετρούμενα εἴδη τῶν ὄντων] καὶ λόγους, διὰ τὸ δηλοῦσθαι πάντα διὰ τούτων.

The Pythagoreans, having devoted themselves to mathematics, and admiring the accuracy of its reasonings, because it alone among human activities knows of proofs, and seeing [equally] the facts about harmony, that they happen on account of numbers, generally admitted [and (seeing) the mathematics of optics depending on (dia)grams], they deemed these (facts of mathematics) and their

Rep. 530, i.e. the circle of Archytas. A *μάταιος πραγματεία* in relation to *λογισμοί* is also mentioned by the Xenophontic Socrates (*Mem.* 4.7.8).—The assertion so often repeated, that Plato differentiated *λογιστική*, as the practical art of calculation, from *ἀριθμητική*, as the theory of number, and that the latter was an achievement of the Pythagoreans (Tannery, *HScH* 381; Heath, *Math.* I 13f; Dodds, *Gorg.* 199; K. Vogel, "Beiträge zur griechischen Logistik," *SbMü*, math.-ph. Kl., 1936, 361f; Becker, *MD* 45), is an error long ago corrected (J. Klein, "Die griechische Logistik und die Entstehung der Algebra," *QSt* 3 [1936] 23ff; A. Wedberg, *Plato's Philosophy of Mathematics* [Stockholm, 1955] 22f). Plato demands mathematics as pure theory for *λογιστική* as well as for *ἀριθμητική*, and there is a practical "arithmetic" as well as a theoretical "art of calculation," or "logistics." (*Phlb.* 56e). The difference between arithmetic and *λογιστική* corresponds to that in everyday language between "counting" and "calculating." (*Charm.* 166a, *Gorg.* 451b; cf. *Rep.* 522e, *Epin.* 978e; the two together, *Rep.* 522c–e, 525a, *Phdr.* 274c, *Hp. mi.* 367a; *λογιστικοί* alone, *Euthyd.* 290c, *Rep.* 525b, *Pol.* 259e, *Tht.* 145a; cf. also *Rep.* 510c: *οἱ περὶ . . . λογισμοὺς . . . ὑποθέμενοι τό τε περιττὸν καὶ τὸ ἄρτιον . . .* with *Tht.* 198a–c: *ἀριθμητική* as the *ἐπιστήμη ἀρτίου καὶ περιττοῦ*). The differentiation of theoretical arithmetic and practical "logistics" was set up by Geminus (by contrast to the "Pythagorean" quadrivium: Procl. *In Eucl.* 38.1ff; Geminus–Anatolius ap. Hero Def. 135.5 p. 98 Heiberg). Olympiodorus foisted it onto Plato (Olympiod. *In Gorg.* 450b p. 31.4ff Norvin; cf. Schol. *Charm.* 165e).

[1] Iam. *Comm. math. sc.* p. 78.8–18; cf. above, ch. I 2, n. 112.

> principles to be, generally, causative of existing things, so that whoever wishes to comprehend the true nature of existing things should turn his attention to these, that is to numbers [and the geometrical entities among existing things] and proportions, because it is by them that everything is made clear.

In the two places where geometry is mentioned the course of the argument is deranged. To begin with the second, λόγους ("proportions") is without an article, and, as regards the sense, is intrusive after the clearly established dichotomy of arithmetic and geometry; and amidst the Aristotelian formulations the phrase τὰ γεωμετρούμενα εἴδη τῶν ὄντων sounds like late Platonism.[2] To delete the offending words gives τοὺς ἀριθμοὺς καὶ λόγους, which belong together and suit the context. In the earlier passage, too, there are several difficulties. The word ἔνισον is peculiar and unexampled,[3] and ὅτι destroys the continuity.[4] τὰ περὶ τὴν ὄψιν μαθήματα is not only unclear,[5] but factually wrong; it is not the fact that mathematics is mathematical or geometrical that moved the Pythagoreans so deeply, but that an everyday concern like music, impinging on us directly through the senses, turns out to conform to mathematical rules. Thus τὰ περὶ τὴν ὄψιν μαθήματα is not truly analogous to τὰ περὶ τὴν ἁρμονίαν. Again, to delete the reference to geometry not only makes ἔνισον superfluous, but gives a smoother and more meaningful sentence. Aristotle, in the *Metaphysics* (985b31), names the connection of music and number as one of the roots of Pythagorean speculation, but never brings this into relation with geometry.[6] Aristoxenus, too, in the one relevant passage remaining to us (fr. 23), speaks only of "concern with numbers," which Pythagoras especially "honored" and advanced. It seems likely, then, that Iamblichus, in a passage drawn from Aristotle, added the references to geometry, so as to emphasize the many-sidedness of Pythagorean μαθήματα, whereas in the original testimony geometry plays no role, in contrast to that of arithmetic.

[2] τὰ γεωμετρούμενα is a school term from late antiquity. See Plut. *Quaest. Plat.* 1001–1002, Procl. *In Eucl.* 211.4, Hero *Deff.* 1 p. 14.19, 136.51 p. 150.17, *Geom.* 2 p. 176.1 Heiberg.

[3] Vitelli suggests ἐπ' ἴσον. Perhaps ἐξ ἴσου?

[4] Festa deletes ὅτι.

[5] Aristotle classifies ὀπτικά with geometry (as ἁρμονικά with arithmetic: *An. post.* 75b16, *Met.* 997b20, *Met.* 1078a14, *An. post.* 78b37, *Phys.* 194a8ff). He speaks of "geometrical optics"; but he never mentions Pythagoreans in this connection.

[6] It has been shown above (ch. I 2, n. 75) that the doctrine of πέρατα was not Pythagorean.

The most important achievement that we know of that was made by a Pythagorean in the field of geometry is the famous solution, by Archytas, of the problem of doubling the cube.[7] Successful accomplishment of this task, talked of for at least a generation but never yet performed, stimulated further action. The improved solutions, from Eudoxus and Menaechmus to Eratosthenes, do not diminish but confirm the credit of the πρῶτος εὑρετής. But the mathematician who was Archytas' immediate predecessor in this regard was Hippocrates of Chios. It was he who had reduced the problem of doubling the cube—or finding $\sqrt[3]{2}$—to that of the "two mean proportionals," and thereby given it a planimetric sense which first made the solution possible.[8] Thus Archytas, in his decisive geometrical achievement, is not the representative of a special, Pythagorean tradition, but of the main tradition of Greek geometry in general, represented in his time by Hippocrates of Chios; and, if he was successful, it was probably *mathematica quadam facultate, non pythagorea.*

The real problem of Pythagorean geometry, therefore, in the history of thought and of mathematics is this: are we to assign the well-attested geometrical accomplishments of Pythagoreans to the time before or after Hippocrates of Chios? Only in the former case could we speak of the Pythagoreans as founders of Greek geometry; in the latter, it is only a matter of individual contributions to the development and perfection of something already there, quite respectable achievements but on an equal basis with others, and themselves dependent on older and more basic work. Only Heidel put the problem in this fashion;[9] "Pythagorean geometry" is usually treated, as though this were perfectly natural, before Hippocrates. It is assumed that he had before him at least the content of the first four books of Euclid, and that these are of Pythagorean origin.[10] This is in spite of the fact that the tradition names Hippocrates as the first author of στοιχεῖα.

Citing Eudemus, Proclus refers two passages in the first book of Euclid to the Pythagoreans—though not to Pythagoras. These are the

[7] Archytas A14 = Eudemus fr. 141 = Eutocius *In Archim.* III² 84 Heiberg. On the problem of duplication of the cube, see Heath, *Math.* I 244ff; Becker, *MD* 75ff. To be sure, Archytas' construction cannot be executed with ruler and compass.

[8] Hippocrates, DK 42.4 = ps.-Eratosth. *Epist. ad. Ptol.* ap. Eutocius *In Archim.* III² 88.17 Heiberg; also Procl. *In Eucl.* 213.7ff (not in DK).

[9] Heidel, *AJP* 1940.

[10] Tannery, *Géom.* 106ff (cf. above, ch. VI 1, n. 44); Rey 240 (despite his skepticism in principle, 230ff); van der Waerden, *SA* 135; with reserve, Heath, *Math.* I 166ff.

proof for the sum of the angles of a triangle[11] and the method of application of areas.[12] The same Eudemus may be the source of a few further reports about discoveries of the Pythagoreans—not of Pythagoras—in Proclus and the scholia to Euclid.[13] The whole fourth book of Euclid is Pythagorean,[14] as is the proposition that 6 equilateral triangles, or 4 squares, or 3 regular hexagons, will fill the plane about a given point;[15] and the three regular solids tetrahedron, cube, and dodecahedron[16] (the two others, octahedron and icosahedron, are ascribed to Theaetetus). A common origin for the tradition about all these propositions is suggested by their close interdependence. The content of the fourth book is the relationship of the regular polygon and the circle; the exhaustion of the plane by regular polygons presupposes the proposition about the sum of the angles of a triangle, and leads naturally to the construction of regular polyhedra; and the solution of quadratic equations by the application of areas is a presupposition of the construction of the regular pentagon as well as of further problems in the construction of polyhedra.

It is very likely that these data are incomplete. Proclus' detailed commentary on Euclid is extant only for the first book, as is true of Pappus' commentary on book 10 (in Arabic translation). The scholia, which of course draw from the commentaries, have a habit of omitting just those details which are useful for the history of mathematics; so that it may have happened that Eudemus' reports of Pythagorean discoveries in the other books of Euclid have been lost. On the other hand, the Pythagoreans, as predecessors of the "divine" Plato, were extremely important for Proclus, who regarded the word *μαθηματική* itself as Pythagorean in origin.[17] Therefore he would surely not have let slip any opportunity to mention Pythagorean achievements. This authorizes us to surmise that Eudemus, and the intermediary sources

[11] Procl. *In Eucl.* 379.2 = Eudemus fr. 136 = DK 58B21 (Eucl. 1.32).

[12] Procl. *In Eucl.* 419.15 = Eudemus fr. 137 = DK 58B20 (Eucl. 1.44); cf. Iam. *Comm. math. sc.* p. 75.21, Plutarch, above, ch. VI 2, n. 7.

[13] It can be shown that the scholia to the first book are mostly from Proclus (cf. e.g. above, ch. VI 2, n. 6), and those to the tenth book from Pappus (cf. e.g. below, n. 57). Both cite Eudemus.

[14] Schol. Eucl. 273.3: *εὕρημα δὲ τοῦτο τὸ βιβλίον τῶν Πυθαγορείων*, 273.13: *τὰ ὅλα δὲ θεωρήματα τοῦ προκειμένου βιβλίου ιζ'* (read *ις'*) *ὄντα Πυθαγορείων εὑρήματα*. This testimonium is missing in DK, and not mentioned by Heath (*Math.* I), van der Waerden (*SA*), or Becker.

[15] Procl. *In Eucl.* 304.11 (cf. Arist. *Cael.* 306b4, Eucl. 1.15, porism).

[16] Schol. Eucl. 654.3: *τρία δὲ τῶν προειρημένων ε' σχημάτων τῶν Πυθαγορείων ἐστίν, ὅ τε κύβος καὶ ἡ πυραμὶς καὶ τὸ δωδεκάεδρον, Θεαιτήτου δὲ τό τε ὀκτάεδρον καὶ τὸ εἰκοσάεδρον*. Cf. ch. I 3, n. 116; on *χροιά*, "plane surface," see ch. I 3, n. 96.

[17] Procl. *In Eucl.* 45.5ff; cf. above, ch. VI 1, n. 119.

for history of mathematics dependent on him,[18] did not have anything more to say about Pythagoreanism in the first book of Euclid. This in turn gives us certain clues to the evaluation of Pythagorean contributions in the Euclidean corpus: there are two Pythagorean theorems in the first book, and other propositions, proofs, and constructions are ascribed to Thales or Oenopides. The fourth book is Pythagorean, as the fifth is from Eudoxus.[19] What we are concerned with, then, is certain individual, restricted portions, to be ranked along with what other mathematicians "discovered"; there is no reason to posit a unique contribution by the Pythagoreans or a substratum of Pythagorean *Elements*. More than once, before Euclid, *Στοιχεῖα* had been written, and each time their structure and methods of proof had been thought through again.[20]

The chronology of these Pythagoreans is not guaranteed by any external testimony;[21] whether we go up to the early fifth century or down to the fourth, everything depends on combinations. Since Theaetetus, who was still very young in 399 B.C. and fell in battle in 369 B.C.,[22] brought to completion the treatment of the regular polyhedra, the "Pythagoreans" in question must be placed earlier. Eudemus names Theaetetus, Archytas, and Eudoxus, mentioning their individual accomplishments,[23] so that the anonymous Pythagoreans probably belong to the time before Archytas. The decisive question of their

[18] Proclus surely did not use Eudemus directly. Tannery (*Géom.* 18ff) concluded that Geminus was the intermediary.

[19] Schol. Eucl. 280.7ff: *τὸ δὲ βιβλίον Εὐδόξου τινὲς εὕρεσιν εἶναι λέγουσι . . .*

[20] The basic stock probably goes back to the *Στοιχεῖα* of Hippocrates of Chios, which were the first of their kind (Eudemus fr. 133 = Procl. *In Eucl.* 66.7: *πρῶτος γὰρ ὁ Ἱπποκράτης τῶν μνημονευομένων καὶ στοιχεῖα συνέγραψεν*). Von Fritz shows (*ABG* 1959, 72ff) that the first books of Euclid underwent a thorough revision after the time of Aristotle, perhaps at the hands of Euclid himself.

[21] Procl. *In Eucl.* 419.15 = Eudemus fr. 137: *ἔστι μὲν ἀρχαῖα, φασὶν οἱ περὶ τὸν Εὔδημον, καὶ τῆς τῶν Πυθαγορείων μούσης εὑρήματα ταῦτα*. This sounds as though Eudemus had been speaking of "ancient" Pythagoreans; but Proclus' next words, *ἀπὸ δὲ τούτων καὶ οἱ νεώτεροι . . .* refer to Apollonius of Perga. Thus the word *ἀρχαῖα* comes from Proclus (or Geminus), for whom the Pythagoreans are simply, as a group, "ancient" (cf. 419.21f, 420.22f).—When von Fritz writes, on Eudemus' fragment 136 (*AnnMath* 1945, 258), "Eudemus . . . attributes to the early Pythagoreans the proof . . ." the word "early" does not correspond to anything in the Greek text.—According to Aristoxenus the "last Pythagoreans" were living about 366/365 B.C. (above, ch. II 5, n. 43).

[22] Cf. Sachs 88ff; von Fritz, *RE* V A 1352f.

[23] Theaetetus, above, ch. VI 2, n. 82; Archytas: Eudemus frr. 60, 65, 141; Eudoxus: Eudemus frr. 133 and 148. The indefinite expression "Pythagoreans" could have the same explanation in Eudemus as for Aristotle (above, ch. I 4): where he cautiously uses this expression, the persons in question themselves referred to "Pythagoras." But he could also mean Archytas; cf. above, ch. I 2, n. 106, on Eudemus fr. 60. It is also conceivable that Theodorus of Cyrene is at least included in the expression (above, ch. VI 1,

relation to Hippocrates of Chios depends on inner criteria, on the relation of concepts, propositions, constructions which seem to be presupposed by the Pythagoreans or by Hippocrates. In a closed system like geometry, so many combinations of the same propositions are possible, and so many proofs for the same conclusions, that it is never possible to establish chronological priority with mathematical certainty.

The application of areas was known to Plato,[24] but Hippocrates of Chios, for a problem soluble by this method, used the method of "inclination" or "verging" (*νεῦσις*); it looks as though the application of areas was at least not fully developed in Hippocrates.[25] A special case of the application of areas is the construction of the golden section. The star-pentagon or "pentagram" and the dedecahedron play a role in Pythagoreanism, but both figures had been known, in a purely empirical way, for centuries,[26] without any mathematical construction. Euclid gives the construction of the golden section, and therefore of the regular pentagon, in book 4, which is ascribed "as a whole" to the Pythagoreans; and it depends on constructions in book 2. Hippocrates

n. 12). In spite of this, Frank's formulation is too sweeping: "What is ordinarily called Pythagorean mathematics is in essentials the work of Archytas, Theaetetus, and Eudoxus" (232). Theaetetus is never called a Pythagorean, and Eudoxus is treated in the tradition as an original thinker.

[24] On the idea and the method of the application of areas, see Heath, *Math.* I 150ff; Becker, *MD* 60ff; van der Waerden, *SA* 118ff. Euclid 2 has a generalized and therefore more complicated form of the application of areas.—*παρατείνειν*, Pl. *Rep.* 527a; a specific problem, Pl. *Meno* 86e (cf. Heath, *Math.* I 298ff; Becker, *Gnomon* 28 [1956] 225). *διορισμός*, to which Plato alludes here, is a method developed by Leon, one of his contemporaries (Eudemus fr. 133 = Procl. *In Eucl.* 66.22). Just as Plato learned from Eudoxus in the field of astronomy (above, ch. IV 2), so in mathematics he learned from his contemporaries, not from "ancient" Pythagoreans.

[25] Heller, *AbhBln* 1958, 9. comes to the conclusion that Hippocrates still did not know the method of application of areas. See also Becker, *MD* 59f.

[26] Above, ch. II 4, n. 81; below, n. 65. That the magical efficacy of the pentagram does not depend on mathematically exact construction is shown by the fact that they are often drawn quite irregular. See, e.g., the Gallic coins in W. Deonna, *Bull. de l'Ass. Pro Aventico* 16 (1954) 47; he also mentions (p. 24 n. 4) an empirical method of casting metallic dodecahedra, with no recourse to Euclidean geometry.—Heath conjectures that the "golden section" was a discovery of the Pythagoreans (*Math.* I 160f, *Eucl.* 2.97ff); cf. Becker, *MD* 63. Heller, *AbhBln* 1958, 9f, tries to reconstruct an earlier *νεῦσις* construction of the regular pentagon. In Eudemus fr. 133 = Procl. *In Eucl.* 67.6 we read, (*Εὔδοξος*) *τὰ περὶ τὴν τομὴν ἀρχὴν λαβόντα παρὰ Πλάτωνος εἰς πλῆθος προήγαγεν καὶ ταῖς ἀναλύσεσιν ἐπ' αὐτῶν χρησάμενος*. Sachs (97, 128f), Sarton (*Hist.* 442f), and Heller (*AbhBln* 1958, 12f) interpret this as referring to the golden section, which would imply that the report of the scholium on Eucl. 4 (above, n. 14) was false. But the formulation in Proclus is so general that this conclusion does not hold up. The topic may be the new, systematic development based on Eudoxus' new theory of proportion (as Heller thinks), or it could be the systematic development of Eucl. 2. In the first 11 of the 14 propositions of the second book the problem is the division (*τέμνειν*) of a line!

surely knew essential parts of the second book,[27] but not necessarily the content of the fourth. Plato seems to know the golden section.[28] The proposition about the angles of a triangle seems to be simple and fundamental; Aristotle knows it in the "Euclidean" form, differing in a small detail from the Pythagorean;[29] still, in what follows in Euclid nothing depends on it except the proposition about the occupation of a plane about a point by regular polygons (above, n. 15). It obviously became important in the time of Aristotle for the fundamental questions of geometry, which lead to the formulation of the Euclidean parallel postulate.[30] The relation of Eudemus' "Pythagoreans" to Hippocrates remains ambiguous.

There is no such ambiguity in the comparison with the achievements which Eudemus ascribes to Oenopides. These constructions are so elementary that Tannery concluded Pythagorean geometry must really have been a strictly guarded secret of the school, and therefore inaccessible to Oenopides.[31] It is stated that Oenopides stole from Pythagoras the determination of the ecliptic, which shows that he must have measured its angle. The usual approximative value is the arc of a regular fifteen-angled figure;[32] and it is with the construction of such a polygon that the fourth book of Euclid, whose origin is Pythagorean, closes. This presupposes Oenopides' result; Pythagorean achievements in geometry are later than Oenopides, and also hardly earlier than

[27] Namely Eucl. 2.12–13 (the extension of the "Pythagorean theorem" to scalene triangles) and 2.14 (the transformation of rectangle into square, construction of mean proportionals, and the geometrical construction of square roots)—the culminating propositions of the second book.

[28] The simplest interpretation of the mathematical allusion at *Hp. ma.* 303b relates it to the golden section (Heath, *Math.* I 304; more complicated: E. Strycker, *Mélanges Boisacq* [Paris, 1937] 317–326).

[29] Heath, *Aristotle* 23ff, on Arist. *Met.* 1051a24, *ἀνάγειν*. The necessary auxiliary line is drawn from the base of the triangle upward in Eucl. 1.32, in Eudemus fr. 136 through the apex of the triangle and parallel to the base.—Geminus (Eutocius on Apollonius, II 170.4 Heiberg) states that the proposition about the sum of the angles of a triangle was first proved separately for equilateral, isosceles, and scalene triangles, *πρότερον ἐν τῷ ἰσοπλεύρῳ παὶ πάλιν ἐν τῷ ἰσοσκελεῖ καὶ ὕστερον ἐν τῷ σκαληνῷ*. This may be spun out of a passage of Aristotle, which mentions this merely as a logical possibility: *οὐδ' ἄν τις δείξῃ καθ' ἕκαστον τὸ τρίγωνον ἀποδείξει ἢ μιᾷ ἢ ἑτέρᾳ ὅτι δύο ὀρθὰς ἔχει ἕκαστον, τὸ ἰσόπλευρον χωρὶς καὶ τὸ σκαληνὲς καὶ τὸ ἰσοσκελές, οὕπω οἶδε τὸ τρίγωνον ὅτι δύο ὀρθαῖς . . .* (*An. post.* 74a25ff). See Heath, *Math.* I 136, *Eucl.* I 319f, following Heiberg. But Becker has shown that the individual proofs can be grasped easily and clearly from considerations of symmetry (*Grdl.* 27, *MD* 39).

[30] See Heath, *Math.* I 339, 358, 375, *Eucl.* I 191; I. Tóth, "Das Parallelenproblem im Corpus Aristotelicum," *Arch. for Hist. of Exact Sciences* 3 (1966) 249–422. Archytas addressed himself to the problem of the infinity of space (Eudemus fr. 65).

[31] Tannery, *Géom.* 86; DK 41.12–14 (Eudemus fr. 138); above, ch. VI 1, nn. 128–130.

[32] I.e. 24° (correct value, 23° 27′ 3″).

Hippocrates of Chios.[33] They belong in the period, approximately, between 430 and 400 B.C.; it remains obscure what individuals are lurking beneath the general appellation of "Pythagorean" (above, n. 23).

The most important "discovery" was that of the application of areas. Scholars now agree that the point of these exercises is primarily algebraic; they provide an equivalent for quadratic equations. In Babylonian mathematics they had been solved algebraically, and the individual examples of the application of areas correspond exactly to the methods developed there. Thus the "geometrical algebra" of the Greeks is revealed as the transposition of Babylonian techniques of calculation into geometrical form.[34] The occasion for the development of this seemingly complicated "geometrical algebra" was the discovery of the irrational, which made impossible, according to Greek views, the solution of quadratic problems by use of number. In fact, the dicovery of the irrational is also ascribed to Pythagoreans, or even to Pythagoras himself; but the actual situation is extremely hard to grasp because of the profusion of ancient legend and allegory, and the modern conjectures they have inspired. The ancients speak of this situation in terms of "secrecy" and "treason," the moderns of the "Grundlagenkrisis der griechischen Mathematik."

Pythagorean lore was at least in part secret, as is attested by Aristotle and Aristoxenus, and as is natural in an archaic social order.[35] On the other hand, Pythagorean "secrecy" was undoubtedly misused in later times, as a carte blanche to permit the publication of forgeries as newly discovered books, and brand the discoveries of later thinkers as plagiarism of Pythagoras.[36] Names that occur in such context are Empedocles,[37] Philolaus,[38] Oenopides,[39] a certain musical theorist

[33] This was Heidel's conclusion, *AJP* 1940, 18.

[34] Fundamental is Neugebauer, *QSt* 3, 245ff; cf. *ExSc* 147ff; S. Gandz, *Osiris* 3 (1938) 460–470; van der Waerden, *MtAnn* 1947–49, 131f; *SA* 124f; *RE* XXIV 286–288; above, n. 24. The expression "geometrical algebra" has been current since Zeuthen. Van der Waerden calls the Pythagoreans the "transmitters of ancient Babylonian algebra to the Greek world" (*MtAnn* 1940–1941, 160; cf. *MtAnn* 1947–1949, 132). Neugebauer is hesitant: "My answer to this question cannot be proved by documentary evidence" (*ExSc* 147); so is G. Junge, *Osiris* 8 (1948) 316–321). A more direct kind of appropriation of Babylonian arithmetic is represented by the Pythagorean number series (above, ch. VI 2, esp. nn. 9 and 43).

[35] Above, ch. II 4, nn. 96 ff.

[36] Above, ch. III 1, nn. 28 ff. Burkert, *Philologus* 1961.

[37] Timaeus *FGrHist* 566F14 = D.L. 8.54 (λογοκλοπία); Neanthes *FGrHist* 84F26 = D.L. 8.55.

[38] Above, ch. III 1, n. 28.

[39] Above, ch. IV 1, n. 37.

called Simos,[40] and Plato.[41] The oldest authorities are Timaeus of Tauromenium and Duris of Samos, who both seem to show a local patriotic partiality to Pythagoras. It is significant that, in the case of Oenopides, Eudemus, the earlier witness, has nothing to say of plagiarism. A more famous and more important story is that of the mathematical treason of Hippasus, bringing with it the impressive anecdote that he was drowned at sea, as punishment; and this same story is bound up with the discovery of the irrational.

The discovery of the problem of the irrational in geometry, and the development of the ability to cope with it, is a fundamental accomplishment of Greek mathematics which holds a lasting fascination for modern historians of science. The tradition of secrecy, betrayal, and divine punishment provided the occasion for the reconstruction of a veritable melodrama in intellectual history. The realization that certain geometrical magnitudes are not expressible in terms of whole numbers is thought of as "une véritable scandale logique," bound to shake the very foundations of the Pythagorean doctrine, which maintained that "everything is number"; for to the Greeks, number and irrationality are mutually exclusive.[42] Thus one comes to speak of a *Grundlagenkrisis*—a crossroads or dilemma as to the very foundations of Greek and Pythagorean mathematics in the fifth century[43] —and to see in the tradition about the death of the "traitor" a reflection of the shock and despair that this discovery must have brought: "O that the irrational

[40] Duris *FGrHist* 76F23 = Por. *VP* 3 = DK 56.2: Pythagoras' son Arimnestus is said to have set up in the sanctuary of Hera (on Samos) a dedicatory monument with the epigram

Πυθαγόρεω φίλος υἱὸς Ἀρίμνηστός μ' ἀνέθηκε,
πολλὰς ἐξευρὼν εἰνὶ λόγοις σοφίας.

τοῦτο δ' ἀνελόντα Σῖμον τὸν ἁρμονικὸν καὶ τὸν κανόνα σφετερισάμενον ἐξενεγκεῖν ὡς ἴδιον. εἶναι μὲν οὖν ἑπτὰ τὰς ἀναγεγραμμένας σοφίας, διὰ δὲ τὴν μίαν, ἣν Σῖμος ὑφείλετο, συναφανισθῆναι καὶ τὰς ἄλλας τὰς ἐν τῷ ἀναθήματι γεγραμμένας. Diels (DK I 445 n.) thinks the *σοφίαι* referred to are the *μεσότητες*, but the seventh of them was not discovered till after Eratosthenes (above, ch. VI 2, n. 92). The word *κανών* suggests rather the calculation of a scale (as Wilamowitz, *Platon* II 94). There were therefore, aside from the Pythagorean musicologists, other *ἁρμονικοί* in competition with them.

[41] Above, ch. III 1, n. 28.

[42] Tannery, *MSc* I 268, *Géom.* 98, *HScH* 259; Arist. *Met.* 1021a5: *ὁ γὰρ ἀριθμὸς σύμμετρος.* In Diophantus the expression occurs that the number sought *γίνεται οὐ ῥητός* (e.g. 4.10); see esp. Hasse–Scholz 65f; below, n. 71.

[43] Hasse–Scholz; cf. Heath, *Math.* I, 155; Brunschvicg, *Étapes* 45ff, *Le rôle du pythagorisme dans l'évolution des idées* (Paris, 1937) 21ff (chapter heading: "Une découverte scandaleuse"); von Fritz, *AnnMath* 1945, 244f; Sarton, *Hist.* 283f; Becker, *MD* 13f, 71ff; Heller, *AbhBln* 1958, 11; Junge, *C&M* 1958, 53f (though at 67f, Junge rejects the idea of "Geheimhaltung aus schlechtem Gewissen," and maintains, correctly, that the Pythagoreans were apparently not upset by the matter).

had never been discovered!"[44] But had the painful fact of its existence really been held as a carefully guarded *arcanum imperii*?[45]

An important prop for this theory of a *Grundlagenkrisis*, and also a useful chronological point of reference, was the interpretation of the polemics of Zeno of Elea as relevant to the history of mathematics. This was inaugurated by Helmut Hasse and Heinrich Scholz, who argued that his critique was directed specifically against some "unclean" (*unsauber*) mathematics of infinitesimals, by means of which the Pythagoreans supposedly attempted to escape the consequences of irrationality.[46] If this is correct, the discovery of the irrational must have taken place before 460 B.C., which would fit in well with the conjectural dating of Hippasus.[47]

More recent interpretation of Eleatic philosophy does not confirm the attempt to place Zeno in the history of mathematics.[48] His arguments are sometimes given a specific mathematical purport[49] which is in no way suggested by his words, and which antiquity did not find in them, assigning him, as it did, to the ranks of the *φυσικοί* rather than the *μαθηματικοί*.[50] Even when one distills out the purely mathematical content of his arguments, and does find the concept of the infinitely small and of the infinite series, still these series converge in a rational value, not in irrational proportions.[51] Certain as it is that Zeno's arguments are relevant to mathematics and even contributed in a certain way to the development of Greek mathematics,[52] still, from the historical point of view ontology is prior to mathematics.[53]

[44] Junge, *C&M* 1958, 53; cf. Heath, Becker, von Fritz *AnnMath* 1945, 260.

[45] "Im schlimmsten Sinn unsaubere Methoden," Hasse–Scholz 10.

[46] Hasse–Scholz 8ff, and J. Zafiropulo, *L'École éléate* (Paris, 1950) 178ff. On the other hand, Mondolfo (*Inf.* 238ff) and Junge (*C&M* 1958, 54ff) think that the fact of irrationality was not concealed but openly admitted, and that this stimulated the development of Zeno's arguments.—Philolaus A26 aims to avoid the irrational, but this is not "unclean" mathematics; it is not mathematics at all.

[47] Von Fritz, *AnnMath* 1945, 245ff; cf. above, ch. II 5, nn. 69–76.

[48] G. E. L. Owen, "Zeno and the Mathematicians," *Proc. of the Arist. Soc.* 58 (1957–58) 199–222, calls Tannery's thesis "an obstructive myth" (212); also see G. Vlastos, *Encyc. of Philos.* VIII (New York, 1967) 376–377.

[49] *εἰ πολλά ἐστιν* (fr. 1, fr. 3) is supposed to mean "if it is legitimate to regard a line as an aggregate of infinitely many, infinitely small 'atomic' lines" (Hasse–Scholz 10); see above, ch. III 3.

[50] Stressed by Heidel, *AJP* 1940, 25 n. 54; Hasse and Scholz maintain (12) that Zeno belongs in the history of mathematics, rather than in that of Sophism.

[51] Emphasized by Becker, *Gnomon* 27 (1955) 267.

[52] Zeno, to some extent, obstructed the development of the calculus of infinitesimals among the Greeks; and Archimedes uses his *ἔφοδος*, which is equivalent to integration, only as a heuristic method, which must be followed by a rigorous proof (see, e.g., van der Waerden, *SA* 224f).

[53] Cf. above, ch. VI 1, n. 138; III 3, n. 54.

Furthermore, it is not attested in any ancient source that Hippasus discovered the irrational, or divulged this knowledge; side by side stand diverse accounts, mutually exclusive, of the secrecy and the publication of Pythagorean mathematics. According to the version of the Pythagorean *mathematici*, obviously recorded by Aristotle,[54] Hippasus was the first to "publish and construct" the "sphere of the twelve pentagons," that is, the dodecahedron; he was drowned at sea as a punishment for this offense, but got the reputation of being the discoverer. According to the contrary version of the *acusmatici*, Hippasus was clearly the man who, by his discovery, inaugurated a new trend in Pythagoreanism, different from the original. There is no direct mention, here, of the irrational, the "betrayal" of which appears in a somewhat different account. Plutarch, who is our oldest witness for this, speaks of the secrecy, and the prohibition of putting doctrines down in writing, in the Pythagorean group: "And when their treatment of the abstruse and mysterious processes of geometry had been divulged to a certain unworthy person, they said the gods threatened to punish such lawlessness and impiety with some signal and widespread calamity."[55] We cannot equate this episode with that of Hippasus; the latter was a Pythagorean, so that his initiation into the "difficult and secret procedures" was therefore not any kind of "divulgement," and his death was not a *κοινὸν κακόν*.

The next witness is Pappus:[56]

> This science (or knowledge) had its origin in the sect (or school) of Pythagoras, but underwent an important development at the hands of the Athenian, Theaetetus ... Indeed the sect (or school) of Pythagoras was so affected by its reverence for these things that a saying became current in it, namely, that he who first disclosed the knowledge of surds or irrationals and spread it abroad among the common herd perished by drowning; which is most probably a parable by which they sought to express their conviction that firstly, it is better to conceal (or veil) every surd, or irrational, or inconceivable in the universe, and, secondly, that the soul which by error or heedlessness discovers or reveals anything of this nature which is

[54] Cf. above, ch. II 5. Iam. *Comm. math. sc.* p. 77.18 = Iam. *VP* 88: *περὶ δ' Ἱππάσου λέγουσιν, ὡς ἦν μὲν τῶν Πυθαγορείων, διὰ δὲ τὸ ἐξενεγκεῖν καὶ γράψασθαι πρώτως σφαῖραν τὴν ἐκ τῶν δώδεκα πενταγώνων ἀπώλετο κατὰ θάλατταν ὡς ἀσεβήσας, δόξαν δὲ λάβοι ὡς εὑρών, εἶναι δὲ πάντα ἐκείνου τοῦ ἀνδρός ...*

[55] Plut. *Numa* 22, tr. Perrin.

[56] I 1, p. 63; I 2, p. 64 Junge–Thomson (cf. above, ch. VI 2, n. 82). Pappus is dated, approximately, 300 A.D.

in it or in this world, wanders (thereafter) hither and thither on the sea of non-identity (i.e. lacking all similarity of quality or accident), immersed in the stream of the coming-to-be and passing-away, where there is no standard of measurement. This was the consideration which Pythagoreans and the Athenian Stranger held to be an incentive to particular care and concern for these things. . .

The scholia to Euclid have preserved an abbreviated version of the Greek text,[57] and Iamblichus, too, knows the tradition Pappus is following. But he also has another, according to which the traitor was only symbolically killed—a tomb was erected with his name. Iamblichus sets the three versions side by side—the symbolic "death" of the betrayer of irrationality, the drowning of the man who revealed the dodecahedron, and the drowning of the one who divulged the fact of irrationality.[58] In addition, Iamblichus has a different story about how Pythagorean geometry became known, without any quarrel or catastrophe: permission was given an impoverished Pythagorean to earn a living by giving lessons in geometry. It is conjectured that this version was originally related to Hippocrates of Chios.[59]

[57] Schol. Eucl. 417.12ff: *τῶν γὰρ Πυθαγορείων λόγος τὸν πρῶτον τὴν περὶ τούτου θεωρίαν εἰς τοὐμφανὲς ἐξαγαγόντα ναυαγίῳ περιπεσεῖν, καὶ ἴσως ἠνίττοντο, ὅτι πᾶν τὸ ἄλογον ἐν τῷ παντὶ καὶ ἄλογον καὶ ἀνείδεον κρύπτεσθαι φιλεῖ, καὶ εἴ τις ἂν ψυχὴ ἐπιδράμοι τῷ τοιούτῳ εἴδει τῆς ζωῆς πρόχειρον καὶ φανερὸν τοῦτο ποιήσηται, εἰς τὸν τῆς γενέσεως ὑποφέρεται πόντον καὶ τοῖς ἀστάτοις ταύτης κλύζεται ῥεύμασιν.* The double *ἄλογον* of the manuscripts must be a corruption; but there were 3 elements in Pappus' sentence, too. Should we perhaps read *ἄπορον* as in Plutarch?—Probably we should read *ἐπιδραμοῦσα* for *ἐπιδράμοι*. Heiberg's conjecture was *εἴ τις . . . ψυχῇ ἐπιδραμὼν . . .*; but in Pappus the subject is "soul."—An interpolation in Elias (*In Arist. Cat.* 125.12; *CAG* XVIII 1) reads: *ὡς καί τις τῶν Πυθαγορείων ἐκδοὺς τὸ μονόβιβλον τὸ περὶ τῶν ἀλόγων γραμμάτων ναυαγίῳ περιέπεσεν, ὡς τὰ ἀπόρρητα ἐξορχησάμενος.* The specific detail about the *μονόβιβλον* probably stems from confusion with the ps.-Aristotelian *De lineis insecabilibus* (*Περὶ ἀτόμων γραμμῶν*).

[58] Iam. *VP* 246f, evidence of their *ἐχεῤῥημοσύνη: τὸν γοῦν πρῶτον ἐκφάναντα* (sic) *τὴν τῆς συμμετρίας καὶ ἀσυμμετρίας φύσιν τοῖς ἀναξίοις μετέχειν τῶν λόγων οὕτως φασὶν ἀποστυγηθῆναι, ὡς . . . καὶ τάφον αὐτοῦ κατασκευασθῆναι . . .* (247) *οἱ δέ φασι καὶ τὸ δαιμόνιον νεμεσῆσαι τοῖς ἐξώφορα τὰ Πυθαγόρου ποιησαμένοις· φθαρῆναι γὰρ ὡς ἀσεβήσαντα ἐν θαλάσσῃ τὸν δηλώσαντα τὴν τοῦ εἰκοσαγώνου σύστασιν·* (this is repetition of the report about Hippasus, Iam. *VP* 88; above, n. 54; cf. Sachs 83, following Rohde, Q 168; thus the specification *τοῦτο δ' ἦν δωδεκάεδρον, ἓν τῶν πέντε λεγομένων στερεῶν σχημάτων, εἰς σφαῖραν ἐντείνεσθαι* is the work of Iamblichus) *ἔνιοι δὲ τὸν περὶ τῆς ἀλογίας καὶ τῆς ἀσυμμετρίας ἐξειπόντα τοῦτο παθεῖν ἔλεξαν.*

[59] Iam. *VP* 89 = *Comm. math. sc.* p. 78.1ff: *λέγουσι δὲ οἱ Πυθαγόρειοι ἐξενηνέχθαι γεωμετρίαν οὕτως· ἀποβαλεῖν τινα τὴν οὐσίαν τῶν Πυθαγορείων, ὡς δὲ τοῦτ' ἠτύχησε, δοθῆναι αὐτῷ χρηματίσασθαι ἀπὸ γεωμετρίας.* This stands in the context of the material taken from Aristotle, but is obviously an interpolation made by some compiler (as is the following sentence; above, ch. VI 1, n. 43); the Aristotelian passage is concerned with the difference between *mathematici* and *acusmatici*, but here one hears simply of "Pythagoreans." To the report of Hippasus' *ἐξενγκεῖν* is added a rival report of the *ἐξενηνέχθαι* of geometry in general, which takes no note of Hippasus. The preceding context has another sentence

Can we reconcile these various versions, in a spirit of compromise, with the thesis that Hippasus discovered or made known the dodecahedron "as well as" ("und ebenso") the irrational;[60] or, as Kurt von Fritz ingeniously suggested, that Hippasus discovered the irrational in the case of the dodecahedron, that is to say, of the regular pentagon?[61] The fact of irrationality, of infinite reciprocal subtraction, is easily seen in the case of the "Golden Section"; but there is no hint, in the tradition, that this was the point of departure for the discovery of the irrational.[62] There are two branches of the story of the traitor: on one hand Hippasus, the dodecahedron, and drowning at sea, and on the other the discovery of irrationality but no name mentioned, and various reports about the nature of the penalty. The explanation of the offender's death as symbolical is found in the report of Nicomachus, who referred to the letter of Lysis;[63] and the motif of drowning, of which Plutarch makes no mention, may also be an accretion to the story of the betrayal of irrationality.

All the accounts involving some kind of mathematical secret and its

(*Comm. math. sc.* 77.24) with the key word *ἐκφέρειν*: *ἐπέδωκε δὲ τὰ μαθήματα, ἐπεὶ ἐξηνέχθησαν, δισσοὶ προάγοντε μάλιστα, Θεόδωρός τε ὁ Κυρηναῖος καὶ Ἱπποκράτης ὁ Χῖος.* The language has peculiarities (plural verb with neuter plural subject; *δισσοί* = *δύο*, an Ionic and poetic usage, in any case not Aristotelian; and an impossible participle in the dual). The content corresponds to Procl. *In Eucl.* 66.4 = Eudemus fr. 133, from whose source Iamblichus probably derived the interpolated passage. (This was noticed by Tannery, *MSc* VII 112, and von Fritz, *AnnMath* 1945, 245, though the latter wrongly attributes to Eudemus the preceding report about Hippasus, which is not closely connected with the sentence in question. See also Heller, *AbhBln* 1958, 7).—Aristotle reports that Hippocrates of Chios lost his property (DK 42.2); and this seems to be the source of this version (Sachs 120.1).—Tannery, *MSc* VII 115ff, *Géom.* 84f, *HScH* 124, takes the report seriously, and suggests that after the catastrophe of the Pythagorean rule, under the pressure of material need, the Pythagoreans published the supposed *Tradition suivant Pythagore*. (See above, ch. VI 1, n. 44.)

[60] Becker, *MD* 71.

[61] Von Fritz, *AnnMath* 1945; following him, Heller, *AbhBln* 1958, 9ff; Junge, *C&M* 1958, 42.

[62] To my knowledge, the connection of the regular pentagon with the irrational is never emphasized in the tradition. The paradigmatic example for the irrational is always the diagonal of the square (below, nn. 75, 81); and von Fritz, in an earlier article (*RE* V A 1813), assumed that the diagonal was "zweifellos" the point of departure for the discovery.

[63] Iam. *VP* 74f (from Nicomachus according to Rohde, Q 138; at Iam. *VP* 253 = Por. *VP* 58, Nicomachus cites the letter of Lysis) and Clem. Al. *Strom.* 5.57 (from Nicomachus?), citing the letter of Lysis, which was addressed to a certain Hipparchus (Hippasus D.L. 8.42; but Clement, Iamblichus, and the MSS of the collection of letters all have Hipparchus). The latter had the temerity to *δαμοσίᾳ φιλοσοφέν*, and is given the threat, if he does not mend his ways, *τέθνακάς μοι*. The complete text is in Hercher, *Epistologr. gr.* p. 601–603; Thesleff, *Texts* 111–114. See Burkert, *Philologus* 1961, 1ff, where it is conjectured that the letter was forged in the 2nd half of the 3rd century B.C., as an introduction to the *Hypomnemata*.

betrayal share the same inherent improbability, which was perceived by Tannery.[64] Fame and profit can only result from an invention if it is welcomed by an expectant public, so that a mathematical discovery is only of interest in mathematical circles—*mathematica mathematicis scribuntur*. Nobody who is not already schooled in mathematical logic is going to be deeply impressed by the fact of irrationality. But if mathematical logic was already in existence, the soil was prepared for further development; and in mathematics it is especially easy for different students to come to identical results. If mathematics had been an exclusive possession of the Pythagoreans, its betrayal would have been meaningless; but if the problems and methods of deductive mathematics were already present outside the Pythagorean circle—a fact not to be doubted—then this ἐκφέρειν could at most hasten a process that was already going on.

This dilemma, however, does not apply in the case of the dodecahedron. As in the case of musical discoveries (ch. V 1), the tradition about Hippasus, though surrounded by legend, makes sense. In the background of the mathematical problem of the dodecahedron there stands the dodecahedron as a cult object. Numerous dodecahedra made of bronze have been found in Gaul and thereabouts; and one made of stone has been found in northern Italy, dating back to prehistoric times.[65] Their significance and use is unclear; the best conjecture seems to be that they were a kind of dice, used for oracular or mantic purposes. In Plato's *Timaeus* the dodecahedron appears unexpectedly as the image of the whole (55c); it is widely supposed that the Pythagorean tradition was in his mind, the one that presupposed the Hippasus story and was not without relation to the Italo-Gallic region.[66] The dodecahedron may well have been important as a σύμβολον in the Pythagorean school, like the pentagram; Hippasus' offense was in analyzing the sacred object, publicly, by mathematical means. Whether he actually made a mathematical construction of it is

[64] Tannery, *Géom.* 82f. In spite of this, he accepted the tradition of secrecy, because of the supposed "primitiveness" of the mathematics of Oenopides (n. 31).

[65] F. Lindemann, "Zur Geschichte der Polyeder und der Zahlzeichen," *SBMü*, math.-ph. Kl., 26 (1897) 625–768, described 28 dodecahedra, including one of steatite from Monte Loffa, northern Italy, dated 9th to 6th century B.C. (cited by Sachs 83f). On the (more than 50) Celtic dodecahedra, see J. de Saint-Venant, *Dodécaèdres perlés en bronze creux ajouré de l'époque gallo-romaine* (Nevers, 1907); L. Saint-Michel, *Bull. de l'Ass. G. Budé* 3rd ser. 4 (1951) 92–116; and W. Deonna, *Bull. de l'Ass. Pro Aventico* 16 (1954) 19–89, who makes it seem probable that they were used in a "jeu divinatoire." Saint-Michel and Deonna ascribe the Celtic dodecahedra to Pythagorean influences, but the prehistoric example would rather indicate that here, again, Pythagoras was dependent on older material.

[66] Cf., e.g., Taylor, *Tim.* 377–378.

uncertain,[67] and it is all the more doubtful whether, in this process, he stumbled onto the problem of the irrational.

It is striking that in the accounts of the "betrayal of the secret of the irrational" the oldest, that of Plutarch, is the least precise, that Pappus speaks only of a legend which arose as a result of the discovery, and that all the reports stress the deep significance rather than the facts of the matter.[68] In Plutarch it is clear that the word ἄρρητος, set in quotation marks, as it were, by λεγόμεναι, is to be understood in a double sense. The "ineffable because irrational" is at the same time the "unspeakable because secret." Carefully guarded secret doctrines (ἄρρητα)—which are dangerous to the uninitiate—played an important part, in Plutarch's day, in all the mysteries and similar organizations.[69] Thus Pythagorean philosophy was, as Nicomachus puts it, ἄρρητος ἐν τοῖς στήθεσι διαφυλαχθεῖσα.[70] Not that it was arbitrarily "forbidden" —ἀπόρρητος—but it was "unsayable." The fascination of the ἄρρητον lies in the pretense to indicate the fundamental limitations of human expression, which are at the same time transcended by the initiate. It is no wonder that this fascination was felt in the realm of mathematics as well; Athenagoras reports of the Pythagorean Lysis the definition, ἀριθμὸν ἄρρητον ὁρίζεται τόν θεόν.[71] This exciting double sense of the word ἄρρητος is what makes the story of the discovery and betrayal of the irrational an *exemplum* for Plutarch, and even more for Pappus, who is probably following some Platonic source. When we see that a name is not mentioned, and that the details vary from one version to another, it is tempting to think that it was precisely the

[67] Sachs 82: "That the mathematical construction of the dodecahedron took place before that of the octahedron—when all the solids were known—is unthinkable." She translates γράψασθαι "sich zeichnen" (83), and supposes that the Pythagoreans knew the dodecahedron only in an empirical way. The expression δωδεκάσκυτοι σφαῖραι in Pl. *Phd.* 110b (cf. Plut. *Quaest. Plat.* 5.1.1003d), shows that (as today) balls for children were made in the shape of dodecahedra, of 12 pieces of leather; this makes it seem unlikely that the dodecahedron was first made known, outside of Pythagorean circles, by Hippasus.

[68] Pappus expressly cites the "Athenian stranger" from Pl. *Leg.* 819d, and in general his exposition is strongly influenced by Platonism.

[69] ἄρρητα ἱερά as early as Herodotus 5.83, 6.135, Aristoph. *Nub.* 302, Eur. *IT* 41, *Bacch.* 472, etc. On the ἄρρητον in Plato, see Friedländer, *Platon* I 63ff = I 59ff Eng. tr. From late antiquity, see for example Plut. *De Is. et Os.* 25: ὅσα τε μυστικοῖς ἱεροῖς περικαλυπτόμενα καὶ τελεταῖς ἄρρητα διασῴζεται; *De an. Procr.* 4.1013c (the argument that the world was created, and therefore not eternal): ὡς τι δεινὸν καὶ ἄρρητον οἰόμενοι δεῖν περικαλύπτειν καὶ ἀρνεῖσθαι, Hippol. *Ref.* 1 praef. 1 (on the Gnostics) ἀποκρύπτειν τὰ ἄρρητα ἑαυτῶν μυστήρια . . .

[70] Nicom. ap. Por. *VP* 57 = Iam. *VP* 252.

[71] Athenag. *Suppl.* 6 = DK 46.4 = p. 114.15 Thesleff. Of course this is as spurious as the other Pythagorean quotations in Athenagoras, but still apparently the earliest extant reference for the expression "irrational number" (see above, n. 42).

ambiguity of the word ἄρρητος which provided the germ of the tradition, or at least that this was an influential factor in its development. One had the story of Hippasus as told by Aristotle, the Lysis letter with its threat of symbolic death, and, in general, the careful secrecy of the Pythagorean group. Pythagorean lore was ἄρρητος like that of the mysteries,[72] and the penalty of betrayal was death. Pythagorean lore was mathematical in nature, and in geometry ἄρρητα play a role which is mysterious to a layman; how easy a step to equate the two senses of ἄρρητος and create the story of the secret of the irrational and its betrayal! The authority of Eudemus fails us twice, in Pappus and in Proclus;[73] but it may underlie the scholia on Euclid.[74] Thus it is possible that he mentioned the fact of the Pythagoreans' discovery, giving the cue for the development of the legend. But the involvement of Hippasus is scarcely better attested than that of Zeno, so that another apparent chronological foothold is lost.

The deep significance of the discovery, so dramatically expressed in the catchword *Grundlagenkrisis*, is not attested in the sources. "Nowhere in the many passages about the irrational in Plato and Aristotle can we detect any reference to a scandal, though it would surely still have been known in their day."[75] In addition, the inherent connection of the problem of the irrational with Pythagorean speculation and philosophy, which some have supposed they saw, is doubtful. The so-called Pythagorean theorem, when applied to a square, would necessarily lead to the irrational $\sqrt{2}$; but Babylonians, Indians, and Chinese knew the theorem without knowing about the irrational. Side numbers

[72] The comparison with the Eleusinian mysteries is drawn in the letter of Lysis, p. 112.3ff Thesleff = Hercher, *Epistolog. gr.* p. 602.4ff = Iam. *VP* 75.

[73] Pappus cites Eudemus for the achievement of Theaetetus (above, ch. VI 2, n. 82), but not in connection with the Pythagoreans (above, n. 56). After the introductory sentence on Pythagoras follows a citation of Plato (*Tht.* 143e); then Apollonius of Perga is mentioned, and Eudemus makes his appearance only after a second Platonic citation (*Tht.* 147c), so that the entire passage cannot be ascribed to him. See above, n. 68.

[74] Schol. Eucl. pp. 415. 7–416.13, characteristically speaking of Πυθαγόρειοι; cf. the introductory sentence of Pappus (above, n. 56): "This science had its origin in the sect of Pythagoras, but underwent an important development at the hands of the Athenian Theaetetus." An expedient of practical geometry was to speak of two different "measures," e.g. of the side and the diagonal of the square, Schol. Eucl. p. 416. 8–10, Arist. *Met.* 1053a14ff (misunderstood by Ross, *Met.* II 283; not explained by Heath, *Aristotle* 218f), Pl. *Parm.* 140b–c.

[75] Reidemeister 30. Van der Waerden had opposed the theory of a *Grundlagenkrisis* (*MtAnn* 1940–1941, 155): "not a philosophical problem, but one that arose within the development of mathematics itself". See also von Fritz, *ABG* 1955, 84f. The expression ἄλογοι ὥσπερ γραμμαί seems almost proverbial as early as Plato's *Republic* (534d). It is an important μάθημα, *Leg.* 819d; cf. Arist. *Met.* 983a19ff: οὐθὲν γὰρ ἂν οὕτως θαυμάσειεν ἀνὴρ γεωμετρικὸς ὡς εἰ γένοιτο ἡ διάμετρος μετρητή.

and diagonal numbers belong in the same context but do they represent a way of understanding or of avoiding the irrational?[76] The arithmetical proof of the irrationality of $\sqrt{2}$ is regarded as early, but it does not emerge naturally as a consequence of the Pythagorean pebble games.[77] Music theory advances as far as the problems of the irrational, but stops there and declares them nonexistent.[78] The irrational belongs to the domain, not of arithmetic, but of geometry.

For the Pythagoreans who were concerned with the number theory described by Aristotle, and for the cosmology summarized in the phrase "everything is number," the irrational has obviously no importance. Eurytus, who cannot be dated earlier than about a generation before Archytas,[79] tried to determine "the number" of specific objects, oblivious that the discovery of the irrational had long before cut the ground from beneath the Pythagorean theory of numbers. What is more, Aristotle, though he knew about it, did not use the fact of irrationality as an argument against the Pythagorean doctrines he criticizes, while the book *On Indivisible Lines* does use irrationality against the "atomic lines" of Xenocrates.[80] Clearly Pythagorean number theory and deductive mathematics lie on two different planes; "all things are number" never means "all magnitudes are commensurable." In Pythagorean number theory the relations of existing things are interpreted, and the "nonexistent" is left out of account.

In attempting to date the discovery of the irrational, we have the known fact that Theodorus of Cyrene proved the irrationality of square roots in the cases between $\sqrt{3}$ and $\sqrt{17}$, so that the proof for $\sqrt{2}$ was known before his time.[81] Democritus seems to have been

[76] Above, ch. VI 2, nn. 14–18.

[77] Above, ch. VI 2, n. 54.

[78] Above, ch. V 1, n. 4.

[79] DK 45. How far these Pythagoreans were from being true mathematicians is shown by the fact that their "units," according to Aristotle, "had magnitude," while even Protagoras presupposes a definition of the point, among geometers, as without extension.

[80] Ps.-Arist. *Lin. ins.* 969b33ff. Junge pointed to Eurytus, in this connection, as early as 1907 (*Symb. Joach.* 230ff); cf. *C&M* 1958, 57ff.

[81] Pl. *Tht.* 147d. As to the method of proof Theodorus may have used, there has been a long discussion, not yet concluded, among modern scholars. Whereas Vogt (*Bibl. Math.* 1909–1910, 101ff) and Junge (*C&M* 1958, 42ff) reconstructed a geometrical proof making use of reciprocal subtraction, and van der Waerden (*SA* 142ff) advocated an algebraic-mathematical method; von Fritz (*RE* V A 1815ff), Cherniss (*Rev. of Metaph.* 4 [1951] 411f), and A. Wasserstein ("Theaetetus and the History of the Theory of Numbers," *CQ* 52 [1958] 165–179) favored an apagogic method analogous to the numerical proof of the irrationality of $\sqrt{2}$ (above, ch. VI 2, n. 47). The irrationality of $\sqrt{2}$ can very easily be proved for the square by the method of reciprocal subtraction (van der Waerden, *MtAnn* 1947–49, 689f, *SA* 127; Becker, *MD* 73f). The resulting διάγραμμα is identical to that of the "side" and "diagonal" numbers, above, ch. VI 2, n. 16.

concerned with the problem;[82] but how far back into the fifth century we should date the discovery is an open question. If we try to determine the position of Hippocrates of Chios, the results are as ambivalent as those relating to the problem of the application of areas (above, n. 25). He obviously has no trouble constructing a line with a length equal to $\sqrt{\frac{3}{2}}$,[83] and $\sqrt[3]{2}$ is also a geometrical problem for him. One finds the square root by constructing the mean proportional; "geometrical algebra" is on the way. But, on the other hand, Hippocrates uses a definition of the equality of ratios which is not applicable to irrational magnitudes.[84] The problem in the theory of proportion which this raises was solved only by Eudoxus;[85] and a passage in the *Academicorum index* seems to allude to a forward step between Hippocrates and Eudoxus: in Plato's time, it says, (*ἦλθεν εἰς κορυφὴν*) . . . *τὰ περὶ [τοὺς ἀρι]θμοὺς προβλήματα, τῶν περὶ Ε[ὔδο]ξον μετάστησάντων τὸν ἀ[φ' Ἵπ]πο[κρά]τους ἀρχαιομόν*.[86]

[82] He has a title (B11p) *περὶ ἀλόγων γραμμῶν καὶ ναστῶν α′ β′ (ναστόν = πυκνόν, ὄν)*. Vogt (*Bibl. Math.* 1909–10, 144ff) contests the reference of this to the problem of the irrational, since he wishes to show that the word *ἄλογος* in the sense "irrational" is post-Platonic, overlooking Pl. *Rep.* 534d. One can only guess how Democritus solved the problem: for a spatial atomism are S. Luria (*QSt* 2 [1932] 106–185) and Heath, *Aristotle* 79f; for the distinction of geometrical space from the atoms (following Simpl. *Phys.* 82.1, Schol. Arist. 469b14) are R. Philippson, *Hermes* 64 (1929) 177ff, and Michel 675 n. 1.

[83] *γραμμὴ ἡμιολία δυνάμει*, Eudemus fr. 140.

[84] Hippocrates takes his departure from the proposition that similar segments are related as the squares of their bases. The proof is that circles are related as the squares of their diameters: *ὡς γὰρ οἱ κύκλοι πρὸς ἀλλήλους ἔχουσιν, οὕτως καὶ τὰ ὅμοια τμήματα· ὅμοια γὰρ τμήματά ἐστι τὰ τὸ αὐτὸ μέρος ὄντα τοῦ κύκλου* (Simpl. *Phys.* 61.11ff; cf. Eucl. 7 def. 21; the exact proof was first offered by Eudoxus; for a conjecture about Hippocrates' proof, see Becker, *Gnomon* 27 [1955] 267 n. 4). This sentence is regarded by Tannery (*MSc* I 399ff), Diels (ed. of Simpl. in *CAG*), and Becker (*QSt* 3, 411–419) as an addition of Simplicius, and Wehrli omits it. The ascription to Eudemus was defended by Rudio (*Bibl. Math.* 3 [1902] 1–62—where he weakens his argument with the claim that *τμῆμα* first means sector, and then in the following sentence segment; refuted by Tannery, *MSc* III 119–130 = *Bibl. Math.* 3 [1902] 342–349). See also the discussion of Heath, *Math.* I 187ff. The citation from Euclid inserted by Simplicius ends before *ὡς γὰρ* . . . The word *ἐδείκνυεν* (p. 61.8) is followed by the indication of the course of proof, which is followed, as a consequence (*διὸ καὶ*, p. 61.14; Wehrli and Diels have the citation of Eudemus take up with *καὶ*, without *διό*), by the sentence about the angles in similar segments. Therefore, the proposition using the definition of proportionality, which is valid only for numbers, not for geometrical magnitudes (Euclid 7 instead of the general theory of proportion in book 5) may be regarded as evidence from Eudemus about Hippocrates. After Zeuthen, van der Waerden called attention to the passage, *MtAnn* 1940–1941, 157f; cf. von Fritz, *ABG* 1959, 60ff, 69f.

[85] See Becker, *MD* 102ff; van der Waerden, *SA* 187ff; Heath, *Math.* I 32ff. A "pre-Eudoxan" theory of proportion, which was applicable to irrational magnitudes, and which operates with reciprocal subtraction, was reconstructed by Becker (*QSt* 2 [1932] 311–333; cf. *Grdl.* 79ff, *MD* 103f; von Fritz, *ABG* 1955, 92; van der Waerden, *MtAnn* 1947–1948, 688f, *SA* 177ff; Heath is skeptical, *Aristotle* 80ff).

[86] Page 16 Mekler. A perfectly certain restoration is hardly possible.

The conquest of the irrational in Greek mathematics, and along with it the development of "geometrical algebra," clearly was a gradual process. There were beginnings before Hippocrates of Chios, but the decisive breakthrough was later. The Pythagoreans to whom Eudemus attributed the application of areas and perhaps also the "discovery" of the irrational may well belong to this later period. Whether it was the same mathematician who first grasped the idea of the *ἀσύμμετρον* and "demonstrated" it by alternate subtraction of side and diagonal of a square, who exploited the superiority of geometry to algebra in the development of the application of areas, and who perceived the difficulty that the *ἄλογοι γραμμαί* do not fit into the calculation (ratio, *λόγος*)—or whether several men worked, one after the other, on these problems—we can no more answer these questions than we can give their names. But in any case, the beginnings of the development that is brought to a climax by Eudoxus are not far from Theodorus of Cyrene.[87] This strictly mathematical development had little relation to the cosmological number theory of men like Philolaus and Eurytus. Pythagoreans made significant contributions to the development of Greek geometry, but the thesis of the Pythagorean foundation of Greek geometry cannot stand, any more than the legend of a great mathematics held secret.

4. NUMBER AND COSMOS

Aristotle represents the Pythagoreans' number theory as arising from their study of exact mathematics,[1] and this relationship may well have seemed obvious, or even necessary. To be sure, there is no evidence, and no way of showing by philological means, that the Pythagoreans

[87] Frank concludes from a passage in Plato's *Laws* (819d) that in his time the problem was quite new, so that before 400 it was "completely unknown among those most keenly interested in scientific matters" (228): it was brought to Athens by Theodorus in 399 B.C. But since at least half a century elapsed between 400 and the composition of the *Laws*, the word *ὀψέ* in 819d cannot be pressed (cf. von Fritz, *AnnMath* 1945, 243 n. 6). Vogt, *Bibl. Math.* 1909–1910, dated the decisive discovery about 410, van der Waerden (*MtAnn* 1947–1949, 153) before 420, and von Fritz, thinking of Hippasus, about 450 (*ABG* 1955, 84 n. 159). A simple point of departure for the discovery was provided by the traditional (partly Babylonian) approximative values for the diagonal of a square. When the Greeks endeavored to find an exact result to replace the approximations, they discovered the fact of incommensurability, in the same way that they recognized the squaring of the circle as a problem.

[1] Above, ch. VI 1, n. 68.

were the originators of mathematics as a deductive science; and it is impossible to discern any relationship between the doctrine of transmigration, which is reliably attested for them, and the science of mathematics. Number does, however, dominate the Pythagoreans' general view of the world. In the relations among numbers they found the essence of musical harmony, and they knew or discovered a number of propositions that are mathematically significant. Is not this sedulous preoccupation with number precisely what we mean by mathematics? Do we not see here, in epitome, the origin and basis of natural science? To this question we must answer no.

Number and mathematical science are by no means equivalent. Numbers go back in origin to the mists of prehistoric times, but mathematical science, properly speaking, did not emerge earlier than sixth- and fifth-century Greece. People knew numbers before mathematics in the strict sense; and it was in the pre-scientific era that the "number mysticism" arose, or "number symbolism" or "numerology," which continues even now to exert a certain influence.[2] No one could overlook the fact that this kind of thing was present in Pythagoreanism; Aristotle names first of all, among the *ὁμοιώματα* which the Pythagoreans thought subsisted between numbers and things, the equation of certain numbers with *δικαιοσύνη*, *ψυχὴ καὶ νοῦς*, and *καιρός* (*Met.* 985b27ff), and only with a "furthermore" goes on to add the mathematical theory of music. It is not necessary to attempt here a complete presentation of Pythagorean number speculation; in particular, the very luxuriant tradition from later antiquity can be left aside,[3] since the demonstrably ancient material, attested by Aristotle, is sufficient to establish the basic facts.

One is *νοῦς* and *οὐσία*;[4] two is *δόξα*;[5] three is the number of the whole—beginning, middle, and end;[6] four is justice—equal times equal—but it is also, in the form of the tetractys, the "whole nature of numbers";[7] five is marriage, as the first combination of odd and even, male and female;[8] seven is opportunity (*καιρός*) and also Athena, as the "virginal" prime number;[9] ten is the perfect number, which

[2] In English the hybrid "numerology" has become standard, though the term "arithmology," introduced by Delatte, would be preferable on philological grounds. There is no question of "mysticism" in the proper sense, involving union of the inner and outer world; and the "symbolism" referred to is not an arbitrary assignment of "signs," but the discovery of apparently natural correspondences and interrelations (*φύσει καὶ οὐ νόμῳ*, Philolaus fr. 9).—There is no comprehensive exposition of ancient number symbolism, like that provided by Dornseiff for letter mysticism (*Das Alphabet in Mystik und Magie*, 2nd ed., Leipzig, 1925). On particular topics, see E. Wölfflin, "Zur Zahlensymbolik," *ALL* 9 (1896) 333–351; H. Usener, "Dreiheit," *RhM* 58 (1903) 1–47, 161–208, 321–362 (cf. *RAC* s.vv. Drei, Dreieck). An exhaustive collection of material is W. H. Roscher, *Die enneadischen und hebdomadischen Fristen und Wochen der ältesten Griechen* (*AbhLpz* 21.4, 1903); *Die Sieben- und Neunzahl im Kultus und Mythus der Griechen* (*AbhLpz* 24.1 1904); *Die Hebdomadenlehren der griechischen Philosophen und Ärzte* (*AbhLpz* 24.6, 1906); *Enneadische Studien* (*AbhLpz* 26.1, 1907); *Die Tesserakonten und Tesserakontenlehren der Griechen und anderer Völker* (*BerLpz* 61 [1909] 17ff); *Die Zahl 50 in Mythus, Kultus, Epos und Taktik der Hellenen und anderer Völker* (*AbhLpz* 33.5, 1919); O. Weinreich, *Triskaidekadische Studien* (Giessen, 1916) 95ff; Germain passim. V. F. Hopper, *Medieval Number Symbolism* (New York, 1938, with a good summary of the ancient phenomena).

[3] For this tradition see, along with Zeller I 495–507, the works of Robbins and de Falco (above, ch. I 3, n. 22). The *Theologumena arithmeticae* gives the most detailed summary, and de Falco's apparatus provides a convenient key to the parallel accounts. Posidonius was involved in the transmission of Pythagorean number doctrine (Theo Sm. 103.16ff, Sext. Emp. *Math.* 7.93ff; above, ch. I 3, nn. 8ff), but the crucial attestations are earlier. Speusippus' book *Περὶ Πυθαγορικῶν ἀριθμῶν* (fr. 4) was doubtless very important; but there seems to have been an indeterminable number of developments and revisions, many in pseudepigrapha (Prorus *Περὶ τῆς ἑβδομάδος*, *Th. ar.* 57.15, Megillus *Περὶ ἀριθμῶν*, *Th. ar.* 34.21; cf. Jerome *Epist.* 49.19). The origin of Pythagorean mathematics in number mysticism is emphasized by Joel, *Geschichte der antiken Philosophie* I (Tübingen, 1921) 358ff; Rey 395; Schuhl, *Essai* 258ff; Junge, *DtMath* 342f; van der Waerden, *SA* 93ff; Zeller (495ff), but cf. 453 n. 3; he treats the symbolism as "development and application" of the number theory, and Burnet refuses to see it as the starting point, regarding the musical discovery as primary (*EGP* 107f).

[4] The data given in Aristotle's treatises are supplemented by Alexander from Aristotle's lost book on the Pythagoreans. At *Met.* 985b30 he speaks of *ψυχὴ καὶ νοῦς*, upon which Alexander comments (39.13), *νοῦν δὲ καὶ οὐσίαν ἔλεγον τὸ ἕν· τὴν γάρ ψυχὴν ὡς τὸν νοῦν εἶπε* (sc. Aristotle). That is to say, Alexander corrects Aristotle; by *ψυχὴ καὶ νοῦς*, he means precisely *νοῦς* (cf. Pl. *Crat.* 400a: *νοῦν καὶ ψυχήν*, referring to Anaxagoras' *Noûs*). This word was all he found attested; at the same time he supplemented Aristotle's testimony with the word *οὐσία*; he is not paraphrasing, but commenting by the citation of further source material—obviously Aristotle's lost book on the Pythagoreans (*ἐδήλωσε* [sc. Aristotle], Alex. *Met.* 38.8).

[5] Arist. *Met.* 990a23, Alex. *Met.* 39.16; *τόλμα*, Alex. *Met.* 74.12 (on Arist. *Met.* 990a22; cf. Plut. *De Is. et Os.* 75, *Th. ar.* 9.6, Plot. 5.1.1, Procl. *In Alc. I* 104e). But here, Alexander's word *φασίν* apparently does not refer to Aristotle (75.27). For *ἀνικία* see Arist. *Met.* 990a24, and cf. *Th. ar.* 34.11, 35.1.

[6] Arist. *Cael.* 268a11. The Orphic verse, according to which Zeus is beginning, middle, and end (*Orph. frag.* 21), is alluded to in the Derveni papyrus, and then in Plato (*Leg.* 715e). The proclamation of the god is prior to the general Pythagorean formulation (cf. also Pl. *Parm.* 145a), showing that in this respect Pythagoreanism is dependent on purely religious, or "Orphic," sources. Tannery (*MSc* II 184ff, *HScH* 390f) and Roscher (*AbhLpz* 1906, 18ff) supposed that there was an Orphic number symbolism earlier than the Pythagorean; but the evidence cited is late (see Delatte, *Litt.* 208ff, on the supposed *Hymn to Number*). Whether the use of number as an effective force went further in Orphism than in other ritual, is quite uncertain.

[7] Arist. *Met.* 985b29, 1078b23, *EN* 1132b23, *MM* 1182a11, Alex. *Met.* 38.8ff; see above, ch. I 3, n. 120, ch. II 4, nn. 154ff.

[8] Arist. *Met.* 1078b23, Alex. *Met.* 39.8; cf. above, ch. I 2, n. 31. The interpretation of 6 as marriage (= 2 × 3) is later (*Th. ar.* 43.5ff, with the parallels cited).

[9] Arist. *Met.* 985b30, 990a23, 1078b22, Philolaus fr. 20 (above, ch. III 2, n. 52). According to Alex. *Met.* 39.3ff, the number 7 is Athena, because among the first 10 numbers "it neither begets another nor is begotten by any." On the connection of gods and numbers, see Xenocrates fr. 15, Plut. *De Is. et Os.* 10, 75, Stob. 1 prooem. 10, *Th. ar.* passim.

comprehends the whole nature of number and determines the structure of the cosmos, and with it ends the symbolic interpretation of numbers.[10]

Numbers are alternately even and odd, and this antithesis is the same as that in the cosmos between "limiting" and "unlimited"; odd numbers correspond to the more highly valued principle, that of "limit," and are also male. One is an exceptional case, being simultaneously even and odd, female and male. Geometrical figures, too, have specific functions and their "character." Philolaus assigns masculine divinities to the triangle, and feminine to the square. It is probably correct that the pentagram was a symbol of recognition among the Pythagoreans, and the dodecahedron seems also to have played some such role.[11]

Of course there was nothing like a complete system; there were overlappings and contradictions of all kinds—*ἀνάγκη πολλὰ συμβαίνειν τὰ αὐτά*, as Aristotle says.[12] One might be tempted to ignore all this, as a side growth on the main stem of Pythagorean wisdom; but more careful examination reveals that this kind of lore is to be recognized not as a branch but as a root, and one which goes very deep.

In primitive cultures, numbers are not abstract concepts with a content which is mathematical and quantitative, but mysterious beings: "each number has its own individual physiognomy, a kind of mystic atmosphere, a 'field of action' peculiar to itself."[13] The comparative material offered by anthropology need not be considered here except

[10] Arist. *Met.* 986a8, *Probl.* 910b31. The later compendia only treat the numbers up to 10, with the exception of Aristid. Quint. 3 p. 121ff M.

[11] Above, ch. I 2, nn. 24–32; the astrological derivation at Sext. Emp. *Math.* 5.7f is later.—Above, ch. IV 3, n. 60; II 4, n. 81; VI 3, n. 65.

[12] Arist. *Met.* 1093a1. See also Zeller I 501.2. As an example, should we regard the "perfect" number 10, which climaxes the series, as even, and therefore unlimited?—The ironic collection of groups of 7 at Arist. *Met.* 1093a13ff is regarded by Roscher (*AbhLpz* 1906, 25ff) as ancient Pythagorean doctrine, which Aristotle took from a written Pythagorean source. But the "Seven against Thebes" sounds like plain mockery, and the "seven vowels" cannot be early Pythagorean, at least in Magna Graecia (*o* for *ω*, *IG* XIV 636, 664, 647, from Metapontum; *ε* for *η*, *IG* XIV 630, 631, 643, 652; *H* = h *IG* XIV 643; but *H* = *η*, *IG* XIV 648, 4th century B.C.). Roscher is too hasty in accepting as "doctrine" what may be no more than jesting.

[13] L. Lévy-Bruhl, *Les fonctions mentales dans les sociétés inférieures* (Paris, 1951[9]), 236, as tr. by L. Clare, *How Natives Think* (London, 1926) 206. The section 235–257 = 225–223 is entitled "Puissance mystique du nombre" ("Primitives' Numeration"). See also Ebert, *Reallex. d. Vorgesch.* XIV 475ff; Rühle in Kittel, *Theol. Wörterb. des NT* (and Eng. tr.), s.v. *ἀριθμός*, with references; A. B. Keith in Hastings, *ERE* IX (1917) 407ff; A. Seidenberg, "The Ritual Origin of Counting," *Archive for Hist. of Exact Sciences* 2 (1962) 1–40.

insofar as we find phenomena similar to the Pythagorean. Not that we start by assuming connections and influences, but that the peculiarities of certain psychological phenomena, more likely to be obscured by concepts of Greek science and mathematics, may be clarified in this manner.

In certain Negro peoples the king of the gods is invoked thrice; the queen four times; the amulet worn by a man has 3 knots, that of a woman 4. The birth of a boy is celebrated after 3 days, that of a girl after 4; a dead son is mourned for 33 days, a daughter 44. If we add that the number 5 plays a part in the marriage ritual, we have 3 as the male number, 4 as the female, and 5 as the number of marriage—among the Negroes of the Sudan![14] This sounds so Pythagorean that one is almost tempted to think of some kind of direct connection. In fact, this possibility cannot be excluded entirely;[15] but the same situation—3 as the man's number, 4 as the woman's—is found in the ritual of Alaskan Eskimos.[16] At other points on the globe's surface, 4 belongs to the man and 3 to the woman; and the numbers 9 and 7 are found for man and woman, as well as 3 and 5, 4 and 5, and 6 and 5. The range of the evidence extends from the South Seas, over Africa, to the American Indians. There are traces of a similar use of 3, 4, and 5 among the Germans;[17] and in Rome a girl was named 8 days, a boy 9 days after birth,[18] while 5 torches were used in the wedding ceremony.[19] Above all, we find that with very few exceptions festivals of the gods were set on odd days of the month.[20]

[14] E. Fettweis, "Ueber das Verhältnis des mathematischen Denkens zum mystischen Denken auf niederen Kulturstufen," *Archeion* 14 (1932) 207–220; "Berührungspunkte der pythagoräischen [sic] Zahlenlehre mit dem Totemismus," *Zeitschr. f. philos. Forsch.* 5 (1950–1951) 179–196.—The Arabs of contemporary Egypt believe that a grave with three cover slabs is that of a man, one with four that of a woman (Fettweis, p. 188).

[15] Fettweis, *Zeitschr. f. philos. Forsch.* 1950–1951, 179; *Archeion* 1932, 210f, referring to Plut. *De Is. et Os.* 56 (3 is Osiris, 4 Isis, 5 Horus). Plutarch combines Egyptian and Greek material; there was certainly some Hellenistic Greek influence in the Sudan.

[16] Fettweis, *Zeitschr. f. philos. Forsch.* 1950–1951, 190.

[17] H. von der Au, "Drei lärren Strömp: Zur Deutung eines Vogelsberger Frauentanzes," *Hess. Bl. f. Volkskunde* 35 (1936) 73ff, on the number 5 and fertility magic; p. 83 on the number 3. See also J. Grimm, *Deutsche Rechtsaltertümer* I[4] (1899) 285ff.

[18] Plut. *Quaest. Rom.* 102, Macrob. *Sat.* 1.16.36, Festus p. 120 M. Plutarch naturally interprets this as Pythagorean; in the 1st century B.C., Castor of Rhodes connected Roman and Pythagorean (*FGrHist* 250F16).

[19] Plut. *Quaest. Rom.* 2. The use of the word *cerio* (*κηρίων*) shows that Greek influence was present.

[20] Exceptions: the Regifugium, the Equirria, and the procession to the Argei (see G. Wissowa, *Religion und Kultus der Römer* [Munich, 1912[2]] 436f, Censorinus 20.4, Festus p. 109 M.). T. Mommsen sees Pythagorean influence (*Die römische Chronologie* [Berlin, 1859[2]] 15.12), as does F. Bömer, *Ovid, Fasten* I (Heidelberg, 1957) 35. W. Aly wishes to use this to date the establishment of the calendar of festivals (*ARW* 33 [1936] 59); and

When Babylonian seers name the 13th, 15th, and 17th days as favorable for setting a ridgepole,[21] they are differentiating the odd numbers, as favorable, from the even. In East Prussia odd days of the month used to be regarded as lucky, in Pomerania the even.[22] In ancient folklore, a remedy for headache was made by crushing an odd number of berries;[23] it was recommended that a hen be set to hatch an odd number of eggs,[24] and that a herd of animals consist of an odd number.[25] Here Pythagorean influence cannot be ruled out,[26] but the basic idea is older, and the practices are independent of Pythagoras.

The number 7 has a special significance in ancient Egyptian medicine,[27] as in that of certain Indian tribes.[28] Solon uses it in dividing human life into periods,[29] as do the Etruscans.[30] Among the Babylonians, each divinity has a specific number which belongs to it—for example, Ishtar has 15, perhaps reminiscent of the 5 which is γάμος.[31]

ancient tradition, relying on just this kind of observations, made Pythagoras the teacher of Numa (on this, see Burkert, *Philologus* 1961). See also above, ch. II 4, nn. 128, 136. On the other hand, Macrobius (*Sat.* 1.13.5) speaks of "secretum hoc et ante Pythagoram parturiente natura." K. Latte, *Römische Religionsgeschichte* (Munich, 1960) 199, called attention to the fact that with the reverse numeration of days, used by the Romans, these festivals come on even-numbered days in the latter half of the month; but it may be that the reverse numeration was not the oldest (A. K. Michels, *The Calendar of the Roman Republic* [Princeton, 1967] 139).—The role of 3, 9, and 27 in the Roman cult of the dead is contrary to Pythagorean doctrine (H. Diels, *Sibyllinische Blätter* [Berlin, 1890] 40ff); cf. below, n. 56. Thus in any case the roots of Roman number symbolism are earlier than Pythagoras.

[21] Meissner II 278f.

[22] *Handwörterbuch des deutschen Aberglaubens* VIII 1641f.

[23] Plin. *HN* 23.156; cf. 24.82, 28.56, 28.33 ("Pythagoras").

[24] Varro *Rust.* 3.9.12, Columella 8.5.8, Plin. *HN* 10.151, 163, 18.231, Pallad. 1.27.1, *Geop.* 14.7.13. See Roscher, *AbhLpz* 1904, 65 n. 153. This is common in Germany: *Handwörterb. des deutschen Abergl.* IV 452.

[25] *Geop.* 18.2.8; also attested for Germany, *Relig. in Gesch. u. Gegenwart* V² 2068.

[26] On Hellenistic pseudepigrapha dealing with *georgica* and related topics, see M. Wellmann, "Die Georgika des Demokritos," *AbhBln* 1921 n. 4; "Die φυσικά des Bolos Demokritos und der Magier Anaxilaos aus Larissa," *AbhBln* 1928 n. 7. Their influence is seen as early as Cato *Agr.*, Wellmann 1921, 34ff.

[27] Papyrus Ebers, ca. 1500 B.C.; Roscher, *AbhLpz* 1906, 108.

[28] L. Lévy-Bruhl, *How Natives Think* (above, n. 13) 212.

[29] Fr. 19 Diehl; often cited in ancient numerology, where the treatment of 7 was generally the fullest.

[30] Varro ap. Cens. 14.6. Varro, who likes to "Pythagorize," is of course not entirely reliable on such a matter. He had himself buried "Pythagorico modo" (Plin. *HN* 35.160).

[31] Meissner 131; cf. also A. Jeremias, *Handbuch der altorientalischen Geisteskultur* (Berlin 1929²) 265f. F. X. Kugler, "Der Ursprung der Zahlsymbole . . . in pythagoreischer Beleuchtung," *Klio* 11 (1911) 481–496, wished to use this as an indication that the whole of Pythagoras' number symbolism was derived from Babylon; but the development is a complex one, and number symbolism can be shown to have existed in Greek culture before Pythagoras (Germain 61ff).—The highest god of the Hittites, the weather god, is represented ideographically by the sign for 10 (*Wörterbuch der Mythologie* ed. H. W. Haussig [Stuttgart, 1961] I 209).

The number 4 plays a dominant role in the cosmology of the American Indians: corresponding to the 4 directions and the 4 winds, the gods are divided into groups of four, and colors, actions, and tribal territories are divided according to the same principle.[32] It is number that orders the universe.

In Chinese thought, number symbolism was developed into an amazingly delicate and complex system, which comprehends cosmos and man, nature and social order.[33] The basic antithesis of Yang and Yin is manifested in the antithesis of odd and even number; and it is odd number which corresponds to the active, male principle, Yang. The significance of the numbers from 1 to 10 is of fundamental importance. They are divided into two groups of five, from 1 to 5 and from 5 to 9, and then brought into connection with the five elements, the points of the compass, the seasons, the main types of taste or flavor, the colors, and, above all, with the five notes of the pentatonic scale. Five is the number of procreation. The numbers are grouped together in a figure made with counters or pebbles—ψῆφοι—which is a representation of the entire universe.[34] The Chinese know the numerical laws of the musical intervals, and express the relative height of a tone in terms of a series of numbers, which is also used in determining lengths for bamboo flutes. But this is not a matter of physical theory, but of analogies with cosmic regularities; in the ratio 3 : 2 or 4 : 3 is expressed the relationship of Yang and Yin;[35] the numbers that occur have a value and significance of their own. Unity, in this context, is not regarded as a number and can be added or subtracted at will,[36] in order to produce a result in which the relationship in question can be expressed in "significant" numbers. Thus, for example, the interval of a fourth can be 81 : 60 instead of 80 : 60; and the arithmetical complications in the calculation of the scale, in which the Pythagoreans got themselves tangled, can readily be avoided. The objective is not, in fact, accuracy, but the revelation of connections and correspondences. The five notes of the pentatonic scale are also arranged in the form of a pentagram.[37]

[32] Lévy-Bruhl, *How Natives Think*, 210; Schuhl, *Essai* 259 n. 5.

[33] For the following see Granet, esp. pp. 209ff; H. Köster, *Symbolik des chinesischen Universismus* (Stuttgart, 1958) 44ff; Fung Yu-Lan, *A History of Chinese Philosophy*, tr. D. Bodde (Princeton, 1952–1953) II 93ff. For references and advice in this field, I am much indebted to Prof. H. Steininger.

[34] Arrangement Ho-t'u, Granet 177f; connected with the Ming t'ang, Granet 178ff.

[35] See above, ch. V 1, n. 64, on superparticular proportions.

[36] Compare the German "acht Tage" for a week, along with "14 Tage" (French "quinze jours") for "2 weeks" (and English "fortnight").

[37] Granet 230ff.

The Chinese knew the "Pythagorean theorem," but used it only with whole numbers. Here again the unit does not count, so that 8, 9, 12 can represent the sides of a right triangle,[38] and 5, 5, 7 those of a right isosceles triangle.[39] The objective, again, is not mathematics, but the aim of making the proportions in the roof of a Chinese house correspond to the proportions of the cosmos.

We read in a text of the Han period which features the study of "emblems and numbers," "Heaven, earth, the yin and yang, and wood, fire, earth, metal and water, make nine; together with man, they make ten. Heaven's number is with this made complete."[40] It is as though we had an ancient Pythagorean text before our eyes. Also, as the Platonic "system of derivation" follows the Pythagorean, in China a philosophy emerged, neo-Confucianism, which is strongly reminiscent of Platonism.[41]

Perhaps one ought not to exclude completely the possibility of direct connections between the Pythagoreans and the Chinese;[42] but whatever tenuous lines of connection between East and West there were before the beginning of the modern age, they are not so important as the basic, underlying idea or attitude. Speculation about numerical relationships in the cosmos is world wide, firmly established in ritual, and capable of being elaborated into a rich and igeniously structured system without either presupposing or giving rise to mathematics in the proper sense of the word. To suppose, therefore, that number speculation is derived from mathematics and that consequently Pythagorean mathematics is primary (as others have done besides Aristotle) is no better than a petitio principii.

The nature and origin of number symbolism has also been treated

[38] $8^2 + 9^2 = 12^2 + 1$; Granet 249ff.

[39] Plato calls 7 the *διάμετρος ῥητὸς πεμπάδος* (*Rep.* 546c); cf. the Pythagorean "side numbers" and "diagonal numbers," above, ch. VI 2, n. 14.

[40] Fung Yu-Lan II 19 (Tung Chung-shu, 2nd century B.C.); cf. the text Ta Tai Lichi, cited by Köster 45f; "Heaven one, earth two, man three."

[41] Fung Yu-Lan II 458 cites Shao Yung, 12th century A.D.: "The Supreme Ultimate is a unity which does not move. It produces a duality, and this duality is spirit . . . Spirit produces numbers, the numbers produce emblems, and the emblems produce implements" (i.e., the objects of the physical universe). This is the series *ἕν, (ἀόριστος) δυάς, ἀριθμοί, εἴδη, αἰσθητά.*

[42] Granet is negative (209ff), but Fung Yu-Lan (II 11) holds such an influence to be possible. Earlier, Père Amiot believed that Pythagoras was dependent on the Chinese (*Mémoires concernant l'histoire, les sciences . . . des Chinois* [Paris, 1776–1791] VI 173 [not accessible to me]). E. Chavannes reversed the relationship (*Les mémoires historiques de Se-ma-Ts'ien* III 2 [Paris, 1895] 630–645, cf. 647 on the "Metonic" cycle). Hellenistic astrology reached as far as Japan.

from the point of view of C. G. Jung's school of psychoanalysis.[43] The mass of data collected is amazing; number symbolism continues to exert a surprising influence even in the subconscious mind of modern man. The fascination of the "perfect" number 3 was exploited by the Third Reich; and the pentagram lives on in the flags and emblems of the United States and the Soviet Union.[44] Thus for the psychologist the numbers are archetypes. To be sure, the connection of mind and symbol does not yet seem to be completely clear or capable of formulation with scientific precision. The meanings of the individual numbers are sometimes almost uniform, but sometimes entirely ambiguous, giving the impression of being arbitrarily interchangeable. It is surely mistaken to derive number symbolism from certain particular natural observations, as Roscher referred the special significance of 7 and 9 to the phases of the moon.[45] It is also clear that certain elementary calculations play an important role. One learns to count and calculate in childhood, and from the beginning the numbers are apprehended as things, with certain characteristics;[46] they preserve this peculiarity even, at an unconscious level, in the mind of the adult. For example, the odd numbers, which cannot be divided evenly, are felt as uncomfortable, even perverse—and therefore, if one knows how to use them, powerful and lucky. A prime number like 7 is especially hard to manage and therefore very significant; and here an additional psychological factor is at work—it is easy to picture in one's mind a group of six, or a hexagon, but much harder to imagine a heptagon. Thus many kinds of threads join to compose this complicated fabric.

Like the numbers, the simpler geometrical shapes have an "archetypal" symbolism. The square symbolizes the earth both for the Chinese and for certain Indian tribes;[47] and Philolaus "dedicates" the angle of the square to the goddesses Rhea, Demeter, and Hestia.[48] But here too the attributions are inconsistent; there is no direct relationship

[43] L. Paneth, *Zahlensymbolik im Unbewusstsein* (Zürich, 1952); see also C. G. Jung, *Aion* (Zürich, 1951) 323f.

[44] Paneth, 45ff, discusses the significance of 5 as the number of vitality and sexuality, without knowing the Pythagorean equation of 5 and marriage. For the pentagram on flags, see De Vogel, *Pythagoras*, 297–299.

[45] Roscher, *AbhLpz* 1904, 67ff. On the other hand, H. Quiring tries to trace the importance of 7 to the planets alone (*Altertum* 4 [1958] 208–214); correctly Junge, *DtMath* 356.

[46] Dornseiff (above, n. 2), 17ff, uses the expression *Kindheitsmystik*.

[47] Lévy-Bruhl, *How Natives Think* (New York, 1966) 114 (Dakota Indians). They also represent the earth by four dots (::).

[48] Above, ch. IV 3, n. 60. Eudoxus adds Hera and Aphrodite.

between the triangle as a female sexual symbol[49] and the male gods to whom the Pythagoreans dedicated the angle of the triangle (above n. 48). It is noteworthy that in prehistoric ornamentation there are examples which anticipate the "tetractys."[50] The ψῆφος figures, too, with their speculative interpretation, "make sense," in a way, as "archetypal" patterns; they "speak to" certain psychic dispositions. Those mysterious dodecahedra mentioned above[51] may be rooted in similar soil.

Greece, too, has its primeval, ritually significant symbolic numbers. Even in the Homeric epics one notices the preference for certain numbers,[52] especially 3 and 9, and also 5 and 7—the odd numbers. The number 8 hardly occurs at all, and also apparently played no part in the ancient Pythagorean tradition.[53] Certain numbers belong to certain gods; the cult of Apollo and that of Dionysus were dominated by the numbers 7 and 9.[54] The significance of 3 in purification ritual was emphasized by Aristotle; as Usener perceived, it is rooted in a primitive conception of number, in which, after 1 and 2, 3 means plurality in general.[55] At the symposium one set out three craters of wine, and poured a libation from the first to Zeus and the Olympians, from the second to the Heroes, and from the third to Zeus Soter or Teleios.[56] Two belongs to the chthonic world, whereas 3 is the number

[49] E.g., the expression δέλτα in Ar. *Lys.* 151, Paus. 2.21.1; A. Stuiber, *RAC* s.v. Dreieck, with refs.; M. Eliade, *The Forge and the Crucible*, tr. S. Corrin (London: Rider, 1962) 185.

[50] A. D. Nock, *AJA* 50 (1946) 142 n. 14; M. Hoernes and O. Menghin, *Urgesch. d. bild. Kunst in Europa* (Vienna, 1925³) 49 (rattle amulets from Tribano, near Padua), 283, 349, 397, 429, 559. See also Stapleton, *Osiris* 1958, 32ff; J. Schwabe, *Antaios* 8 (1967) 444 (pottery from Samarra, *East and West* 15 [1964–1965] 72); J. Meysing, *Rev. des sciences religieuses* 39 (1965) 218.

[51] Above, ch. VI 3, n. 65.

[52] See Germain, passim.

[53] Cf. nn. 4–10. For Homer, Germain 8f; on the other hand, 8 becomes important in late antiquity and in Christianity: F. J. Dölger, *Antike und Christentum* 4 (1934) 153–182.

[54] Roscher, *AbhLpz* 1904, 5ff, 54ff. For example, the Carneia lasted 9 days; 9 men gather in each of 9 "sunshades" (tent-like structures: Ath. 4.141e); Apollo was born on the seventh day of the month, and he is ἑβδομαγέτας at Aesch. *Sept.* 800. There is a 7-day, sevenfold beating with κράμβη ἑπτάφυλλος at the Pharmakos ritual (Hipponax fr. 11; IX 49f Diehl–Beutler; cf. Nilsson I 107ff). There are 7 bunches of grapes and 7 dolphins on the famous Dionysus cup of Exekias.

[55] Above, n. 2; Arist. *Cael.* 268a14; cf. *Od.* 11.28. Reemphasizing 3, one gets 9 and 27, which are important in the cult of the dead (Soph. *OC* 479ff, Germain 38ff; on Roman phenomena, above, n. 20).

[56] K. Kircher, *Die sakrale Bedeutung des Weins im Altertum* (Giessen, 1910) 17ff, 34ff; Nilsson, *Op.* I 428ff. Ζεὺς τέλειος, Eur. fr. 148, Ar. fr. 526, Philochorus *FGrHist* 328F87 (with Pythagorean interpretation). In Pythagorean doctrine, one is enjoined to bring odd-numbered offerings to the gods of heaven, even to those of the underworld (Plut. *Numa*

of completion or fulfillment. If one is tempted to see here the early onset of Pythagorean interpretation, still even in Homer himself, δοιή means "doubt," or "confusion"; and it is not far from here to δόξα. In wrestling, a match has been ended, since time immemorial, when one contestant has won three falls (τριάζειν). The special status of the first four numbers, as a group—the tetractys—is reflected even in the fact that they alone are declined, after Indo-European tradition.

According to Aristotle,[57] the Athenians were divided, in early times, into 4 phylae and 12 phratries each made up of 30 families, on the model of the division of the year into seasons, months, and days. Numerical order unites society and cosmos; and even if this were to be put down as fourth-century speculation, it may well be based on something older. The number 7 is in Solon's view the one according to which human life is to be divided into periods; and neither its interpretation as καιρός nor its important role in Greek medicine can be traced to Pythagoras as originator.[58] The most popular expression of the inferior status of even numbers came in the theory that 7-month and 9-month babies can live, but 8-month babies cannot.[59] It is asserted in the Hippocratic book on the subject that the first movement of the embryo in the womb takes place after 3 months in the case of boys and 4 in the case of girls.[60] The formation of the embryo takes 30 days for boys and 42 for girls; and the postpartum discharge lasts a proportionate length of time.[61] Thus 3 is the male and 4 the female number, which in the second example is conjoined with a period measured in sevens. There is some Pythagorean material in the Hippocratic writings;[62] but the

14, Por. *VP* 38, Iam. *VP* 156, Schol. A *Il.* 23.171; Plato alludes to this, *Leg.* 717a; Serv. *Aen.* 3.305, *Buc.* 5.66, 8.75, ascribes it—probably following Varro—to the Romans). But this is not universally valid for either Greek ritual (above, n. 55) or Roman (above, n. 20).

[57] *Ath. Pol.* fr. 5 = *Lexicon Patm.* p. 152 Sakkelariou.

[58] Above, n. 29. For the material in the Hippocratic corpus about "critical days," see Roscher, *AbhLpz* 1906, 55–84. Hippocrates (*Epid.* 1.26) distinguishes a number of κρίσιμοι ἐν ἀρτίῃσιν from another ἐν περισσῇσιν. See also Heraclitus A18, B126a (whose genuineness is very doubtful, because of the dual, quite irregular for Ionic), Alcmaeon A15, Empedocles A83, fr. 153a, Hippo A16, Aristox. fr. 23. Criticism of *Pythagorici numeri*: Cels. *Med.* 3.4.

[59] Modern medicine does not confirm this doctrine, at least in such an extreme form. Of course, ancient physicans were very seldom able to determine precisely the time of conception.

[60] Hippoc. *Nat. puer.* 21, VII 510 L., *Steril.* 233, VIII 446 L.; differently Diogenes o. Apollonia A26, B9.

[61] *Nat. puer.* 18, VII 500 L.; but according to Diocles and Strato the embryo is formed in the case of girls by the sixth, in that of boys by the seventh hebdomad (Strato, frr. 97–98 Wehrli)—here the girls are quicker, but they still have an even number.

[62] Above, ch. III 2, nn. 113 ff; III 3 nn. 63, 86.

forms ὁ τρίπους and ἡ τράπεζα are attested even in Mycenaean Greek, and there is no obvious reason for the variation in gender in nouns from the same root, except that gender is determined by number. The birthday of Apollo is celebrated on the seventh, that of Artemis on the sixth day of the month.[63] A man, says Hesiod,[64] should marry at about 30,

ἡ δὲ γυνὴ τέτορ' ἠβώοι, πέμπτῳ δὲ γαμοῖτο.

A girl has four years to herself after she reaches puberty, and her marriage is to take place in the fifth. So the numbers 3, 4 and 5, as those of man, woman, and marriage, are present in early Greek folk custom.

All this goes to show that the curious numerical correspondences, known as Pythagorean as early as Aristotle, are not a late development or a trivialization of a Pythagorean philosophy or mathematics, but have their roots in primitive ideas of number. The numerical symbols for male, female, opinion, marriage, opportunity, and "the whole" are earlier in origin than the time of Pythagoras; they were present not only in oriental lore but in the language and customs of the Greeks themselves. The notion that numbers have a "metamathematical," cosmic significance, and that they reveal the principle of the order of the world and of human life, is not any kind of scientific or philosophical insight, but a readily comprehensible characteristic of pre-mathematical thinking about number. Pythagorean number symbolism is therefore much older than any natural science, mathematics, or astronomy that Pythagoras or his pupils could be imagined to have practiced. It has nothing to do with science in our sense—which is to say, the Greek sense—of the word; it neither presupposes this nor

[63] D.L. 2.44; Deubner, *Attische Feste* (Berlin, 1932) 179, 201, 209.

[64] *Op.* 695, 698. Naturally, 40 cannot be the usual age for women to marry, nor, hardly, can 17 for men; it is all the more remarkable how 3, 4, and 5 are worked into the formulation.—A provision in Plato's *Laws* is that a man may be punished with blows until the age of 30, a woman till 40 (932b–c, cf. 845c). According to the *Republic* (460e), a woman may have children from the twentieth to the fortieth year, a man from the twenty-fifth to the fifty-fifth.—Perhaps it is worth noting that though groups of 4 are uncommon in ritual, it is in women's cults that they do appear (Germain 51; the Heraea at Olympia, Paus. 5.16; Demeter Chthonia at Hermione, Paus. 2.35.7; the four daughters of Celeus, *Hymn. Hom. Cer.* 109f).—One hopes that, on the third day of the month, a child born will turn out to be a boy; for a girl born on this day would turn out to be unfeminine in character (Schol. BT on *Il.* 8.39, *Suda* s.v. *Τριτογένεια*). Naturally, there are exceptions: Heracles and Hermes were born on the fourth.—One resists the temptation to mention the 3 Doric and 4 Ionian phylae.

advances it. The Pythagorean doctrine that "all is number" grows directly out of "archetypal" number symbolism, which in one degree or another is worldwide in occurrence.

Of course, number symbolism can be combined with scientific knowledge. Both in China and among the Pythagoreans, it took over mathematical music theory; and in the "harmony of the spheres" we have an adaptation of mathematical musicology to the newly discovered, scientific understanding of the cosmic order. But this cannot mean more than an application and confirmation of a basic idea which was already present and was open to any kind of enrichment. Number is not quantity and measurability, but order and correspondence, the articulation of life in rhythmical pattern, and the perspicuous depiction of the whole as the sum of its parts. To see a "consistently quantitative view of the world"[65] in Pythagorean number theory is a mistake.

One cannot help recognizing how closely this number symbolism is connected with the realm of the *acusmata*. In both cases the origin lies in primeval custom, found in somewhat similar form among the most varied peoples and cultures; and in both cases we find the circumscription of life and thought by forms and formulas, which are laid on the Pythagorean by authority and whose explanation and interpretation are secondary in importance. Number in fact means restraint; counting is performed in successive acts, and thus time itself, composed of successive events, is number.[66] The great mass of the *acusmata* had to do with sacrificial ritual, and its methods and times, the καιροί,[67] so that correct piety (εὐσέβεια) depends on knowing number. While this reveals the orderly arrangement of time, such a point of view may lead also to thoughts of the recurrence of the same, of periodic transmigration, or even to the recurrence of the same condition of the world.[68] To the four seasons correspond the four ages of man's life;[69] the

[65] Frank 72: "konsequent quantitative Weltanschauung."

[66] It is not a coincidence that in the oldest references τετράς, δεκάς, τριακάς are mostly used in a temporal sense, of the 4th, 10th, and 30th day (Hes. *Op.* 794, 798, *Hymn. Hom. Merc.* 19; differently in *Il.* 2.126). Seidenberg (above, n. 13) propounds a thesis that counting had its origin in a "creation ritual." This is unprovable, but his collection of material on ritual counting is worth attention.

[67] Iam. *VP* 85 (following Aristotle: above, ch. II 4, n. 5): τὰ δὲ πλεῖστον ἔχοντα μῆκος, περί τε θυσίας καθ' ἑκάστους τοὺς καιροὺς πῶς χρὴ ποιεῖσθαι. A portion of the exposition, based on later sources, is at Iam. *VP* 152.

[68] Eudemus fr. 88, Dicaearchus ap. Por. *VP* 19.

[69] This is frequent in later sources: the "tripartitum," D.L. 8.10, Ov. *Met.* 15.199–213. For parallels see Delatte, *Vie* 110. Perhaps Alcmaeon has this in mind in fr. 2, if it means that, while the year is continually renewed, the winter of man's life is not followed by a new spring (above, ch. III 3, n. 97). Cf. the riddle of the Sphinx solved by Oedipus, and Hippoc. *Nat. hom.* 2.

correspondence of part and whole, of man and cosmos, is mirrored in number. To the order of time corresponds also the order of space.[70] There are some indications that the ancient idea of macrocosm and microcosm was known to the Pythagoreans;[71] we "enumerate"[72] the parts in either realm, and it is number that reveals their correspondence. Thus the multiplicity of the world is reduced to clearly articulated groups, whose mutual relationship is known to the "wise man"; and he "honors" this orderly arrangement in his practical activity.

It is not clear to what extent the idea of number was dominant in the *acusmata*. Only two of the individual precepts we have speak of number: the catechetical "What is wisest? Number" and the reference to the tetractys. In this word there seem to be condensed a great many connotations. Four has its own value and significance as the number of "right" ordination and adjustment (above, n. 32); the triangular figure, made up of ψῆφοι, speaks for itself as a graphic and diagrammatic representation of order. That the first four numbers add up to ten, a number that had drawn special attention from very early times, is immediately evident from the figure; and it may well have seemed the supreme manifestation of the mysterious power of the tetractys that the same first four numbers express the basic ratios of the musical intervals.

In any case, there is no break in level between the number symbolism, even in its cosmic sense, and the other *acusmata*. Number, ritual, and the doctrines about the soul, motifs which compete in the later course of intellectual history, coincide in Pythagoreanism—in a completely pre-scientific realm. If this content of the Pythagorean tradition is unmistakably older than Pythagoras, then it is quite improbable, and would have to be proven beyond doubt, that all of this worked its way into Pythagoreanism by some sort of roundabout route, rather than coming by way of the master's own teaching. Our conclusion must be that in number symbolism, as in the *acusmata*, we are dealing with doctrine of Pythagoras himself.

The question remains, wherein lay the incontestable fame of the man and his influence on generations of disciples; must he not have contributed something basically new, for example by the "recognition

[70] Hesiod measures cosmic distance in time (*Th.* 722f), Anaximander in linear terms (above, ch. IV 1, nn. 55–56).

[71] Above, ch. I 2, nn. 45ff; III 2, nn. 150, 162.

[72] καταριθμοῦμεν, Diod. 1.11 (Hecataeus of Abdera?), describing the "Egyptian" doctrine of microcosm and macrocosm.

of the universal regularity, to be comprehended by means of number," through which the word cosmos acquired "a deeper meaning, and one of decisive importance for the history of philosophy"?[73] Mathematical astronomy, after some stirrings in the fifth century, was only brought to full development by Eudoxus; music theory was at first more a number game than a science; and the "philosophiae naturalis principia mathematica," in the sense of Newton, were never attainable to the Greeks, even though in the *Timaeus* it seems that Plato dreamed of something of this kind.[74] Modern perspectives distort our view of the ancient "wisdom" of Pythagoras, as in fact it had soon become distorted in antiquity. An impressive achievement can consist in the synthesis, systematization, and consistent development of something preexisting, to say nothing of the unaccountable workings of a personality which is not dependent on originality or the independent importance of specific thoughts or doctrines, but of itself enhances the significance of meditation and teaching. Pythagoras was no lonely, unrecognized thinker, but an extremely renowned and successful "sage," the founder of a society which lasted a long time. Further, is it not a great achievement if a strong personality, by virtue of its charismatic power, gives new life to what is old, and preserves its spiritual forces through a period of turmoil and change? The schemes and projects of science always fall short; there remains an unexplained residue to cause disquiet and the yearning for completeness. To round this out into a comprehensive knowledge which could satisfy all the aspects of a man's being, this ideal was regarded as accomplished by Pythagoras. Perhaps it is here that we should see his great importance; because of him there remained alive something of the ancient lore, pushed to one side by the growth of science but never really dispensable, powers of the past which could preserve a feeling of security in an increasingly secular and material world.

It has long been known that conscious and unconscious, rational and irrational impulses, logic and mysticism, interpenetrate in a complicated and nearly inextricable fashion. As Kepler discovered his second planetary law in "Pythagorean" manipulation of regular polyhedra, so one might find it obvious that precisely the pre-philosophical lore of Pythagoras provided the stimulus for Pythagorean science. But not only does the cosmic significance of number come from pre-logical

[73] Kranz, *ABG* 1955, 32: "eine tiefere, und zwar für die Philosophiegeschichte entscheidend wichtige Bedeutung."

[74] See Becker, *Fs. Gadamer* 12.

number symbolism, but, even in that which Aristotle presents as the philosophy of the Pythagoreans, there emerges again and again a spirit and method directly opposite to that of exact mathematics, so that the latter cannot have arisen from the activities of the Pythagoreans. It is not an unbroken unity of science and religious-ethical teaching that we find in the Pythagorean tradition, but a groping attempt to mediate between two levels, to transpose an ancient interpretation of the world into the language of the recently founded *φυσιολογία*. In this transposition, apparently, the opposition of "limiting" and "unlimited" first attained an important part, even though the high valuation placed on "limit" is a derivative from the ancient lore. This made it possible to show a relationship between essential characteristics of being and the world of number. Other agreements were sought and happily found—significant numbers in the calculation of the scale, regularities in the pebble diagrams, orderly behavior among the stars. But there is no steady continuity or consistency; different kinds of material were present, partly retained, like the ancient numerical symbols, or the mere juxtaposition of analogous ideas,[75] and partly added, like the medical doctrines in Philolaus (A27–28), which do not have any necessary relation to his cosmological ideas. The *acusmata* and similar regulations were passed on, without further justification, as Plato attests Philolaus did (*Phd.* 61d–e). Thus this Pythagorean philosophy was a synthesis closely tied to the conditions of the time, and without any lasting validity. The nature of the tradition authorizes us to regard it as largely the work of Philolaus, following after some similar attempts of Hippasus.

But even Philolaus and his pupils must have been persuaded that in their thoughts they were only following and carrying out the insights of Pythagoras, and presenting in different terms what he had long ago known and meant. New interpretation of this sort is inevitable, if an authority is to maintain his position as intellectual history progresses. Pythagoras was the wisest of all men, the tradition said. For an age that still knew no science, *γόητες* and *τελεσταί* were the "sages," and Pythagoras belongs in this context, as we can see from the oldest witnesses, whether they mock or admire him. But only a few decades later, it was impossible to accept this; so the image of Pythagoras must change. If Pythagoras was "wise," he was "wise" in the spirit of the new age, certainly not a *γόης* but a scientist. The "sage" acquires his

[75] Philolaus fr. 13.

knowledge by his relation to the gods, whereas the scientist bases his on deductive proof. The "sage" works immediately, in his own person, whereas the scientist writes books. The "sage" knows the roads through Heaven and Underworld, whereas the scientist measures cosmic distances in stades. The "sage" interprets the signs of the zodiac, whereas the scientist calculates the movements of the heavenly bodies in advance. Number lore in particular must inevitably appear in a very different light; what had been a symbolic ordering and classification of the multiplicity of phenomena becomes, in hindsight, mathematics, for a nonmathematical idea of number is no longer acceptable. The delight of the pragmatic historian in a *πρῶτος εὑρέτης* helped to transmute this interpretation into a historical datum.

Plato forced the divergent factors of rationality and morality, of physical science and speculative interpretation, into a new synthesis,[76] though he is quite consciously continuing ancient tradition. All the more, it was Plato's pupils who placed themselves under the authority of "the ancients," and as a consequence saw Pythagoras in the light of what Plato himself had been the first to do. The pre-Socratic interpretation of Philolaus, inadequate in many respects, was supplanted by a kind of Platonism, which was regarded in succeeding centuries as the "true" Pythagoreanism. Since the content was mostly just passed along with no other justification than authority, and was accepted without further examination, it corresponded in its function to the unified conception of the world characteristic of an earlier, pre-scientific

[76] Number symbolism belongs, for Plato, in the realm of myth; and this is true, in particular, of the famous "nuptial number" of *Rep.* 545e. (See esp. Heath, *Math.* I 305ff; A. Diès, *Le nombre de Platon* [Paris, 1936]; more recently, A. Ahlvers, *Zahl und Klang bei Platon* [Bern, 1952] 11ff; M. Denkinger, "L'énigme du nombre de Platon," *REG* 68 [1955] 38–76; F. von Ehrenfels, *AGP* 44 [1962] 240–244. Clearchus had already commented on the passage, as shown by frr. 3–4 Wehrli.) The Muses present the exposition of the nuptial number *παίζουσαι καὶ ἐρεσχηλοῦσαι*, as Plato has it (*Rep.* 545c; this is the merriment of the superior; cf. *Phdr.* 236b, *Phlb.* 53c, *Leg.* 885c). What follows is not nonsense but the indication of meaningful mathematical connections. A relationship with Pythagoreanism is probable (above, ch. VI 2, n. 13; it is hard to decide whether Alex. *Met.* 75.27ff is genuine tradition or a secondary reconstruction). But this does not mean that whoever has discovered the number 12,960,000, or some other number, will have grasped the secret of the cosmos, and put himself in a position to prevent its downfall. Its decline is not to be arrested by human means (546b). The veil of riddle suggests an infinite task; it is myth, in the sense of groping for truth at the threshold of the ineffable. Even in decline and destruction, it is not chaos or blind conjecture that rules, but an eternal, intelligent order; and mathematical knowledge can enable us to grasp this in some small degree. This is the interpretation the Platonist Eratosthenes also gives to the legend of the "Delian problem," of duplication of the cube: the god's intention was, through the insoluble problem, to inspire interest in the study of geometry (Theo Sm. 2.3ff).

age. Again, since a doctrine whose appeal is to faith is normally presented in as ancient garb as possible, the name of Pythagoras sometimes obscured that of Plato. The tradition of Pythagoras as a philosopher and scientist is, from the historical point of view, a mistake. But the fascination that surrounded, and still surrounds, the name of Pythagoras does not come, basically, from specific scientific connotations, or from the rational method of mathematics, and certainly not from the success of mathematical physics. More important is the feeling that there is a kind of knowing which penetrates to the very core of the universe, which offers truth as something at once beatific and comforting, and presents the human being as cradled in a universal harmony. In the figure of Pythagoras an element of pre-scientific cosmic unity lives on into an age in which the Greeks were beginning, with their newly acquired method of rational thought, to make themselves masters of their world, to call tradition into question, and to abandon long-cherished beliefs. The price of the new knowledge and freedom was a loss in inner security; the paths of rational thought lead further and further in different directions, and into the Boundless. There the figure of the ancient Sage, who seemed still to possess the secret of unity, seemed more and more refulgent. Thus after all, there lived on, in the image of Pythagoras, the great Wizard whom even an advanced age, though it be unwilling to admit the fact, cannot entirely dismiss.

Abbreviations

Abbreviations generally follow the standard list of the *Oxford Classical Dictionary* (that in the second edition, of 1970, is substantially identical with that of the first). The following list includes some works frequently cited, some departures from the usage of *OCD*, and some items omitted from it. For Greek authors, see also the list in Liddell-Scott-Jones, *Greek–English Lexicon* (Oxford, 1925–40, with Supplement, 1968), and for journals, the list in *L'année philologique*.

1. PERIODICALS AND COLLECTIVE WORKS

AA	*Archäologischer Anzeiger*
AA	*Acta antiqua Academiae Scientiarum Hungaricae* (only cited for works of Szabó; see bibliography)
A&A	*Antike und Abendland*
AAHG	*Anzeiger für die Altertumswissenschaft*, hrsg. von der Oesterreichischen Humanistischen Gesellschaft
ABG	*Archiv für Begriffsgeschichte*
AbhBln	*Abhandlungen der Preussischen Akademie der Wissenschaften*, Berlin
AbhLpz	*Abhandlungen der Sächsischen Gesellschaft der Wissenschaften*, Leipzig
AbhMainz	*Abhandlungen der geistes- und sozialwissenschaftlichen Klasse der Akademie der Wissenschaften und der Literatur*, Mainz
AC	*L'antiquité classique*
AGP	*Archiv für Geschichte der Philosophie*
AJA	*American Journal of Archaeology*
AJP	*American Journal of Philology*
ALL	*Archiv für lateinische Lexikographie und Grammatik*
AM	*Athenische Mitteilungen*
ANET	*Ancient Near Eastern Texts*, ed. J. B. Pritchard
AnzAW	*Anzeiger für die Altertumswissenschaft*

AtcR	*Atene e Roma*
Arch. delt.	*Ἀρχαιολογικὸν δελτίον*
Arch. eph.	*Ἀρχαιολογικὴ ἐφημερίς*
ARW	*Archiv für Religionswissenschaft*
CAG	*Commentaria in Aristotelem graeca*, Berlin (cited by page and line)
CAH	*Cambridge Ancient History*
CIG	*Corpus inscriptionum graecarum*
C&M	*Classica et medievalia*
CMG	*Corpus medicorum graecorum*
CP	*Classical Philology*
CQ	*Classical Quarterly*
CR	*Classical Review*
CRAI	*Comptes rendues de l'Académie des Inscriptions et Belles-lettres*
DK	Hermann Diels, *Die Fragmente der Vorsokratiker*, 6th ed. by Walther Kranz (Berlin: Weidmann, 1951–1952; repr. of the 5th ed., 1934–1937, with *Nachträge*; later editions are reprints of this)
Dox.	Hermann Diels, *Doxographi graeci* (Berlin, 1879; repr. 1958)
FGrHist	Felix Jacoby, *Die Fragmente der griechischen Historiker* (Berlin: Weidmann, Leiden: Brill, 1923–)
FHG	Car. and Theod. Müller, *Fragmenta historicorum graecorum* (Paris, 1841–1870)
GGN	*Göttingische gelehrte Nachrichten*
GRBS	*Greek, Roman, and Byzantine Studies*
HSCP	*Harvard Studies in Classical Philology*
HTR	*Harvard Theological Review*
IF	*Indogermanische Forschungen*
IG	*Inscriptiones graecae*
JAW	*Jahresbericht über die Fortschritte der klassischen Altertumswissenschaft*, founded by Conrad Bursian
JHI	*Journal of the History of Ideas*
JHS	*Journal of Hellenic Studies*
Kern, *Orph. frag.*	Otto Kern, *Orphicorum fragmenta* (Berlin: Weidmann, 1922; repr. 1963)
LSJ	Henry George Liddell and Robert Scott, *A Greek-English Lexicon*, new ed. by Henry Stuart Jones and Roderick McKenzie (Oxford: Oxford Univ. Pr., 1925–1940); suppl., ed. E. A. Barber et al. (Oxford, 1968)
MH	Museum Helveticum

NED	*New English Dictionary* (Oxford, 1903)
NGG	*Nachrichten der Göttinger Gelehrten Gesellschaft*
NJb	*Neue Jahrbücher für das klassische Altertum, Geschichte und deutsche Literatur*
Orph. Frag.	see Kern
PhilosQ	*Philosophical Quarterly*
PhW	*Philologische Wochenschrift*
PPF	Hermann Diels, *Poetarum philosophorum fragmenta* (Berlin, 1901)
QSt	*Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, B: *Studien* (Berlin, 1931ff)
RAC	*Reallexikon für Antike und Christentum*
RE	*Realencyclopädie der classischen Altertumswissenschaft*
REA	*Revue des études anciennes*
REG	*Revue des études grecques*
RhGr	*Rhetores graeci*, ed. Leonhard [von] Spengel
RhM	*Rheinisches Museum*
RHR	*Revue de l'histoire des religions*
RivFil	*Rivista di filologia e d'istruzione classica*
RM	*Römische Mitteilungen*
Roscher, *Lex.*	W. H. Roscher (ed.), *Ausführliches Lexikon der griechischen und römischen Mythologie* (Leipzig: Teubner, 1884–1937)
RPh	*Revue de philologie*
SBBln	*Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin*
SBHeid	*Sitzungsberichte der Heidelberger Akademie der Wissenschaften*
SBLpz	*Sitzungsberichte der sächsischen Akademie der Wissenschaften*, Leipzig
SBMü	*Sitzungsberichte der Bayerischen Akademie der Wissenschaften*, Munich
SBWien	*Sitzungsberichte der Akademie der Wissenschaften in Wien*
SEG	*Supplementum epigraphicum Graecum*
SIG	*Sylloge inscriptionum graecarum*, ed. Wilhelm Dittenberger (3rd ed., Leipzig, 1915–1923; repr. Hildesheim: Olms, 1960)
StudIt	*Studi italiani di filologia classica*
SVF	*Stoicorum veterum fragmenta*, coll. Hans von Arnim (Leipzig, 1903–1921)
TAPA	*Transactions of the American Philological Association*

UCPCP	*University of California Publications in Classical Philology*
WS	*Wiener Studien*
ZNTW	*Zeitschrift für die neutestamentliche Wissenschaft*

2. ANCIENT AUTHORS

Ach. *Is.*	Achilles Tatius, *Introductio in Aratum*, ed. Ernst Maass (*Commentariorum in Aratum reliquiae*, Berlin, 1898)
Ael. *NA*	Aelian, *De natura animalium*
Ael. *VH*	Aelian, *Varia historia*
Aët.	*Ἀετίου Περὶ τῶν ἀρεσκόντων συναγωγή*, *Dox.*, 267–444
Alex. *Met.*	Alexander Aphrodisiensis, *In Aristotelis Metaphysica commentaria*, ed. Michael Hayduck (Berlin, 1891; *CAG* I; cited by page and line)
Anat.	Anatolius, *Sur les dix premiers nombres*, ed. J. L. Heiberg, in *Annales internationales d'histoire, Congrès de Paris 1900* (Paris, 1901–1902; cited by page and line)
Anon. Phot.	Anonymus Photii, Phot. cod. 249 (Thesleff, *Texts*, pp. 237–242; see also bibliography s.v. Immisch)
A.P.	*Anthologia Palatina*
Ap. *H.m.*	Apollonius, *Mirabilia*, ed. Otto Keller, *Naturalium rerum scriptores graeci* I (Leipzig, 1877), 43ff.
Apul. *Plat.*	Apuleius, *De Platone et eius dogmate*
Arist. *EE*	Aristotle, *Ethica Eudemia*
Arist. *EN*	Aristotle, *Ethica Nicomachea*
Arist. fr.	*Aristotelis fragmenta*, ed. Valentin Rose (Leipzig, 1886); *Aristotelis dialogorum fragmenta*, ed. Richard Walzer (Florence, 1934; repr. Hildesheim: Olms, 1963); *Aristotelis fragmenta selecta*, ed. W. D. Ross (Oxford: Oxford Univ. Pr., 1955)
Arist. *Met.*	Aristotle, *Metaphysica*
Arist. *MM*	Aristotle, *Magna moralia*
Aristid. Quint.	Aristides Quintilianus, *De musica*, ed. R. P. Winnington-Ingram (Leipzig: Teubner, 1963; cited by Meibom's pagination)
Aristox. *Harm.*	Aristoxenus, *Elementa harmonica*, ed. Rosetta da Rios (Rome: Typis Publicae Officinae Polygraphicae, 1954; cited by Meibom's pagination)

Ascl. *Met.*	Asclepius, *In Aristotelis Metaphysicorum libros A–Z commentaria*, ed. Michael Hayduck (Berlin, 1888; *CAG* VI 2; cited by page and line)
Ath.	Athenaeus, *Deipnosophistarum libri XV*
Boeth. *Ar.*	Boethius, *De institutione arithmetica*, ed. Gottfried Friedlein (Leipzig, 1867)
Boeth. *Mus.*	Boethius, *De institutione musica*, ed. Gottfried Friedlein (Leipzig, 1867)
Cens.	Censorinus, *De die natali*
Chalcid. *In. Tim.*	Chalcidius, *Timaeus a Calcidio translatus commentarioque instructus*, ed. J. H. Waszink (Leiden: Brill, 1962; *Plato Latinus* 4)
Cic. *Nat. d.*	Cicero, *De natura deorum*
Claud. Mam. *De statu an.*	Claudianus Mamertus, *De statu animae*
Clem. Al. *Protr.*	Clemens Alexandrinus, *Protrepticus*, ed. Otto Stählin (Leipzig: Hinrichs, 1905)
Clem. Al. *Strom.*	Clemens Alexandrinus, *Stromateis*, ed. Otto Stählin (Leipzig: Hinrichs, 1906–1909); I^3, II^2 ed. L. Früchtel (Berlin: Akademie-Verlag, 1960, 1970)
Damasc. *Princ.*	Damascius, *De principiis*, ed. C. A. Ruelle (Paris, 1889; cited by volume, page, and line)
Diod.	Diodorus Siculus, *Bibliotheca historica*
D.L.	Diogenes Laertius (see bibliography s.v. Delatte, *Vie*)
Eucl.	Euclid (when no title is given, ref. is to *Elementa*)
Eust.	Eustathius, *Commentarii ad Homeri Iliadem et Odysseam*
Geop.	*Geoponica*, ed. Heinrich Beckh (Leipzig, 1895)
Greg. Naz. *Ep.*	Gregorius Nazianzenus, *Epistulae*
Harpocr.	Harpocration
Hdt.	Herodotus
Hebd.	*Die hippokratische Schrift Von der Siebenzahl*, ed. W. H. Roscher (Paderborn: Schöningh 1913)
Hippoc.	Hippocrates, *Oeuvres complètes*, ed. Emile Littré (Paris, 1839–1861; cited by volume and page)
Hippol. *Ref.*	Hippolytus, *Refutatio omnium haeresium*, ed. Paul Wendland (Leipzig: Hinrichs 1916)
Hypomn.	*Hypomnemata* (*Πυθαγορικὰ ὑπομνήματα*, Anonymus Alexandri; D.L. 8.25-33)
Iam. *Comm. math. sc.*	Iamblichus, *De communi mathematica scientia*, ed. Nicola Festa (Leipzig: Teubner, 1891)
Iam. *In Nic.*	Iamblichus, *In Nicomachi Arithmeticam introductionem*, ed. Hermenegildus Pistelli (Leipzig, 1894; cited by page and line)

Iam. *Protr.*	Iamblichus, *Protrepticus*, ed. Hermenegildus Pistelli (Leipzig: Teubner 1888)
Iam. *VP*	Iamblichus, *De vita Pythagorica*, ed. Ludwig Deubner (Leipzig: Teubner 1937)
Joseph. *Ap.*	Josephus, *Contra Apionem*
Lact. Plac. *Theb.*	Lactantius Placidus, *Commentarii in Statii Thebaida*
Macrob. *Sat.*	Macrobius, *Convivia primi diei Saturnaliorum*
Macrob. *Somn. Sc.*	Macrobius, *Commentarius in Ciceronis Somnium Scipionis*
Max. Tyr.	Maximus Tyrius
Nicom. *Ar.*	Nicomachi Geraseni Pythagorei *Introductionis arithmeticae libri II*, ed. Richard Hoche (Leipzig, 1866)
Nicom. *Ench.*	Nicomachus, *Harmonicum enchiridium*, ed. Karl von Jan (*Musici scriptores graeci*, Leipzig, 1895) 210–265
Nicom. *Exc.*	*Excerpta ex Nicomacho*, ibid. 266–282
Nicom. *Th. ar.*	see *Th. ar.*
Olympiod. *In Alc.*	Olympiodorus, *In Platonis Alcibiadem commentarii*, ed. L. G. Westerink (Amsterdam: North Holland, 1956)
Olympiod. *In Gorg.*	Olympiodorus, *In Platonis Gorgiam commentaria*, ed. William Norvin (Leipzig: Teubner, 1936; repr. Hildesheim: Olms, 1966)
Olympiod. *In Meteor.*	Olympiodorus, *In Aristotelis Meteorologica commentarii*, ed. Wilhelm Stüve (Berlin, 1900; *CAG* XII 2)
Olympiod. *In Phd.*	Olympiodorus, *In Platonis Phaedonem commentaria*) ed. William Norvin (Leipzig: Teubner, 1913,
Ov. *Fast.*	Ovid, *Fasti*
Ov. *Met.*	Ovid, *Metamorphoses*
Paus.	Pausanias
Philo *Op.*	Philo, *De opificio mundi*
Philop. *De an.*	Philoponus, *In Aristotelis De anima libros commentaria*, ed. Michael Hayduck (Berlin, 1897; *CAG* XV; cited by page and line)
Philop. *Phys.*	Philoponus, *In Aristotelis Physica commentaria*, ed. Hieronymus Vitelli (Berlin, 1887–1888; *CAG* XVI–XVII; cited by page and line)
Philostr. *VA*	Philostratus, *Vita Apollonii Tyanensis*
Pi.	Pindar (fragments cited according to Schroeder's numeration)
Pl.	Plato

Por. *Abst.*	Porphyry, *De abstinentia*, in *Opuscula selecta*, ed. August Nauck (Leipzig, 1886²; repr. 1963) 83–270
Por. *De antr. nymph.*	Porphyry, *De antro nympharum*, ibid. 55–81
Por. *In Ptol.*	Porphyry, *Kommentar zur Harmonielehre des Ptolemaios*, ed. Ingemar Düring (Göteborg: Elander 1932; cited by page and line)
Por. *VP*	Porphyry, *Vita Pythagorae*, in *Opuscula selecta*, ed. August Nauck (Leipzig, 1886²) 17–52
Procl. *In Eucl.*	Proclus, *In Euclidem commentarius*, ed. Gottfried Friedlein (Leipzig, 1873; cited by page and line)
Procl. *In Remp.*	Proclus, *In Platonis Rempublicam commentarii*, ed. Wilhelm Kroll (Leipzig: Teubner, 1899–1901; cited by volume, page, and line)
Procl. *In Tim.*	Proclus, *In Platonis Timaeum commentaria*, ed. Ernst Diehl (Leipzig: Teubner, 1903–1906; cited by volume, page, and line)
Procl. *Theol. Pl.*	Proclus, *In Platonis theologiam libri VI*, ed. Aemilius Portus (Hamburg, 1618)
Ptol. *Harm.*	*Die Harmonielehre des Klaudios Ptolemaios*, ed. Ingemar Düring (Göteborg; Elander, 1930)
Ptol. *Synt.*	Ptolemy, *Syntaxis mathematica* (*Almagest*)
Schol. A *Il.*	*Scholia in Homeri Iliadem*, ed. Wilhelm Dindorf, vols. I–II (Oxford, 1874)
Schol. Ap. Rh.	*Scholia in Apollonium Rhodium vetera*, ed. Carl Wendel (Berlin: Weidmann, 1935)
Schol. Arat.	*Commentariorum in Aratum reliquiae*, ed. Ernst Maass (Berlin, 1898)
Schol. Arist.	*Scholia in Aristotelem*, coll. C. A. Brandis (Berlin, 1836; *Aristotelis Opera*, ed. Immanuel Bekker, vol. IV, repr. DeGruyter, 1961)
Schol. B *Il.*	*Scholia in Homeri Iliadem*, ed. Wilhelm Dindorf, vols. III-IV (Oxford, 1877)
Schol. Eucl.	*Scholia in Elementa*, in *Euclidis Elementa*, ed. J. L. Heiberg, vol. V (Leipzig, 1888) 71–738 (cited by page and line)
Schol. Lucian	*Scholia in Lucianum*, ed. Hugo Rabe (Leipzig: Teubner 1906)
Schol. *Od.*	*Scholia in Homeri Odysseam*, ed. Wilhelm Dindorf (Oxford, 1855)
Schol. Pl.	*Scholia Platonica*, ed. W. C. Greene (Haverford: American Philological Association, 1938)

Schol. Soph.	*Scholia in Sophoclis tragoedias vetera*, ed. P. N. Papageorgius (Leipzig, 1888)
Schol. T *Il.*	*Scholia in Homeri Iliadem*, vols. V–VI, ed. Ernst Maass (Oxford, 1887–1888)
Sext. Emp. *Math.*	Sextus Empiricus, *Adversus mathematicos*
Sext. Emp. *PH*	Sextus Empiricus, *Πυρρώνειοι ὑποτυπώσεις*
Simpl. *Cael.*	Simplicius, *In Aristotelis De caelo commentaria*, ed. J. L. Heiberg (Berlin, 1894; *CAG* VII; cited by page and line)
Simpl. *De an.*	Simplicius, *In Aristotelis De anima commentaria*, ed. Michael Hayduck (Berlin, 1882; *CAG* XI; cited by page and line)
Simpl. *Phys.*	Simplicius, *In Aristotelis Physica commentaria*, ed. Hermann Diels (Berlin, 1882–1895; *CAG* IX, X; cited by page and line)
Stob.	Ioannes Stobaeus, ed. Curt Wachsmuth and Otto Hense (Berlin: Weidmann, 1884–1912; repr. 1958)
Syrian. *Met.*	Syrianus, *In Aristotelis Metaphysica commentaria*, ed. Wilhelm Kroll (Berlin, 1902; *CAG* VI 1; cited by page and line)
Tert. *An.*	Tertullian, *De anima*
Th. ar.	[Iamblichus], *Theologumena arithmeticae*, ed. Vittorio de Falco (Leipzig: Teubner, 1922; cited by page and line)
Themist. *An.*	Themistius, *In libros Aristotelis De anima paraphrasis*, ed. Richard Heinze (Berlin, 1899; *CAG* V 3; cited by page and line)
Themist. *Or.*	Themistius, *Orationes*, ed. Wilhelm Dindorf (Leipzig, 1832)
Themist. *Phys.*	Themistius, *In Aristotelis Physica paraphrasis*, ed. Heinrich Schenkl (Berlin, 1900; *CAG* V 2; cited by page and line)
Theodoret	Theodoretus Cyrensis, *Graecarum affectionum curatio*
Theo Sm.	Theo Smyrnaeus, ed. Eduard Hiller (Leipzig, 1878; cited by page and line)
Tim. Locr.	Timaeus Locrus, ed. C. F. Hermann (Leipzig, 1852; *Platonis Dialogi* IV 407–421); now see Walter Marg, in Thesleff, *Texts*, 203–225
Xen. *Mem.*	Xenophon, *Memorabilia*
Xen. *Symp.*	Xenophon, *Symposium*

(Modern works cited by author or by author and short title)

Adam: James Adam (ed.), *The Republic of Plato* (Cambridge: Cambridge Univ. Pr., 1902).

Andrissi: G. L. Andrissi, "Su alcuni punti controversi dell'astronomia antica: Il sistema di Filolao," *Scientia* 72 (1942) 1–13.

Bechtel, *GrD*: Friedrich Bechtel, *Die griechischen Dialekte* (Berlin: Weidmann, 1921–1924).

Becker, *Fs. Gadamer*: Oskar Becker, "Die Aktualität des pythagoreischen Gedankens," in *Die Gegenwart der Griechen im neueren Denken: Festschrift H. G. Gadamer* (Tübingen: Mohr, 1960) 7–30.

Becker, *Grdl.*: ———, *Grundlagen der Mathematik in geschichtlicher Entwicklung* (Freiburg-Munich: Albert, 1954).

Becker, *MD*: ———, *Das mathematische Denken der Antike* (Göttingen: Vandenhoeck & Ruprecht, 1957).

Becker, *QSt* 1: ———, "Die diairetische Erzeugung der platonischen Idealzahlen," *QSt* 1 (1931) 464–501.

Becker, *QSt* 3: ———, "Die Lehre vom Geraden und Ungeraden im neunten Buch der Euklidischen Elemente," *QSt* 3 (1936) 533–553.

Becker, *QSt* 4: ———, "*Παραμηκεπίπεδοι ἀριθμοί*: Die Zahlen von der form n.n. (n + 1) bei Nikomachos von Gerasa, mit einem Anhang über die platonische Hochzeitszahl," *QSt* B 4 (1938) 181–192.

Becker, *ZwU*: ———, *Zwei Untersuchungen zur antiken Logik* (Wiesbaden: Harassowitz, 1957).

Berger, *Erdkunde*: Ernst Hugo Berger, *Geschichte der wissenschaftlichen Erdkunde der Griechen* (Leipzig: Veit, 1903²).

Bertermann: Guilelmus Bertermann, *De Iamblichi Vitae Pythagoricae fontibus* (Diss. Königsberg: Hartung, 1913).

Bidez, *Eos*: Joseph Bidez, *Eos, ou Platon et l'orient* (Brussels: Hayez, 1945).

Bidez-Cumont, *Mages*: Joseph Bidez and Franz Cumont, *Les mages hellénisés* (Paris: Les Belles Lettres, 1938).

Boeckh: August Boeckh, *Philolaos des Pythagoreers Lehren nebst den Bruchstücken seines Werkes* (Berlin, 1819).